Workshop on Decoherence in Quantum Dynamical Systems
held at ECT in Trento, from April 26th–30th 2010

Decoherence of electron waves by irreversible electromagnetic interaction with the charges in a resistive plate

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Ladies and Gentlemen,
further information on the viewgraphs you will find in the following papers: F. Hasselbach: Progress in electron- and ion-interferometry. Report on Progress in Physics **73**(2010) 016101-43
P. Sonnentag (Dissertation): Ein Experiment zu kontrollierten Dekohõrenz im Elektroneninterferometer (in german)
P. Sonnentag and F. Hasselbach: Measurement of decoherence of electron waves and visualization of the quantum-classical transition. Phys. Rev. Lett. **98**(2007) 200402-1-4

Outline:

- **Introduction**
- Decoherence – virtual and real
- Main principle and theory of the experiment
- Experimental set-up
- Measurements and results
- Determination of the ‘coherent energy width’ of the beam
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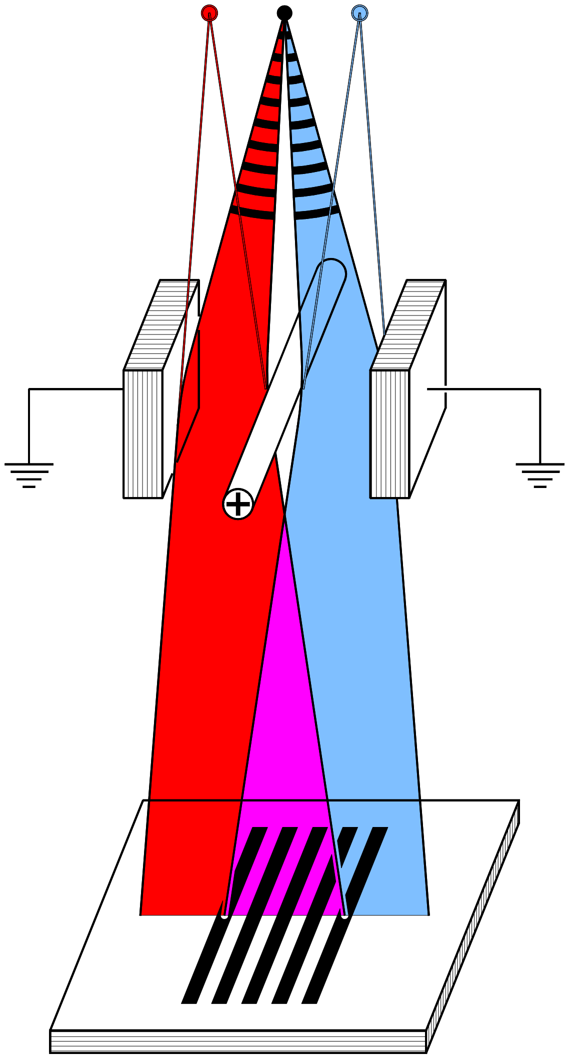
Möllenstedt biprism

wave-front splitting



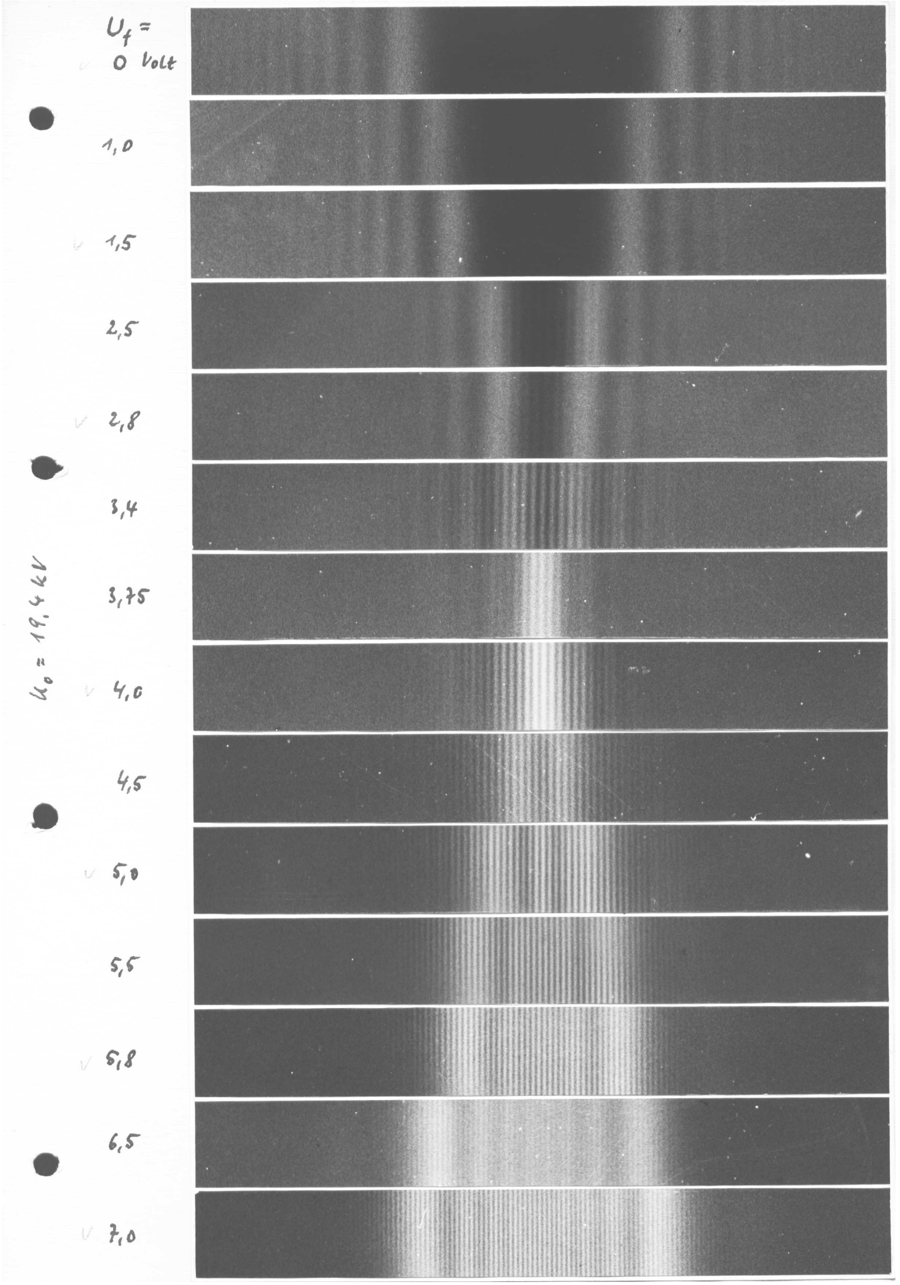
electron source

biprism

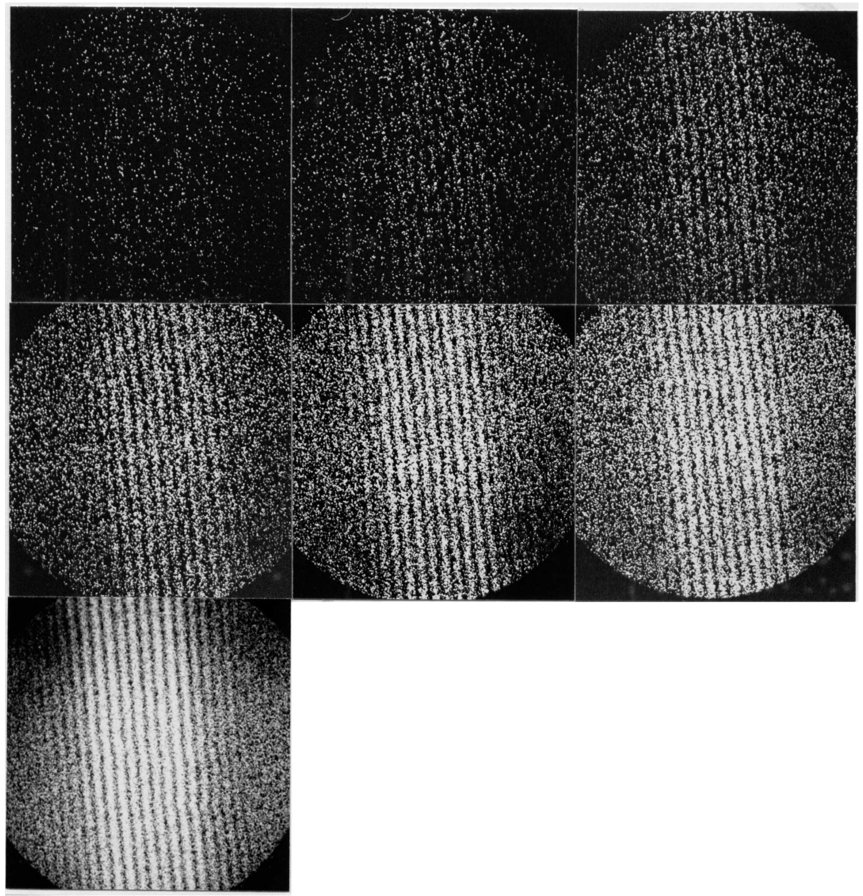
interference fringes

G. Möllenstedt and H. Düker 1955

From H. Dükers PHD-thesis (1955):



Build-up of interference fringes out of single electrons:



(for the first time: Merli, Missiroli, Pozzi 1976)
[Hasselbach, Wohland 1977]

