Non-Markovian quantum dynamics: a stochastic Schrödinger equation approach

ECT* WORKSHOP

DECOHERENCE IN QUANTUM DYNAMICAL SYSTEMS

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Outline

1. **SDEs in Quantum Mechanics**
2. **SDEs for open quantum systems: stochastic unravellings**
3. **½ spin example**
4. **SDEs for non-Markovian systems**
5. **Non-Markovian generalization of the Joos-Zeh model**

SDEs in Quantum Mechanics

SDE = Stochastic differential equation

SDE in Hilbert spaces: random corrections to the unitary evolution given by the Schrödinger equation.

Use in quantum mechanics:

1. **Continuous quantum Measurement theory:** describe the effect of a continuous measurement on a quantum system

2. **Collapse models:** solve the measurement problem

3. **Open quantum systems:** effective description of the interaction system with environment

Stochastic unravellings

Lindblad equation (Markovian evolution)

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \gamma \sum_n A_n \rho(t) A_n - \frac{\gamma}{2}\{A^2, \rho(t)\}$$

Stochastic unravelling $\rho(t) = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H dt + \sqrt{\gamma}\sum_n (\xi A_n - \xi_R \langle A_n\rangle_t) dW_t^{(n)} - \frac{\gamma}{2}\sum_n (A_n^2 - 2\xi\xi_R A_n \langle A_n\rangle_t + \xi_R^2 \langle A_n\rangle_t^2) dt\right] |\psi_t\rangle$$

$\xi_R = Re[\xi]$

$\xi \in \mathbb{C}, \quad |\xi| = 1$

$\langle A_n\rangle_t = \langle\psi_t|A_n|\psi_t\rangle$

n independent standard
Wiener processes
=
White noises
=
Markovian noises
(δ-correlation in time)

Two special cases

There are infinitely many stochastic unravellings. Two are particularly useful

1. **ξ = 1: collapse equation** (= continuous quantum jumps)

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H dt + \sqrt{\gamma} \sum_n (A_n - \langle A_n \rangle_t) dW_t^{(n)} - \frac{\gamma}{2} \sum_n (A_n - \langle A_n \rangle_t)^2 dt \right] |\psi_t\rangle$$

Collapse to a common eigenstate of A_n

2. **ξ = i: random quantum potential** (quantum jumps analogy?)

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H dt + i\sqrt{\gamma} \sum_n A_n dW_t^{(n)} - \frac{\gamma}{2} \sum_n A_n^2 dt \right] |\psi_t\rangle$$

“Itô term”. It disappears from solutions

Advantages

1. **Computational:** Size of the problem

   Density matrix ~ $N^2/2$

   State vector ~ N

   In some cases: easier to solve for state vector and average over a few samples

2. **Visualization:** It allows to think in terms of a state vector under a unitary evolution + external random potential

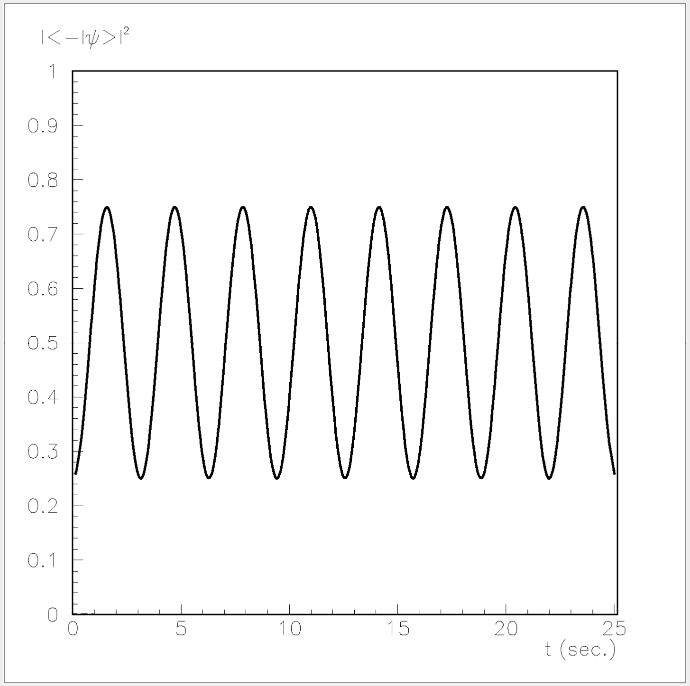
3. **SDEs vs Quantum Jumps approach:** More advanced mathematical tools

½ spin example

$$d|\psi_t\rangle = \left[-i\omega\sigma_x dt + \sqrt{\gamma}(\sigma_z - \langle\sigma_z\rangle_t)dW_t - \frac{\gamma}{2}(\sigma_z - \langle\sigma_z\rangle_t)^2 dt\right]|\psi_t\rangle$$

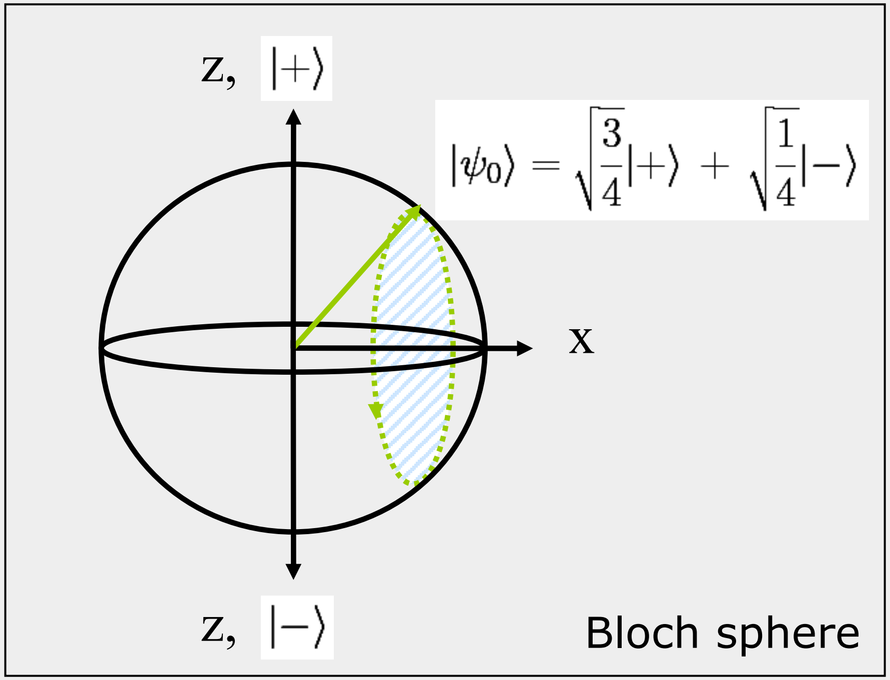
In the standard quantum case:

$$\frac{d}{dt}\rho_t = -i[\sigma_x, \rho_t] - \frac{\gamma}{2}[\sigma_z, [\sigma_z, \rho_t]]$$



