Decay analysis with reservoir structures

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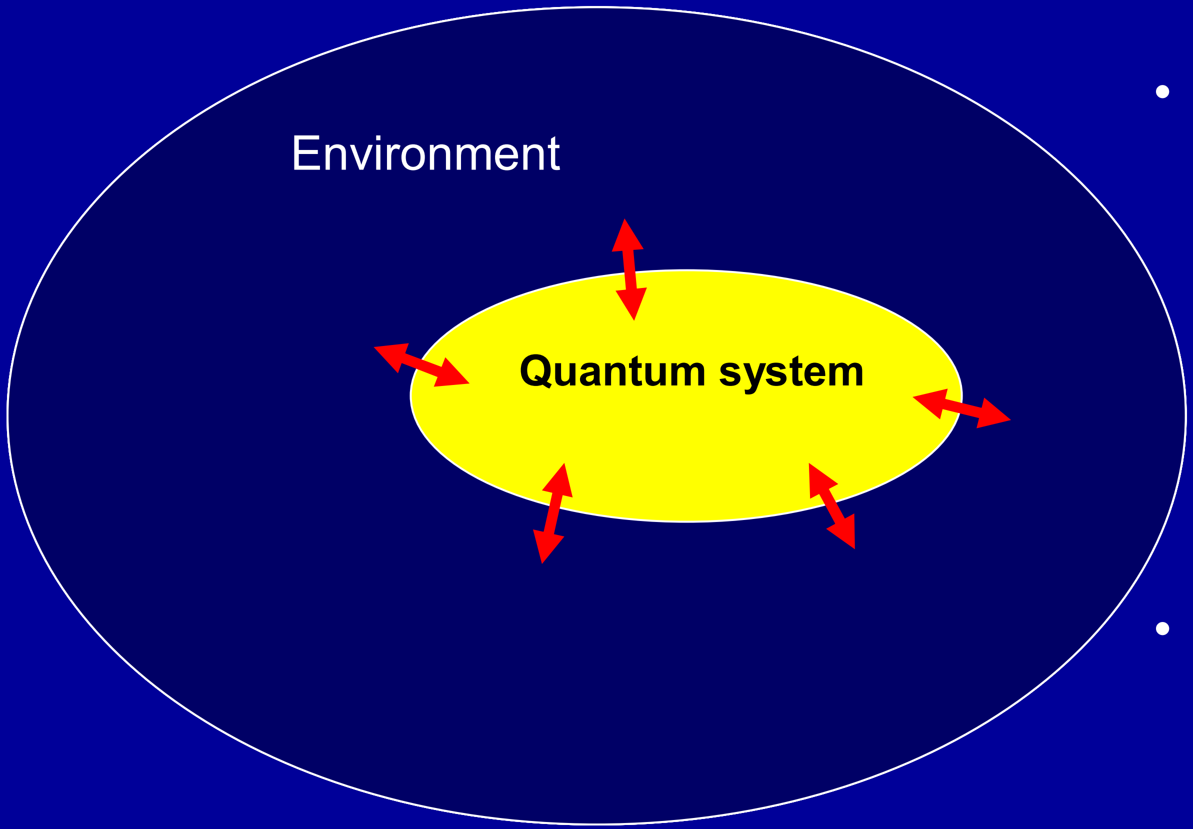
Contents:

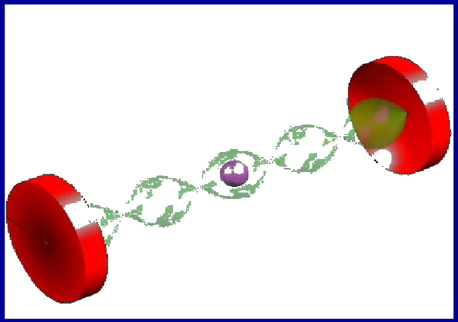
- Introduction and some standard approaches
- Pseudomode method
- Connection via Fano diagonalization
- Where is the memory of a reservoir (quantum trajectories and psedomodes)?
- Dynamic reservoir structures

Quantum system with environment

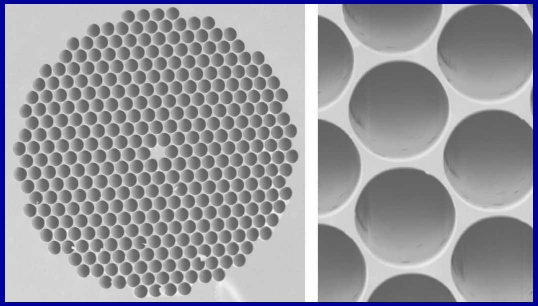


Quantum optical paradigms:

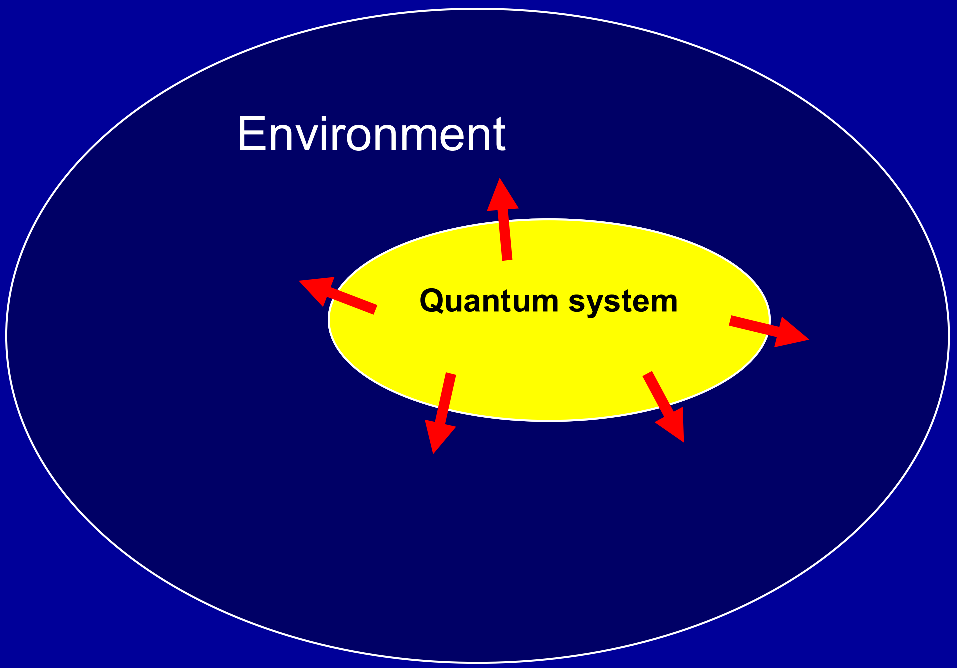
- Cavity system



- Photonic band-gap system



Fermi Golden rule

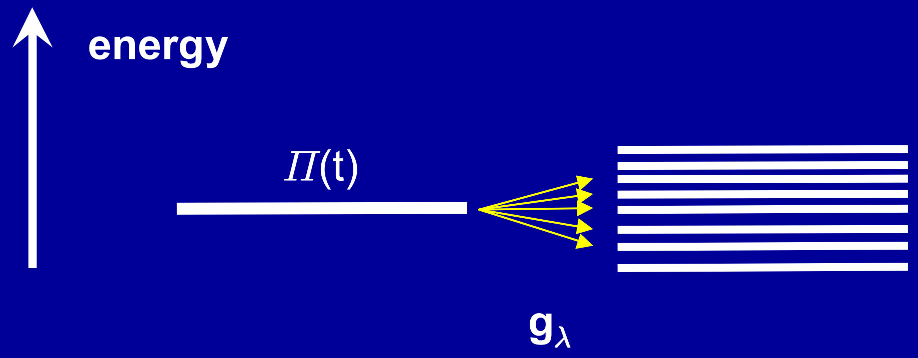


- Golden rule

$$\Gamma_a = \frac{2\pi}{\hbar} |\hbar g_\lambda|^2 \frac{\rho_\lambda}{\hbar}$$

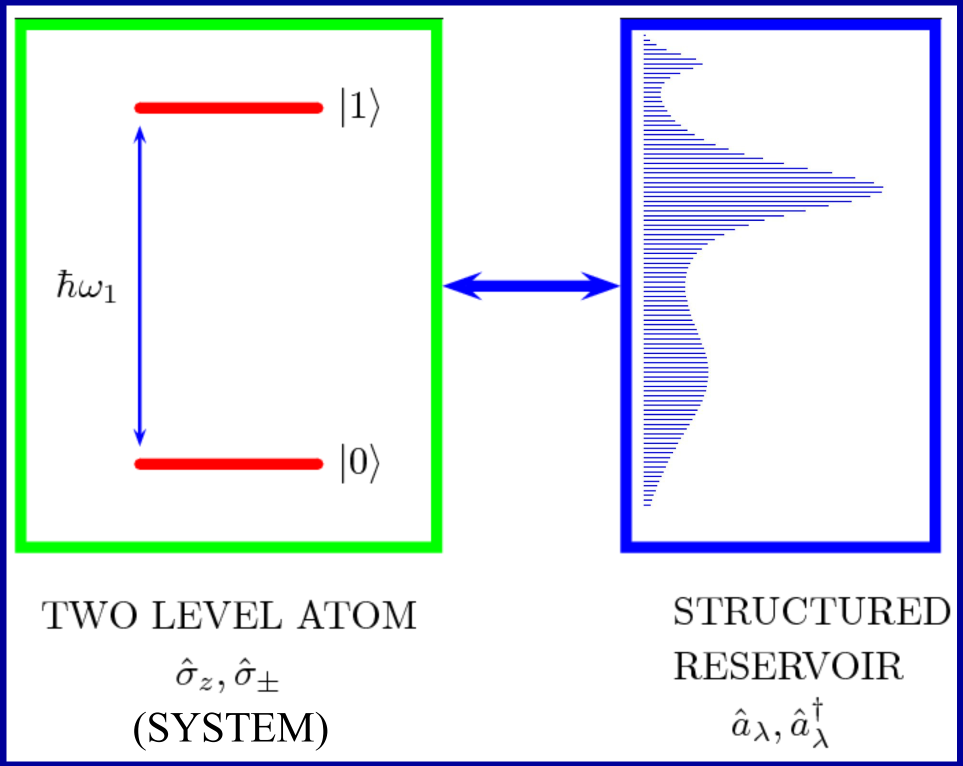
- Population decay

$$\Pi(t) = \Pi(0)\, e^{-\Gamma_a t}$$



A first decay problem

energy



- TLS
- RWA

Initial condition:

$$|\Psi(0)\rangle = |1\rangle|\ldots 0_\lambda \ldots\rangle$$

- Empty initial bath $\Rightarrow$ Restricted Hilbert space
- Direct numerical simulation possible (nb. recurrences)

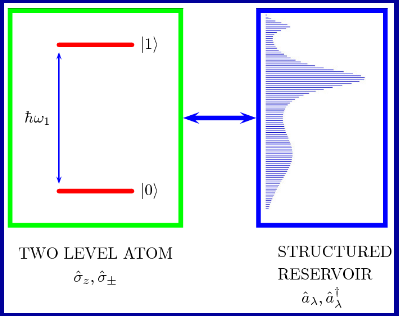
$$\hat{H} = \sum_\lambda \hbar\omega_\lambda \hat{a}_\lambda^\dagger \hat{a}_\lambda + \frac{1}{2}\hbar\omega_1 \left(\hat{\sigma}^+\hat{\sigma}^- - \hat{\sigma}^-\hat{\sigma}^+\right) + \sum_\lambda \left(\hbar g_\lambda^* \hat{a}_\lambda \hat{\sigma}^+ + \text{h.c.}\right)$$

Amplitude equations for dynamics

- State vector:

$$|\Psi(t)\rangle = \tilde{c}_a(t)\mathrm{e}^{-i\omega_1 t}|1\rangle|\ldots 0_\lambda \ldots\rangle + \sum_\lambda \tilde{c}_\lambda(t)\mathrm{e}^{-i\omega_\lambda t}|0\rangle|\ldots 1_\lambda \ldots\rangle$$

- Complex amplitude equations ($\Delta_\lambda = \omega_\lambda - \omega_1$):



$$i\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_a = \sum_\lambda g^*_\lambda \mathrm{e}^{-i\Delta_\lambda t}\tilde{c}_\lambda$$

$$i\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_\lambda = g_\lambda \mathrm{e}^{i\Delta_\lambda t}\tilde{c}_a$$

- Integro-differential equation for atomic amplitude

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_a(t) = -\int_0^t \mathrm{d}\tau \tilde{G}(\tau)\tilde{c}_a(t-\tau)$$

with memory kernel

$$\tilde{G}(\tau) = \sum_\lambda |g_\lambda|^2 \mathrm{e}^{-i\Delta_\lambda \tau} = \int d\omega_\lambda \rho(\omega_\lambda)|g_\lambda|^2 \mathrm{e}^{-i\Delta_\lambda \tau}$$

- Reservoir structure function: we let $\rho_\lambda |g_\lambda|^2 = \frac{\Omega^2}{2\pi} D(\omega_\lambda)$

Digression: Weisskopf-Wigner theory

- The bath is flat (or 'fairly' flat):

$$\begin{aligned}\tilde{G}(\tau) &= \sum_\lambda |g_\lambda|^2 \mathrm{e}^{-i\Delta_\lambda \tau} = \int d\omega_\lambda \rho(\omega_\lambda)|g_\lambda|^2 \mathrm{e}^{-i\Delta_\lambda \tau} \\ &\rightarrow \rho |g|^2 \int d\omega_\lambda \mathrm{e}^{-i\Delta_\lambda \tau} = \rho |g|^2 \, 2\pi\delta(\tau)\end{aligned}$$

Integro-differential equation for atomic amplitude

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_a(t) &= -\int_0^t \mathrm{d}\tau \tilde{G}(\tau)\tilde{c}_a(t-\tau) \\ &= -2\pi\rho|g|^2 \int_0^t \mathrm{d}\tau \delta(\tau)\tilde{c}_a(t-\tau) \\ &= -\pi\rho|g|^2 \tilde{c}_a(t) \qquad \text{n.b. factor } 1/2 \\ &\equiv -\frac{\Gamma_a}{2}\tilde{c}_a(t)\end{aligned}$$

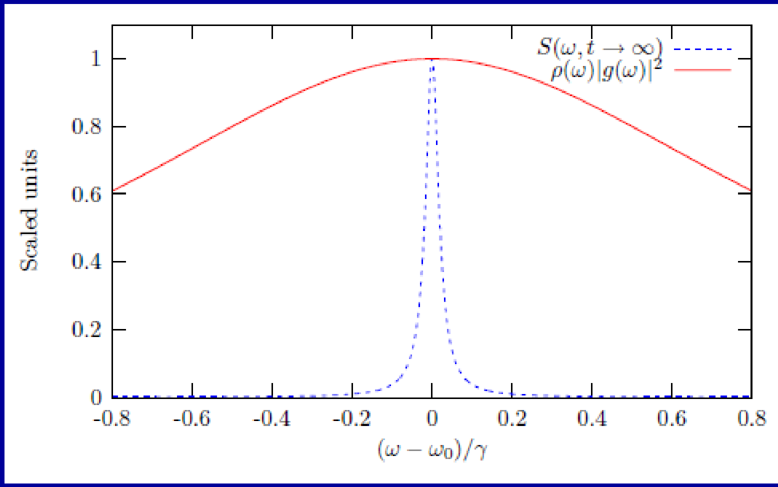
With population decay rate: $\Gamma_a = 2\pi |g_\lambda|^2 \rho_\lambda$

Excitation of bath modes

- Define excitation spectrum as $S(\omega_\lambda) = \rho_\lambda \mid c_\lambda|^2$
- Use ODE: $i\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_\lambda = g_\lambda e^{i\Delta_\lambda t}\tilde{c}_a$
- For W-W:

$$
\begin{aligned}
c_\lambda(t\to\infty) &= \frac{g_\lambda^*}{(\omega_\lambda-\omega_0)+i\Gamma_a/2} \\
S(\omega, t\to\infty) &= \frac{\rho(\omega)|g(\omega)|^2}{(\omega-\omega_0)^2+(\Gamma_a/2)^2} \\
&\sim \frac{\rho(\omega_0)|g(\omega_0)|^2}{(\omega-\omega_0)^2+(\Gamma_a/2)^2} \\
&\sim \frac{\Gamma_a/2}{\pi\left[(\omega-\omega_0)^2+(\Gamma_a/2)^2\right]}.
\end{aligned}
$$

- Width governed by decay rate
- Applies also for smooth structure



Digression: Master Equations

$$\frac{\partial\hat{\rho}(t)}{\partial t} = -i\left[\widehat{H}_I(t),\hat{\rho}(t)\right]$$

Schrödinger Eqn. for system+bath; $\rho=|\Psi\rangle\langle\Psi|$

$$\widehat{H}_I(t) = \sum_\lambda \left(g_\lambda\hat{\sigma}^+\hat{a}_\lambda e^{-i(\omega_\lambda-\omega_0)t} + g_\lambda^*\hat{\sigma}^-\hat{a}_\lambda^\dagger e^{i(\omega_\lambda-\omega_0)t}\right)$$

Integrate and iterate …

$$\hat{\rho}(t) = \hat{\rho}(t_0) - i\int_{t_0}^{t}\left[\widehat{H}_I(t'),\hat{\rho}(t')\right]\,dt'$$

$$\frac{\partial\hat{\rho}(t)}{\partial t} = -i\left[\widehat{H}_I(t),\hat{\rho}(t_0)\right] - \int_{t_0}^{t}\left[\widehat{H}_I(t),\left[\widehat{H}_I(t'),\hat{\rho}(t')\right]\right]\,dt'$$

Approximation 1:

$$\hat{\rho}(t) \approx \rho_S(t)\otimes\rho_B$$

Bath is “large”, unaltered by interaction

Trace over bath for system operator

$$\frac{\partial \hat{\rho}_S(t)}{\partial t} \approx -iTr_B\left\{\left[\widehat{H}_I(t), \hat{\rho}_S(t_0)\otimes\hat{\rho}_B\right]\right\} - Tr_B\left\{\int_{t_0}^{t}\left[\widehat{H}_I(t), \left[\widehat{H}_I(t'), \hat{\rho}_S(t')\otimes\hat{\rho}_B\right]\right] dt'\right\}.$$

Insert interaction Hamiltonian (+ make RWA)

$$\frac{\partial \hat{\rho}_S(t)}{\partial t} \approx -\int_0^t \sum_\lambda |g_\lambda|^2 \left\{ e^{-i(\omega_\lambda-\omega_0)(t-t')}\left[\hat{\sigma}^+\hat{\sigma}^-\hat{\rho}_S(t') - \hat{\sigma}^-\hat{\rho}_S(t')\hat{\sigma}^+\right] + e^{i(\omega_\lambda-\omega_0)(t-t')}\left[-\hat{\sigma}^-\hat{\rho}_S(t')\hat{\sigma}^+ + \hat{\rho}_S(t')\hat{\sigma}^+\hat{\sigma}^-\right]\right\}dt'$$

Approximation 2: For a “broad” reservoir structure ρ_S(t ') $\approx$ ρ_S(t)
Integral contributes around t’=0; short correlation time for bath

$$\frac{\partial \hat{\rho}_S(t)}{\partial t} \approx \Gamma_a\left\{\hat{\sigma}^-\hat{\rho}_S(t)\hat{\sigma}^+ - \frac{1}{2}\hat{\sigma}^+\hat{\sigma}^-\hat{\rho}_S(t) - \frac{1}{2}\hat{\rho}_S(t)\hat{\sigma}^+\hat{\sigma}^-\right\}$$

This master equation is a weak coupling limit of PM theory.

