Dissipation in Multidimensional Quantum Tunneling and Subbarrier Fusion

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ECT* Workshop “Decoherence in QM Systems

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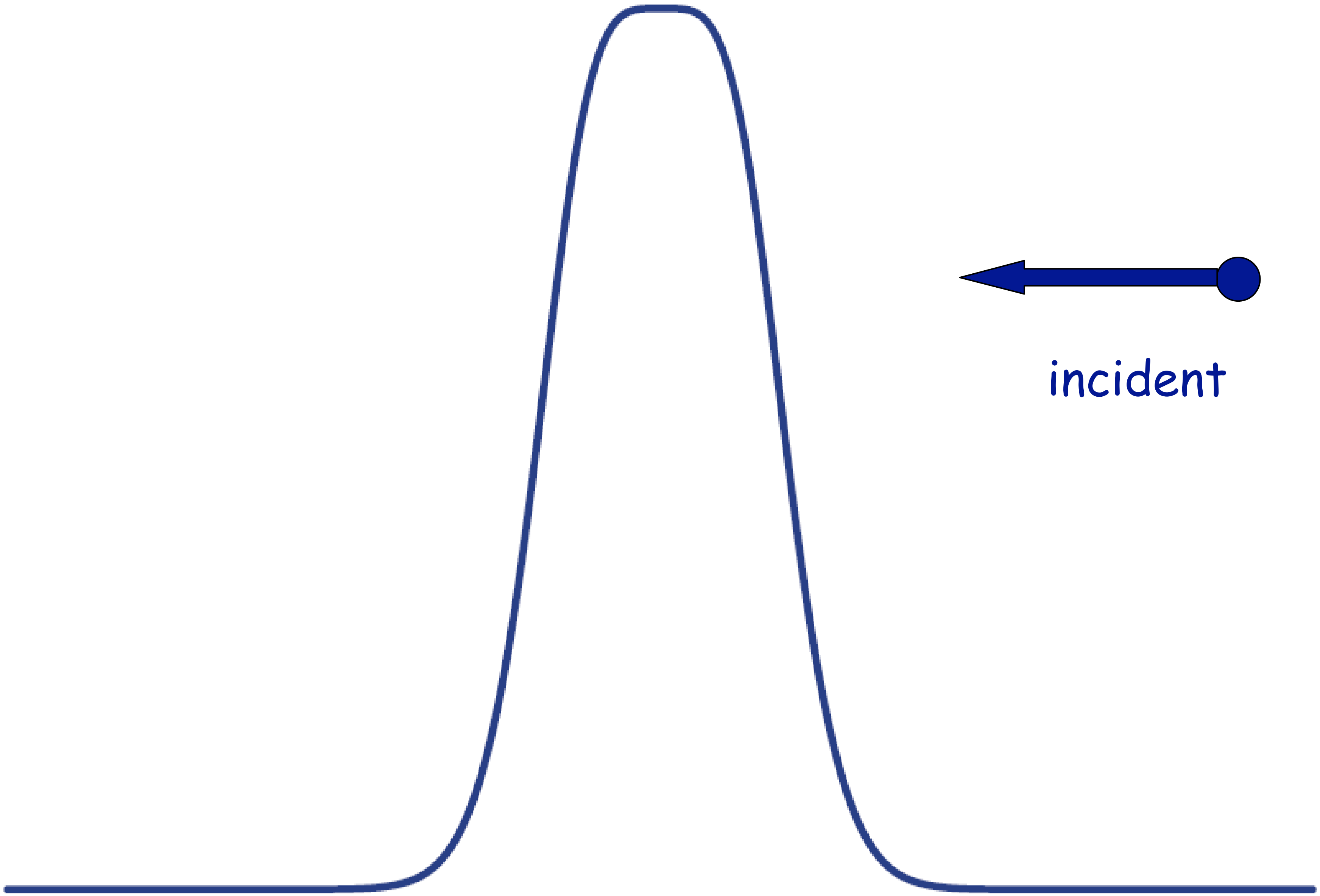
One-dimensional quantum tunneling is simple!

☺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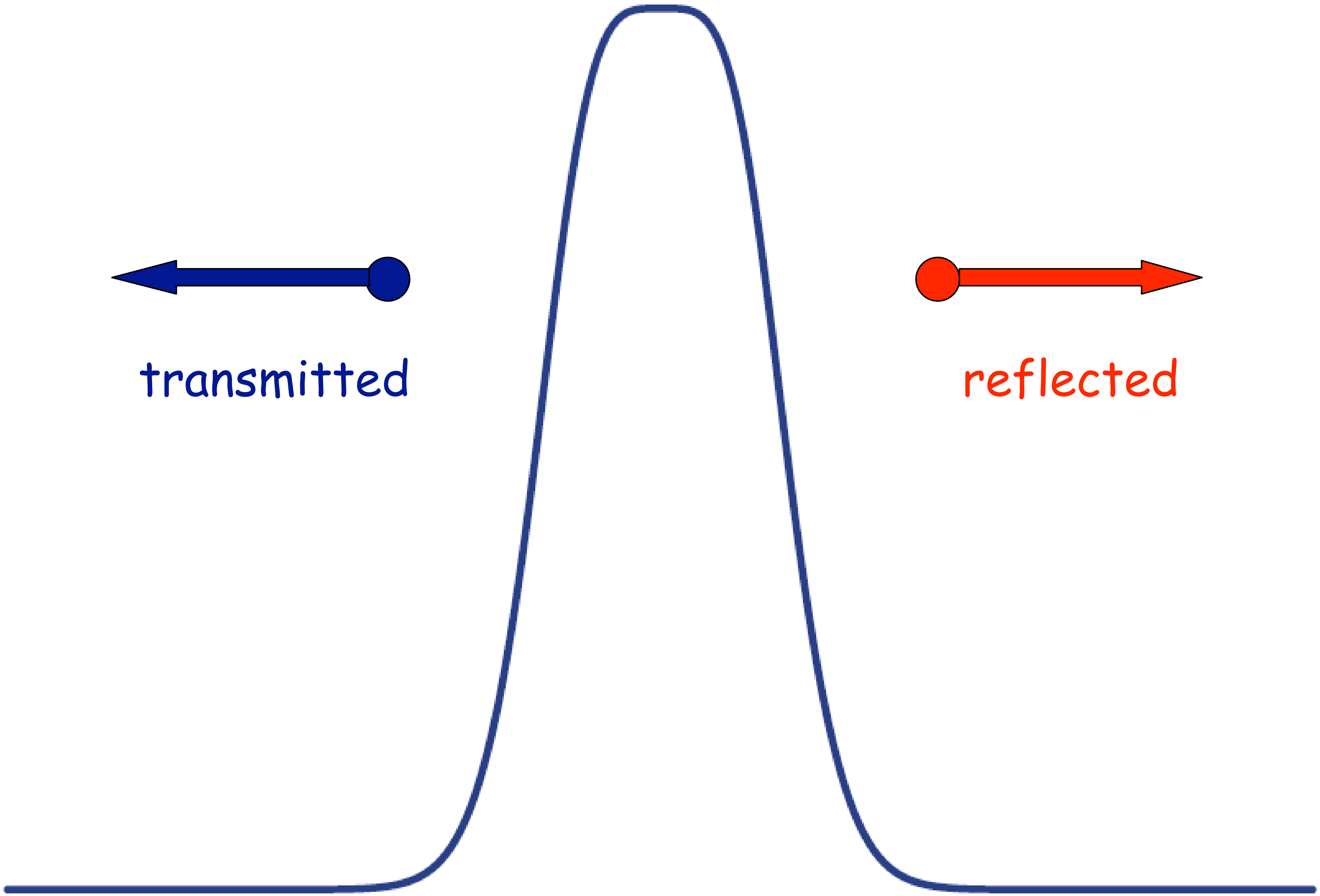
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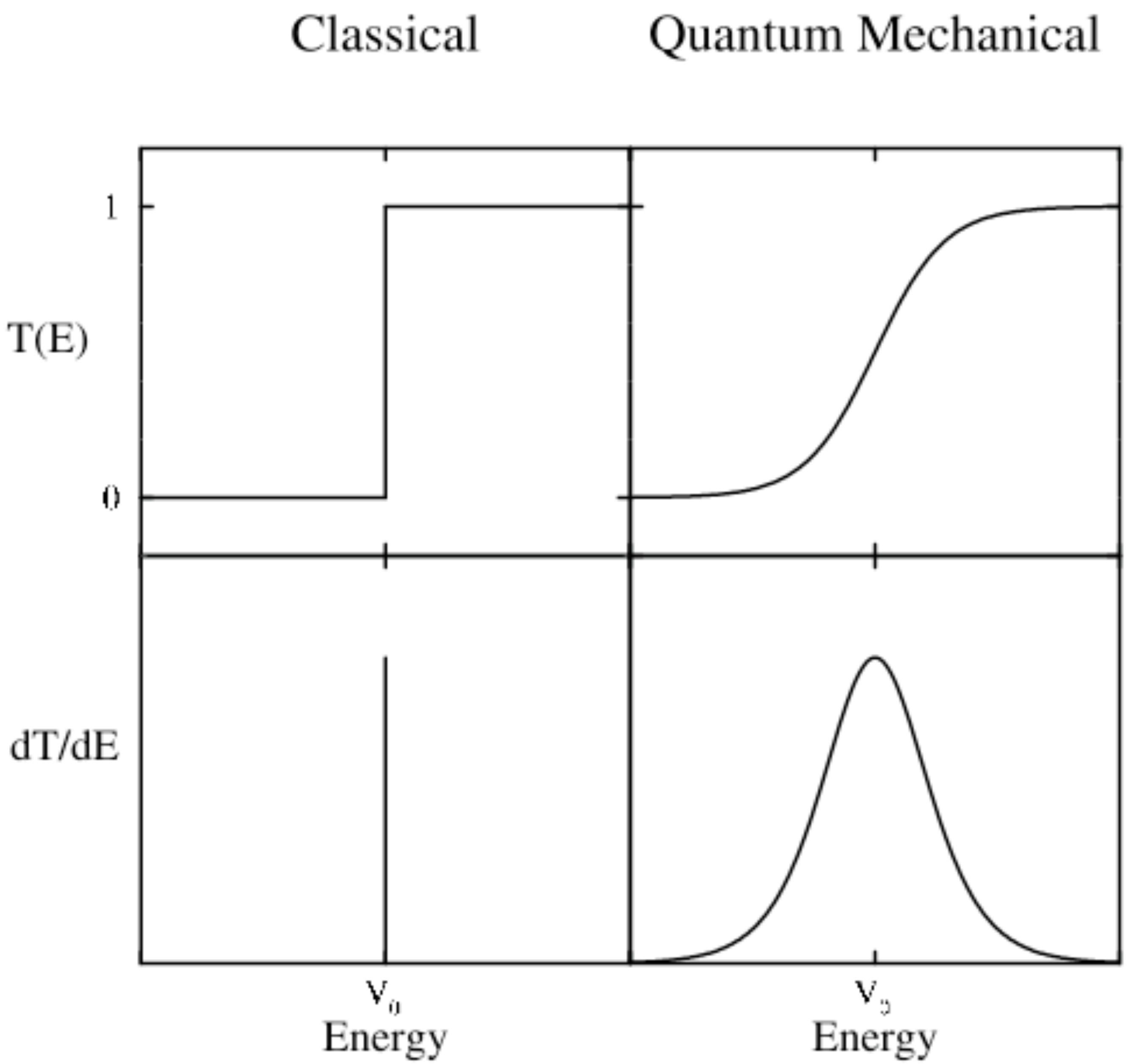


