Radiative capture reactions

N.K. Timofeyuk

University of Surrey

Radiative capture reactions:

$A_1 + A_2 \rightarrow A + \gamma$

The most important stellar capture reactions:

$(N,Z) + p \rightarrow (N,Z+1) + \gamma$

$(N,Z) + n \rightarrow (N+1,Z) + \gamma$

$(N,Z) + \alpha \rightarrow (N+2,Z+2) + \gamma$

Examples:

Stationary hydrogen burning:

$p(p,e^+\nu)d(p,\gamma)^3He(^3He,2p)^4He(^3He,\gamma)^7Be(p,\gamma)^8B(\beta^+)2^4He$

Explosive hydrogen burning:

$^{11}C(p,\gamma)^{12}N$; $^{17}F(p,\gamma)^{18}Ne$; $^{18}F(p,\gamma)^{19}Ne$; $^{20}Na(p,\gamma)^{21}Mg$; $^{21}Na(p,\gamma)^{22}Mg$

CNO cycle:

$^{12}C(p,\gamma)^{13}N\ (\beta^+)^{13}C(p,\gamma)^{14}N(p,\gamma)^{15}O(\beta^+)^{15}N(p,\alpha)^{12}C$

Helium burning:

$^{12}C(\alpha,\gamma)^{16}O(\alpha,\gamma)^{20}Ne(\alpha,\gamma)^{24}Mg$

Neutron capture reactions:

$^{12}C(n,\gamma)^{13}C$; $^{14}C(n,\gamma)^{15}C$; $^{20}Ne(n,\gamma)^{21}Ne$; $^{22}Ne(n,\gamma)^{23}Ne$

$A_1 + A_2 \rightarrow A + \gamma$

Initial state $A_1 + A_2$: is a continuum state with
Kinetic energy: E_i
Momentum: $\boldsymbol{k}_i$
Angular momentum J_1 and J_2
Projection of angular momentum ν_1 and ν_2
Wave function of $A_1 + A_2$ $\Psi^{\nu 1 \nu 2}$ $H\Psi^{\nu_1\nu_2} = E_i\Psi^{\nu_1\nu_2}$

Final state $A + \gamma$: A is bound (a discrete state) and the photon is in the continuum
Binding energy of $A_1 + A_2$: E_f
Momentum of nucleus A: $\boldsymbol{k}_f$
Angular momentum and parity of nucleus A $J_f \pi_f$
Wave function of A $\Psi^{Jf\,Mf\pi f}$ $H\Psi^{J_f M_f} = E_f \Psi^{J_f M_f}$

Energy conservation : $E_i = E_f + E_\gamma$; $E_\gamma = \hbar k_\gamma c$

The starting point to calculate transition per unit time from initial state $|i\rangle$ to final state $|f\rangle$ is the **Fermi's golden rule.**
It assumes that the transition occur due to a perturbation and
is given, to first order in the perturbation, by

$$T_{i\to f} = \frac{2\pi}{\hbar}\left|\left\langle i\left|H'\right|f\right\rangle\right|^2 \rho$$

where ρ is the density of final states (number of states per unit of energy) and $\langle i|H'|f\rangle$ is the matrix element of the perturbation H' between the final and initial states.
Fermi's golden rule is valid when the initial state has not been significantly depleted by scattering into the final states.

Cross sections for capture reactions

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 \, m_i m_f \, \frac{p_f}{p_i} \left| T_{i\to f} \right|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{k_\gamma}{2\pi\hbar} \frac{1}{(2I_1+1)(2I_2+1)} \sum_{M_1 M_2 M_f} \left| \left\langle \Psi^{J_f M_f \pi_f} \left| H' \right| \Psi^{M_1 M_2} \right\rangle \right|^2$$

H′ is the interaction between γ and nucleus A and contains products of nuclear and electromagnetic currents.

H′ can be expanded into electric $M^E_{\lambda\mu}$ and magnetic $M^M_{\lambda\mu}$ multipole operators.

σ = 0 , electric multipoles

$$M^{E}_{\lambda\mu} = e\sum_{i=1}^{A}\left(\frac{1}{2}-\tau_{i3}\right) r_i^{\lambda} Y_{\lambda\mu}\left(\hat{r}_i\right)$$

σ = 1 , magnetic multipoles

$$M^{M}_{\lambda\mu} = \mu_N \sum_{i=1}^{A}\left[\left(\frac{1}{2}-\tau_{i3}\right)\frac{2\vec{l}_i}{\lambda+1} + g_p\left(\frac{1}{2}-\tau_{i3}\right)\vec{s}_i + g_n\left(\frac{1}{2}+\tau_{i3}\right)\vec{s}_i\right]\cdot\vec{\nabla} r_i^{\lambda} Y_{\lambda\mu}\left(\hat{r}_i\right)$$

τ_{i3} is the isospin projection; $\mu_N = e\hbar/2m_Nc$ is the Bohr magneton,
g_p and g_n are gyromagnetic factors of proton and neutron respectively.
$Y_{\lambda\mu}$ is the spherical function.

$$\sigma(E_i) = \frac{2J_f+1}{(2I_1+1)(2I_2+1)}\sum_{\lambda\sigma l_i J_i I}\frac{k_\gamma^{2\lambda+1}}{(2l_i+1)}\frac{8\pi(\lambda+1)}{\hbar\lambda(2\lambda+1)!!^2}\left|\left\langle \Psi^{J_f\pi_f}\left\| M^{\sigma}_{\lambda}\right\| \Psi^{J_i\pi_i}_{l_i I}(E_i)\right\rangle\right|^2$$

Angular momentum coupling

lI scheme

$$\vec{J}_1 + \vec{J}_2 = \vec{I}$$

$$\vec{l} + \vec{I} = \vec{J}$$

lj scheme

$$\vec{l} + \vec{J}_2 = \vec{j}$$

$$\vec{j} + \vec{J}_1 = \vec{J}$$

Transition from one scheme to another:

$$\varphi_{lI} = \sum_j (-)^{J_1 - J_2 - I} \sqrt{(2I+1)(2j+1)}\, W(J_1 J_2 J l, I j)\, \varphi_{lj}$$

where W is the Racah coefficient.

Selection rules for matrix elements $\left\langle \Psi^{J_f \pi_f} \left\| M^{\sigma}_{\lambda} \right\| \Psi^{J_i \pi_i}_{l_i I}(E_i) \right\rangle$

$| J_i - J_f | \le \lambda \le J_i + J_f$

$\pi_i \pi_f = (-)^{\sigma+\lambda}$

For $T_3=0$ $\Delta T = \pm 1$, which means that E1 transitions are forbidden in N=Z nuclei if isospin impurities are absent

Reaction	J_1	J_2	l_i	J_i	$\sigma\lambda$	J_f	
d(p,γ)^{3}He	1^+	$\frac{1}{2}^+$	0	$\frac{1}{2}^+$	M1,E2	$\frac{1}{2}^+$	*suppressed?*
				$3/2^+$	M1,E2		*suppressed?*
			1	$\frac{1}{2}^-$	E1		
^{7}Be(p,γ)^{8}B	$3/2^-$	$\frac{1}{2}^+$	0	1^-	E1	2^+	
				2^-	E1		
d(^{4}He,γ)^{6}Li	1^+	0^+	0	1^+	M1	1^+	*suppressed?*
			1	0^-	E1		*suppressed?*
			1	1^-	E1		*suppressed?*
			1	2^-	E1		*suppressed?*
			2	1^+	E2		
			2	2^+	E2		
			2	3^+	E2		

Theoretical methods and models to calculate radiative capture

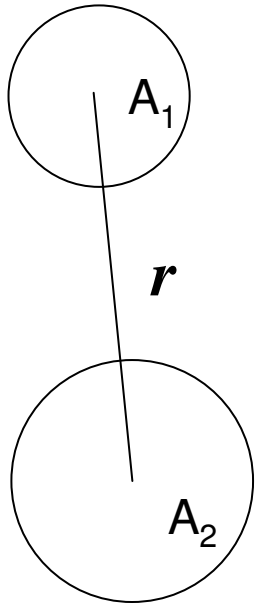
$$\sigma(E_i) = \frac{2J_f + 1}{(2I_1 + 1)(2I_2 + 1)} \sum_{\lambda \sigma l_i J_i I} \frac{k_\gamma^{2\lambda+1}}{(2l_i + 1)} \frac{8\pi(\lambda + 1)}{\hbar \lambda (2\lambda + 1)!!^2} \left| \left\langle \Psi^{J_f \pi_f} \left\| M_\lambda^\sigma \right\| \Psi_{l_i I}^{J_i \pi_i}(E_i) \right\rangle \right|^2$$

