Decoherence in Low-Energy Nuclear Collisions ?

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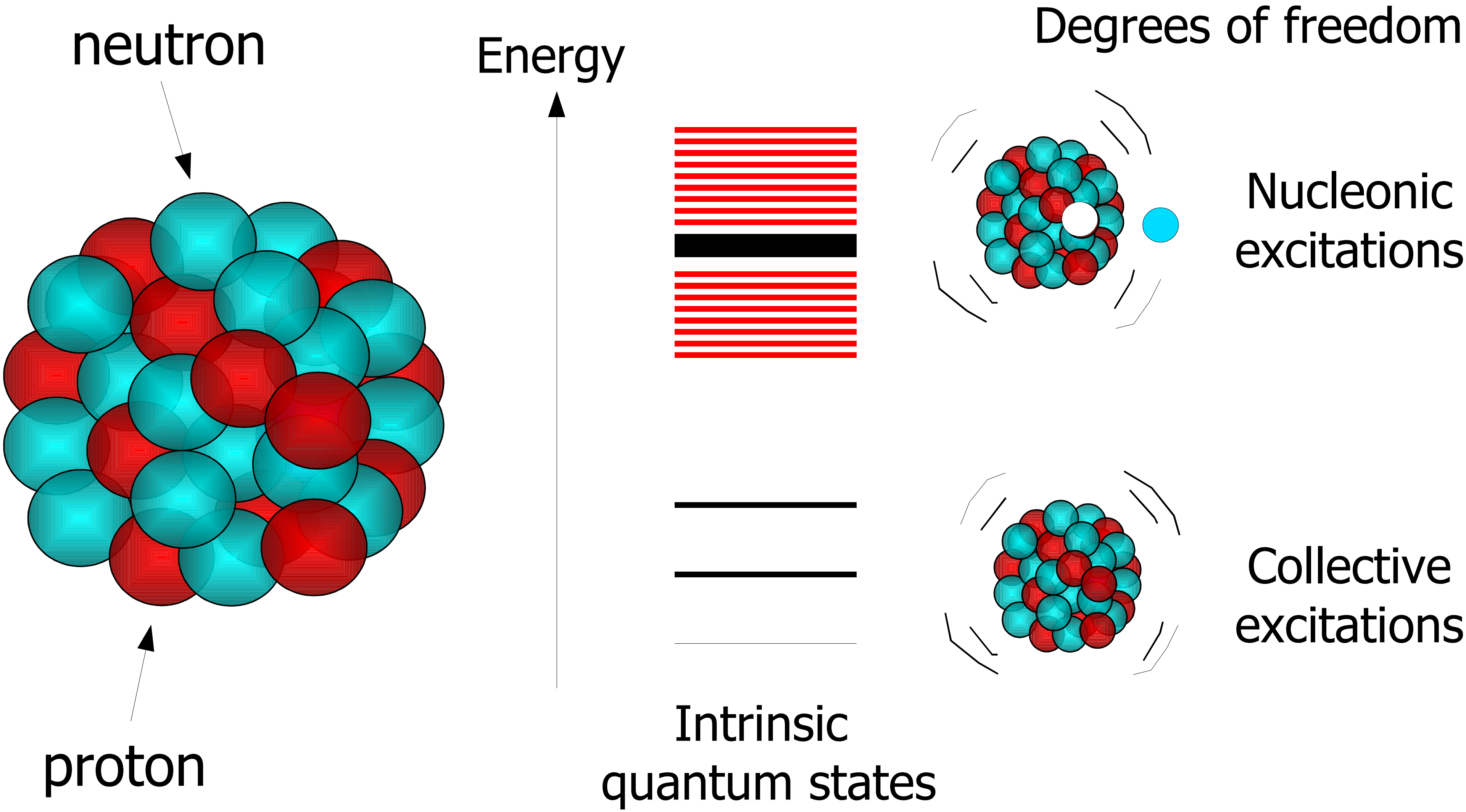
Outline

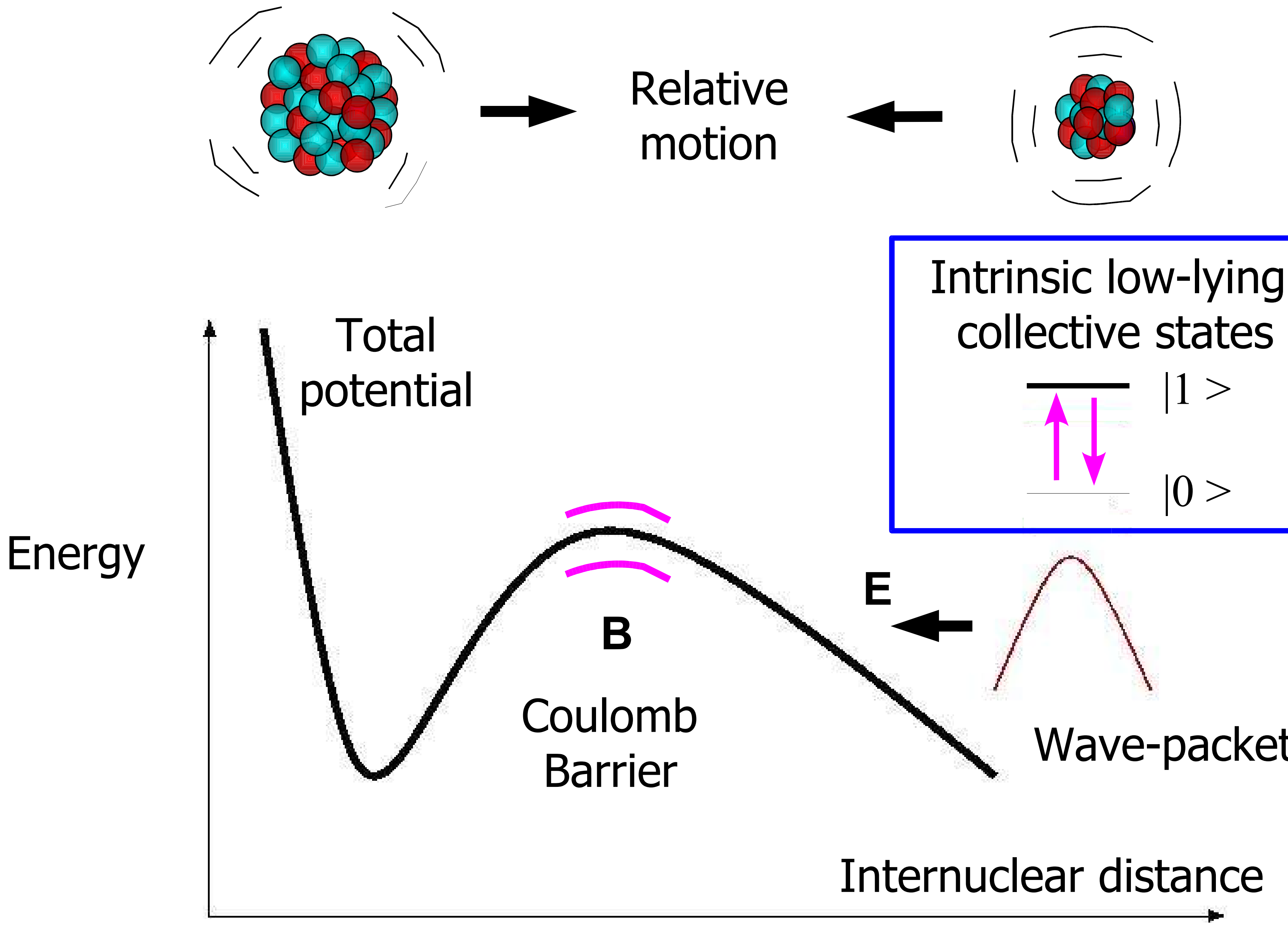
- Introduction
  - Nuclear structure & collision dynamics
  - Coherent coupled channels model
  - Checking theory against measurements
- Quantum decoherence in nuclear collisions
  - Picture & main ideas
  - Decoherence in the complex-potential model ?
  - Coupled-channels density-matrix approach
- Summary

Composite Atomic Nucleus



How do these excitations affect the nuclear collision dynamics ?