End digression

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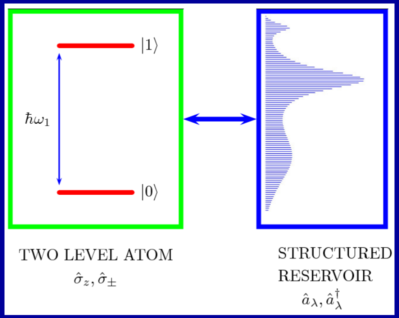
Pseudomode development

- Coupled amplitude equations ($\Delta_\lambda = \omega_\lambda - \omega_1$):

$$i\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_a = \sum_\lambda g_\lambda^* \mathrm{e}^{-i\Delta_\lambda t}\tilde{c}_\lambda$$
$$i\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_\lambda = g_\lambda \mathrm{e}^{i\Delta_\lambda t}\tilde{c}_a$$



- Integro-differential equation for atomic amplitude

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_a(t) = -\int_0^t \mathrm{d}\tau \tilde{G}(\tau)\tilde{c}_a(t-\tau)$$

with memory kernel

$$\tilde{G}(\tau) = \sum_\lambda |g_\lambda|^2 \mathrm{e}^{-i\Delta_\lambda \tau} = \int d\omega_\lambda \rho(\omega_\lambda)|g_\lambda|^2 \mathrm{e}^{-i\Delta_\lambda \tau}$$

- Reservoir structure function D(ω) or $\rho|g|^2$: we let

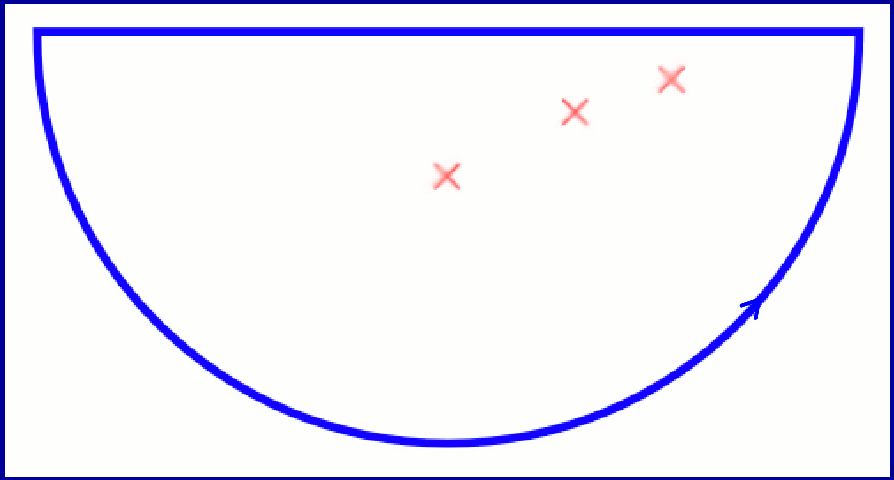
$$\rho_\lambda |g_\lambda|^2 = \frac{\Omega^2}{2\pi} D(\omega_\lambda)$$

Normalisation: $$\frac{1}{2\pi}\int D(\omega)d\omega = 1$$

B.M. Garraway, Phys. Rev. A 55, 2290 (1997).

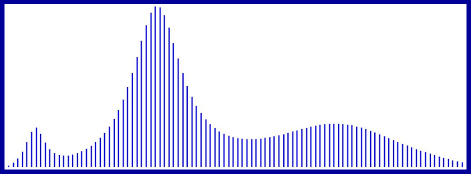
- System behaviour depends on the reservoir structure function $D(\omega_\lambda)$ with weight $\Omega^2 = \int d\omega_\lambda\, \rho(\omega_\lambda)\, |g_\lambda|^2$

- Pseudomode idea based on considering poles of $D(\omega_\lambda)$ in the lower half complex ω_λ plane.
  Poles at z_1, z_2, z_3 …., residues r_1, r_2, r_3 …



- Extend ω to $-\infty$
- Evaluate contour for any meromorphic function
- Kernel $\rightarrow$

$$\tilde{G}(\tau) = -i\Omega^2 \sum_l r_l e^{-i(z_l - \omega_1)\tau}$$



Could solve for atomic $c_1(t)$, but instead …

- New kernel for same integro-differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{c}_a(t) = -\int_0^t \mathrm{d}\tau \tilde{G}(\tau)\tilde{c}_a(t-\tau) \qquad \tilde{G}(\tau) = -i\Omega^2 \sum_l r_l e^{-i(z_l-\omega_1)\tau}$$

- Introduce effective amplitudes b_l and then

$$i\frac{dc_a(t)}{dt} = \omega_1 c_a(t) + \sum_l \mathcal{K}_l b_l(t)$$

$$i\frac{db_l(t)}{dt} = z_l b_l(t) + \mathcal{K}_l c_a(t)$$

Where $\mathcal{K}_l = \Omega\sqrt{-ir_l}$ are PM couplings

- Atom-pseudomode system satisfies simple equations
- Pseudomodes replace continuum structure
- An exact description (within RWA …)
- Can derive exact master equations: examples follow...

Single pole (l=1)

$$D(\omega) = \frac{\Gamma}{(\omega-\omega_1)^2 + (\Gamma/2)^2} \qquad z_1 = \omega_1 - i\Gamma/2$$

$$\begin{aligned} i\frac{d}{dt}\tilde{c}_a &= \sum_\lambda g_\lambda \mathrm{e}^{-i\Delta_\lambda t}\tilde{c}_\lambda \\ i\frac{d}{dt}\tilde{c}_\lambda &= g_\lambda \mathrm{e}^{i\Delta_\lambda t}\tilde{c}_a \end{aligned}$$

$$\begin{aligned} i\frac{d}{dt}c_a &= \omega_0 c_a + \Omega_0 b_1 \\ i\frac{d}{dt}b_1 &= (\omega_1 - i\Gamma/2)b_1 + \Omega_0 c_a \end{aligned}$$

Pseudomode master equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = -i\left[H_0, \hat{\rho}\right] - \frac{\Gamma}{2}\left(\hat{a}^\dagger\hat{a}\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a}^\dagger\hat{a}\right)$$

with

$$H_0 = \omega_0(\hat{\sigma}_z + 1)/2 + \omega_c\hat{a}^\dagger\hat{a} + \Omega_0\left(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right)$$

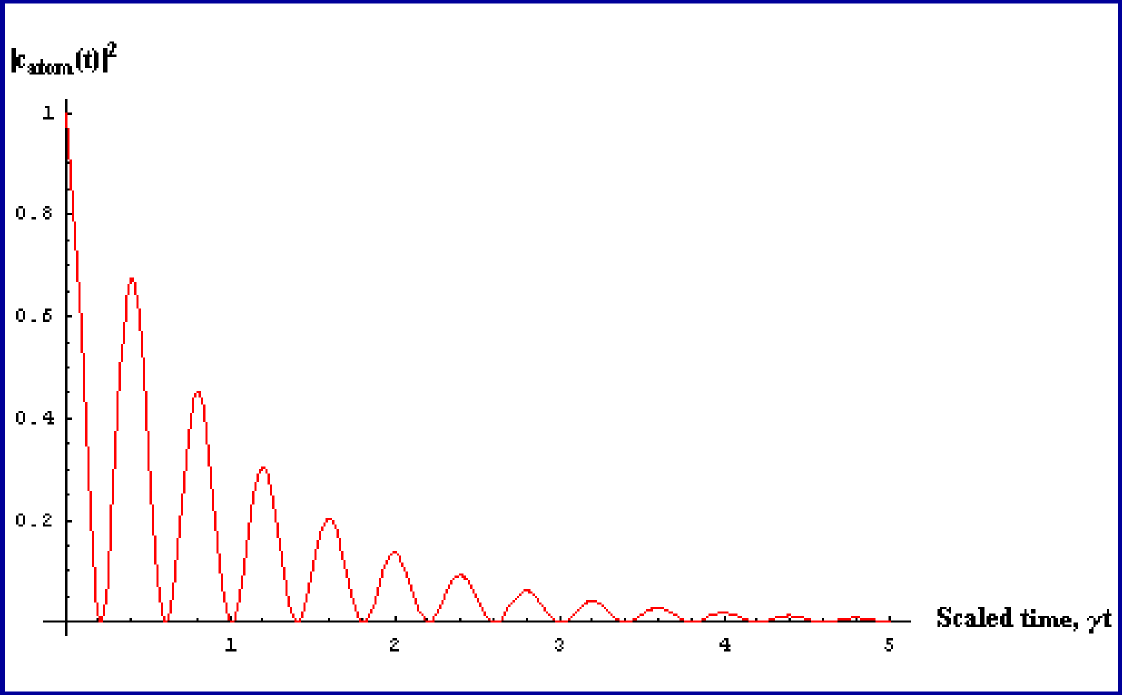
ρ is an enlarged atomic density matrix

Damped Rabi Oscillations

Solve the single pole pseudomode equations to give (resonance):