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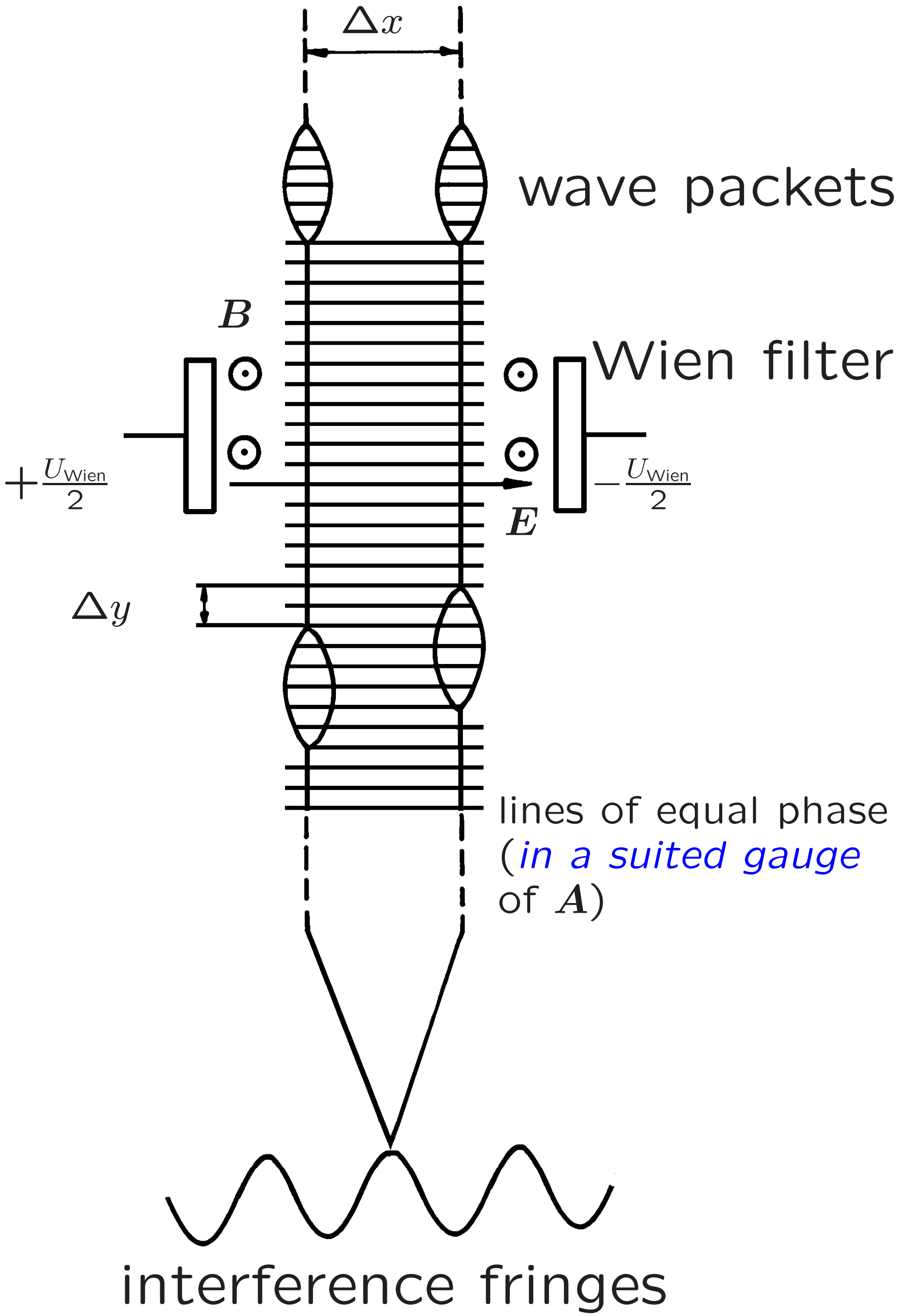
Decoherence

quantum mechanical regime $\Longleftrightarrow$ interference fringes

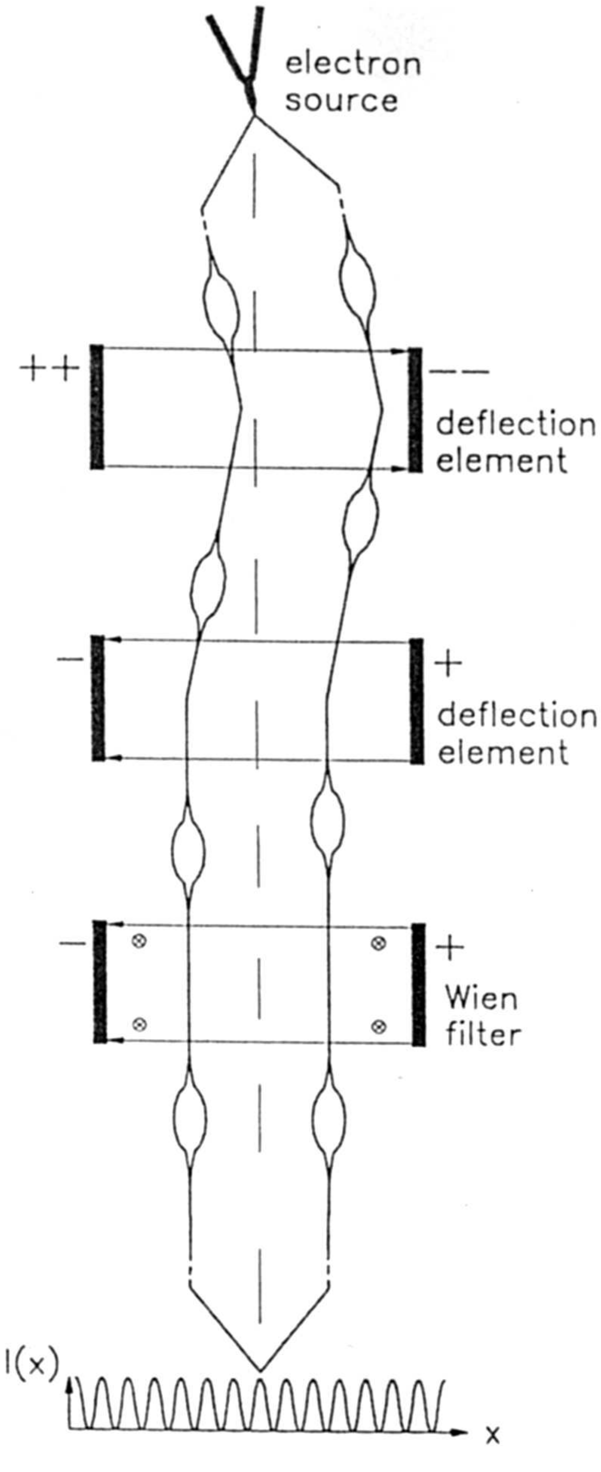
Interaction/entanglement with the environment $\Longrightarrow$ disappearing fringes $\Longrightarrow$ onset of decoherence

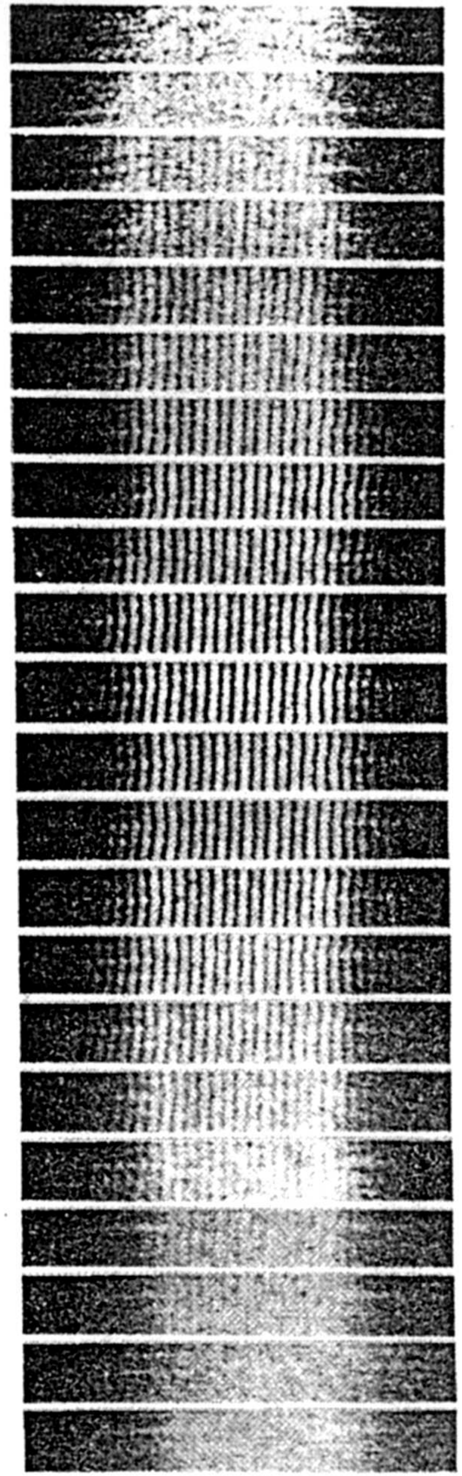
reversible interaction — irreversible interaction
virtual decoherence — decoherence

Virtual decoherence in an electron interferometer by a Wien-filter (crossed electric and magnetic fields)



Δx
wave packets
$\boldsymbol{B}$
Wien filter
$+\frac{U_{\text{Wien}}}{2}$
$-\frac{U_{\text{Wien}}}{2}$
$\boldsymbol{E}$
Δy
lines of equal phase
(*in a suited gauge*
of $\boldsymbol{A}$)
interference fringes


electron source
++
--
deflection element
-
+
deflection element
-
+
Wien filter
I(x)
x



Decoherence

Decoherence is the **emergence of classical features** of a quantum system, resulting from its unavoidable – and in general **irreversible** – **interaction with the environment**.