- R-matrix
- Potential model
- Microscopic models
- Ab-initio calculations

Potential model

The interaction between nucleons from A_1 and A_2 is replaced by potential $V(r)$ so that

$$H=H_1+H_2+T_{rel}+V(r)$$



Entrance channel wave function:

$$\Psi^{\nu_1\nu_2}(\boldsymbol{k}) = \sum_{J_i M_i l_l I} (J_1\nu_1 J_2\nu_2 \mid I\nu)(l_i m I\nu \mid J_i M_i)$$

$$\times \varphi_{l_i I}^{J_i}(r)\left[\left[\Psi^{J_1\pi_1}\otimes\Psi^{J_2\pi_2}\right]_I \otimes Y_{l_i}(\hat{r})\right]_{M_i}^{J_i} Y_{l_i m}^{*}(\hat{\boldsymbol{k}})$$

Exit channel wave function:

$$\Psi^{J_f M_f \pi_f} = \varphi_{l_f I}^{J_f}(r)\left[\left[\Psi^{J_1\pi_1}\otimes\Psi^{J_2\pi_2}\right]_I \otimes Y_{l_f}(\hat{r})\right]_{M_f}^{J_f}$$

$$H_1\Psi^{J_1M_1} = E_1\Psi^{J_1M_1}$$

$$H_2\Psi^{J_2M_2} = E_2\Psi^{J_2M_2}$$

$$\left(T_{rel}+V(r)\right)\varphi_{l_i I}^{J_i}(r) = E_i\,\varphi_{l_i I}^{J_i}(r)$$

In the electromagnetic multipole operators can be rewritten as a sum of internal and relative terms as well:

$$M^{\sigma}_{\lambda\mu} = M^{\sigma}_{\lambda\mu}(\boldsymbol{r}) + M^{\sigma}_{\lambda\mu}(\xi_{A_1}) + M^{\sigma}_{\lambda\mu}(\xi_{A_2})$$

This equation is exact for $\lambda \leq 2$. For higher order terms, crossed terms may contribute.

For electric transitions:

$$\left\langle \Psi^{J_f\pi_f}_{l_f I_f} \left\| M^{E}_{\lambda}(\boldsymbol{r}) \right\| \Psi^{Ji\pi_i}_{l_i I_i} \right\rangle = \frac{e}{\sqrt{4\pi}} \left(Z_1 \left(\frac{A_2}{A} \right)^{\lambda} + Z_2 \left(-\frac{A_1}{A} \right)^{\lambda} \right) \left((2J_i+1)(2\lambda+1)(2l_i+1) \right)^{1/2}$$

$$\times (-)^{l_i+J_i+I_i} \, \delta_{I_i I_f} \left\langle l_i 0 \lambda 0 \middle| l_f 0 \right\rangle \begin{Bmatrix} J_i & J_f & \lambda \\ l_f & l_i & I_i \end{Bmatrix} \int_0^{\infty} \varphi^{J_f\pi_f}_{l_f I_f}(r) r^{\lambda+2} \varphi^{J_i\pi_i}_{l_i I_i}(r) dr$$

If $\varphi^{J_f\pi_f}_{l_f I_f}(r)$ and $\varphi^{J_i\pi_i}_{l_i I_i}(r)$ are orthogonal then the contribution from internal electromagnetic operators is absent.

Magnetic transition M1 contains integral $\int_0^{\infty} \varphi^{J_f\pi_f}_{l_f I_f}(r) r^2 \varphi^{J_i\pi_i}_{l_i I_i}(r) dr$ which is zero if initial and final wave funuctions are orthogonal

Choice of the A_1+A_2 potential

Phenomenological potential

$$V(r) = V_c(r) + V_{coul}(r) + V_{so}(r)$$

Central

Woods-Saxon potential $V_c(r) = V_0 / (1+\exp((r - R)/a))$

$R = r_0 A_2^{1/3}$ or $R = r_0 (A_1^{1/3} + A_2^{1/3})$

r_0 is the radius and a is the diffusseness

Spin-orbit potential

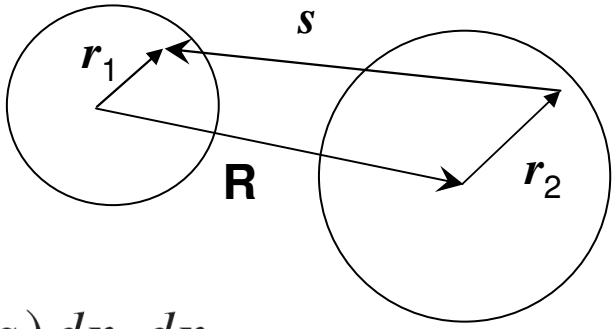
$$V_{SO}(r) = -\frac{\lambda}{c^2 M}\frac{1}{r}\frac{dV_c}{dr}\vec{l}\cdot\vec{s}$$

Coulomb potential: uniformly charged sphere

$$V_{coul}(r) = \frac{Z_1 Z_2 e^2}{2R_c}\left(3 - \left(\frac{r}{R_c}\right)^2\right) \quad \text{for } r \le R_c$$

$$= \frac{Z_1 Z_2 e^2}{R_c} \quad \text{for } r > R_c$$

Folding potential for $a + A$



$$V(R) = \lambda V_F(R)$$

$$= \lambda \int \int \rho_a(\mathbf{r}_1)\rho_A(\mathbf{r}_2)\, v_{\text{eff}}(E, \rho_a, \rho_A, s)\, d\mathbf{r}_1 d\mathbf{r}_2$$

$$s = |\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1|$$

Densities ρ_a and ρ_A can be derived from

- measured charge distributions (for stable nuclei)
- model calculations (e.g. Hartree-Fock)

λ can be obtained from

- fitting to bound states energies
- fitting to thermal total cross sections (for neutrons)
- fitting to resonance energies

DDM3Y (density dependent) NN effective potential

A.M.Kobos, B.A.Brown, R.Lindsay, G.R.Satchler, Nucl.Phys. A425, 205 (1984)

$v_{eff}(E,\rho_a,\rho_A,\ s) = v_{M3Y}(E,s)\ \ f\,(E,\rho_a+\rho_A)$

$v_{M3Y}(E,s) = 7999\ \exp(-4s)/4s - 2134\ \exp(-2.5s)/2.5s + J_{00}(E)\delta(s)$

Exchange part

$J_{00}(E) = -276\ (1 - 0.005E/A_a)$ (MeV· fm^3)

Density dependent part:

$f\,(E,\rho) = C(E)(1+\alpha(E)e^{-\beta(E)\rho})$

Coefficients $C(E)$,$\alpha(E)$ and $\beta(E)$ are determined by fitting volume integral of $v_{eff}(E,\rho_a,\rho_A,\ s)$ to the strength of the real part of a G-matrix effective interaction obtained from Bruekner-Hartree-Fock calculations for nuclear matter of various densities and at various energies.

Two-body wave functions and their asymptotics

$$\left(T_{rel} + V(r)\right) \varphi_{l_i I}^{J_i}(r) = E_i \varphi_{l_i I}^{J_i}(r)$$

In intital (scattering state) $E_i > 0$.
At $r \to \infty \qquad V(r) \to V_{coul}(r)$

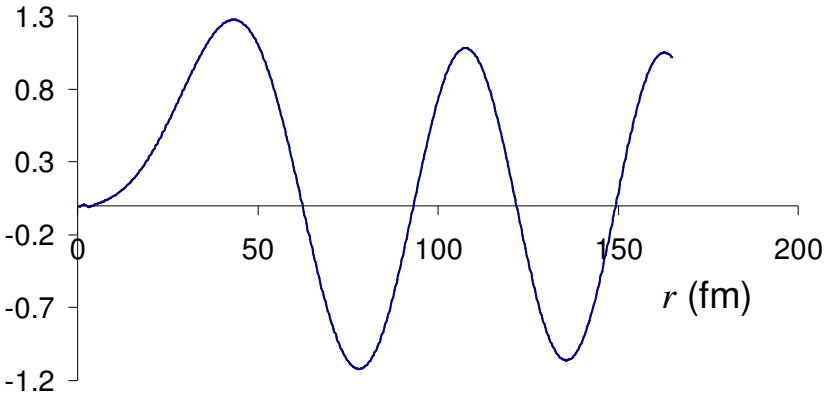
$$\varphi_{l_i I}^{J_i}(r) \to \sqrt{\frac{4\pi(2l_i+1)}{v}} \frac{1}{k_i r} \left(F_{l_i}(k_i r) + \left(G_{l_i}(k_i r) + iF_{l_i}(k_i r) \right) e^{i\delta_{J_i}} \sin\delta_{J_i} \right)$$

where $k_i = (2\mu E_i)^{1/2}$
F is the regular and G is irregular at the origin Coulomb wave functions
δ is the phase shift.