Classical
Quantum Mechanical
T(E)
1
0
dT/dE
V_0
Energy
V_0
Energy

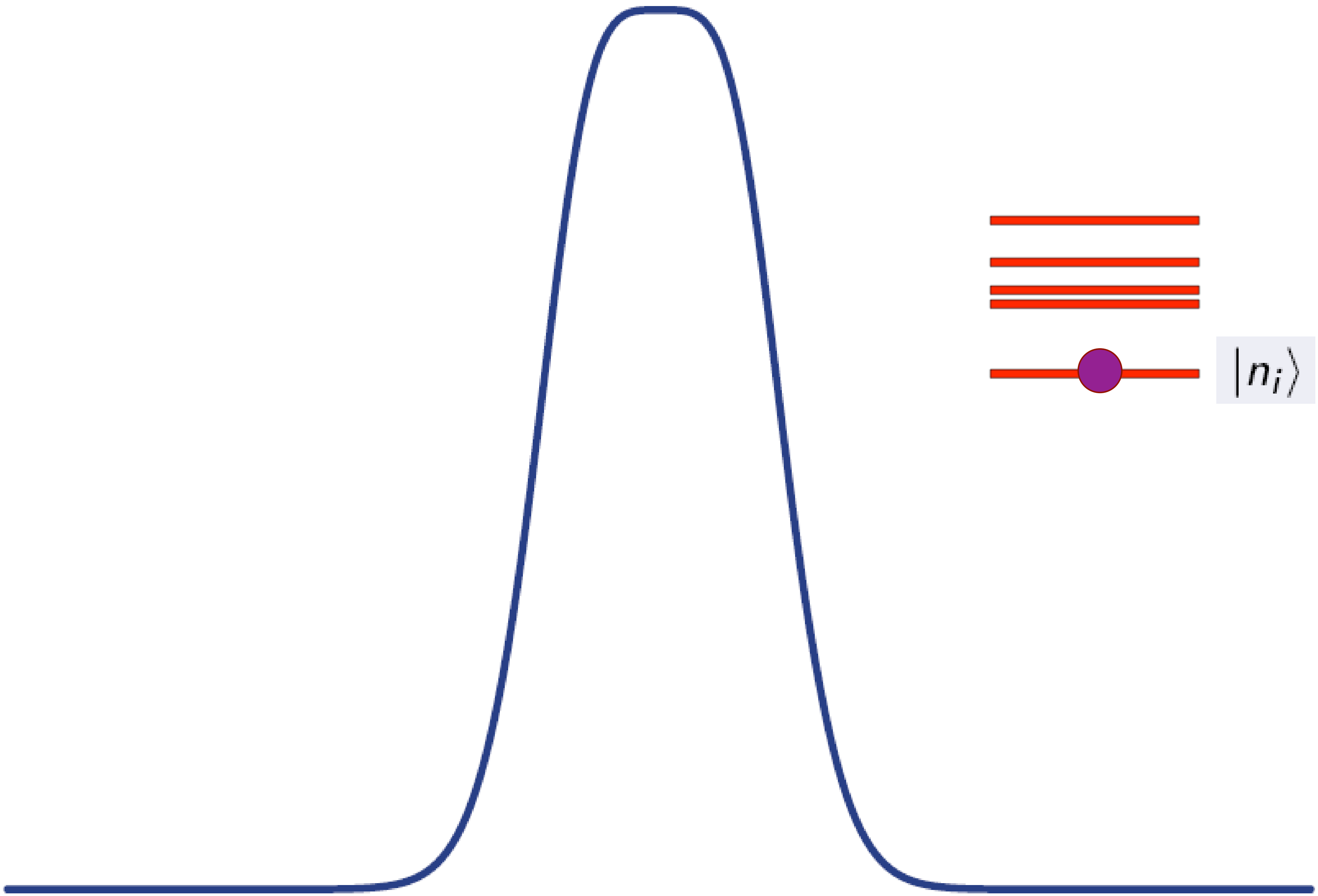
What we do:

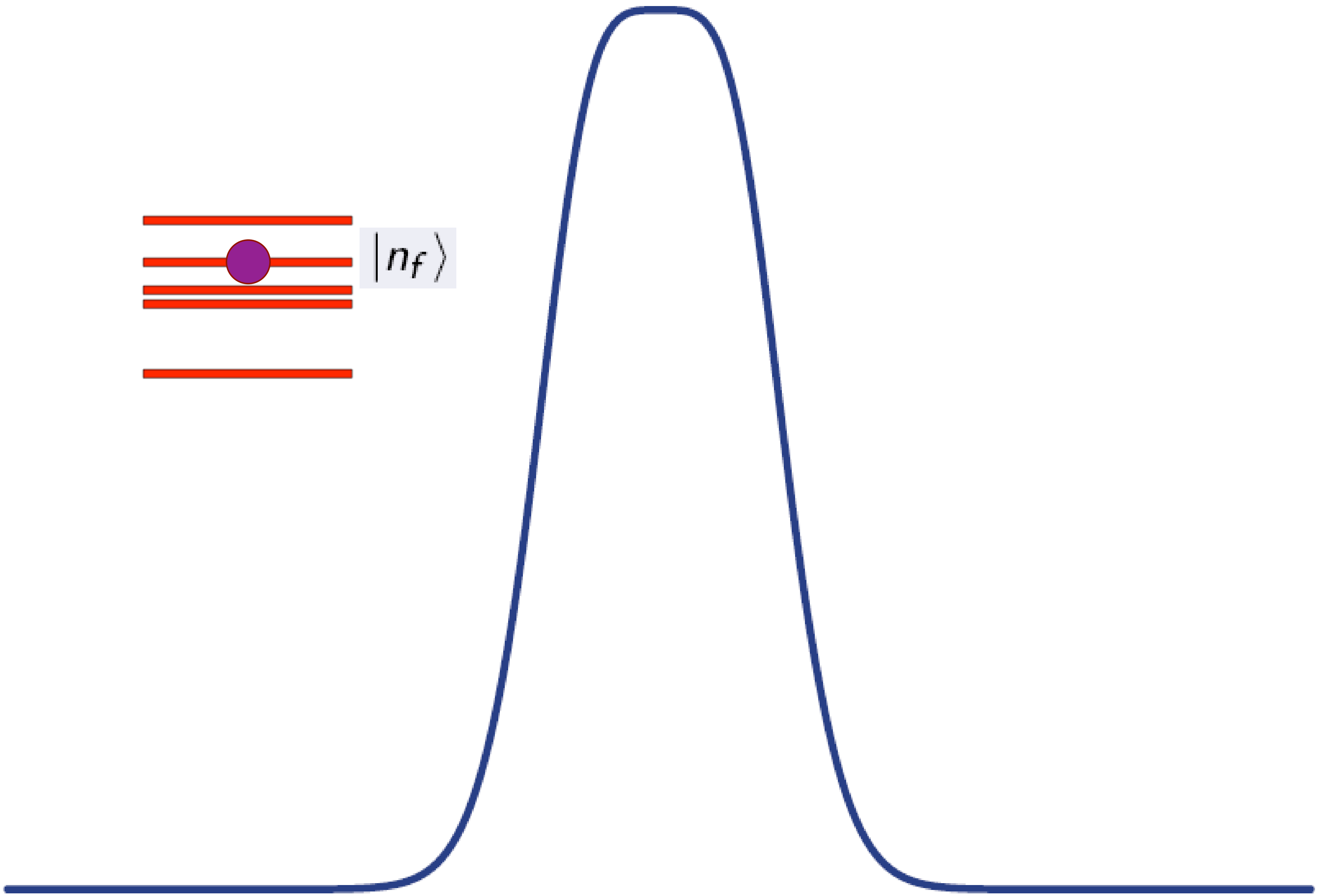
$$\hat{H} = \frac{\mathbf{P}^2}{2M} + V(R) + H_0(\mathbf{q}) + V_{\rm int}(\mathbf{q}, \mathbf{R})$$

