

# Coherence & Decoherence in Ultrafast Molecular Processes

Non-Markovian Approaches based upon a  
Hierarchical Effective-Mode Decomposition

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# Topics

- 1 Ultrafast Processes in Molecular Systems
  - Hierarchically Structured Environments
  - Photochemistry and Conical Intersections
  - Energy and Charge Transfer: Examples

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- ① Ultrafast Processes in Molecular Systems
  - Hierarchically Structured Environments
  - Photochemistry and Conical Intersections
  - Energy and Charge Transfer: Examples
- ② Model Hamiltonians & Quantum Dynamics
  - Vibronic Coupling and Lattice Models
  - Quantum Dynamics Approaches
  - Effective-Mode Models

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## ① Ultrafast Processes in Molecular Systems

Hierarchically Structured Environments  
Photochemistry and Conical Intersections  
Energy and Charge Transfer: Examples

## ② Model Hamiltonians & Quantum Dynamics

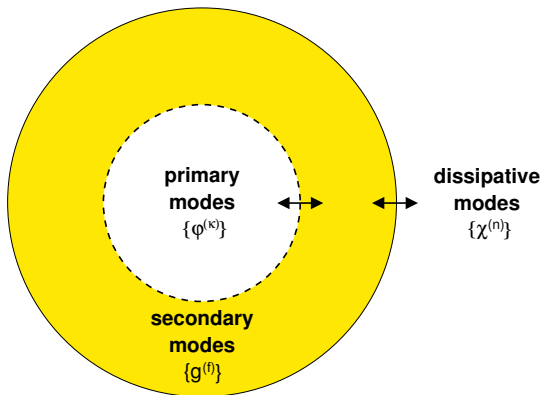
Vibronic Coupling and Lattice Models  
Quantum Dynamics Approaches  
Effective-Mode Models

## ③ Implications of Effective-Mode Picture

Spectral Densities  
Connection to Reduced-Dynamics Approach  
Transfer Processes in Fluctuating Environments

## Dynamics & Reactivity in Molecular Systems

- **often markedly non-Markovian**: subsystem and bath (solvent, cluster, protein environment) evolve on similar time scales
- **primary zone**: solute or chromophore carrying initial excitation
- **secondary zone**: “first solvent shell”, i.e., zone where system-bath correlations are dominant
- **dissipative zone**: external bath which can often be described by Markovian models



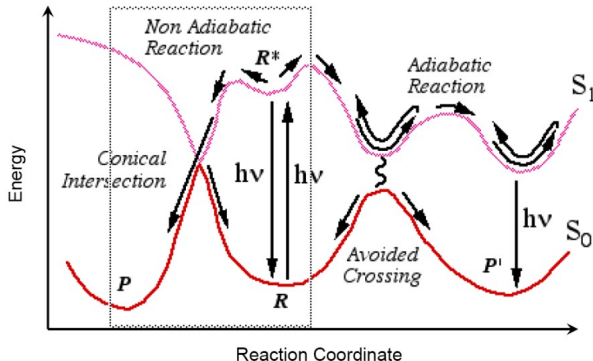
## Effective-Mode Picture, in a Nutshell

- construct **effective, Brownian oscillator type modes** for secondary zone:

$$\hat{H}_{SB} = \hat{h}_S \sum_i c_n \hat{x}_n \longrightarrow c_{\text{eff}} \hat{h}_S \hat{X}_{\text{eff}} + \text{eff-residual bath coupling}$$

- carry out explicit dynamics in the **augmented space of primary + effective/secondary modes**
- **approximate dissipative effects exerted by residual modes** – in the simplest case, by Ohmic friction, or else by **truncated Mori chains**
- by-product of the analysis: **hierarchy of reduced spectral densities**
- **non-Markovian equations for the primary subspace** can be formulated – but are not the most practical way to proceed
- **This approach is most useful for processes where short-time dynamical effects dominate, e.g., in photochemistry**

# Photochemistry: Femtosecond Events in Excited States

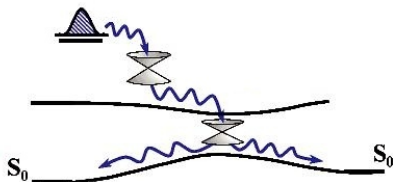


A. Piryatinski, <http://cnls.lanl.gov/External/people/AndreiPiryatinski.php>

- ultrafast processes, non-exponential decays, interfering dynamical pathways
- many processes require (non-adiabatic) quantum dynamics
- direct quantum propagation is often the method of choice

## Landmark Topology: Conical Intersections (Coln's)

Coln = photochemical funnel



Schultz et al., J. Am. Chem. Soc. 125, 8098 (2003)

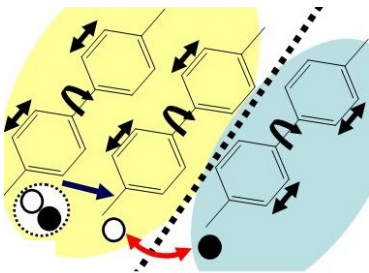
- Double cone topology at degeneracy
- Coln topology highly anharmonic
- Extreme breakdown of the Born-Oppenheimer approximation
- Ultrafast decay (fs to ps scale)
- Coln's are ubiquitous (Truhlar/Mead: "Principle of non-rareness of Coln's")
- Polyatomic molecules; Jahn-Teller effect in solids

Köppel, Domcke, Cederbaum, Adv. Chem. Phys. 57, 59 (1984)  
*Conical Intersections*, Eds. Yarkony, Köppel, Domcke (2004)



## Photochemistry of “Complex” Systems

- polyatomic molecules
- solute-solvent systems
- biological chromophores & photoswitches
- extended, multi-chromophoric systems
- molecular nano-scale assemblies



- delocalized excitations
- ultrafast processes (fs–ps)
- quantum coherence & decoherence (vibrational and electronic)
- special topologies, e.g., conical intersections

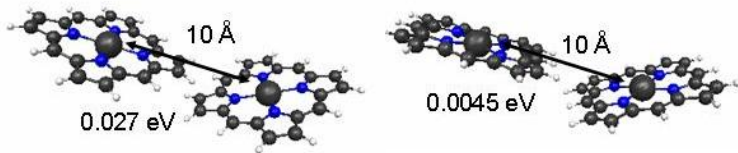
## Excitons, EET, Charge Transfer

- exciton formation, delocalization & trapping
  - Frenkel excitons:  $|\Psi_{\text{exciton}}\rangle = \sum_n^{\text{exc}} c_n |\Phi_n\rangle$  with  $n_{\text{exc}} \sim 5-10$ , where  $|\Phi_n\rangle =$  configuration with the  $n$ th monomer excited
  - trapping due to exciton-phonon interactions
- excitation energy transfer (EET, exciton migration)<sup>1</sup>
  - coherent vs. non-coherent (Förster limit) transfer
  - examples: EET in semiconducting polymers, light-harvesting systems, DNA, carbon nanotube (CNT) and quantum dot (QD) assemblies
- charge separation/transfer (exciton dissociation)<sup>1</sup>
  - examples: photovoltaic materials, e.g., semiconducting polymers, CNT-porphyrin assemblies, photosynthetic reaction center

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<sup>1</sup>often involve non-adiabatic dynamics

## Example 1: Excitation Energy Transfer (EET)



Tamura, Mallet, Oheim, Burghardt, J. Phys. Chem. C, 113, 7548 (2009)

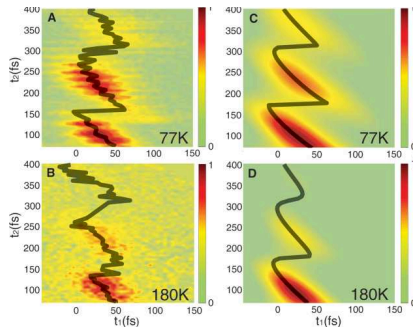
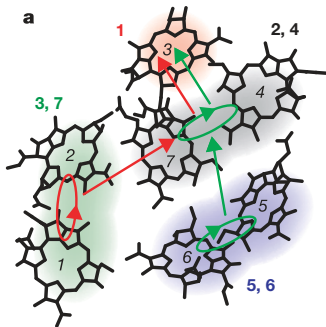
$$H = \sum_i \frac{\omega_i}{2} \left( p_i^2 + x_i^2 \right) + \begin{pmatrix} \kappa_i^{(1)} x_i^{(1)} & V_{12}^{\text{Coulomb}} \\ V_{12}^{\text{Coulomb}} & \kappa_i^{(2)} x_i^{(2)} \end{pmatrix}$$

$$V_{12}^{\text{Coulomb}} = \int d\mathbf{r}_D d\mathbf{r}_A \frac{\rho_D^{(eg)}(\mathbf{r}_D) \rho_A^{(ge)}(\mathbf{r}_A)}{|\mathbf{r}_D - \mathbf{r}_A|}$$