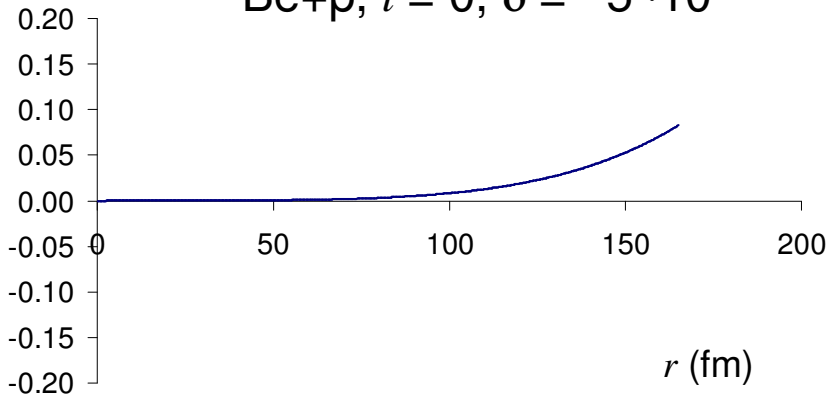
The normalization of this wave function is determined by orthogonality of the continuum wave function and is related to the defition of the cross section.

Scattering wave functions at E_i = 20 keV

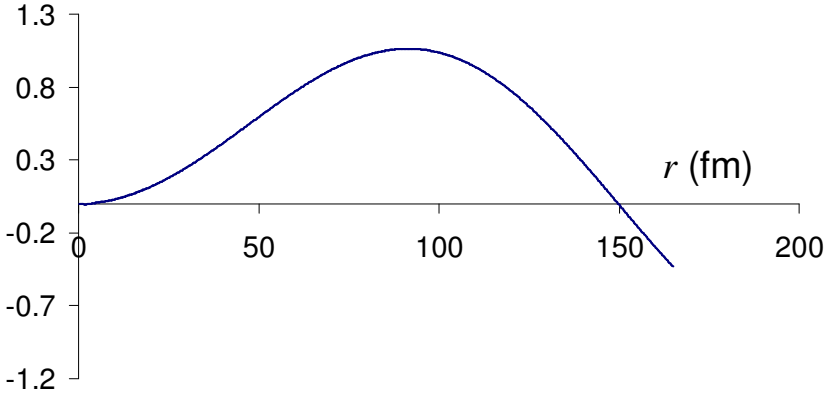
^{3}He+α, l = 0, δ = 5 ·10^{-4}



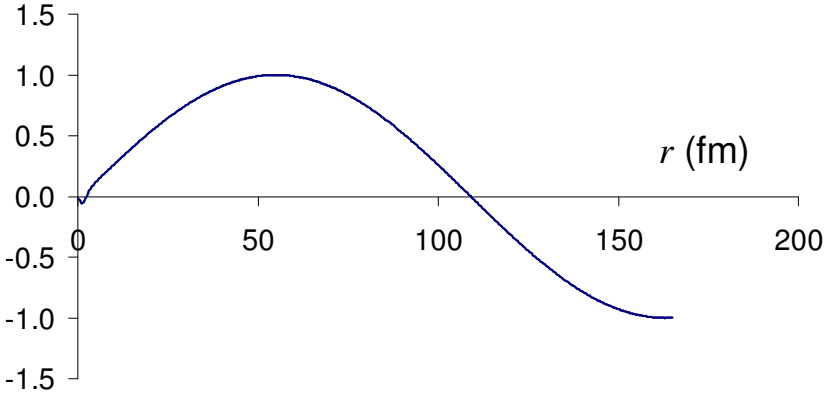
^{7}Be+p, l = 0, δ = –5 ·10^{-12}



^{14}C+n, l = 1, δ = 3 ·10^{-6}



^{7}Li+n, l = 0, δ= – 2.5 ·10^{-2}



Final (bound) states

$$\left(T_{rel}+V(r)\right)\varphi_{l_f I}^{J_f}(r)=E_f\,\varphi_{l_f I}^{J_f}(r)$$

In the final bound state $E_f < 0$.
At $r \to \infty$ $\quad V(r) \to V_{coul}(r)$

$$\varphi_{l_f I}^{J_f}(r) \to b_{l_f I}^{J_f}\,\frac{W_{-\eta_f, l_f+1/2}(2k_f r)}{r}$$

where $k_f = (2\mu E_f)^{1/2}$, $\eta = Z_1 Z_2 e^2/(\hbar^2 v)$ is the Sommerfeld parameter,
W is the Whittaker function
b is the (single-particle) asymptotic normalization coefficient (ANC)
φ is normalized to unity.

In fact, the normalization of φ should be the same as the normalization of the projection of the wave function of A into product of the wave functions of A_1 and A_2 $\quad \left[\left[\Psi^{J_1\pi_1}\otimes\Psi^{J_2\pi_2}\right]_I\otimes Y_{l_f}(\hat{r})\right]_{M_f}^{J_f}$

Overlap integral $$\mathcal{I}_{l_f I}(r) = \left\langle \Psi^{J_f M_f \pi_f} \left| \left[\left[\Psi^{J_1 \pi_1} \otimes \Psi^{J_2 \pi_2} \right]_I \otimes Y_{l_f}(\hat{r}) \right]^{J_f}_{M_f} \right. \right\rangle$$

is the projection of the wave function of the final state of nucleus A into the product of the wave functions of A_1 and A_2.

The norm of this overlap is called spectroscopic factor

$$S_{l_f I} = \frac{A!}{A_1!\,A_2!} \int_0^\infty \left(\mathcal{I}_{l_f I}(r) \right)^2 r^2 dr$$

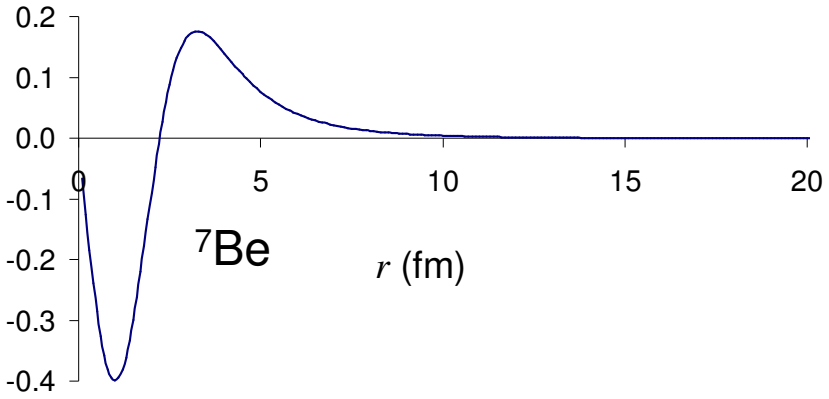
When calculating the radiative capture the final two-body wave function $\varphi^{J_f}_{l_f I}(r)$ is replaced by $\sqrt{S_{l_f I}}\,\varphi^{J_f}_{l_f I}(r)$

The asymptotic part is therefore $$\sqrt{S_{l_f I}}\, b^{J_f}_{l_f I} \frac{W_{-\eta_f, l_f + 1/2}(2k_f r)}{r} = C^{J_f}_{l_f I} \frac{W_{-\eta_f, l_f + 1/2}(2k_f r)}{r}$$

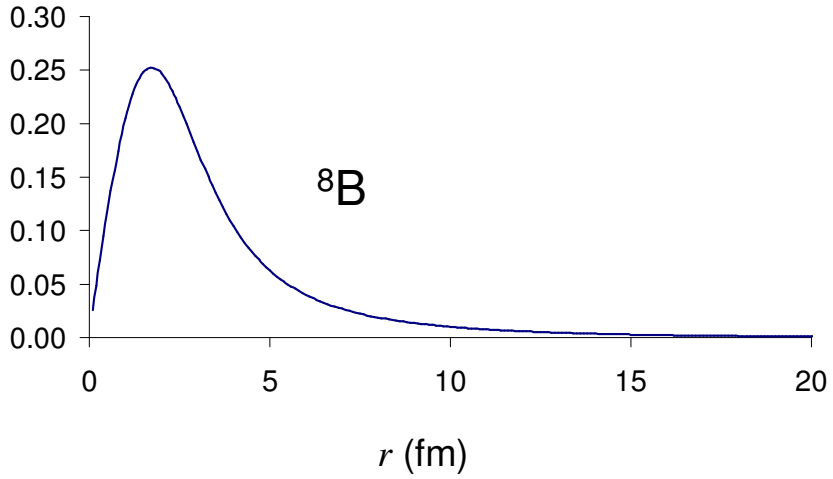
where C is the ANC $$C^{J_f}_{l_f I} = \sqrt{S_{l_f I}}\, b^{J_f}_{l_f I}$$