Barrier + an absorbing potential

"environment"

Sometimes it is easier to think of an idealized problem…


$|n_i\rangle$


$|n_f\rangle$

Calculating the fusion cross-section:

$$\hat{H} = \frac{\mathbf{P}^2}{2M} + V(R) + H_0(\mathbf{q}) + V_{\mathrm{int}}(\mathbf{q}, \mathbf{R})$$

$$\sigma(E) \propto \sum_f \left| \left\langle f \left| \frac{1}{E - \hat{H}} \right| i \right\rangle \right|^2$$

Eigenchannels, assume H_0 << E

$$\sigma(E) \propto \sum_f \left| \left\langle f \left| \frac{1}{E - \frac{\mathbf{P}^2}{2M} - V(R) - V_{\mathrm{int}}(\mathbf{q}, \mathbf{R})} \right| i \right\rangle \right|^2$$

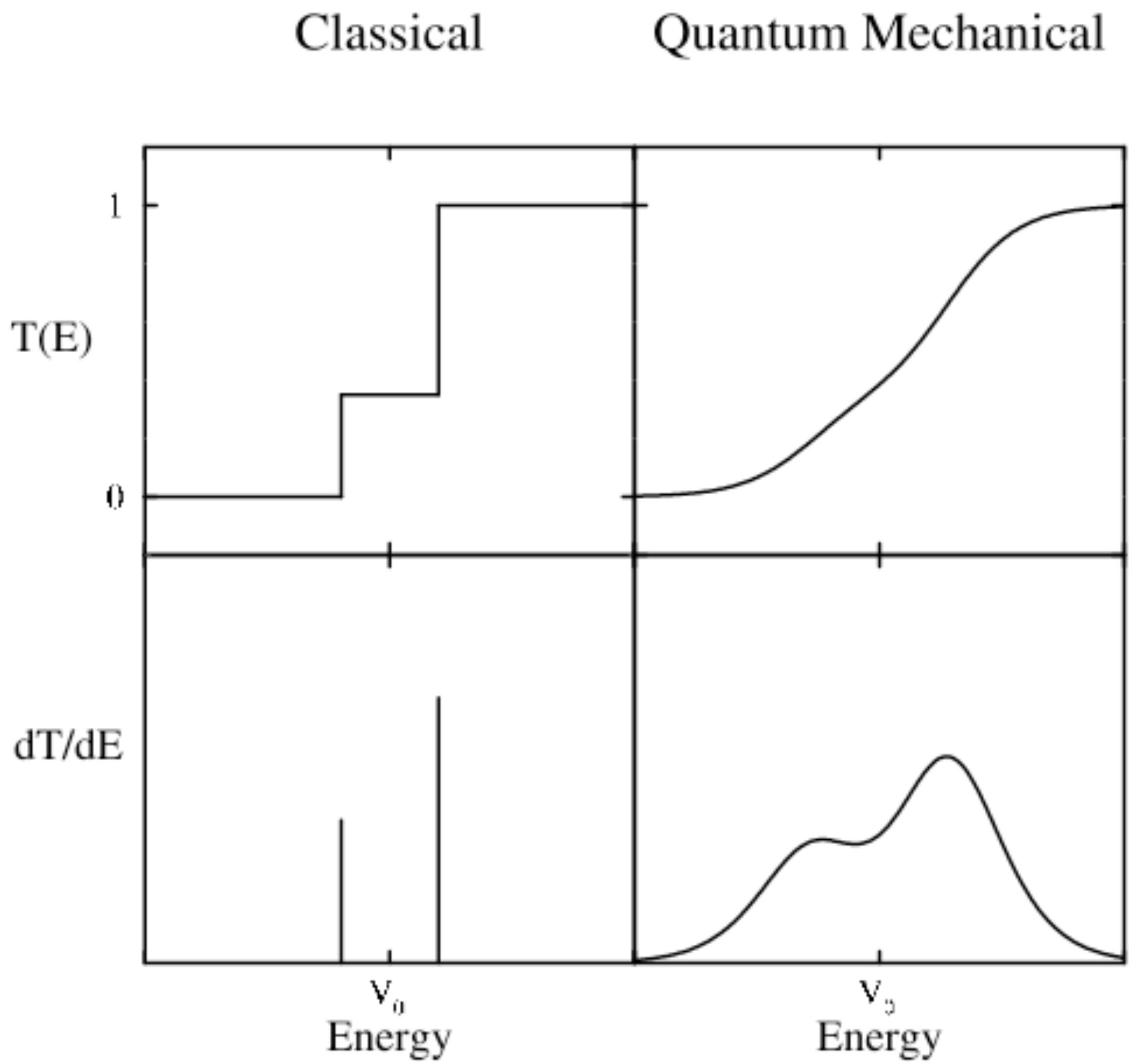
Eigenchannels, assume $H_0 \ll E$

$$\sigma(E) \propto \sum_f \left| \left\langle f \left| \frac{1}{E - \frac{\mathbf{P}^2}{2M} - V(R) - V_{\rm int}(\mathbf{q}, \mathbf{R})} \right| i \right\rangle \right|^2$$

$$\sigma \propto \sum_f \left| \left\langle f \left| \frac{1}{E - \frac{\mathbf{P}^2}{2M} - V(R) - V_{\rm int}(\mathbf{q}, \mathbf{R})} \left(\int d^3q |q\rangle\langle q| \right) \right| i \right\rangle \right|^2$$

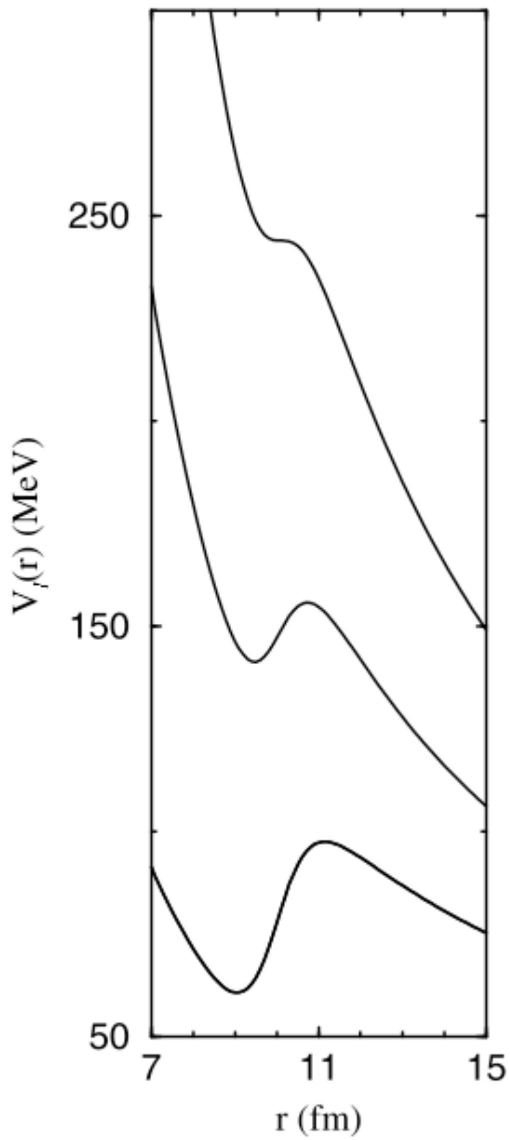
If $\sum_f |f\rangle\langle f| = 1$

$$\sigma_{\rm total} = \int d^3q |\Psi_i(q)|^2 \sigma(\text{calc. w/ pot. } V(R) + V_{\rm int}(q, \mathbf{R}))$$