- inter-monomer couplings via **transition densities**
- generalization of Förster rate theory & transition dipole approximation
- coherent regime: photosynthesis, artificial light-harvesting systems

# “Coherence Dynamics in Photosynthesis: Protein Protection of Excitonic Coherence”

Lee, Cheng, Fleming, Science 316, 1462 (2007)

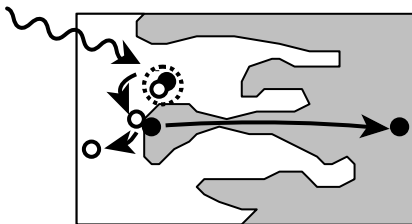


- one would expect an extremely rapid dephasing (decoherence):  $\tau_{\text{dec}} < 50$  fs
- but observed coherence lifetimes are  $\sim 600$  fs to 1 ps

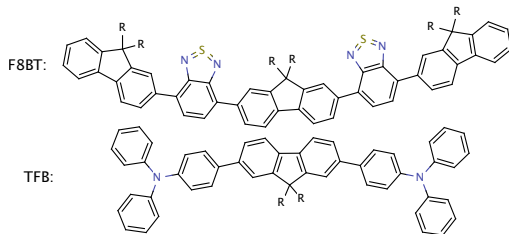
## Example 2: Exciton Dissociation at a Polymer Interface (Heterojunction)

exciton = electron + hole

→ photovoltaic devices, organic light-emitting diodes (OLED's), ...

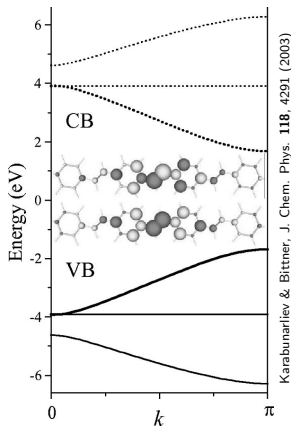


Peumans, Uchida, Forrest, Nature **125**, 8098 (2003)

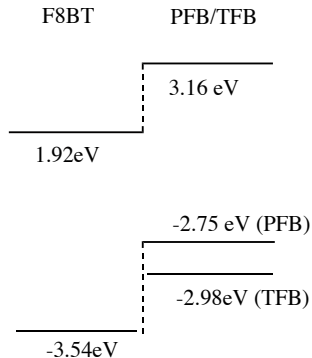


molecular-level understanding of interactions & dynamics  
at the polymer interface is required

## Zeroth-Order Picture of a Heterojunction



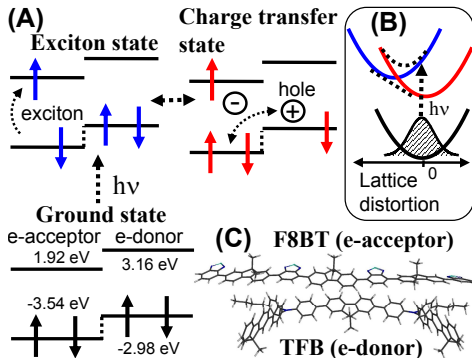
polymer/polymer interface:



- HOMO/LUMO  $\leftrightarrow$  valence/conduction band
- 1st bound excited state: singlet exciton ( $^1B_u^-$  in PPV); Frenkel type exciton
- @junction: compare band offset vs. exciton binding energy ( $\epsilon_B \sim 0.5$  eV)

## Objective: More Detailed Perspective of Ultrafast Events

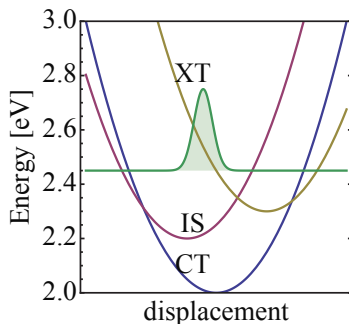
TFB:F8BT  
heterojunction



Tamura, Bittner, Burghardt, JCP 126, 021103 (2007)

- initial photogeneration of an exciton state (XT, bright state)
- exciton decay to an interfacial charge transfer state (CT, exciplex)
- the XT  $\rightarrow$  CT transition is mediated by electron-phonon coupling

## 3-State Electron-Phonon Coupling Model



parameterization for TFB:F8BT:  
 polymer lattice model based on dimer;  
 TDDFT and semi-empirical (PM3) calculations  
 + Wannier-function representation

Bittner et al., JCP 122, 214719 (2005)

$$H = \sum_i^{N \sim 30} H_i = \sum_i \frac{\omega_i}{2} (p_i^2 + x_i^2) + V_i^{\text{lin}}$$

$$V_i^{\text{lin}} = \begin{pmatrix} \kappa_i^{(1)} x_i & \lambda_i^{(12)} x_i & \lambda_i^{(13)} x_i \\ \lambda_i^{(12)} x_i & \kappa_i^{(2)} x_i & \lambda_i^{(23)} x_i \\ \lambda_i^{(13)} x_i & \lambda_i^{(23)} x_i & \kappa_i^{(3)} x_i \end{pmatrix}$$

state 1 = exciton (XT) state

state 2 = charge transfer (CT) state

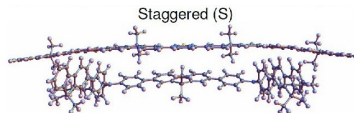
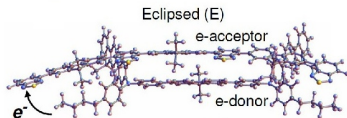
state 3 = intermediate (IS) state

phonons = C=C stretch + ring torsions

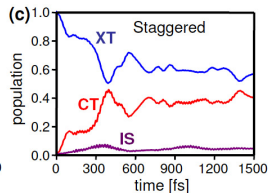
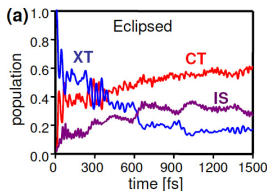
Tamura, Ramon, Bittner, Burghardt, J. Phys. Chem. B 112, 10269 (2008)



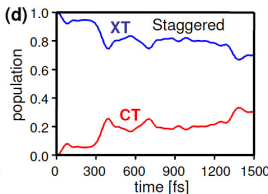
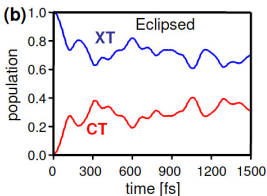
# Quantum Dynamics of Exciton Dissociation



3-state



2-state



- 3-state 28-mode model
- MCTDH calculations
- sample over relevant interface configurations
- intermediate states play a key role
- qualitative agreement with time-resolved photoluminescence

Tamura, Ramon, Bittner, Burghardt,  
 Phys. Rev. Lett. **100**, 107402 (2008)

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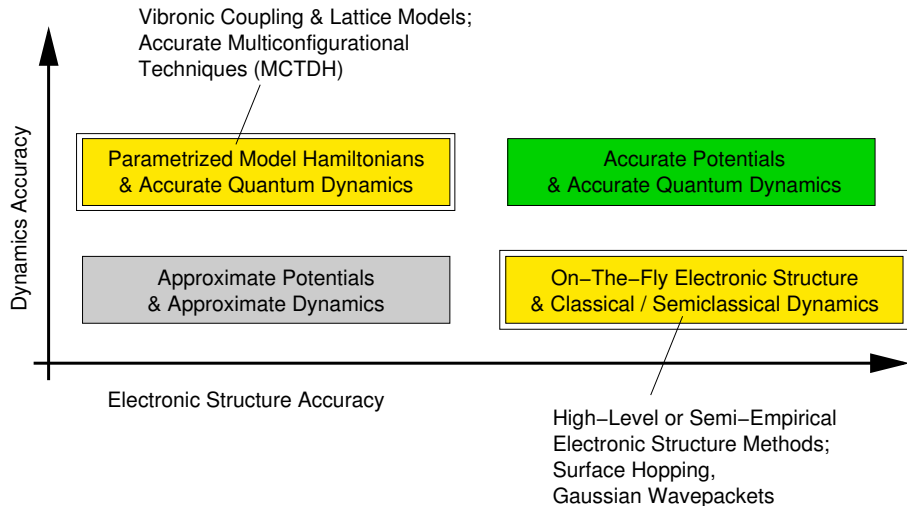
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## Methods for “Large” Systems



