Quantum decoherence and entanglement in two-mode open systems

Aurelian Isar

Department of Theoretical Physics
National Institute of Physics and Nuclear Engineering
Bucharest-Magurele, Romania
isar@theory.nipne.ro

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Outline

- In the framework of the th. of open systems (OS) based on completely positive q. dynamical semigs - master eq. for 1 harmonic oscillator interacting with environment (thermal bath) - degree of quantum decoherence (QD)
  - master equation for 2 uncoupled harmonic oscillators interacting with environment (thermal bath) - degree of QD
- Peres–Simon cond. for separability of 2-mode Gaussian states - generation and evolution of entanglement in terms of the covariance matrix for a Gaussian input state
- depending on values of diffusion, dissipation coeffs and temperature of environment, the state keeps its initial type: separable or entangled, or entanglement sudden birth or death (collapse or revival of entanglement)
- depending on the environment coeffs and T, the initial state evolves asymptotically to an entangled or separable equilibrium bipartite state, indep. of the type of initial state
  $\rightarrow$ logarithmic negativity - degree of entanglement

Entanglement and decoherence

- reduced dynamics of OS is irreversible and satisfies a forward-in-time composition law: it is described by a so-called q. dynamical semig. that incorporates the dissipative and noisy effects due to the environment
- environment acts as a source of decoherence: in general, the corresponding reduced dynamics irreversibly transforms pure states (one-dimensional projections) into statistical mixtures (density matrices)
- entanglement (purely q. correlations) of a state of two ss. embedded in a same heat bath - it is generally expected that it would be destroyed by decoherence effects
- however, this is not the only possibility: if suitably engineered, the environment can entangle an initial separable state of two dynamically independent ss: the reason is that, although not directly interacting between themselves, there can be an environment mediated generation of q. correlations between 2 ss. immersed in it

Quantum decoherence (QD)

- Alicki: irreversible, uncontrollable and persistent formation of a quantum correlation (entanglement) of the system with its environment (damping of coherences present in the quantum state, when off-diagonal elements of the density matrix decay below a certain level, so that this density matrix becomes approximately diagonal)
- strongly depends on the interaction between system and its external environment
- *harmonic oscillator interacting with an environment* in the framework of theory of open quantum systems
- we determine the *degree of QD* for a harmonic oscillator in a thermal bath
- it is found that the system manifests a QD which is *more and more* significant in time

Lindblad-Kossakowski theory

- Lindblad-Kossakowski *axiomatic* formalism is based on quantum dynamical semigroups (complete positivity property is fulfilled)
- irreversible time evolution of an open system is described by the following general q. *Markovian master equation* for the density operator $\rho(t)$:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_j ([V_j\rho(t), V_j^\dagger] + [V_j, \rho(t)V_j^\dagger])$$

- H - Hamiltonian of the system
- V_j, $V_j^\dagger$ - operators on the Hilbert space of H (they model the environment)

Master equation for damped h.o.

- V_1 and V_2 - linear polynomials in q and p (equations of motion as close as possible to the *classical* ones) and H - general quadratic form

$$H = H_0 + \frac{\mu}{2}(qp + pq), \quad H_0 = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}q^2$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_0, \rho]$$

$$-\frac{i}{2\hbar}(\lambda + \mu)[q, \rho p + p\rho] + \frac{i}{2\hbar}(\lambda - \mu)[p, \rho q + q\rho]$$

$$-\frac{D_{pp}}{\hbar^2}[q, [q, \rho]] - \frac{D_{qq}}{\hbar^2}[p, [p, \rho]] + \frac{D_{pq}}{\hbar^2}([q, [p, \rho]] + [p, [q, \rho]])$$

Diffusion and dissipation coeffs

- *fundamental constraints* $D_{pp} > 0, D_{qq} > 0$,

$$D_{pp}D_{qq} - D_{pq}^2 \geq \frac{\lambda^2 \hbar^2}{4}$$

- when the asymptotic state is a *Gibbs* state
$\rho_G(\infty) = e^{-\frac{H_0}{kT}} / \mathrm{Tr} e^{-\frac{H_0}{kT}}$,

$$D_{pp} = \frac{\lambda + \mu}{2} \hbar m \omega \coth \frac{\hbar \omega}{2kT}, D_{qq} = \frac{\lambda - \mu}{2} \frac{\hbar}{m\omega} \coth \frac{\hbar \omega}{2kT},$$

$$D_{pq} = 0$$

$$(\lambda^2 - \mu^2) \coth^2 \frac{\hbar \omega}{2kT} \geq \lambda^2, \ \ \lambda > \mu$$