Classical
Quantum Mechanical
T(E)
1
0
dT/dE
V0
Energy
V0
Energy

Subbarrier Fusion - Experimental Observables



$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E),$$

$$\langle \ell(E) \rangle = \frac{\sum_{\ell=0}^{\infty} \ell \sigma_\ell(E)}{\sum_{\ell=0}^{\infty} \sigma_\ell(E)}.$$

$$\sigma_\ell(E) = \frac{\pi \hbar^2}{2\mu E}(2\ell + 1) T_\ell(E),$$

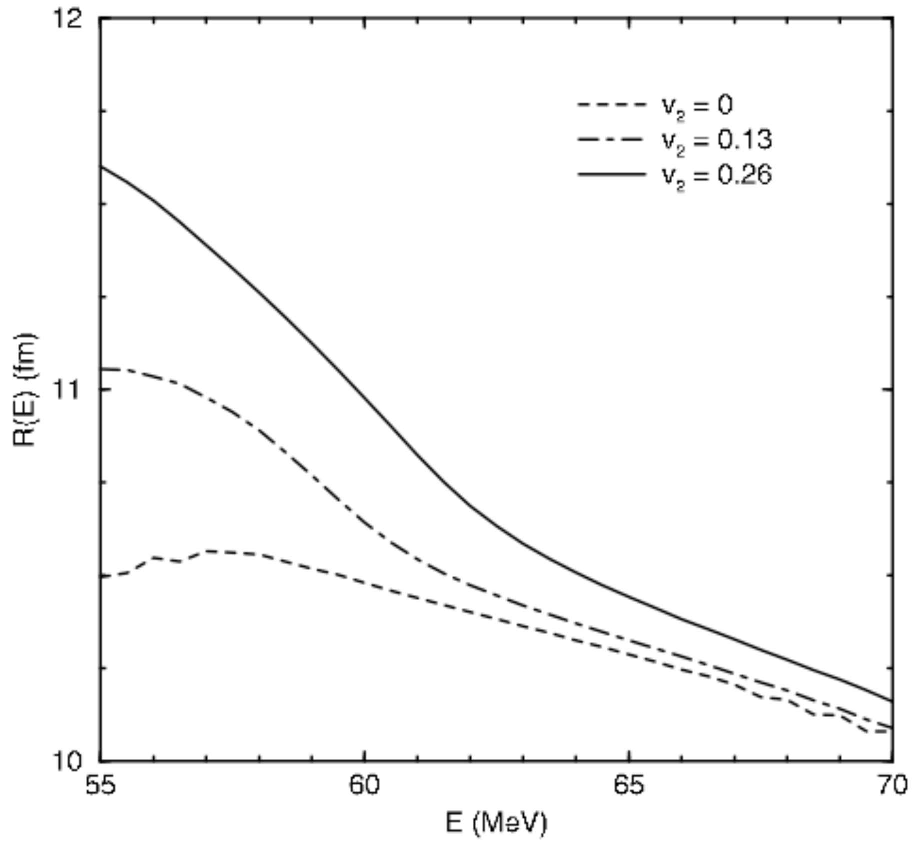
One-dimensional Model

$$T_\ell(E) = \left[1 + \exp\sqrt{\frac{2\mu}{\hbar^2}}\int_{r_{1\ell}}^{r_{2\ell}} dr\left[V_0(r) + \frac{\hbar^2\ell(\ell+1)}{2\mu r^2} - E\right]^{1/2}\right]^{-1}.$$

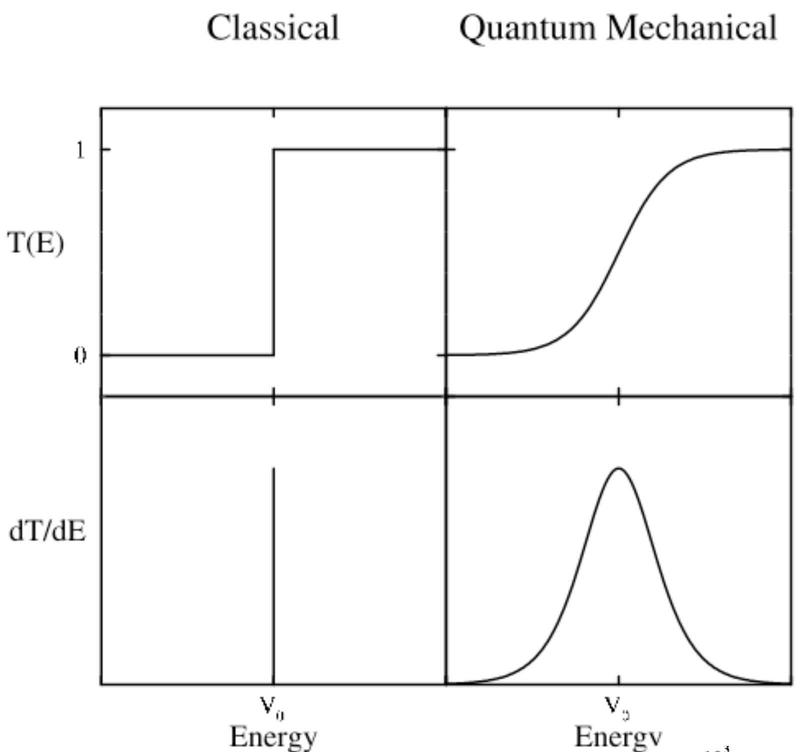
$$T_\ell \simeq T_0\left[E - \frac{\ell(\ell+1)\hbar^2}{2\mu R^2(E)}\right]$$

$$E\sigma(E) = \pi R^2(E)\int_0^E dE' T_0(E')$$

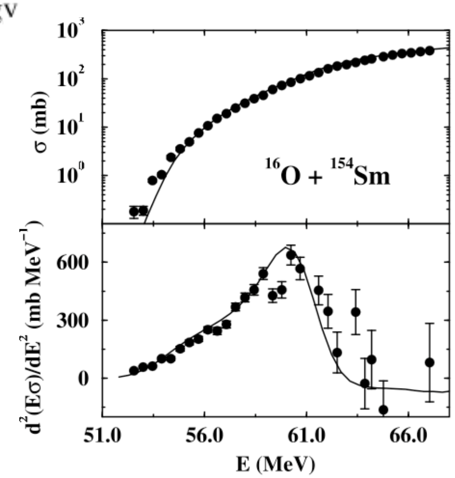
$$\frac{dT_0(E)}{dE} \sim \frac{1}{\pi R^2(E)}\frac{d^2}{dE^2}(E\sigma(E)) + \mathcal{O}(\frac{dR}{dE}).$$

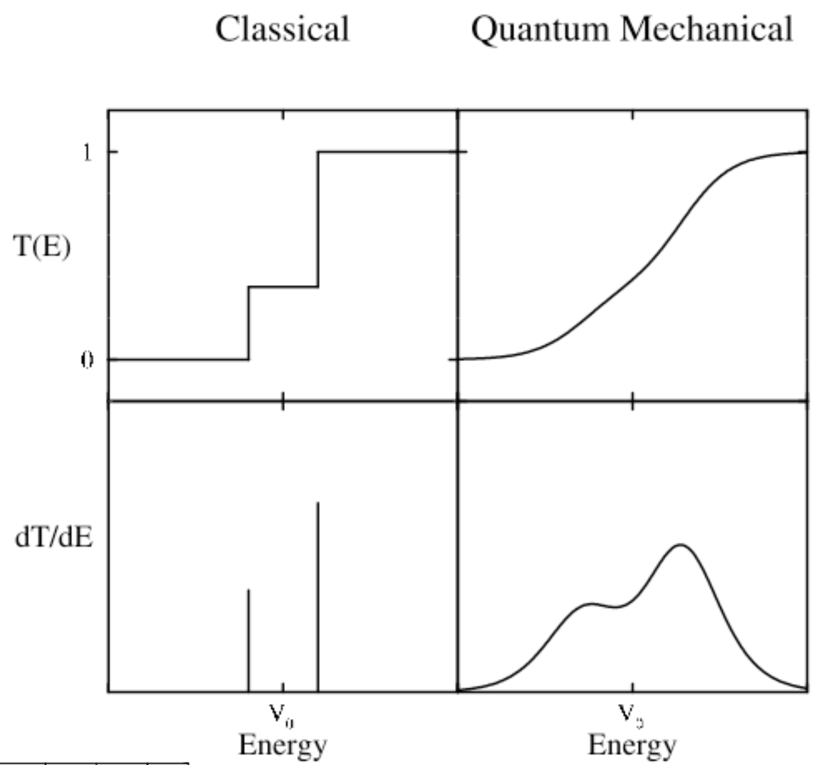


Classical versus Quantum Tunneling



One-dimensional





Multi-dimensional

Rowley, Satchler, and Stelson, 1997

What may be missing from coupled-channels calculations?