Zeh 1970, Zurek 1981, Joos und Zeh 1985, ...

main ideas:

- interaction with the environment – object under consideration is not a closed system (Zeh 1970)
- unobserved degrees of freedom
- entanglement (Schrödinger 1926)

During the formation of decoherence:
off-diagonal elements of the reduced density matrix of the object with respect to the so-called pointer basis decay exponentially:

$$\widehat{\rho}_{\text{object}_{i,j}}(t) = \widehat{\rho}_{\text{object}_{i,j}}(0) \cdot \exp\left(-\frac{t}{\tau_{\text{d}}}\right) \qquad (i \neq j)$$

τ_{d} ... decoherence time

coherence decreases exponentially with time*

*H.-D. Zeh: The reduced density matrix is a useful tool in the theory of decoherence. However it has the disadvantages of (1) depending on an artificial choice of subsystems, (2) not distinguishing between reversible (virtual) and irreversible (real) decoherence and (3) not distinguishing between proper and improper mixtures (ensembles and entaglement).

Experiments on decoherence by

- Pfau et al. emission of photons
- Chapman et al. single photons scattered from atoms
- Kokorowski et al. Multiple photon decoherence in an atom interferometer
- Brune et al. Progressive decoherence of the ‘meter’ in a quantum measurement
- Myatt et al. single trapped ions, coupled to engineered high temperature amlitude- and phase-reservoir, quantum noise- and spontaneous-emission reservoir
- Hackermüller et al., Hornberger et al. decoherence induced by the emission of radiation from C-70 molecules and due to collisions with gas molecules

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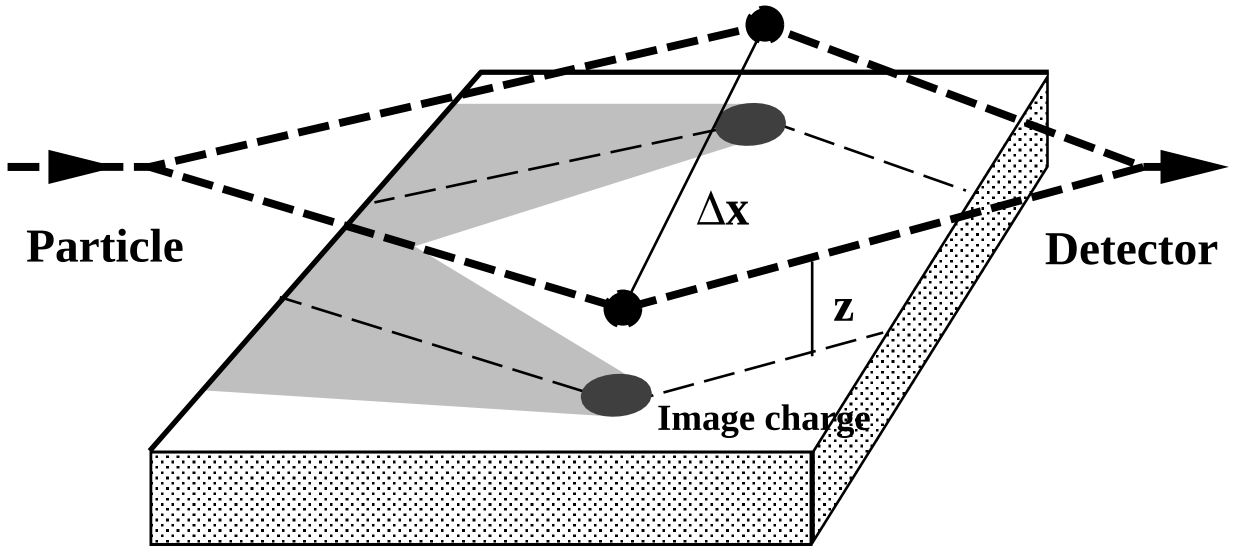
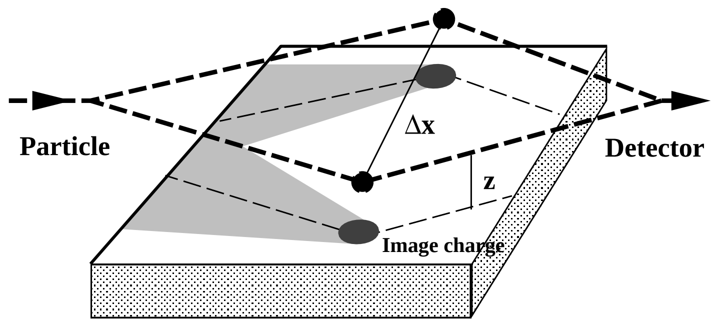
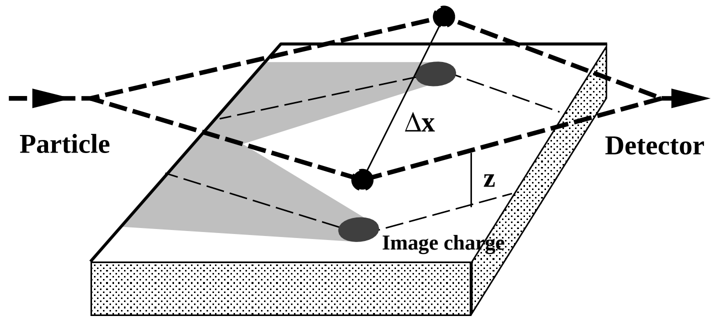
Scheme of the experiment

J. R. Anglin and W. H. Zurek: A precision test of decoherence. Proc. XXXIst Rencontres de Moriond (1996), 263–270. quant-ph/9611049
J. R. Anglin, J. P. Paz, and W. H. Zurek: Deconstructing decoherence. Phys. Rev. A **55** (1997) 4041–4053
P.Machnikowski: Calculation using the many-body quantum description of the electron gas. Phys. Rev. B **73**(2005) 155109
R. Alicki et al. Optimal strategy for a single q-bit gate and the trade-off between opposite types of decoherence Phys. Rev. A **70** (2004) 010501(R)-1–(R)4





- electron beam is split into two parts
- both beams travel over a plate made of a highly resistive material at the same, small height
- $\rightsquigarrow$ interaction between electron (object) and plate (‘environment’):
- electron induces charge inside the plate
- along with the beam electron, induced charges move through the plate
- $\rightsquigarrow$ currents inside plate
- currents encounter ohmic resistance
- $\rightsquigarrow$ dissipation:
- Joule heating; quantum mechanically: disturbance of the state of the electron and phonon gas inside the plate, being *different* for ‘left’ and ‘right’ path of the electron (heating at different *locations*) $\rightsquigarrow$ *which-path information*, particle-like behaviour



- $\rightsquigarrow$ *entanglement* between beam electron and plate
- disturbance is *irreversible*, a record of the electron's path *remains*
- beams are recombined $\rightsquigarrow$ interference contrast is reduced
- microscopic particle without inner degrees of freedom $\rightsquigarrow$ decoherence time sufficiently large
- truly macroscopic and ohmic environment
- strength of decoherence is adjustable over a wide range, from negligible to nearly complete decoherence
- *continuous* transition from quantum to classical behaviour
- dependence of decoherence on *two* parameters can be tested
- $\rightsquigarrow$ "precision test of decoherence"

Decoherence time and visibility:
classical calculation according to Anglin & Zurek:

relaxation time for *spatial motion*: $\tau_{\mathrm{r}} = \frac{v}{|\dot{v}|}$

from power loss P (Boyer 1974):

$$P = -\frac{e^2\, \rho\, v^2}{16\, \pi\, z^3} \quad \text{for} \quad z \gg 4\, \pi\, \varepsilon_0\, \rho\, v$$

$$P = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m}{2}\, v^2\right) = m\, v\, \dot{v}$$

relation between relaxation time and decoherence time following from a linear model: Zurek 1986, Joos 1996, Breuer & Petruccione 2002, ...(with *different* prefactors):

$$\tau_{\mathrm{d}} = \frac{\hbar^2}{m\, k_{\mathrm{B}}\, T\, (\Delta x)^2} \cdot \tau_{\mathrm{r}} = \frac{1}{2\, \pi}\left(\frac{\lambda_T}{\Delta x}\right)^2 \cdot \tau_{\mathrm{r}} = \frac{4\, h^2\, z^3}{\pi\, e^2\, k_{\mathrm{B}}\, T\, \rho\, (\Delta x)^2}$$

with the thermal de Broglie wavelength at temperature T
(*different* prefactors of λ_T in literature):