Bound state wave functions

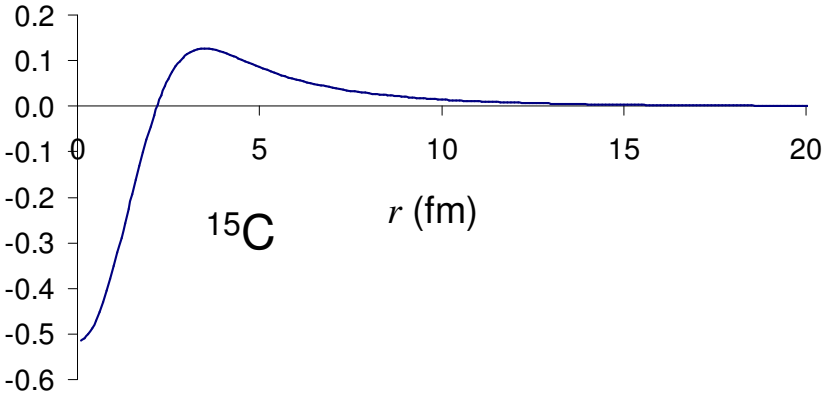
^{3}He+α, $l_f = 1$, $E_f = -1.59$ MeV



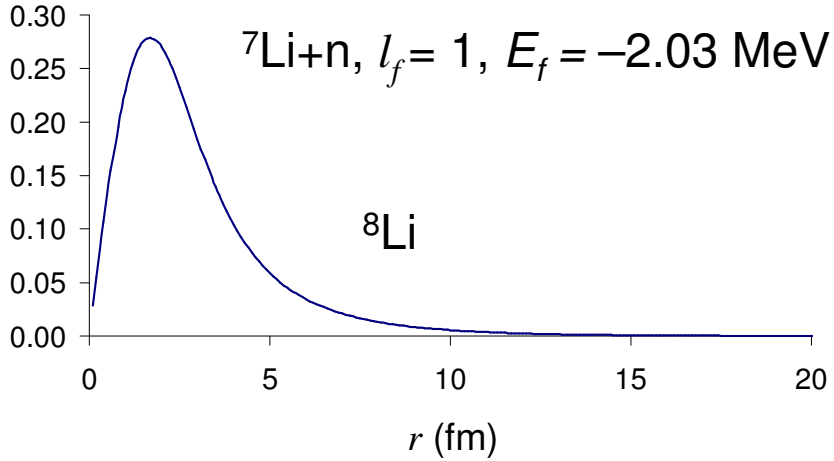
^{7}Be+p, $l_f = 1$, $E_f = -0.137$ MeV



^{14}C+n, $l_f = 0$, $E_f = -1.2$ MeV

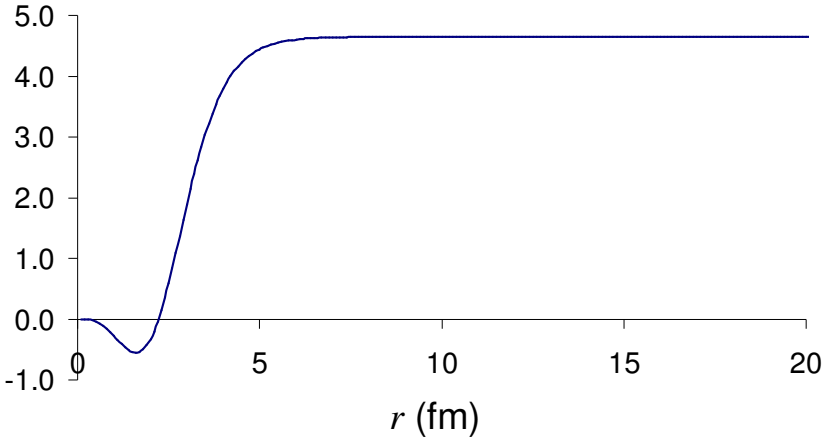


^{7}Li+n, $l_f = 1$, $E_f = -2.03$ MeV

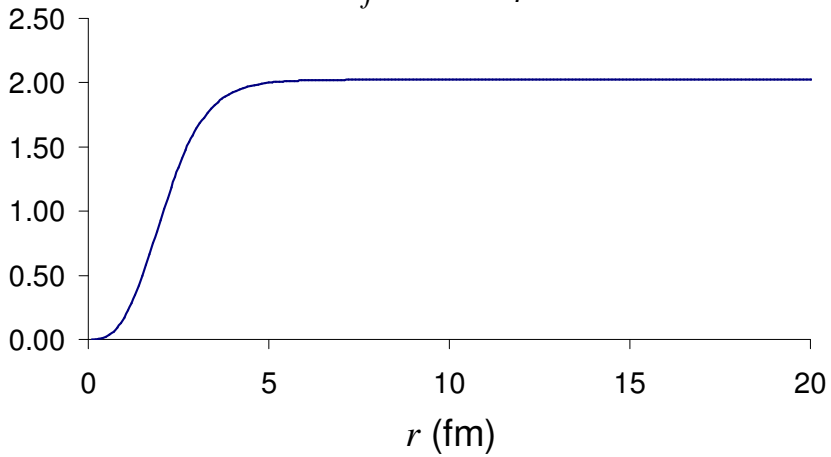


$$\varphi_{l_f I}^{J_f}(r) / \left(W_{-\eta_f, l_f + 1/2}(2k_f r) / r \right)$$

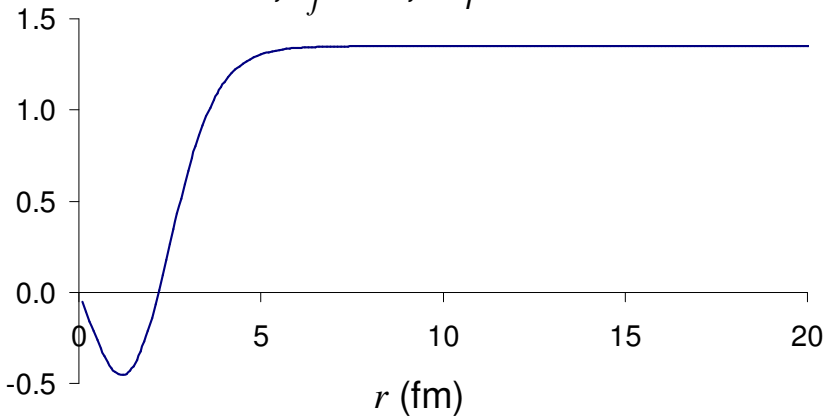
^{3}He+α, $l_f = 1$, $E_f = -1.59$ MeV



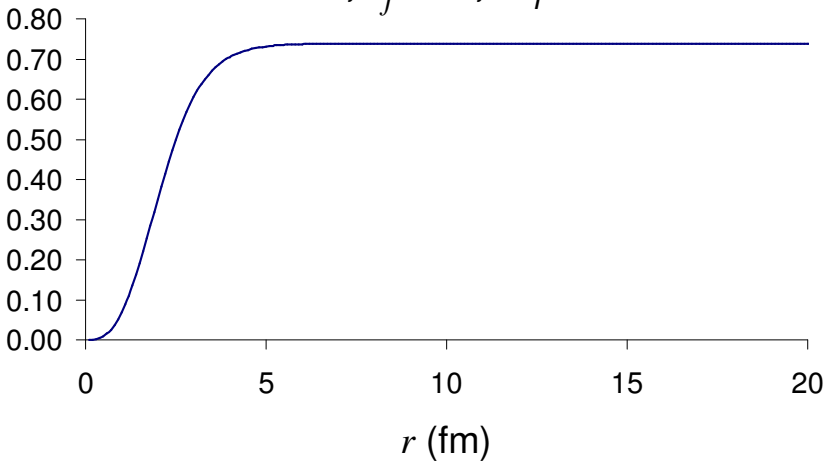
^{7}Be+p, $l_f = 1$, $E_f = -0.137$ MeV



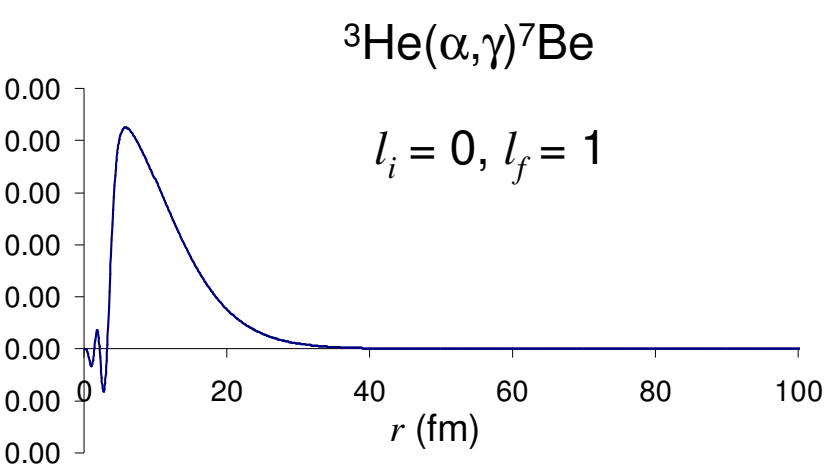
^{14}C+n, $l_f = 0$, $E_f = -1.2$ MeV

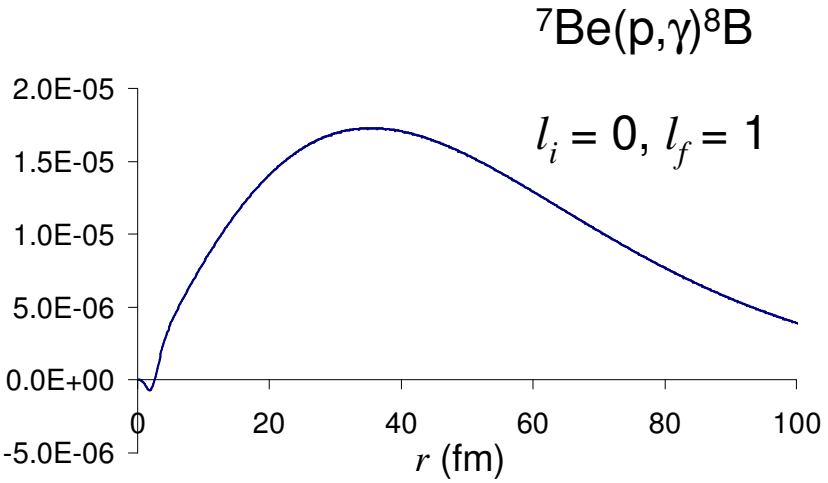


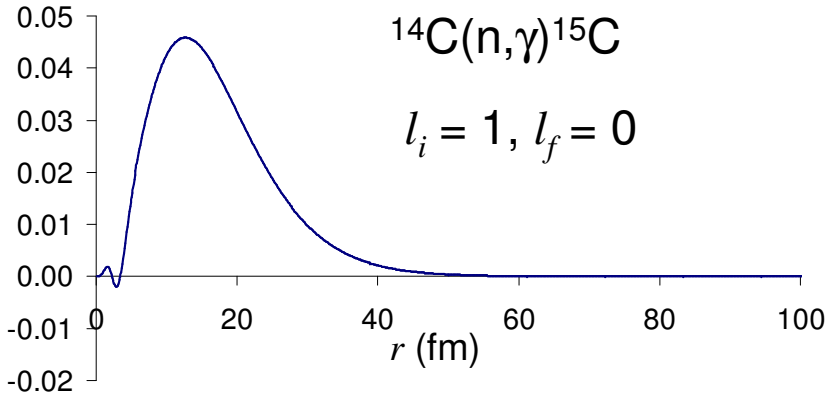