Evolution Eq. in coordinate rep.

$$\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m}(\frac{\partial^2}{\partial q^2} - \frac{\partial^2}{\partial q'^2})\rho - \frac{im\omega^2}{2\hbar}(q^2 - q'^2)\rho$$

$$-\frac{1}{2}(\lambda + \mu)(q - q')(\frac{\partial}{\partial q} - \frac{\partial}{\partial q'})\rho$$

$$+\frac{1}{2}(\lambda - \mu)[(q + q')(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'}) + 2]\rho$$

$$-\frac{D_{pp}}{\hbar^2}(q - q')^2\rho + D_{qq}(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'})^2\rho$$

$$-2iD_{pq}\hbar(q - q')(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'})\rho$$

Physical signification

- first two terms generate a purely *unitary* evolution (usual Liouvillian evolution)
- third and forth terms - *dissipative* (damping effect: exchange of energy with environment)
- last three terms: *noise (diffusive)* (*fluctuation* effects)
- D_{pp} : diffusion in p + generates *decoherence* in q: it reduces the off-diagonal terms, responsible for correlations between spatially separated pieces of the wave packet
- D_{qq} : diffusion in q + generates *decoherence* in p
- D_{pq} : "anomalous diffusion" term - does *not* generate decoherence)

Initial Gaussian wave function

- *correlated coherent state (CCS)* or squeezed CS (special class of pure states, which realizes equality in generalized uncertainty relation)

$$\Psi(q) = \left(\frac{1}{2\pi\sigma_{qq}(0)}\right)^{\frac{1}{4}}$$

$$\times \exp\left[-\frac{1}{4\sigma_{qq}(0)}\left(1 - \frac{2i}{\hbar}\sigma_{pq}(0)\right)(q - \sigma_q(0))^2 + \frac{i}{\hbar}\sigma_p(0)q\right],$$

$$\sigma_{qq}(0) = \frac{\hbar\delta}{2m\omega}, \sigma_{pp}(0) = \frac{\hbar m\omega}{2\delta(1 - r^2)}, \sigma_{pq}(0) = \frac{\hbar r}{2\sqrt{1 - r^2}}$$

Parameters and variances

- δ - *squeezing* parameter (measures the spread in the initial Gaussian packet), r, $|r| < 1$ - correlation coefficient at time $t = 0$
- for $\delta = 1$, $r = 0$ CCS - *Glauber* coherent state - σ_{qq} and σ_{pp} denote the dispersion (variance) of the coordinate and momentum, respectively, and σ_{pq} denotes the correlation (covariance) of the coordinate and momentum
- in the case of a *thermal bath*

$$\sigma_{qq}(\infty) = \frac{\hbar}{2m\omega} \coth \frac{\hbar\omega}{2kT}, \quad \sigma_{pp}(\infty) = \frac{\hbar m\omega}{2} \coth \frac{\hbar\omega}{2kT},$$

$$\sigma_{pq}(\infty) = 0$$

Density matrix

$$< q|\rho(t)|q' > = (\frac{1}{2\pi\sigma_{qq}(t)})^{\frac{1}{2}} \exp[-\frac{1}{2\sigma_{qq}(t)}(\frac{q+q'}{2} - \sigma_q(t))^2$$

$$-\frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}(q-q')^2 + \frac{i\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)}(\frac{q+q'}{2} - \sigma_q(t))(q-q')$$

$$+\frac{i}{\hbar}\sigma_p(t)(q-q')] \quad - \text{ general Gaussian form}$$

- thermal bath, $t \to \infty$ (stationary solution)

$$< q|\rho(\infty)|q' > = (\frac{m\omega}{\pi\hbar \coth \epsilon})^{\frac{1}{2}} \exp\{-\frac{m\omega}{4\hbar}[\frac{(q+q')^2}{\coth \epsilon}$$

$$+(q-q')^2 \coth \epsilon]\}, \quad \epsilon \equiv \hbar\omega/2kT$$

Degree of quantum decoherence

$\Sigma = (q + q')/2, \Delta = q - q',$
$\alpha = \frac{1}{2\sigma_{qq}(t)}, \gamma = \frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}, \beta = \frac{\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)}$

$$\rho(\Sigma, \Delta, t) = \sqrt{\frac{\alpha}{\pi}} \exp[-\alpha\Sigma^2 - \gamma\Delta^2 + i\beta\Sigma\Delta]$$

