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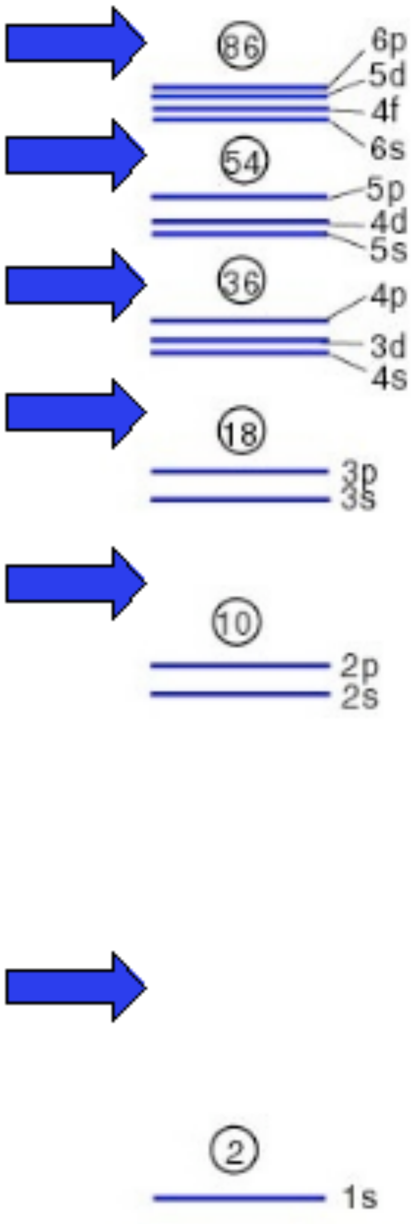
Shell Model for Open Quantum Systems

- Formulation of the problem
- Shell Model for weakly bound and unbound states
- Salient continuum-coupling phenomena – few examples

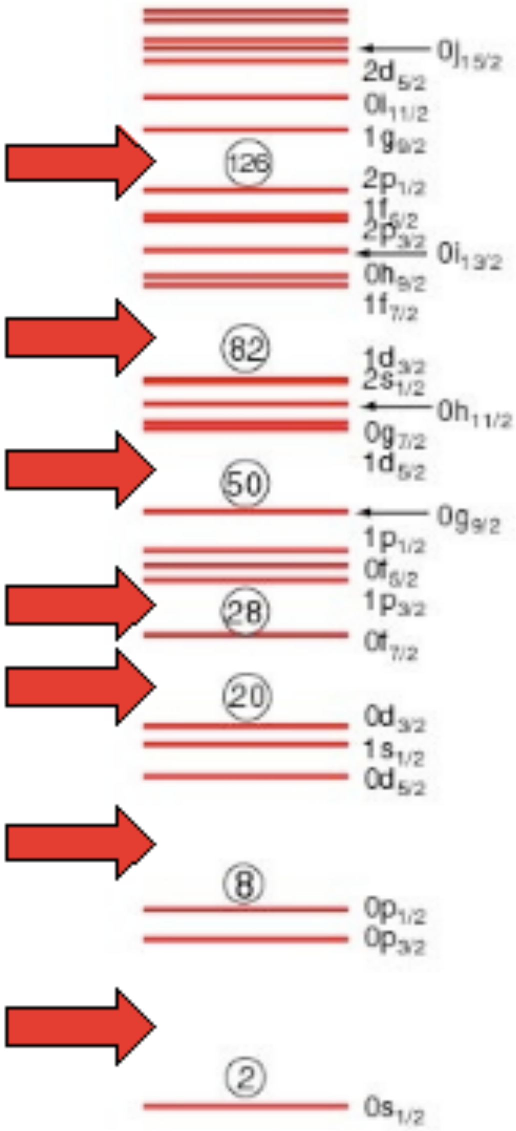
Decoherence in Quantum Dynamical Systems, April 26-30, 2010, ECT*

Atom: electron shells

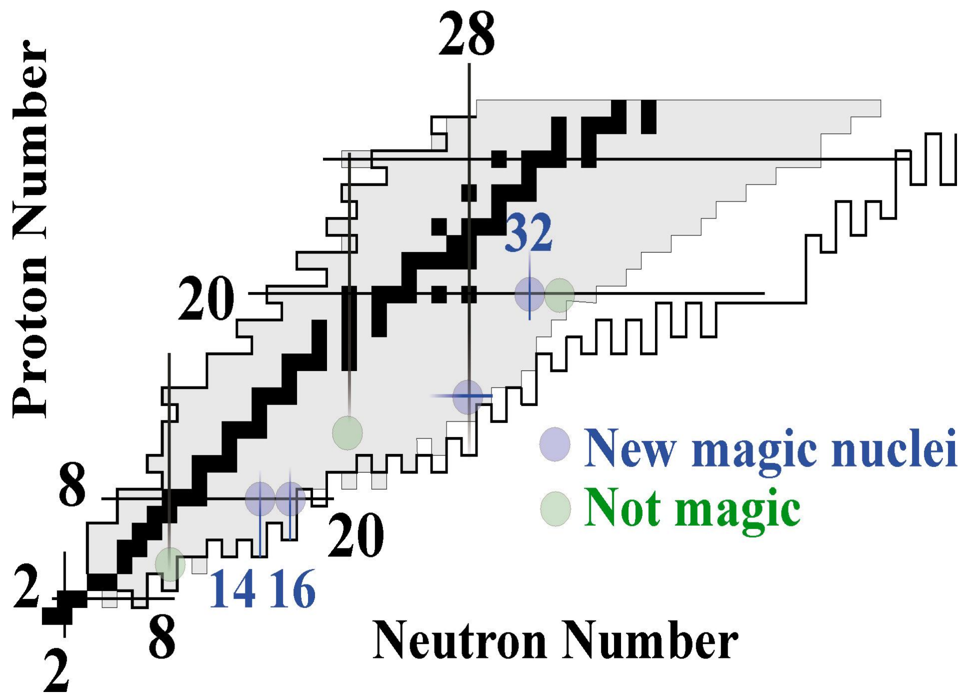


Shell Model of Atoms

Nucleus: proton/neutron shells



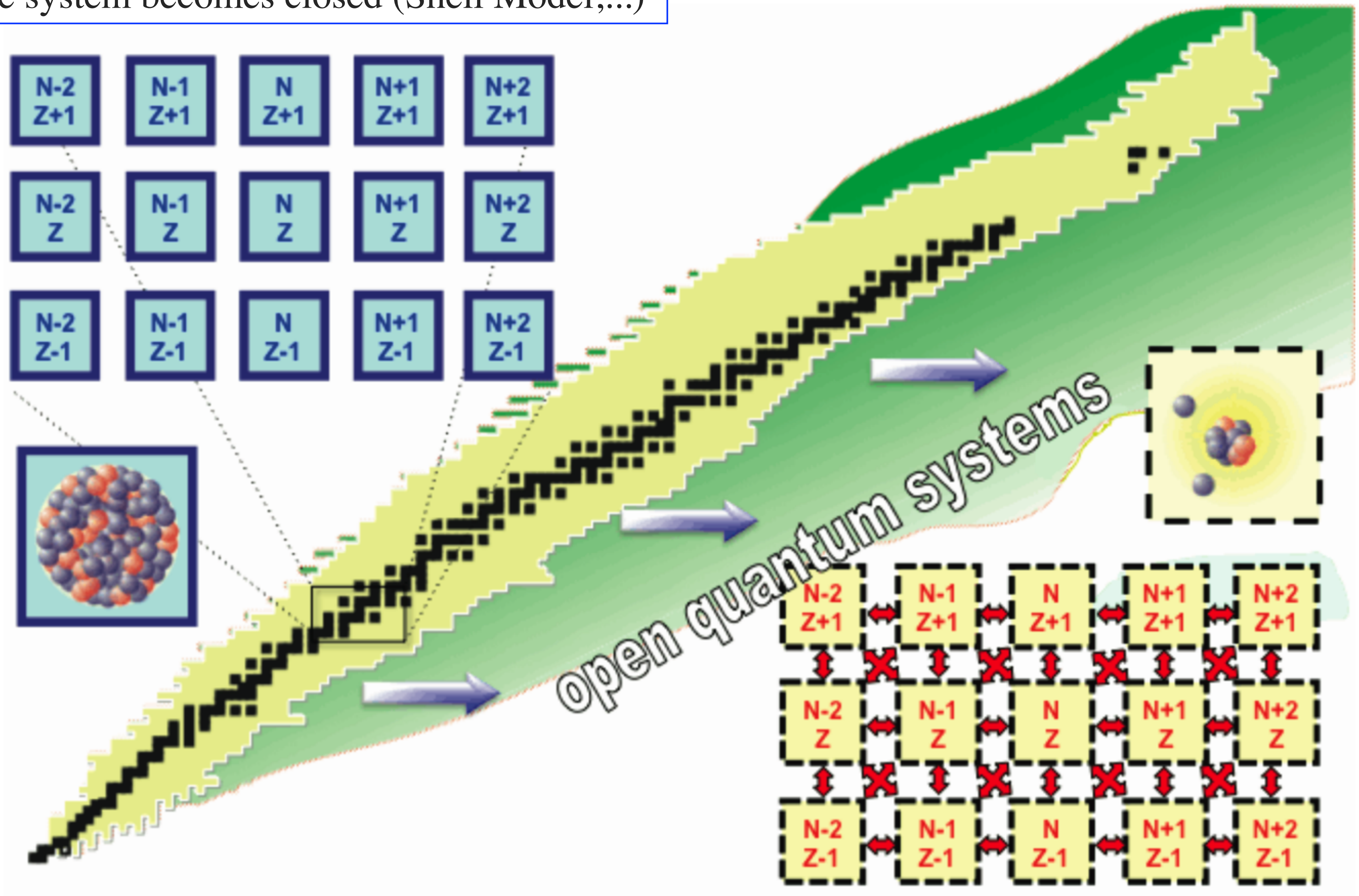
Shell Model of Nuclei



No shell closure for N=8 and 20 for drip-line nuclei;
new shells at 14, 16, 32… (‘monopole migration’)

Old paradigms, universal ideas, are not correct: near the drip lines, nuclear structure may be dramatically different

If continuum space is not considered,
the system becomes closed (Shell Model,...)