^{7}Li+n, $l_f = 1$, $E_f = -2.03$ MeV

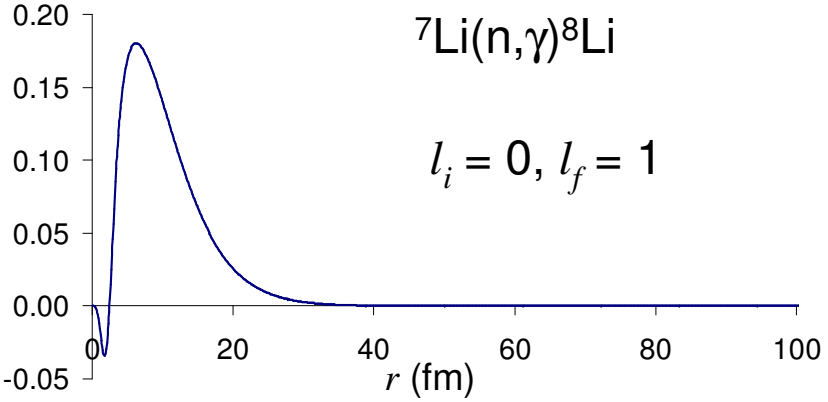


Integrand $\varphi_{l_f I_f}^{J_f \pi_f}(r) r^{\lambda+2} \varphi_{l_i I_i}^{J_i \pi_i}(r)$ for the E1 capture amplitude at E=20 keV





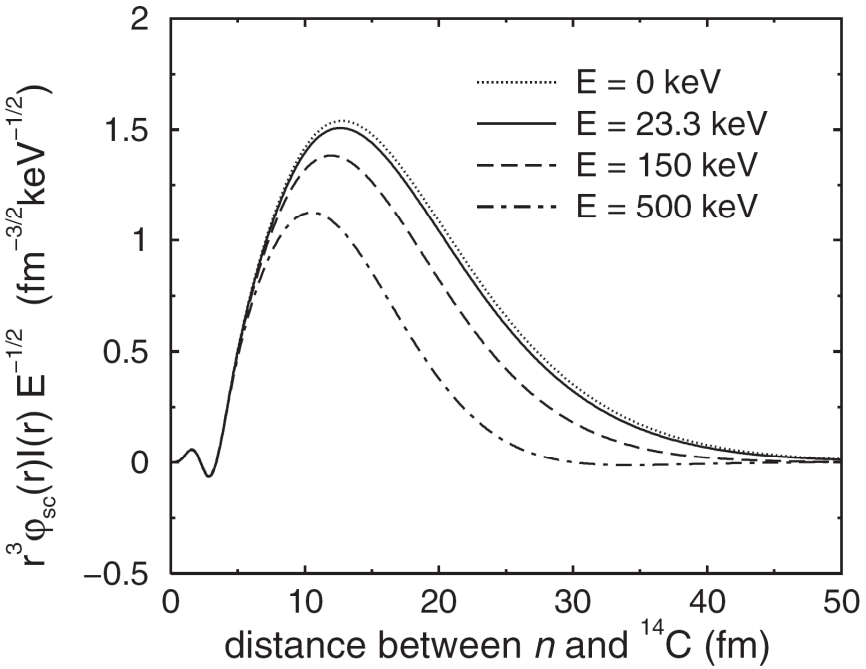




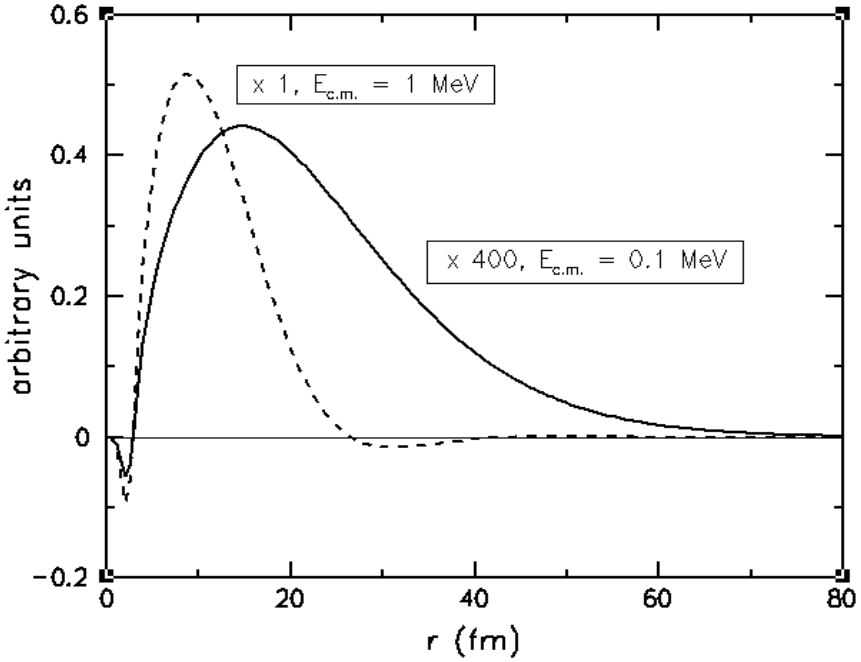
Integrand $\varphi_{l_f I_f}^{J_f \pi_f}(r) r^{\lambda+2} \varphi_{l_i I_i}^{J_i \pi_i}(r)$ for the E1 capture

amplitude for other energies

$^{14}C(n,\gamma)^{15}C$



$^{11}C(p,\gamma)^{12}N$



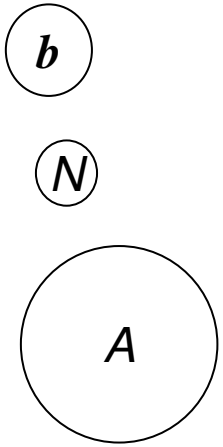
For many capture reactions the most important contributions to the reaction amplitude come from large distances between A_1 and A_2. Therefore, to predict the capture cross sections we need only the wave functions in asymptotic region. More precisely, we need