$$\lambda_T = \frac{h}{\sqrt{2\, \pi\, m\, k_{\mathrm{B}}\, T}}$$

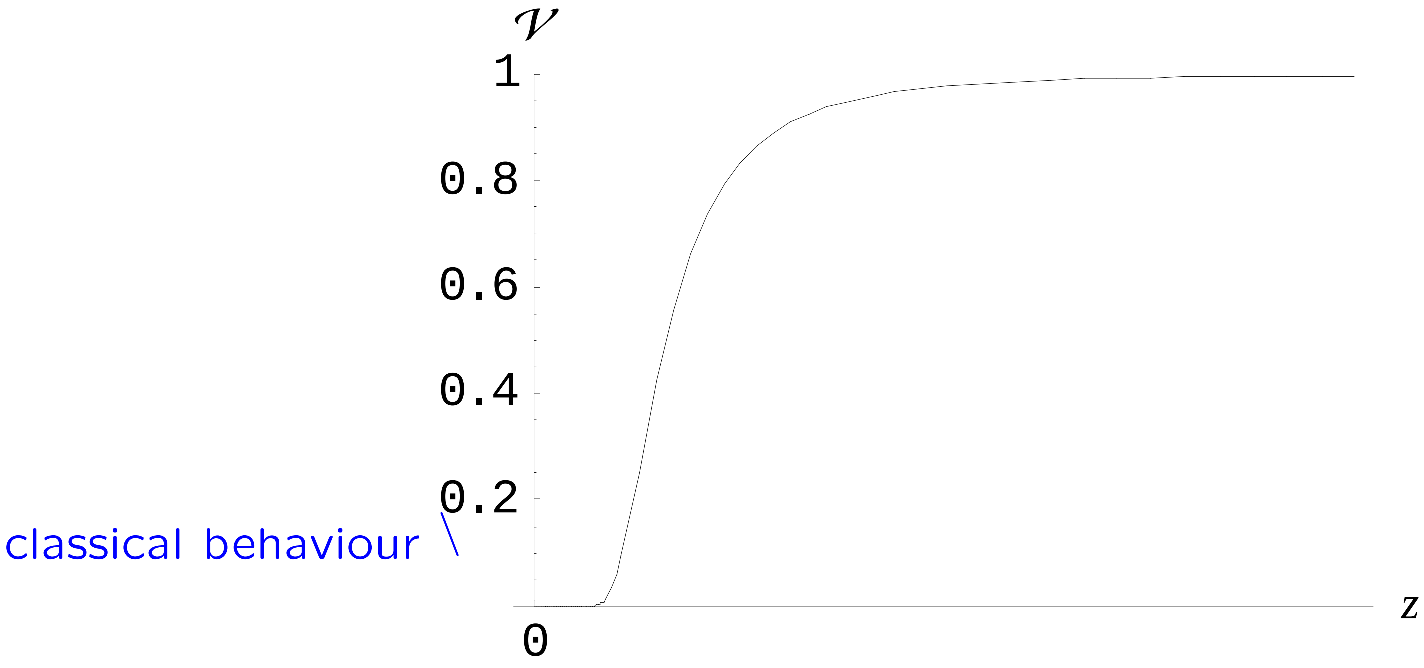
Visibility according to Anglin & Zurek:

time of flight over plate (length L):

$$t_{\text{flight}} = \frac{L}{v}$$

$$\mathcal{V} := \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \exp\left(-\frac{t_{\text{flight}}}{\tau_{\text{d}}}\right) = \exp\left(-\frac{\pi\, e^2\, k_{\text{B}}\, T\, \rho\, L\, (\Delta x)^2}{4\, h^2\, v\, z^3}\right)$$

quantum behaviour



Classical theory:

Energy losses due to dc resistance and Joule heating:

Formation of the screening image charge is dissipationless (adiabatic), involving only virtual transitions and is therefore reversible. The dissipative process is due to carrier scattering as the image charge moves in the resistive material. Which path information is stored in lattice excitations.

Quantum description:

Already the formation of image charge (dissipative electron-hole pair formation around the Fermi surface) is highly dissipative even in the absence of carrier-phonon scattering.

Quantitative comparison for noble metals: Decoherence is by many orders of magnitude higher than that resulting from resistive dissipation. The ohmic resistivity effect is of minor importance.

Visibility according to Machnikowski:

many-body quantum description of the electron gas,
overlap between the spectral density of the reservoir fluctuations and the spectral function related to the unperturbed evolution of the system

$$\mathcal{V} \approx \exp\left(-f_{\text{theor.}} \cdot \frac{\pi}{16} \frac{(\Delta x)^2}{z^2}\right)$$

for *semiconductors*: $f_{\text{theor.}} = ?$

semiconductors: $f_{\text{fit}} \sim 4,4$

Theory (Theories) of the experiment

- Anglin & Zurek: classical calculation of dissipation rate $\rightsquigarrow$ relaxation time $\rightsquigarrow$ decoherence time
- P.Machnikowski: calculation using the many-body quantum description of the electron gas
  [P. Machnikowski: Physical Review B **73** 155109]
- Y. Levinson: calculation using quantum electromagnetic field fluctuations
  [Y. Levinson: Journal of Physics A **37** (2004) 3003–3017;
  arxiv: quant-ph/0312184]
- S. Chaturvedi, I. Marzoli, R.F. O'Connell & W.P. Schleich: calculation using quantum Langevin equation in the framework of quantum carpets
  [preprint]

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Special demands for the interferometer and the beam path for the decoherence experiment:

1. large lateral separation Δx between the two parts of the beam

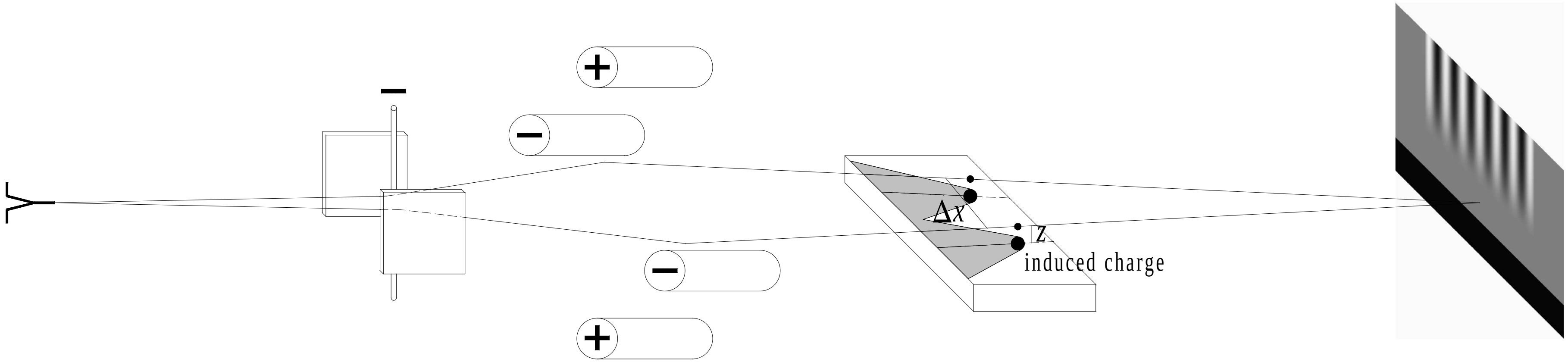
2. small height z above the plate

3. beams parallel to the surface of the plate

4. both parts of the beam at the same height z above the decoherence plate

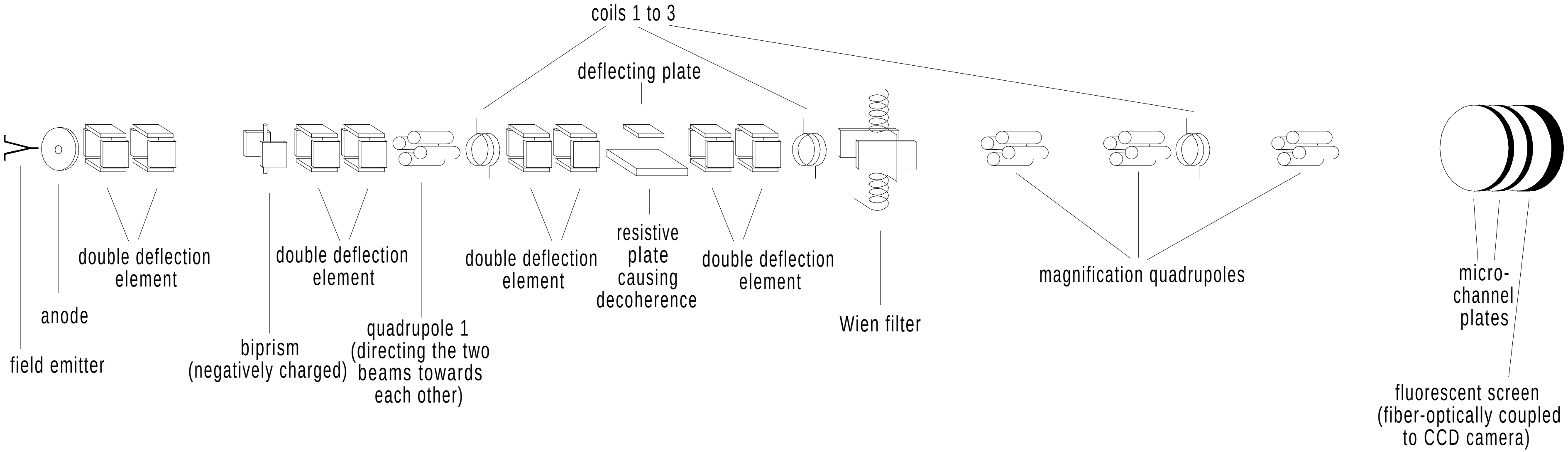
Realization of the decoherence experiment

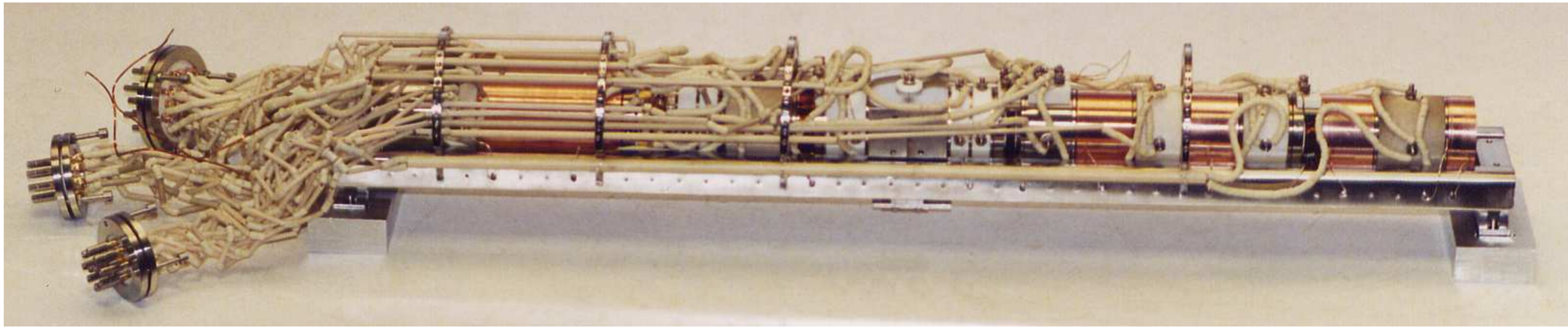
- beam split into two parts and made to diverge by *negatively* charged biprism filament
- beams are recombined by electrostatic quadrupole (electrodes in the plane of the two beams negatively charged)