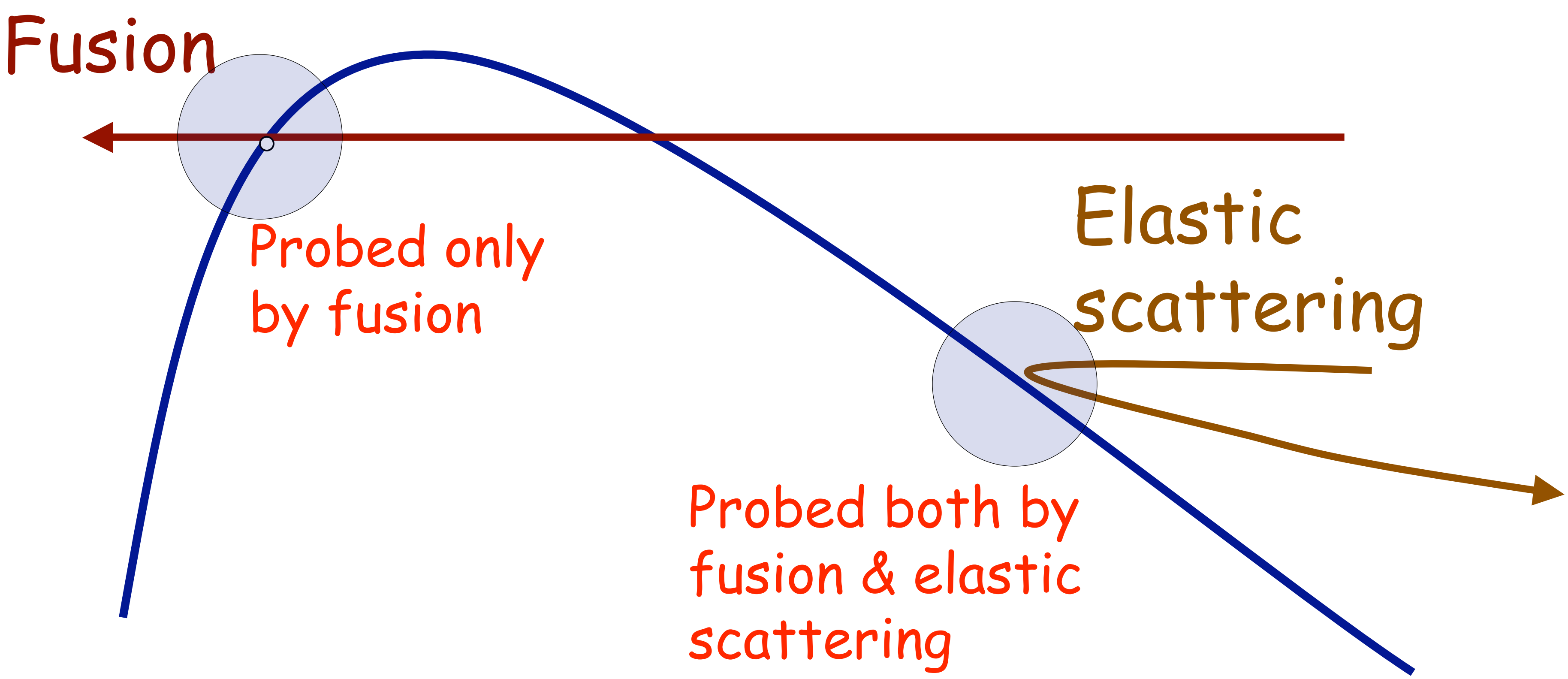
$$\sigma \propto \sum_f |\langle f|\, G(E)\, |i\rangle|^2 = \langle i| G(E)^* G(E) |i\rangle$$

$$\sum_n |n\rangle\langle n| = 1$$

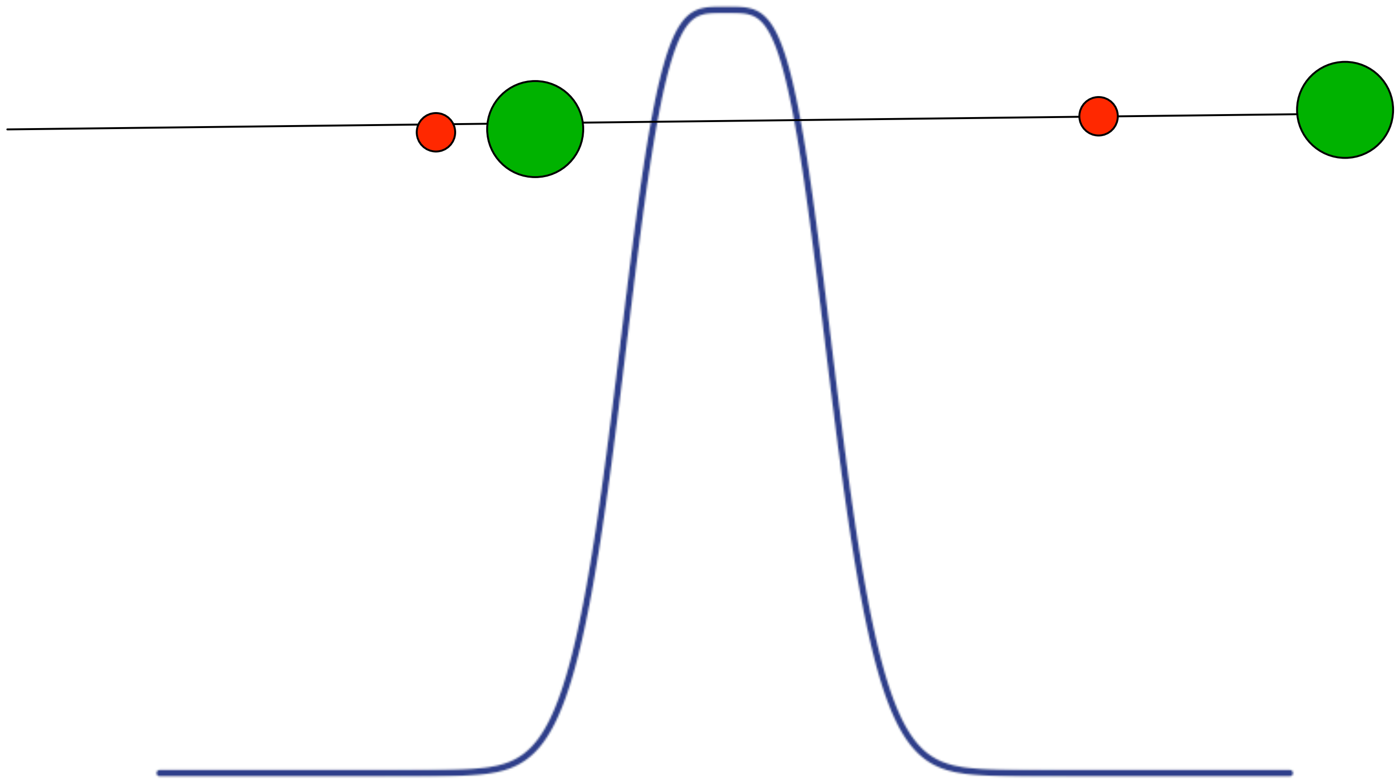
But in a practical coupled-channels calculation we have

