The nuclear many-body problem: an open quantum systems perspective

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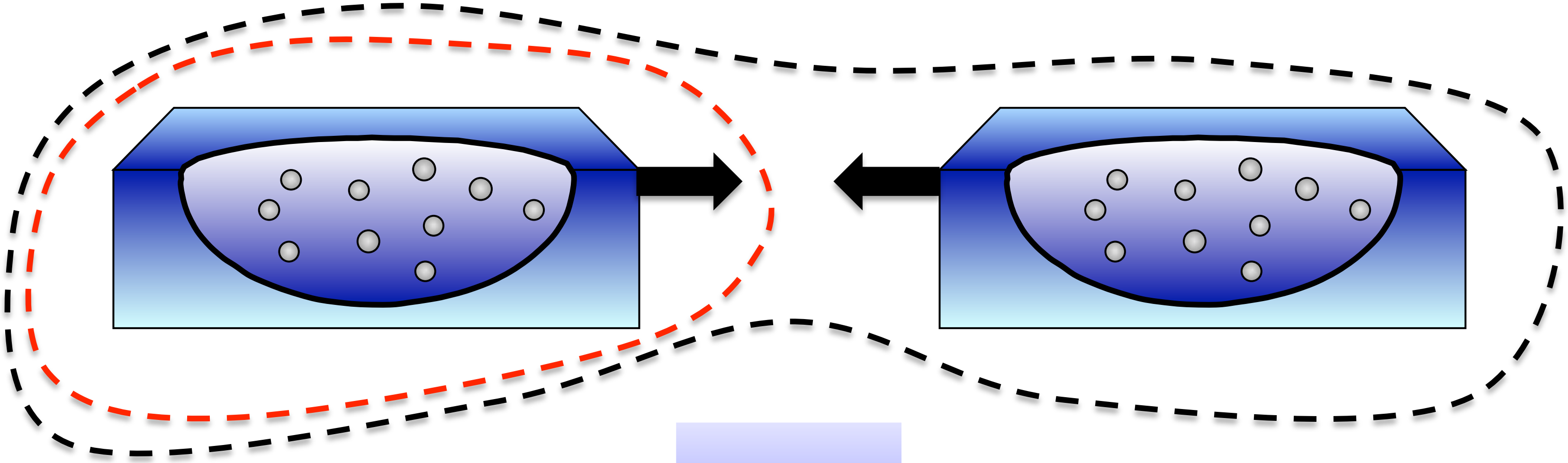
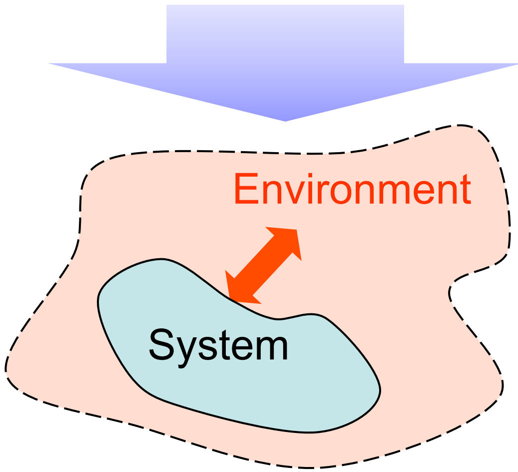
Trento, "Decoherence…" -April 2010

laboratoire commun CEA/DSM CNRS/IN2P3

The nuclear many-body problem as an open quantum object

Generalities: Reduction of information





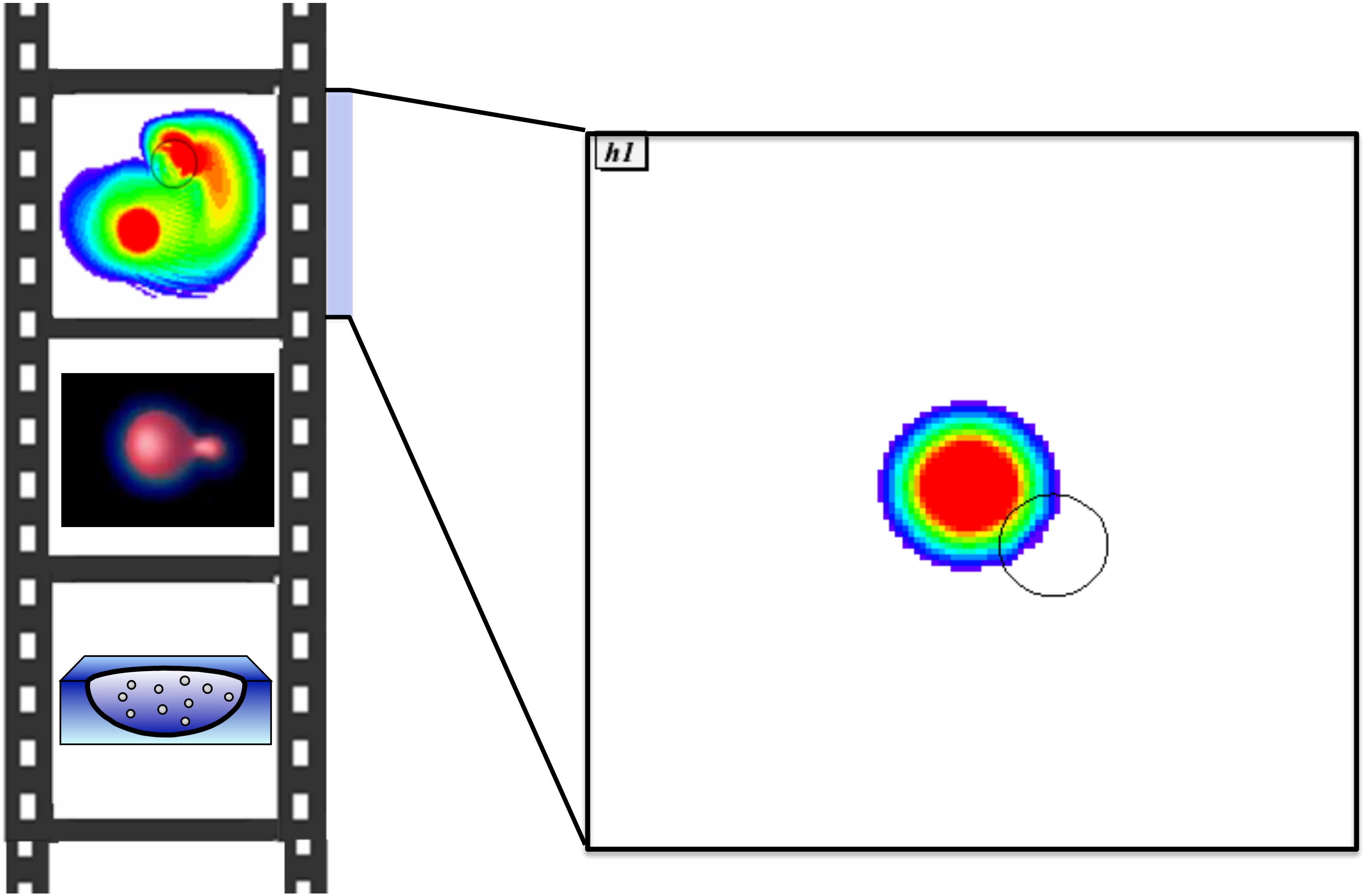
"Few" relevant degrees of freedom needs to be selected (System)

Illustrations discussed here

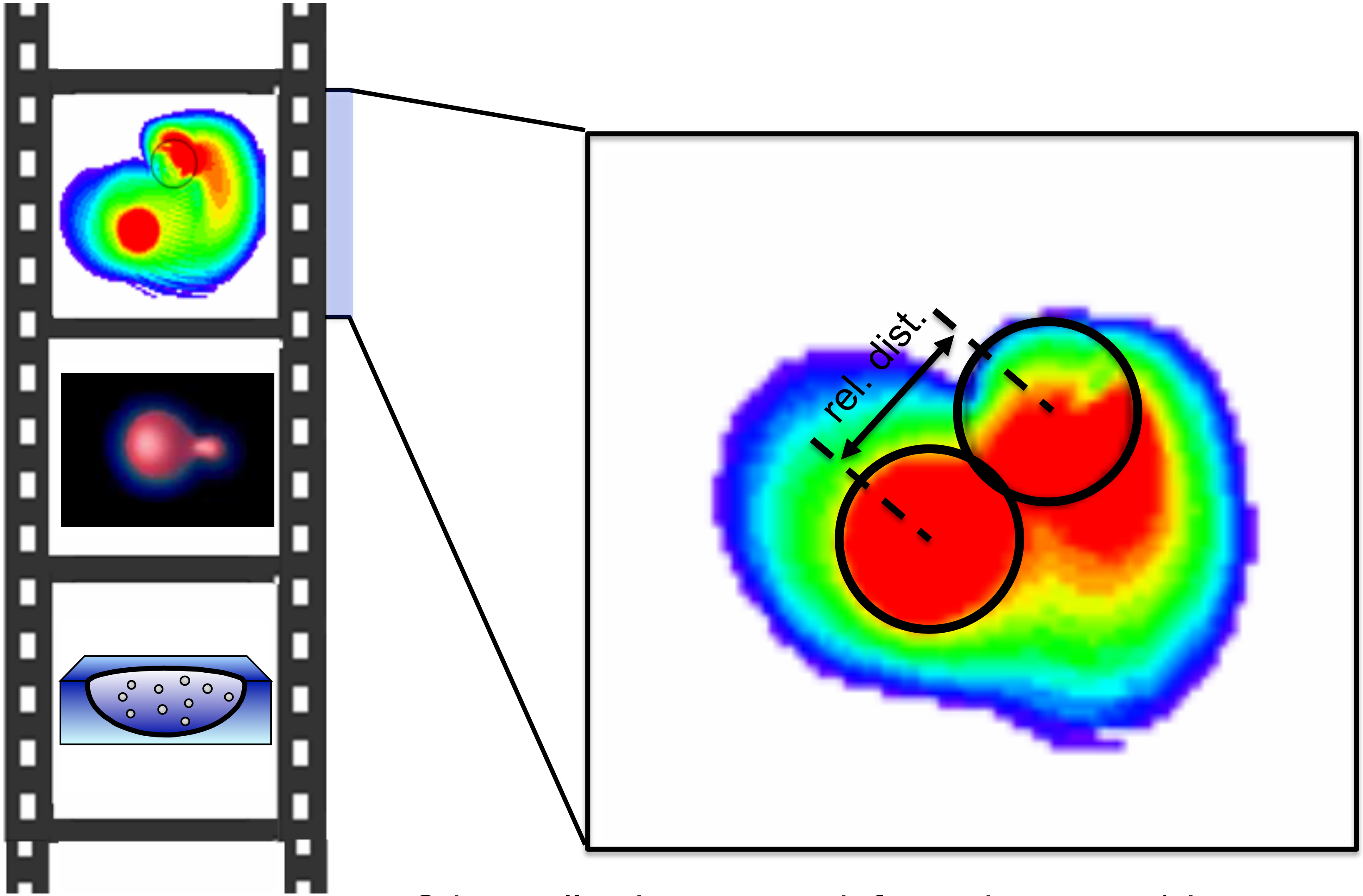
- **Fusion reactions: the role of open channels (discrete and continuous)**
  - **System: collective space** — **Env: intrinsic degrees of freedom**
- **The nuclear many-body problem**
  - **System: one-body observables** — **Env: two-body and higher correlations**