Network of coupled many-body systems

When can we talk about "existence" of an unbound system?

$$T_{1/2} = \ln 2 \frac{\hbar}{\Gamma}, \qquad \hbar = 6.58 \cdot 10^{-22}\ \mathrm{MeV} \cdot \mathrm{sec}$$

$$T_{s.p.} \approx 3 \cdot 10^{-22}\ \mathrm{sec} = 3\,\mathrm{baby\,sec}\,.$$

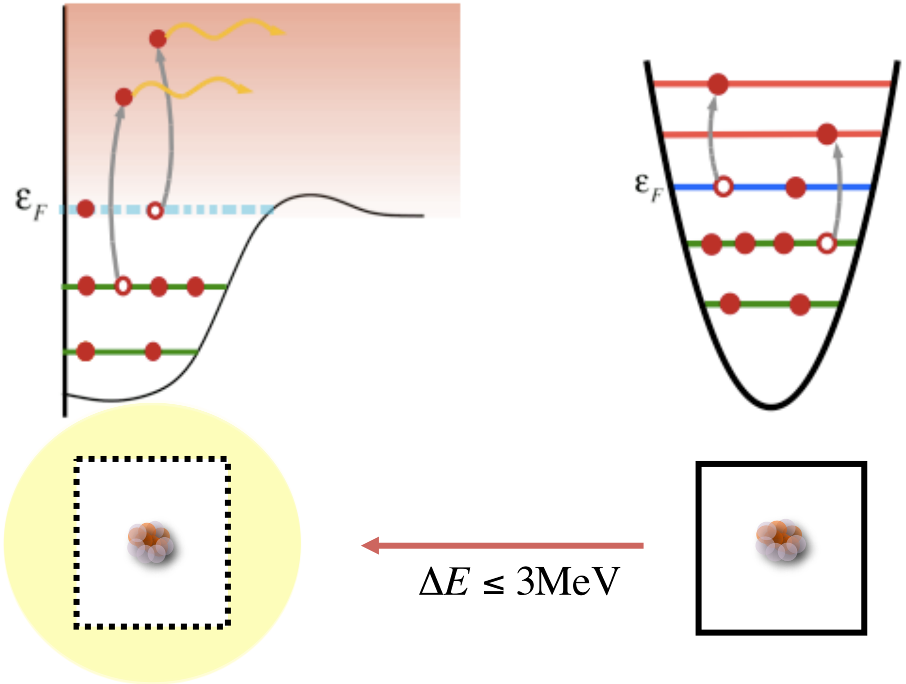
A typical time associated with the s.p. motion

$$T_{1/2} >> T_{s.p.}$$

$$\Rightarrow \quad \Gamma << 1 MeV$$

Open Quantum System
Coupling with continuum taken into account

Closed Quantum System
No coupling with external continuum



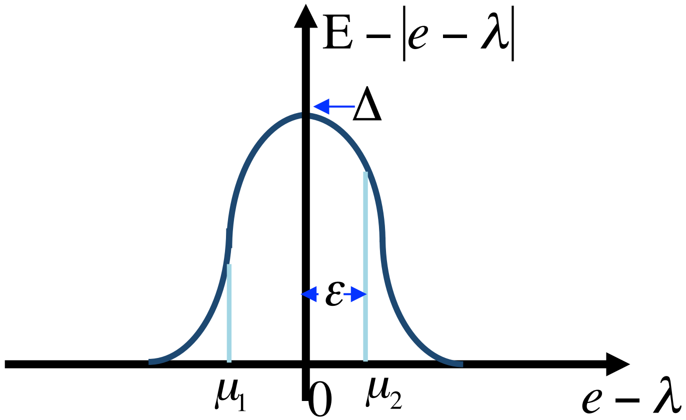
Threshold effects: at the intersection between mean-field and collective energy scales

Paradigm of Nuclear Shell Model is incorrect

The nucleus is a correlated open quantum many-body system
Environment: the continuum of decay channels

Instability of a single-particle motion in a potential with a pair deformation

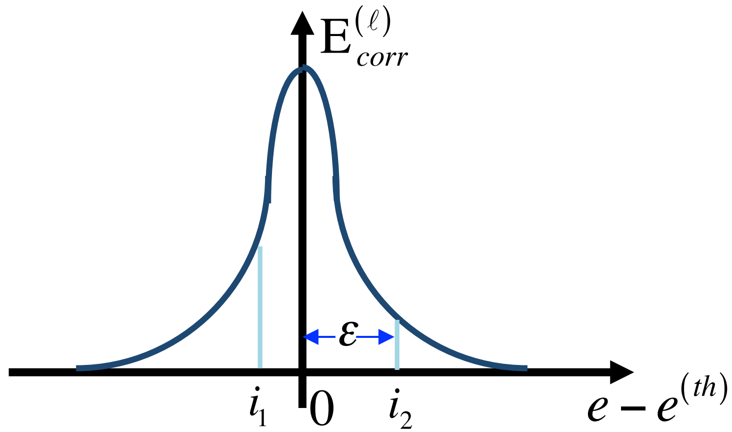
Pairing correction to single-particle eigenstates



Admixture of single-particle configurations with $e > \lambda$

Instability of SM eigenstates at channel threshold

Continuum coupling correction to shell model eigenstates



Admixture of many-body continuum states with $E > E_{th}$

Shell Model in the Complex Energy Plane (Gamow Shell Model)
(Rigged Hilbert space formulation)

Quasi-stationary open quantum system extension of the SM for well-bound systems

$$i\hbar\frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t) \qquad \Phi(r,t) = \tau(t)\Psi(r)$$

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \longrightarrow \tau(t) = \exp\left(-i\left(e - i\frac{\Gamma}{2}\right)\right)$$

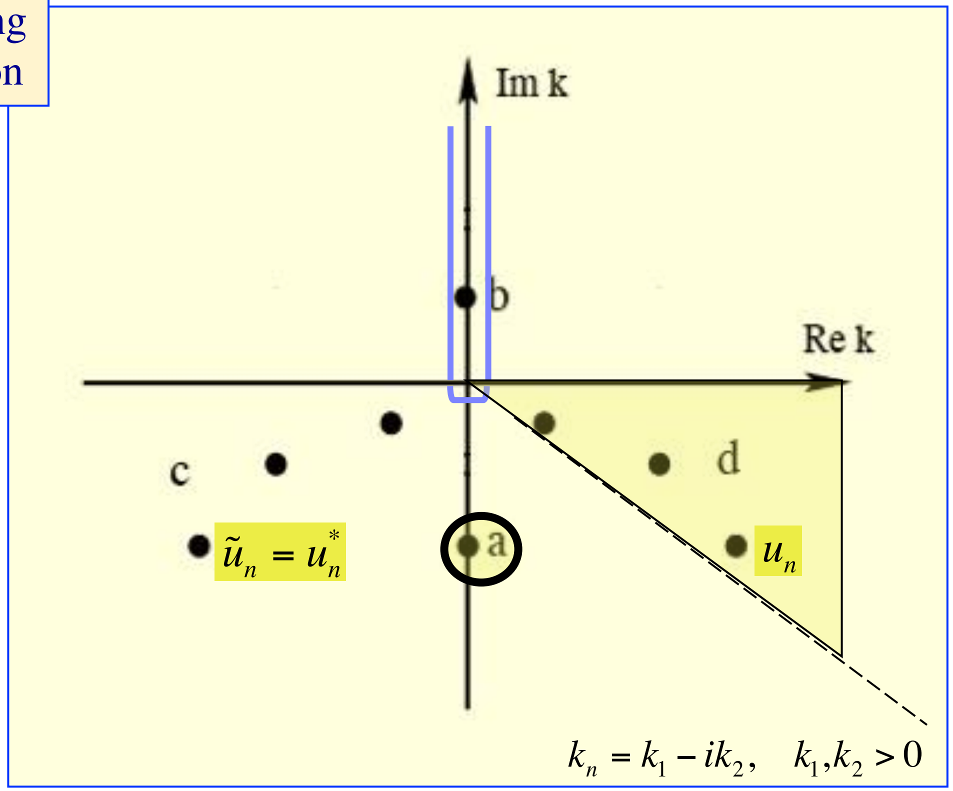
$$\Psi(0,k) = 0 \ , \begin{cases} \Psi(\vec{r},k) \underset{r\to\infty}{\to} O_l(kr) \\ \Psi(\vec{r},k) \underset{r\to\infty}{\to} I_l(kr) + O_l(kr) \end{cases}$$

outgoing solution

$$k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$$ (poles of the S-matrix)

Bound states	$k_n = i\kappa_n$
Antibound states	$k_n = -i\kappa_n$
Resonances	$k_n = \pm\gamma_n - i\kappa_n$

Only bound states are integrable!



Resonances are genuine intrinsic properties of quantum systems but they do not belong to the Hilbert space

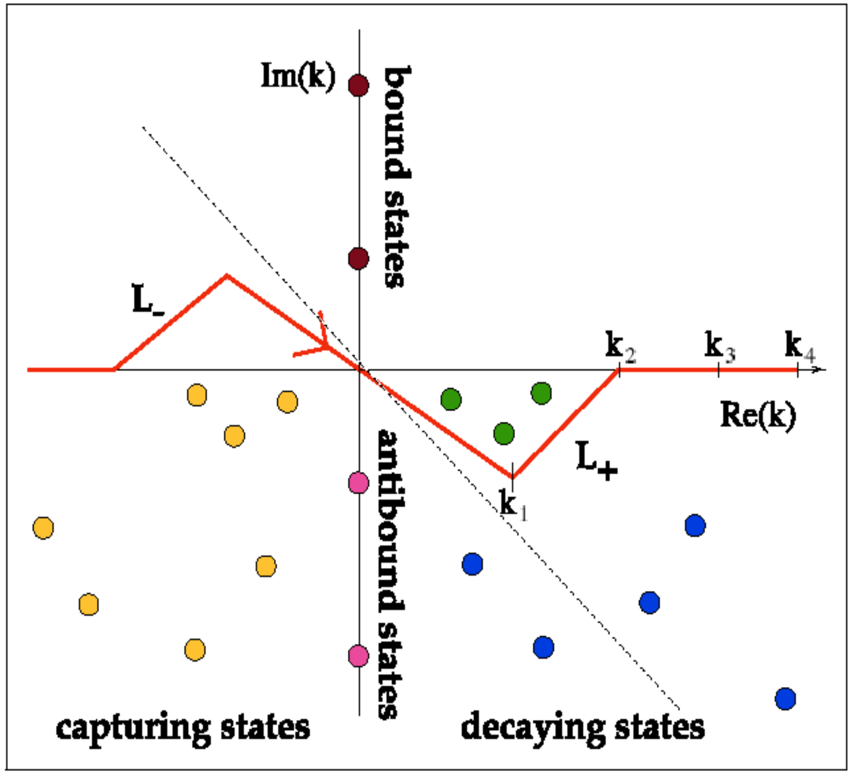
Completness relation for one-body states:

T. Berggren, Nucl. Phys. A109, 265 (1968)

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1$$

- $\sum_n$: bound, anti-bound, and resonance states
- $\int_{L_+}$: non-resonant continuum

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \sum_{i=1}^{N_d} |u_i\rangle\langle\tilde{u}_i| \cong 1 \quad ; \quad \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$



Completness relation for many-body states:

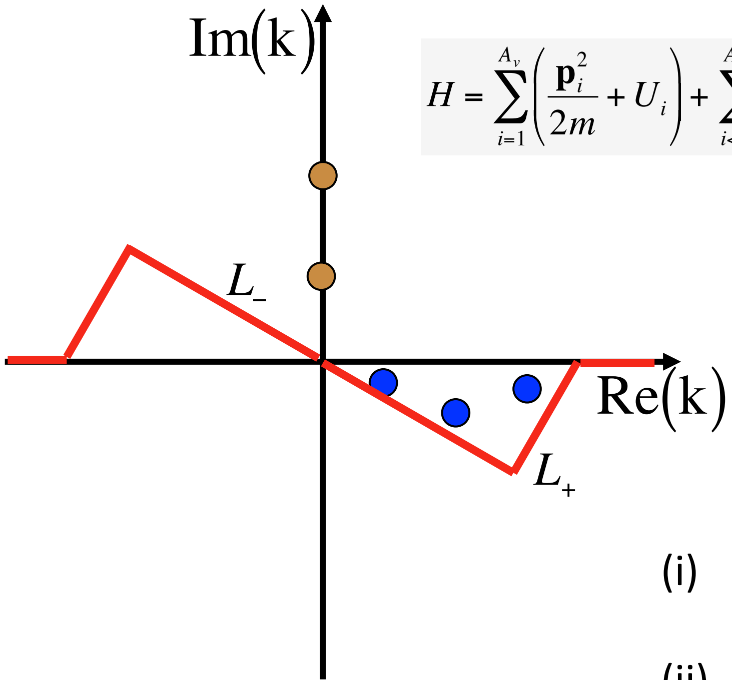
$$\sum_k |SD_k\rangle\langle SD_k| \cong 1$$

Euclidean inner product → RHS inner product

$$\langle u_n|u_n\rangle = \int_0^\infty dr\, u_n^*(r) u_n(r) \xrightarrow[r\to\infty]{} e^{2k_2 r}$$

$$\langle \tilde{u}_n|u_n\rangle = \int_0^\infty dr\, \tilde{u}_n^*(r) u_n(r) \xrightarrow[r\to\infty]{} e^{2ir(k_1 - ik_2)}$$

Gamow Shell Model



$$H = \sum_{i=1}^{A_v} \left(\frac{\mathbf{p}_i^2}{2m} + U_i \right) + \sum_{i<j}^{A_v} \left(V_{ij} + \frac{\mathbf{p}_i \mathbf{p}_j}{(A_c + 1)m} \right)$$

$$H \rightarrow [H]_{ij} = [H]_{ji}$$

(i) two-step diagonalization: $|\Psi_0^{(J)}\rangle \rightarrow \{|\Psi^{(J)}\rangle\}$

- selection of states: $\max\{|\langle \Psi_i^{(J)} | \Psi_0^{(J)} \rangle|\}$

(ii) Density Matrix Renormalization Group method

J. Rotureau et al., PRL 97, 110603

SM respecting unitarity in weakly-bound/unbound states is built on a skeleton of the S-matrix and the many-body completness relation

N. Michel, W. Nazarewicz, M. Ploszajczak, T. Vertse,
J. Phys. G: Nucl. Part. Phys. (Topical Review) 36, 013001 (2008)

Rigged Hilbert space: the natural framework to formulate quantum mechanics

In mathematics, a rigged Hilbert space (Gel'fand triple, nested Hilbert space, or equipped Hilbert space) is a construction designed to link the distribution and square-integrable aspects of functional analysis. Such spaces were introduced to study spectral theory in the broad sense. They can bring together the 'bound state' (eigenvector' and 'continuous spectrum', in one place. The also provide a natural formulation of the Dirac ket-formalism and time-asymmetric aspects of quantum world.

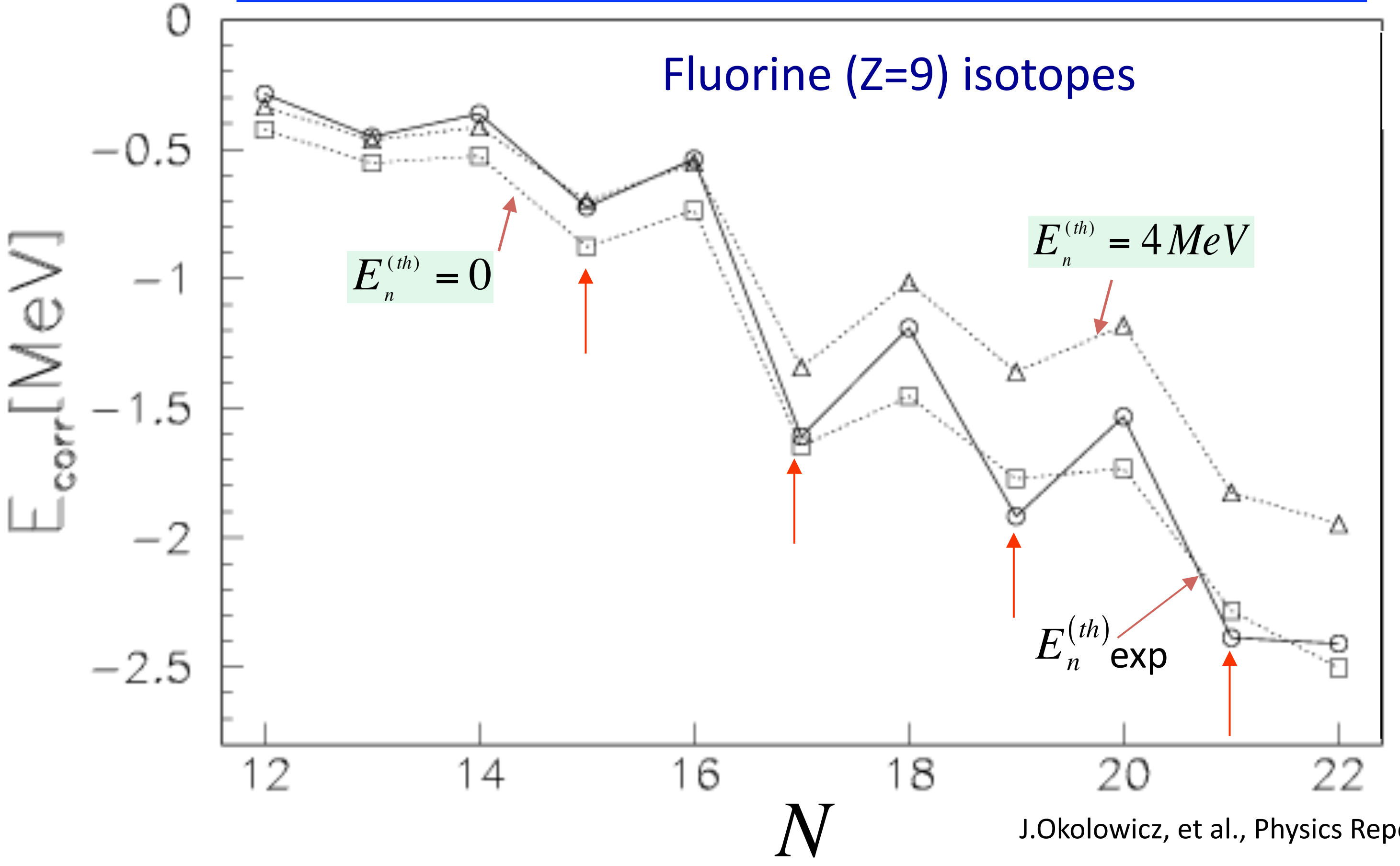
Mathematical foundations in the 1960s by Gel'fand et al. who combined Hilbert space with the theory of distributions. Hence the rigged Hilbert space, rather than the Hilbert space alone, is the natural mathematical setting of Quantum Mechanics.

I.M. Gel'fand and N.J. Vilenkin, Generalized Functions, vol. 4: Some Applications of Harmonic Analysis. Rigged Hilbert Spaces Academic Press, New York, 1964

Salient continuum-coupling phenomena

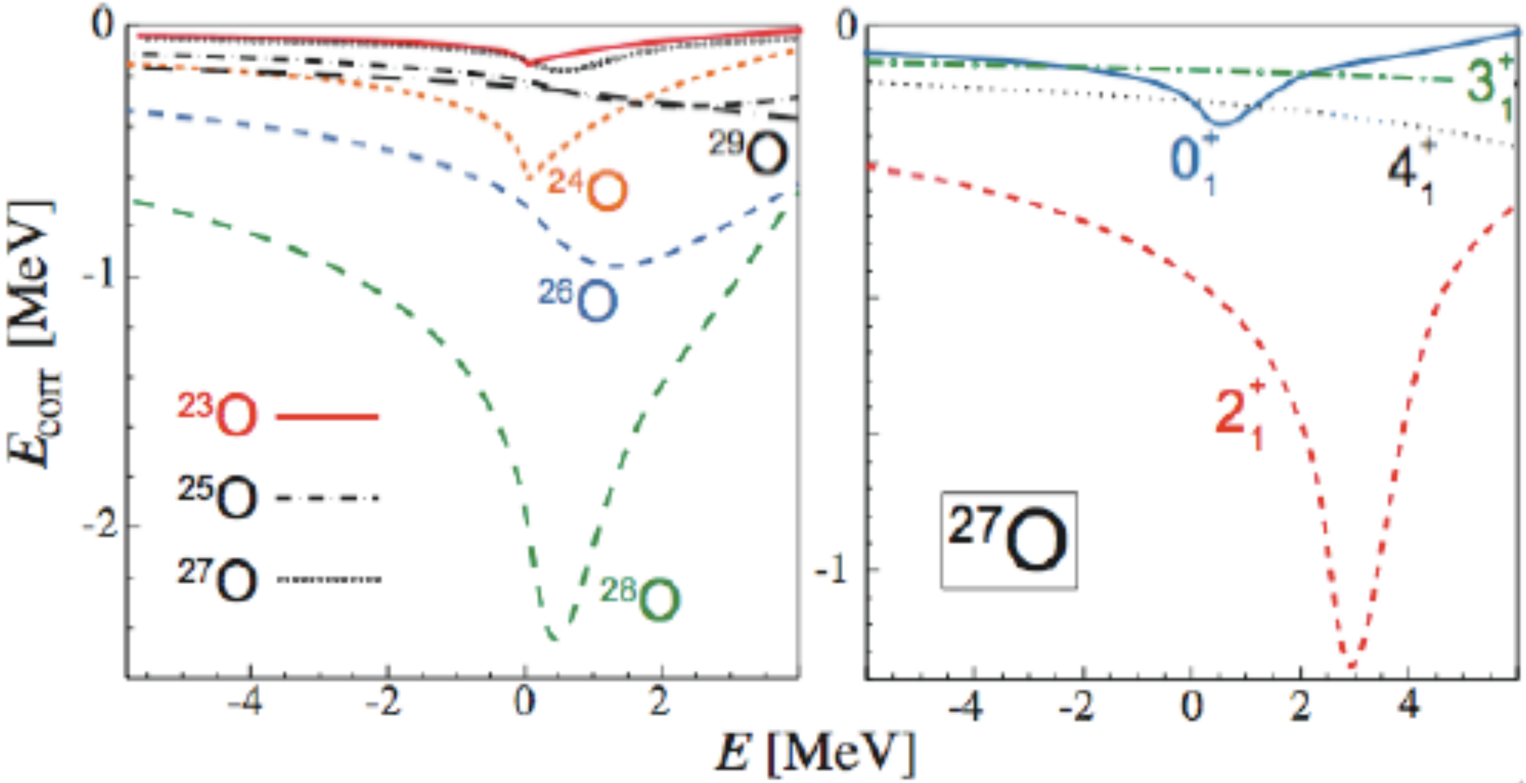
- Anti-odd-even staggering in odd-Z (N) isotopic chains
- Threshold effects in configuration mixing (spectroscopic factors, mirror symmetry breaking, asymptotic normalization coefficients of 1N-overlap integrals,...)
- Alignment of near-threshold states with the decay channel
- Segregation of time-scales in the continuum
- Phase rigidity variations
- Level degeneracies