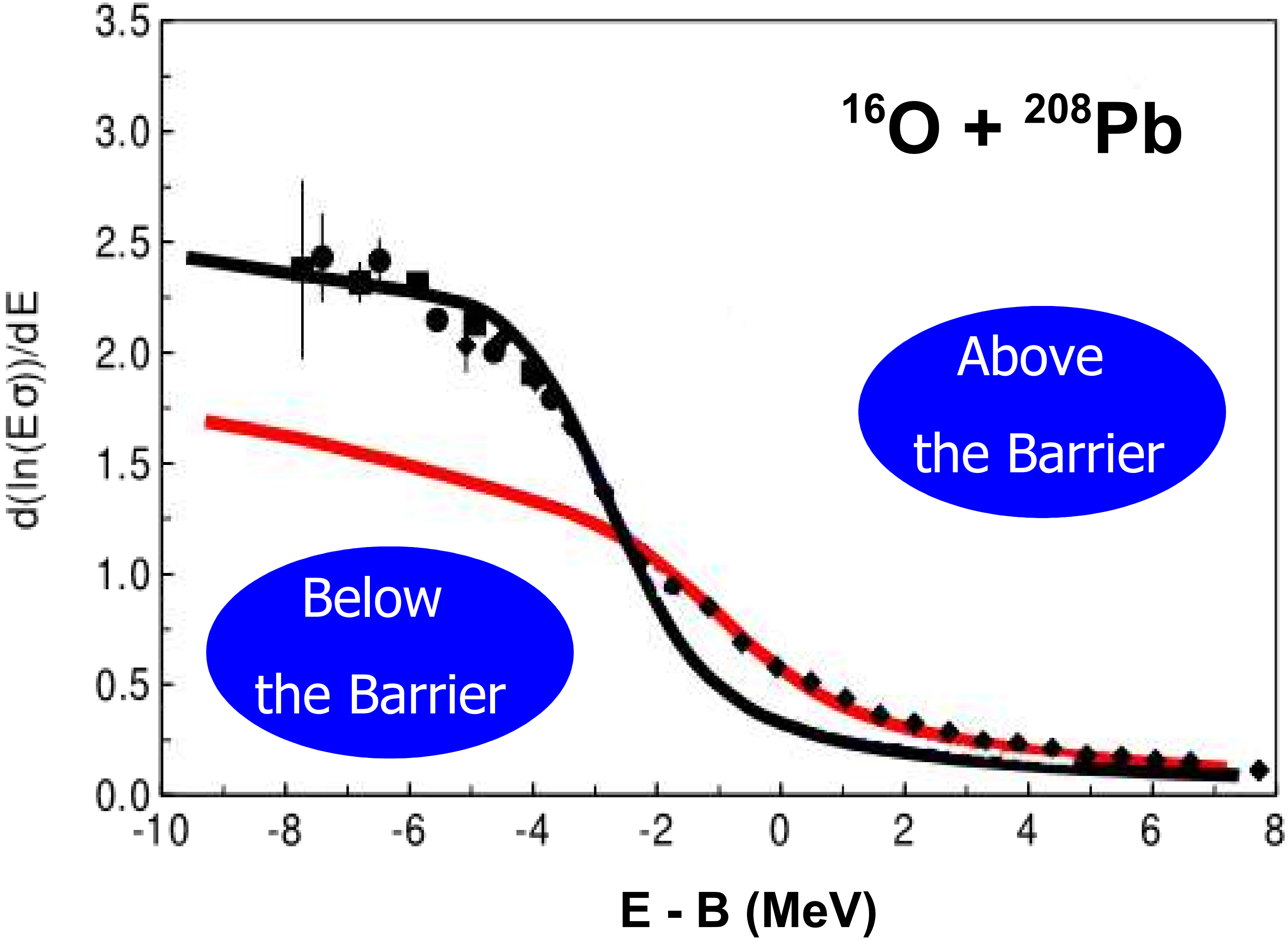
Low-Energy Collision Dynamics: Coherent Quantum Description



Failure of the Coherent Quantum Description

Coupling Assisted Quantum Tunnelling: Nuclear Fusion

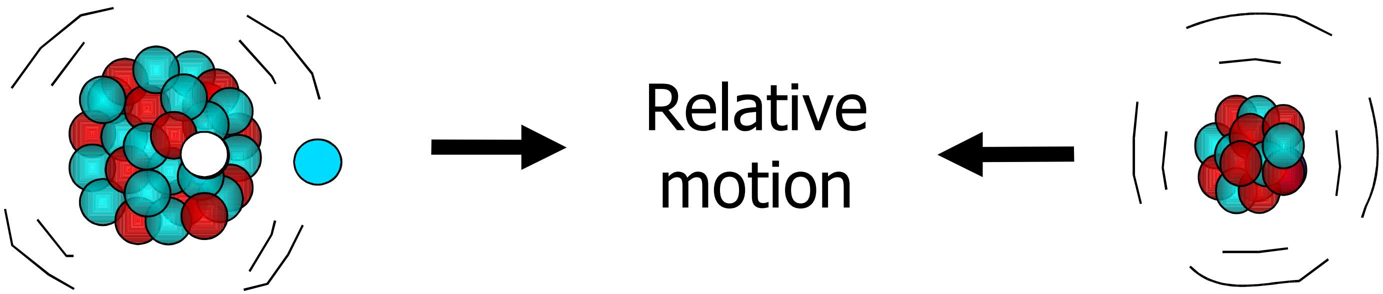
Measure of Fusion Probability

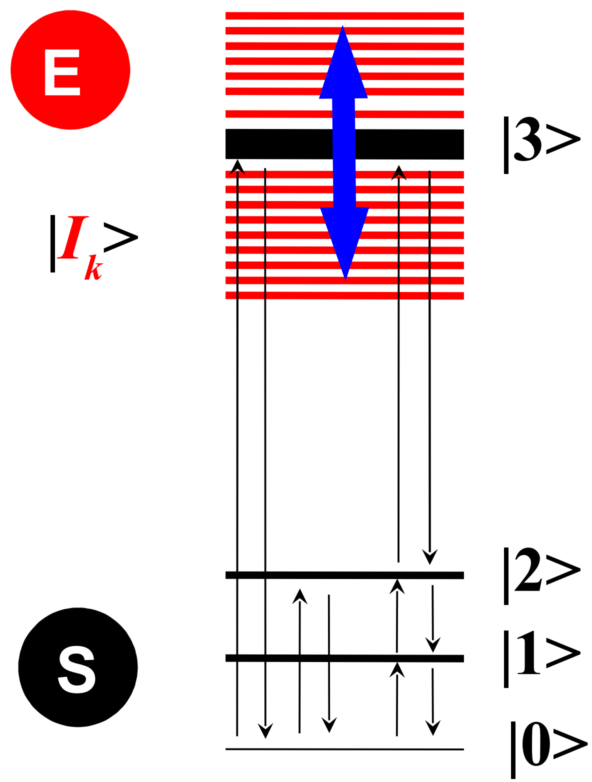


Dasgupta, Hinde, AD-T, Bouriquet, Low, Milburn & Newton, PRL 99 (2007) 192701

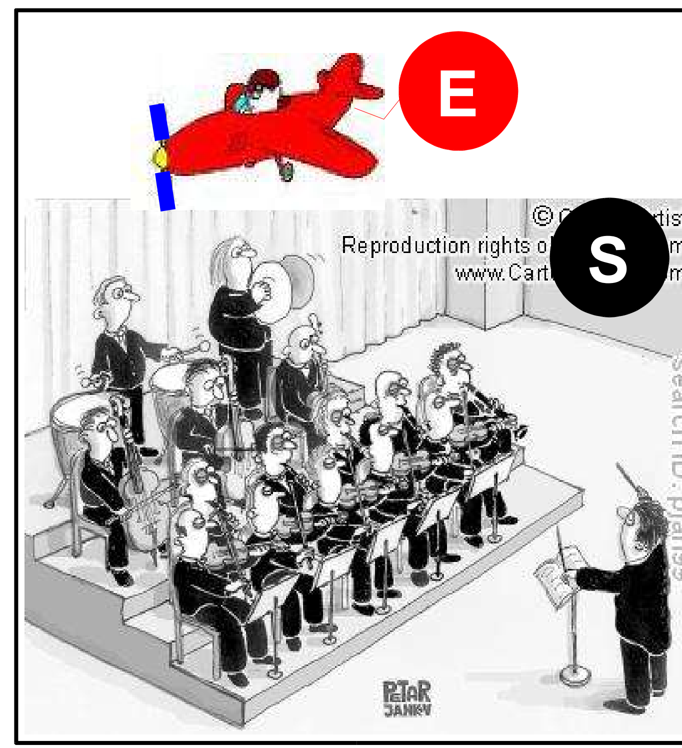
Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604