(for zero initial mean values of q and p)
- representation-independent measure of the degree of QD :
ratio of the dispersion $1/\sqrt{2\gamma}$ of the off-diagonal element to the
dispersion $\sqrt{2/\alpha}$ of the diagonal element

$$\delta_{QD}(t) = (1/2)\sqrt{\alpha/\gamma} = \hbar/2\sqrt{\sigma(t)}$$

Schrödinger generalized uncert. f.

$$\sigma(t) \equiv \sigma_{qq}(t)\sigma_{pp}(t) - \sigma_{pq}^2(t)$$

$$\sigma(t) = \frac{\hbar^2}{4}\{e^{-4\lambda t}[1 - (\delta + \frac{1}{\delta(1 - r^2)})\coth\epsilon + \coth^2\epsilon]$$

$$+ e^{-2\lambda t}\coth\epsilon[(\delta + \frac{1}{\delta(1 - r^2)} - 2\coth\epsilon)\frac{\omega^2 - \mu^2\cos(2\Omega t)}{\Omega^2}$$

$$+ (\delta - \frac{1}{\delta(1 - r^2)})\frac{\mu\sin(2\Omega t)}{\Omega} + \frac{2r\mu\omega(1 - \cos(2\Omega t))}{\Omega^2\sqrt{1 - r^2}}]$$

$$+ \coth^2\epsilon\}$$

- underdamped case ($\omega > \mu$, $\Omega^2 \equiv \omega^2 - \mu^2$)

Discussion of QD (1)

- δ_{QD} decreases, and therefore QD increases, with t and T, i.e. the density matrix becomes more and more diagonal and the contributions of the off-diagonal elements get smaller and smaller
- the degree of purity decreases and the degree of mixedness increases with t and T
- for $T = 0$ the asymptotic (final) state is pure and δ_{QD} reaches its initial maximum value 1
- limit of long times

$$\sigma(\infty) = \frac{\hbar^2}{4} \coth^2 \epsilon,$$

$$\delta_{QD}(\infty) = \tanh \frac{\hbar\omega}{2kT},$$

- high T :

$$\delta_{QD}(\infty) = \frac{\hbar\omega}{2kT}$$

Discussion of QD (2)

- $\delta_{QD} = 0$ when the quantum coherence is completely lost
- a pure state undergoing unitary evolution is highly coherent: it does not lose its coherence, i.e. off-diagonal coherences never vanish - then $\delta_{QD} = 1$ and there is no QD
- only if $\delta_{QD} < 1$, there is a significant degree of QD, when the magnitude of the elements of the density matrix in the position basis are peaked preferentially along the diagonal $q = q'$
- $\delta_{QD} < 1$ is of the order of unity for long enough time, so that we can say that the considered system interacting with the thermal bath manifests QD - dissipation promotes quantum coherences, whereas fluctuation (diffusion) reduces coherences and promotes QD; the balance of dissipation and fluctuation determines the final equilibrium value of δ_{QD}
- the quantum system starts as a pure state (Gaussian form) and this state becomes a quantum mixed state during the irreversible process of QD

Decoherence time scale (1)

- diffusion in momentum occurs at the rate set by D_{pp}
- in the macroscopic limit, when $\hbar$ is small compared to other quantities with dimensions of action, such as $\sqrt{D_{pp} < (q - q')^2 >}$, the term in master eq. containing $D_{pp}/\hbar^2$ dominates and induces the following evolution of the density matrix:

$$\frac{\partial \rho}{\partial t} = -\frac{D_{pp}}{\hbar^2}(q - q')^2 \rho$$

- the diagonal ($q = q'$) terms remain untouched
- in the case of a thermal bath, we obtain

$$t_{deco} = \frac{2\hbar}{(\lambda + \mu)m\omega\sigma_{qq}(0)\coth\epsilon}, \quad \epsilon \equiv \frac{\hbar\omega}{2kT}$$

where we have taken $(q - q')^2$ of the order of the initial dispersion in coordinate $\sigma_{qq}(0)$