$$\gamma = 0$$

$$\omega = 1$$

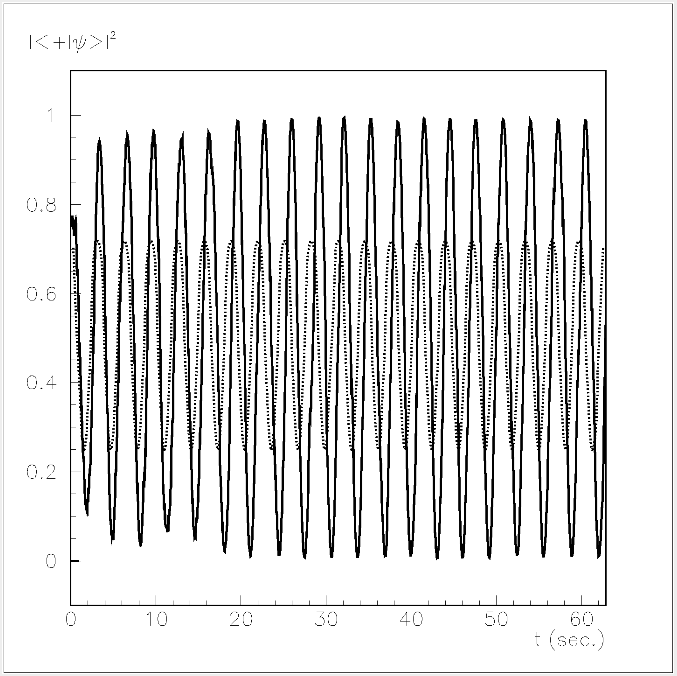


A. Bassi and E. Ippoliti: *Phys. Rev. A 69, 012105 (2004).*

Case γ< ω

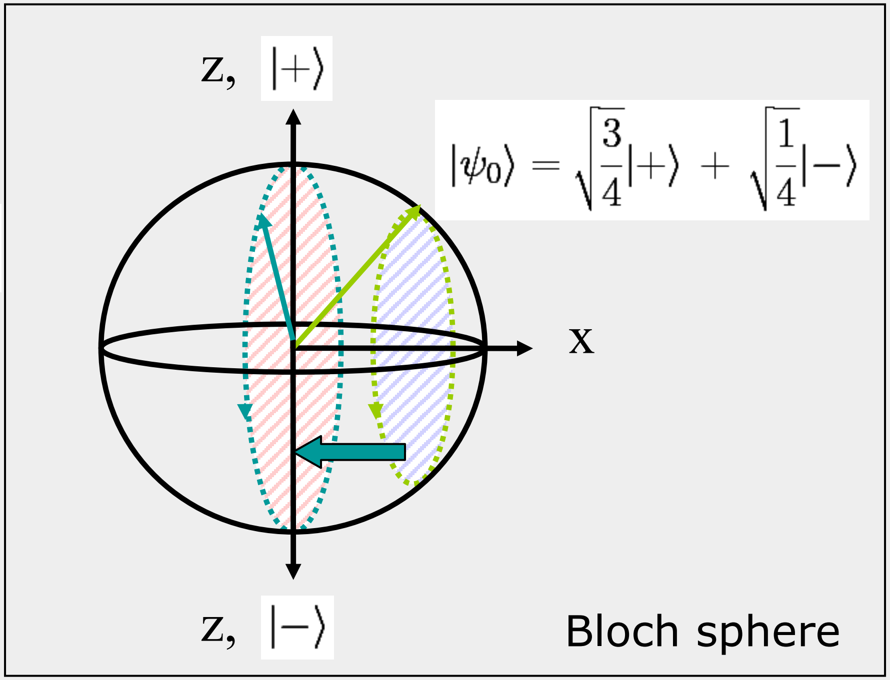
In the standard quantum case: Hamiltonian stronger than other terms

Two effects:



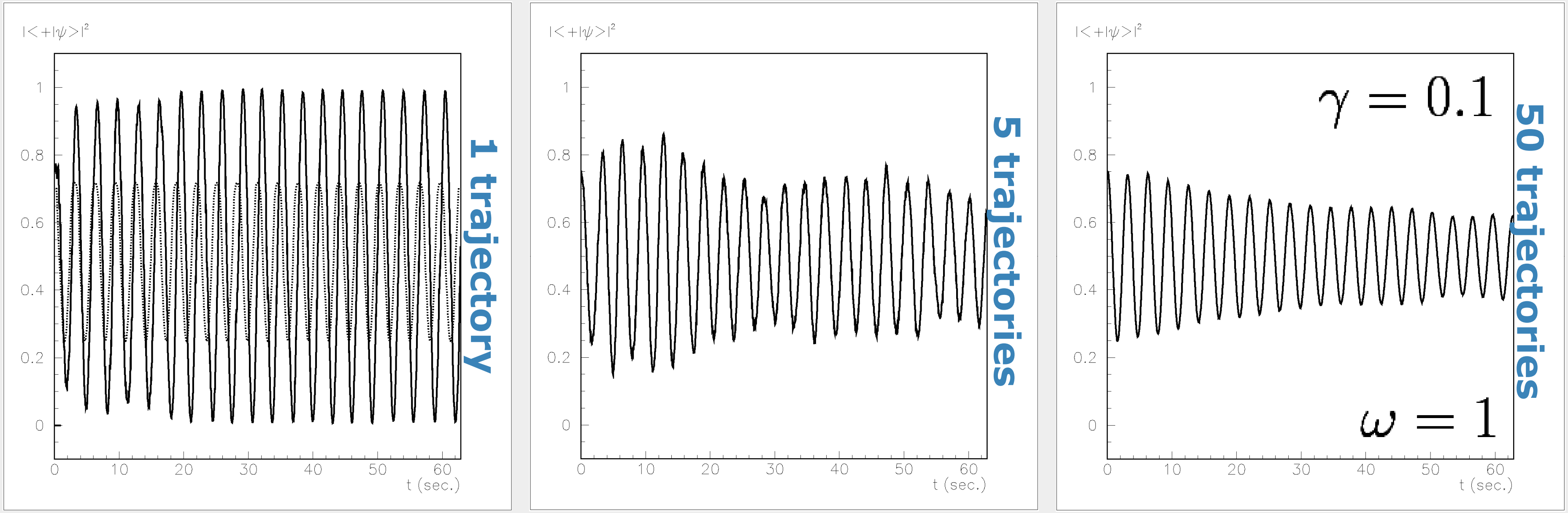
$$\gamma = 0.1$$

$$\omega = 1$$



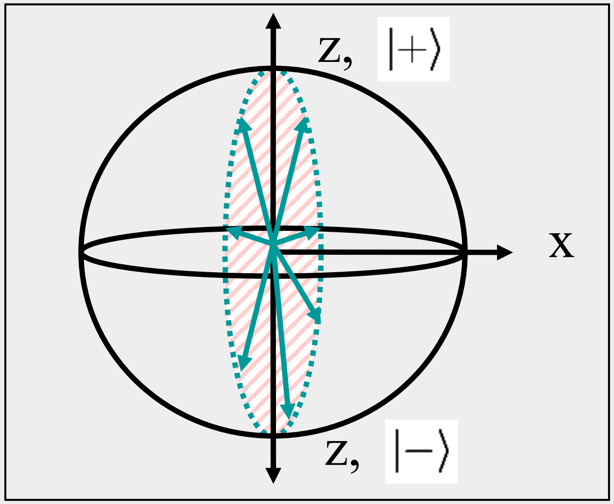
1. The rotation plane drifts towards the yz plane of the Bloch sphere