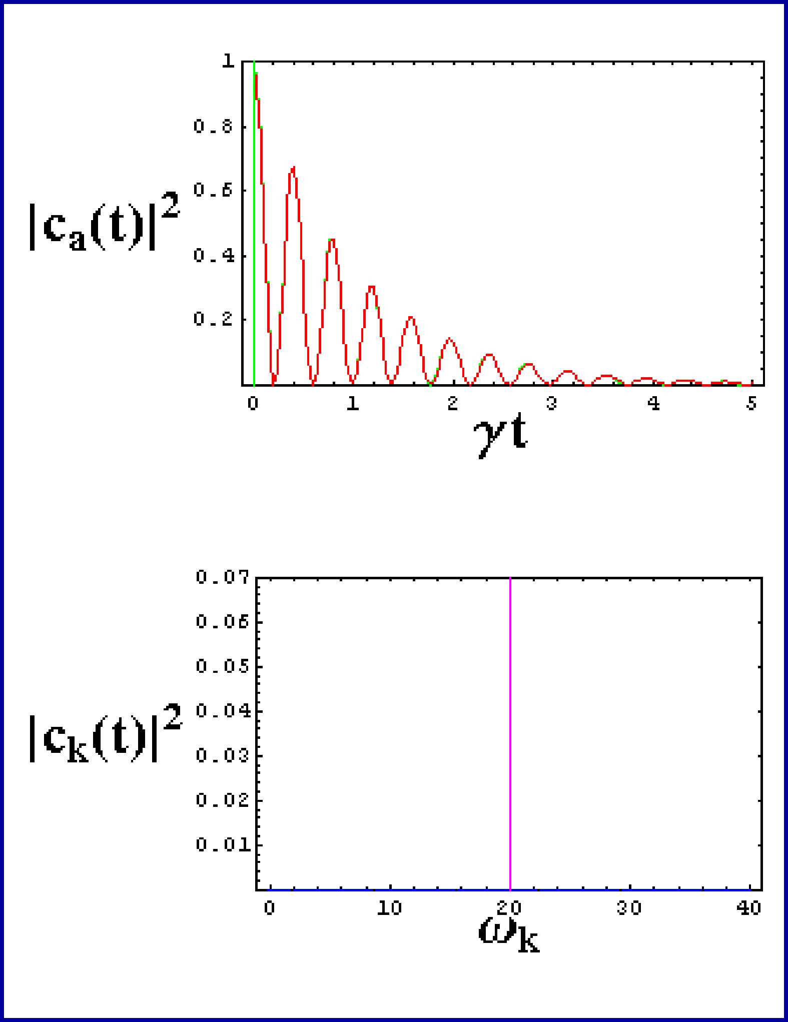
$$c_a(t) \approx e^{-\Gamma t/4} \cos\left(\frac{\Omega}{2} t\right) \quad \text{where} \quad \Omega = \sqrt{\Omega_0^2 - (\Gamma/2)^2}$$

$$|c_a(t)|^2 \approx \frac{1}{2} e^{-\Gamma t/2} \left(1 + \cos(\Omega t)\right)$$



- Exact and approximate solutions possible
- Numerical solution of equations with full bath possible, too.
- We can extract the bath c_k …

Vacuum Rabi Splitting

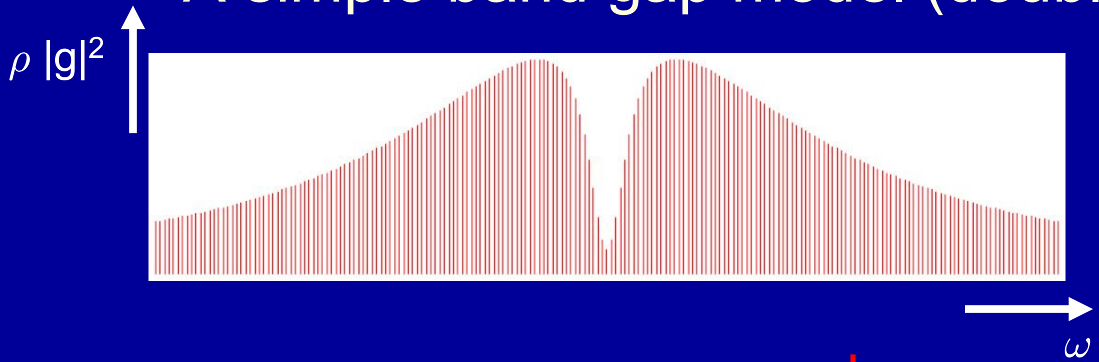


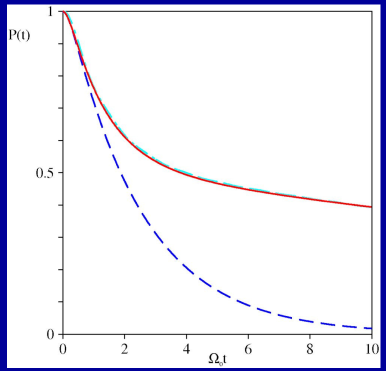
- Single pole example
- Excitation oscillates backwards and forwards between atom and reservoir.
- Reservoir excitation is an idealised spectrum.
- Final splitting is the Rabi frequency Ω
- Width of final peaks is $\Gamma/2$ (=γ)
- Each Rabi oscillation increases the number of peaks in reservoir spectrum by one.
- Finite numerical bath has a recurrence time =1/(level-spacing)

What do we learn?

- We can solve this example easily, e.g. by Laplace transform of the Integro-differential Eqn. – do we need to talk about "pseudomodes"?
- Answer: Yes – for other problems - if we want to talk about master equations.
- No perturbation theory was needed
- Link to a master equation: is this general …?

A simple band gap model (double pole)





DOS:

$$D(\omega) = W_1 \frac{\Gamma_1}{(\omega-\omega_c)^2 + (\Gamma_1/2)^2} - W_2 \frac{\Gamma_2}{(\omega-\omega_c)^2 + (\Gamma_2/2)^2}$$

Master equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = -i\left[H_0, \hat{\rho}\right] - \frac{\Gamma_1'}{2}\left(\hat{a}_1^\dagger \hat{a}_1 \hat{\rho} - 2\hat{a}_1\hat{\rho}\hat{a}_1^\dagger + \hat{\rho}\hat{a}_1^\dagger\hat{a}_1\right) - \frac{\Gamma_2'}{2}\left(\hat{a}_2^\dagger \hat{a}_2 \hat{\rho} - 2\hat{a}_2\hat{\rho}\hat{a}_2^\dagger + \hat{\rho}\hat{a}_2^\dagger\hat{a}_2\right)$$

with

$$\begin{aligned} H_0 = {} & \omega_0(\hat{\sigma}_z + 1)/2 + \omega_c \hat{a}_1^\dagger \hat{a}_1 + \omega_c \hat{a}_2^\dagger \hat{a}_2 \\ & + \frac{\sqrt{W_1 W_2}(\Gamma_1 - \Gamma_2)}{2}\left(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger\right) + \Omega_0\left(\hat{a}_2^\dagger \hat{\sigma}_- + \hat{a}_2 \hat{\sigma}_+\right) \end{aligned}$$

$$\Gamma_1' = \Gamma_1\Gamma_2\left(\frac{W_1}{\Gamma_1} - \frac{W_2}{\Gamma_2}\right), \qquad \Gamma_2' = W_1\Gamma_1 - W_2\Gamma_2$$

- Atom-pseudomode system satisfies Lindblad form master equations (effect of augmentation)
- Pseudomodes can be coupled
- Can be problems e.g. with branch cuts
- Multiple excitations ...