- resistive material: n-doped silicon (doped with phosphorus), $\rho = 1{,}5 \cdot 10^{-2}\,\Omega\,\mathrm{m}$
- length of plate: 10 mm
- room temperature $T \approx 295\,\mathrm{K}$

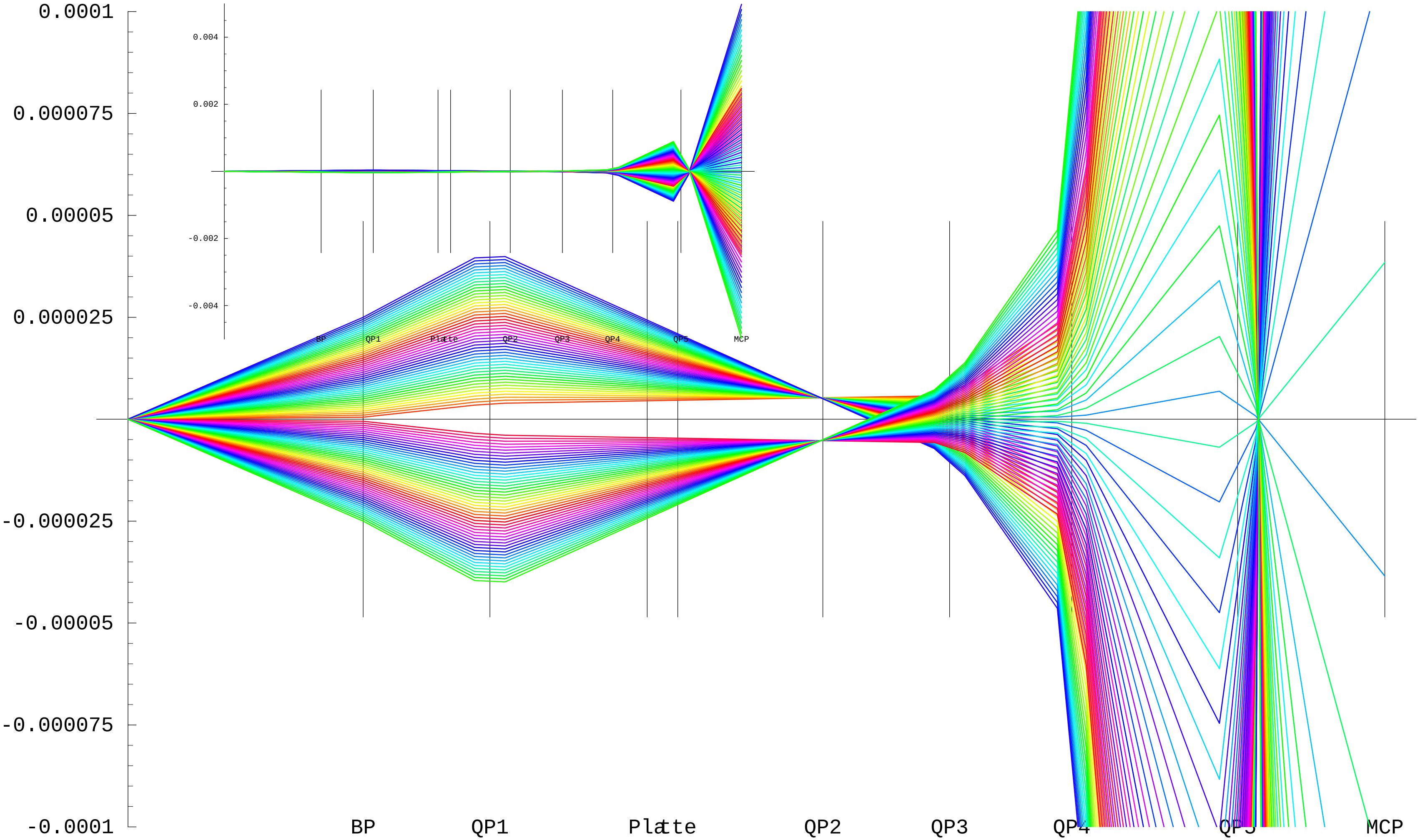
Overall set-up of the interferometer





Simulation of the beam path via transfer matrices

biprism voltage $U_\mathrm{f} = -0{,}655\,\mathrm{V}$, both partial beams shown:



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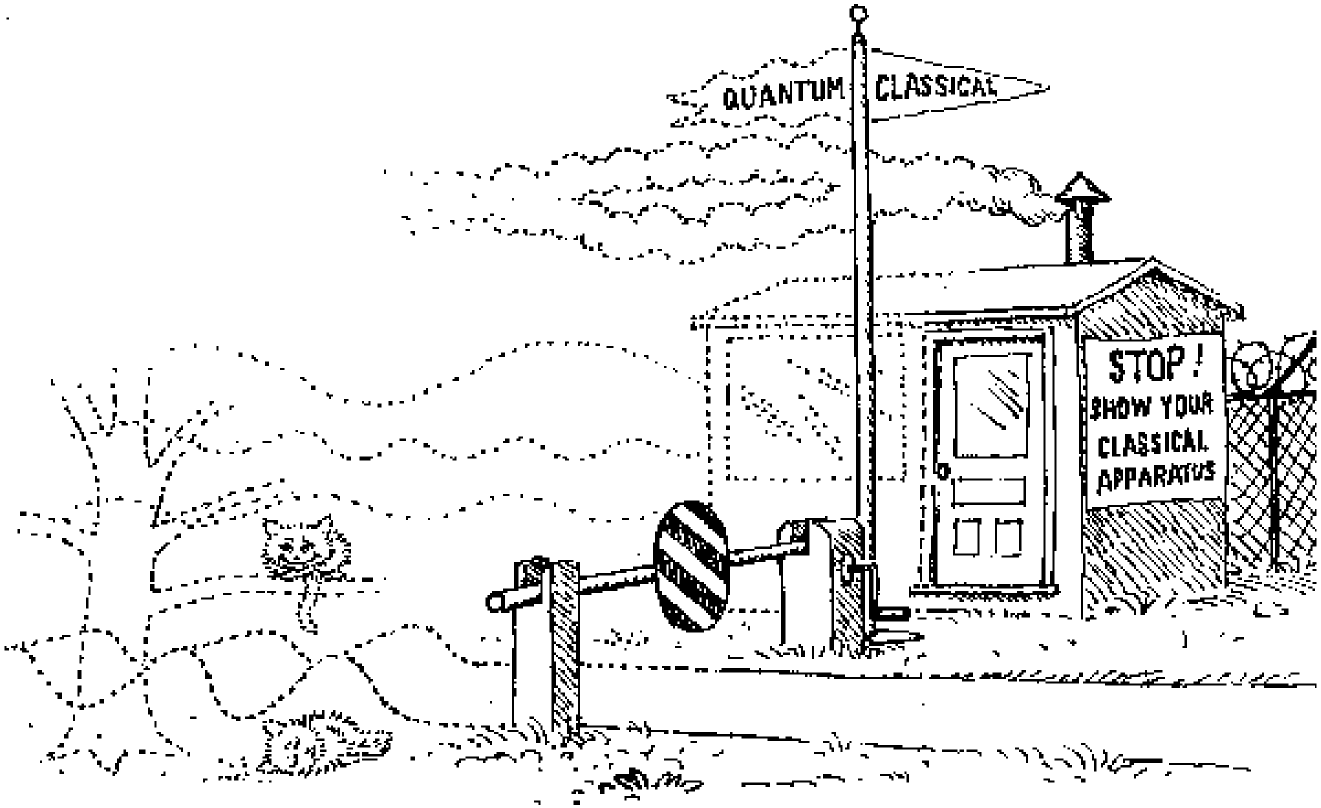
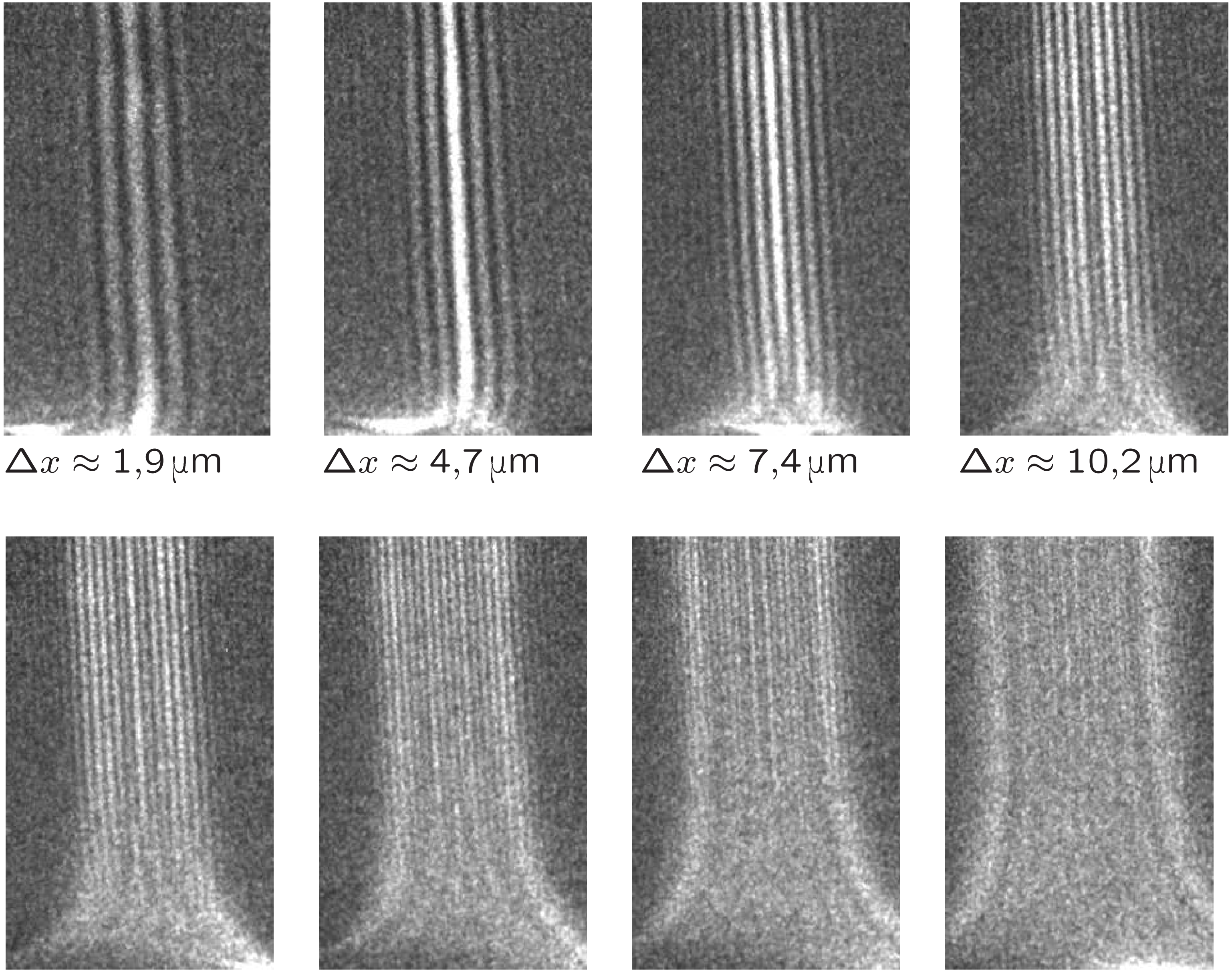
What does the quantum-classical border look like? NOT like this:



(known since the first experiments on controlled decoherence)

Now we have real ‘photos’ of this ‘border’ (NOT really a ‘border’!)



Decoherence as a function of height z of the beams over the plate for increasing lateral separations Δx of the beams.

Analysis of the experimental data

CCD pictures were normalized using dark and flat field correction.

Height z determined by mechanically moving the plate.

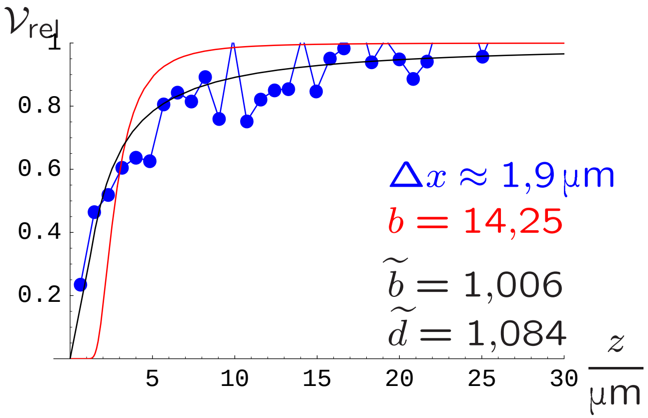
lateral separation Δx between the interfering beams determined from simulation of the beam path.

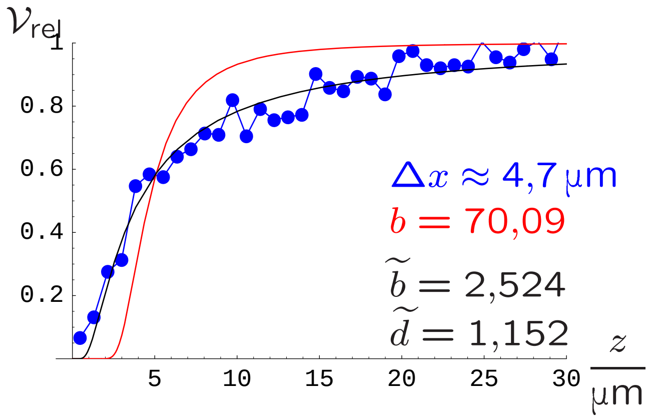
Determination of the visibility (loss) *caused by decoherence*:

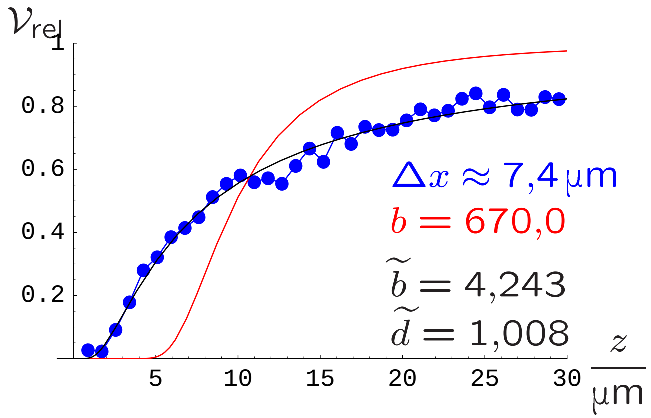
- intensity averaged over 10 pixels in height
- background of scattered electrons subtracted: determined from intensity in shadow area
- modulation by Fresnel diffraction removed (approximately): division by mean intensity over one fringe period
- dividing visibility by the visibility far away from the plate (at the same lateral separation)

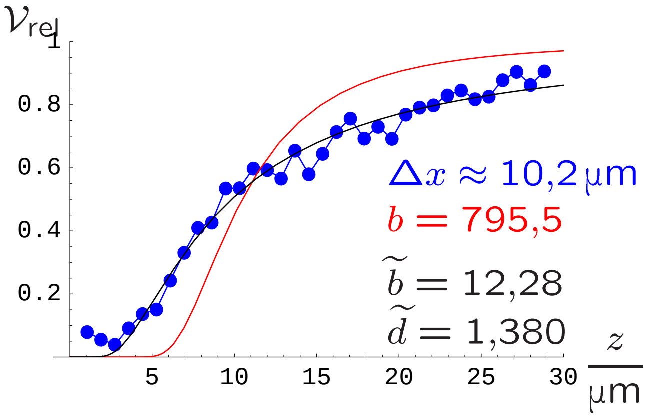
  $\rightsquigarrow$ effect of angular coherence, of the modulation transfer function of the imaging system, of an inclination of the fringes with respect to the camera's columns, of a possible longitudinal shift (being *independent of height*) of the wave packets against each other (longitudinal coherence), of a possible intensity difference (being *independent of height*) between the beams, etc. eliminated

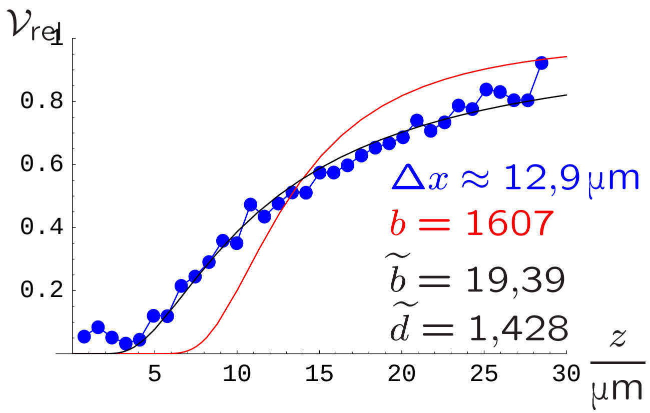
Visibility as a function of height z and comparison with the calculation by Anglin & Zurek

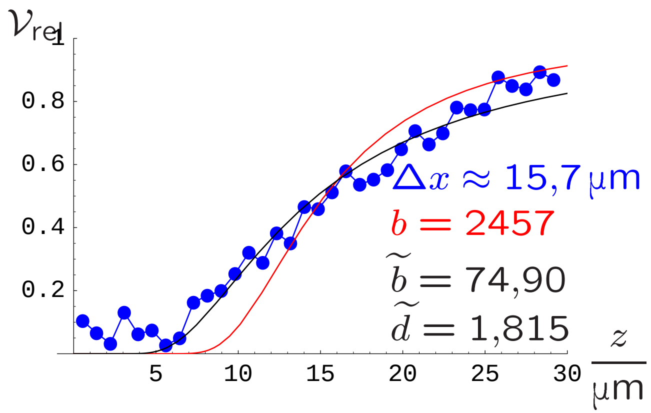


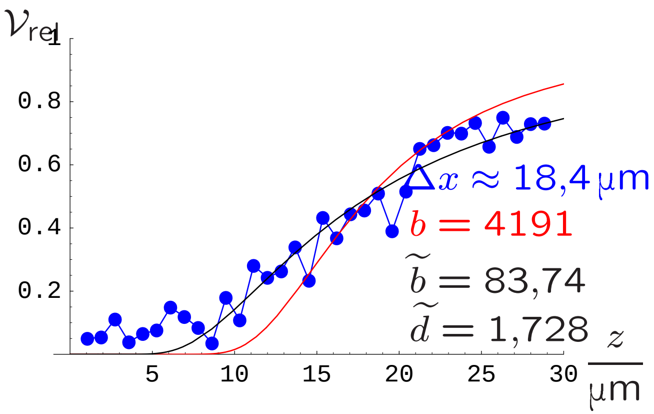












measurement results;