The nuclear many-body problem as an open quantum object

(i) Macroscopic reduction

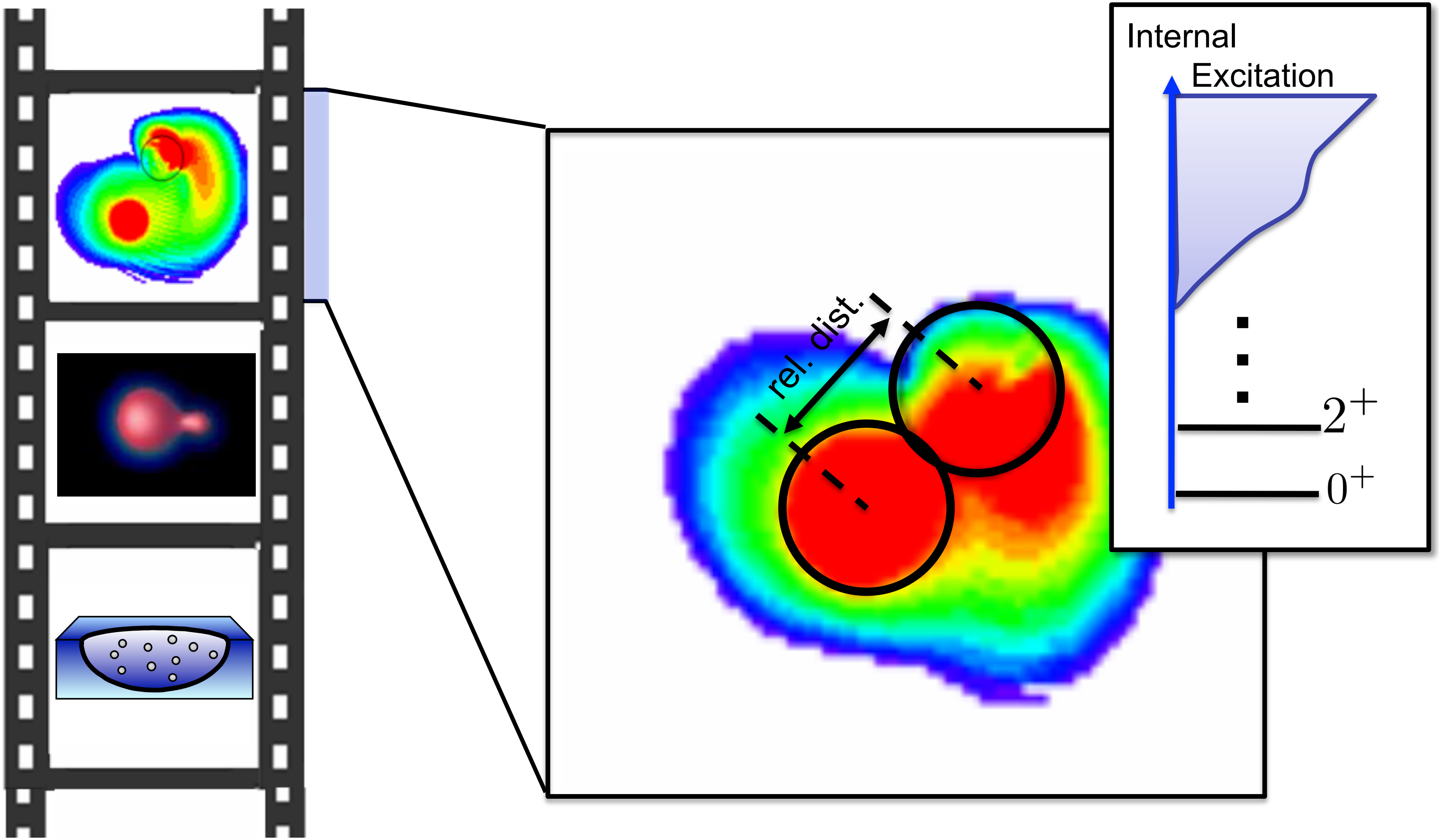


(i) Macroscopic reduction



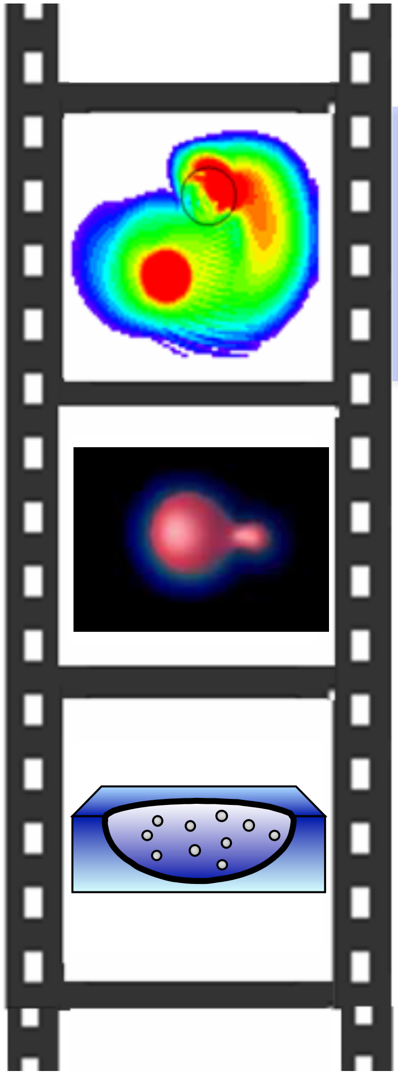
Other collective space: deformation, mass/charge asymmetry …

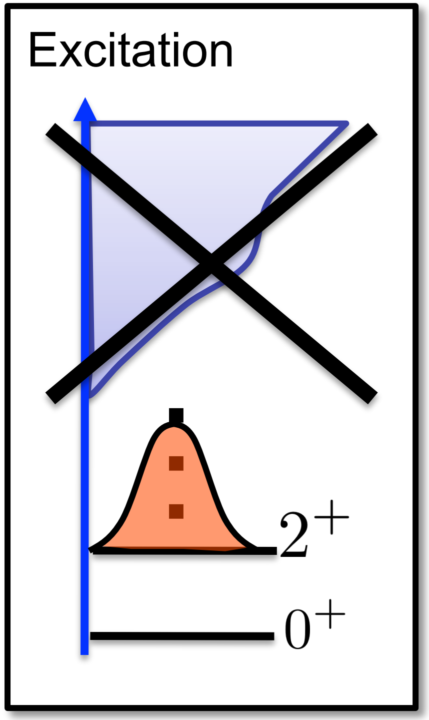
Open channels : discrete internal excitations



One of the difficulty is to treat both discrete and continuous channels in a common framework

Stochastic semi-classical treatment of discrete channels





Collective Motion + Coupling

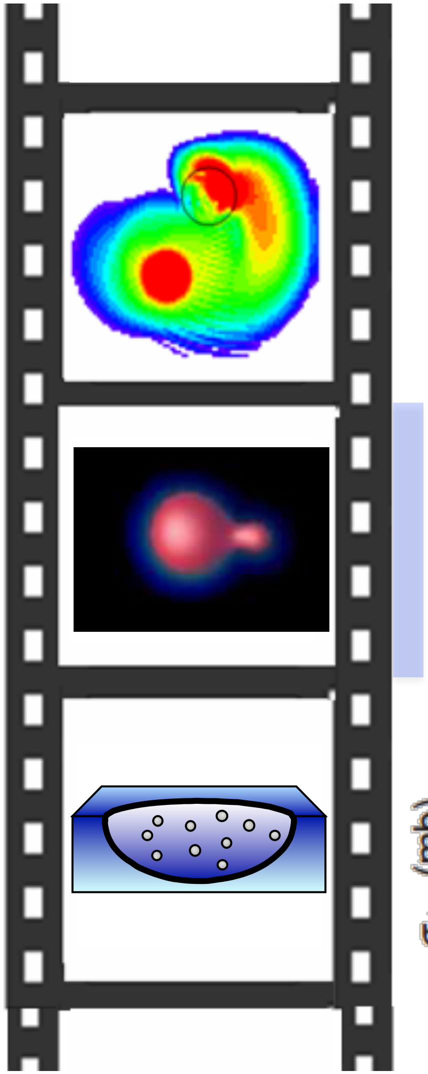
$$H = \frac{P^2}{2\mu} + \frac{l(l+1)\hbar^2}{2\mu R^2} + V_C(R) + V_N(R, \Omega, \alpha_{i\lambda}) + \sum_{i=1}^{2} \sum_{\lambda=0}^{N-1} \left[\frac{\Pi_{i\lambda}^2}{2D_{i\lambda}} + \frac{1}{2} C_{i\lambda} \alpha_{i\lambda}^2 \right],$$

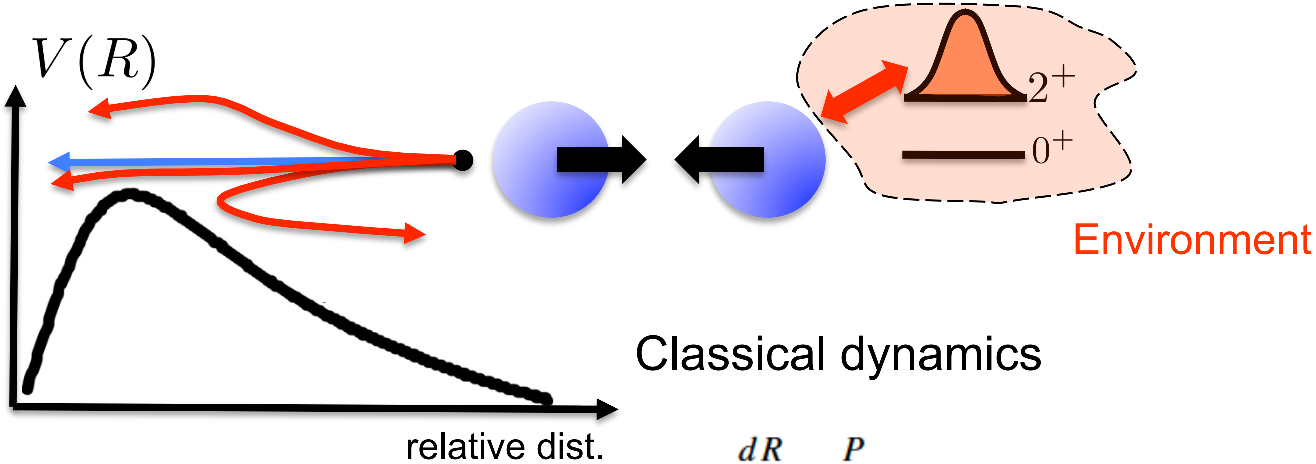
Discrete Channels

Esbensen et al, PRL 41 (1978)

- Initial Phase-space sampling of zero point motion
- Classical dynamics of system+environment
  With stochastic initial condition

Stochastic semi-classical treatment of discrete channels





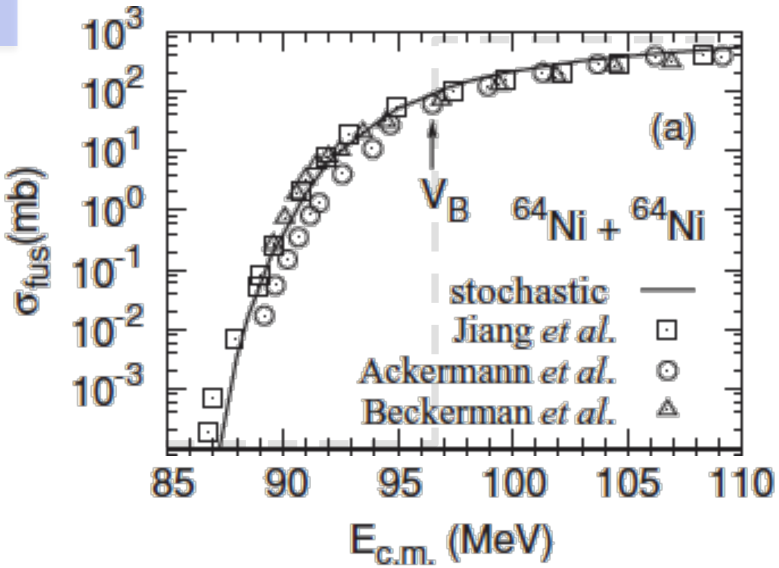
Classical dynamics

$$\frac{dR}{dt} = \frac{P}{\mu},$$

$$\frac{dP}{dt} = -\frac{dV_C(R)}{dR} - \frac{\partial V_N(R,\Omega,\alpha_{i\lambda})}{\partial R} + \frac{l(l+1)\hbar^2}{\mu R^3}$$

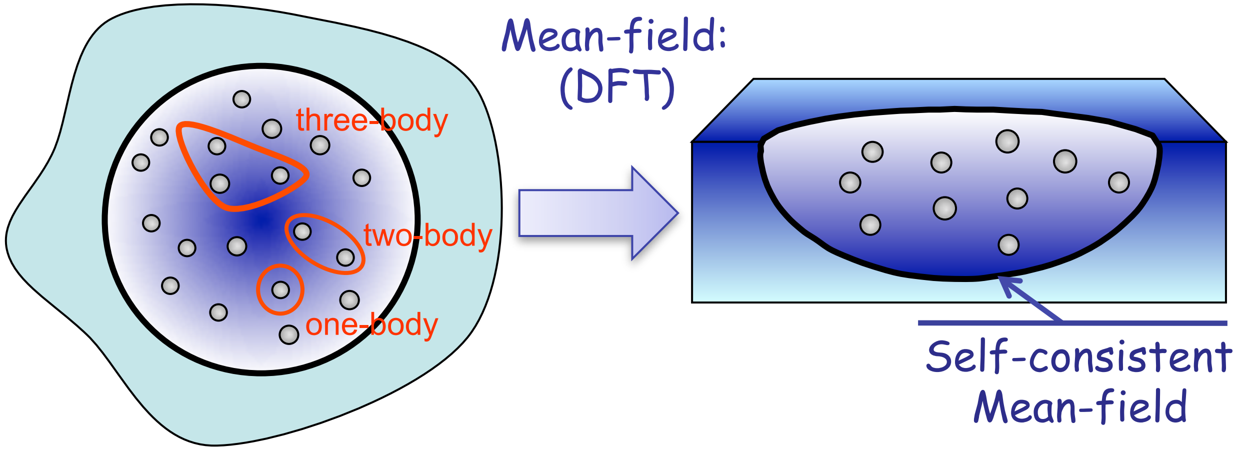
$$\frac{d\alpha_{i\lambda}}{dt} = \frac{\Pi_{i\lambda}}{D_{i\lambda}},$$

$$\frac{d\Pi_{i\lambda}}{dt} = -\frac{\partial V_N(R,\Omega,\alpha_{i\lambda})}{\partial \alpha_{i\lambda}} - C_{i\lambda}\alpha_{i\lambda}.$$