Continuum-coupling correction to binding energies



J.Okolowicz, et al., Physics Reports 374, 271

Anti-odd-even staggering of E_{corr}



Strong, near-threshold variations of the continuum-coupling energy correction for certain states ('Hermitian' configuration mixing)

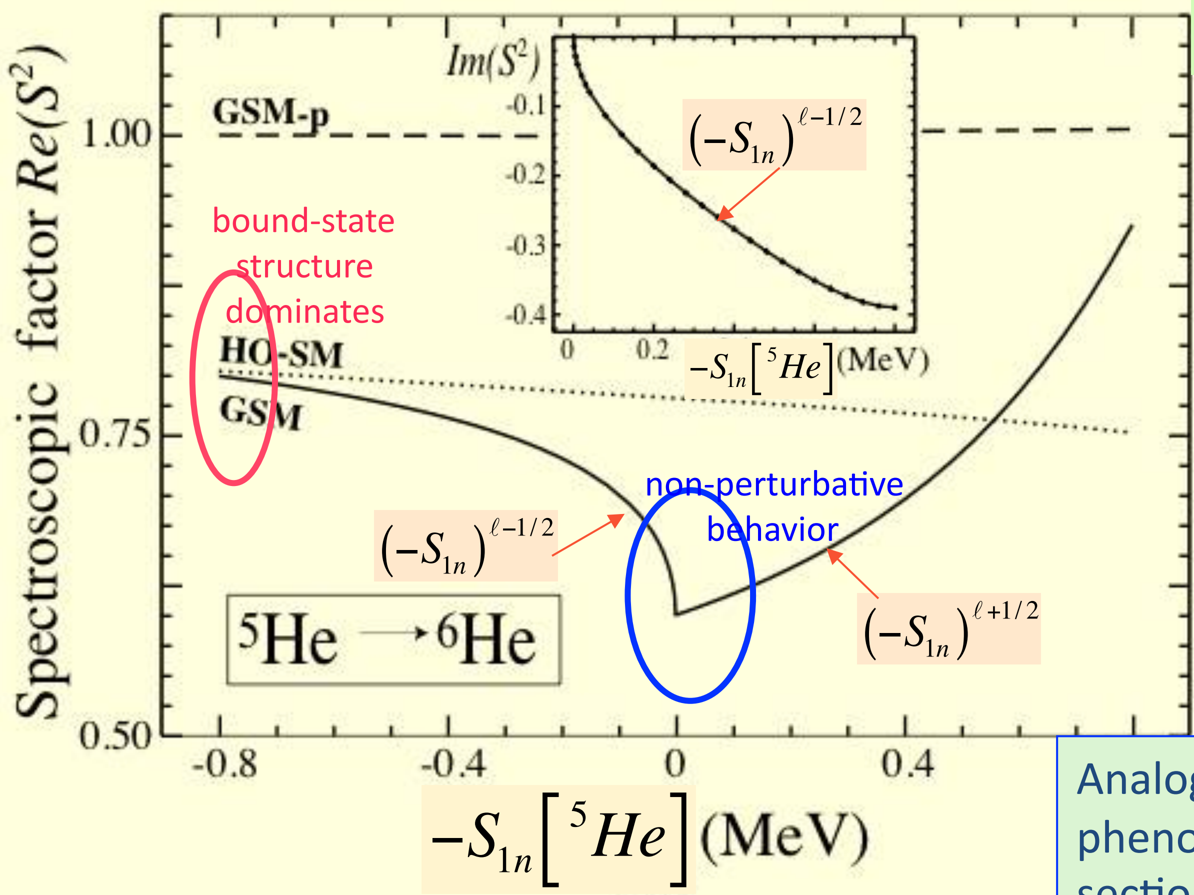
Configuration mixing in weakly bound/unbound many-body states

$$S^2 \equiv \int u_{\ell j}^2(r)dr = \sum\!\!\!\!\int_{\mathcal{B}} \langle \widetilde{\Psi_A^{J_A}} || a_{\ell j}^+(\mathcal{B}) || \Psi_{A-1}^{J_{A-1}} \rangle^2$$

independent of the s.p. basis

$$\left\langle {}^6\mathrm{He(g.s.)} \middle| \left[{}^5\mathrm{He(g.s.)} \otimes p_{3/2} \right]^{0^+} \right\rangle$$

bound — weakly bound/unbound



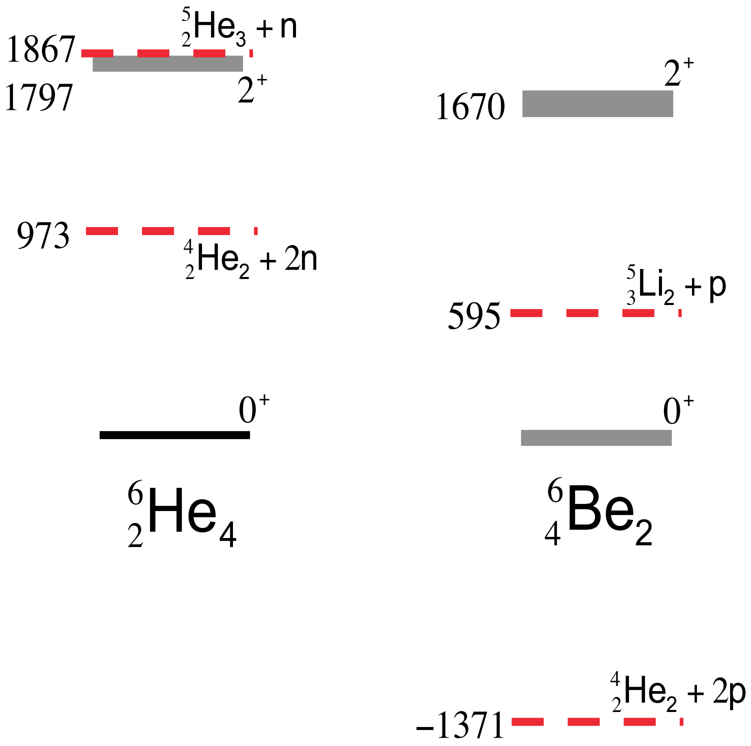
Analogy with Wigner threshold phenomenon for reaction cross-sections E.P. Wigner, Phys. Rev. 73, 1002

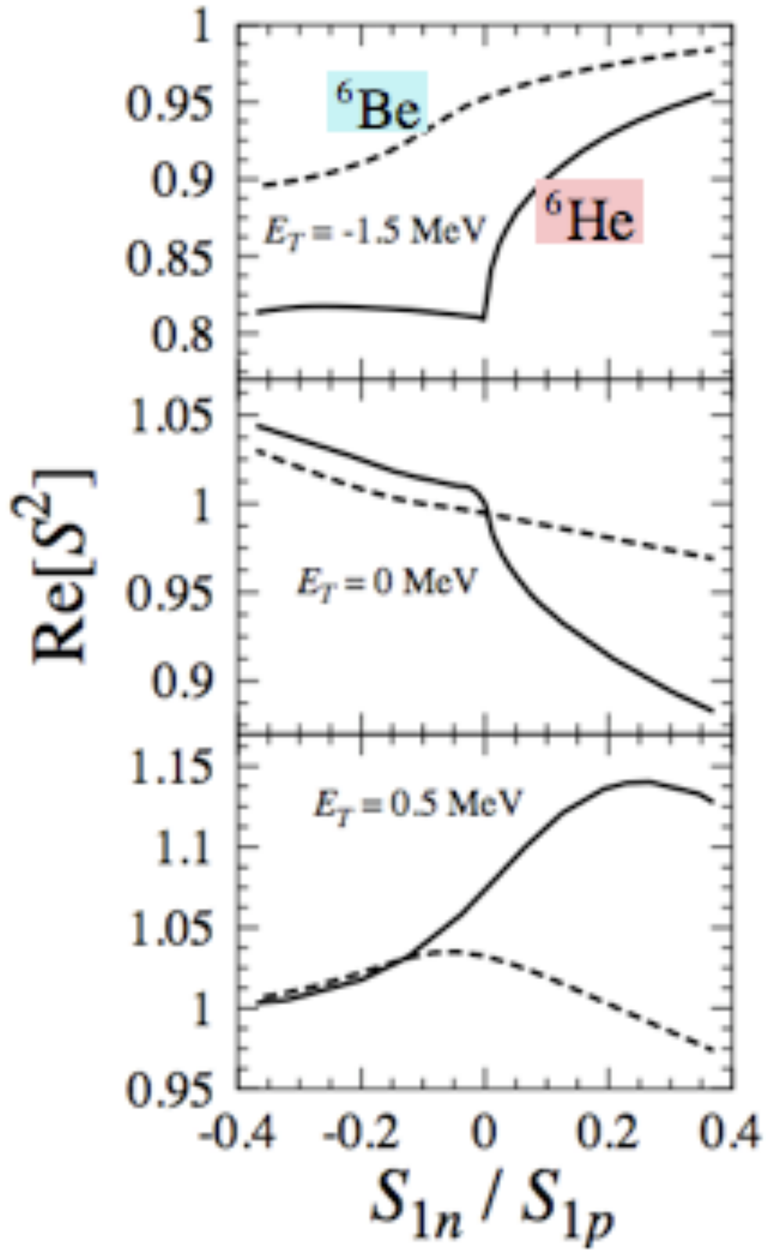
N. Michel, et al., Phys. Rev. C75, 031301(R)

$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \quad \Longleftrightarrow \quad (-S_n)^{\ell-1/2} \quad \text{for} \quad S_n < 0$$

$$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1} \quad \qquad \quad (-S_n)^{\ell+1/2} \quad \text{for} \quad S_n > 0$$