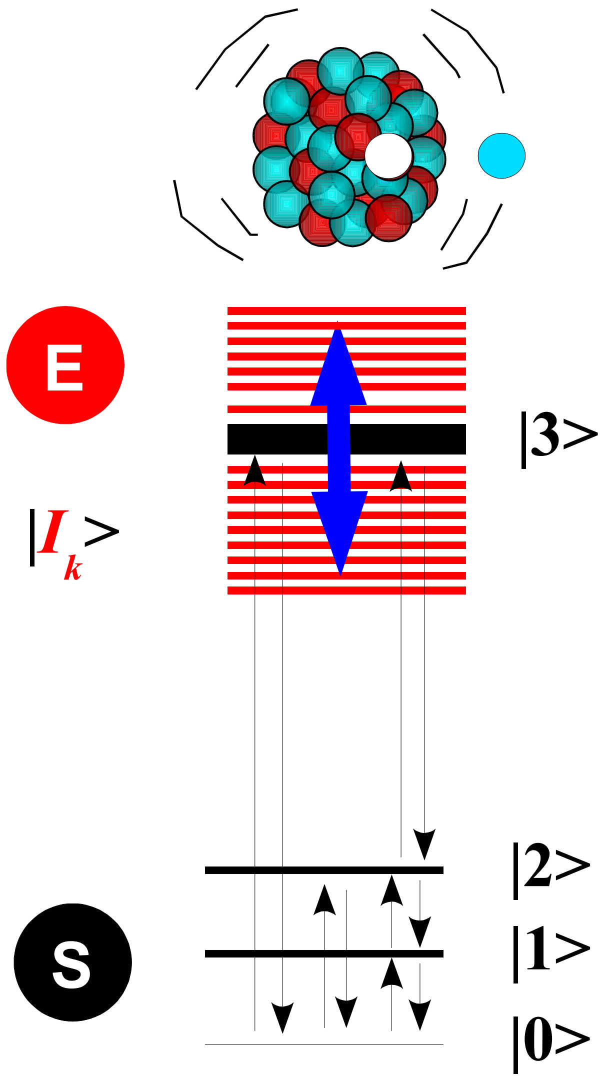


Intrinsic
quantum states



Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604



Intrinsic quantum states

$\partial\hat{\rho}/\partial t = [\hat{\mathcal{L}}_H + \hat{\mathcal{L}}_D]\hat{\rho}\ , \quad \hat{\rho}(0) = \hat{\rho}_0$ Master equation

$\hat{\mathcal{L}}_H\hat{\rho} = -i[\hat{H},\hat{\rho}]/\hbar$ Schrödinger description

$\hat{\mathcal{L}}_D\hat{\rho} = \sum_k \left(\hat{C}_k\hat{\rho}\hat{C}_k^\dagger - \frac{1}{2}[\hat{C}_k^\dagger\hat{C}_k, \hat{\rho}]_+\right)$

$\hat{C}_{Ij} = \sqrt{\Gamma_{Ij}}|I\rangle\langle j|$

Decoherence & Absorption

In Practice (e.g, Pesce & Saalfrank, 1998)

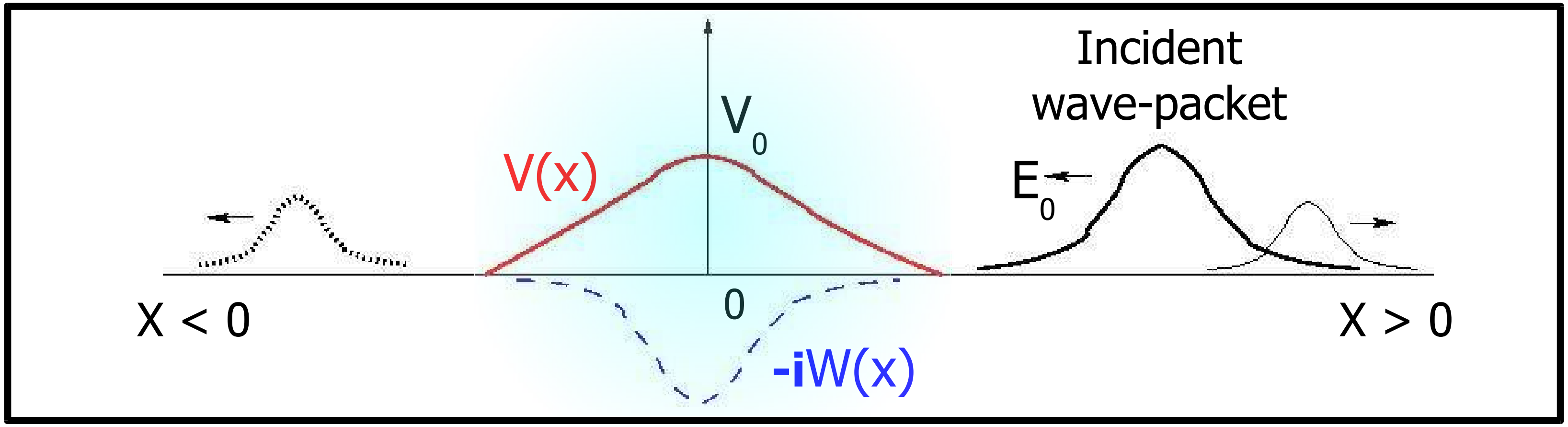
$\hat{\rho}(t) = \sum_{ij,rs} |r)|i\rangle\ \rho_{ij}^{rs}(t)\ \langle j|(s|\ , \quad \rho_{ij}^{rs}(0) = \rho_{00}^{rs}(0) = g_0(r)g_0^*(s)$

$|i\rangle, i = 1,\ldots N$ Intrinsic (energy) basis

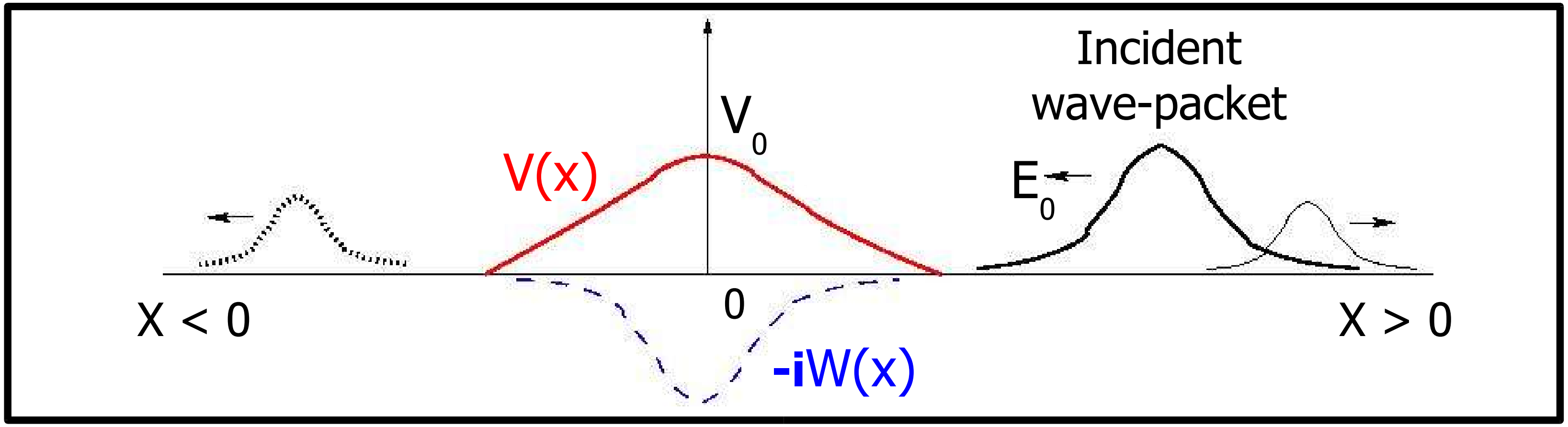
$|r), r = 1,\ldots M$ Coordinate (grid) basis