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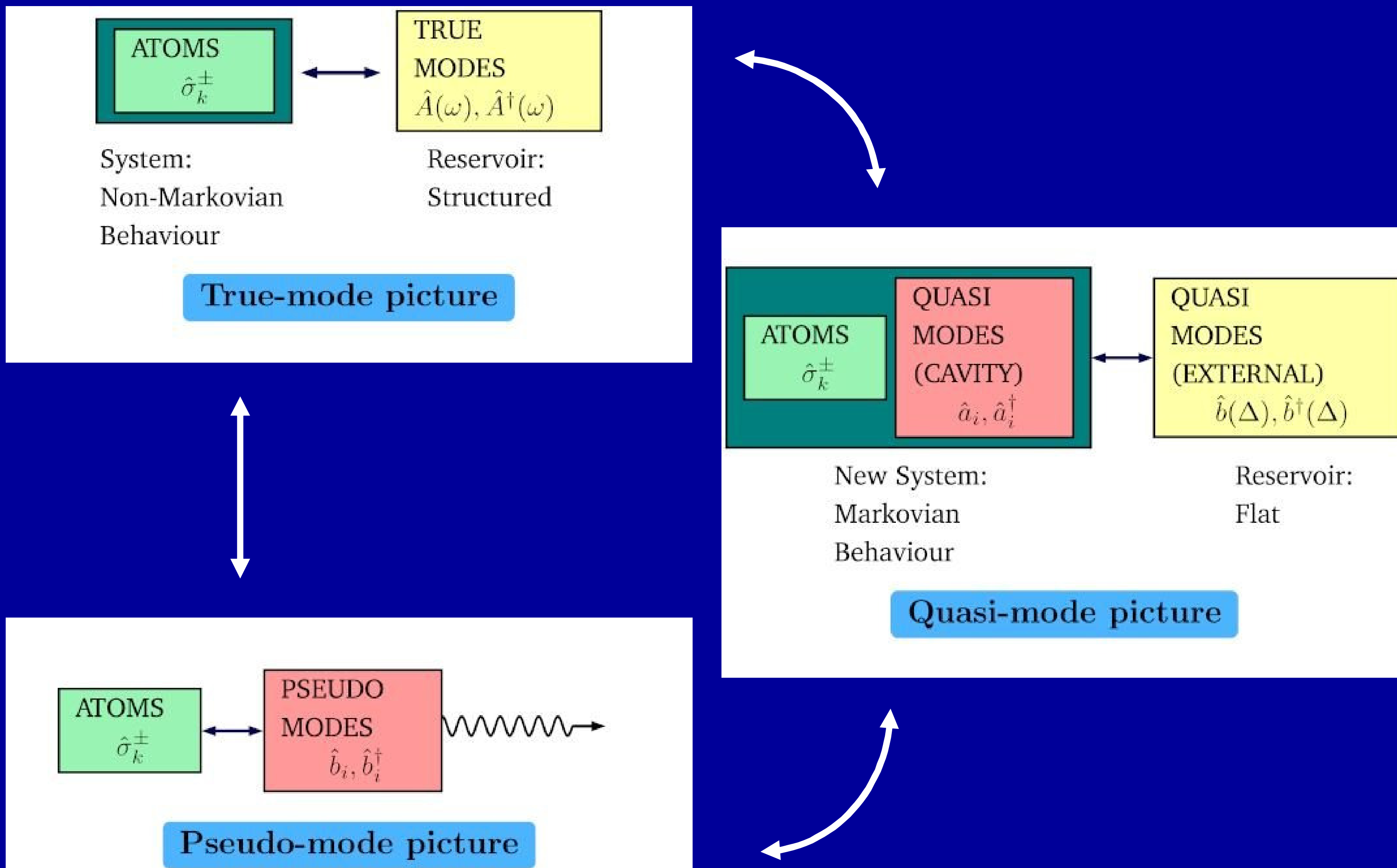
Three approaches to system description



Fano treatment: fields

True Mode Annihilation Operator – connection to internal and external QMs

$$\hat{A}(\omega) = \sum_i \alpha_i(\omega)\hat{a}_i + \int d\Delta \rho_c(\Delta)\beta(\omega,\Delta)\hat{b}(\Delta)$$

Quasi-mode annihilation operators in terms of true modes:

$$\hat{a}_i = \int d\omega\, \rho(\omega)\alpha_i^*(\omega)\hat{A}(\omega)$$

$$\hat{b}(\Delta) = \int d\omega\, \rho(\omega)\beta^*(\Delta,\omega)\hat{A}(\omega)$$

$$[\hat{A}(\omega), \hat{A}^\dagger(\omega')] = \delta(\omega - \omega')/\rho(\omega)$$

$$\left[\hat{A}(\omega), \hat{H}_F\right] = \hbar\omega\hat{A}(\omega)$$

The inverse problem

- Given a density of states ρ can we construct a pseudomode master equation?
- Represent actual ρ g^2 by model with poles (pseudomodes)
- Errors at high frequency (off resonant) in the model become discrepancies at long time scales.

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Time-local MEs and the pseudomodes

$$|\Psi(t)\rangle = \tilde{c}_1(t)\mathrm{e}^{-i\omega_1 t}|1\rangle|\ldots 0_\lambda \ldots\rangle + \sum_\lambda \tilde{c}_\lambda(t)\mathrm{e}^{-i\omega_\lambda t}|0\rangle|\ldots 1_\lambda \ldots\rangle$$

Non-Markovian dynamics in time-local Lindblad form (Breuer Petruccione *Theory of open quantum systems*, 2001)

$$\frac{d\rho_A}{dt} = \frac{S(t)}{2i}[\sigma_+\sigma_-, \rho_A] + \gamma(t)\left[\sigma_-\rho_A\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_A\}\right]$$

$$S(t) = -2\,\mathrm{Im}\left\{\frac{\dot{c}_1(t)}{c_1(t)}\right\}, \quad \gamma(t) = -2\,\mathrm{Re}\left\{\frac{\dot{c}_1(t)}{c_1(t)}\right\}.$$

Preserves trace, positivity

Quantum jump simulations

$$\dot{\rho}(t) = \frac{1}{i\hbar}[H_S, \rho(t)] + \sum_j \Gamma_j C_j \rho(t) C_j^\dagger - \frac{1}{2}\sum_j \Gamma_j \{C_j^\dagger C_j, \rho(t)\}.$$

Deterministic evolution:

$$H = H_S - \frac{i\hbar}{2}\sum_j \Gamma_j C_j^\dagger C_j.$$

$$|\phi_\alpha(t+\delta t)\rangle = \left(1 - \frac{iH\delta t}{\hbar}\right)|\psi_\alpha(t)\rangle.$$

$$|\psi_\alpha(t)\rangle \rightarrow |\psi_\alpha(t+\delta t)\rangle = \frac{|\phi_\alpha(t+\delta t)\rangle}{\||\phi_\alpha(t+\delta t)\rangle\|}.$$

Jump process:

$$|\psi_\alpha(t)\rangle \rightarrow |\psi_\alpha(t+\delta t)\rangle = \frac{C_j|\psi_\alpha(t)\rangle}{\|C_j|\psi_\alpha(t)\rangle\|}.$$

Jump probability

$$p_\alpha^j(t) = \Gamma_j \delta t \langle\psi_\alpha(t)|C_j^\dagger C_j|\psi_\alpha(t)\rangle.$$

Complete time step:

$$\overline{\sigma_\alpha(t+\delta t)} = (1-p_\alpha)\frac{|\phi_\alpha(t+\delta t)\rangle\langle\phi_\alpha(t+\delta t)|}{1-p_\alpha} + \sum_j p_\alpha^j \frac{C_j|\psi_\alpha(t)\rangle\langle\psi_\alpha(t)|C_j^\dagger}{\langle\psi_\alpha(t)|C_j^\dagger C_j|\psi_\alpha(t)\rangle}.$$

Non-Markovian quantum jumps (Piilo et al 2008)

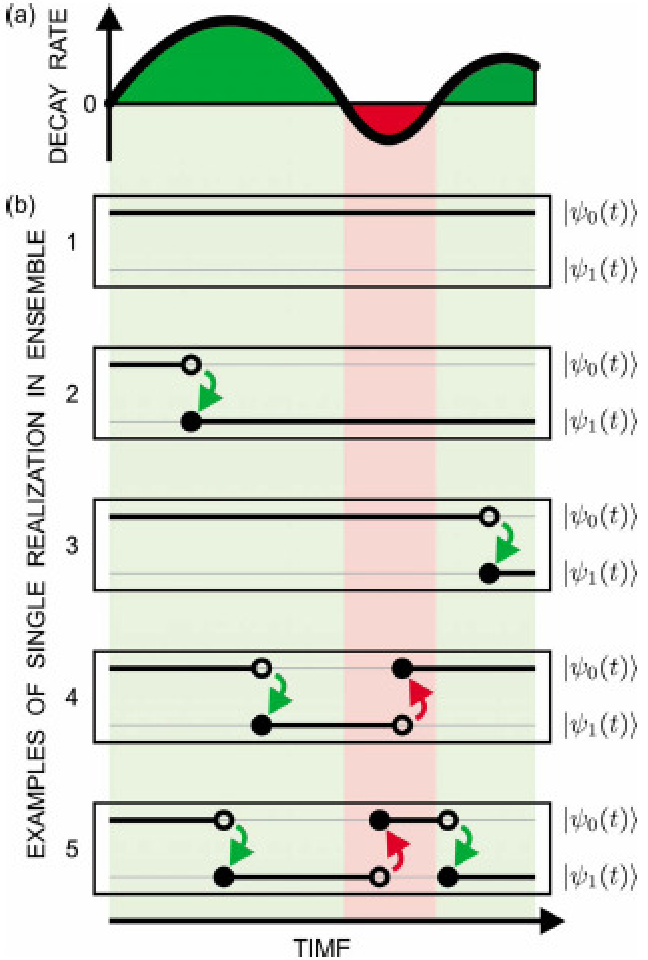


FIG. 10. (Color online) (a) Sketch of a time-dependent decay rate with periods of positive and negative values (arbitrary units). (b) Examples of single realizations encountered in the ensemble. The system is assumed to be such that there is only one decay channel and two physically different states: the initial state $|\psi_0(t)\rangle$ and the target state of a quantum jump $|\psi_1(t)\rangle$ (deterministic evolution is given by thin horizontal lines). The state of an ensemble member at the given time is indicated by the thick line. Quantum jumps from $|\psi_0\rangle$ to $|\psi_1\rangle$ (arrows down) occur at random times dur-

Why needed?

$$\frac{d\rho_A}{dt} = \frac{S(t)}{2i}[\sigma_+\sigma_-, \rho_A] + \gamma(t)\left[\sigma_-\rho_A\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_A\}\right]$$

$\gamma(t)$ can be negative!