Configuration mixing in mirror systems



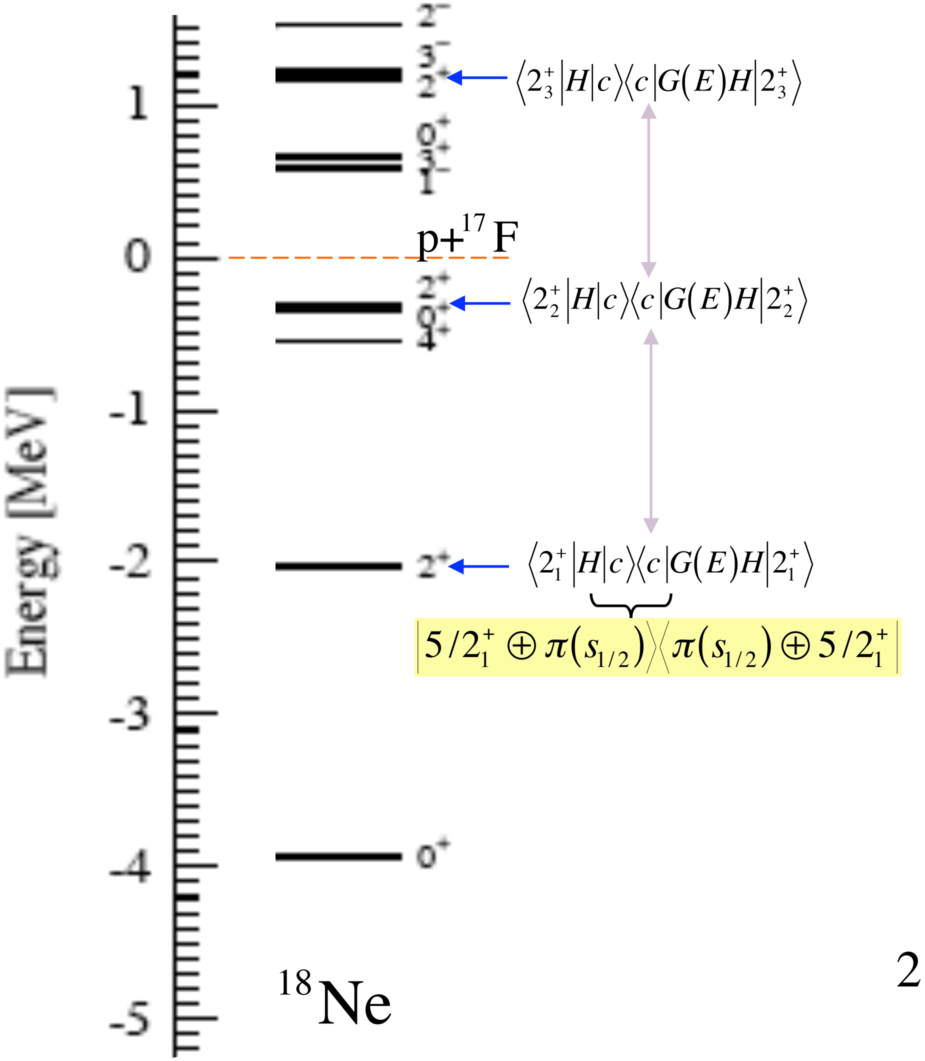


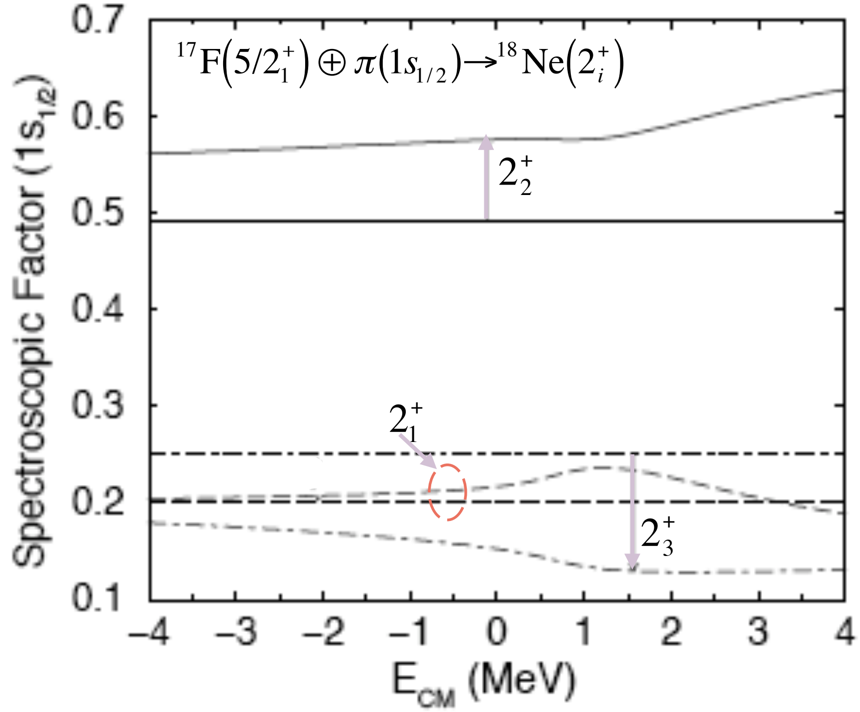
For experimental S_{1n}/S_{1p} , S_{2n}/S_{2p}:

$$S^2_{^6\mathrm{He}}\left[0^+\right] = 0.88 - i0.386$$

$$S^2_{^6\mathrm{Be}}\left[0^+\right] = 1.057 - i0.181$$

Alignment of near-threshold states with decay channels

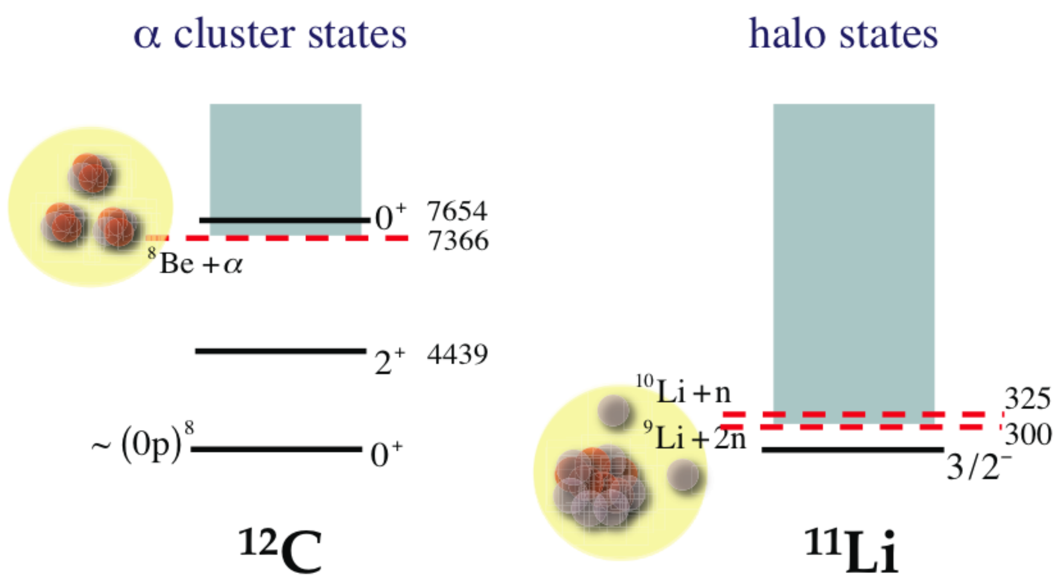




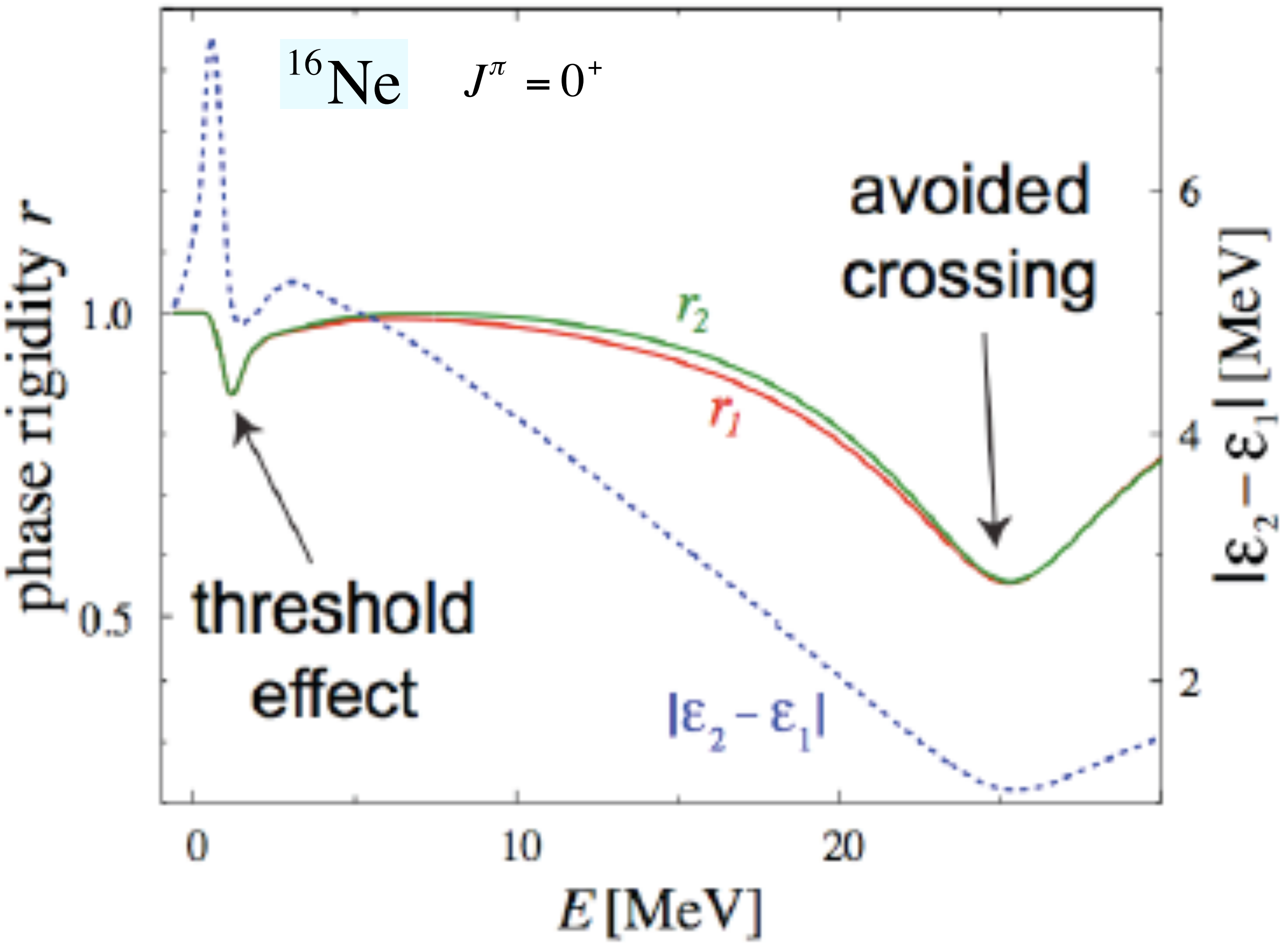
2_2^+ aligns with the decay channel $\mathrm{p}+{}^{17}\mathrm{F}(5/2_1^+)$