Absence of Decoherence in the Optical Potential Model

AD-T, accepted as a Rapid Communication in PRC



Optical Potential Model



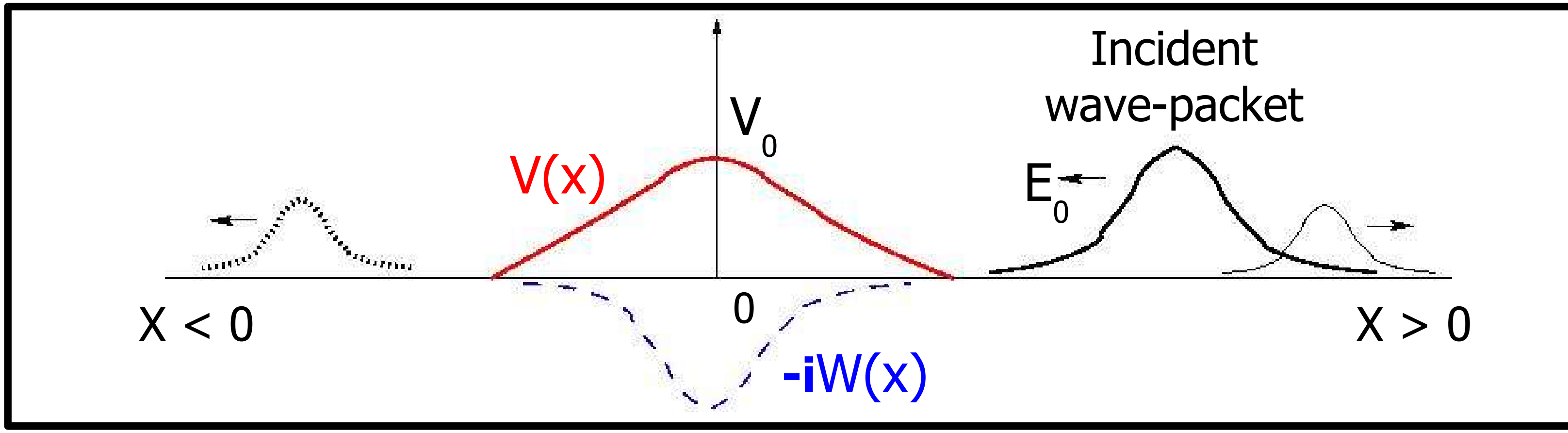
$$\hat{\rho}_0 = |\chi_0\rangle\langle\chi_0|$$

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}(\hat{H}_{eff}\hat{\rho} - \hat{\rho}\hat{H}^{\dagger}_{eff})$$

$$\hat{H}_{eff} = \hat{H}_s - i\,W(x)$$

$$\hat{H}_s = \hat{T} + V(x)$$

Optical Potential Model



$$\rho_0(x, x') = \chi_0(x)\, \chi_0^*(x')$$

$$\dot{\rho}_{xx'} = -\frac{i}{\hbar}\, [\, \hat{H}_s, \hat{\rho}\,]_{xx'} + (\mathcal{L}_D\, \hat{\rho})_{xx'},$$

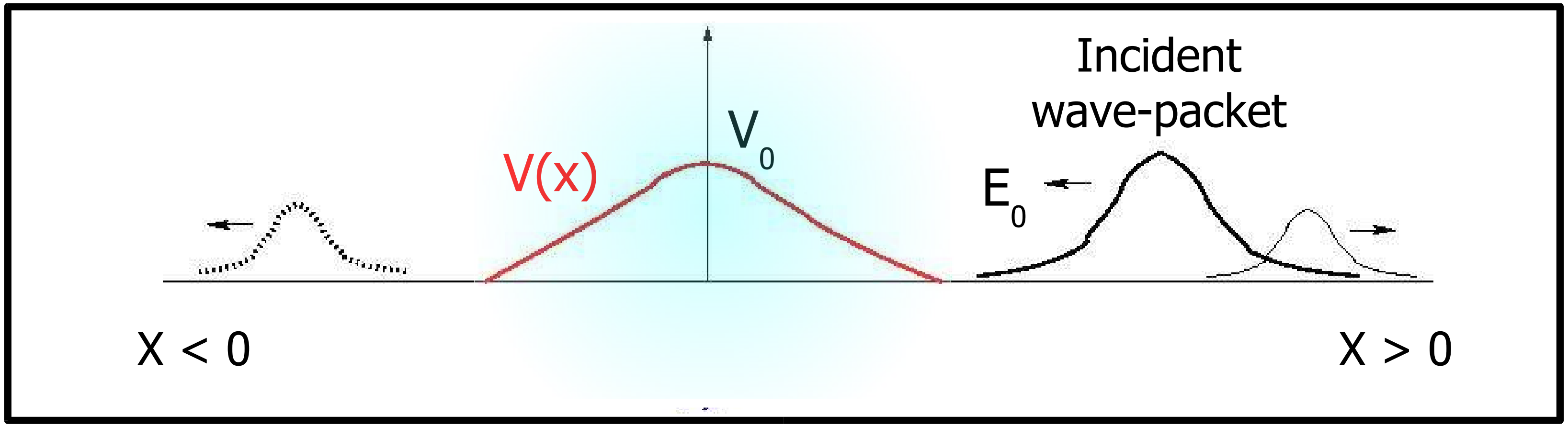
$$\hat{H}_s = \hat{T} + V(x)$$

$$(\mathcal{L}_D\, \hat{\rho})_{xx'} = -\frac{1}{\hbar}\, (\, W(x) + W(x')\,)\, \rho_{xx'}$$

Huisinga et al, J. Chem. Phys. **110** (1999) 5538

Kosloff, Ann. Rev. Phys. Chem. **45** (1994) 145

Lindblad Dissipative Dynamics



$$\dot{\rho}^{11}_{xx'} = -\frac{i}{\hbar}[\hat{H}_s, \hat{\rho}]^{11}_{xx'} + (\mathcal{L}_D\,\hat{\rho})^{11}_{xx'},$$

$$\dot{\rho}^{22}_{xx'} = (\mathcal{L}_D\,\hat{\rho})^{22}_{xx'},$$

$$(\mathcal{L}_D\,\hat{\rho})^{kl}_{xx'} = \delta_{kl}\sum_{j=1}^{2}\sqrt{\gamma^{kj}_{xx}}\,\rho^{jj}_{xx'}\sqrt{\gamma^{kj}_{x'x'}} - \frac{1}{2}\sum_{j=1}^{2}(\gamma^{jk}_{xx} + \gamma^{jl}_{x'x'})\,\rho^{kl}_{xx'},$$

The absorption rate to state $|2\rangle$ is given by $\gamma^{21}_{xx} = W(x)/\hbar$

Measure of Coherence

K. Blum, Density Matrix Theory and Applications (2nd Edition, Plenum Press, 1996) p. 39

For a *pure* state described by the state vector $|\chi\rangle$:

$$\hat{\rho} = |\chi\rangle\langle\chi|, \text{ and } \mathrm{Tr}(\hat{\rho}) = \langle\chi|\chi\rangle.$$

$$\hat{\rho}^2 = |\chi\rangle\langle\chi|\chi\rangle\langle\chi|, \text{ and } \mathrm{Tr}(\hat{\rho}^2) = \langle\chi|\chi\rangle\,\langle\chi|\chi\rangle = [\mathrm{Tr}(\hat{\rho})]^2.$$

Hence, $\boxed{\mathrm{Tr}(\hat{\rho}^2)/[\mathrm{Tr}(\hat{\rho})]^2 = 1}$, for nonzero values of $\mathrm{Tr}(\hat{\rho})$.