$$\sum_{\text{states included}} |n\rangle\langle n| < 1$$

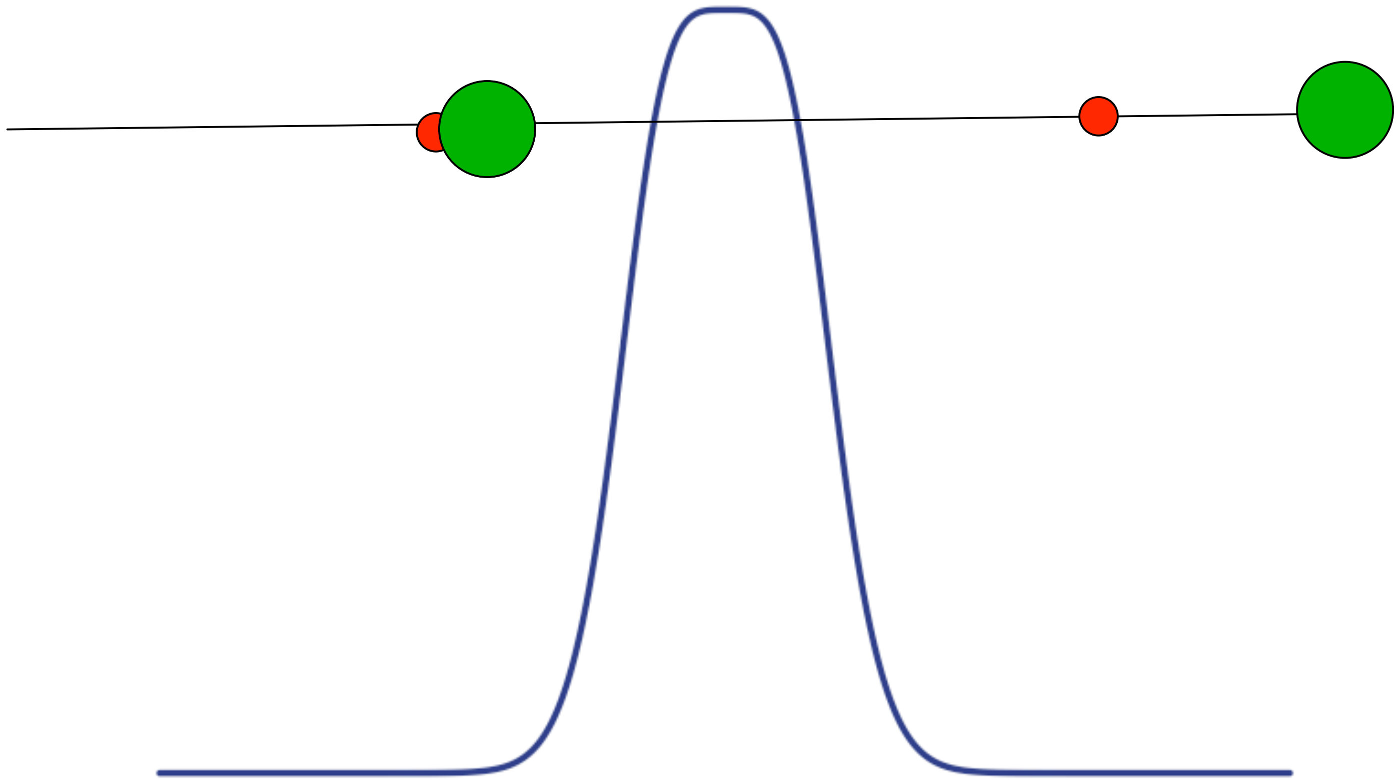
An outstanding question: Why is the diffuseness for both fusion and quasi-elastic scattering equal to 1.5 to 2 times the diffuseness for elastic scattering?



For asymmetric systems Coulomb force is relatively weaker; hence the tail of the nuclear potential can "turn over" the sum, forming the barrier at a relatively large separation:



On the other hand for symmetric systems Coulomb force is relatively stronger; hence it takes more of the nuclear potential to "turn over" the sum, forming the barrier at very close separations:



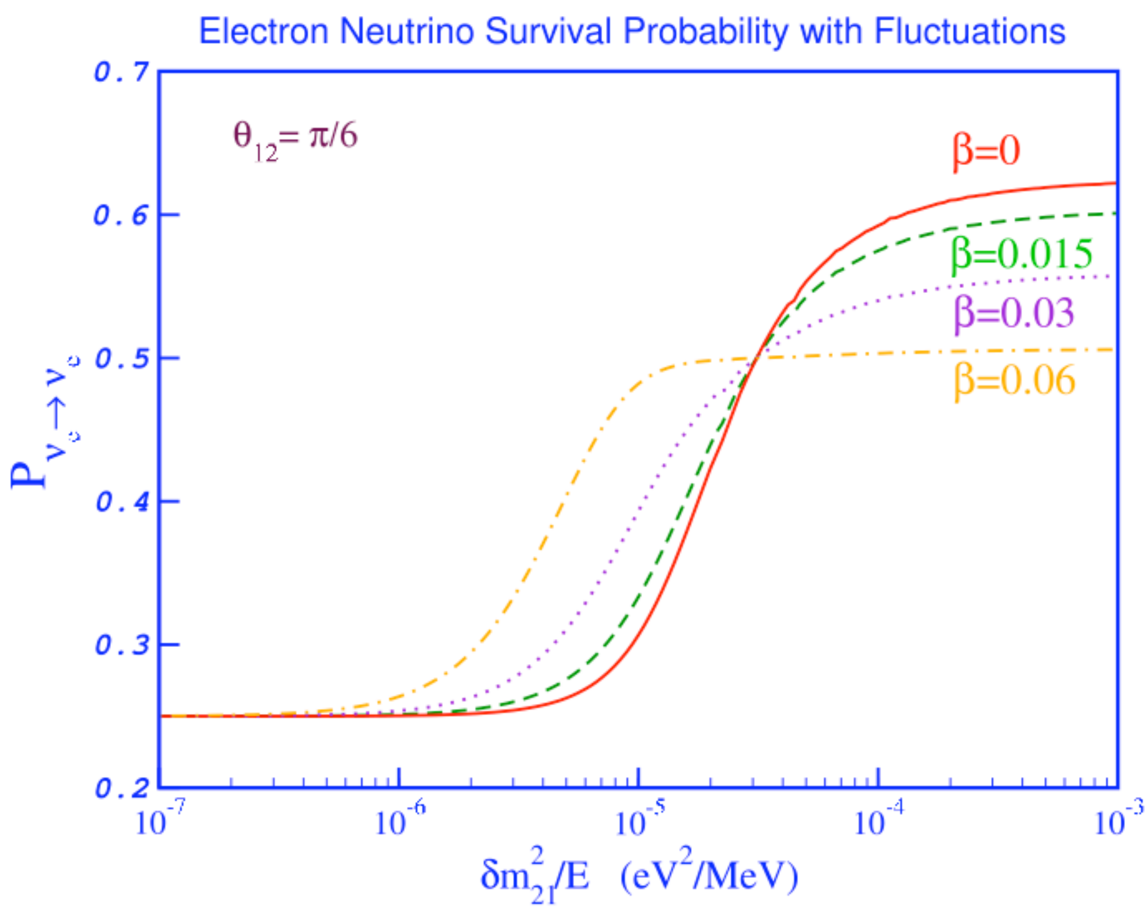
Another example: Neutrino Oscillations in fluctuating electron background

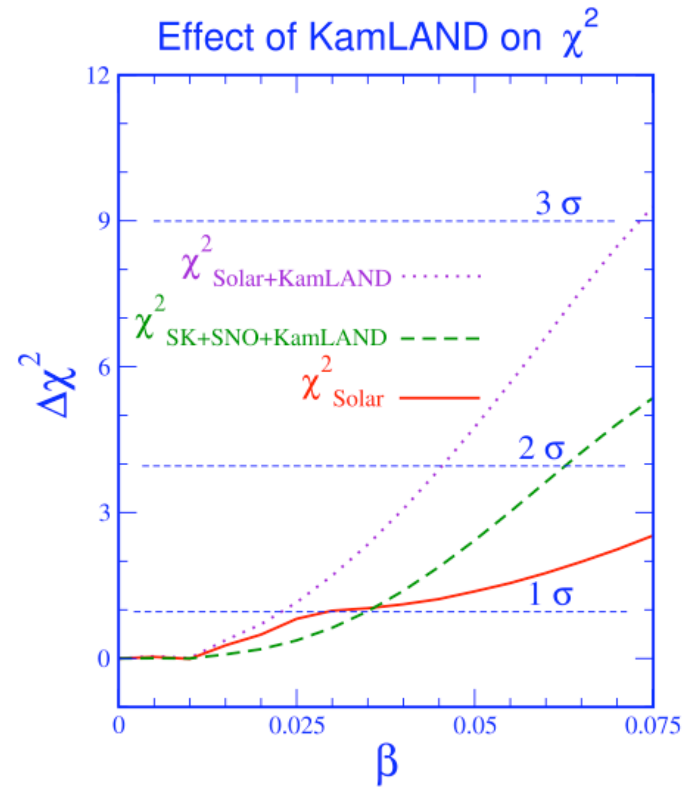
$$H = \left(-\frac{\delta m^2}{4E} \cos 2\theta + \frac{G_F}{\sqrt{2}} [N_e(r) + N_e^r(r)] \right) \sigma_z + \left(\frac{\delta m^2}{4E} \sin 2\theta \right) \sigma_x$$

$$\langle N_e^r(r) \rangle = 0$$

$$\langle N_e^r(r) N_e^r(r') \rangle = \beta^2 N_e(r) N_e(r') \exp(-|r - r'|/\tau_c)$$