R. Chatterjee et al., Nucl. Phys. A764 (2006) 528

Halo and cluster states close to the threshold



Phase rigidity and configuration mixing

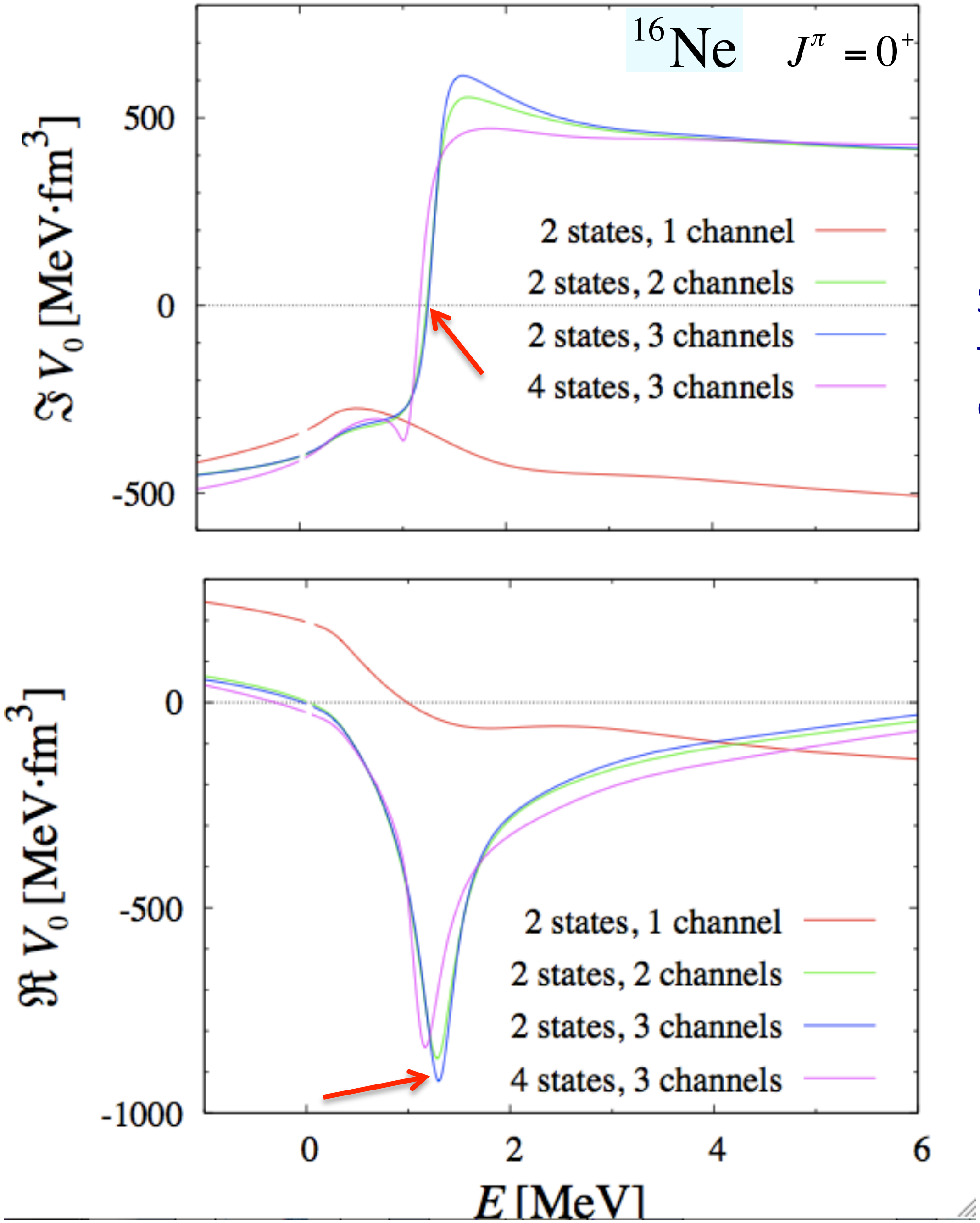


$$r = \left|\rho^2\right|$$

$$\rho = \frac{\int d\mathbf{r}\,\psi(\mathbf{r})^2}{\int d\mathbf{r}\left|\psi(\mathbf{r})\right|^2} = e^{2i\varphi}\,\frac{\int d\mathbf{r}\left|\psi_r(\mathbf{r})\right|^2 - \left|\psi_i(\mathbf{r})\right|^2}{\int d\mathbf{r}\left|\psi_r(\mathbf{r})\right|^2 + \left|\psi_i(\mathbf{r})\right|^2}$$

P.W. Brouwer, Phys. Rev. E68, 046205 (2003)
S.A. van Langen et al., Phys. Rev. E55, R1 (1996)

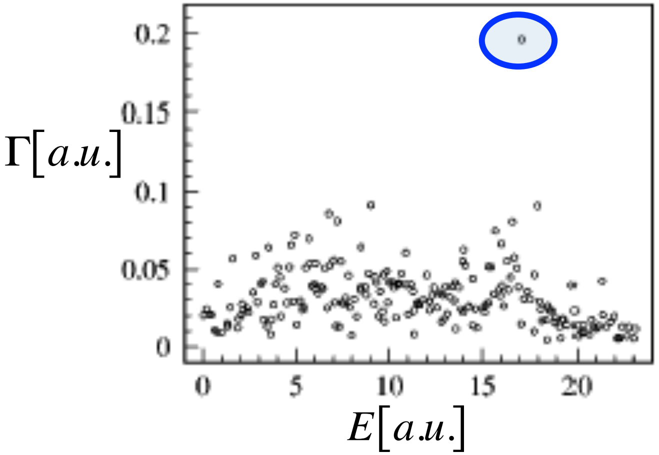
Exceptional strings close to the threshold



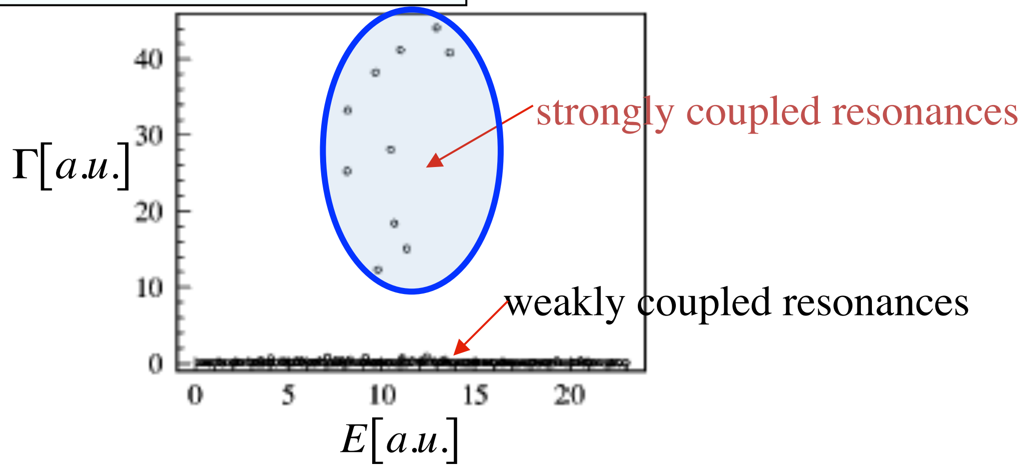
Strong configurations mixing near threshold due to the proximity of exceptional strings

Segregation of time scales in the continuum

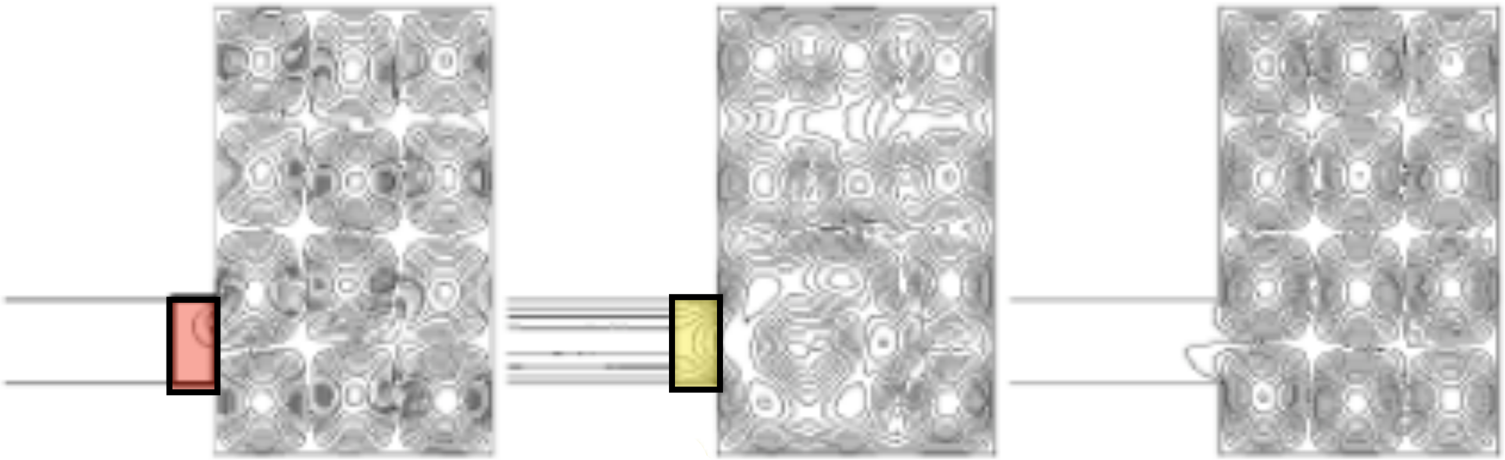
$J^{\pi} = 0^{+}, T = 0$ states in ^{24}Mg , 10 channels