- t_{deco} is very much shorter than relaxation time $\rightarrow$ in the macroscopic domain QD occurs very much faster than relaxation
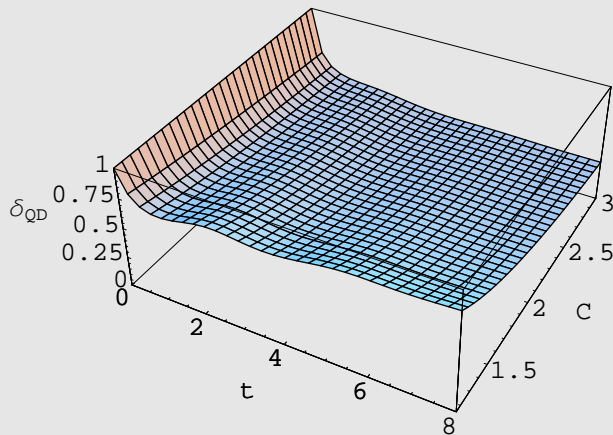
Figures (1)

Figure: δ_{QD} on T ($C \equiv \coth \hbar\omega/2kT$) and t
($\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0$).
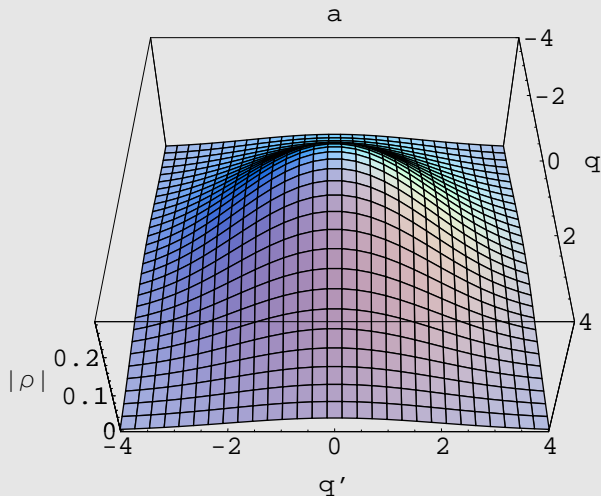
Figures (2)

Figure: ρ in $q-$ representation ($\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0$) at $t = 0$.
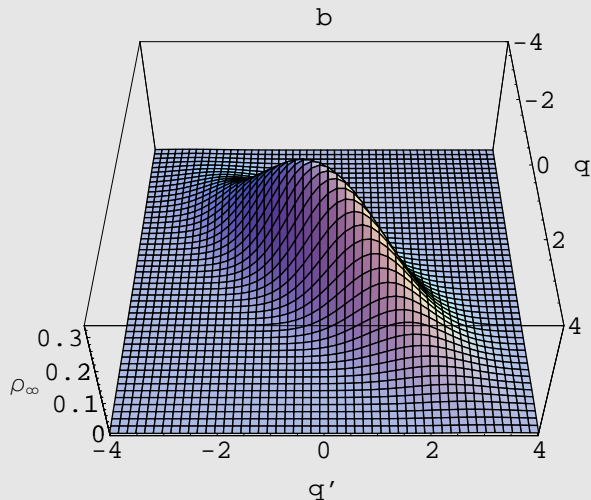
Figures (3)

Figure: ρ in $q-$ representation ($\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0$) at $t \to \infty$ and $C = 3$.
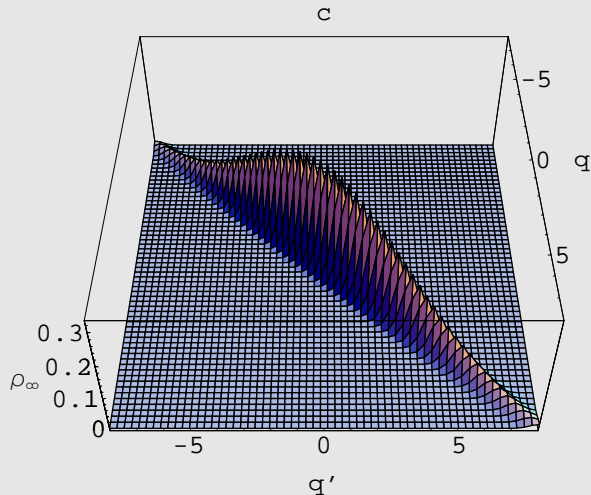
Figures (4)

Figure: ρ in $q-$ representation ($\lambda = 0.2, \mu = 0.1, \delta = 4, r = 0$) at $t \to \infty$ and $C = 20$.