Simulation problems: negative probabilities

… use new NMQJ approach

Time-local MEs and the pseudomodes

Breuer Petruccione master equation

$$\frac{d\rho_A}{dt} = \frac{S(t)}{2i}[\sigma_+\sigma_-,\rho_A] + \gamma(t)\left[\sigma_-\rho_A\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho_A\}\right]$$

$$S(t) = -2\,\mathrm{Im}\left\{\frac{\dot{c}_1(t)}{c_1(t)}\right\}, \quad \gamma(t) = -2\,\mathrm{Re}\left\{\frac{\dot{c}_1(t)}{c_1(t)}\right\}.$$

Pseudomode master equation
(Lorentzian structure)

$$\frac{d\rho}{dt} = -i[H_0,\rho] - \frac{\Gamma}{2}[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a],$$

$$H_0 = \omega_0\sigma_+\sigma_- + \omega_c a^\dagger a + \Omega_0[a^\dagger\sigma_- + a\sigma_+]$$

Density matrix ρ is in an enlarged space.
Trace out the PM:

$$\frac{d\rho_A}{dt} = \frac{A(t)}{2i}[\sigma_+\sigma_-,\rho_A] + B(t)\left[\sigma_-\rho_A\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho_A\}\right],$$

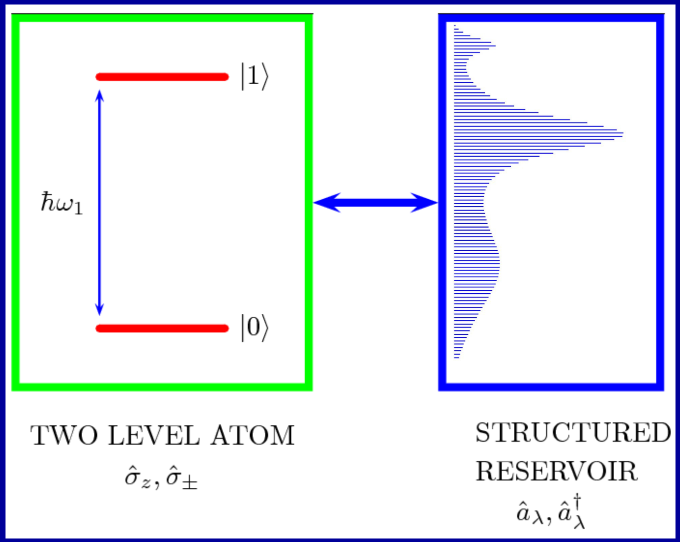
The connection

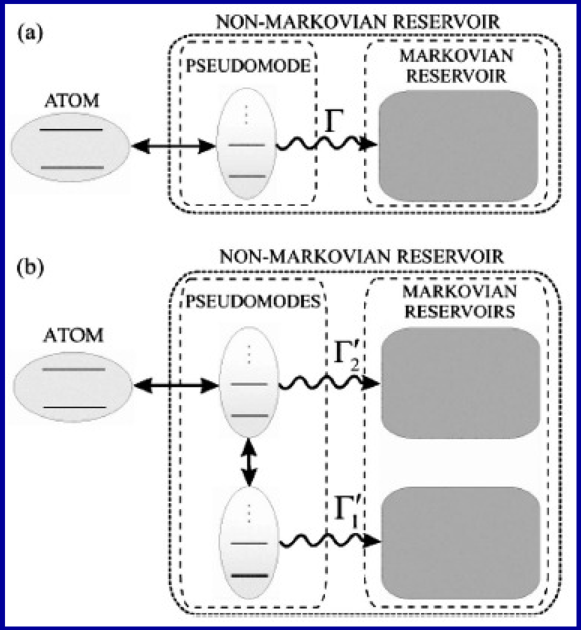
$$A(t) = S(t) \qquad B(t) = \gamma(t)$$

Link to pseudomodes yields interpretation with $\frac{d|b_1(t)|^2}{dt} + \Gamma|b_1(t)|^2 = \gamma(t)|c_1(t)|^2$

Normal QJ simulation possible.

Mazzola et al. Phys. Rev. A **80**, 012104 (2009)





Environment division:

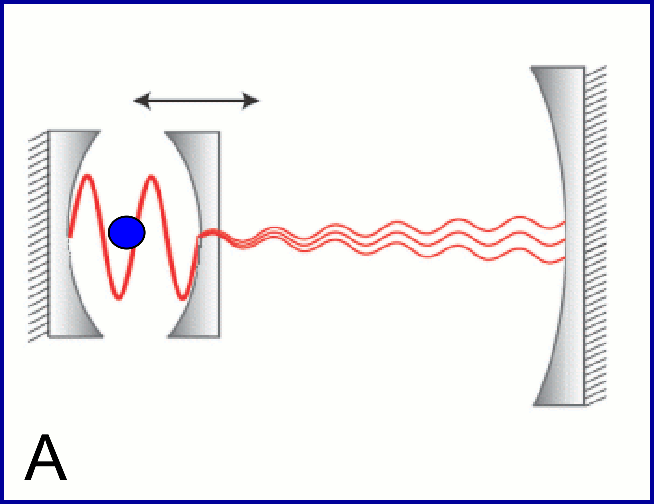
- Memory part
- Non-memory part

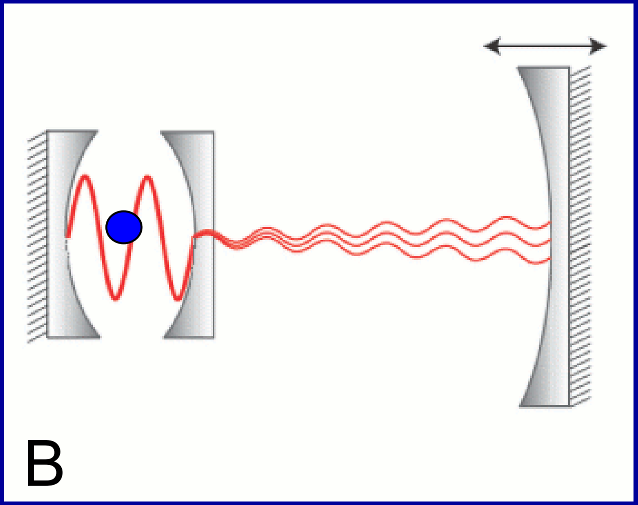
Contents:

- Introduction and some standard approaches
- Pseudomode method
- Connection via Fano diagonalization
- Where is the memory of a reservoir (quantum trajectories and psedomodes)?
- Dynamic reservoir structures

Time-dependent reservoir structures

- Cavity realisation: A time dependent position of the mirror affects mode structure





A: Essentially straightforward and previously studied (GSA+…).

B: Could lead to a different kind of dynamic structure. Quantify and explore.

$$\hat{\mathcal{H}}_I(t) = \sum_k g_k(t) \Bigg[\hat{\sigma}^- \hat{b}_k^\dagger \exp\left(-i\int_0^t [\omega_0 - \omega_k(\tau)]\, d\tau\right) + \hat{\sigma}^+ \hat{b}_k \exp\left(+i\int_0^t [\omega_0 - \omega_k(\tau)]\, d\tau\right) \Bigg]$$

Atomic dynamics

Solve the Schrödinger equation to give:

$$
\begin{aligned}
i\frac{\partial c_a(t)}{\partial t} &= \sum_k g_k(t) \exp\left(+i\int_0^t [\omega_0 - \omega_k(\tau)]\, d\tau\right) c_k(t) \\
i\frac{\partial c_k(t)}{\partial t} &= g_k(t) \exp\left(-i\int_0^t [\omega_0 - \omega_k(\tau)]\, d\tau\right) c_a(t)
\end{aligned}
$$