Case γ< ω

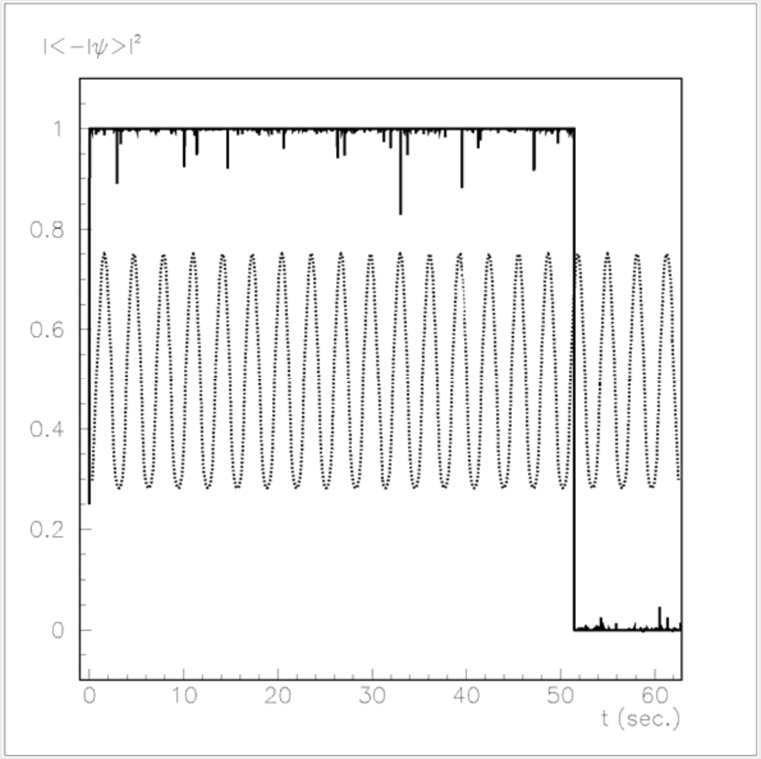


2. Relative phases of the different trajectories randomly change in time.
   Randomization increases in time.

Linked to decoherence



Case γ> ω



$$\gamma = 50 \qquad \omega = 1$$

Collapse probability ≈ Born rule

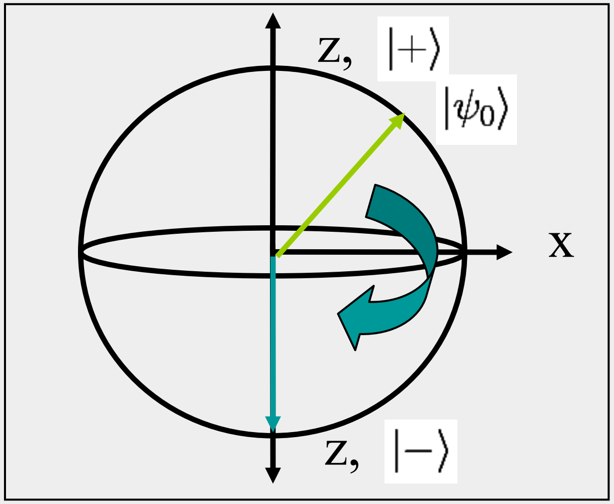
Reduction: state vector jumps to one of the two eigenstates of σ_z.

Reduction time = the smaller, the bigger γ.

Persistence of collapse: the state vector remains in one eigenstate the longer, the greater the value of γ.

Eventually, the state will jump to the eigenstate, than back, …

This effect is more rare, for larger γ.



Non-Markovian dynamics

No general theory so far developed.

Three approaches:

1. **Fundamental description:** effective equations from microscopic dynamics $\longrightarrow$ non-Markovian quantum Brownian motion

2. **Mathematical analysis:** general structures from fundamental requirements $\longrightarrow$ generalization of Lindblad structure

3. **Phenomenological approach:** "guess" a reasonable form of the equations

We follow the third approach, with SDEs.

IDEA: replace the white noise with a colored noise

Non-Markovian SDEs

Linearized non-Markovian equation

$$\frac{d}{dt}|\psi_t\rangle = \left[-\frac{i}{\hbar}H + \sqrt{\lambda}qw_t - 2\sqrt{\lambda}q\int_0^t ds\,\alpha(t,s)\frac{\delta}{\delta w_s}\right]|\psi_t\rangle$$

Noise's correlation function

L. Diosi and W. T. Strunz: Phys. Lett. A 235, 569 (1997).

1. **Quantum measurement theory:** non-Markov generalization of the continuous measurement of the particle's position

2. **Collapse models:** non-Markov spontaneous collapse of the wave function

3. **Open quantum systems:** non-Markov interaction of the particle with the environment, via its position = non-Markov generalization of the **Joos-Zeh model**

Free particle solution

Green's function

$$\psi_t(x) \;=\; \int_{-\infty}^{+\infty} dx_0 \; G(x,t;x_0,0)\;\psi_0(x_0)$$

Path integration

$$G(x,t;x_0,0) \;=\; \int_{q(0)=x_0}^{q(t)=x} \mathcal{D}[q]\exp[S(q)]$$

Non standard action

$$S(q) \;=\; \int_0^t ds\left[\frac{im}{2\hbar}\dot{q}(s)^2 + \sqrt{\lambda}\,q(s)w(s) - \lambda q(s)\int_0^t dr\; q(r)\alpha(s,r)\right]$$

A. Bassi and L. Ferialdi: Phys. Rev. Lett. 103, 050403 (2009).

Green's function

$$G(x,t;x_0,0) = \sqrt{\frac{m}{2i\pi\hbar t u(t)}} \exp\left[-A_t(x_0^2 + x^2) + B_t x_0 x + C_t x_0 + D_t x + E_t\right]$$

Time-translation invariant correlation function $\alpha(t,s) = \alpha(|t-s|)$

Same structure as in the white noise case

$$A_t = \frac{i\hbar}{2m}\dot{f}_t(0) \qquad B_t = \frac{i\hbar}{m}\dot{f}_t(t)$$

➡ Deterministic functions

$$C_t = -\frac{i\hbar}{2m}\dot{h}_t(0) + \frac{\sqrt{\lambda}}{2}\int_0^t ds\, w_s f_t(s) \quad D_t = \frac{i\hbar}{2m}\dot{h}_t(t) + \frac{\sqrt{\lambda}}{2}\int_0^t ds\, w_s f_t(t-s)$$

$$E_t = \frac{\sqrt{\lambda}}{2}\int_0^t ds\, w_s h_t(s)$$

➡ Random functions depending on the noise

Green's function

Two unknown functions

$$\frac{im}{2\hbar}\ddot{h}_t(s) + \lambda \int_0^t dr\, \alpha(s,r) h_t(r) \;=\; \frac{\sqrt{\lambda}}{2} w_s$$

$$\frac{im}{2\hbar}\ddot{f}_t(s) + \lambda \int_0^t dr\, \alpha(s,r) f_t(r) \;=\; 0$$

➡ In general, not exactly solvable

Exponential correlation function $\alpha(t,s) = \frac{\gamma}{2} e^{-\gamma|t-s|}$

The second-order integro-differential equations can be transformed into fourth-order ordinary differential equations

➡ All functions explicitly known

Gaussian wave functions

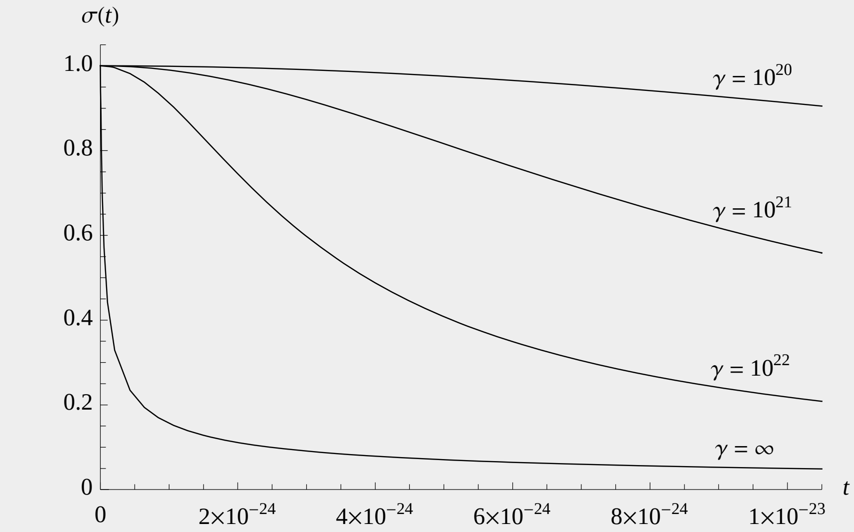


FIG. 1. Time evolution of the **spread in position,** for small times.

γ = ∞: **white-noise case.**

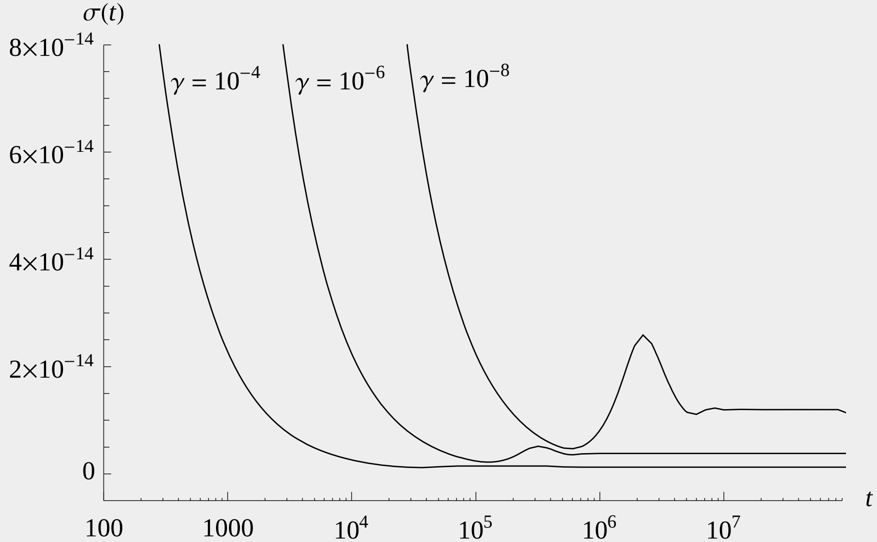


FIG. 2. Time evolution of the **spread in position,** for large times.