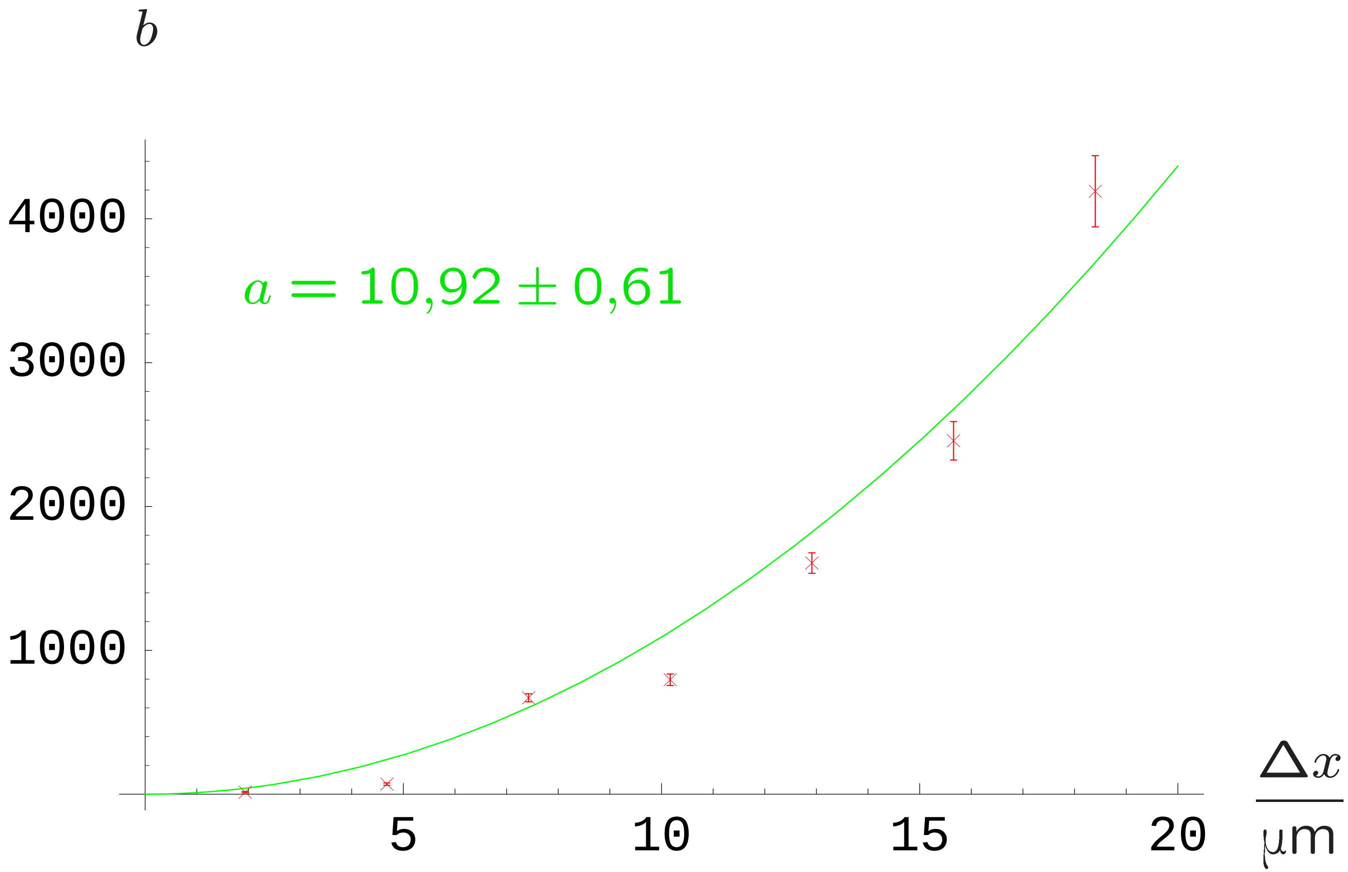
fit with $\exp\left(-\frac{b}{(z/\mu m)^3}\right)$;

fit with $\exp\left(-\frac{\tilde{b}}{(z/\mu m)^{\tilde{d}}}\right)$

Comparison with the calculation by Anglin & Zurek: decoherence as a function of lateral separation Δx

prefactor b of $-\frac{1}{(z/\mu\text{m})^3}$ in the exponent of visibility as a function of Δx:

fit with function $b = a \cdot (\Delta x/\mu\text{m})^2$



Comparison with the calculation by Anglin & Zurek: quantitative strength of decoherence

relative visibiliy: $\mathcal{V}_{\text{rel}} = \exp\left(-\frac{b}{(z/\mu\text{m})^3}\right) = \exp\left(-\frac{a\cdot(\Delta x/\mu\text{m})^2}{(z/\mu\text{m})^3}\right)$

experimental value: $a = 10{,}9^{+13{,}6}_{-5{,}8}$

theoretical values:

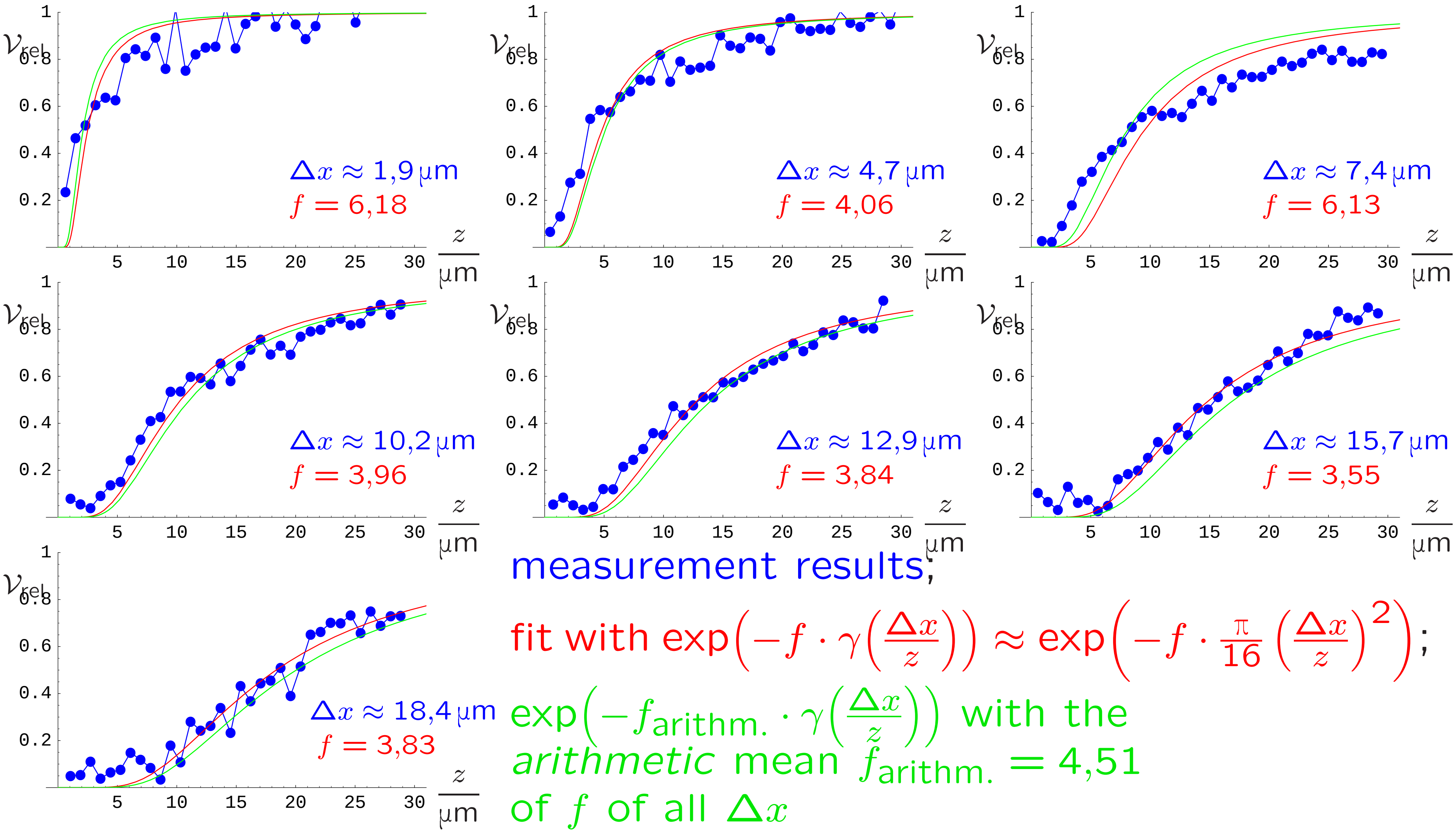
$$a_{\text{theor.}} = \frac{\pi\, e^2\, k_{\text{B}}\, T\, \rho\, L}{4\, h^2\, v \cdot 1\,\mu\text{m}} = 1159 \pm 353$$

resp.