Eliminate Bath Modes

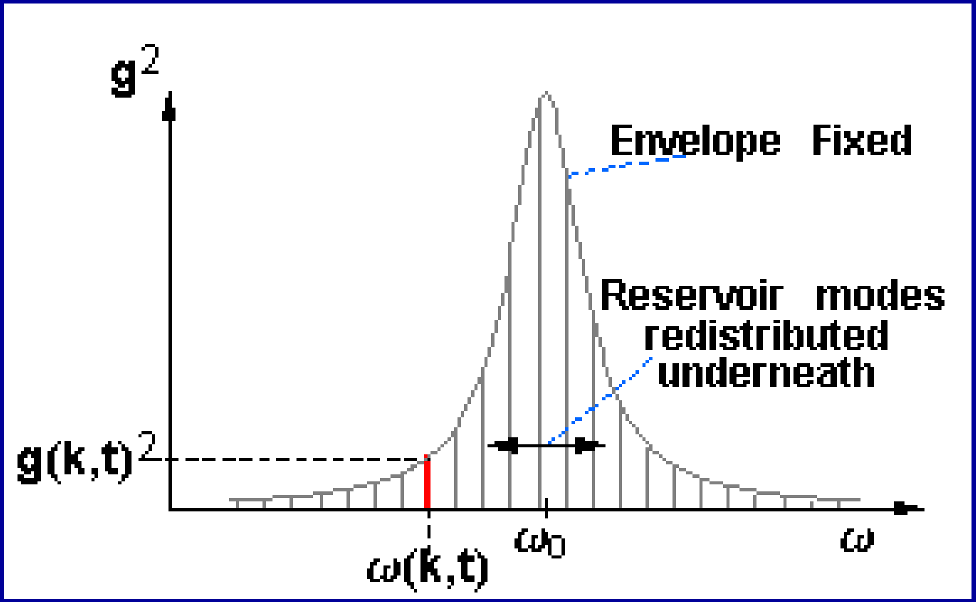
$$
\frac{\partial c_a(t)}{\partial t} = -\int_0^t K(t,t')c_a(t')\, dt'
$$

Memory Kernel

$$
K(t,t') = \sum_k g_k(t) g_k(t') \exp\left(+i\int_{t'}^t [\omega_0 - \omega_k(\tau)]\, d\tau\right)
$$

A chirped bath model

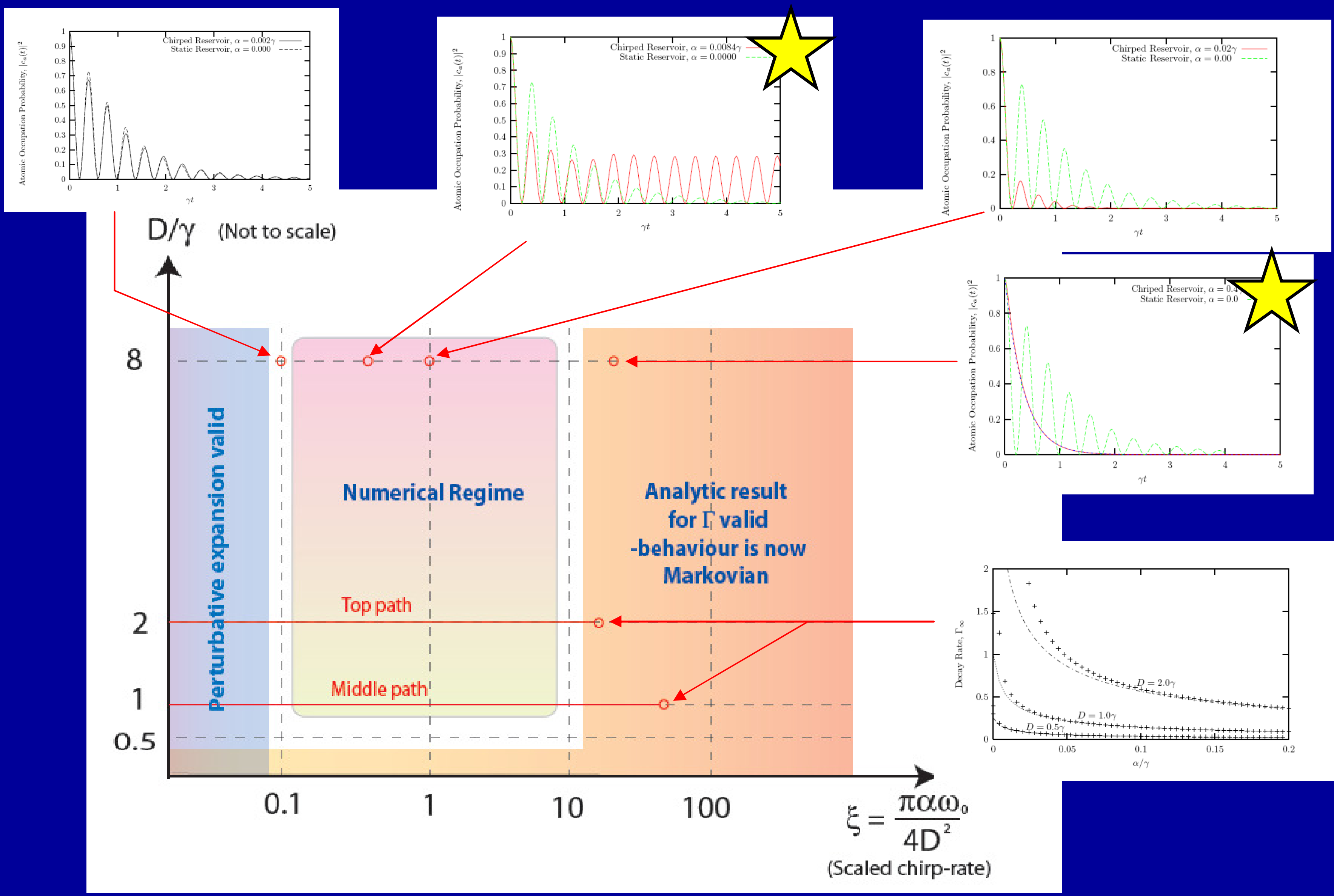
Macroscopic structure remains static while a ‘wind’ of modes pass through the resonance ⇒ linear chirp of bath modes: $\omega(k,t) = \omega_k + \chi t$



$$\rho_k g_k(t) g_k(t') = \frac{(\Omega_0/2)^2 \gamma}{\pi \sqrt{[\gamma^2 + (\omega_0 - \omega_k(t))^2]\,[\gamma^2 + (\omega_0 - \omega_k(t'))^2]}}$$

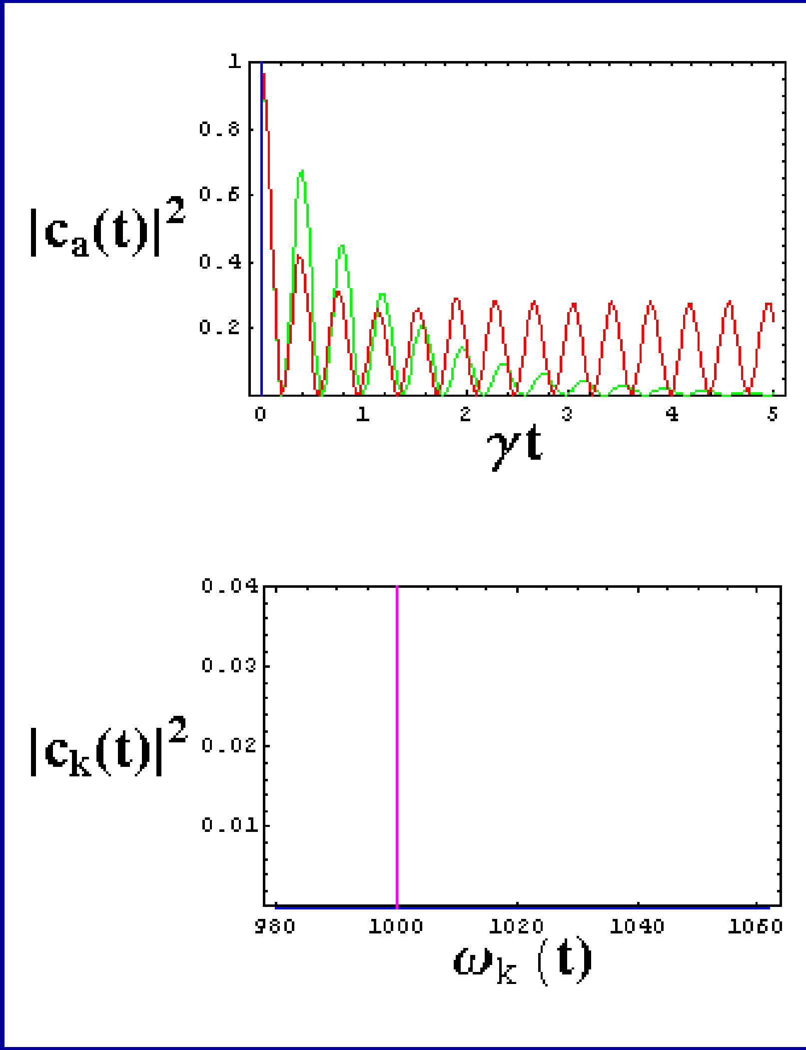
- **A new feature of this model is the presence of TWO-TIME reservoir structure function**
- **Branch cuts ⇒ mostly numerical approach …**

Different regimes of interest



Linington & Garraway, J Phys B (2006), $D \rightarrow \Omega_0/2$

Population Recycling



…if the mode-frequencies are modulated at this characteristic rate, then the left-hand Rabi peak may be repeatedly brought back onto resonance.

Rabi-oscillations become stable due to recycling of reservoir population.