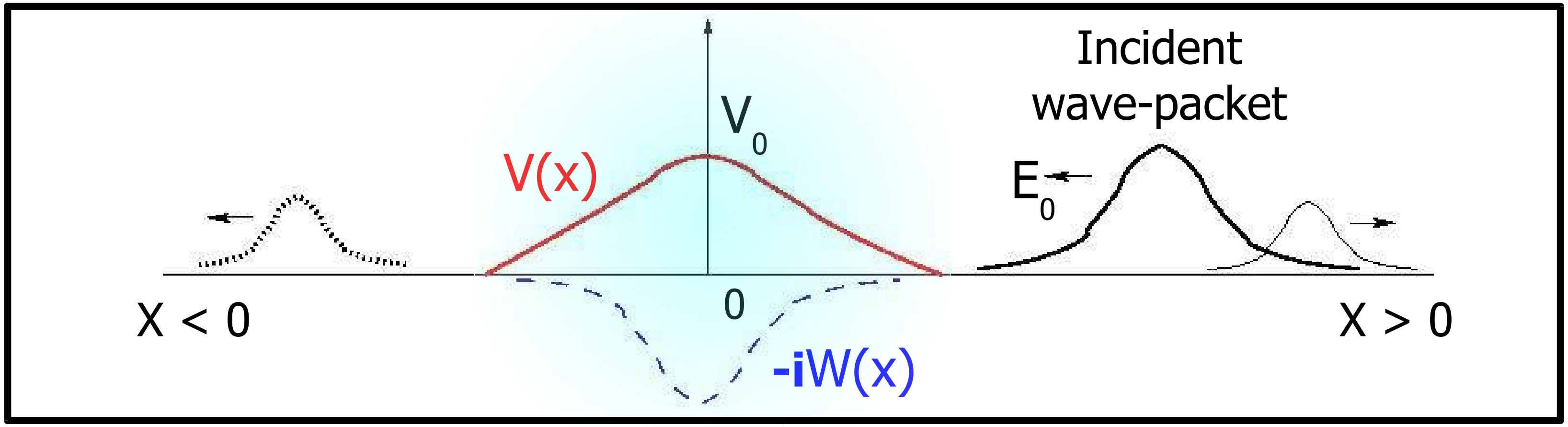
For a *mixed* state, there is no single state vector describing the system:

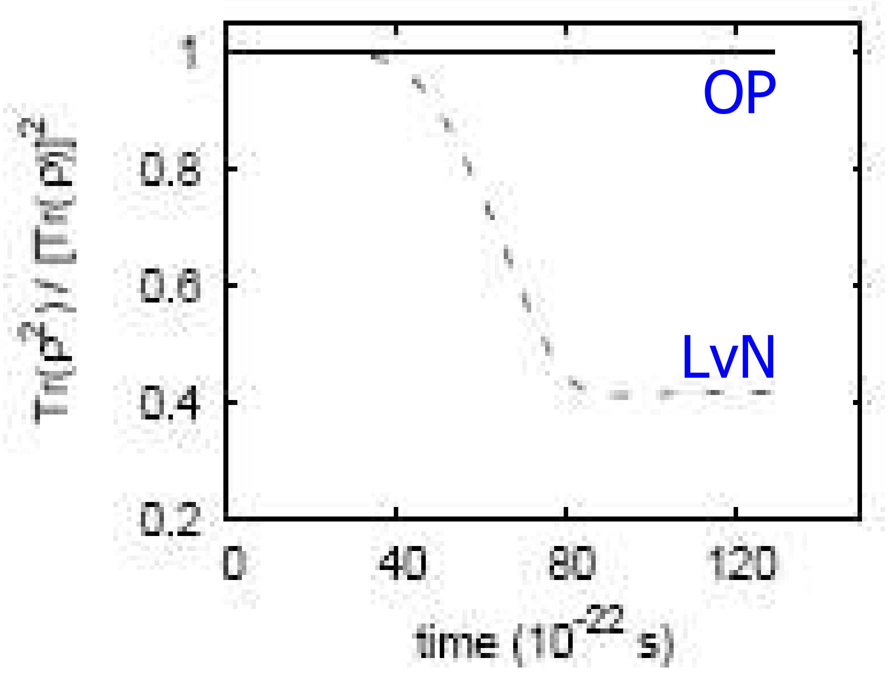
$$\boxed{\mathrm{Tr}(\hat{\rho}^2)/[\mathrm{Tr}(\hat{\rho})]^2 < 1}.$$

The transition from a pure state to a mixed state is caused by decoherence.

Absence of Decoherence in the Optical Potential Model

AD-T, accepted as a Rapid Communication in PRC





Absence of Decoherence in the Optical Potential Model

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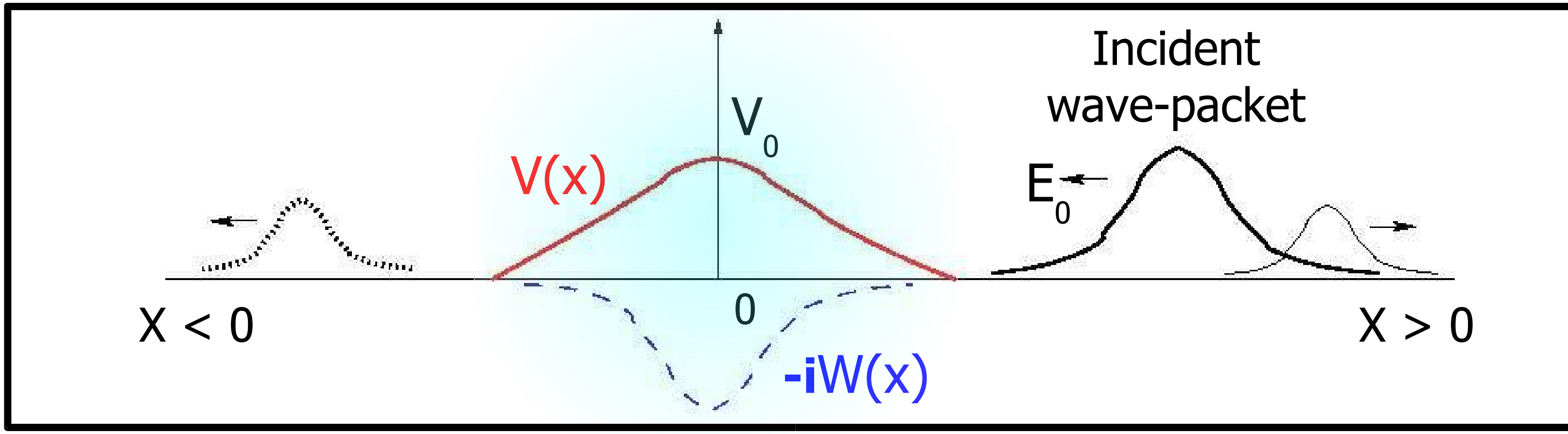
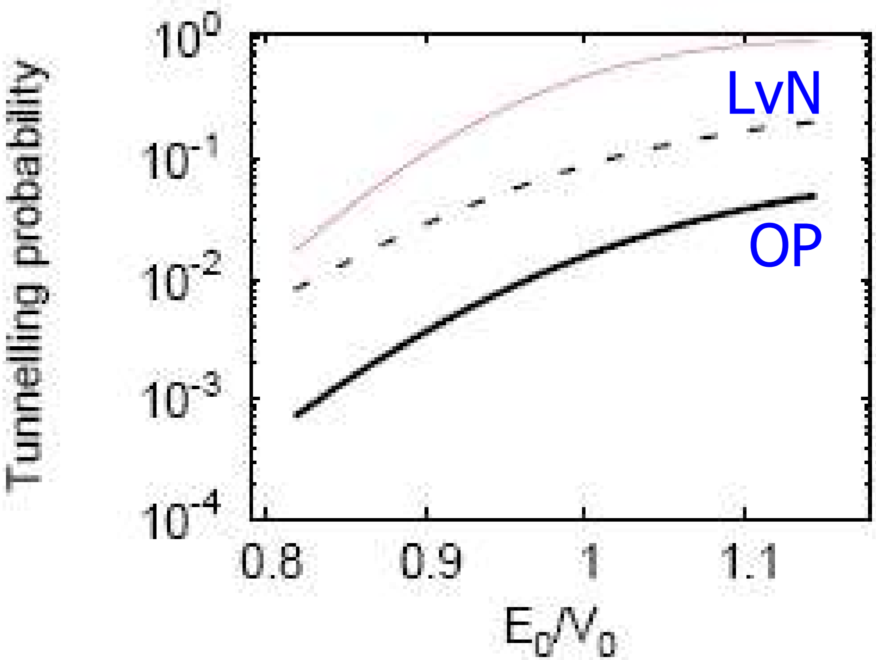
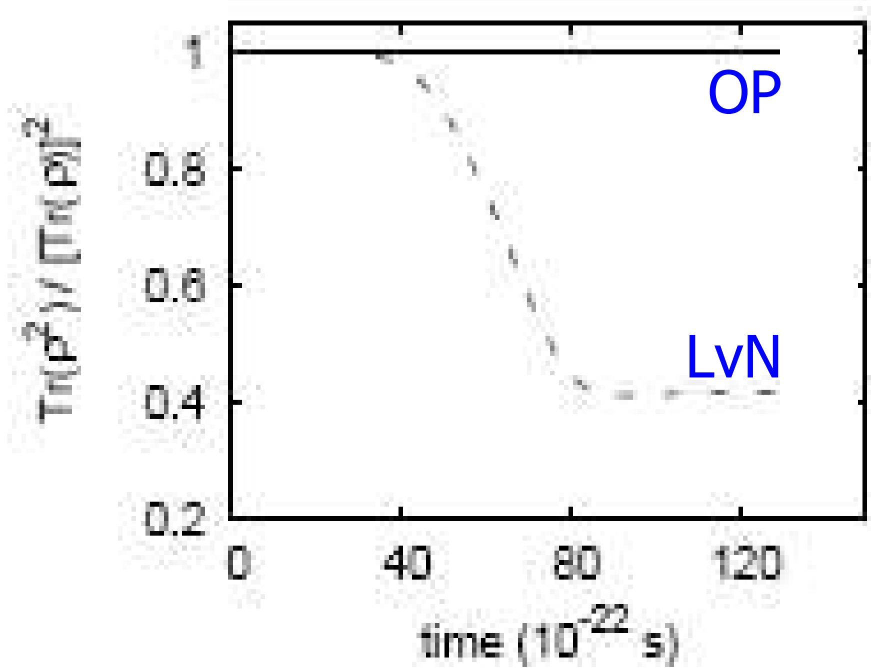
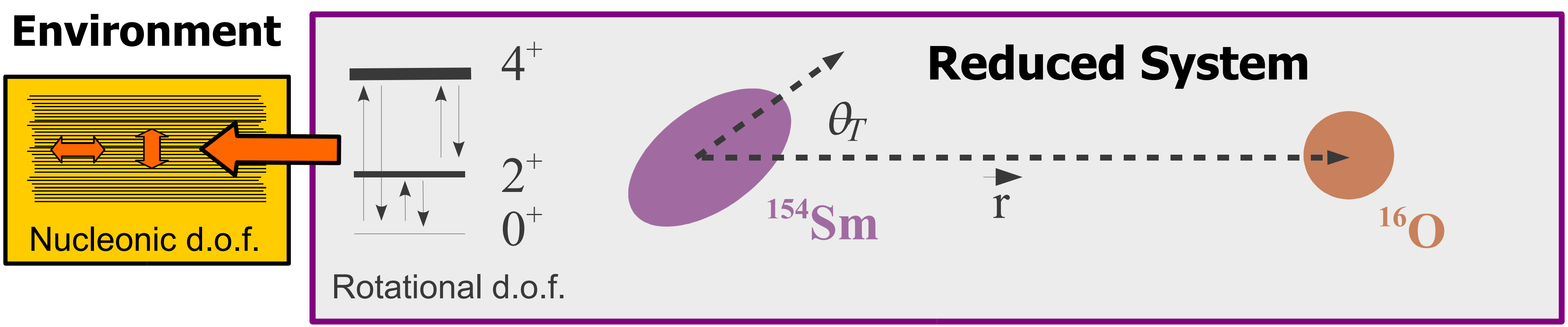