$$a_{\text{theor.}} = \frac{e^2\, k_{\text{B}}\, T\, \rho\, L}{8\, \pi\, h^2\, v \cdot 1\,\mu\text{m}} = 58{,}73 \pm 17{,}9$$

(different proportionality constants between decoherence time and relaxation time were used by Anglin & Zurek)

Comparison with the calculation by Machnikowski



measurement results;

fit with $\exp\left(-f \cdot \gamma\left(\frac{\Delta x}{z}\right)\right) \approx \exp\left(-f \cdot \frac{\pi}{16}\left(\frac{\Delta x}{z}\right)^2\right)$;

$\exp\left(-f_{\text{arithm.}} \cdot \gamma\left(\frac{\Delta x}{z}\right)\right)$ with the *arithmetic* mean $f_{\text{arithm.}} = 4{,}51$ of f of all Δx

Other visibility-reducing effects had to be ruled out:

- angular coherence
- longitudinal coherence
- difference in intensity between the partial beams
- time-dependent charging of dust particles on plate surface
- decoherence due to vacuum fluctuations†
- energy difference between interfering partial beams

†[L.H. Ford 1993; Breuer & Petruccione 2001, Mazzitelli, Paz & Villanueva 2003; ...]: very weak effect

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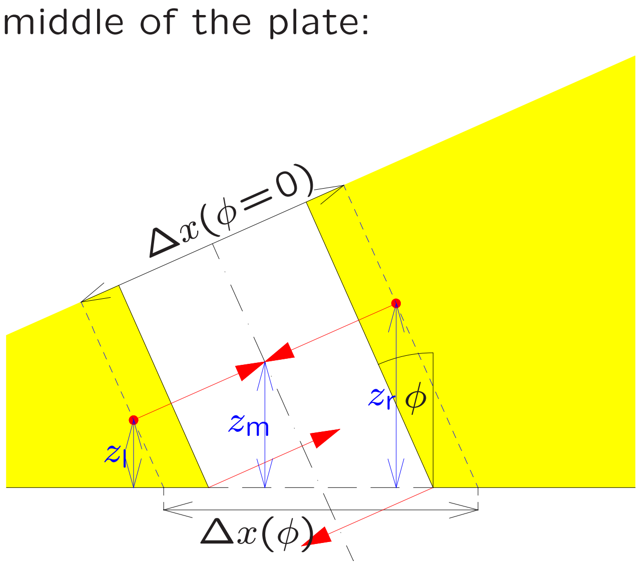
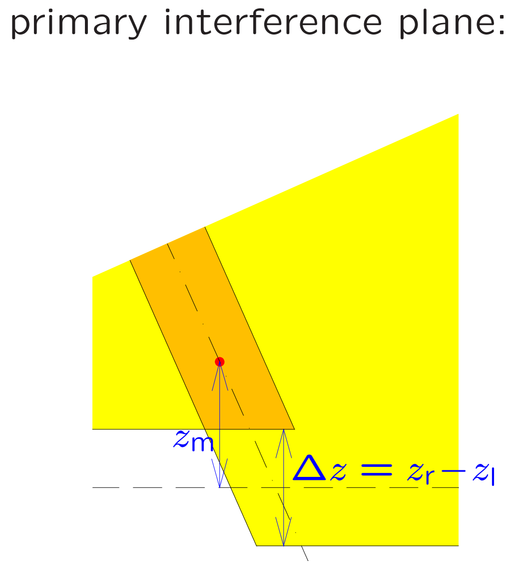
Energy difference between the partial beams

arrangement of the beams can be rotated around the optical axis using an electromagnetic coil

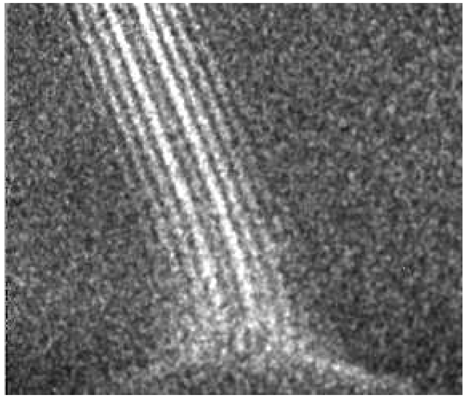
different heights above the plate
$\rightsquigarrow$ *different* energy losses
$\rightsquigarrow$ energy difference between the partial beams





Coherent energy width of field-emitted electrons

$z \approx 20\,\mu\text{m}$



$z \approx 0\,\mu\text{m}$

$\phi = 1{,}4\,^\circ$
$\Delta x(\phi = 0) \approx 10\,\mu\text{m}$
interference is visible for $z_\text{m} = 5\,\mu\text{m}$
$\leadsto$ $\Delta z = 0{,}24\,\mu\text{m}$; energy losses: $\delta\mathcal{E} = 32\,\text{meV}$ resp. $\delta\mathcal{E} = 35\,\text{meV}$

if exposure time can be assumed as infinite: interference only between components of the same energy
$\leadsto$ 'coherent energy width' of the field-emitted electron beam is *at least* 3 meV !

(so far known only: from uncertainty relation and time of flight (or length of the interferometer) follows: *at least* $10^{-8}\,\text{eV}$ [Nicklaus & Hasselbach 1993])

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Outlook Further possibilities:

- larger separation $\Delta x \rightsquigarrow$ saturation of decoherence?
- other materials for the plate (differently doped semiconductors $\rightsquigarrow$ further parameter which can be varied *continuously*; metals $\rightsquigarrow$ decision between theories of Anglin & Zurek and of Machnikowski; superconductor $\rightsquigarrow$ does decoherence vanish?)
- different temperatures T, different length L of plate, different electron energy/velocity v
- different combination of ρ and $v \rightsquigarrow$ test the exact formula for power loss
- different choice of ρ, T and $z \rightsquigarrow$ fulfill the conditions of Levinson's calculation
- different charge and mass $\rightsquigarrow$ ions $\rightsquigarrow$ also inner degrees of freedom
- smaller thicknesses of the plate
- dependence on beam intensity / mean time interval between succeeding electrons?
- more precise determination of 'coherent energy width': $\Delta x = 0$, quantitative analysis of visibility as a function of the angle of rotation
- is there a difference of the 'coherent energy width' between field emission and thermionic emission of electrons? Measurement of the 'coherent energy width' as a function of temperature (a few K up to thermionic emission) at good UHV

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Summary

- decoherence investigated in a conceptually very simple system:
- object is a single free elementary particle
- truly macroscopic ohmic environment
- interaction due to electric field of a charge
- here decoherence can be understood easily in terms of which-path information
- real *'photos'* of the quantum-classical border
- *continuous* transition from quantum to classical behaviour $\leadsto$ confirmation of the general theory of decoherence
- dependence on two parameters investigated: height z above the plate and lateral separation Δx of the beams
- comparison with Anglin & Zurek's calculations: $1/z^3$-dependence (of negative logarithm of visibility) not exactly confirmed, $(\Delta x)^2$-dependence very well confirmed, numerical strength of decoherence up to 2 orders of magnitude weaker than predicted
- comparison with Machnikowski's calculations: $(\Delta x)^2/z^2$-dependence very well confirmed, so far no exact theoretical value known of the prefactor for semiconductors
- other reasons than decoherence for the observed reduction of visibility could be ruled out
- 'coherent energy width' of the field-emitted electron beam is at least 3 meV

Energy loss due to Joule heating:

power loss: Boyer 1974:

$$P = -\frac{e^2\,\rho\,v^2}{16\,\pi\,z^3} \quad \text{for} \quad z \gg 4\,\pi\,\varepsilon_0\,\rho\,v$$

more exactly: Tomassone & Widom 1997, Schaich 2001:

$$P = -\frac{e^2\,\rho\,v^2}{16\,\pi\,z^3} \cdot \int_0^\infty \frac{u\,\mathrm{e}^{-u}}{\left(1 + \left(\frac{\varepsilon_0\,\rho\,v}{z}\right)^2 u^2\right)^{\frac{3}{2}}}\,\mathrm{d}u$$

for plates of small thickness D: Boyer 1996:

$$\left(\frac{1}{16\,z^3} + \sum_{j=1}^{\infty} \frac{1}{(2\,z + 2\,j\,D)^3}\right) \quad \text{instead of} \quad \frac{1}{16\,z^3}$$

Energy loss due to Joule heating

energy loss:

$$\delta\mathcal{E} = P \cdot t_{\text{flight}} = P \cdot \frac{L}{v}$$

length of plate $L = 10\,\text{mm}$;

acceleration voltage $U_\text{B} = 1{,}7\,\text{kV} \rightsquigarrow v = 8\,\% \cdot c$:

$\delta\mathcal{E}$ in eV	gold: $\rho = 2{,}2 \cdot 10^{-8}\,\Omega\,\text{m}$		n-doped silicon (with P) with $\rho = 1{,}5 \cdot 10^{-2}\,\Omega\,\text{m}$	
	according to Boyer	according to Tomassone & Widom, Schaich	according to Boyer	according to Tomassone & Widom, Schaich
$z = 50\,\mu\text{m}$	$-1{,}4 \cdot 10^{-10}$	$-1{,}4 \cdot 10^{-10}$	$-9{,}3 \cdot 10^{-5}$	$-8{,}9 \cdot 10^{-5}$
$z = 10\,\mu\text{m}$	$-1{,}7 \cdot 10^{-8}$	$-1{,}7 \cdot 10^{-8}$	$-1{,}2 \cdot 10^{-2}$	$-7{,}4 \cdot 10^{-3}$
$z = 1\,\mu\text{m}$	$-1{,}7 \cdot 10^{-5}$	$-1{,}7 \cdot 10^{-5}$	$-11{,}6$	$-0{,}58$