White-noise case = straight line at $1.27\text{x}10^{-15}$ m.

Average values

Position

$$\frac{d}{dt}\mathbb{E}[\langle q\rangle_t] \;=\; \frac{1}{m}\,\mathbb{E}[\langle p\rangle_t]$$

Momentum

$$\frac{d}{dt}\mathbb{E}[\langle p\rangle_t] \;=\; 0$$

➡ Like in the Markovian case

Energy

$$\frac{d}{dt}\mathbb{E}[\langle H\rangle_t] \;=\; \frac{\lambda\hbar^2}{m}\int_0^t ds\;\alpha(t,s)$$

Exponential correlation function

$$\mathbb{E}[\langle H\rangle_t] \;=\; \mathbb{E}[\langle H\rangle_0] + \frac{\lambda\hbar^2}{m}\left(t + \frac{e^{-\gamma\,t}-1}{\gamma}\right)$$

Non-Markovian master equation

Equation for the state vector

$$\frac{d}{dt}|\psi_t\rangle = \left[-\frac{i}{\hbar}H + \sqrt{\lambda}q w_t - 2\sqrt{\lambda}q\int_0^t ds\,\alpha(t,s)\frac{\delta}{\delta w_s}\right]|\psi_t\rangle$$

Statistical operator = ensemble of states

$$\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$$

Problem: how to compute averages

Having the explicit expression of the Green's function, the problem is solved

Harmonic oscillator

… after a long calculation

$$
\begin{aligned}
\frac{d}{dt}\rho_t &= -\frac{i}{\hbar}[H,\rho(t)] - \gamma\int_0^t ds\,\alpha(t,s)\cos\omega(t-s)\,[q,[q,\rho_t]] \\
&+ \frac{\gamma}{m\omega}\int_0^t ds\,\alpha(t,s)\sin\omega(t-s)[q,[p,\rho_t]], \qquad H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2
\end{aligned}
$$

1. **White noise case:** it reduces to the Joos-Zeh model
2. **Time dependent functions:** explicit expression
3. **New term:** q–p commutator
4. **Positivity is preserved!**

Comparison with Hu-Paz-Zhang

Hu-Paz-Zhang

$$
\begin{aligned}
\frac{d}{dt}\rho_t(x,y) \quad = \quad & \frac{i\hbar}{2m}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\rho_t(x,y)-\frac{i}{\hbar}\frac{m\omega^2}{2}\left(x^2-y^2\right)\rho_t(x,y)-\frac{i}{\hbar}\frac{m}{2}\delta\Omega^2(t)\left(x^2-y^2\right)\rho_t(x,y) \\
& -\Gamma(t)(x-y)\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}\right)\rho_t(x,y)-\frac{m}{\hbar}\Gamma(t)h(t)(x-y)^2\rho_t(x,y) \\
& -i\Gamma(t)f(t)(x-y)\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)\rho_t(x,y)
\end{aligned}
$$

Our case

Frequency shift

$$
\begin{aligned}
\frac{d}{dt}\rho_t(x,y) \quad = \quad & \frac{i\hbar}{2m}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\rho_t(x,y)-\frac{i}{\hbar}\frac{m\omega^2}{2}\left(x^2-y^2\right)\rho_t(x,y)-\cancel{\frac{i}{\hbar}\frac{m}{2}\delta\Omega^2(t)\left(x^2-y^2\right)\rho_t(x,y)} \\
& \cancel{-\Gamma(t)(x-y)\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}\right)}\rho_t(x,y)-\frac{m}{\hbar}\Gamma(t)h(t)(x-y)^2\rho_t(x,y) \\
& -i\Gamma(t)f(t)(x-y)\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)\rho_t(x,y)
\end{aligned}
$$

Dissipative term

Conclusions

1. Interest in stochastic evolutions for quantum mechanical systems ranges over a large variety of contexts
2. SDEs are a very powerful mathematical tool for dealing with such situations (in the Markovian case)
3. SDEs suggest how to generalize the dynamics to the non-Markovian case, at least phenomenologically
4. Technical details in the articles …

Case with ξ=1 (collapse)

More complicated equations.

The CSL Model

G.C. Ghirardi, P. Pearle and A. Rimini, *Phys. Rev. A* 42, 78 (1990).

$$\boxed{d|\psi_t\rangle = \Big[\underbrace{-\frac{i}{\hbar}H\,dt}_{\text{Quantum Hamiltonian}} + \underbrace{\sqrt{\lambda}\int d^3x(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)dW_t(\mathbf{x}) - \frac{\lambda}{2}\int d^3x(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)^2 dt}_{\text{NEW COLLAPSE TERMS}}\Big]|\psi_t\rangle}$$

NEW COLLAPSE TERMS ➡ **New Physics**

$N(\mathbf{x}) = a^\dagger(\mathbf{x})a(\mathbf{x})$ particle density operator, $\langle N(\mathbf{x})\rangle_t = \langle\psi_t|N(\mathbf{x})|\psi_t\rangle$ **nonlinearity**

$W_t(\mathbf{x}) =$ noise, $\mathbb{E}[W_t(\mathbf{x})] = 0$, $\mathbb{E}[W_t(\mathbf{x})W_s(\mathbf{y})] = \delta(t-s)e^{-(\alpha/4)(\mathbf{x}-\mathbf{y})^2}$ **stochasticity**

$\lambda \sim 10^{-17}\,\mathrm{s}^{-1}$ collapse strength $r_C = 1/\sqrt{\alpha} \sim 10^{-5}$ cm correlation length

---

Mass proportional CSL model: $\lambda \longrightarrow \lambda\left(\frac{m}{m_N}\right)^2$, $m_N =$ nucleon mass

Usefulness of collapse models

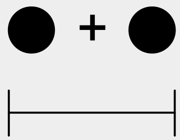
1. **Collapse models as a solution of the measurement problem of Quantum Mechanics.** These models offer a paradox-free description of quantum measurements (and of all physical processes).

2. **Collapse models as a rival theory of Quantum Mechanics.** Important, in order to give a quantitative meaning to experiments testing quantum linearity. They are an alternative theory, which makes different predictions, to which these experiments can be compared.

3. **Collapse models as phenomenological models of an underlying pre-quantum theory.** If quantum mechanics is not exact, and spontaneous collapse-type effects are seen in experiments, these model may offer a direction to look for a new theory.

Collapse rate

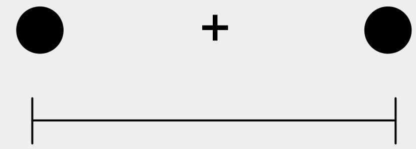
Small superpositions



$\ll r_C$

No collapse

Large superpositions



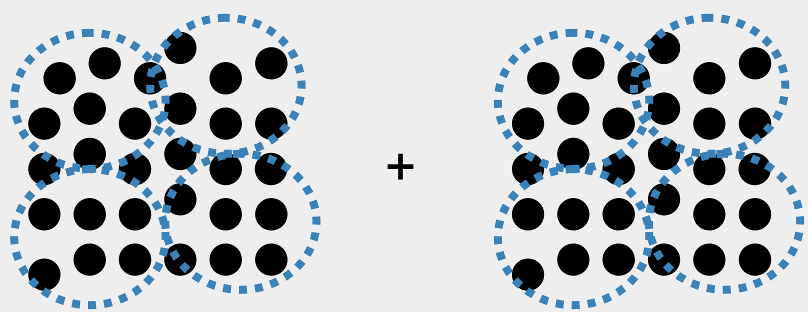
$\geq r_C$

Collapse

$$\boxed{\mathbf{\Gamma = \lambda n^2 N} \quad (\text{rate} = s^{-1})}$$

n = number of particles within r_C

N = number of such clusters

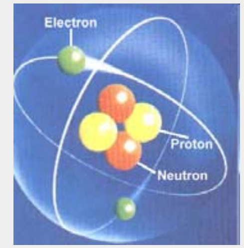


2. Lower bounds

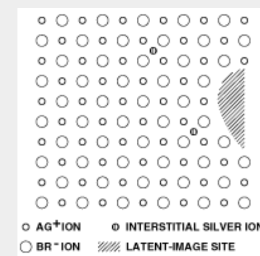
$\lambda \sim 10^{-17}\,\mathrm{s}^{-1}$: QUANTUM / CLASSICAL