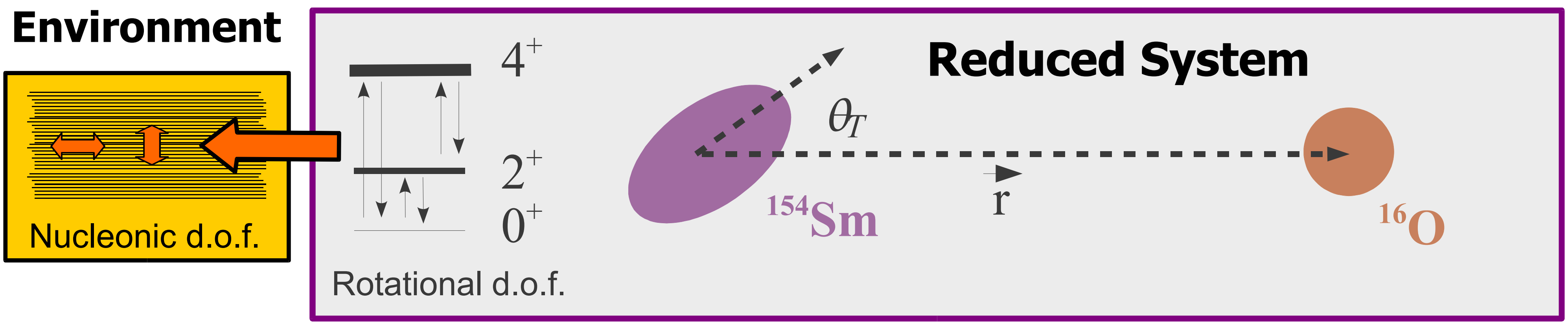


Decoherence significantly affects quantum tunnelling, and thus scattering as well

Coupled-Channels Density-Matrix Approach



Coupled-Channels Density-Matrix Approach



$$|\chi\rangle = \sum_{LJM} \psi_{k_0}(r)\,|0\,L;JM\rangle \;\Rightarrow\; \hat{\rho}_0 = |\chi\rangle\langle\chi|$$

$$\hat{\rho}_0 = \sum_{\alpha,\alpha',rs} |r)|\alpha\rangle\, \rho^{rs}_{\alpha\alpha'}(t=0)\, \langle\alpha'|(s|,$$

where $\alpha \equiv (IL;JM)$, $|\alpha\rangle$ and $|r)$ are the coupled angular momentum basis and the discrete grid-basis describing the internuclear separations, respectively.

$$\rho^{rs}_{\alpha\alpha'}(t=0) = N^2 \exp\left[-\frac{(r-r_0)^2}{2\sigma^2}\right] e^{ik_0\, r} \times \exp\left[-\frac{(s-r_0)^2}{2\sigma^2}\right] e^{-ik_0\, s}\, \delta_{I\,0}\, \delta_{I'\,0},$$

where N is determined from the normalization condition $\sum_{r\alpha} \rho^{rr}_{\alpha\alpha} = 1$.

Coupled-Channels Density-Matrix Approach

Equations of Motion

S

$$
\begin{aligned}
i\hbar\,\dot{\rho}^{rs}_{\alpha\alpha'} \;=\; & \sum_t \left(T^{rt}\,\rho^{ts}_{\alpha\alpha'} - \rho^{rt}_{\alpha\alpha'}\,T^{ts} \right) \\
& + \left[U_\alpha(r) - U_{\alpha'}(s) \right] \rho^{rs}_{\alpha\alpha'} \\
& + \sum_\beta \left[V_{\alpha\beta}(r)\,\rho^{rs}_{\beta\alpha'} - \rho^{rs}_{\alpha\beta}\,V_{\beta\alpha'}(s) \right] \\
& + \left(\varepsilon_\alpha - \varepsilon_{\alpha'} \right) \rho^{rs}_{\alpha\alpha'} \\
& + i\hbar \left\{ \delta_{\alpha\alpha'} \sum_\mu \sqrt{\Gamma^{rr}_{\alpha\mu}}\,\rho^{rs}_{\mu\mu}\,\sqrt{\Gamma^{ss}_{\alpha\mu}} \right. \\
& \left. - \frac{1}{2} \sum_\mu \left(\Gamma^{rr}_{\mu\alpha} + \Gamma^{ss}_{\mu\alpha'} \right) \rho^{rs}_{\alpha\alpha'} \right\}
\end{aligned}
$$

E

$$
\begin{aligned}
\dot{\rho}^{rs}_{\bar{\alpha}\bar{\alpha}'} \;=\; & \delta_{\bar{\alpha}\bar{\alpha}'} \sum_\mu \sqrt{\Gamma^{rr}_{\bar{\alpha}\mu}}\,\rho^{rs}_{\mu\mu}\,\sqrt{\Gamma^{ss}_{\bar{\alpha}\mu}} \\
& - \frac{1}{2} \sum_\mu \left(\Gamma^{rr}_{\mu\bar{\alpha}} + \Gamma^{ss}_{\mu\bar{\alpha}'} \right) \rho^{rs}_{\bar{\alpha}\bar{\alpha}'}
\end{aligned}
$$

Expectation value of an observable: $\langle \hat{\mathcal{O}}(t) \rangle = \dfrac{\mathrm{Tr}[\hat{\mathcal{O}}\,\hat{\rho}(t)]}{\mathrm{Tr}[\hat{\rho}(t)]}$

Coupled-Channels Density-Matrix Approach

Asymptotic Observables

The probability for producing the target in state (I, M_I) with the relative coordinate in the direction $\hat{r}'$:

$$\frac{dW}{d\Omega}(I, M_I) = \sum_q C^{JM}_{LmIM_I} Y_{Lm}(\hat{r}') \mathcal{S}_{\gamma\lambda}(t_f) \times C^{J'M'}_{L'm'IM_I} Y^*_{L'm'}(\hat{r}'),$$