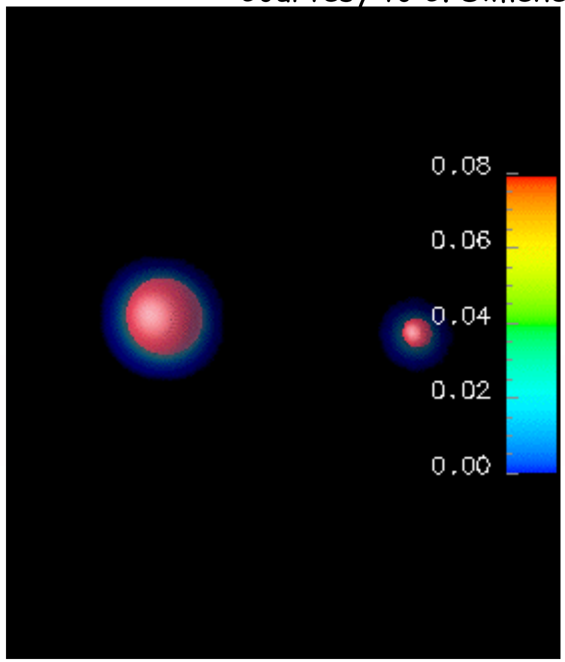


Ayik, Yilmaz,DL, PRC81 (2010)

Microscopic reduction

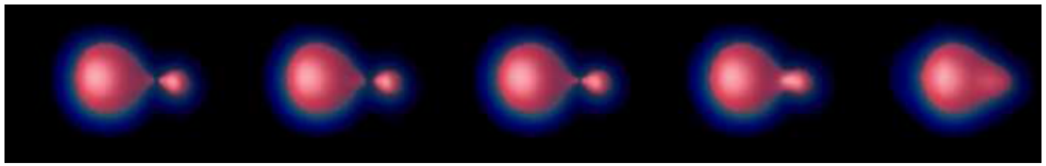


Courtesy to C. Simenel

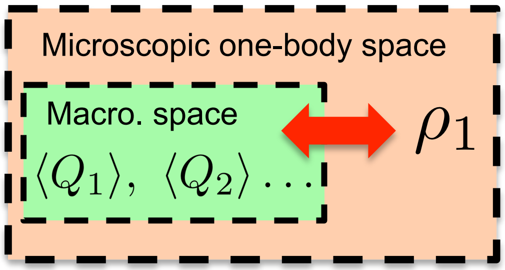


"Simple" Trial state:

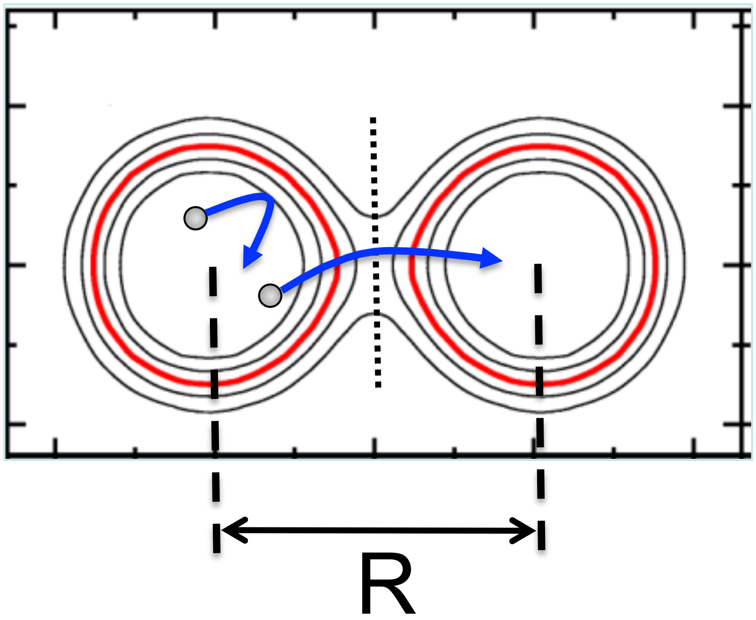
$$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots) \longrightarrow \begin{aligned} |\Phi_{HF}\rangle &= \Pi a^+_\alpha |0\rangle \\ i\hbar \frac{d}{dt}\rho_1 &= [h_{\rm MF}, \rho_1], \end{aligned}$$



Selection of few relevant degrees of freedom:



Expected One-body origin of dissipation

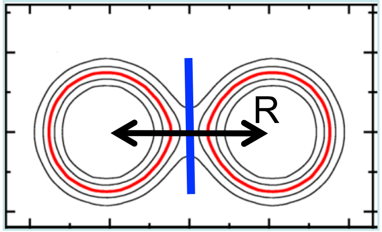


-transfer of particle

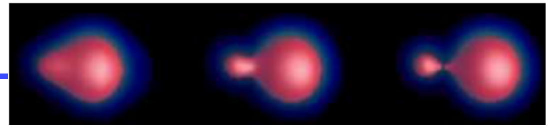
-reflection of particles

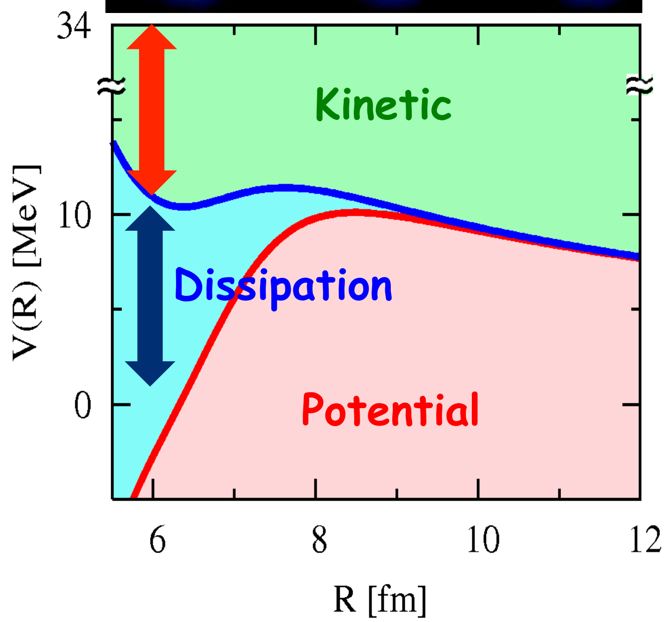
Macroscopic reduction: dissipation

Washiyama, DL, PRC78 (2008).
Washiyama, DL, Ayik, PRC79 (2009).

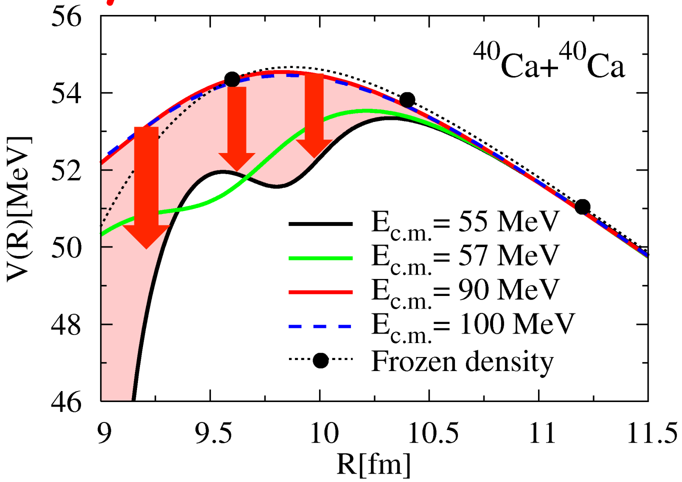


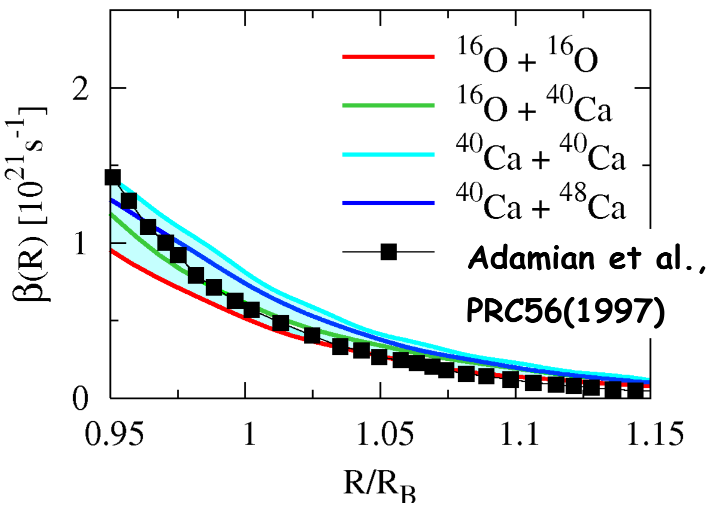
$$\frac{dP}{dt} = -\frac{dV}{dR} - \gamma(R)\frac{dR}{dt}$$





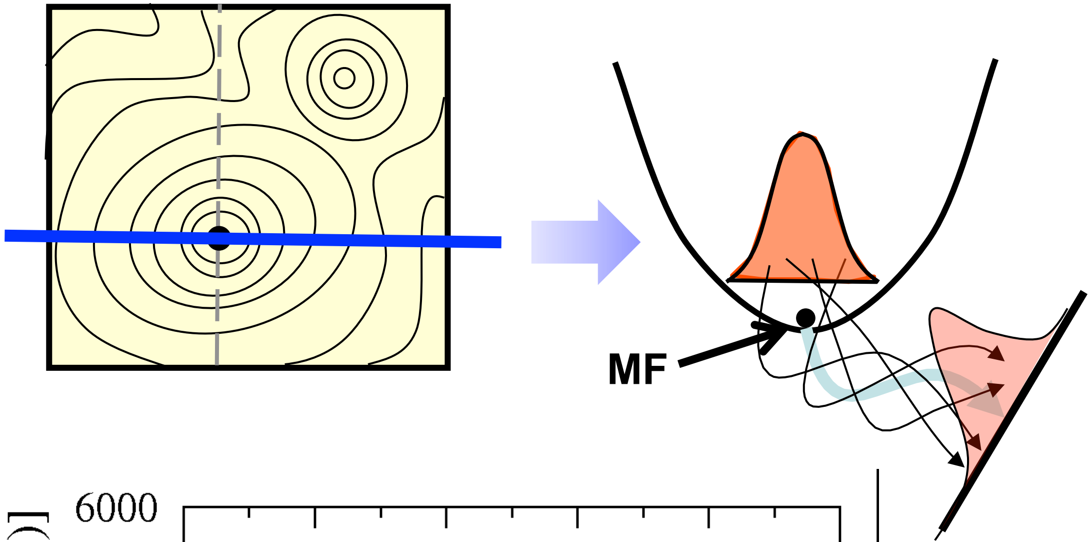
Dynamical Reduction effect



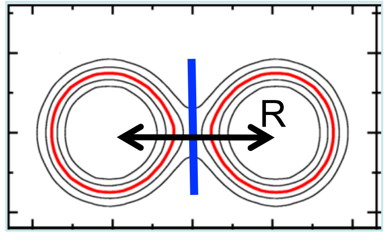


Dissipation ⇨ Internal Excitation

Ayik, PLB 658, (2008).