Markovian master equation

- in axiomatic formalism based on completely positive q. dyn. semigs, irreversible time evolution of an OS is described by the gen. q. Markovian master eq. for the density operator (Schrödinger rep.)

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_j (V_j^\dagger[\rho(t), V_j] + [V_j^\dagger, \rho(t)]V_j)$$

- for an op. A (Heisenberg rep.)

$$\frac{dA(t)}{dt} = \frac{i}{\hbar}[H, A(t)] + \frac{1}{2\hbar}\sum_j (V_j^\dagger[A(t), V_j] + [V_j^\dagger, A(t)]V_j)$$

- H - Hamiltonian of the open q. system
- $V_j, V_j^\dagger$ - operators defined on the Hilbert space of H (model the interaction of the open system with the environment)

Complete positivity and entanglement

- the physical meaning of complete positivity can mainly be understood in relation to the existence of entangled states, the typical example being given by a vector state with a singlet-like structure that cannot be written as a tensor product of vector states

- positivity property guarantees the physical consistency of evolving states of single systems, while complete positivity prevents inconsistencies in entangled composite systems

- therefore the existence of entangled states makes the request of complete positivity necessary

- the positivity of the states of the compound system will be preserved only if the dyn. semig. of the subs. is completely positive.

Operators

- q. dyn. semigs that preserve in time Gaussian form of the states: H - polyn. of second degree in coordinates x, y and momenta p_x, p_y of the two q. os and $V_j, V_j^\dagger$ - polyns. of first degree in canonical observables ($j = 1, 2, 3, 4$):

$$V_j = a_{xj} p_x + a_{yj} p_y + b_{xj} x + b_{yj} y$$

- we are interested in discussing correlation effect of environment – assume that the 2 ss are indep. (do not interact directly - uncoupled identical h. os.)

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m\omega^2}{2}(x^2 + y^2)$$

- dyn. semig. implies positivity of the matrix formed by the scalar products of the vectors $\mathbf{a}_x, \mathbf{a}_y, \mathbf{b}_x, \mathbf{b}_y$ (their entries are the components $a_{xj}, a_{yj}, b_{xj}, b_{yj}$, respectively)

Environment coefficients

- matrix of environment coeffs (D_{xx}, D_{xp_x},... and λ real) ($\hbar = 1$)

$$\begin{pmatrix} D_{xx} & -D_{xp_x} - i\frac{\lambda}{2} & D_{xy} & -D_{xp_y} \\ -D_{xp_x} + i\frac{\lambda}{2} & D_{p_x p_x} & -D_{yp_x} & D_{p_x p_y} \\ D_{xy} & -D_{yp_x} & D_{yy} & -D_{yp_y} - i\frac{\lambda}{2} \\ -D_{xp_y} & D_{p_x p_y} & -D_{yp_y} + i\frac{\lambda}{2} & D_{p_y p_y} \end{pmatrix}$$

- the principal minors of this matrix are positive or zero
- constraints on coefficients (from Cauchy-Schwarz ineq.)

$$D_{xx} D_{yy} - D_{xy}^2 \geq 0, \ D_{xx} D_{p_x p_x} - D_{xp_x}^2 \geq \frac{\lambda^2}{4},$$

$$D_{xx} D_{p_y p_y} - D_{xp_y}^2 \geq 0, \ D_{yy} D_{p_x p_x} - D_{yp_x}^2 \geq 0,$$

$$D_{yy} D_{p_y p_y} - D_{yp_y}^2 \geq \frac{\lambda^2}{4}, \ D_{p_x p_x} D_{p_y p_y} - D_{p_x p_y}^2 \geq 0$$

Equations of motion

- bimodal covariance matrix

$$\sigma(t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{xp_x} & \sigma_{p_xp_x} & \sigma_{yp_x} & \sigma_{p_xp_y} \\ \sigma_{xy} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{xp_y} & \sigma_{p_xp_y} & \sigma_{yp_y} & \sigma_{p_yp_y} \end{pmatrix}$$

$$\frac{d\sigma}{dt} = Y\sigma + \sigma Y^{\mathrm{T}} + 2D, \quad Y = \begin{pmatrix} -\lambda & 1/m & 0 & 0 \\ -m\omega^2 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1/m \\ 0 & 0 & -m\omega^2 & -\lambda \end{pmatrix}$$