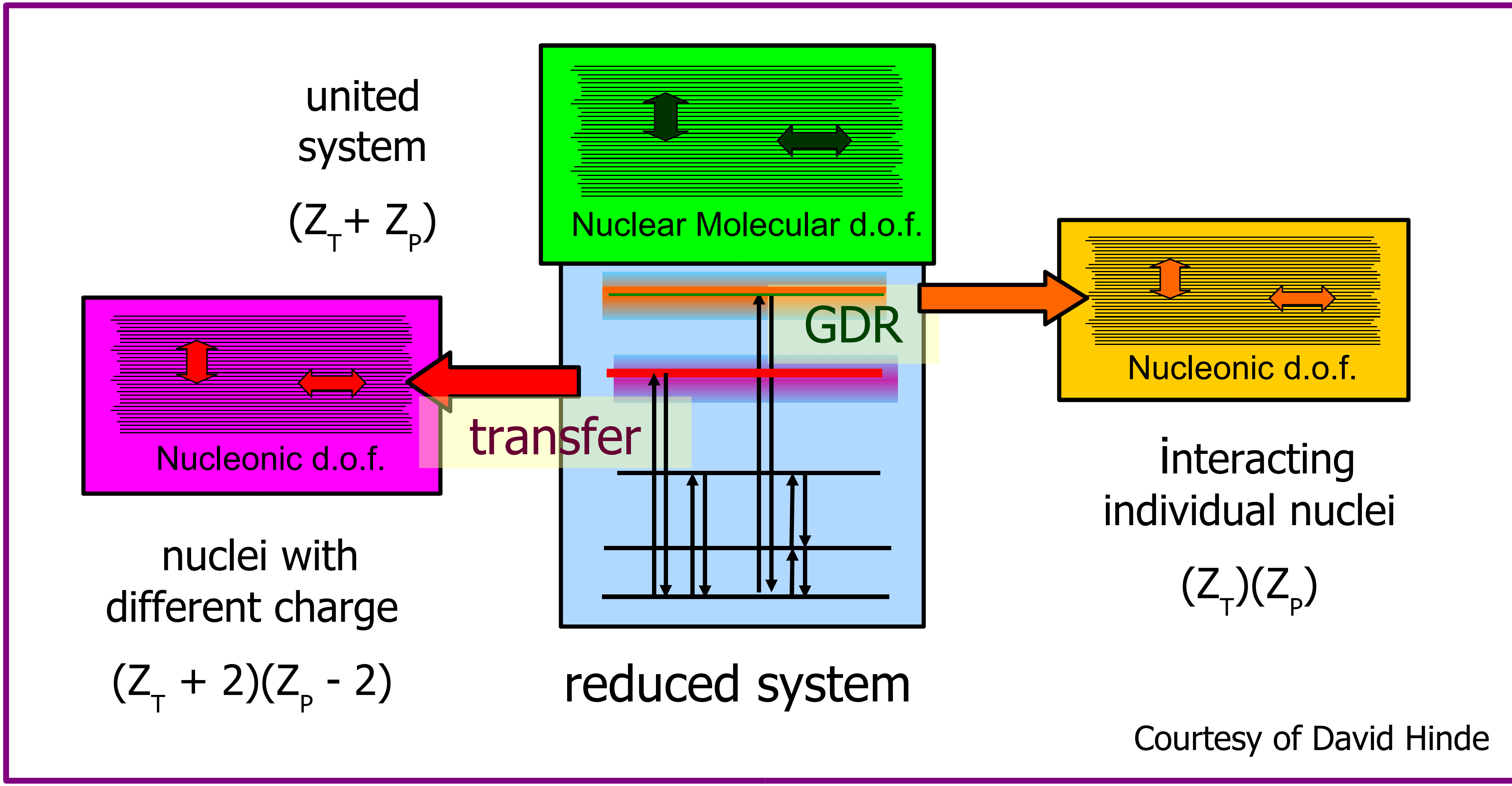
where $q \equiv (L, m, J, M, L', m', J', M')$, $\gamma \equiv (IL; JM)$, $\lambda \equiv (IL'; J'M')$, and $\mathcal{S}_{\gamma\lambda}(t_f) = \sum_{r'} \rho^{r'r'}_{\gamma\lambda}(t_f)$.

Integrating over all directions $\hat{r}'$ of solid angles, and summing over all M_I, the total probability for producing the target in state I (population) is obtained:

$$W(I) = \sum_{M_I} \sum_{LmJM} (C^{JM}_{LmIM_I})^2 \mathcal{S}_{\gamma\gamma}(t_f)$$

Coupled-Channels Density-Matrix Approach

Different Environments



Track decoherence and absorption through different environments

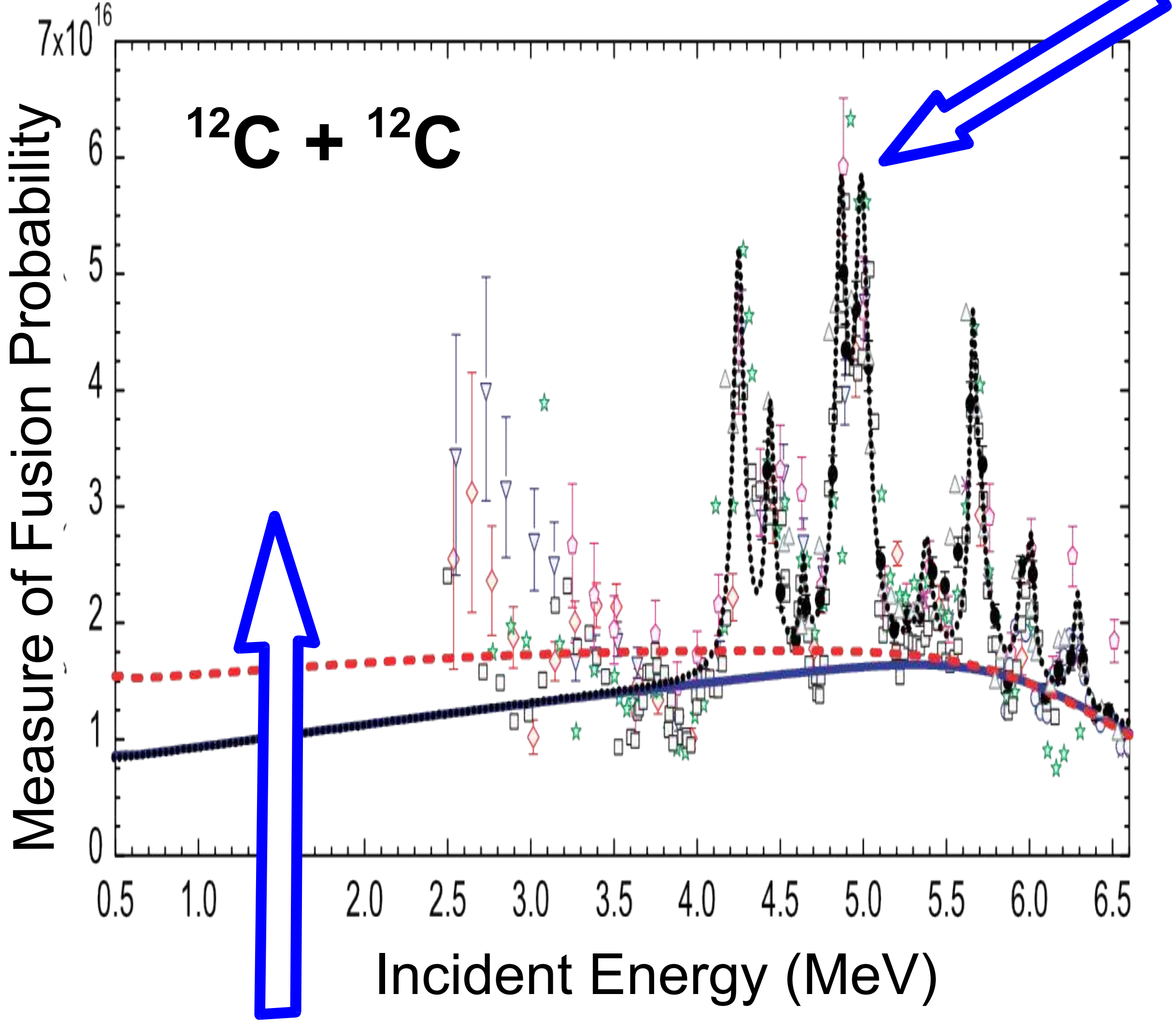
Environments are specific to particular degrees of freedom