Application to fusion

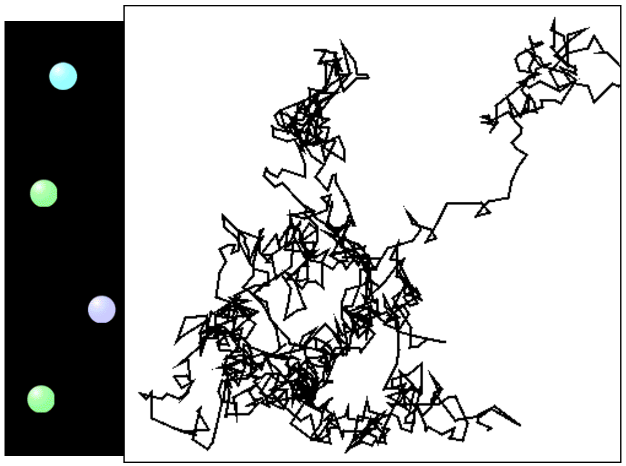


Mean-field

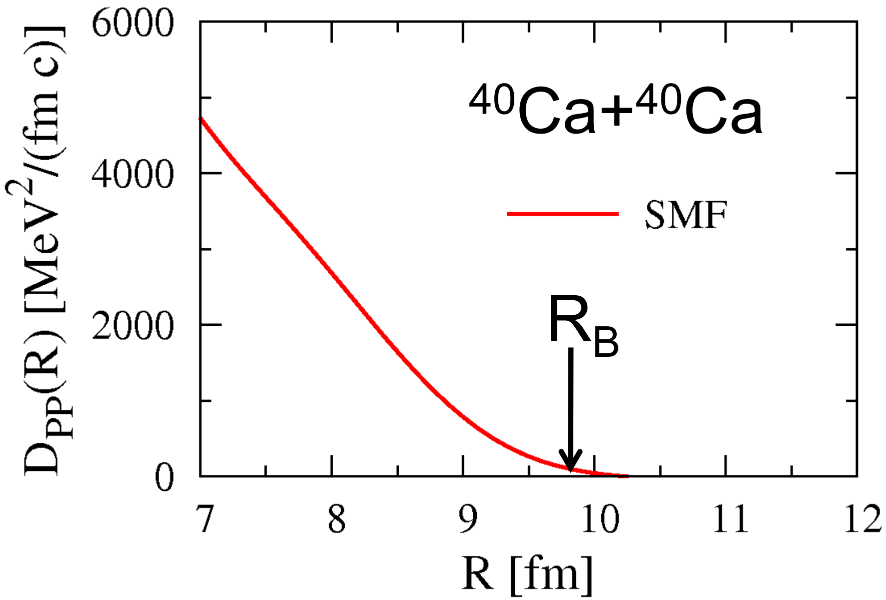
$$\frac{d}{dt}P = -\frac{d}{dR}U(R) - \gamma(R)\dot{R}$$

Mean-field+Initial fluct.

$$\frac{d}{dt}P^{\lambda} = -\frac{d}{dR^{\lambda}}U(R^{\lambda}) - \gamma(R^{\lambda})\dot{R}^{\lambda} + \xi_P^{\lambda}(t)$$

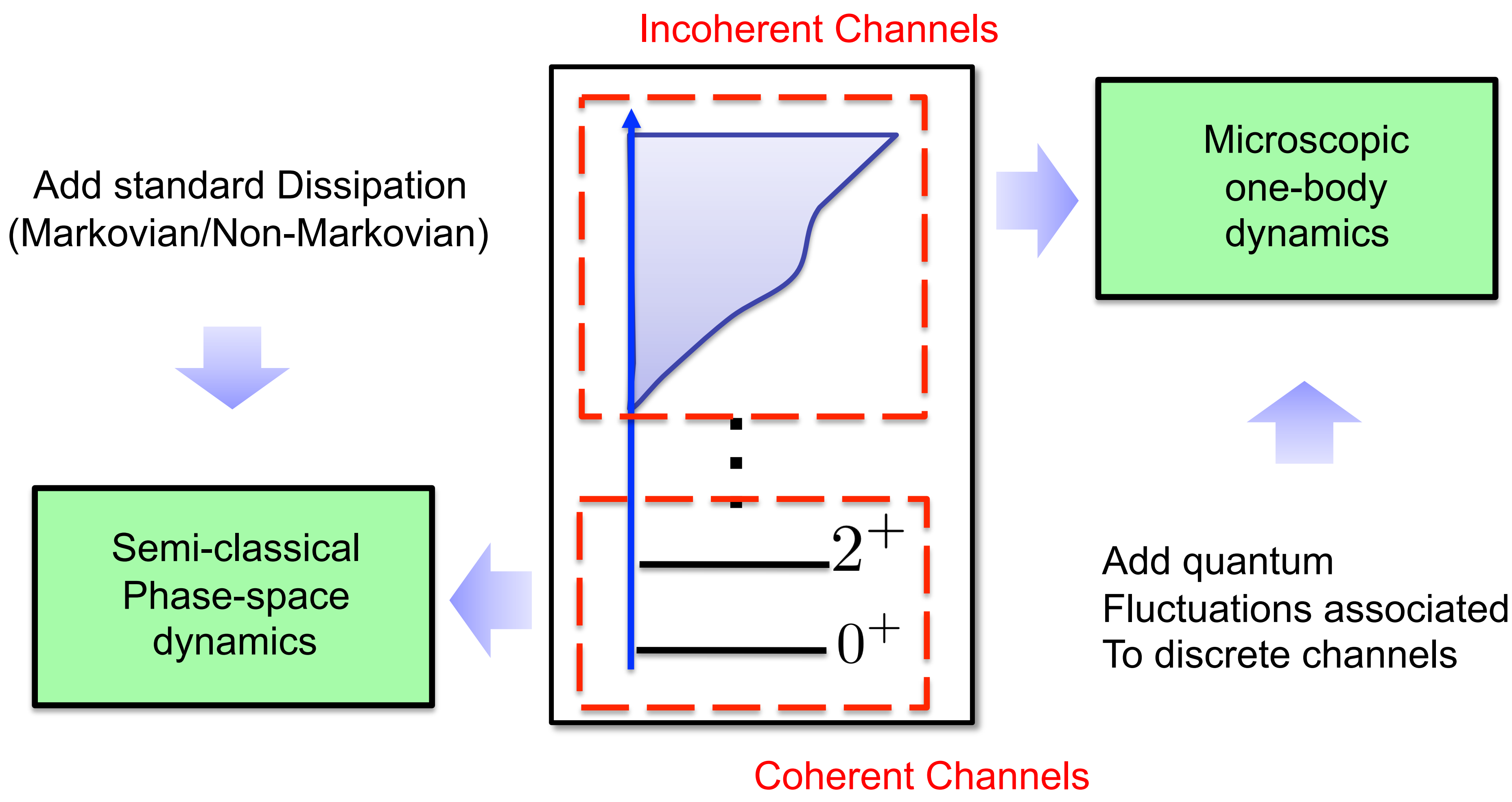


$$\overline{\xi_P^{\lambda}(t)\xi_P^{\lambda}(t')} = 2\delta(t-t')D_{PP}(R)$$

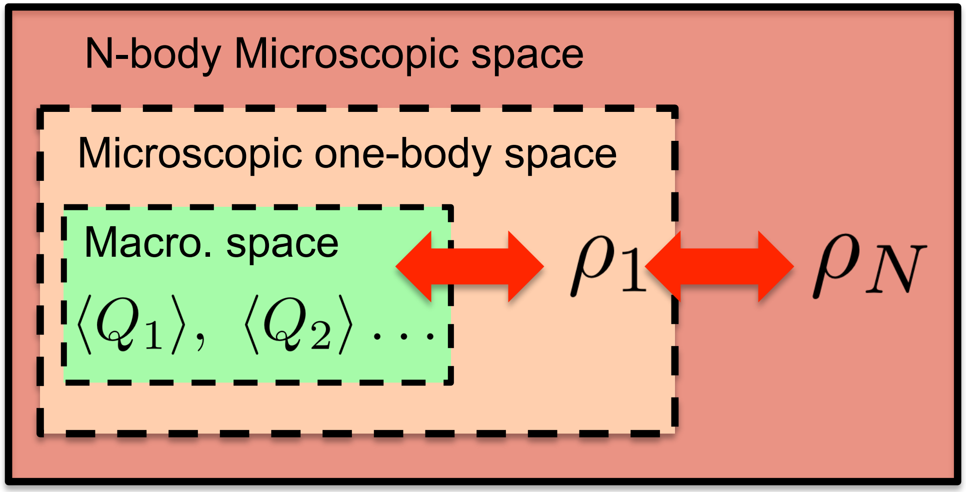


Ayik, Washiyama, DL, PRC (2009)

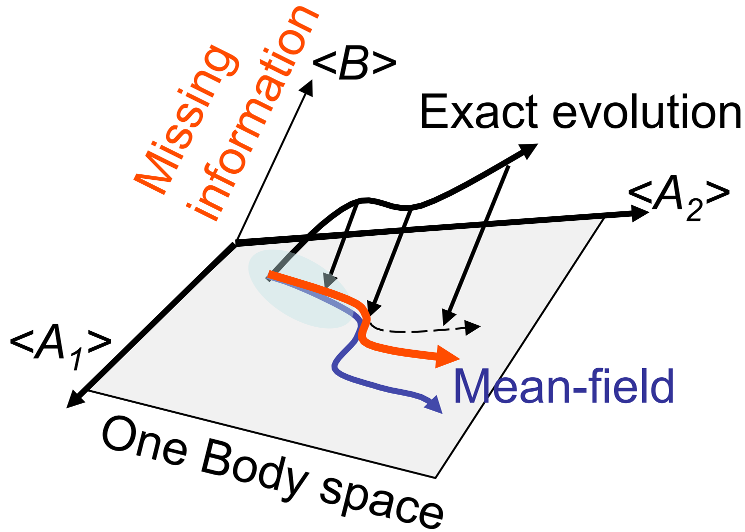
What next?


Incoherent Channels
Add standard Dissipation
(Markovian/Non-Markovian)
Microscopic
one-body
dynamics
Semi-classical
Phase-space
dynamics
2^+
0^+
Add quantum
Fluctuations associated
To discrete channels
Coherent Channels

Part II
Mapping many-body systems
To
Open quantum systems



Y. Abe et al, Phys. Rep. 275 (1996)
D. Lacroix et al, Progress in Part. and Nucl. Phys. 52 (2004)



Short time evolution

$$i\hbar\frac{d}{dt}\rho_1 = [h_{MF},\rho_1] + Tr_2\,[v_{12}, C_{12}]$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\rho_{12} = [h_{\mathrm{MF}}(1)+h_{\mathrm{MF}}(2), \rho_{12}] + (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

Correlation
$$C_{12} = \rho_{12} - (\rho_1\rho_2)_A$$

Approximate long time evolution+Projection (Nakajima-Zwanzig)

$$i\hbar\frac{d}{dt}\rho_1 = [h_{MF},\rho_1] + Tr_2\,[v_{12}, C_{12}]$$

with

$$C_{12}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} U_{12}(t,s)\,F_{12}(s)\,U_{12}^{\dagger}(t,s)\,ds + \delta C_{12}(t)$$

projected two-body effect

Propagated initial correlation

Dissipation (Extended TDHF)

$$i\hbar\frac{d}{dt}\rho = [h_{MF},\rho] + K(\rho)$$

Dissipation and fluctuation

$$i\hbar\frac{d}{dt}\rho = [h_{MF},\rho] + K(\rho) + \delta K(\rho)$$

Random initial condition

Non-Markovian effects

$$i\hbar\frac{\partial}{\partial t}\rho_1 = [h_1[\rho],\rho_1] + \frac{1}{2}\mathrm{Tr}_2[\bar{v}_{12}, C_{12}]$$

with

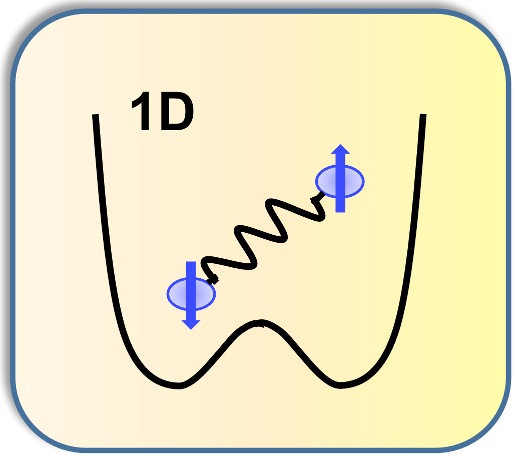
$$C_{12}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} U_{12}(t,s)\, F_{12}(s)\, U_{12}^{\dagger}(t,s)\, ds + \delta C_{12}(t)$$

$$(1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

Non-Markovian master equation

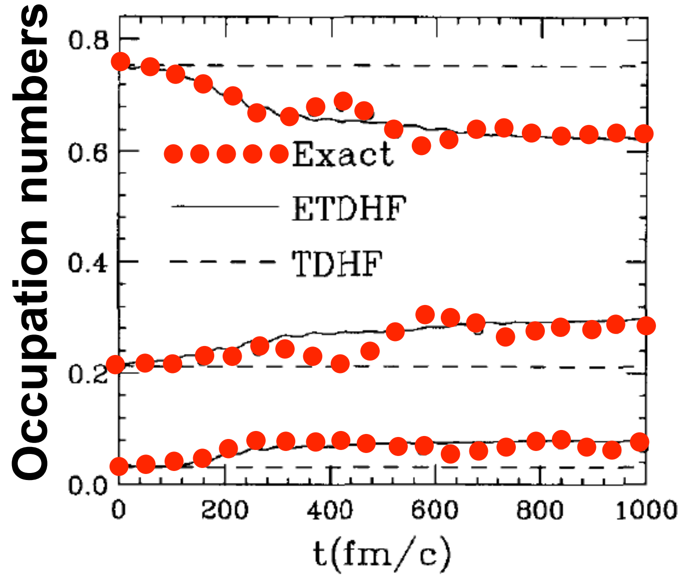
$$\frac{d}{dt}n_\lambda(t) = \int_{t_0}^{t} dt' \left\{ \bar{n}_\lambda(t')\,\mathcal{W}_\lambda^{+}(t,t') - n_\lambda(t')\,\mathcal{W}_\lambda^{-}(t,t') \right\}$$