# Model Hamiltonians

- multi-mode vibronic coupling models (quasi-diabatic)

Taylor expansion around reference geometry (Coln; FC point)

$$H = (T_N + V_0)\mathbf{1} + V$$

$$V_{nn'} = \epsilon_{nn'} + \sum_i \xi_i^{nn'} x_i + \sum_{i,j} \gamma_{ij}^{nn'} x_i x_j + \dots$$

Köppel, Domcke, Cederbaum, Adv. Chem. Phys. (1984)

- lattice models

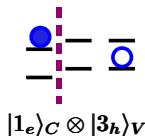
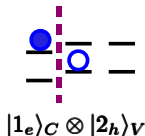
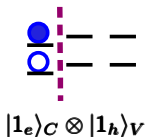
site-site interactions + electron(exciton)-phonon coupling, e.g.,

$$H = \sum_n E_0 a_n^\dagger a_n - J(a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n) + \frac{p_n^2}{2M} + \frac{W}{2}(q_{n+1} - q_n)^2 + \chi a_n^\dagger a_n (q_{n+1} - q_n)$$

A. S. Davydov, Phys. Scr. 20, 387 (1979), Groves & Silbey, J. Chem. Phys. 52, 2099 (1970)

- in both cases: *ab initio* or semiempirical parametrization

## Electronic Structure – Lattice Model



- singly-excited electron-hole ( $e-h$ ) config's:  $|\mathbf{n}\rangle = |n_e \bar{n}'_h\rangle = |n_e\rangle_C \otimes |\bar{n}'_h\rangle_V$  (Wannier function basis)
- set up Hamiltonian in this basis:

$$H_{\text{el}} = \sum_{\mathbf{mn}} \left( F_{\mathbf{mn}} + V_{\mathbf{mn}} \right) a_{\mathbf{m}}^{\dagger} a_{\mathbf{n}}$$

- creation and annihilation op's for  $e-h$  pairs:  $a_{\mathbf{n}}^{\dagger}|0\rangle = |\mathbf{n}\rangle = |n_e \bar{n}'_h\rangle$
- $F_{\mathbf{mn}}/V_{\mathbf{mn}}$  = single/two-particle matrix elements

# Lattice Model Including Electron-Phonon Coupling

$$H = H_{\text{el}} + H_{\text{el-ph}} + H_{\text{ph}}$$

$$= \sum_{\mathbf{nm}} (F_{\mathbf{mn}} + V_{\mathbf{mn}}) a_{\mathbf{m}}^{\dagger} a_{\mathbf{n}} + \sum_{\mathbf{nm}} \sum_{k\alpha} \left( \frac{\partial F_{\mathbf{mn}}}{\partial q_{k\alpha}} \right) a_{\mathbf{m}}^{\dagger} a_{\mathbf{n}} q_{k\alpha} + \sum_{\ell\alpha} \frac{1}{2} \left( \omega_{\ell\alpha}^2 q_{\ell\alpha}^2 + p_{\ell\alpha}^2 \right)$$

↓  
 diagonalize  $H_0 = H_{\text{el}} + H_{\text{ph}}$   
 & select typically 2-3 lowest  
 states

$$H = \sum_i 1/2 \left( \omega_i^2 x_i^2 + p_i^2 \right) + V_i^{\text{lin}}$$

 where 
$$V_i^{\text{lin}} = \begin{pmatrix} \kappa_i^{(1)} x_i & \lambda_i^{(12)} x_i & \lambda_i^{(13)} x_i \\ \lambda_i^{(12)} x_i & \kappa_i^{(2)} x_i & \lambda_i^{(23)} x_i \\ \lambda_i^{(13)} x_i & \lambda_i^{(23)} x_i & \kappa_i^{(3)} x_i \end{pmatrix}$$

Karabunarliev & Bittner, J. Chem. Phys. **118**, 4291 (2003), Groves & Silbey, J. Chem. Phys. **52**, 2099 (1970)

# Linearized Models & Con Topology\*

$$V_{\text{Con}}(x_t, x_c) = V_0(x_t^0, x_c^0)$$

$$+ \begin{pmatrix} \kappa^{(1)} \Delta x_t & \lambda \Delta x_c \\ \lambda \Delta x_c & \kappa^{(2)} \Delta x_t \end{pmatrix}$$

$$\Delta x_t = x_t - x_t^0$$

tuning mode

$$\Delta x_c = x_c - x_c^0$$

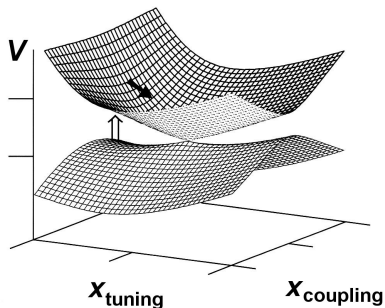
coupling mode

2 dimensions: Con point

3 dimensions: Con seam

N dimensions: (N-2) dimensional  
 intersection space

- \* quasi-diabatic linear vibronic coupling (LVC) form
  - \* can be embedded in a correct representation of the overall potential via regularized diabatic states
- Köppel et al., J. Chem. Phys. **110**, 9371 (1999); **115**, 2377 (2001)



## Quantum Dynamics Approaches

- ① explicit, multidimensional dynamics for the full system + bath space:  
 wavefunction  $\psi_{SB}(t)$  or density operator  $\hat{\rho}_{SB}(t) = \sum_n p_n |\psi_{n,SB}(t)\rangle \langle \psi_{n,SB}(t)|$
- ② reduced dynamics (master equation) methods:  $\hat{\rho}_S(t) = \text{Tr}_B \hat{\rho}_{SB}(t)$
- ③ intermediate methods: explicit treatment of **subsystem + effective-mode ( $E$ ) part of the bath** + master equation for residual ( $R$ ) bath:

$$\begin{aligned} \frac{\partial \hat{\rho}_{SE}}{\partial t} &= -\frac{i}{\hbar} [\hat{H}_{SE}, \hat{\rho}_{SE}(t)] + \hat{L}_{\text{diss}}^{(R)} \hat{\rho}_{SE}(t) \quad ; \quad \hat{\rho}_{SE}(t) = \text{Tr}_R \hat{\rho}_{SER}(t) \\ \hat{L}_{\text{diss}}^{(R)} \hat{\rho}_{SE} &= -i \frac{\gamma}{\hbar} [\hat{X}_E, [\hat{P}_E, \hat{\rho}_{SE}]_+] - \frac{2\gamma M k T}{\hbar^2} [\hat{X}_E, [\hat{X}_E, \hat{\rho}_{SE}]] \end{aligned}$$

- efficient multiconfigurational methods for  $S + E$ , suitable for 10-100 modes
- Caldeira-Leggett type master equations for residual ( $R$ ) modes



# Multiconfigurational Methods (MCTDH & Co)

**dissipative  
modes**  
 $\{\chi^{(n)}\}$



**primary  
modes**  
 $\{\varphi^{(\kappa)}\}$

**secondary  
modes**  
 $\{g^{(f)}\}$

$$\Psi(r, t) = \sum_J A_J(t) \Phi_J(r, t)$$

$$\text{with } \Phi_J(r, t) = \prod_{\kappa=1}^M \varphi_{j_\kappa}^{(\kappa)}(r_\kappa, t)$$

Multi-Configuration Time-Dependent Hartree

Meyer et al., CPL **165**, 73 (1990), Beck et al., Phys. Rep. **324**, 1 (2000)

Gaussian-based hybrid method: G-MCTDH

$$\Phi_J(r, t) = \underbrace{\prod_{\kappa=1}^M \varphi_{j_\kappa}^{(\kappa)}(r_\kappa, t)}_{\text{primary nodes}} \underbrace{\prod_{\kappa=M+1}^P g_{j_\kappa}^{(\kappa)}(r_\kappa, t)}_{\text{secondary modes}}$$