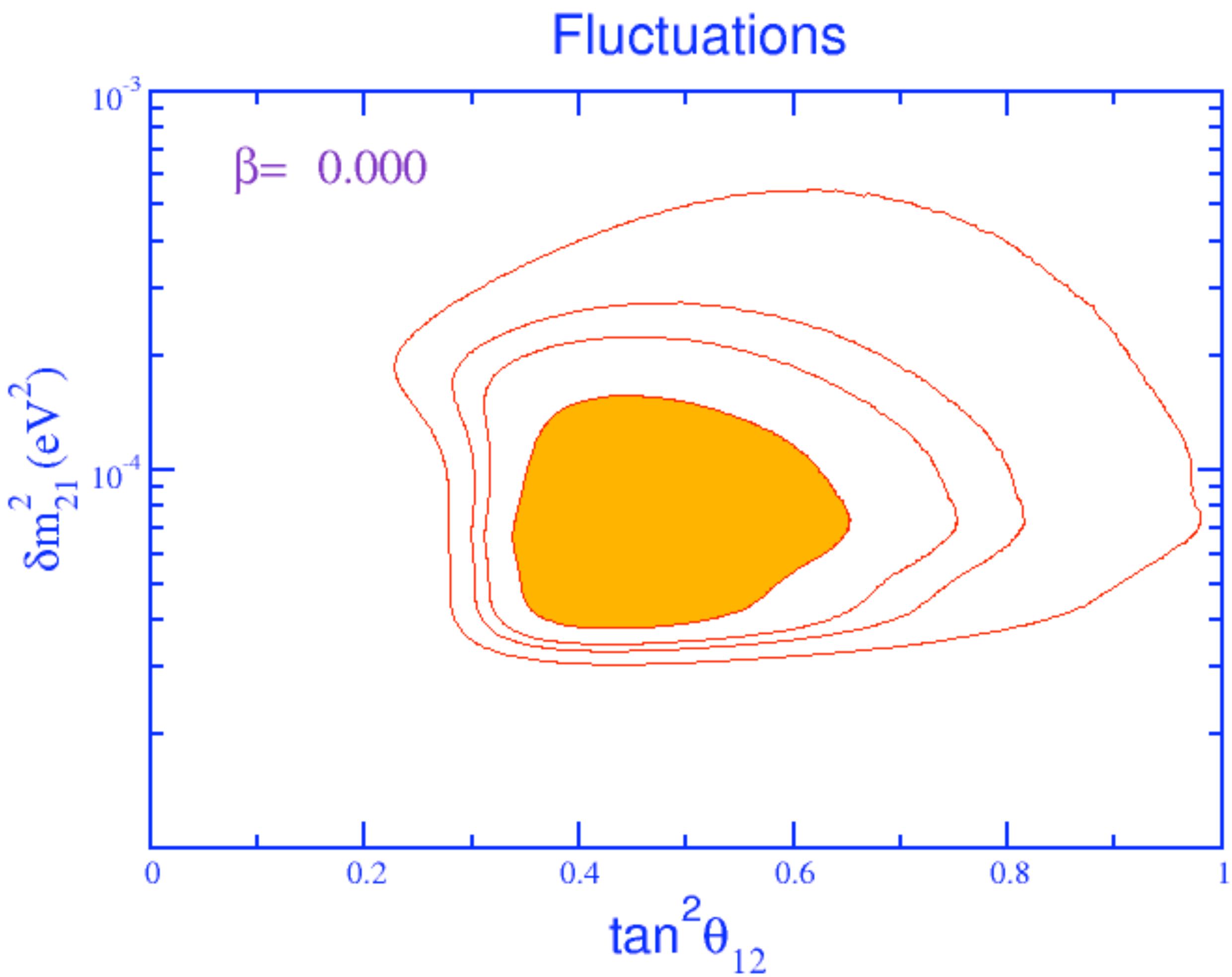
Does the solar density fluctuate?



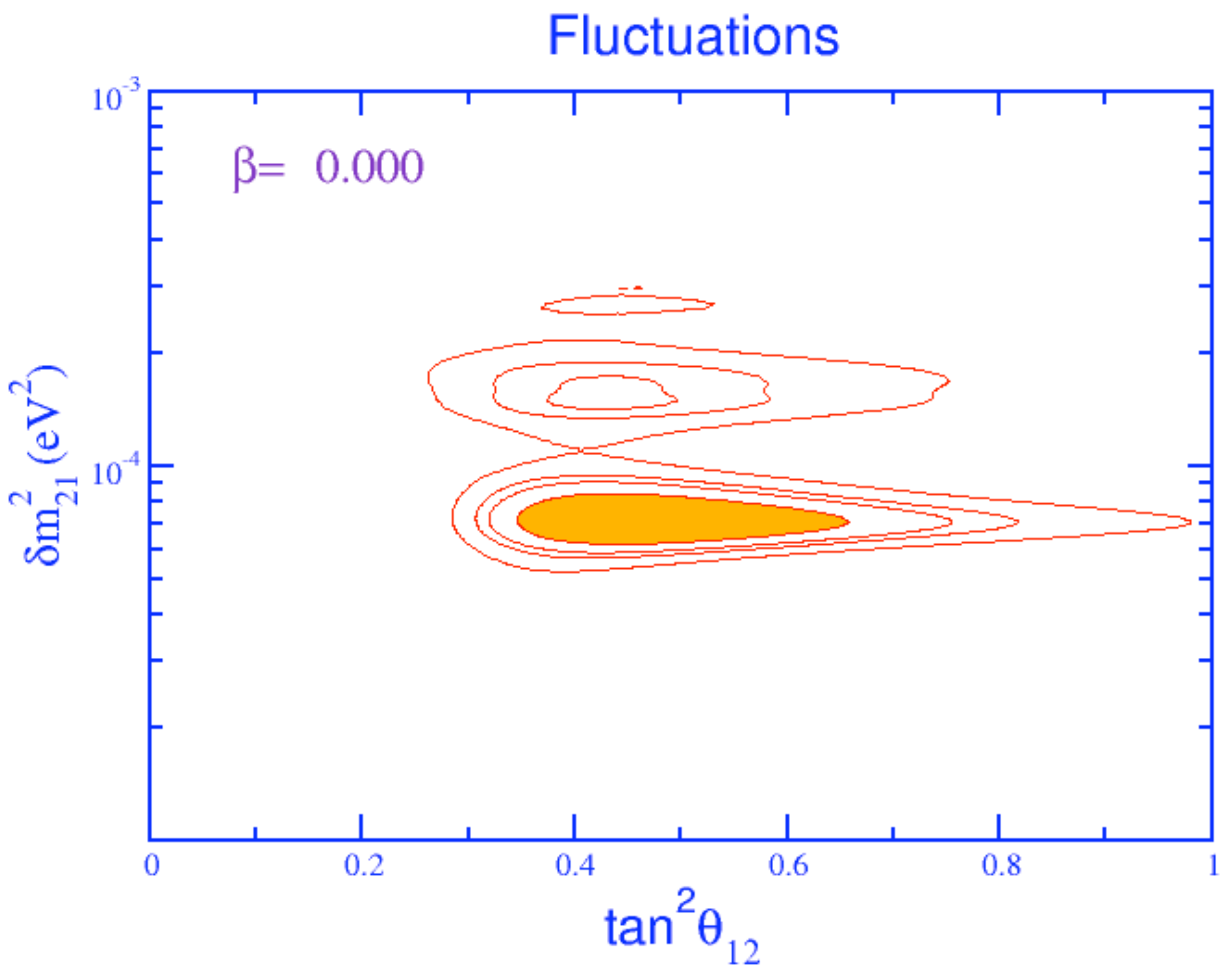


Balantekin and Yuksel,
PRD 68, 013006 (2003)

Solar data only



Solar + KamLAND



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$$\lim_{\tau_c \to \infty} \langle \hat{\rho}(r) \rangle = \frac{1}{\sqrt{2\pi\beta^2}} \int_{-\infty}^{+\infty} dx \exp\left(-x^2/(2\beta^2)\right) \hat{\rho}(r, x)$$

A Simple Model of "Environment" - Balantekin & Takigawa

$$H_0 = \hbar\omega \left(a_0^\dagger a_0 \right) + \sum_{i=1}^{m} \hbar\omega_i \left(b_i^\dagger b_i \right) + \hbar\kappa \sum_i^m \left(a_0^\dagger b_i + a_0 b_i^\dagger \right)$$

$$H_{\rm int} = \alpha_0 f(R)(a_0^\dagger + a_0)$$

Bogoliubov transformation

$$\tilde{a}_j = \chi_{j1} a_0 + \sum_{i=1}^{m} \chi_{j,i+1} b_i$$

$$\chi_{j1} = \left(1 + \sum_{i=1}^{m} \frac{\kappa^2}{(\tilde{\omega}_j - \omega_i)^2} \right)^{-1/2}$$

$$\chi_{j,i+1} = \frac{\kappa}{\tilde{\omega}_j - \omega_i} \chi_{j1}$$

$$\tilde{H}_0 = \sum_{j=1}^{m+1} \hbar\tilde{\omega}_j \tilde{a}_j^\dagger \tilde{a}_j$$

$$\tilde{\omega}_j - \omega_0 = \kappa^2 \sum_{i=1}^{m} \frac{1}{\tilde{w}_j - \omega_i}$$