Example: two interacting fermions in 1dimension

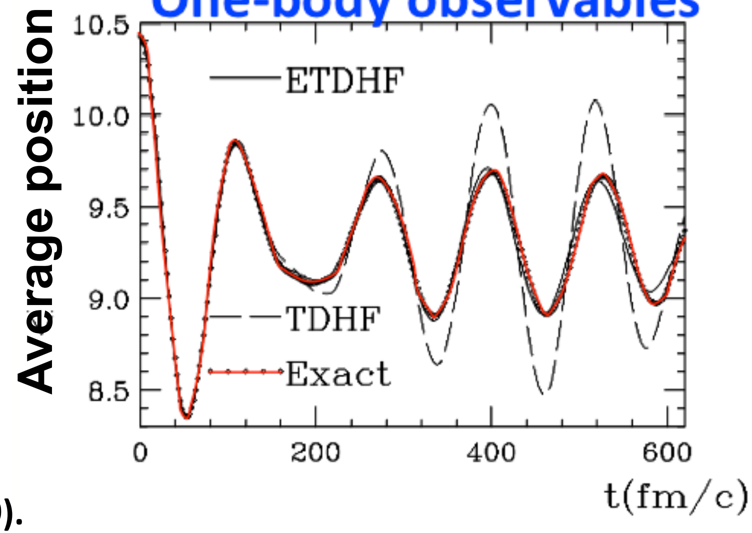


DL, Chomaz, Ayik, Nucl. Phys. A (1999).

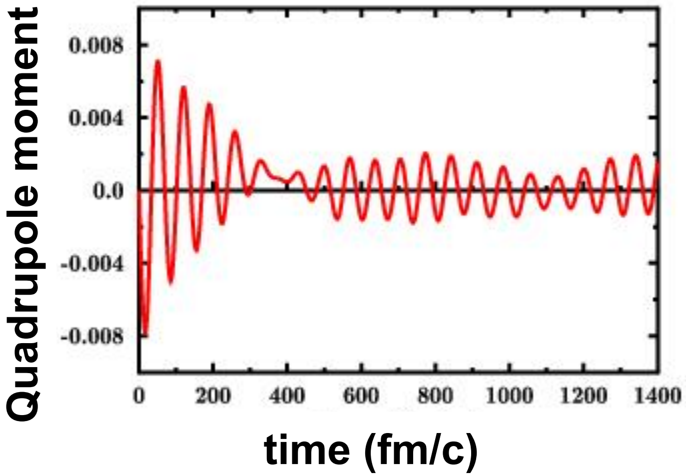
Occupation number evolution



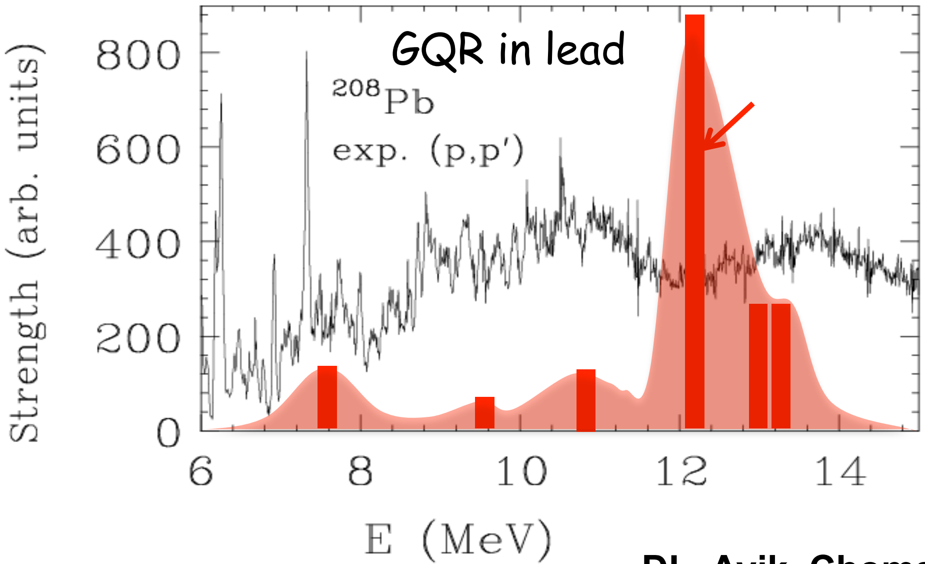
One-body observables



application to collective motion



Giant Quadrupole resonances



- Mean energy is OK
- Damping (dissipation) and fragmentation is missed

DL, Ayik, Chomaz, Prog. Part and Nucl. Phys. (2004)

Non-Markovian dynamics beyond mean-field

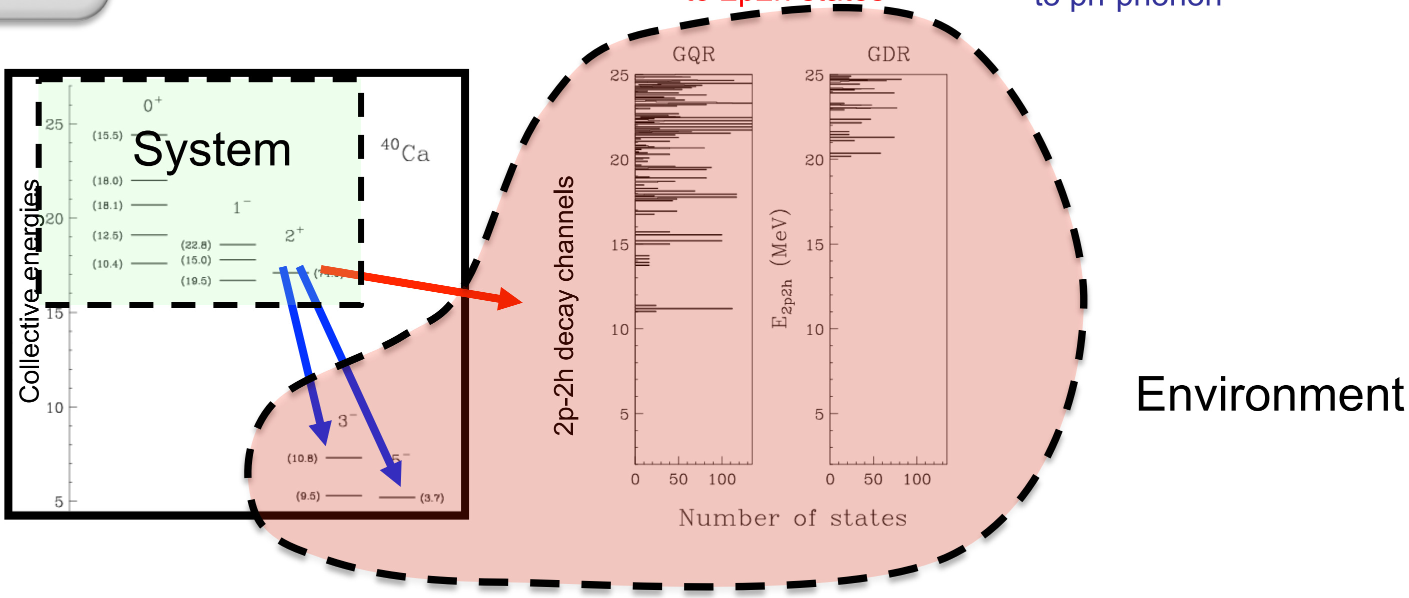
application to collective motion

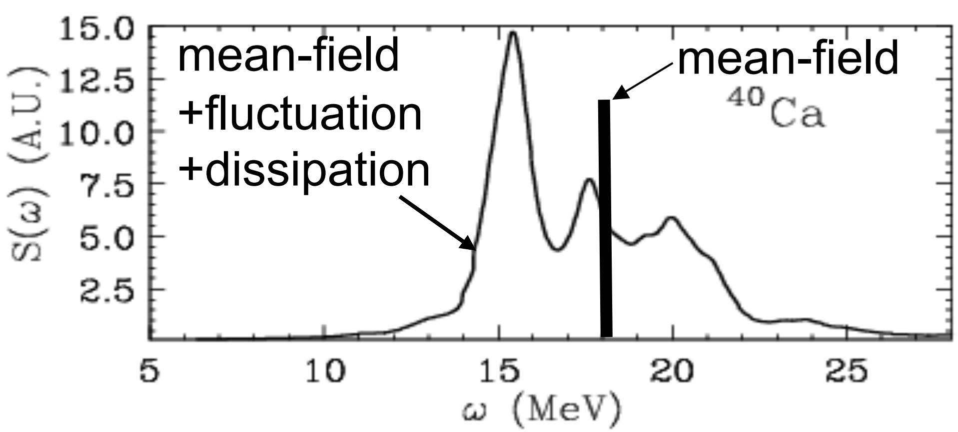
$$i\hbar\frac{\partial}{\partial t}\rho^{(n)} - [h(\rho^{(n)}),\rho^{(n)}] = K_I(\rho^{(n)}) + \delta K^{(n)}(t)$$

Mean-field

Coupling to 2p2h states

Coupling to ph-phonon





Incorporate dissipation in many-body system

Not so easy to use in Large amplitude Collective motion

Markovian limit, quantum-diffusion and stochastic Schrödinger Equation

GOAL: Restarting from an uncorrelated state $D = |\Phi_0\rangle \langle \Phi_0|$ we should:

1-have an estimate of $D = |\Psi(t)\rangle \langle \Psi(t)|$

2-interpret it as an average over jumps between "simple" states

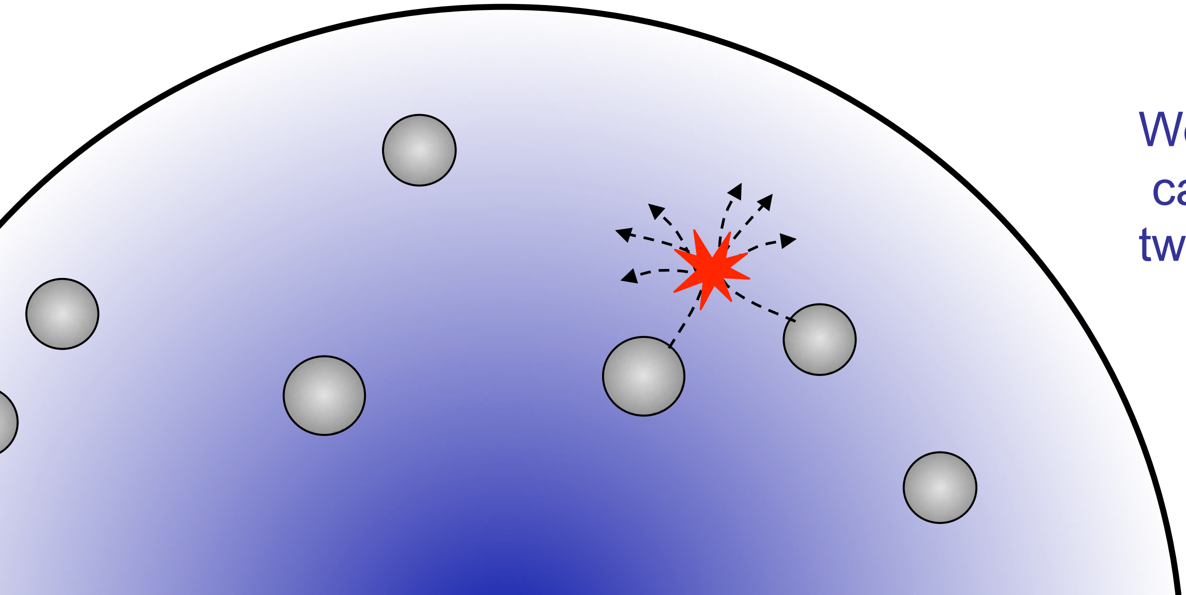
Weak coupling approximation : perturbative treatment