High chirp-rate – Markov limit

(Analytic approach)

In the high-chirp limit (ξ >> 1), each bath mode is effectively coupled to the atom for only a very short time; there is no time for memory effects.

$$\frac{\partial c_a(t)}{\partial t} \approx -c_a(t)\,\frac{\Omega_0^2\gamma}{4\pi}\int_0^t dt'\int_{-\infty}^{\infty} d\omega_k \frac{\exp\left[+i\left(\omega_0-\omega_k\right)t-\frac{i\chi}{2}t^2\right]}{\sqrt{\left(\gamma^2+\left(\omega_0-\omega_k-\chi t\right)^2\right)}} \times \frac{\exp\left[-i\left(\omega_0-\omega_k\right)t'+\frac{i\chi}{2}t'^2\right]}{\sqrt{\left(\gamma^2+\left(\omega_0-\omega_k-\chi t'\right)^2\right)}}$$

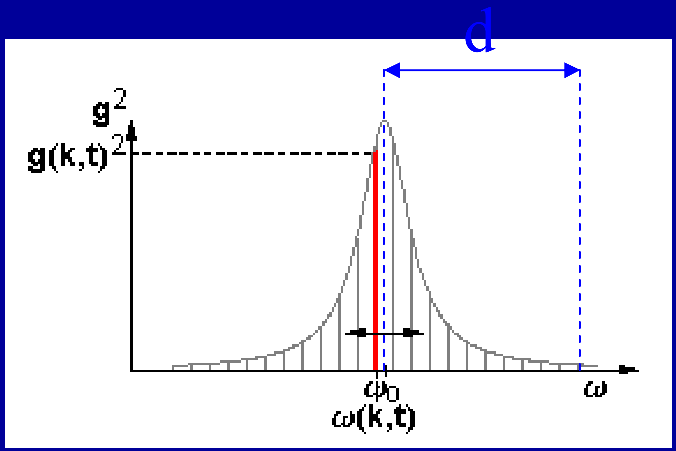
$$\frac{\partial c_a(t)}{\partial t} \approx -c_a(t)\Gamma(t)$$

Approximate Markovian form – even for strong coupling

$$\Gamma_\infty = \lim_{t\to\infty}\{\Gamma(t)\} = \frac{\Omega_0^2\gamma}{8\pi\alpha\omega_0}\left|K_0\left(\frac{i\gamma^2}{4\alpha\omega_0}\right)\right|^2$$

Can extract energy from the cavity on a very short time-scale!

Case 2: Oscillating frequency manipulation



$$\omega_k(t) = \omega_k + d\sin(\Omega t)$$

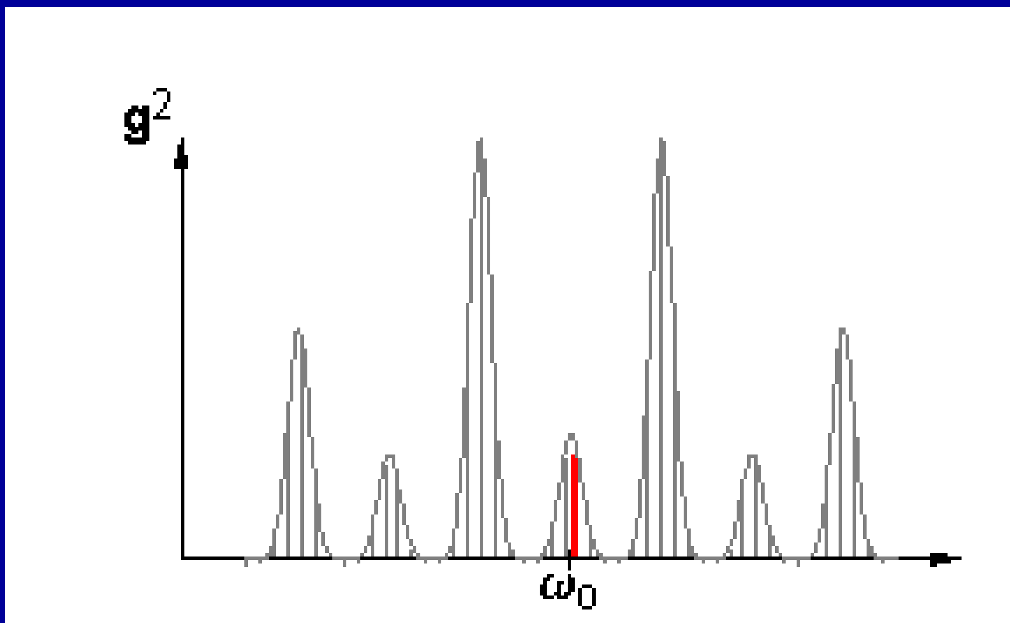
Non-linear phase-term:

$$\exp\left(+i[\omega_0 - \omega_k]t + i\frac{d}{\Omega}[1-\cos(\Omega t)]\right)$$

Can be Fourier-decomposed as a sum of simple phase-factors:

$$\sum_{l=-\infty}^{\infty} (-i)^l J_l(d/\Omega)\exp\left(i[\omega_k - \omega_0 - l\Omega]t + i\frac{d}{\Omega}\right)$$

Two equivalent ways of thinking about the problem…

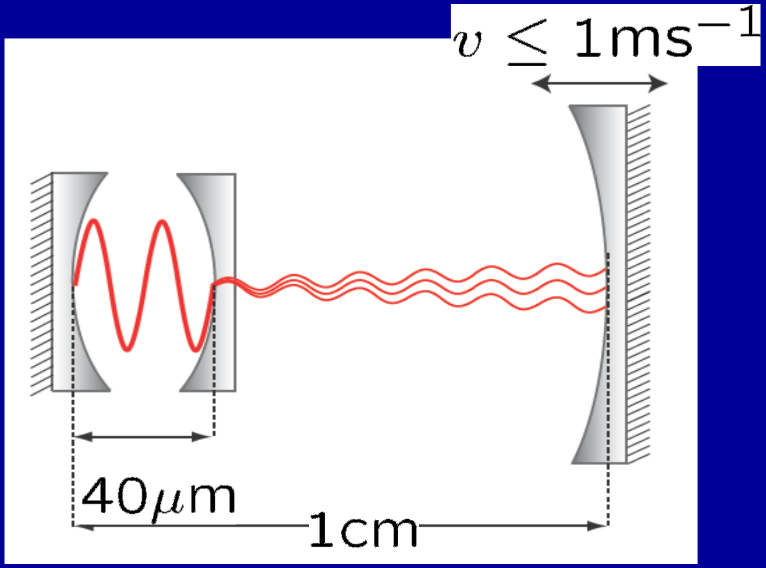


…either the atom is coupled to a collection of baths with time-dependent frequencies and static couplings.

…or the atom is coupled to a single bath with time-dependent couplings and static frequencies…

$$\hat{\mathcal{H}}_I(t) = \sum_k g_k(t)\left[\hat\sigma^-\hat b_k^\dagger \exp\left(-i\int_0^t[\omega_0-\omega_k(\tau)]d\tau\right) + \hat\sigma^+\hat b_k \exp\left(+i\int_0^t[\omega_0-\omega_k(\tau)]d\tau\right)\right]$$

Observable effects



Modulation frequency:

$\Omega/2\pi \leq 10^7\text{s}^{-1}$

Linewidth:

$\gamma/2\pi \leq 10^6\text{s}^{-1}$

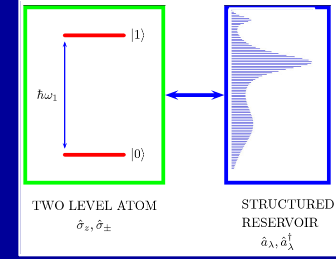
Coupling strength:

$D/2\pi \leq 4 \times 10^7\text{s}^{-1}$

*(Numbers borrowed from PRL **93**, 233603 (2004) – Kimble group)*

Weak coupling: Inhibition of decay-rate by a factor of up to 1000

Strong coupling: Enhancement of decay-rate by up to a factor 15, (or modest suppression).



Summary

- Pseudomodes:
  - Pseudomode: the part of the reservoir structure which retains a memory.
  - Meromorphic reservoir structures lead to standard master equations
  - Multiple excitations covered and checked
  - No representation when branch cuts present – does it matter?
  - Expect application to local density of states (reservoir structure)
- Time dependent reservoir structures:
  - At present pseudomodes only useful in perturbative limit
  - High chirp → Markovian decay (which can be fast!)
  - Enhanced and inhibited decay via χ seen for linear and oscillatory chirp
  - Rich behaviour …

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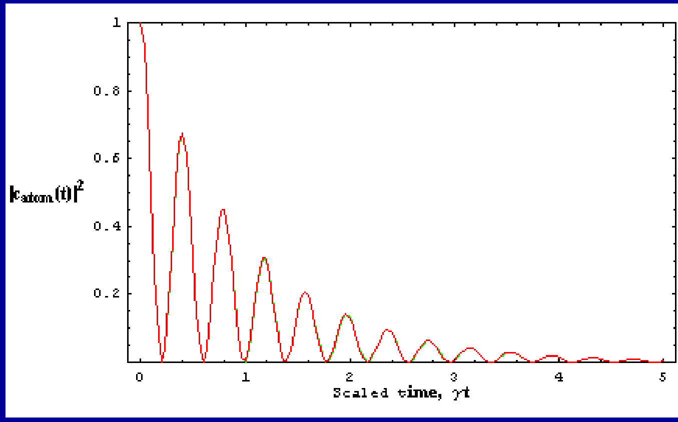

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