D - matrix of diffusion coefficients

$$D = \begin{pmatrix} D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y} \\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y} \\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y} \\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y} \end{pmatrix}$$

Time-dependent solution

$$\sigma(t) = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{xp_x} & \sigma_{p_xp_x} & \sigma_{yp_x} & \sigma_{p_xp_y} \\ \sigma_{xy} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{xp_y} & \sigma_{p_xp_y} & \sigma_{yp_y} & \sigma_{p_yp_y} \end{pmatrix}$$

$$\sigma(t) = M(t)(\sigma(0) - \sigma(\infty))M^{\mathrm{T}}(t) + \sigma(\infty),$$

$M(t) = \exp(tY)$, $\lim_{t\to\infty} M(t) = 0$ (Y must only have eigenvalues with negative real parts)

$$Y\sigma(\infty) + \sigma(\infty)Y^{\mathrm{T}} = -2D.$$

Degree of quantum decoherence - two-mode case

$$\Sigma_x = (x + x')/2, \Sigma_y = (y + y')/2, \Delta_x = x - x', \Delta_y = y - y'$$

$$\rho(\Sigma_x, \Sigma_y, \Delta_x, \Delta_y, t) = N \exp[-A_1\Sigma_x^2 - B_1\Sigma_y^2 - C_1\Sigma_x\Sigma_y$$
$$-A_2\Delta_x^2 - B_2\Delta_y^2 + C_2\Delta_x\Delta_y$$
$$+iA_3\Sigma_x\Delta_x + iB_3\Sigma_y\Delta_y + iC_3\Sigma_x\Delta_y + iC_4\Sigma_y\Delta_x] \quad (1)$$

(for zero initial mean values of x, y and p_x, p_y)
- representation-independent measure of the degree of QD :
ratio of the dispersion $\frac{1}{\sqrt{2A_2}}$ of the off-diagonal element to the

dispersion $\sqrt{\frac{2}{A_1}}$ of the diagonal element

$$\delta_{QD}(t) = \frac{1}{2}\sqrt{\frac{A_1}{A_2}} = \frac{1}{2}\sqrt{\frac{B_1}{B_2}}$$

(symmetry in s. 1 and 2 $\rightarrow$ single degree of QD)

2-mode QD

- starting with an initial coherent state $\delta_{QD}(0) = 1$, $\delta_{QD}(t)$ is decreasing with increasing time until it reaches a final asymptotic non-zero value for non-zero T
- normalization factor N and time dependent factors A_j, B_j, C_j - can be expressed as functions of the elements of the inverse of $\sigma(t)$

Environment induced entanglement

Two-mode Gaussian state is entirely specified by its covariance matrix σ, which is a real, symmetric and positive matrix

$$\sigma = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}$$

(A, B and C are 2×2 matrices).

- theory of open quantum systems allows couplings via the environment between uncoupled oscillators - diffusion coefficients can simulate an interaction between uncoupled oscillators: indeed, Gaussian states with det $C \geq 0$ are separable states, but for det $C < 0$, it may be possible that the states are entangled

Evolution of entanglement

- partial transposition criterion: a state results entangled iff operation of partial transposition does not preserve its positivity (PPT criterion)
- for Gaussian states, nec. and suf. criterion for separability: $S \geq 0$ (Simon)

$$S \equiv \det A \det B + (\frac{1}{4} - |\det C|)^2$$

$$-\text{Tr}[AJCJBJC^{\text{T}}J] - \frac{1}{4}(\det A + \det B)$$

$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the 2×2 symplectic matrix

- since the two oscillators are identical, it is natural to consider environments for which $D_{xx} = D_{yy}$, $D_{xp_x} = D_{yp_y}$, $D_{p_xp_x} = D_{p_yp_y}$, $D_{xp_y} = D_{yp_x}$ - then both unimodal covariance matrices are equal, $A = B$, and the entanglement matrix C is symmetric

Time evolution of entanglement

- for a thermal environment characterized by temperature T

$$m\omega D_{xx} = \frac{D_{p_x p_x}}{m\omega} = \frac{\lambda}{2}\coth\frac{\omega}{2kT}, \quad D_{xp_x} = 0, m^2\omega^2 D_{xy} = D_{p_x p_y}$$

(corresponds to the case when the asymptotic state is a Gibbs state)

- for Gaussian states, the measures of entanglement of bipartite systems are based on some invariants constructed from the elements of the covariance matrix - logarithmic negativity