intermediate coupling



strong coupling

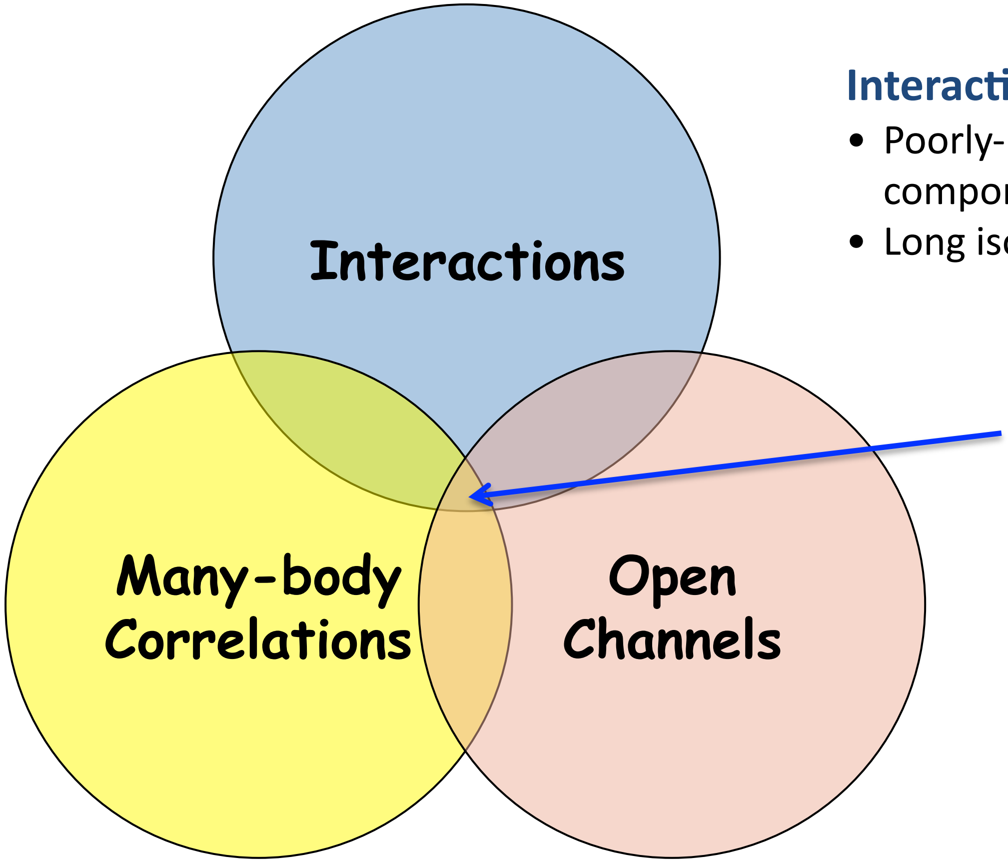


closed cavity

'Bound' states in the continuum

opened cavity

Physics of open nuclei is demanding !



Interactions
- Poorly-known spin-isospin components come into play
- Long isotopic chains ***crucial***

^{7}H, ^{11}Be, ^{42}Si, ^{45}Fe, ^{101}Sn, ^{141}Ho

Configuration interaction
- Mean-field concept often ***questionable***
- Asymmetry of proton and neutron Fermi surfaces gives rise to new couplings
- New collective modes; polarization effects

Open channels
- Nuclei are ***open quantum systems***
- Exotic nuclei have low-energy decay thresholds
- Coupling to the continuum important
  - Virtual scattering
  - Unbound states
  - Impact on in-medium Interactions

Thank you!

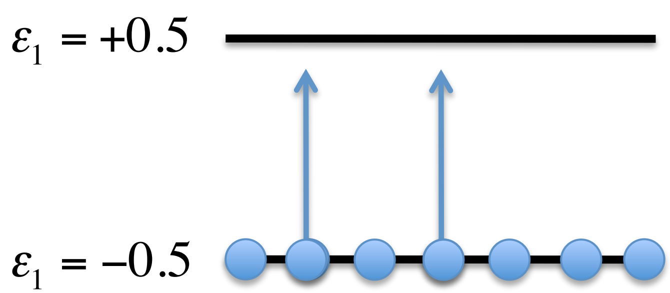
Part II

Quantum tunneling in the driven many-body system

P. Kaminski, M.P., R. Arvieu, Nucl. Phys. A579, 144 (1994)

Tunneling is a non-perturbative phenomenon of purely quantum origin

Static SU(2) model



$$\hat{H} = \sum_{k=1}^{2} \varepsilon_k \left(\sum_{n=1}^{N} a_{nk}^{+} a_{nk} \right) - \frac{1}{2} \sum_{k,l=1}^{2} V_{kl} \left(\sum_{n=1}^{N} a_{nk}^{+} a_{nl} \right)^2$$

$$V_{kl} = V\left(1 - \delta_{kl}\right) \qquad V \geq 0$$

$$G_{kl} = \sum_{n=1}^{N} a_{nk}^{+} a_{nl} \Rightarrow \begin{cases} K_0 = \frac{1}{2}\left(G_{22} - G_{11}\right) \\ K_{+} = G_{21} \\ K_{-} = G_{12} \end{cases}$$

$$\hat{H} = \varepsilon K_0 - \frac{1}{2} V\left(K_{+}^2 + K_{-}^2\right) \qquad \varepsilon = \varepsilon_2 - \varepsilon_1$$

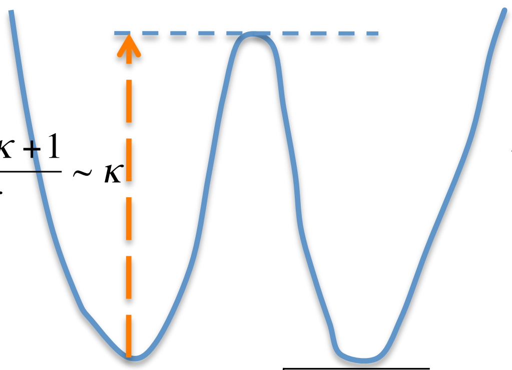
Classical limit

$$\left|\psi_{SD}(z)\right\rangle \equiv |z\rangle = \exp\left(z^{*}G_{21}\right)|0\rangle \Rightarrow \lim_{N\to\infty}\Psi_{g.s.}^{(SU(2))} \equiv \Psi_{HF}^{(SU(2))}$$

$$\left(z,z^{*}\right) \to \left(\beta = \frac{1}{\sqrt{1+z^{*}z}}, \beta^{*} = \frac{1}{\sqrt{1+z^{*}z}}\right) \to \left(q = \sqrt{\frac{1}{2}}\left(\beta+\beta^{*}\right), p = \sqrt{\frac{1}{2}}\left(\beta-\beta^{*}\right)\right)$$

$$\mathcal{H}_{cl} = \frac{\left\langle\psi_{SD}\right|\hat{H}\left|\psi_{SD}\right\rangle}{N\varepsilon} = -\frac{1}{2} + \frac{1}{2}(1-\kappa)q^{2} + \frac{1}{2}(1+\kappa)p^{2} + \frac{1}{4}\kappa\left(q^{4}-p^{4}\right) \qquad \kappa = \frac{V(N-1)}{\varepsilon}$$

$$\Delta V = \frac{\kappa^{2}-8\kappa+1}{4\kappa} \sim \kappa$$



$$\pm q_{eq} = \pm\sqrt{(\kappa-1)/\kappa}$$

$$V(q) = \mathcal{H}_{cl}(q,p=0) = \frac{1}{4}\kappa q^{4} + \frac{1}{2}(1-\kappa)q^{2} - \frac{1}{2}$$

The time-dependent SU(2) model

$$V(t) = V_0 + \frac{\alpha\varepsilon}{N-1}\sin^9(\beta t) \Leftrightarrow \kappa(t) = \kappa_0 + \alpha\sin^9(\beta t)$$

Driving does not change the symmetry of the unperturbed (static) Hamiltonians: $\hat{H},\mathcal{H}_{cl}$

Solution:

$$\begin{cases} \hat{H}(t) = H\left(t + \frac{2\pi n}{\beta}\right) \\ \mathcal{H}_{cl}(t) = \mathcal{H}_{cl}\left(t + \frac{2\pi n}{\beta}\right) \end{cases} \qquad n = 0,\pm1,\pm2,\ldots$$

Floquet theorem:

$$\Psi(t) = \sum_k c_k \Omega_k(t) = \sum_k c_k \exp(-ie_k t)\Phi_k(t)$$

quasi-energies

$$\Phi_k(t) = \Phi_k\left(t + \frac{2\pi n}{\beta}\right) \qquad n = 0,\pm1,\pm2,\ldots$$

quasi-energy eigenstates

$$\Omega_k(t) \equiv \exp(-ie_k t)\Phi_k(t) = \exp(-ie_{k,l} t)\Phi_{k,l}(t) \qquad e_{k,l} = e_k + l\beta$$

For each **k** there exists an infinite number of quasi-energies $e_{k,l}$, however number of independent quasi-energy classes is always equal to the number of eigenenergies in the corresponding stationary case

$$\begin{cases} \Omega_k(t) = \sum_n \Phi_{k,n} e^{in\beta t} \exp(ie_k t) \\ \hat{H}(t) = \sum_n \hat{H}_{(n)} e^{in\beta t} \end{cases} \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

⇓

$$\sum_l \left(\hat{H}_{(n-l)} + n\beta\delta_{l,n} \right) \Phi_{k,l} = e_k \Phi_{k,n} \quad n,l = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Tunneling of wave packets

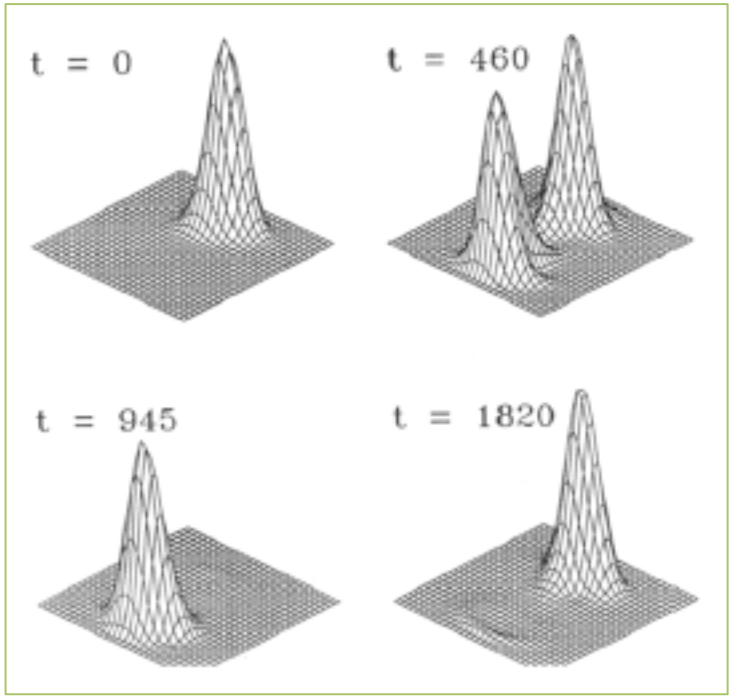
In the absence of time-dependent driving:

$$\left|\Psi(t=0)\right\rangle = \left|q_{eq}, p=0\right\rangle \cong \sqrt{\frac{1}{2}}\left(\left|\Psi_0^{(+)}\right\rangle + \left|\Psi_0^{(-)}\right\rangle\right) \Rightarrow \left|\Psi(t)\right\rangle \cong \sqrt{\frac{1}{2}}\left(\exp\left(-iE_0^{(+)}t\right)\left|\Psi_0^{(+)}\right\rangle + \exp\left(-iE_0^{(-)}t\left|\Psi_0^{(-)}\right\rangle\right)\right)$$

the wave packet oscillates coherently with the frequency: $T_{osc} = 2\pi / \left|E_0^{(+)} - E_0^{(-)}\right|$