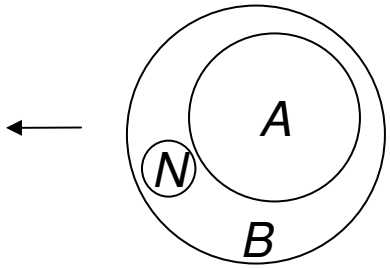
- ANCs (for final bound states)
- Phase shifts (for initial scattering states)

ANCs can be determined from

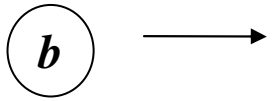
- Peripheral transfer reactions
  - (d,t) and (d,^{3}He),
  - reactions with heavy ions, where absorption is large,
  - some (d,p) reactions
- Coulomb breakup

Peripheral transfer reaction $a + A \rightarrow b + B$





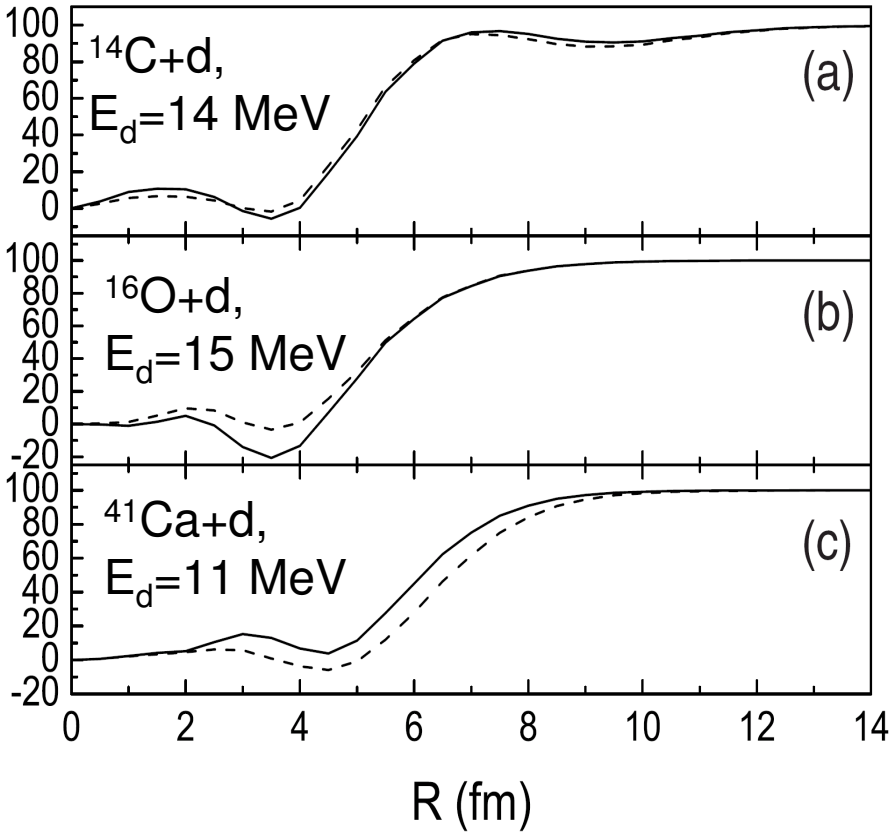
$$C = S_{\exp} b^2$$



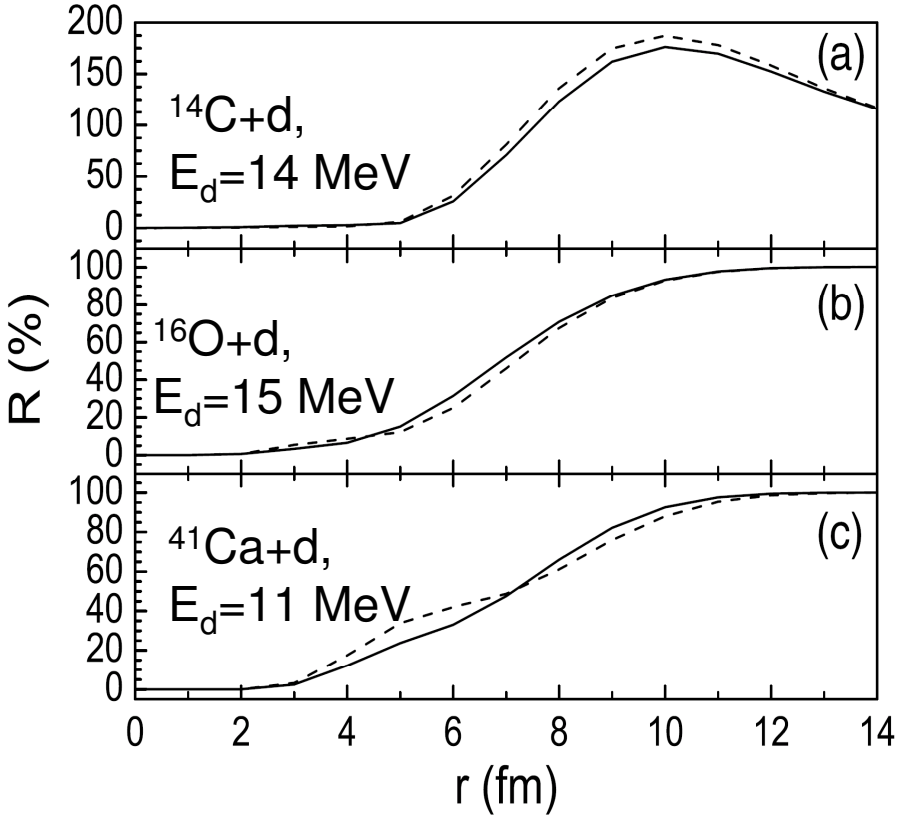
$$S_{\exp} = \frac{experimental\ cross\ section}{theoretical\ cross\ section\ (S_{theor} = 1)}$$

Contribution to the A(d,p)B reaction amplitude

D.Y.Pang, F.M.Nunes, A.M.Mukhamedzhanov, Phys. Rev. **C 75**, 024601 (2007)

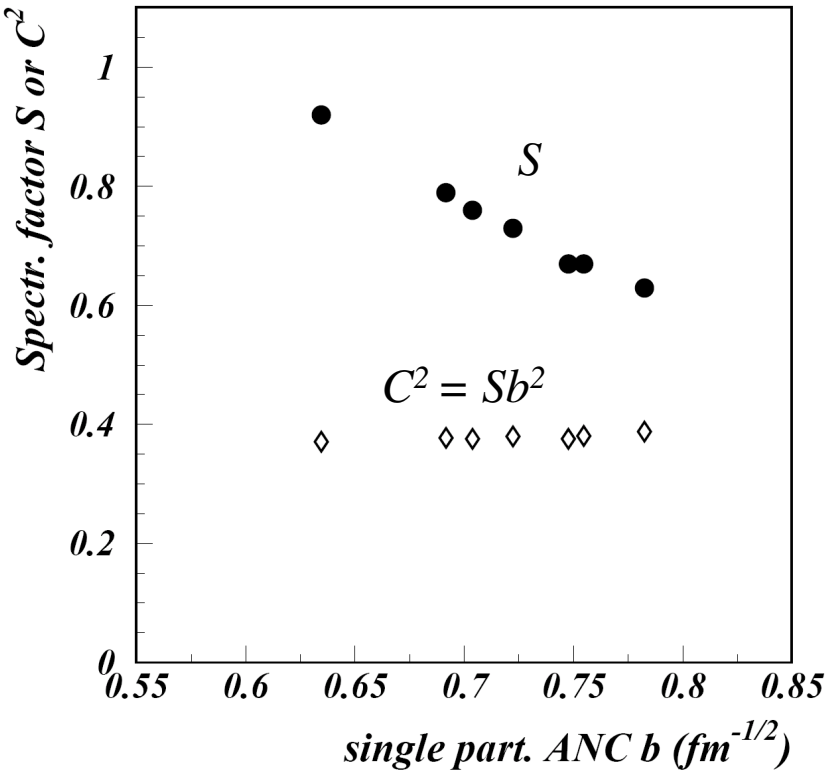


Cut off in the distance between *d* and *A*



Cut off in the distance between *n* and *A*

Peripherality test



For peripheral reactions the ANC does not depend on the choice of the geometry of the potential well used to calculate the single-particle wave function but the spectroscopic factor does.