Reinhard and Suraud, Ann. of Phys. 216 (1992)

$$|\Psi(t')\rangle = |\Phi(t')\rangle - \frac{i}{\hbar}\int \delta v_{12}(s)\,|\Phi(s)\rangle\, ds - \frac{1}{2\hbar^2} T\left(\int\int \delta v_{12}(s)\delta v_{12}(s')\,ds\,ds'\right)|\Phi(s)\rangle$$

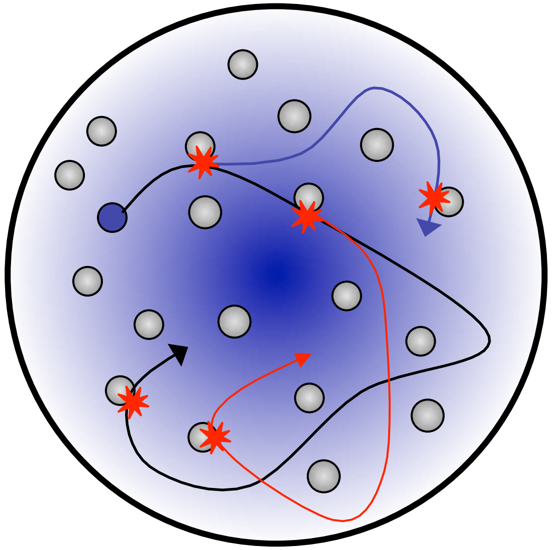
Residual interaction in the mean-field interaction picture

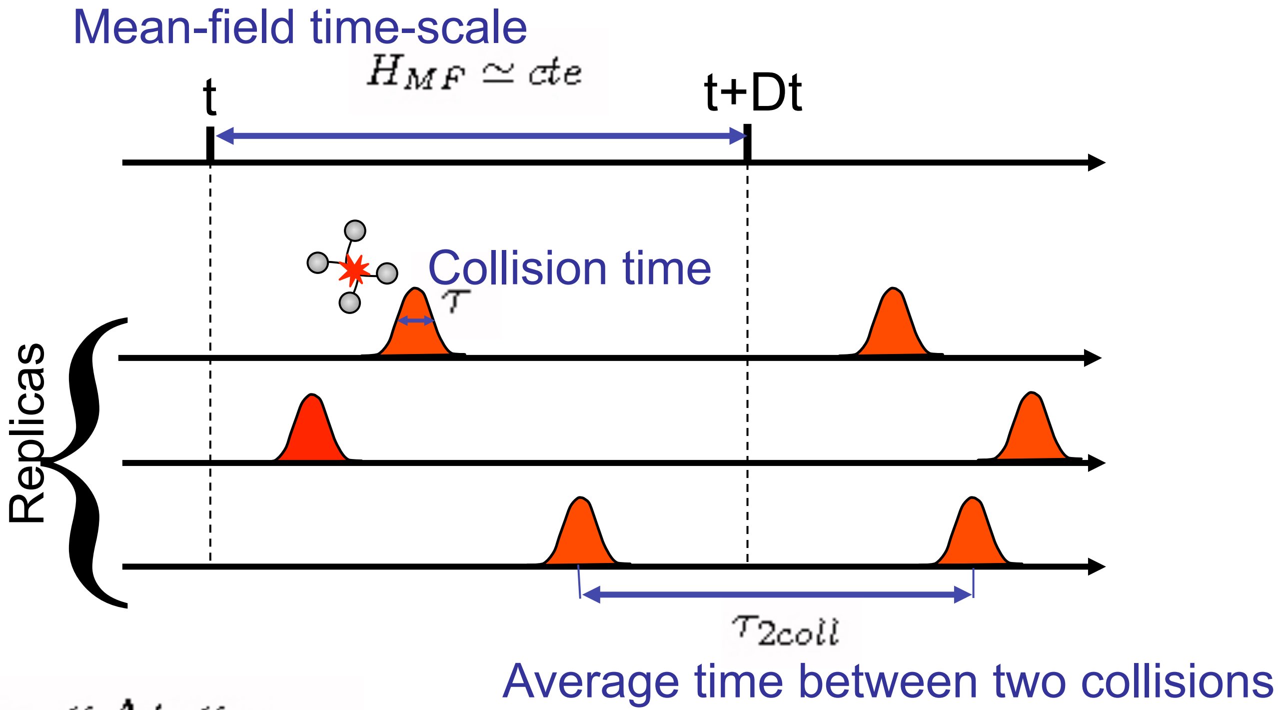
Statistical assumption in the Markovian limit :



We assume that the residual interaction can be treated as an ensemble of two-body interaction:

$$\begin{cases} \overline{\delta v_{12}(s)} = 0 \\ \overline{\delta v_{12}(s)\delta v_{12}(s')} \propto \overline{\delta v_{12}^2(s)}\, e^{-(s-s')^2/2\tau^2} \end{cases}$$





Hypothesis : $\tau \ll \Delta t \ll \tau_{2coll}$

Average Density Evolution:

$$\Rightarrow \quad \overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$$

Dissipation: link between Extended TDHF and Lindblad Eq.

One-body density
Master equation
step by step

$$\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$$

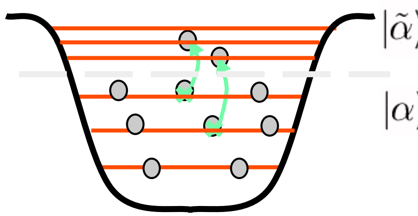
Initial simple state

$$D = |\Phi\rangle\langle\Phi|$$
$$\rho = \sum_\alpha |\alpha\rangle\langle\alpha|$$

$$i\hbar\frac{d}{dt}\rho = [h_{MF}, \rho] - \frac{\tau}{2\hbar^2}\mathcal{D}(\rho)$$

with $\langle j|\mathcal{D}|i\rangle = \overline{\left\langle\left[\left[a_i^+ a_j, \delta v_{12}\right], \delta v_{12}\right]\right\rangle}$

2p-2h nature
of the interaction



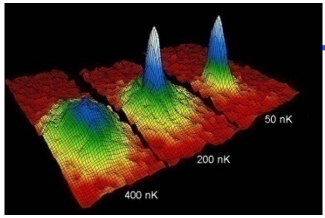
$$\mathcal{D}(\rho) = Tr_2\,[v_{12}, C_{12}]$$

with $C_{12} = (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$

Separability of the
interaction $v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2)$

$$\mathcal{D}(\rho) = \sum_k \gamma_k\,(A_k A_k \rho + \rho A_k A_k - 2A_k\rho A_k)$$

- Dissipation contained in Extended TDHF is included
- The master equation is a Lindblad equation
- Associated SSE *DL, PRC73 (2006)*



1D bose condensate with gaussian two-body interaction

N-body density: $D = |N:\alpha\rangle\langle N:\alpha|$

SSE on single-particle state :

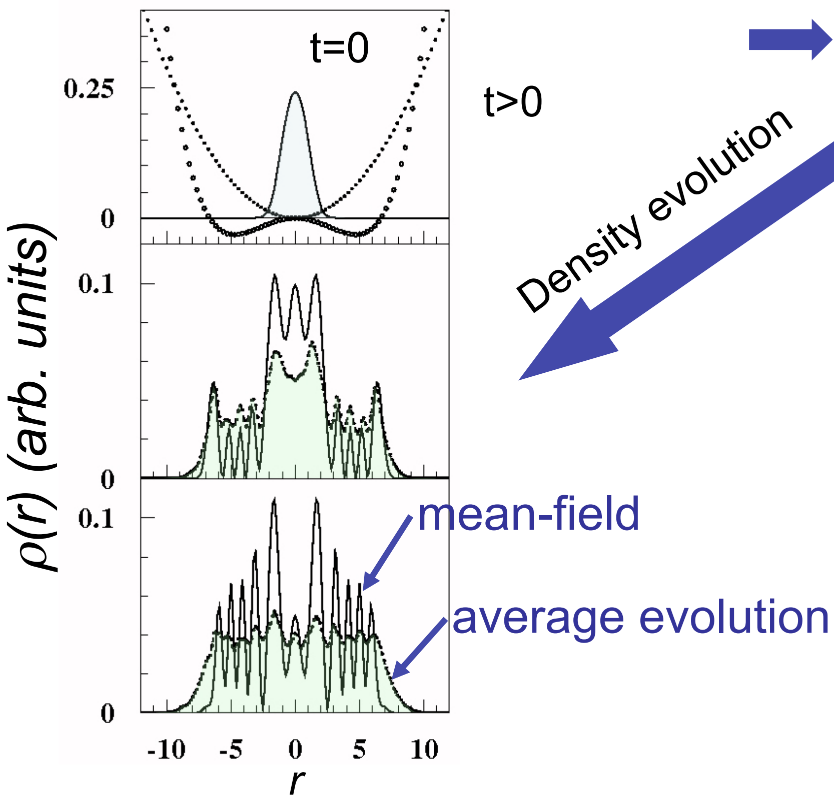
$$d|\alpha\rangle = \left\{ \frac{dt}{i\hbar} h_{MF}(\rho) + \sum_k dW_k (1-\rho) A_k - \frac{dt\tau}{2\hbar^2} \sum_k \gamma_k \left[A_k^2 \rho + \rho A_k \rho A_k - 2 A_k \rho A_k \right] \right\} |\alpha\rangle$$

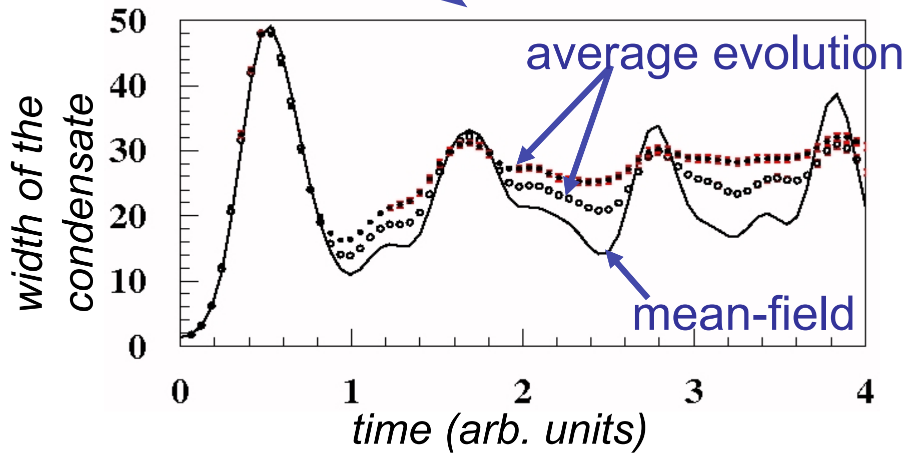
with $dW_k dW_{k'} = -\frac{dt\tau}{\hbar^2}\gamma_k \delta_{kk'}$