Burghardt, Meyer, Cederbaum, JCP **111**, 2927 (1999)

Burghardt, Giri, Worth JCP **129**, 174104 (2008)

## Variationally Optimized Dynamics

$$\Psi(r_1, \dots, r_P, t) = \sum_{j_1} \dots \sum_{j_P} A_{j_1 \dots j_P}(t) \prod_{\kappa=1}^M \phi_{j_\kappa}^{(\kappa)}(r_\kappa, t) \prod_{\kappa=M+1}^P g_{j_\kappa}^{(\kappa)}(r_\kappa, t)$$

$$g_j^{(\kappa)}(r_\kappa, t) = \exp \left[ r_\kappa \cdot a_j^{(\kappa)}(t) \cdot r_\kappa + \xi_j^{(\kappa)}(t) \cdot r_\kappa + \eta_j^{(\kappa)}(t) \right]$$

multidimensional Gaussian functions:

- quasi-classical motion
- on-the-fly dynamics
- analytical integrals

Dirac-Frenkel **variational principle**:

$$\langle \delta \Psi | H - i \frac{\partial}{\partial t} | \Psi \rangle = 0 \longrightarrow \text{dynamical equations (symplectic structure)}$$

- up to 50-100 modes – exponential scaling problem (  $\sim fN^{f+1}$  ) is alleviated

# Dynamical Equations

Burghardt, Meyer, Cederbaum, JCP 111, 2927 (1999)

coefficients:

$$i\dot{A} = S^{-1} \left[ H - i\tau \right] A$$

spf's (primary modes):

$$i\dot{\phi}^{(\kappa)} = \left( \hat{1} - \hat{P}^{(\kappa)} \right) \left[ \rho^{(\kappa)} \right]^{-1} \hat{H}^{(\kappa)} \phi^{(\kappa)}$$

gwp's (secondary modes):

$$i\dot{\Lambda}^{(\kappa)} = \left[ C^{(\kappa)} \right]^{-1} Y^{(\kappa)}$$

$$C_{j\alpha, l\beta}^{(\kappa)} = \rho_{jl}^{(\kappa)} \left\langle \frac{\partial g_j^{(\kappa)}}{\partial \lambda_{j\alpha}^{(\kappa)}} \left| (\hat{1} - \hat{P}^{(\kappa)}) \right| \frac{\partial g_l^{(\kappa)}}{\partial \lambda_{l\beta}^{(\kappa)}} \right\rangle \quad ; \quad Y_{j\alpha}^{(\kappa)} = \sum_l \left\langle \frac{\partial g_j^{(\kappa)}}{\partial \lambda_{j\alpha}^{(\kappa)}} \left| (\hat{1} - \hat{P}^{(\kappa)}) \hat{H}_{jl}^{(\kappa)} \right| g_l^{(\kappa)} \right\rangle$$

- evolution under multiconfigurational mean-field Hamiltonian
- coupled, variational equations for Gaussian parameters
- correlations between primary vs. secondary subspace

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## Reduced-Dimensional Models for Large Systems

e.g.  $N$  secondary-zone modes coupled to electronic subsystem (primary zone)

$$H = \sum_{i=1}^N H_0^{(i)} + \begin{pmatrix} \kappa_i^{(1)} x_i & \lambda_i x_i \\ \lambda_i x_i & \kappa_i^{(2)} x_i \end{pmatrix} \xrightarrow{?} H_{\text{eff}} = \sum_{i=1}^{n_{\text{eff}}} \tilde{H}_0^{(i)} + \begin{pmatrix} K_i^{(1)} X_i & \Lambda_i X_i \\ \Lambda_i X_i & K_i^{(2)} X_i \end{pmatrix}$$

$N$  potentially very large!

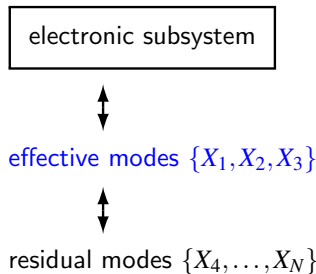
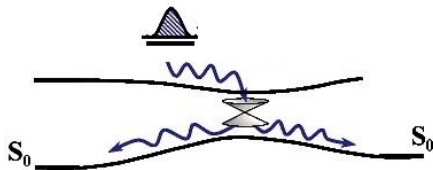
$n_{\text{eff}}$  small

- approximation should be valid on short time scales
- $X_i$ 's are collective modes – “generalized reaction coordinates”

# Conical Intersection (CoIn): Three Effective Modes

Precursors in solid state physics:  
 “cluster modes”  
 “interaction modes”

(O'Brien 1971, Toyozawa, Inoue 1966)

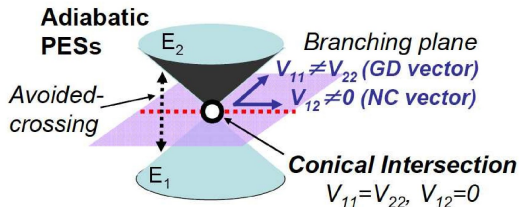


- effective modes describe short-time dynamics exactly
- for  $n$  electronic states:  
 $n(n+1)/2$  effective modes

Cederbaum, Gindensperger, Burghardt, Phys. Rev. Lett., **94**, 113003 (2005)  
 Burghardt, Gindensperger, Cederbaum, Mol. Phys. **104**, 1081 (2006)  
 Gindensperger, Burghardt, Cederbaum, J. Chem. Phys. **124**, 144104, 144105 (2006)

## Why **Three** Effective Modes?

$$H = \sum_{i=1}^N H_i = \sum_{i=1}^N \frac{\omega_i}{2} (p_i^2 + x_i^2) + V_i^{\text{lin}} \quad V_i^{\text{lin}} = \frac{1}{2} \kappa_i^{(+)} x_i + \begin{pmatrix} \frac{1}{2} \kappa_i^{(-)} x_i & \lambda_i x_i \\ \lambda_i x_i & -\frac{1}{2} \kappa_i^{(-)} x_i \end{pmatrix}$$



$$X_+ = \sum_i \kappa_i^{(+)} x_i \quad \text{shift}$$

$$X_- = \sum_i \kappa_i^{(-)} x_i \quad \text{energy gap}$$

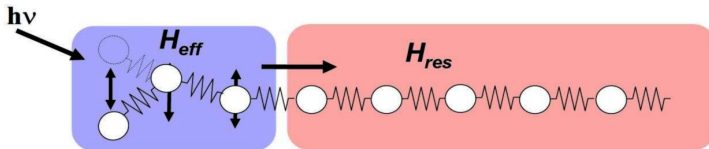
$$X_\Lambda = \sum_i \lambda_i x_i \quad \text{coupling}$$

- $X_-$ ,  $X_\Lambda$  span the branching plane (cf. Atchity & Xantheas & Ruedenberg (1999))
- $X_+$  lies in the  $(N-2)$ -dimensional intersection space

## Additional Transformations in Residual Space

→ Hierarchical Electron-Phonon (HEP) Model:

effective modes + chain(s) of residual modes



- the chain “unravels” the dynamics as a function of time
- more precisely: truncation at the order  $n$  (i.e.,  $3n + 3$  modes) conserves the Hamiltonian moments (cumulants) up to the  $(2n + 3)$ rd order

Tamura, Bittner, Burghardt, JCP **127**, 021103 (2007), Gindensperger, Cederbaum, JCP **127**, 024107 (2007)



# Hierarchical electron-phonon (HEP) model

$$\hat{H}^{(n)} = \hat{H}_{\text{eff}} + \sum_{l=1}^n \hat{H}_{\text{res}}^{(l)}$$

$$\hat{H}_{\text{res}}^{(l)} = \sum_{i=3l+1}^{3l+3} \frac{\Omega_i}{2} (\hat{P}_i^2 + \hat{X}_i^2) \hat{1} + \sum_{i=3l+1}^{3l+3} \sum_{j=i-3}^{i-1} d_{ij} \left( \hat{P}_i \hat{P}_j + \hat{X}_i \hat{X}_j \right) \hat{1}$$

$$\mathbf{d} = \begin{pmatrix} \bullet & \bullet & \bullet & \square & & & & \\ \bullet & \bullet & \bullet & \square & \square & & & \\ \bullet & \bullet & \bullet & \square & \square & \square & & \\ \square & \square & \square & \bullet & \bullet & \bullet & \square & \\ \square & \square & \square & \bullet & \bullet & \bullet & \square & \square \\ \square & \square & \square & \bullet & \bullet & \bullet & \bullet & \square \\ \square & \square & \square & \bullet & \bullet & \bullet & \bullet & \bullet & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