Example:

$^{8}Li \rightarrow {}^{7}Li + n$ from $^{12}C(^{8}Li,^{7}Li)^{13}C$

L.Trache *et al*, Phys.Rev. C 67, 062801 (2003)

ANC method:

Direct peripheral transfer reactions A(a,b)B allow to determine ANCs and then to use then to calculate non-resonant radiative capture cross sections.

First suggestion was to use reaction ^{7}Be(^{3}He,d)^{8}B to obtain ^{7}Be(p,γ)^{8}B
*(A.M. Mukhamedzhanov, R.E. Tribble and N.K. Timofeyuk, Phys. Rev. C**51**, 3472 (1995))*

Now several reactions have been determined with the ANC method,

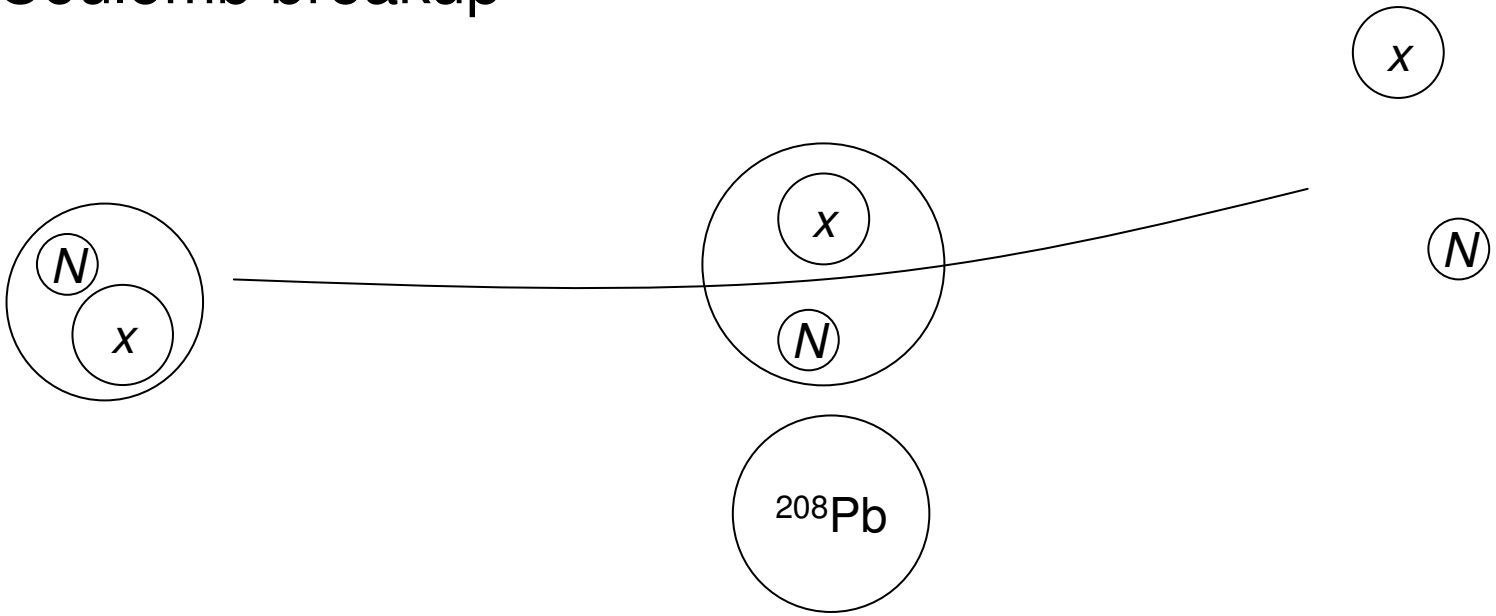
for example: $^{7}Be(p,\gamma)^{8}B$ *from* $^{9}Be(^{7}Be,^{8}B)^{10}B$ *and* $^{14}N(^{7}Be,^{8}B)^{13}C$

$^{11}C(p,\gamma)^{12}N$ *from* $^{14}N(^{11}C,^{12}N)^{13}C$

$^{14}N(p,\gamma)^{15}O$ *from* $^{14}N(^{3}He,d)^{15}O$ *etc.*

ANC method is also applicable to determine peripheral neutron capture reactions of astrophysical interest, for example, $^{12}C(n,\gamma)^{13}C(1/2^{+})$
(N.Imai, N.Aoi, S.Kubono, D.Beaumel et al, Nucl.Phys. A688, 281c (2001))

Coulomb breakup



Cross sections for Coulomb breakup at some kinematical conditions depend mostly on the asymptotic part of the x-N wave function. Therefore, ANCs can be determined from such reactions.

Energy dependence of capture cross sections.

Charged particles

Energy dependence is mainly determined by the entrance channel wave function $F_l(kr,\eta)$. At small stellar energies

$$F_l(kr,\eta) \to \sqrt{\pi kr}\,\exp\left(-\pi\eta\right) I_{2l+1}(kr)$$

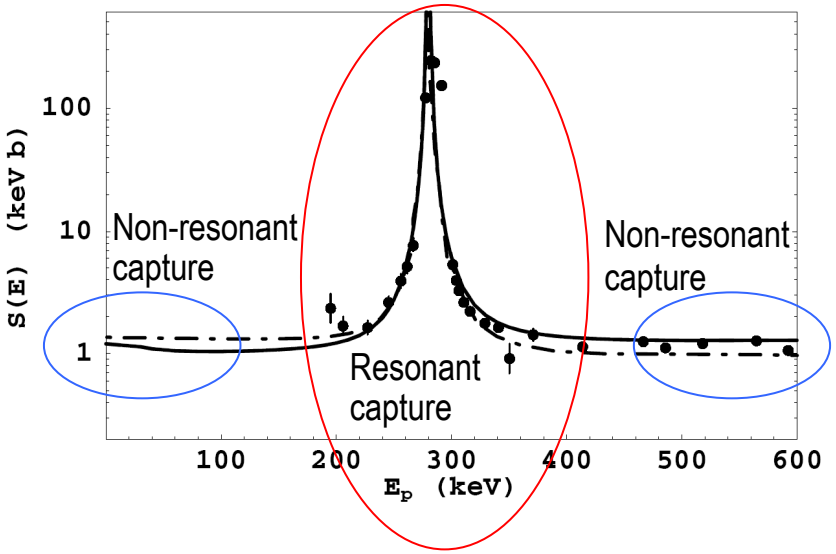
where I is the modified Bessel function.

Since $\eta \sim 1/E^{1/2}$, the main dependence on energy is concentrated in the exp(-πη) factor so that

$$\sigma(E)=\exp(-2\pi\eta)S(E)/E,$$

where $S(E)$ is the astrophysical factor.

Energy dependence of $S(E)$



^{14}N(p,γ)^{15}O(6.793 MeV)

Non-resonant capture near E = 0 can be expanded onto Taylor series

$$S(E) = S(0) + S'(0)E + \frac{1}{2}S''(0)E^2 + \ldots$$

Analytical expressions for $S(0)$, $S'(0)$ and $S''(0)$ are given in

D.Baye, Phys. Rev. ***C*** *61, 025801 (2000)* and

A.M. Mukhamedzhanov and F.Nunes, Nucl. Phys. ***A*** *708, 437 (2002)*

$^{7}Be(p,\gamma)^{8}B$

C.Angulo et al, Nucl. Phys. ***A*** *719, 310c (2003)*

$$S(E) = S(0)(1 + s_1 E)$$
$$S(0) \simeq 38\,\bar{C}^2(1 - 0.0013\bar{a}_0)$$
$$s_1 = -1.81(1 + 0.0087\bar{a}_0)$$

where $\bar{C}^2$ is the sum of ANCs with channel spins $I = 1$ and $I = 2$,

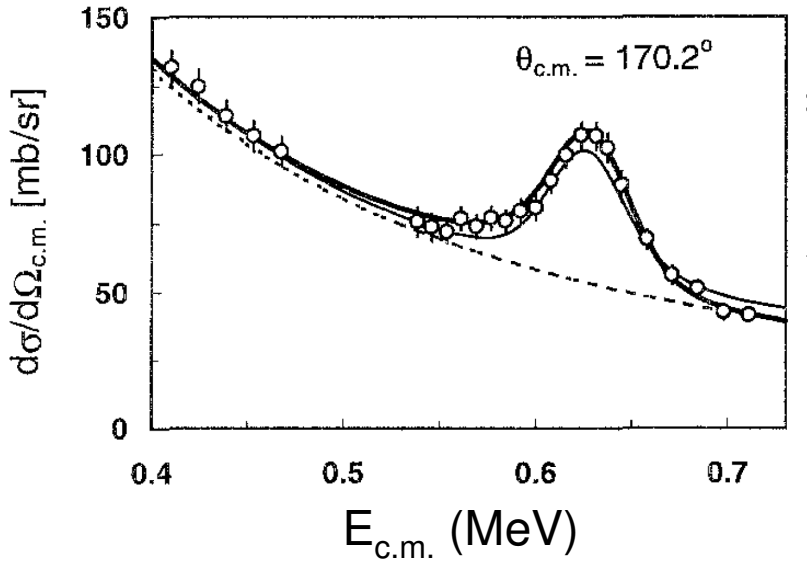
a_0 is the scattering length

$$a_0 = -\lim_{E\to\infty} \frac{\delta_0(E)}{k}$$

a_0 is found from R-matrix analysis of p+^{7}Be elastic scattering

$a_0(I = 1)$ = 25 ± 9 fm

$a_0(I = 2)$ = -7 ± 3 fm



Neutron capture

$$\varphi_{l_i I}^{J_i}(r) \to \sqrt{\frac{4\pi(2l_i+1)}{v}}\frac{1}{k_i r}\left(F_{l_i}(k_i r)+\left(G_{l_i}(k_i r)+iF_{l_i}(k_i r)\right)e^{i\delta_{J_i}}\sin\delta_{J_i}\right)$$