→ The numerical effort is fixed by the number of A_k

Density evolution

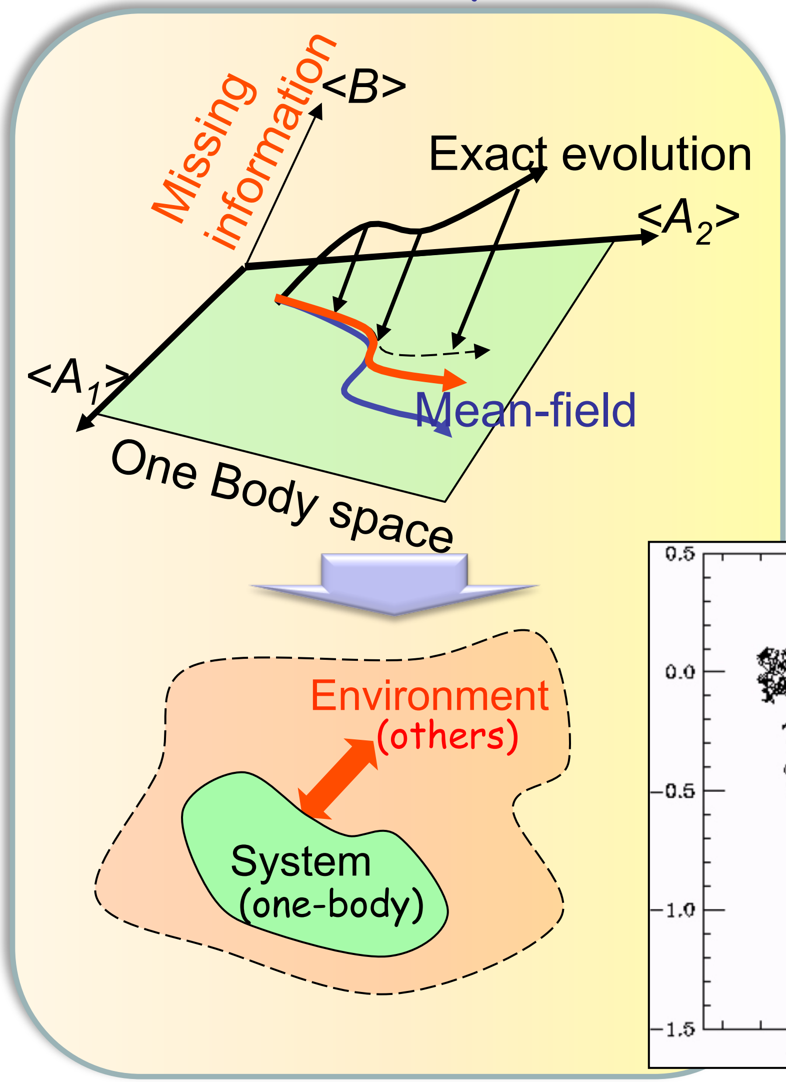
Condensate size



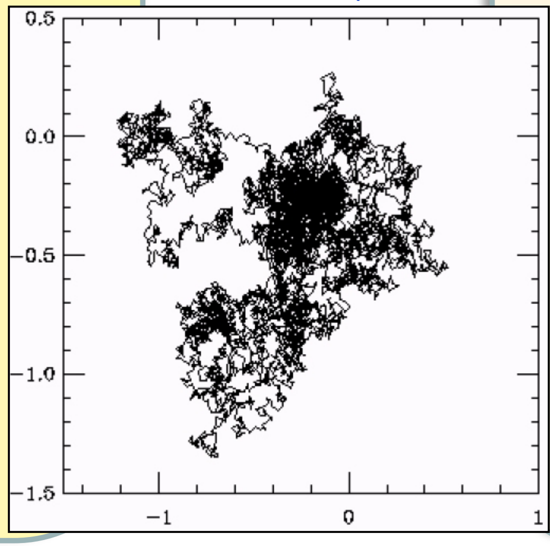


Self-interacting vs Open Quantum systems

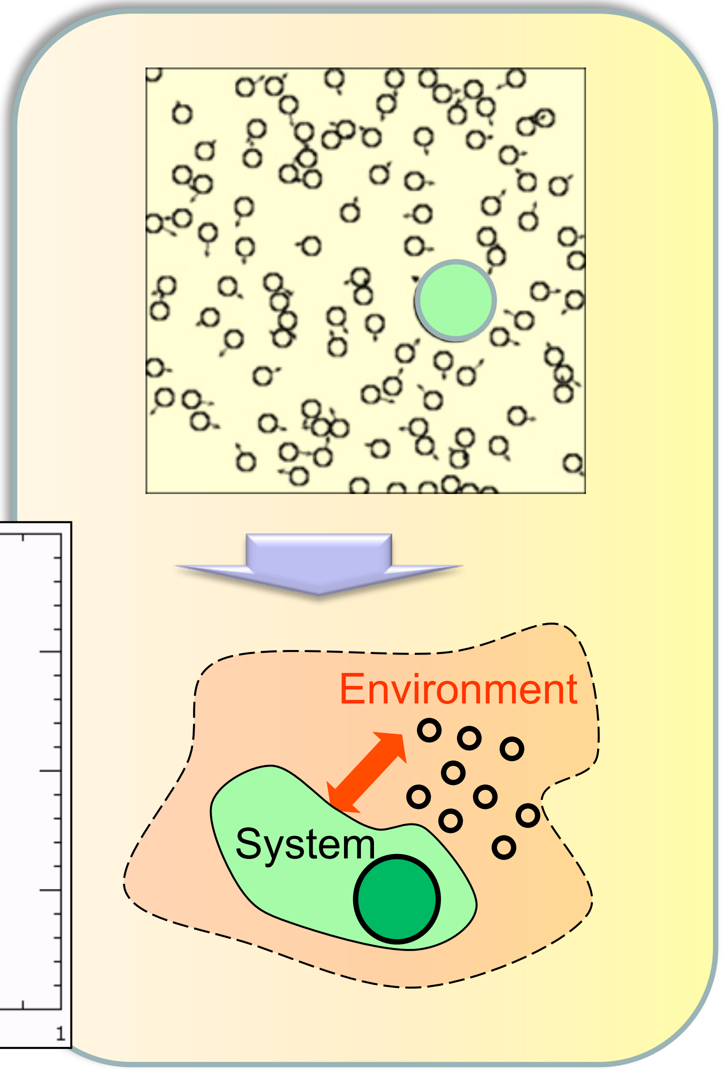
N-body



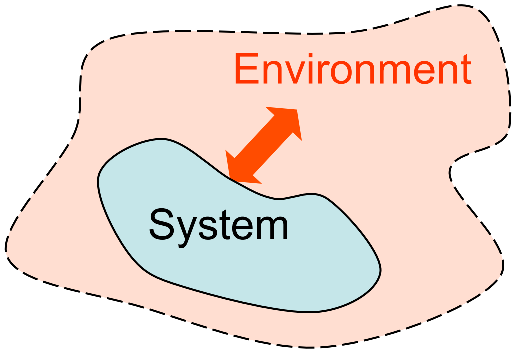
Brownian motion



Open systems



Towards Exact stochastic methods for N-body and Open systems



$$H = H_{\rm S} + H_{\rm E} + H_{\rm Coup}$$

$$H = H_{\rm MF} + H_{\rm Corr}$$

Projection

Lindblad master Eq.
+quantum Diffusion

$$\rho_S = \overline{|\phi\rangle\langle\phi|}$$

Lindblad master Eq.
+quantum Diffusion

$$\rho_S = \overline{|\phi\rangle\langle\phi|}$$

Gardiner and Zoller, *Quantum noise* (2000)
Breuer and Petruccione, *The Theory of Open Quant. Syst. (2002).*

Quantum Monte-Carlo (Exact)

Stoch. master Eq.
+ quantum Diff.

$$D = \overline{\rho_S \otimes \rho_E}$$

$$\rho_S = \overline{|\phi_1\rangle\langle\phi_2|}$$

Stoch. master Eq.
+ quantum Diff.

$$D = \overline{\prod \rho_i}$$

$$D = \overline{|\phi_1\rangle\langle\phi_2|}$$

(G. Hupin talk)

More insight in mean-field dynamics:

Exact state $|\Psi(t)\rangle$ → Trial states $\begin{cases} |Q(t)\rangle \\ |Q+\delta Q\rangle = e^{\sum_\alpha \delta q_\alpha A_\alpha} |Q\rangle \end{cases}$

The approximate evolution is obtained by minimizing the action:

$$S = \int_{t_0}^{t_1} ds \langle Q | i\hbar \partial_t - H | Q \rangle$$

Included part: average evolution

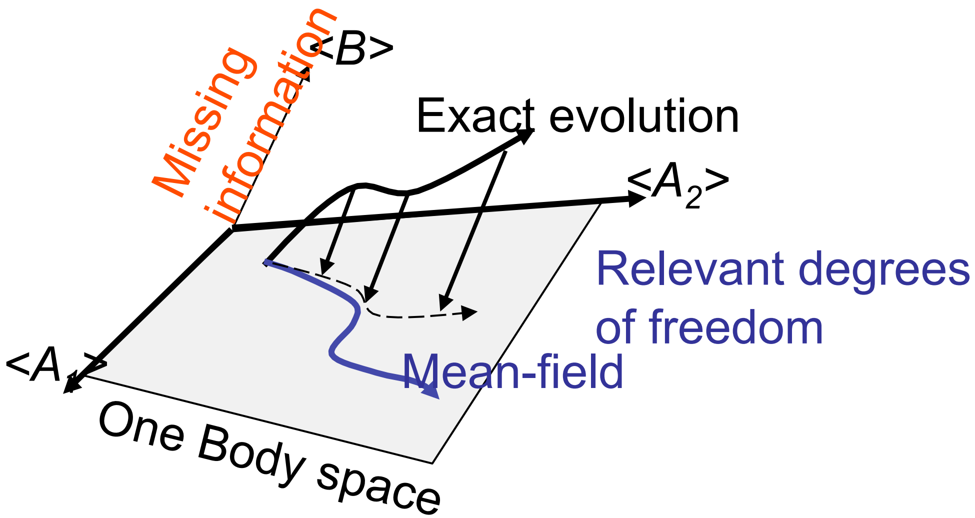
$$i\hbar \frac{d\langle A_\alpha \rangle}{dt} = \langle [A_\alpha, H] \rangle$$ → exact Ehrenfest evolution

$$H = \mathcal{P}_1 H + (1 - \mathcal{P}_1) H$$

Missing part: correlations

$$|dQ\rangle = \sum_\alpha dq_\alpha A_\alpha |dQ\rangle = \frac{dt}{i\hbar} \mathcal{P}_1(t) H |Q\rangle$$

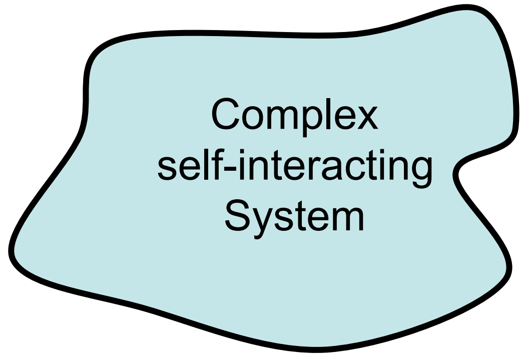
→ $$i\hbar \frac{d\langle A_\alpha A_\beta \rangle}{dt} \neq \langle [A_\alpha A_\beta, H] \rangle$$



Hamiltonian splitting

$$H = \mathcal{P}_1 H + (1 - \mathcal{P}_1) H$$

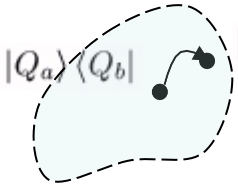
System Environment



The idea is now to treat the missing information as the *Environment* for the Relevant part (*System*)

Existence theorem : Optimal stochastic path from observable evolution

D. Lacroix, Ann. of Phys. 322 (2007).



Theorem :

One can always find a stochastic process for trial states such that $\overline{\langle A_\alpha\rangle},\ \overline{\langle A_\alpha A_\beta\rangle},\cdots\overline{\langle A_{\alpha_1}A_{\alpha_2}\cdots A_{\alpha_k}\rangle}$ *evolves exactly over a short time scale.*

with

$$|Q_a+\delta Q_a\rangle = e^{\sum_\alpha \delta q_\alpha^{[a]} A_\alpha}|Q_a\rangle$$
$$|Q_b+\delta Q_b\rangle = e^{\sum_\alpha \delta q_\alpha^{[b]} A_\alpha}|Q_b\rangle$$

Valid for $D=|Q_a\rangle\langle Q_b|$

or $D=\dfrac{|Q_a\rangle\langle Q_b|}{\langle Q_b\,|\,Q_a\rangle}$

In practice

Mean-field level

$$\begin{cases}\delta q_\alpha^{[a]} = \delta q_\alpha^a\\ \delta q_\alpha^{[b]*} = \delta q_\alpha^{b\,*}\end{cases} \quad\Leftarrow\quad i\hbar\frac{d}{dt}\langle A_\alpha\rangle = \langle[A_\alpha,H]\rangle$$

Mean-field + noise

$$\begin{cases}\delta q_\alpha^{[a]} = \delta q_\alpha^a + \delta\xi_\alpha^{[2]}\\ \delta q_\alpha^{[b]*} = \delta q_\alpha^{b\,*} + \delta\eta_\alpha^{[2]}\end{cases} \quad\Leftarrow\quad \begin{aligned} i\hbar\frac{d\overline{\langle A_\alpha\rangle}}{dt} &= \overline{\langle[A_\alpha,H]\rangle}\\ i\hbar\frac{d\overline{\langle A_\alpha A_\beta\rangle}}{dt} &= \overline{\langle[A_\alpha A_\beta,H]\rangle}\end{aligned}$$