$\lambda \sim 10^{-8}\,\mathrm{s}^{-1}$: QUANTUM / CLASSICAL

Microscopic world (few particles)



Latent image formation (~ 5-6000 particles)

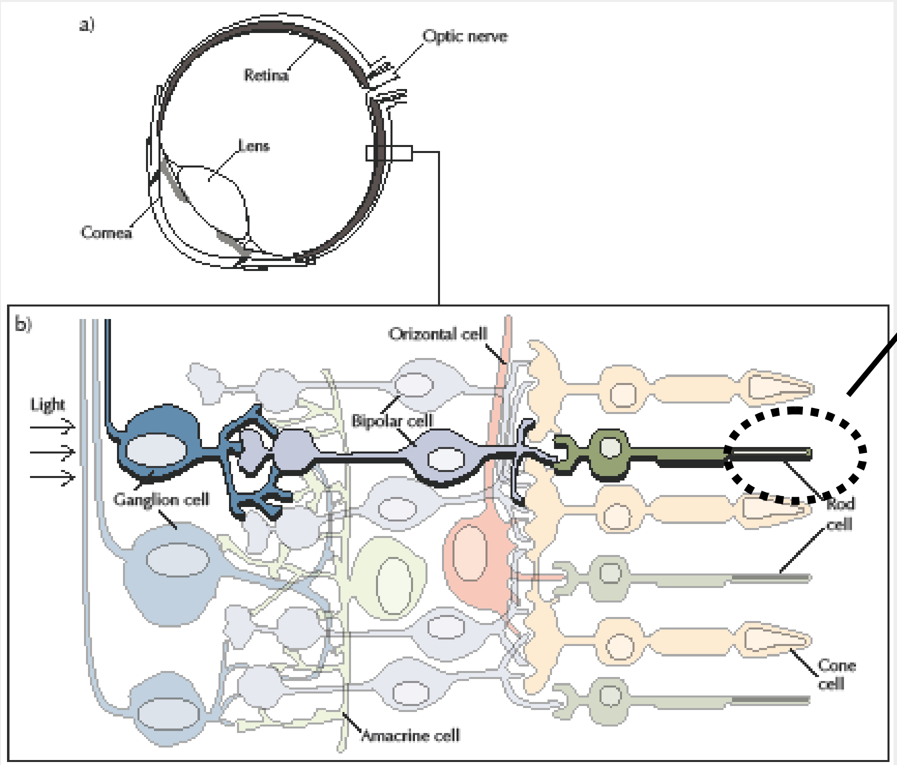


$$\begin{aligned}
\gamma + AgBr &\rightarrow Ag^{+} + Br^{-} \\
&\rightarrow Ag^{+} + Br + e^{-} \quad (e^{-} \text{ trapped by a grain}) \\
&\rightarrow Ag + Br \quad (Ag \text{ trapped by a grain})
\end{aligned}$$

Macroscopic world (10^{13} particles)

n increasing

Collapse in the eye



Threshold of vision: ~6 photons

photon absorbed by the rhodopsin

cis-trans transf. of the rhodopsin

interaction with ~20 transducins

α-subunit splits, binding to a PDE

PDE activated

PDE hydrolyzes ~100 cGMP to GMP

Closure of ~300 ionic channels

~10 Na^+/channel blocked

Time: ~100ms

α-subunit of the transducin: $n \sim 3.9 \times 10^4, \quad N \sim 20$

Other terms give similar a contribution

$$\lambda \sim 1.4 \times 10^{-7}\,\mathrm{s}^{-1}$$

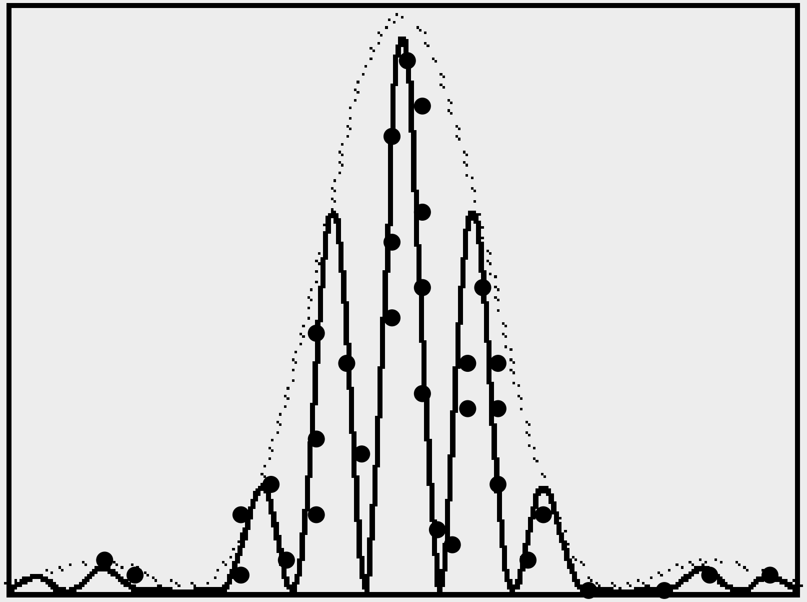
The collapse occurs when ~10^4-10^5 particles are involved

3. Upper bounds

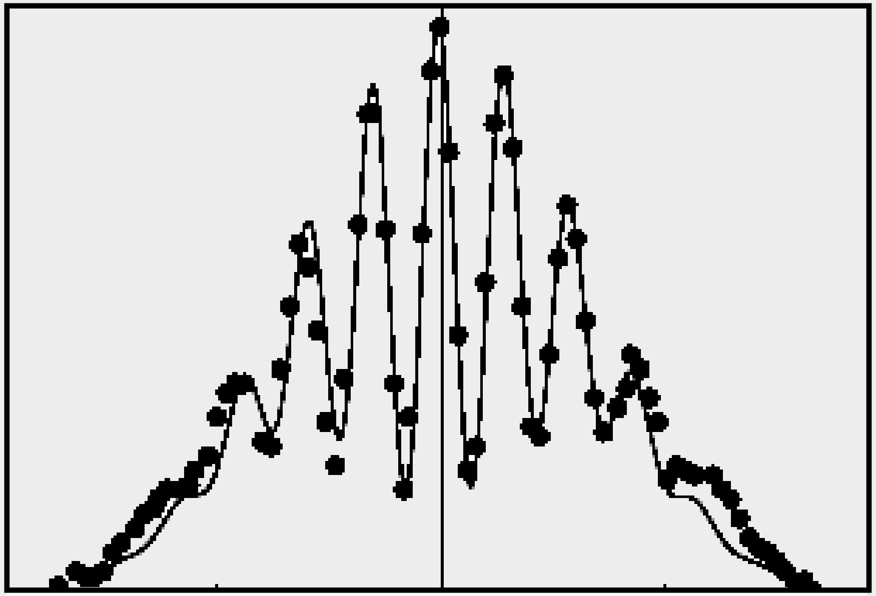
Destruction of quantum interference

The nonlinear terms work against the superposition principle.