Assume

$$\omega_i = i\Delta, \quad i = 0, \pm 1, \pm 2, \ldots$$

$$\frac{\pi\kappa}{\Delta} \gg 1$$

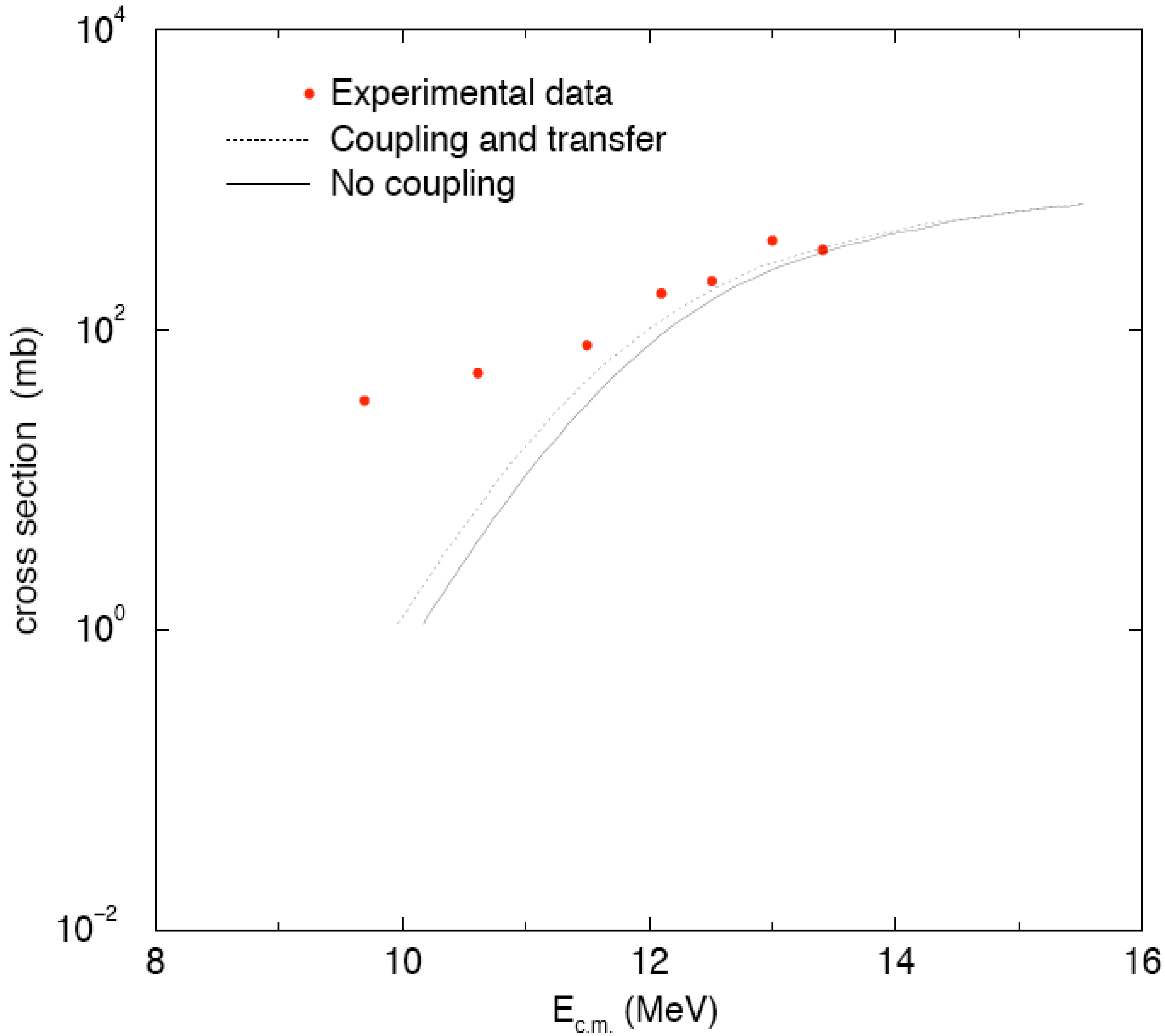
Strength distribution

$$J(\tilde{\omega}_j) = \frac{\chi_{j1}^2}{\Delta} = \frac{1}{2\pi} \frac{\Gamma}{(\tilde{\omega}_j - \omega_0)^2 + (\Gamma/2)^2}$$

Note that the strength distribution is not Ohmic: $J(\omega) \neq \eta\omega$

Subbarrier Fusion of ^{9}Li with ^{70}Zn

Data: Loveland, *et al.* PRC **74**, 064609 (2006) measured using ISAC facility at TRIUMF

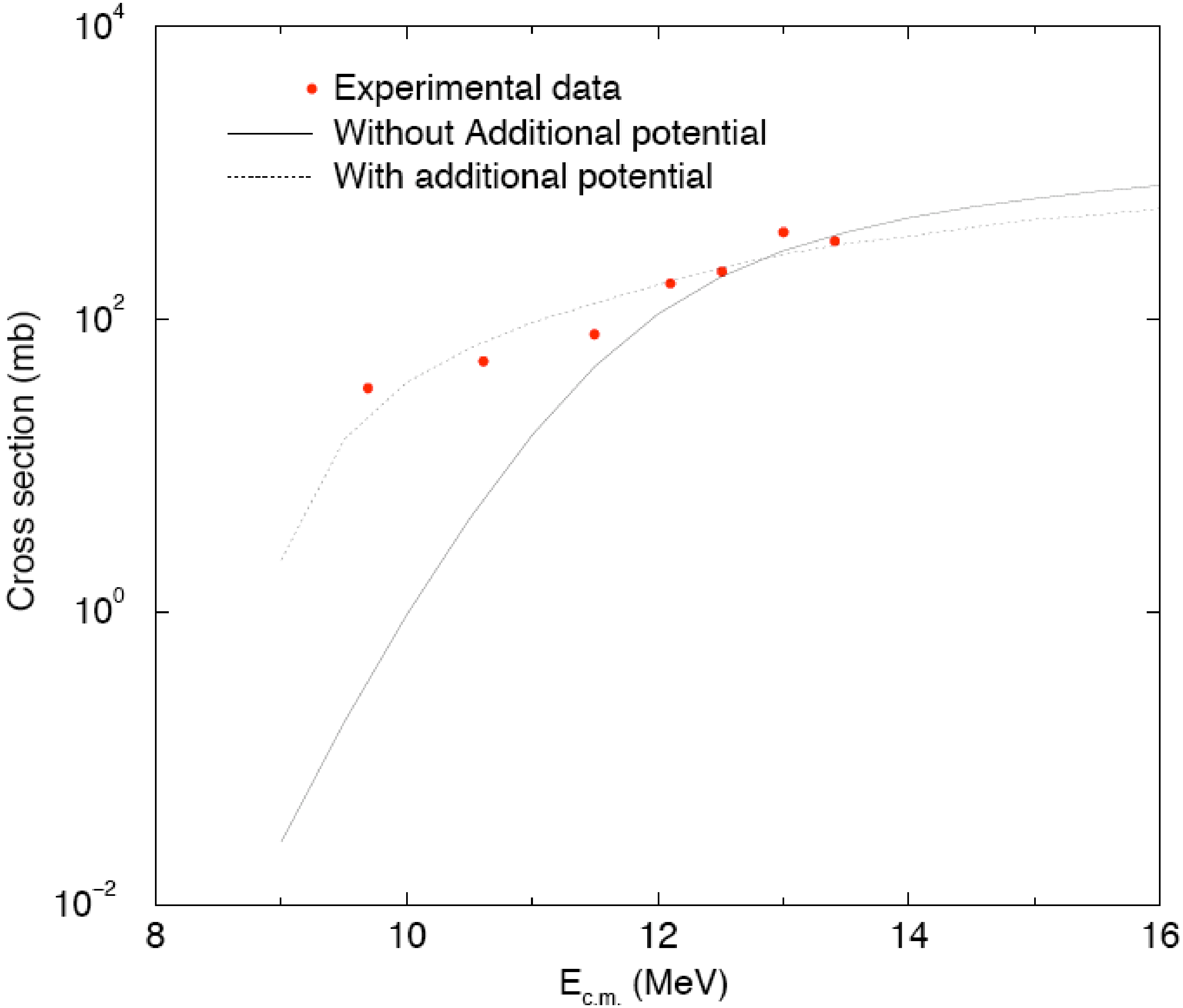


Calculation: Kocak and Balantekin using CCFULL

Could ^{9}Li be capturing two neutrons from ^{70}Zn prior to tunneling since ^{11}Li is also stable?

Add a small potential to describe this two-neutron transfer:

$$V_{2n} = V_d \frac{d}{dr}\left(\frac{1}{1+e^{\frac{r-R}{a}}}\right)$$



Experimental data
Without Additional potential
With additional potential
Cross section (mb)
10^4
10^2
10^0
10^{-2}
8
10
12
14
16
$E_{c.m.}$ (MeV)

Questions

• For asymmetric systems the barrier is outside the region where nuclei touch. Multidimensional barrier penetration is conceptually well-defined. Do we really understand the fusion of such nuclei? What is the large diffuseness telling us?

• What happens when nuclei fuse at energies well-below the barrier? What physics does the very shallow potentials needed to fit the data mimic?

• Do we understand how we should theoretically formulate the fusion of unstable nuclei? What can we learn by studying fusion of nuclei off the line of stability?

• We need data for the fusion of exotic nuclei, both below and above the Coulomb barrier. Such data would open a new chapter in the study of multidimensional quantum tunneling.