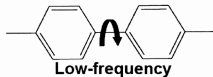
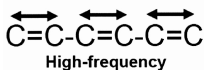
$$\mathbf{d} = \begin{pmatrix} \bullet & \bullet & \bullet & \square & & & & \\ \bullet & \bullet & \bullet & \square & \square & & & \\ \bullet & \bullet & \bullet & \square & \square & \square & & \\ \square & \square & \square & \bullet & \bullet & \bullet & \square & \square & \square & \dots \\ \square & \square & \square & \bullet & \bullet & \bullet & \square & \square & \square & \dots \\ \square & \square & \square & \bullet & \bullet & \bullet & \square & \square & \square & \dots \\ \square & \square & \square & \bullet & \bullet & \bullet & \bullet & & & \\ \square & \square & \square & \bullet & \bullet & \bullet & \bullet & \bullet & & \\ \square & \square & \square & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

chain + Markovian closure

Tamura, Bittner, Burghardt, JCP 126, 021103 (2007); Gindensperger, Köppel, Cederbaum, JCP 126, 034106 (2007)

# Effective Modes for TFB:F8BT Heterojunction

## Vibrational modes



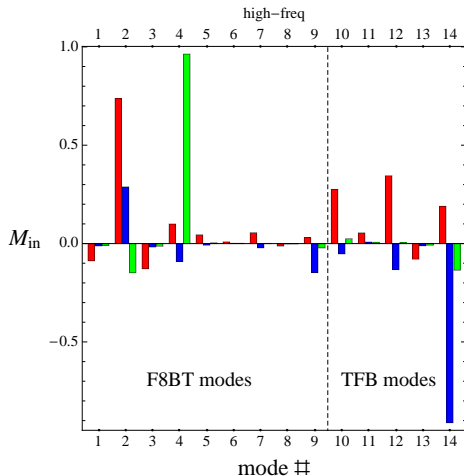
HEP hierarchy / 2-state model:

( $X_1, X_2, X_3$ ): high-frequency

( $X_4, X_5, X_6$ ): low-frequency

( $X_7, X_8, X_9$ ): high-frequency

• phonon branches appear in alternation!

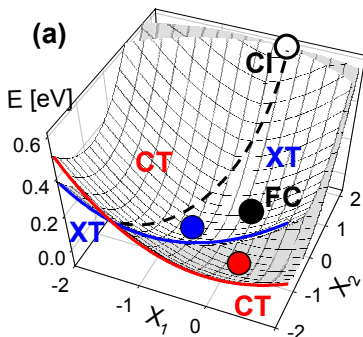


Tamura, Bittner, Burghardt, JCP **126**, 021103 (2007),  
 Pereverzev, Bittner, Burghardt, JCP **131**, 034104 (2009)

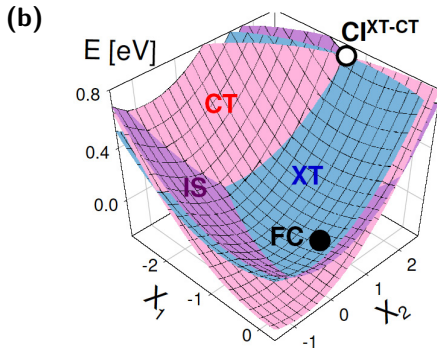
## Nonadiabatic Coupling Region

representation in the branching space coordinates ( $X_1, X_2$ )

2-state XT/CT model



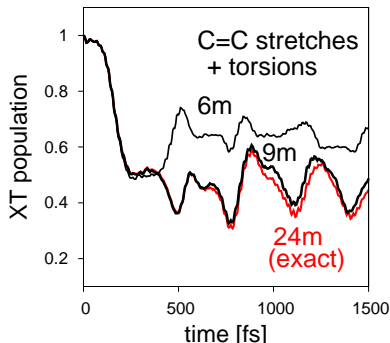
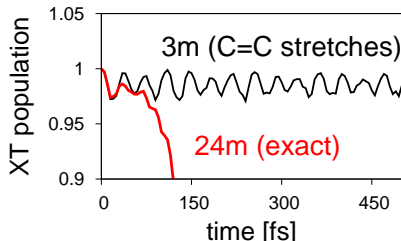
3-state XT/CT/IS model



The dynamics happens “below” a conical intersection

## 2-State Model: Effective-Mode Dynamics (Cont'd)

Tamura, Bittner, Burghardt, JCP **126**, 021103 (2007), JCP **127**, 034706 (2007)

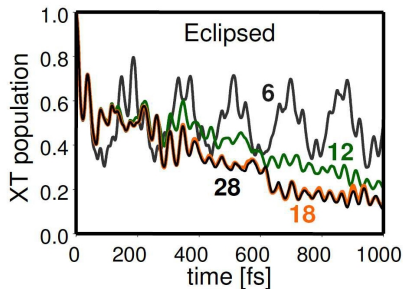


- 3 effective modes: **not** a good approximation on relevant time scale
- 6 effective modes: qualitatively correct  $\rightarrow$  **torsional modes are crucial!**
- 9 effective modes: very close to exact 24-mode result

## 3-State Effective-Mode Model

Tamura, Ramon, Bittner, Burghardt, J. Phys. Chem. B 112, 495 (2008), Phys. Rev. Lett. 100, 107402 (2008)

$$H_{\text{eff}} = H_0 + \sum_{i=1}^6 \begin{pmatrix} K_i^{(1)} X_i + D_i X_i & \Lambda_i^{(12)} X_i & \Lambda_i^{(13)} X_i \\ \Lambda_i^{(12)} X_i & K_i^{(2)} X_i - D_i X_i & \Lambda_i^{(23)} X_i \\ \Lambda_i^{(13)} X_i & \Lambda_i^{(23)} X_i & K_i^{(3)} X_i \end{pmatrix} + \text{bilinear couplings}$$



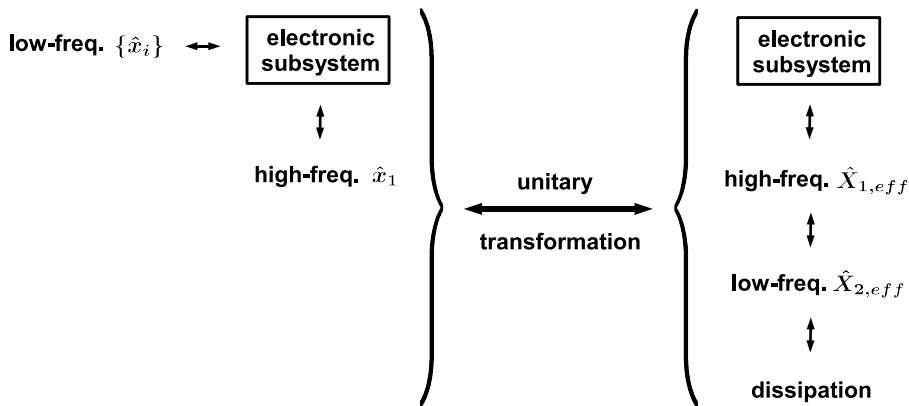
$X_1 \dots X_6$ : high-frequency

$X_7 \dots X_{12}$ : low-frequency

$X_{13} \dots X_{18}$ : high-frequency

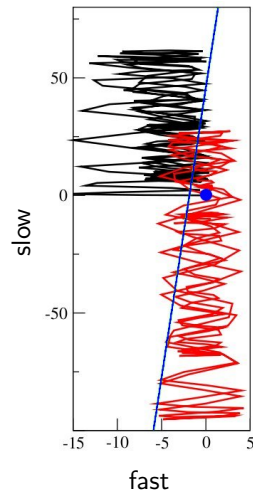
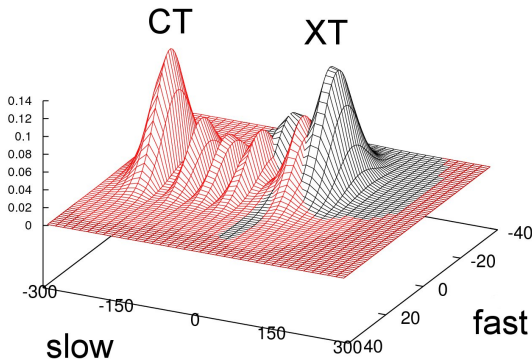
→ same pattern as 2-state!