In interference experiments, one should see a reduction of interference fringes



Prediction of quantum mechanics
(no environmental noise)



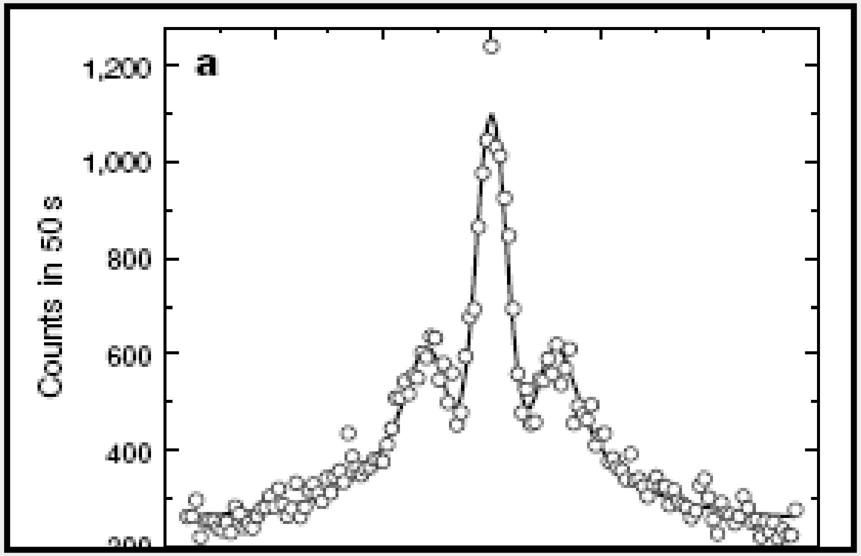
Prediction of collapse models
(no environmental noise)

Upper bounds

Destruction of quantum interference

Diffraction of macro-molecules:

- **C_{60} (720 AMU)**
  M. Arndt et al, *Nature* 401, 680 (1999)
- **C_{70} (840 AMU)**
  L. Hackermüller et al, *Nature* 427, 711 (2004)
- **$C_{30}H_{12}F_{30}N_2O_4$ (1,030 AMU)**
  S. Gerlich et al, *Nature Physics* 3, 711 (2007)



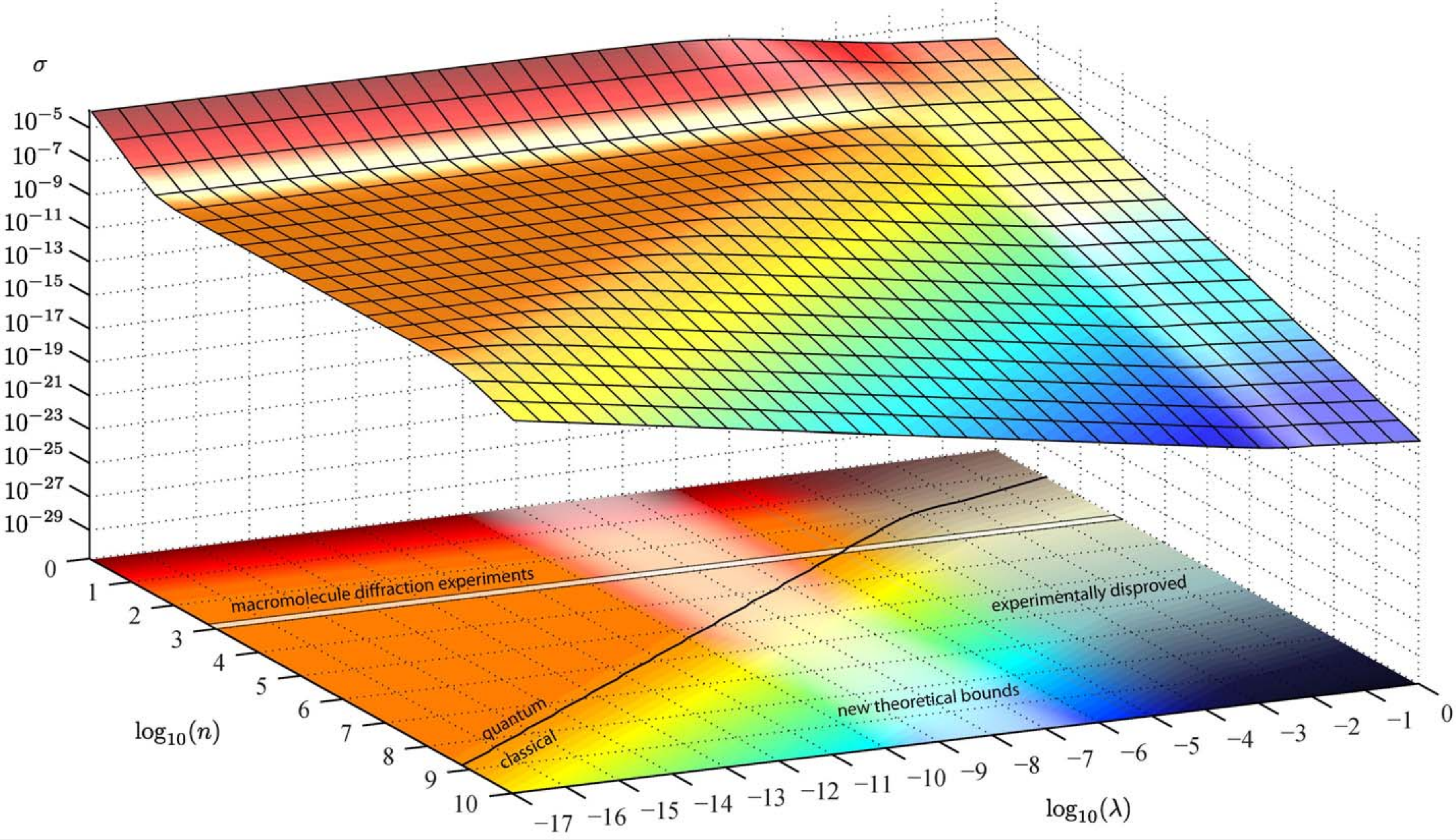
C_{60} diffraction experiment

Future experiments

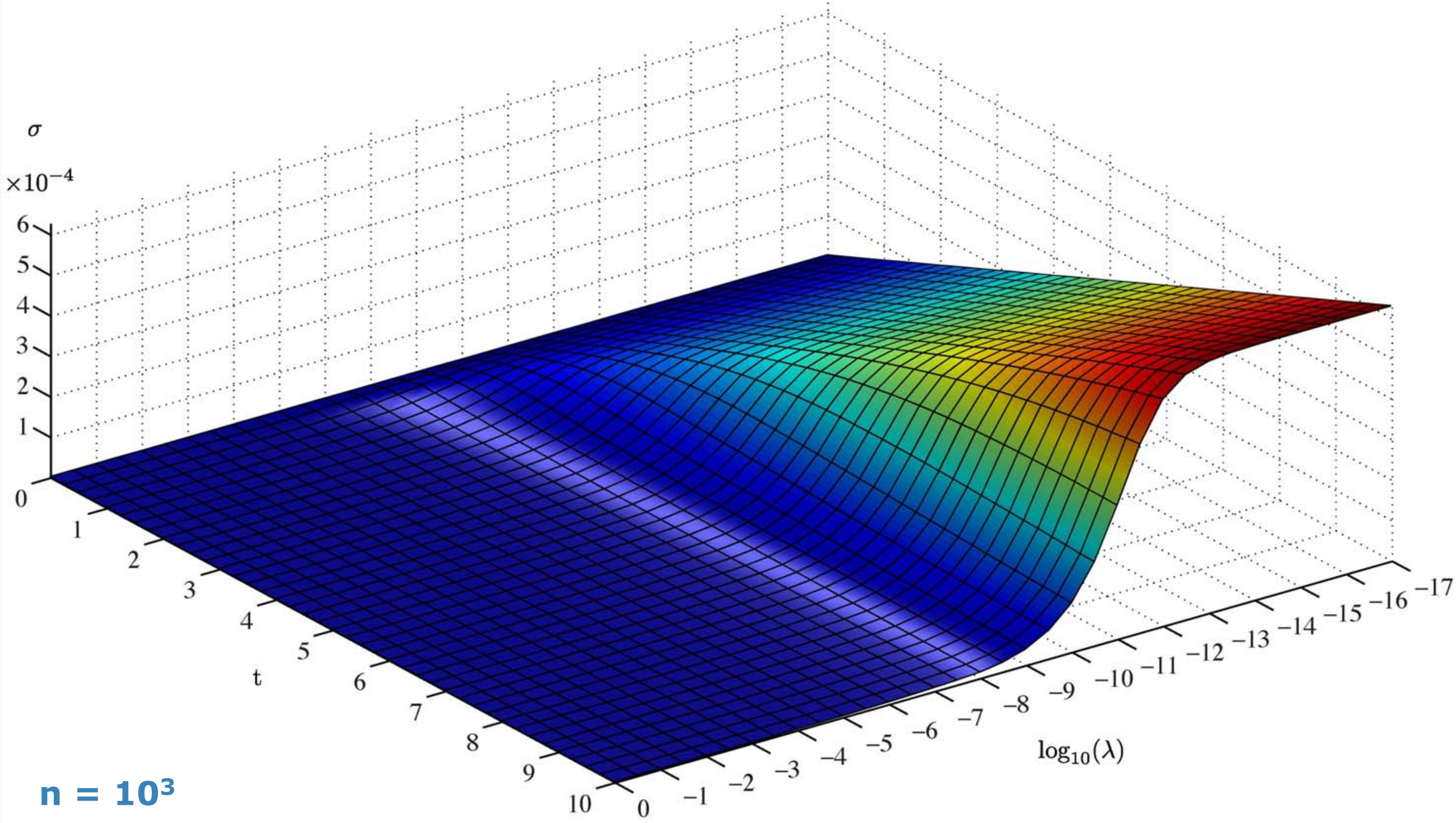
They include much larger molecules (**~11,000 a.m.u.,** possibly up to **1,000,000 a.m.u**.). A three orders of magnitude increase in the number of particles would become interesting