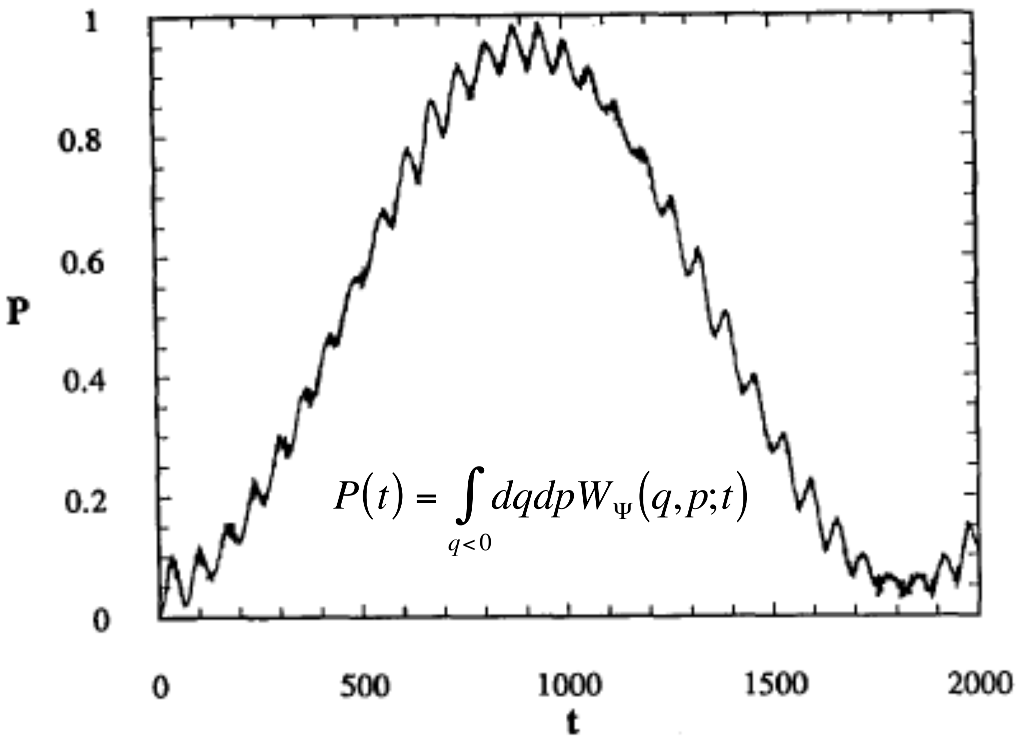
Evolution of the wave packet in the presence of time-periodic driving depends on the spectrum of quasi-energies

$$W_\Psi(q,p;t) = \left|\left\langle \Psi_{SD}(q,p) \middle| \Psi(t)\right\rangle\right|^2 \equiv \left|\left\langle q,p \middle| \Psi(t)\right\rangle\right|^2$$



$$\kappa(t) = \kappa_0 + \alpha \sin^9(\beta t)$$
$$\kappa_0 = 5$$
$$\alpha = 4.75$$
$$\beta = 9 \approx 1.3\omega_0 \quad \omega_0 = \sqrt{2\left(\kappa_0^2 - 1\right)}$$

Tunneling remains coherent – small number of important quasi-energy $\left\{e_{k,l}\right\}$ eigenstates in $\Psi_{SD}\left(q_{eq}, p=0\right)$



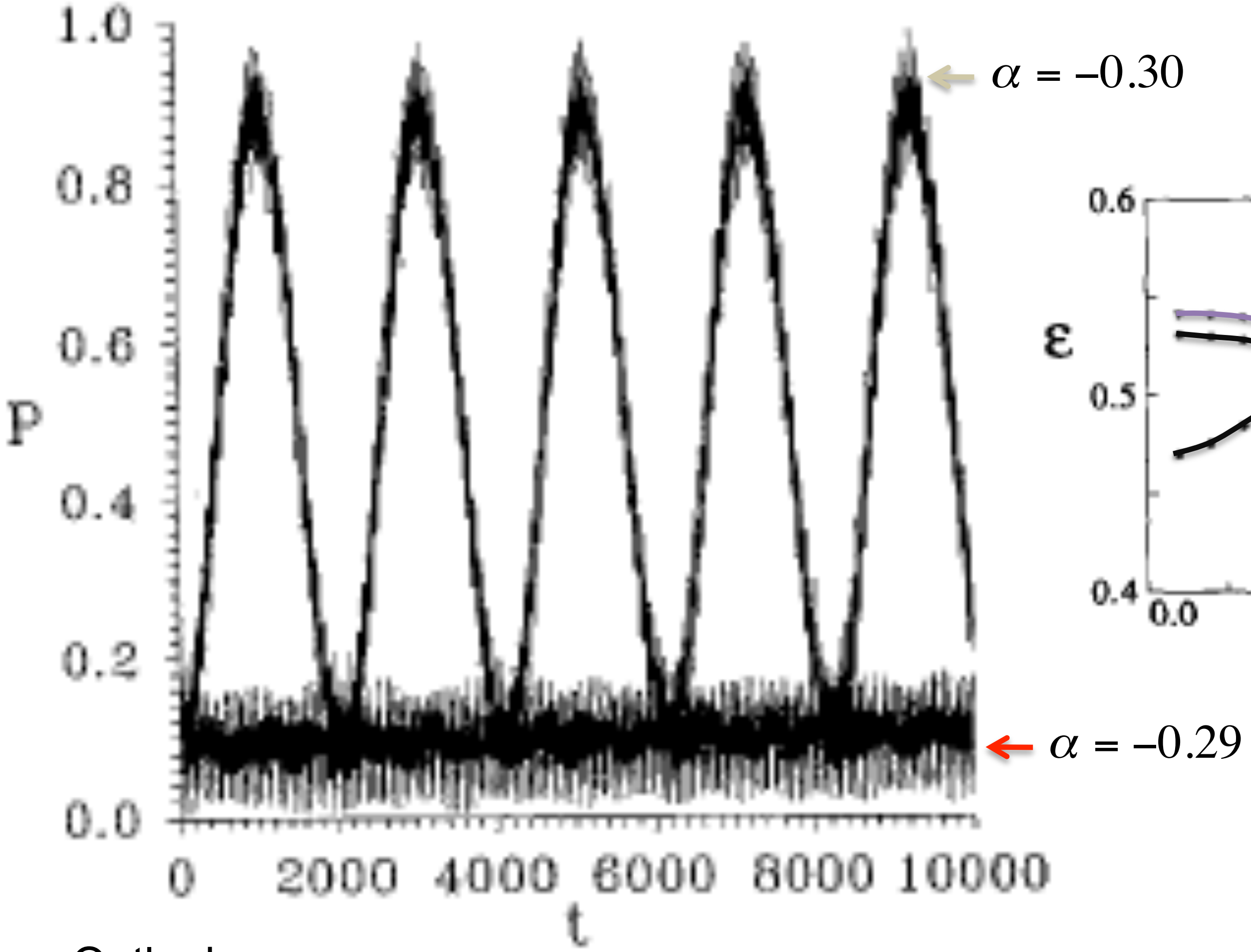
→ $T_{osc} = 1822$

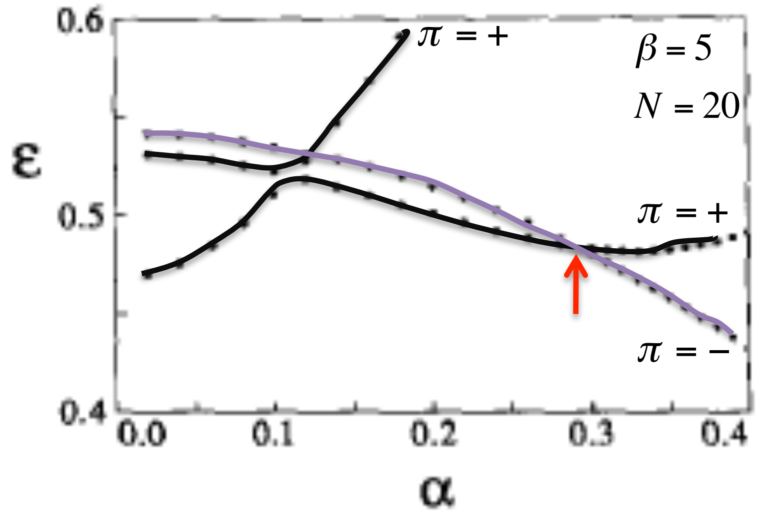
→ $T_{osc} = 63$

π	ϵ_k	$\lvert\langle \Phi_k \lvert q_{eq} \rangle\rvert^2$
+	−4.328873	0.10×10^{-5}
+	−3.436012	0.15×10^{-5}
+	→ 1.524816	0.25×10^{-1}
+	−0.882124	0.63×10^{-6}
+	−0.792667	0.14×10^{-1}
+ ($\pi = +$)	0	0.97×10^{-3}
+	0.792667	0.40×10^{-4}
+	→ 0.882124	0.45
+	1.524816	0.12×10^{-6}
+	3.436012	0.20×10^{-2}
+	4.328873	0.50×10^{-2}
−	−4.444826	0.28×10^{-4}
−	−2.775628	0.17×10^{-3}
−	→ 1.427964	0.36×10^{-1}
−	−0.885572	0.57×10^{-6}
− ($\pi = -$)	−0.793964	0.20×10^{-4}
−	0.793964	0.29×10^{-1}
−	→ 0.885572	0.43
−	1.427964	0.32×10^{-6}
−	2.775628	0.60×10^{-4}
−	4.444826	0.65×10^{-2}

The quasi-energy spectrum depends on the external environment and can be tuned

Coherent suppression of the tunneling





Outlook:

- Tunneling in systems with GOE (GOU) and Poisson quasi-energy spectra statistics
- Fast, incoherent transitions in ‘chaotic’ time-periodic driven many-body systems

Could we control nuclear decay rates in a foreseeable future?