# Minimal Model: High-Frequency Mode + Low-Frequency Bath



—→ connections to models of electron transfer (Sumi-Marcus etc.)

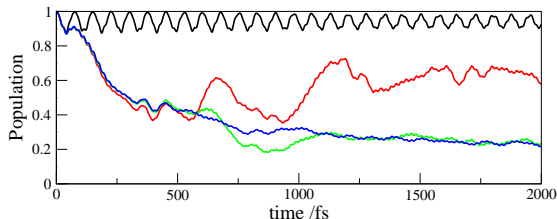
## Why Do the Torsional (Slow) Modes Play a Key Role?



- XT-CT transition induced by low-frequency motion
- location of **avoided-crossing seam** decisive

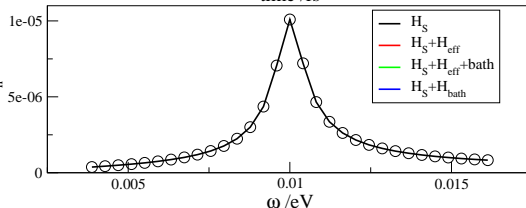
Hughes, Christ, Burghardt, J. Chem. Phys., **131**, 024109 (2009)

# High-Frequency Mode + Eff. Low-Frequency Mode + Dissipation



spectral density

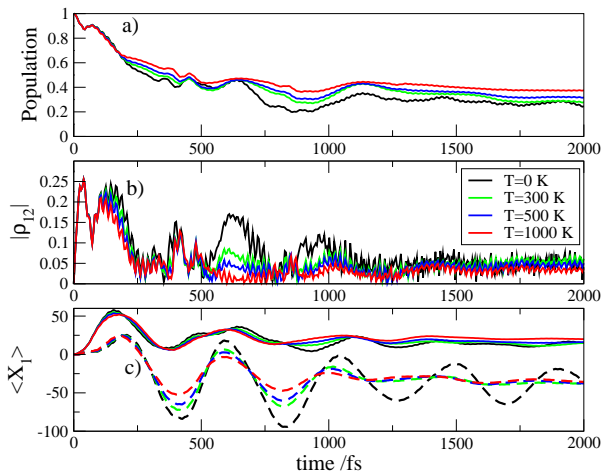
$$J(\omega) = \frac{2\gamma\omega D^2}{(\Omega^2 - \omega^2)^2 + 4\gamma^2\omega^2}$$



- **Brownian-oscillator picture:** effective low-frequency mode + dissipation
- note: very strong friction can quench the transfer



# Electronic Coherence Preserved on Short Time Scales



- electronic coherence evolves concerted with low-frequency motion

## Bottom Line – Vibronic-Coupling Mechanism

- XT-CT transfer mechanism
  - high-frequency modes largely determine coupling to electronic subsystem but exhibit essentially diabatic dynamics
  - low-frequency modes induce XT–CT transition (cf. Bixon-Jortner theory: resonant decay via quasi-continuum of vibronic states)
- coherent nature of the transfer
  - collective modes dominate on short time scales
  - hierarchical electron-phonon model: sequential correlations
  - as a result, coherence can persist on the relevant transfer time scale
  - dissipation acts with a delay
- even though we've studied a model Hamiltonian, these conclusions should have some general validity

# Topics

## ① Ultrafast Processes in Molecular Systems

Hierarchically Structured Environments  
Photochemistry and Conical Intersections  
Energy and Charge Transfer: Examples

## ② Model Hamiltonians & Quantum Dynamics

Vibronic Coupling and Lattice Models  
Quantum Dynamics Approaches  
Effective-Mode Models

## ③ Implications of Effective-Mode Picture

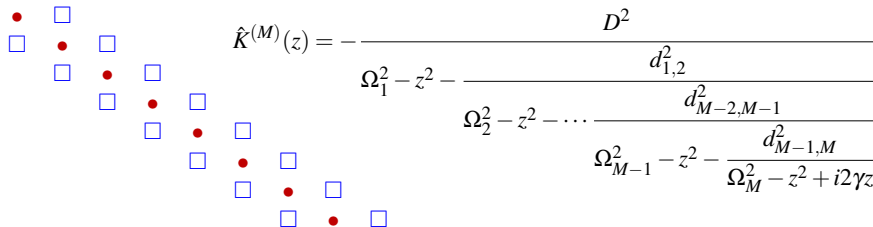
Spectral Densities  
Connection to Reduced-Dynamics Approach  
Transfer Processes in Fluctuating Environments

# Spectral Densities

- generalized Brownian oscillator models
- construct a **series of approximate spectral densities** based upon HEP model
- continued fraction form (cf. Mori chain)

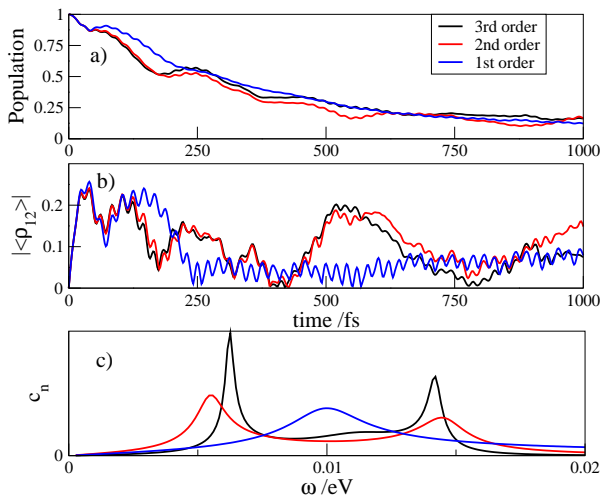
$$J^{(M)}(\omega) = \lim_{\varepsilon \rightarrow 0^+} \text{Im } K^{(M)}(\omega - i\varepsilon)$$

Garg et al., J. Chem. Phys. 83, 4491 (1985), Hughes, Christ, Burghardt, J. Chem. Phys. 131, 024109 (2009)



$$\hat{K}^{(M)}(z) = - \frac{D^2}{\Omega_1^2 - z^2 - \frac{d_{1,2}^2}{\Omega_2^2 - z^2 - \dots - \frac{d_{M-2,M-1}^2}{\Omega_{M-1}^2 - z^2 - \frac{d_{M-1,M}^2}{\Omega_M^2 - z^2 + i2\gamma z}}}}$$

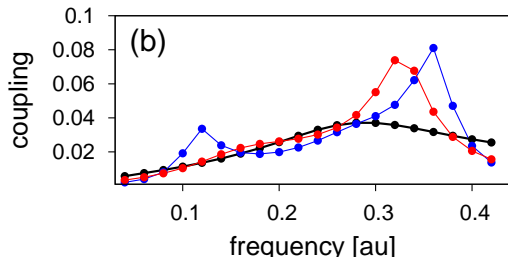
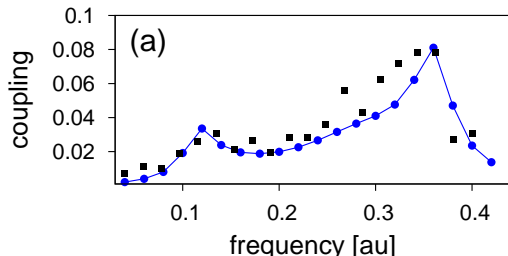
# Hierarchy of Approximate Spectral Densities



- approximation =  $M$ th order Mori chain + Markovian closure
- this is equivalent to an  $M$ th order spectral density
- the environment's spectral density is successively resolved as a function of time

Hughes, Christ, Burghardt, J. Chem. Phys. 131, 024109, 124108 (2009)