	Distance (orders of magnitude) from the standard CSL value	**Distance (orders of magnitude) from the enhanced value**
Diffraction of macro-molecules	**12-13**	**3-4**

Destruction of quantum interference



Time evolution of the spread



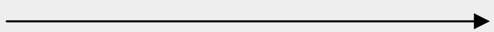
n = 10³

Upper bounds

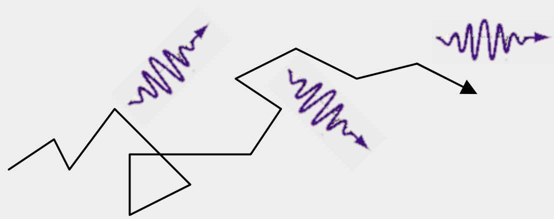
Spontaneous emission of radiation

FREE PARTICLE

1. Quantum mechanics



2. Collapse models

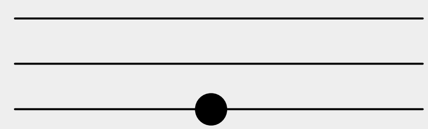


$$\frac{d\Gamma_k}{dk} = \frac{e^2 \lambda \hbar}{2\pi^2 \epsilon_0 m^2 c^3 k}$$

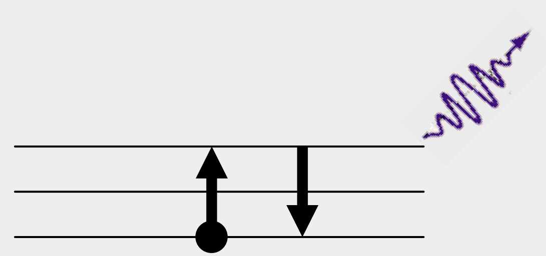
Q. Fu, *Phys. Rev. A* 56, 1806 (1997)

BOUND STATE

1. Quantum mechanics



2. Collapse models



$$\frac{d\Gamma_k}{dk} = 2\left[1 - \frac{1}{(1+(ka_0/2)^2)^2}\right] \frac{e^2 \lambda \hbar}{2\pi^2 \epsilon_0 m^2 c^3 k}$$

S.L. Adler, F. Ramazanoglu, *J. Phys. A* 40, 13395 (2007)

Upper bounds

Spontaneous emission of radiation

Comparison with experimental data

The original CSL models (with the weak value for λ) is ruled out!

In the **mass-proportional** model (noise having a gravitational origin?), one assumes

$$\lambda \rightarrow \lambda \left(\frac{m}{m_N} \right)^2$$

which implies, for example:

$$\frac{d\Gamma_k}{dk} = \frac{e^2 \lambda \hbar}{2\pi^2 \epsilon_0 m^2 c^3 k} \rightarrow \frac{e^2 \lambda \hbar}{2\pi^2 \epsilon_0 m_N^2 c^3 k}$$

TABLE I. Experimental upper bounds and theoretical predictions of the spontaneous radiation by free electrons in Ge for a range of photon energy values.

Energy (keV)	Expt. upper bound (counts/keV/kg/day)	Theory (counts/keV/kg/day)
11	0.049	0.071
101	0.031	0.0073
201	0.030	0.0037
301	0.024	0.0028
401	0.017	0.0019
501	0.014	0.0015

Q. Fu, *Phys. Rev. A* 56, 1806 (1997)

Compatibility is restored

Upper bounds

Spontaneous emission of radiation

Current upper bound on the *mass proportional CSL model*, coming from spontaneous X-ray emission

So far, this is the strongest known upper bound.

If one takes non-white noises into account (non-Markovian dynamics)

$$\left.\frac{d\Gamma_k}{dk}\right|_{\text{colored}} = \gamma(\omega_k) \left.\frac{d\Gamma_k}{dk}\right|_{\text{white}}$$

	Distance (orders of magnitude) from the standard CSL value	Distance (orders of magnitude) from the enhanced value
Spontaneous X-ray emission from Ge	**6**	**-2**

γ = Fourier transform of the correlation function of the noise.

S.L. Adler, F. Ramazanoglu, *J. Phys. A* 40, 13395 (2007)

Cutoff at frequencies ~ 10^{18} s^{-1} sufficient for compatibility with known data

S.L. Adler, F. Ramazanoglu, ibid.

Cutoff at frequencies c/r_C ~ 10^{15} s^{-1}

A. Bassi and G.C. Ghirardi, *Phys. Rep.* 379, 257 (2003)

Upper bounds on the parameter λ

Laboratory experiments	Distance (in orders of magnitude) from standard CSL value	Cosmological data	Distance (in orders of magnitude) from standard CSL value
Fullerene diffraction experiments	**3-4**	Dissociation of cosmic hydrogen	**9**
Decay of supercurrents (SQUIDs)	**6**	Heating of Intergalactic medium (IGM)	**0**
Spontaneous X-ray emission from Ge	**-2**	Heating of protons in the universe	**4**
Proton decay	**10**	Heating of Interstellar dust grains	**7**

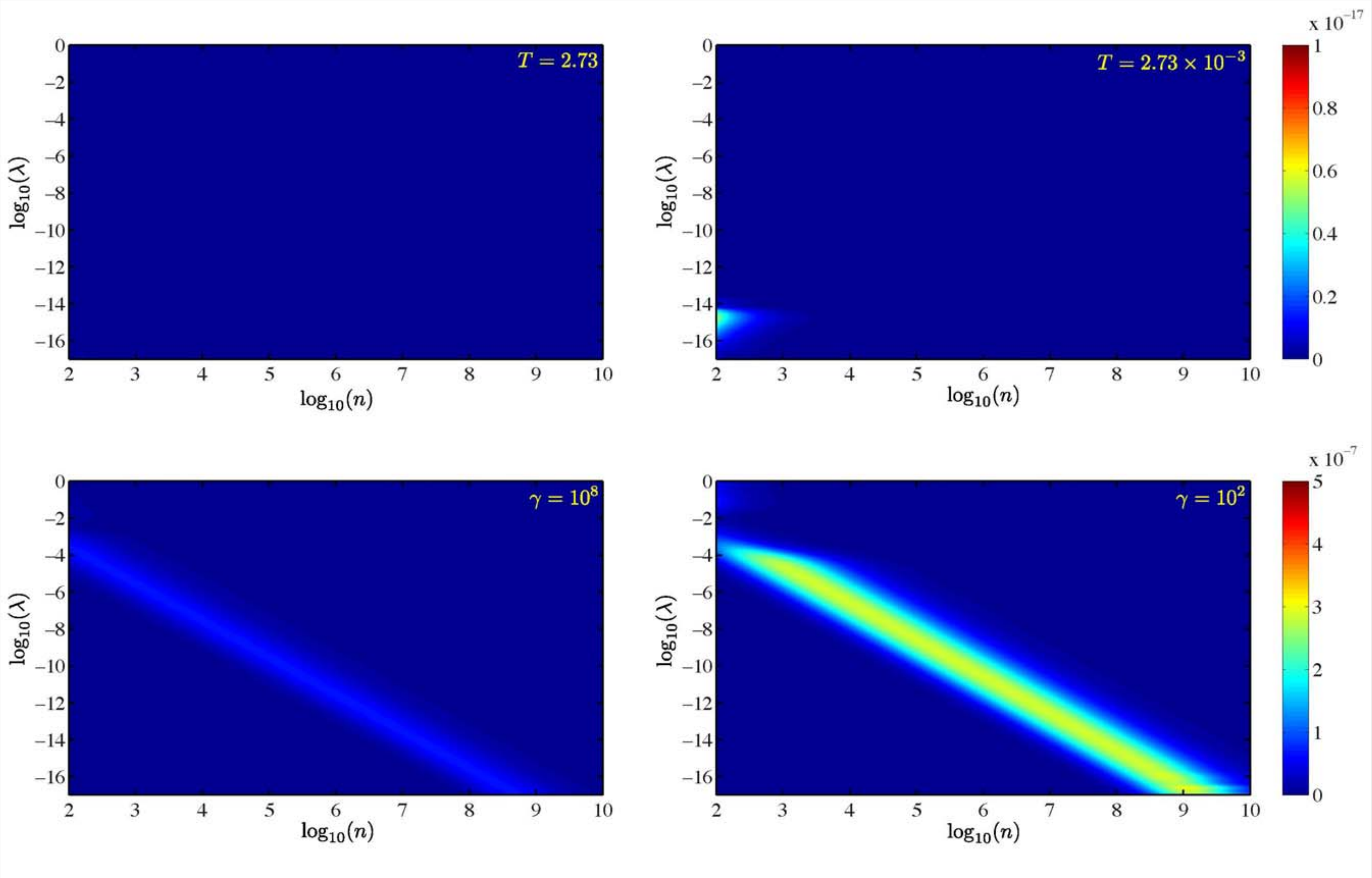
S.L. Adler and A. Bassi, *Science* 325, 275 (2009)