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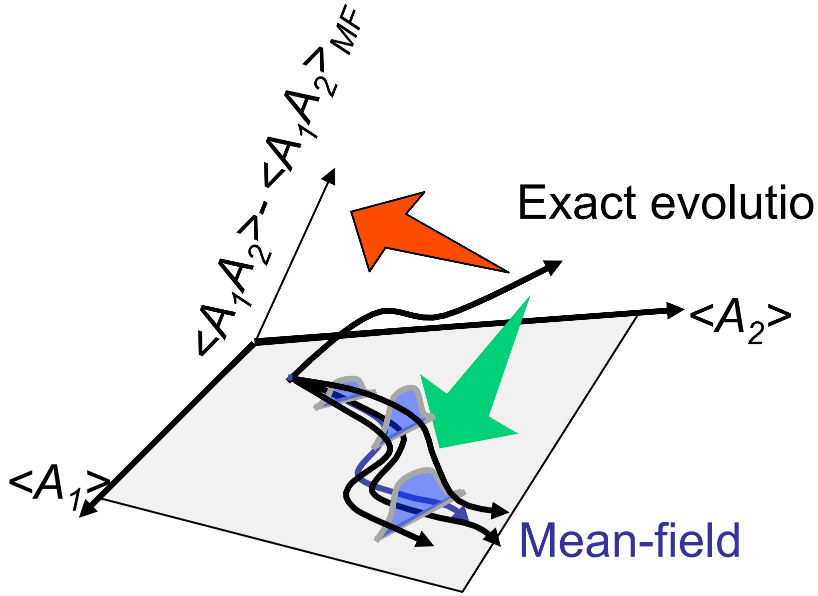
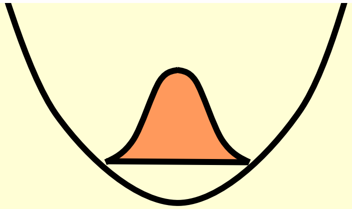


illustration: simulation of the free wave spreading with “quasi-classical states”

$t=0$

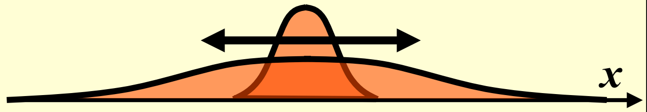
$$H_0 = \hbar\omega(a^+a + \tfrac{1}{2})$$



$\langle x^2 \rangle = \frac{1}{2\eta}$ with $\eta = \frac{m\omega}{\hbar}$

$t>0$

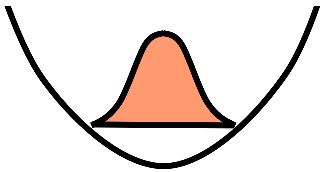
$$H = \frac{p^2}{2m} = -\frac{\hbar\omega}{4}(a^+ - a)^2$$



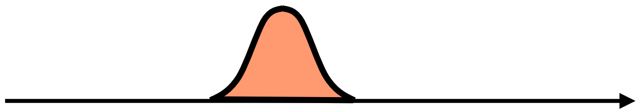
$$\langle x^2 \rangle = \frac{1}{2\eta} + \frac{\hbar^2\eta}{2m^2}t^2$$

Reduction of the information: I want to simulate the expansion with Gaussian wave-function having fixed widths. $\langle x^2 \rangle = cte, \quad \langle p^2 \rangle = cte$

Mean-field evolution:



$t>0$



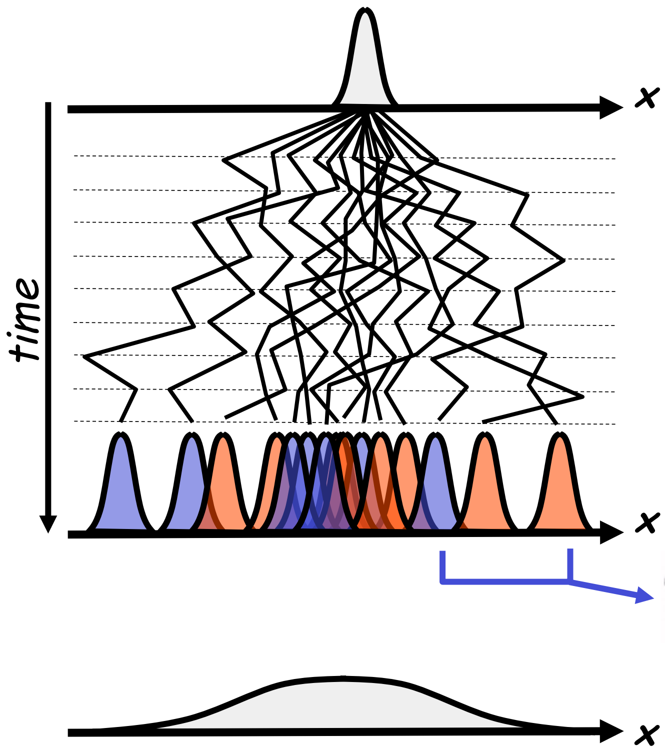
Relevant/Missing information:

Relevant degrees of freedom	Missing information	Trial states
$\langle x \rangle, \quad \langle p \rangle$	$\langle x^2 \rangle, \quad \langle p^2 \rangle, \quad \langle xp \rangle$	$\lvert Q + \delta Q \rangle = e^{\sum_\alpha \delta q_\alpha A_\alpha} \lvert Q \rangle$
$\langle a^+ \rangle, \quad \langle a \rangle$	$\langle a^{+2} \rangle, \quad \langle a^2 \rangle, \quad \langle a^+ a \rangle$	Coherent states: $\lvert \alpha + d\alpha \rangle = e^{d\alpha a^+} \lvert \alpha \rangle$

Guess of the SSE from the existence theorem

Densities

$$D = \frac{|\alpha\rangle\langle\beta|}{\langle\beta|\alpha\rangle}$$ with $$\langle\beta + \mathrm{d}\beta| = \langle\beta|\mathrm{e}^{\mathrm{d}\beta^* a}$$ $$|\alpha + \mathrm{d}\alpha\rangle = \mathrm{e}^{\mathrm{d}\alpha a^+}|\alpha\rangle$$



$$\mathrm{Tr}(Dx^2) = \frac{1}{2\eta} + X^2$$

$$\overline{\mathrm{Tr}(Dx^2)} = \frac{1}{2\eta} + \frac{\hbar^2\eta}{2m^2}t^2$$

Stochastic c-number evolution from Ehrenfest theorem

$$\begin{cases} \mathrm{d}\alpha = \overline{\mathrm{d}\alpha} + \mathrm{d}\xi^{[2]}, \\ \mathrm{d}\beta^* = \overline{\mathrm{d}\beta^*} + \mathrm{d}\eta^{[2]} \end{cases}$$

mean values

$$\overline{d\langle a\rangle} = \overline{d\alpha}$$
$$\overline{d\langle a^+\rangle} = \overline{d\beta^*}$$

fluctuations

$$\overline{d\langle a^2\rangle} = 2\alpha\overline{d\alpha} + \overline{d\xi^{[2]}d\xi^{[2]}}$$
$$\overline{d\langle a^{+2}\rangle} = 2\beta^*\overline{d\beta^*} + \overline{d\eta^{[2]}d\eta^{[2]}}$$

Nature of the stochastic mechanics

$$\begin{cases} X = \frac{1}{\sqrt{2\eta}}(\alpha + \beta^*), \\ P = \mathrm{i}\hbar\sqrt{\frac{\eta}{2}}(\beta^* - \alpha) \end{cases} \Rightarrow \begin{cases} \mathrm{d}X = \frac{P}{m}\mathrm{d}t + \mathrm{d}\chi_1 \\ \mathrm{d}P = \mathrm{d}\chi_2, \end{cases}$$

with $$\overline{\mathrm{d}\chi_1\,\mathrm{d}\chi_2} = \frac{\hbar^2\eta}{2m}\mathrm{d}t$$

the quantum wave spreading can be simulated by a classical brownian motion in the complex plane

D. Lacroix, Ann. Phys. 322 (2007)

Starting point: $H=\sum_{ij}\langle i|T|j\rangle a_i^+ a_j + \frac{1}{2}\sum_{ijkl}\langle ij|v_{12}|lk\rangle a_i^+ a_j^+ a_l a_k$

$D_{ab} = |\Phi_a\rangle\langle\Phi_b|$ with $\langle\Phi_b\,|\,\Phi_a\rangle = 1$

$\rho_1 = \sum |\alpha_i\rangle\langle\beta_i|$

Observables $\langle j|\rho_1|i\rangle = \langle a_i^+ a_j\rangle$

Fluctuations $\langle ij|\rho_{12}|kl\rangle = \langle a_k^+ a_l^+ a_j a_i\rangle$

Ehrenfest theorem ➡ BBGKY hierarchy

$$i\hbar\frac{d}{dt}\rho_1 = [h_{MF},\rho_1],$$

$$i\hbar\frac{d}{dt}\rho_{12} = [h_{MF}(1)+h_{MF}(2),\rho_{12}] + (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

$$v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2)$$

Stochastic one-body evolution

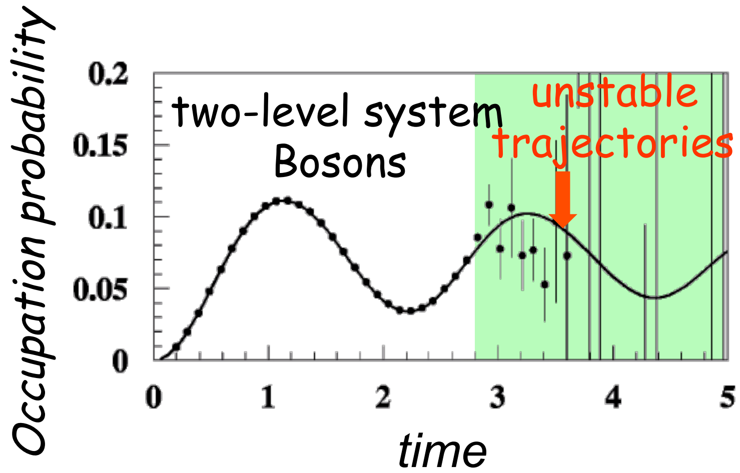
$$d\rho_1 = [h_{MF},\rho_1] + \sum_\lambda d\xi_\lambda^{[2]}(1-\rho_1)O_\lambda\rho_1 + \sum_\lambda d\eta_\lambda^{[2]}(1-\rho_1)O_\lambda\rho_1$$

with $\overline{d\xi_\lambda^{[2]}d\xi_{\lambda'}^{[2]}} = -\overline{d\eta_\lambda^{[2]}d\eta_{\lambda'}^{[2]}} = \delta_{\lambda\lambda'}\frac{dt}{i\hbar}$

- The method is general.
  the SSE are deduced easily

  ➡ extension to Stochastic TDHFB
  DL, arXiv nucl-th 0605033
- The mean-field appears naturally and the interpretation is easier
- the numerical effort can be reduced by reducing the number of observables

but…



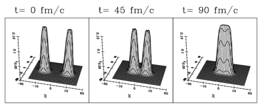
Summary, stochastic methods for Many-Body Fermionic and bosonic systems

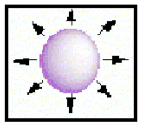
Approximate evolution

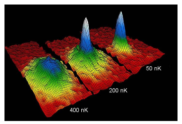
Mean-field	Simplified QJ	Generalized QJ	Exact QJ	variational QJ
$D = \|\Phi\rangle\langle\Phi\|$	$D = \overline{\|\Phi_1\rangle\langle\Phi_2\|}$	$D = \overline{\|\Phi\rangle\langle\Phi\|}$	$D = \overline{\|\Phi_1\rangle\langle\Phi_2\|}$	$D = \overline{\|Q_1\rangle\langle Q_2\|}$ $\|Q_1\rangle = \|q_1, \cdots, q_N\rangle$
~~Fluctuation~~ ~~Dissipation~~	Fluctuation ✓ ~~Dissipation~~	Fluctuation ✓ Dissipation ✓	Everything ✓	Partially everything ✓

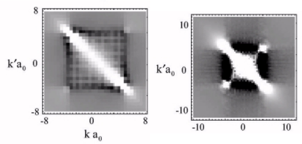
Numerical issues

Mean-field	Simplified QJ	Generalized QJ	Exact QJ	variational QJ
	Flexible	Fixed	Fixed	Flexible









Numerical instabilities