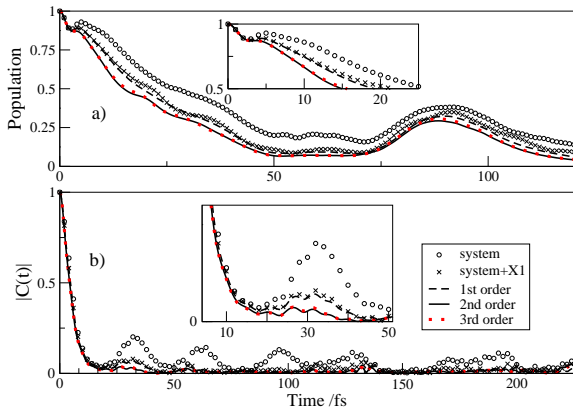
## Spectral Densities, Cont'd



- fit an arbitrary spectral density to a  $K$ th order Mori chain model
- construct successive  $M$ th order,  $M < K$ , approximations
- here,  $K = 3$ ,  $M = 2$ ,  $M = 1$
- dynamics according to  $M$ th order Mori chain with Markovian closure

## $S_2$ - $S_1$ CoIn in Pyrazine: 4+20 Mode Model

tuning-mode bath,  $H_{\text{int}} = \sum_{i=1}^{N_B} \kappa_i^{(-)} x_i \sigma_z \equiv K X_1 \sigma_z$ : 1 effective mode per order  $M$



- 20-mode bath fitted to  $K = 3$  Mori chain + Markovian closure
- construct successive  $M$ th order,  $M < K$ , approximations
- here,  $M = 2, 3$  in very good agreement

## Coln's: Extension to Correlated Spectral Densities

$$\hat{H} = \hat{H}_0 + \hat{H}_{SB} = \hat{H}_0 + \sum_{i=1}^{N_B} \left[ \kappa_{B,i}^{(+)} \hat{x}_{B,i} \hat{1} + \kappa_{B,i}^{(-)} \hat{x}_{B,i} \hat{\sigma}_z + \lambda_{B,i} \hat{x}_{B,i} \hat{\sigma}_x \right]$$

$$\mathbf{J}(\omega) = - \sum_{n=1}^{N_B} \begin{pmatrix} -\kappa_n^{(-)2} & -i\kappa_n^{(-)}\kappa_n^{(+)} & \kappa_n^{(-)}\lambda_n \\ i\kappa_n^{(-)}\kappa_n^{(+)} & -(\kappa_n^{(-)2} + \lambda_n^2) & -i\lambda_n\kappa_n^{(+)} \\ \kappa_n^{(-)}\lambda_n & i\lambda_n\kappa_n^{(+)} & -\lambda_n^2 \end{pmatrix} \delta(\omega - \omega_{B,n})$$

- $\mathbf{J}$  written in the basis of subsystem Heisenberg operators  $\{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$
- three system-bath coupling mechanisms (tuning, coupling, shift)
- diagonal vs. cross-correlated spectral densities



## Connection to Reduced-Dynamics Approach

- spectral densities are related to correlation functions,

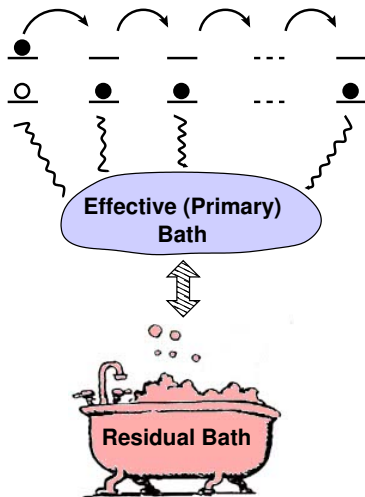
$$\mathcal{C}^{(2,2n)}(t) = \int_{-\infty}^{\infty} d\omega J_{\text{eff}}^{(M=n+1)}(\omega) \frac{\exp(-i\omega t)}{\exp\left(\frac{\omega}{2k_B T}\right) - 1}$$

- correlation functions appear in a 2nd-order master equation for the subsystem density operator, here for the interaction  $\hat{H}_{SB} = \sum_i^N c_i \hat{x}_i \hat{\sigma}_z \equiv D \hat{X}_1 \hat{\sigma}_z$ ,

$$\begin{aligned} \frac{\partial \hat{\rho}_S}{\partial t} = & -i \hat{L}_S(t) \hat{\rho}_S(t) - \int_{t_0}^t dt' \left[ \hat{\sigma}_z(t), \hat{\sigma}_z(t') \hat{\rho}_S(t') \right] \mathcal{C}_B^{(2,k)}(t, t'; t_0) \\ & + \int_{t_0}^t dt' \left[ \hat{\sigma}_z(t), \hat{\rho}_S(t') \hat{\sigma}_z(t') \right] \mathcal{C}_B^{(2,k)}(t', t; t_0) \end{aligned}$$

- beyond 2nd order, memory kernel can be developed in a **double perturbation theory**, w.r.t. the interaction strength and the Mori-chain order

# Transfer Processes in Fluctuating Environments



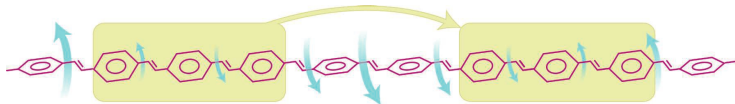
Martelli & Hughes & Burghardt, to be submitted

## Excitation transfer vs. trapping & dissipation & decoherence

- transition between coherent and non-coherent (Förster) transfer
- long-lived excitonic coherences?
- site-local vs. shared (correlated) modes: significant differences
- extend to include thermal gradients

Bittner, Goj, Burghardt, Chem. Phys., in press.

# Exciton Migration & Long-Lived Coherences

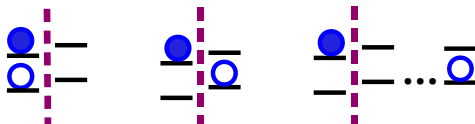


Collini, Scholes, Science 323, 369 (2009), Bredas & Silbey, Science 323, 348 (2009)

$$V^{\text{lin}} = \sum_i \begin{pmatrix} \kappa_i^{(1)} x_i & \lambda_i^{(12)} x_i & & & \\ \lambda_i^{(12)} x_i & \kappa_i^{(2)} x_i & \lambda_i^{(23)} x_i & & \\ & \lambda_i^{(23)} x_i & \kappa_i^{(3)} x_i & \lambda_i^{(34)} x_i & \\ & & \lambda_i^{(43)} x_i & \kappa_i^{(3)} x_i & \lambda_i^{(45)} x_i \\ & & & \ddots & \ddots & \ddots \end{pmatrix}$$

- coherent “surfing” rather than Förster-type hopping?
- role of [site-local](#) vs. [shared \(correlated\)](#) modes
- do shared modes contribute to the conservation of electronic coherence?

# Heterojunction Dynamics: Charge Separation + Photocurrent

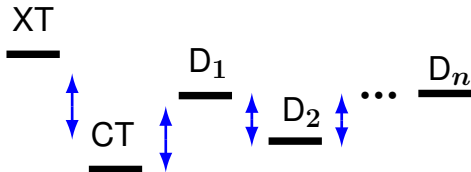


XT

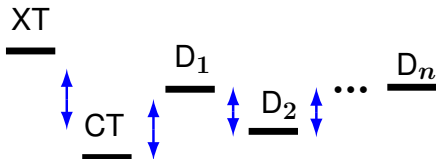
CT

$D_1 \dots D_n$

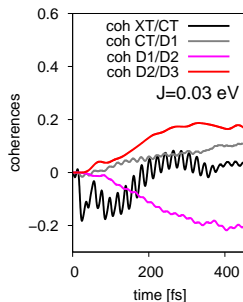
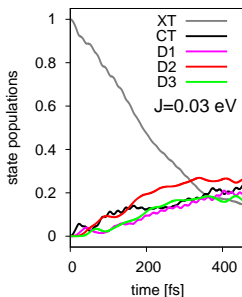
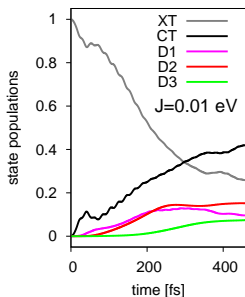
- tuning by low-frequency modes
- tuning by conformational fluctuations
- both factors could on average ensure transfer efficiency



## Detailed Picture of Exciplex Break-Up



- 5 states: XT, CT,  $D_1$  ...  $D_3$
- 4 effective tuning modes
- 30-mode residual bath



# Summary

## ① Molecular-Level Approaches for Extended Systems

- charge transfer, excitation energy transfer
- combine model Hamiltonians & electronic structure information
- alternative/complementary to QM/MM + on-the-fly approaches

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- collective modes dominate on short times scales
- structured environments are gradually explored by the quantum subsystem
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- Gaussian-based multiconfigurational methods
- effective-mode methods
- approximate spectral densities



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- S. Zhao, P. Ramanathan, F. Martelli (Group ENS)

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*Energy Transfer Dynamics in Biomaterial Systems*

This book presents a collection of 14 review articles that cover the key topics addressed in the workshop *Energy Flow Dynamics in Biomaterial Systems* which was held in October 2007 in Paris. These reviews illustrate the many facets of today's theoretical picture of electronic and vibronic dynamics and transport phenomena in biological, biomimetic, and molecular electronic systems. Part I focuses on excitation energy transfer in photosynthetic reaction centers and other multichromophoric systems, part II gives a tour d'horizon of DNA research, and part III addresses molecular electronics and quantum transport in organic materials. Finally, parts IV and V cover recent methodological developments in open system dynamics and hybrid quantum-classical methods. The scope of the book is deliberately broad in terms of physical systems studied and yet unified in the use of quantum dynamical methods to describe transient and often ultrafast energy and charge transfer events in complex systems.