➡ **Present day technology allows for crucial tests.**

4. A cosmological noise field?

	Markovian models (white noise) **All frequencies appear with the same weight**	**non-Markovian models (colored noise)** **The noise can have an arbitrary spectrum**
Models without dissipation (q-coupling) **Only the noise acts on the wave function**	**GRW / CSL** **QMUPL** L. Diosi, *Phys. Rev. A* 40, 1165 (1989).	**non-Markovian CSL** P. Pearle, in *Perspective in Quantum Reality* (1996) S.L. Adler and A. Bassi, *Journ. Phys. A* 41, 395308 (2008). arXiv: 0807.2846 **non-Markovian QMUPL** A. Bassi and L. Ferialdi, *arXiv: 0901.1254*
Models with dissipation ([q+ip]-coupling) **Noise and wave function act on each other**	**Thermal QMUPL model** A. Bassi, E. Ippoliti and B. Vacchini, *J. Phys. A* 38, 8017 (2005). ArXiv: quant-ph/0506083	? **The "true" model?**

Comparison between models



Conclusion

Two messages:

1. **Threshold micro-macro (quantum-classical) for 10^4-10^5 particles**

   Present-day technology allows for crucial tests of the superposition principle.

   Collapse models provide quantitative estimates.

2. **A random cosmological field with “typical” features for temperature and spectrum can induce an efficient collapse of the wave function**

   The collapse as a physical process, caused a background cosmological field

   Underlying deeper lever theory?

Open questions

1. Collapse models assume the existence of a **random field filling space.** What is the origin of such a field? Does it have a **gravitational** nature? Can it be connected e.g. to **dark energy/matter?**

   (S.L. Adler and A. Bassi: *J. Phys. A* 41, 395308, 2008)

2. The coupling between the random field and the wave function is **anti-Hermitian**: what is the origin of this non-standard coupling? Could it be cosmological?

3. Collapse models appear as **phenomenological models of an underlying pre-quantum theory**: what does this theory look like?

   (Adler, "Quantum Theory as an Emergent Phenomenon", C.U.P. 2004)

4. What are the most promising **experiments**, which can detect possible violations of quantum mechanics, as predicted by collapse models?

   (*Sience*, 1st July issue, 2005)

A dedicated experiment

Spontaneous emission of radiation

Spontaneous X-ray emission from Ge offers the strongest upper bound.

This suggests that a **dedicated experiment** which tests collapse models, thus the superposition principle of Quantum Mechanics, should look in this direction.

Main difficulty: one needs to isolate the experimental setup very well.

Solution: underground experiment.

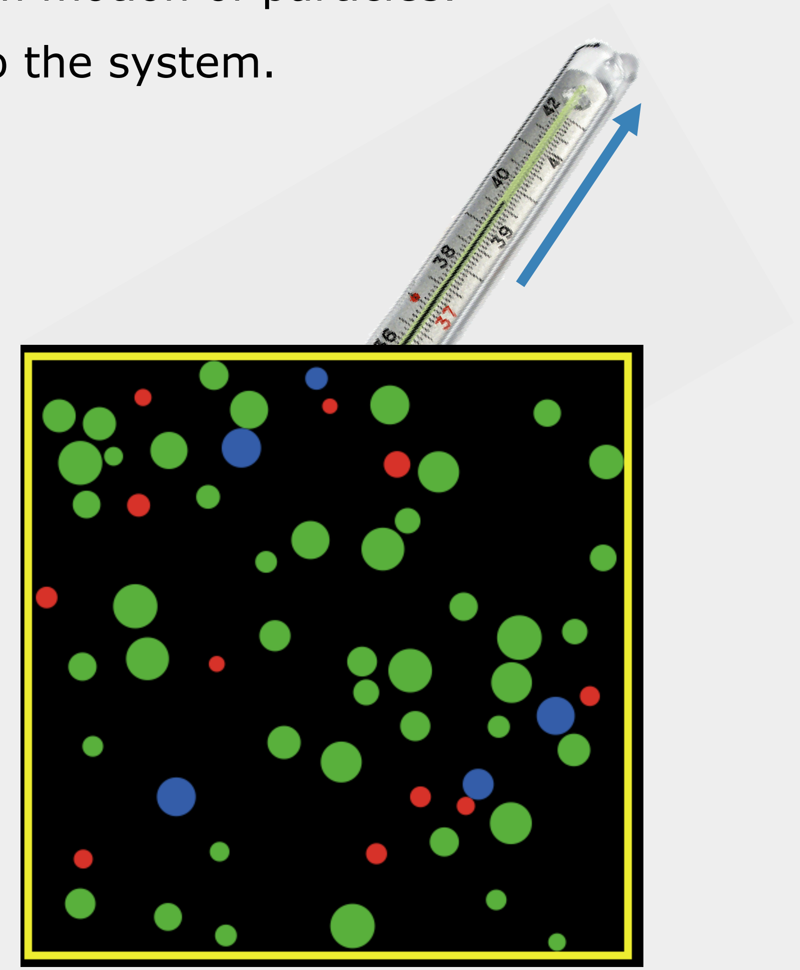
Collaboration with the INFN-LNF laboratories in Frascati, which have also underground facilities (Gran Sasso).

Upper bounds

Energy non-conservation

The stochastic terms induce a random motion of particles.

The noise pumps energy into the system.



For one nucleon (GRW's value)

$$\frac{dE}{dt} = \frac{\lambda \alpha \hbar^2}{4m} \simeq 10^{-25} \text{eV s}^{-1}$$

$$1 \text{ eV increase in } 10^{18} \text{ yr}$$

For a gas (GRW's value)

$$\text{Temperature increase: } 10^{-15} \text{ K/yr}$$

G.C. Ghirardi, A. Rimini, T. Weber, *Phys. Rev. D* 34, 470 (1986)

Upper bounds

Energy non-conservation

Cosmological observations

The smart thing to do is to look at large structures in the universe.

The larger the system, the bigger the spontaneous-collapse effect.

So far, cosmological data are compatible with collapse models.

S.L. Adler, *Jour. Phys. A* 40, 2935 (2007), arXiv:quant-ph/0605072

Cosmological data	Distance (orders of magnitude) from the standard CSL value	Distance (orders of magnitude) from the enhanced value
Dissociation of cosmic hydrogen	17	9
Heating of the Intergalactic medium (IGM)	8	0
Heating of protons in the universe	12	4
Heating of Interstellar dust grains	15	7