Application: Understanding fusion of astrophysically-important collisions at low energies

T. Spillane et al., PRL **98** (2007) 122501

E.F. Aguilera et al., PRC **73** (2006) 064601

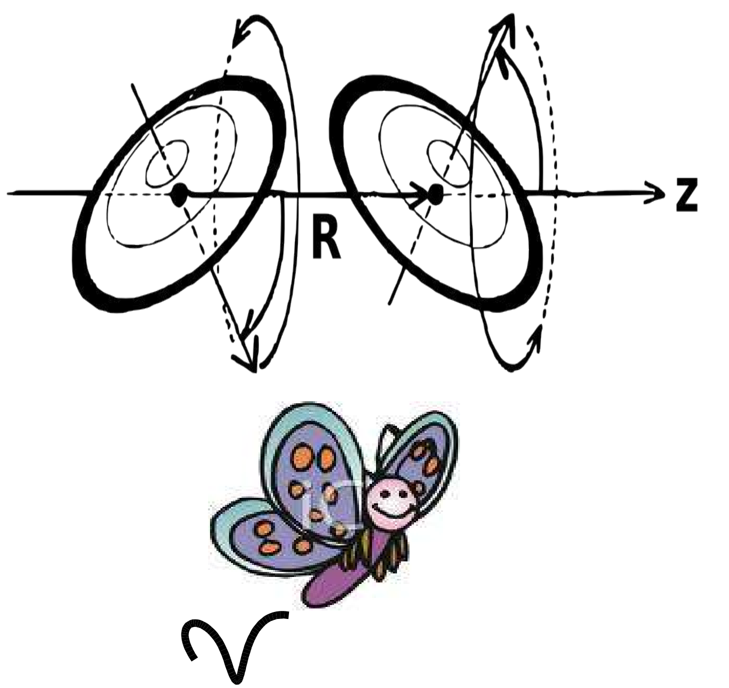
Origin of the resonances ?



Fusion probability at astrophysical energies ?

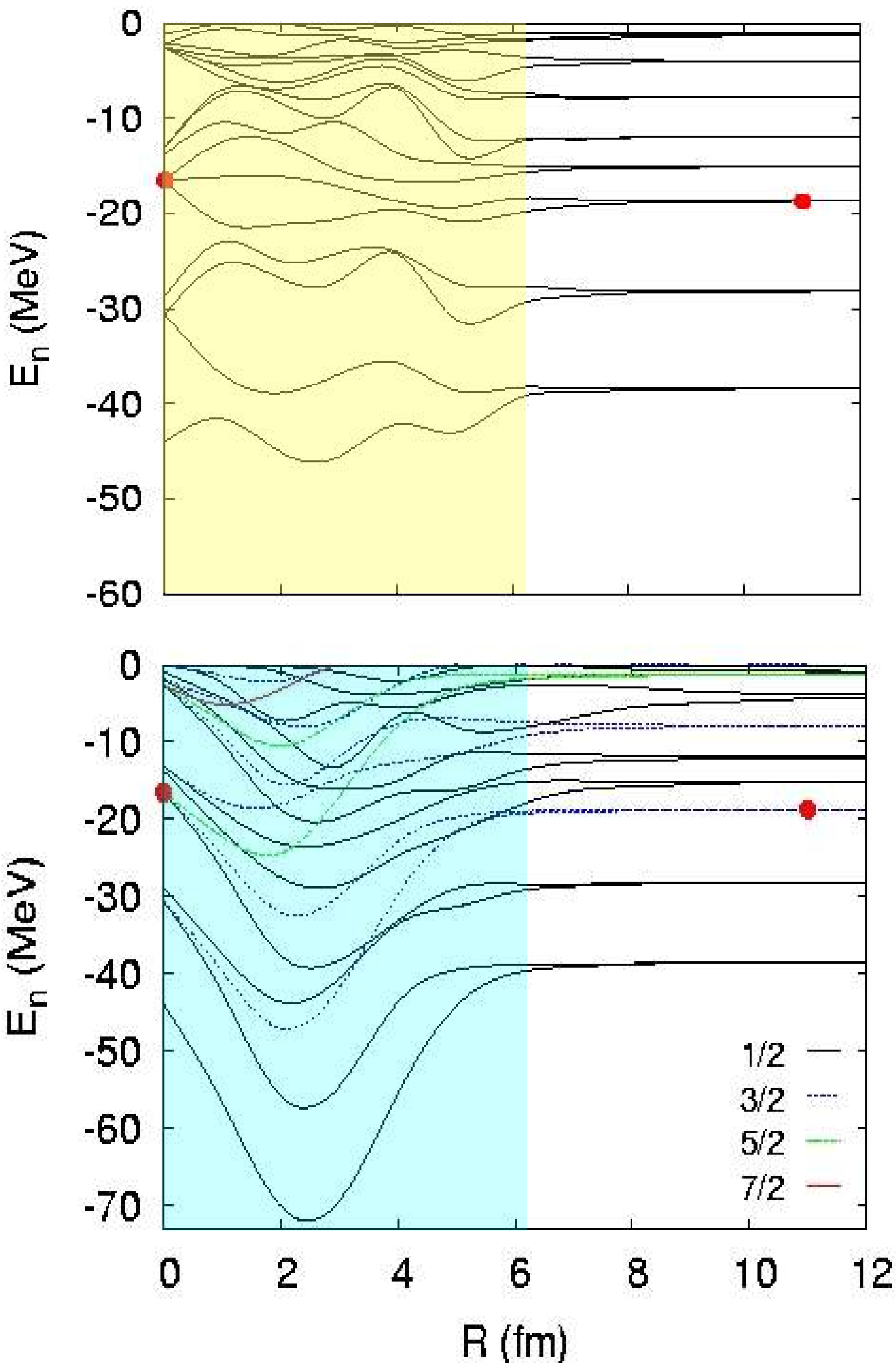
Complex excitation modes

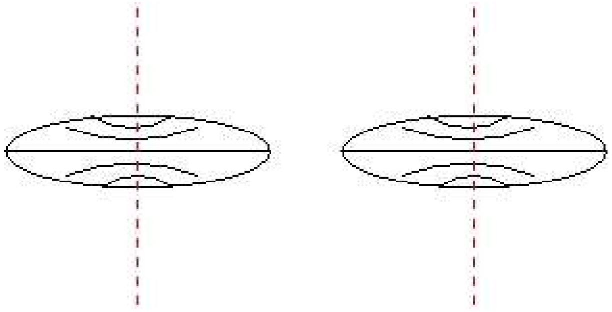
in dinuclear system



AD-T, PRL 101 (2008) 122501

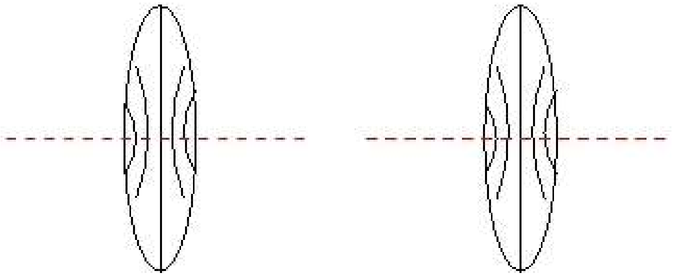
Neutron molecular shell structure of two interacting deformed ^{12}C





$$V = \sum_{s=1}^{2} e^{-i\mathbf{R}_s \hat{k}} \, \hat{U}(\Omega_s) \, V_s \, \hat{U}^{-1}(\Omega_s) \, e^{i\mathbf{R}_s \hat{k}}$$

$$V_s \approx \sum_{\nu\mu}^{N} |s\nu\rangle \, V^s_{\nu\mu} \, \langle s\mu|$$



AD-T, PRL 101 (2008) 122501

Summary

- **Coupled-channels density-matrix approach**, which will quantify the **importance of quantum decoherence** in various areas of nuclear reaction theory

- **Decoherence should always be explicitly included** when modelling low-energy nuclear collision dynamics with a limited set of (relevant) degrees of freedom

Application: Unified quantum description of reaction processes of neutron-rich, weakly-bound nuclei