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Energy Transfer Dynamics in Biomaterial Systems

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David A. Micha  
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# Energy Transfer Dynamics in Biomaterial Systems

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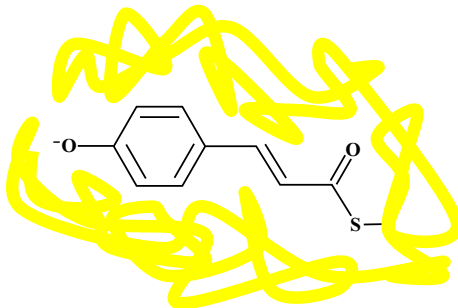
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# Appendix

Quantum Solute + Classical Mesoscopic Environment

# Quantum-Classical, Multi-Scale Approach



- e.g., photoswitches in solution, EET in polar/polarizable environments, ...
- complementary approach to QM/MM-MD type methods

# Quantum-Classical Liouville Equation

$$\boxed{\text{quantum solute}} + \boxed{\text{classical mesoscopic environment}}$$

- quantum-classical Liouville equation, for  $\hat{f}_{\mathbf{r}\mathbf{p}} = \sum_{nm} f_{nm}(\mathbf{r}, \mathbf{p}) |n\rangle \langle m|$ ,

$$\frac{\partial \hat{f}_{\mathbf{r}\mathbf{p}}}{\partial t} = -\frac{i}{\hbar} [\hat{H}_{\mathbf{r}\mathbf{p}}, \hat{f}_{\mathbf{r}\mathbf{p}}] + \frac{1}{2} \left( \{ \hat{H}_{\mathbf{r}\mathbf{p}}, \hat{f}_{\mathbf{r}\mathbf{p}} \} - \{ \hat{f}_{\mathbf{r}\mathbf{p}}, \hat{H}_{\mathbf{r}\mathbf{p}} \} \right)$$

- propagate either the single-particle **phase-space functions**  $f_{nm}(\mathbf{r}, \mathbf{p}, t)$
- or propagate the corresponding **hydrodynamic quantities**:  
local density  $\rho_{nm}(\mathbf{r}, t)$ , current density  $\mathbf{g}_{nm}(\mathbf{r}, t)$ <sup>1</sup>

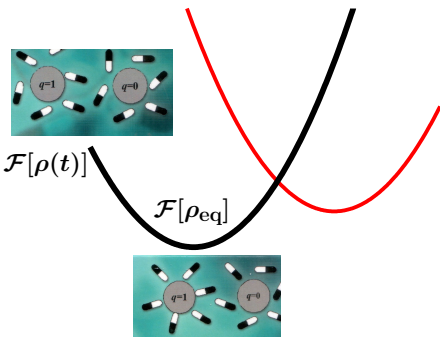
Burghardt & Bagchi, Chem. Phys. **329**, 343 (2006)

<sup>1</sup>local density:  $\rho_{nm}(\mathbf{r}, t) = \int d\mathbf{p} f_{nm}(\mathbf{r}, \mathbf{p}, t)$ , current:  $\mathbf{g}_{nm}(\mathbf{r}, t) = \int d\mathbf{p} \mathbf{p} f_{nm}(\mathbf{r}, \mathbf{p}, t)$

# Classical Dynamical Density Functional Theory (DDFT)

$$\frac{\partial}{\partial t} \rho(\mathbf{r}, t) = -\frac{1}{m} \nabla_r \cdot \mathbf{g}(\mathbf{r}, t)$$

$$\frac{\partial}{\partial t} \mathbf{g}(\mathbf{r}, t) = -\rho(\mathbf{r}, t) \nabla_r \frac{\delta \mathcal{F}[\rho]}{\delta \rho} + \Xi_{\text{diss}}[\mathbf{g}]$$

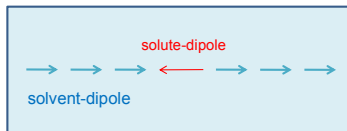


- coupled equations for local density  $\rho(\mathbf{r}, t)$  and current density  $\mathbf{g}(\mathbf{r}, t)$
- free energy functional  $\mathcal{F}[\rho(\mathbf{r}, t)] = kT \ln \Lambda^3 \rho(\mathbf{r}, t) - \int d\mathbf{r}' C_2(\mathbf{r} - \mathbf{r}') \delta \rho(\mathbf{r}', t)$
- transpose to quantum-classical setting: coupled equations for  $\{\rho_{nm}, \mathbf{g}_{nm}\}$ :  
dynamics on coupled Marcus parabolas!

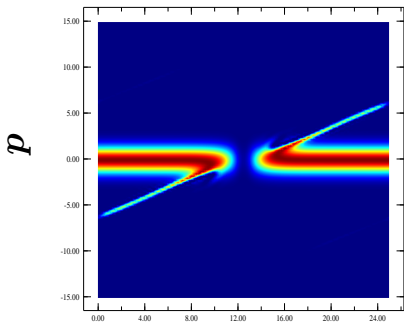
Burghardt & Bagchi, Chem. Phys. **329**, 343 (2006)



# Translational Solvation in 1D: Classical



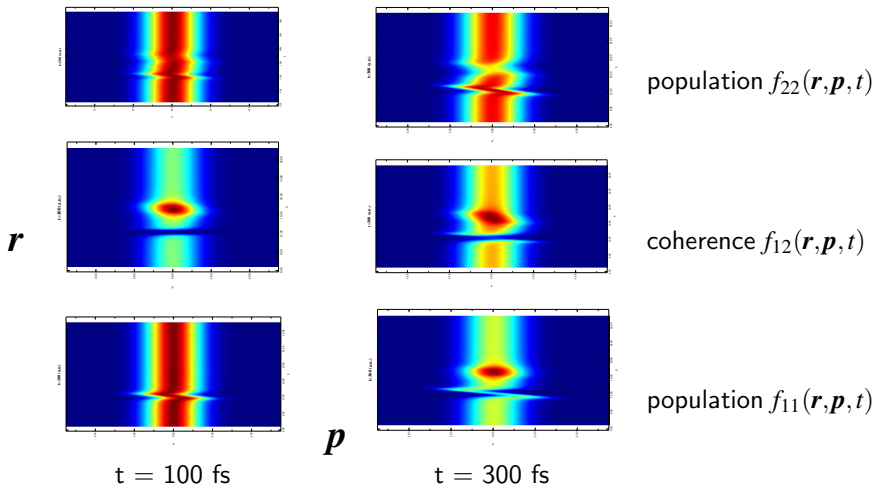
1D model: pure  
translational motion



phase-space density  $f(\mathbf{r}, \mathbf{p}, t)$

- initial condition: homogeneous density  $f(\mathbf{r}, \mathbf{p}, 0) = \rho_0 \exp(-\mathbf{p}^2/4mT)$
- repulsive electrostatic solute-solvent interaction: “dip” in solvent density

# Translational Solvation in 1D: Quantum-Classical (2-State)



- initial condition: homogeneous density for state 1:  $f_{11}(\mathbf{r}, \mathbf{p}, 0) = \rho_0(\mathbf{r})f_M(\mathbf{p})$

Hughes, Ramanathan, Burghardt, to be submitted