- for a Gaussian density operator, logarithmic negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix

Logarithmic negativity

- $L = \max\{0, -\log_2 2\tilde{\nu}_-\}$, where $\tilde{\nu}_-$ is the smallest of the two symplectic eigenvalues of the partial transpose $\tilde{\sigma}$ of the 2-mode covariance matrix σ :

$$2\tilde{\nu}_{\mp}^2 = \tilde{\Delta} \mp \sqrt{\tilde{\Delta}^2 - 4\det\sigma} \qquad (2)$$

- symplectic invariant (seralian) $\tilde{\Delta} = \det A + \det B - 2\det C$.
-

$$L(t) = -\frac{1}{2}\log_2[4f(\sigma(t))], \qquad (3)$$

$$f(\sigma(t)) = \frac{1}{2}(\det A + \det B) - \det C$$

$$- \left(\left[\frac{1}{2}(\det A + \det B) - \det C\right]^2 - \det\sigma(t)\right)^{1/2} \qquad (4)$$

- it determines the strength of entanglement for $L(t) > 0$, and if $L(t) \leq 0$, then the state is separable
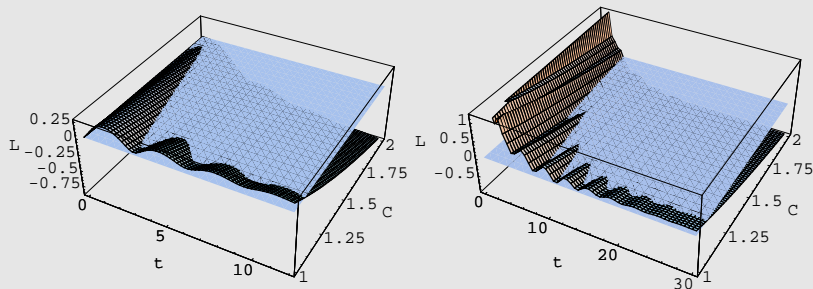
Fig1

Figure: Logarithmic negativity L versus time t and temperature T ($C \equiv \coth \frac{\hbar\omega}{2kT}$) for $\lambda = 0.1$, $D_{xy} = 0$, $D_{xp_y} = 0.049$ for initial uni-modal squeezed state with $\sigma_{xx}(0) = 3/4$, $\sigma_{p_x p_x}(0) = 1/3$, $\sigma_{xp_x}(0) = 0$. Left: separable state with $\sigma_{xy}(0) = \sigma_{p_x p_y}(0) = \sigma_{xp_y}(0) = 0$. Right: entangled state with $\sigma_{xy}(0) = 1/2$, $\sigma_{p_x p_y}(0) = -1/2$, $\sigma_{xp_y}(0) = 0$.

Fig2

Figure: Logarithmic negativity L versus time t and temperature T ($C \equiv \coth \frac{\hbar\omega}{2kT}$) for $\lambda = 0.1$, $D_{xy} = 0$, $D_{xp_y} = 0.049$ for initial Gaussian mixed state with $\sigma_{xx}(0) = 1$, $\sigma_{p_x p_x}(0) = 1/2$, $\sigma_{x p_x}(0) = 0$.
Left: separable state with $\sigma_{xy}(0) = \sigma_{p_x p_y}(0) = \sigma_{x p_y}(0) = 0$. Right: entangled state with $\sigma_{xy}(0) = 1/2$, $\sigma_{p_x p_y}(0) = -1/2$, $\sigma_{x p_y}(0) = 0$.
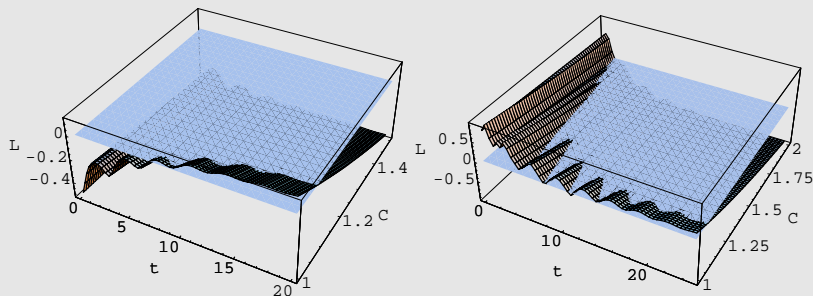
Asymptotic entanglement

- elements of the asymptotic matrices $A(\infty) = B(\infty)$:

$$m\omega\sigma_{xx}(\infty) = \frac{\sigma_{p_x p_x}(\infty)}{m\omega} = \frac{1}{2}\coth\frac{\omega}{2kT}, \quad \sigma_{xp_x}(\infty) = 0$$

and of entanglement matrix $C(\infty)$:

$$\sigma_{xy}(\infty) = \frac{m^2(\lambda^2 + \omega^2)D_{xy} + m\lambda D_{xp_y}}{m^2\lambda(\lambda^2 + \omega^2)}$$

$$\sigma_{xp_y}(\infty) = \sigma_{yp_x}(\infty) = \frac{\lambda D_{xp_y}}{\lambda^2 + \omega^2}$$

$$\sigma_{p_x p_y}(\infty) = \frac{m^2\omega^2(\lambda^2 + \omega^2)D_{xy} - m\omega^2\lambda D_{xp_y}}{\lambda(\lambda^2 + \omega^2)}$$

Asymptotic entanglement

- in the limit of large times:

$$S(\infty) = \left(\frac{1}{4}(\coth^2 \frac{\omega}{2kT} - 1) - \frac{m^2\omega^2 D_{xy}^2}{\lambda^2} + \frac{D_{xp_y}^2}{\lambda^2 + \omega^2} \right)^2$$
$$- \frac{D_{xp_y}^2}{\lambda^2 + \omega^2} \coth^2 \frac{\omega}{2kT}$$

- for environments characterized by such coefficients that the expression $S(\infty)$ is strictly negative, the asymptotic final state is entangled

Asymptotic entanglement

- particular case: $D_{xy} = 0$ - for a given temperature T, we obtain that $S(\infty) < 0$, i.e. the asymptotic final state is entangled, for the following range of values of D_{xp_y}:

$$\coth \frac{\omega}{2kT} - 1 < \frac{2D_{xp_y}}{\sqrt{\lambda^2 + \omega^2}} < \coth \frac{\omega}{2kT} + 1$$

- coefficients have to fulfill also the constraint

$$\frac{\lambda}{2} \coth \frac{\omega}{2kT} \geq D_{xp_y}$$

- if the coefficients do not fulfil the double inequality, then $S(\infty) \geq 0$ and the asymptotic state is separable
- asymptotic logarithmic negativity:

$$L(\infty) = -\log_2 \left[\left| \coth \frac{\omega}{2kT} - \frac{2D_{xp_y}}{\sqrt{\lambda^2 + \omega^2}} \right| \right]$$

- it does not depend on the parameters of the initial Gaussian state

Summary and conclusions (1)

- we have studied QD with the Markovian Kossakowski-Lindblad Eq. for an 1-dim. h. o. and 2-dim. h.o. in interaction with a thermal bath in the framework of the theory of OQS based on q. dynamical semigs
- the system manifests a QD which increases with time and T, i.e. the density matrix becomes more and more diagonal at higher T (loss of quantum coherence); at the same time the degree of purity decreases and the degree of mixedness increases with T
- QD is responsible for washing out the quantum interference effects which are desirable to be seen as signals in some experiments
- in QI processing and computation we are interested in understanding the specific causes of QD because one wants to prevent decoherence from damaging q. states and to protect the information stored in these states from the degrading effect of the interaction with the environment

Summary and conclusions (2)

- when 2 ss are immersed in an environment, then, besides and at the same time with QD phenomenon, the external environment can also generate a q. E of the 2 ss
- using Peres–Simon nec. and suff. cond. for separability of 2-mode Gaussian states, we studied generation and evolution of E - depending on values of diffusion and dissipation coeffs describing environment, or T, the state keeps its initial type: separable or entangled, or E sudden birth or birth (collapse or revival of E) or periodic E sudden birth and death
- depending on the environment coeffs, initial state evolves asymptotically to an entangled or separable equilibrium bipartite state, independent of the type of the initial state
- we calculated the logarithmic negativity characterizing the degree of E of the asymptotic state

Summary and conclusions (3)

- entanglement can be maintained for a definite time or a certain amount of entanglement survives in asymptotic long-time regime
- control of entanglement in OS
- existence of quantum correlations between the two systems is the result of competition between E and QD

[1] A. Isar, Int. J. Q. Information **6**, 689 (2008)
[2] A. Isar, Open Systems & Information Dynamics, **16**, 205 (2009)

Thank You!