For $\eta = 0 \quad F_l(x) = x\, j_l(x)$

Therefore at $E \to 0$

$\sigma_{n\gamma} \sim k^{2l}/v$

$\sigma_{n\gamma}$ increases for $l = 0$ and decreases for $l > 0$ when E $\to$ 0

It is possible to introduce a neutron analog of the astrophysical S-factor

(*D. Baye, Phys. Rev. C* ***70****, 015801 (2004)*)

$$\mathcal{S}(E) = k^{-2l} v\, \sigma(E)$$

$$\mathcal{S}(E) \approx \mathcal{S}(0)(1 + s_1 E + s_2 E^2)$$

Resonant capture

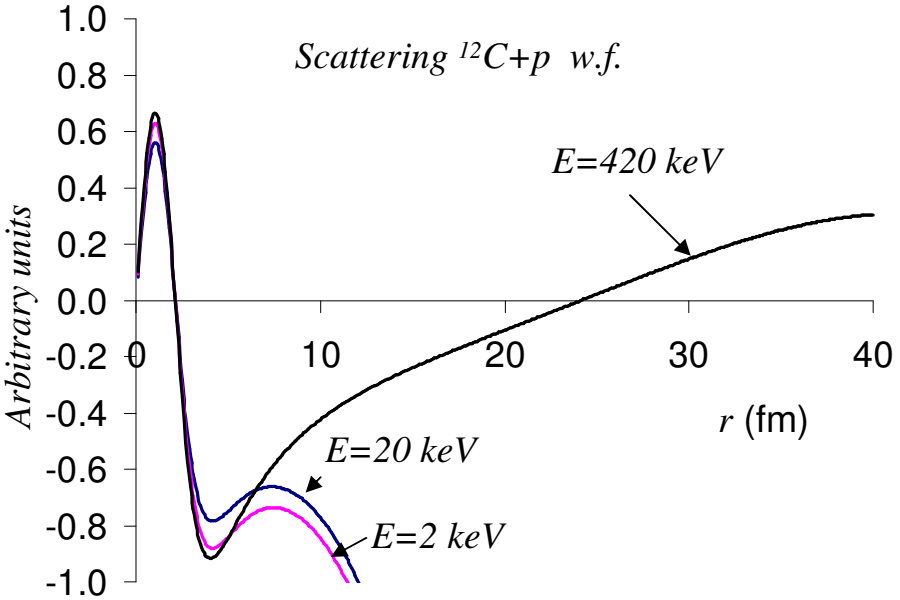
At $E = E_{res}$ $\delta = \pi/2$ and outside the nuclear interior

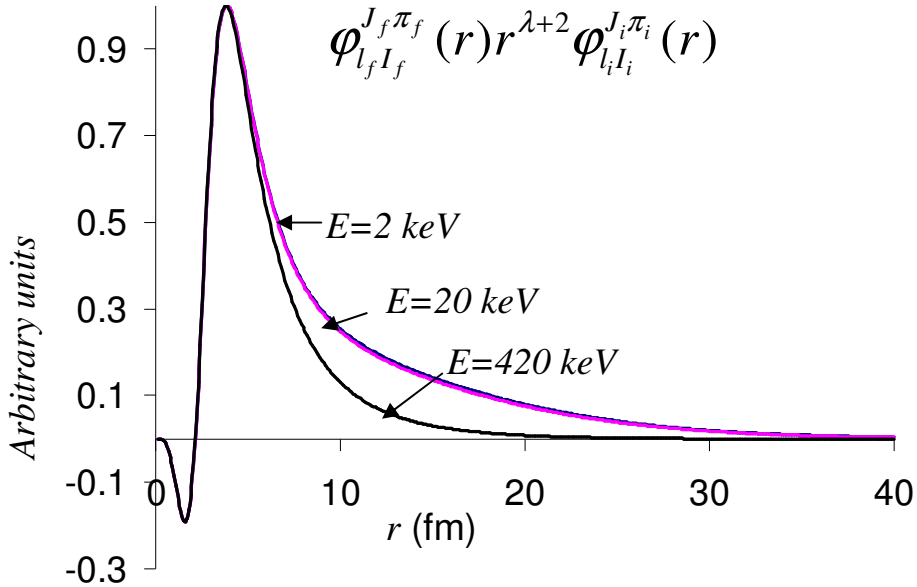
$$\varphi_{l_i I}^{J_i}(r) \to \sqrt{\frac{4\pi(2l_i+1)}{v}}\frac{1}{k_i r}\left(F_{l_i}(k_i r)+\left(G_{l_i}(k_i r)+iF_{l_i}(k_i r)\right)e^{i\delta_{J_i}}\sin\delta_{J_i}\right)$$

$$= \sqrt{\frac{4\pi(2l_i+1)}{v}}\frac{G_{l_i}(k_i r)}{k_i r}$$

$$G_l(x) \sim x^{-l}, \quad x \to 0$$

$^{12}C(p,\gamma)^{13}N$ E1 capture from s-wave E_R = 421 keV





What to do about resonance capture?

- Calculate cross sections in the potential model if the resonance width is reproduced
- R-matrix analysis
- Calculate cross section at E_R and assume that around this resonance the energy dependence is of Breit-Wigner type. At $E=E_R$ the amplitude has physical meaning of probability of the electromagnetic decay from initial to final state.
- Try to separate internal and external contributions and to fix them using some physical assumptions.
  - P. Descouvemont, *Theoretical models for nuclear astrophysics (2004)*
  - X. Tang et al, *Phys. Rev. C* ***67****, 015804 (2003)*
- Use predictions from microscopic models

The Breit-Wigner formula for an isolated single-level resonance

(C.Rolfs and W.S. Rodney, Cauldrons in the Cosmos, University of Chicago Press (1988)

$$S_{\rm res}(E) = \pi \frac{(\hbar c)^2}{2\mu c^2} \frac{(2J+1)}{(2J_1+1)(2J_2+1)} \frac{\Gamma_p(E)\Gamma_\gamma(E)}{(E-E_R)^2 + (\Gamma_{\rm tot}/2)^2} \exp\left(\frac{E_G}{E}\right)^{1/2}$$

$$\Gamma_p(E) = \Gamma_p(E_R) \frac{\exp[-(E_G/E)^{1/2}]}{\exp[-(E_G/E_R)^{1/2}]},$$

E_G is the Gamow energy

$$\Gamma_\gamma(E) = \Gamma_\gamma(E_R) \frac{(Q+E)^{2l+1}}{(Q+E_R)^{2l+1}},$$

Q is the *Q*-value for the (p,γ) reaction

Reaction rate is then

$$N_A < \sigma v > \;=\; N_A \left(\frac{8}{\pi \mu m_N (k_B T)^3}\right)^{\frac{1}{2}} \int \sigma(E)\, E\, \exp(-E/k_B T)\, dE$$

$$N_A \langle \sigma v \rangle_{\rm res} = N_A \left(\frac{2\pi}{\mu k T}\right)^{3/2} \hbar^2 \omega\gamma \exp(-E_R/kT),$$

where the resonance strength $\omega\gamma$ is given by

$$\omega\gamma = \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma_{\rm tot}}.$$

The case when resonant and non-resonant capture do not interfere

$$\sigma(E_i) = \frac{2J_f + 1}{(2I_1 + 1)(2I_2 + 1)} \sum_{\lambda \sigma l_i J_i I} \frac{k_\gamma^{2\lambda+1}}{(2l_i + 1)} \frac{8\pi(\lambda + 1)}{\hbar \lambda (2\lambda + 1)!!^2} \left| \left\langle \Psi^{J_f \pi_f} \left\| M_\lambda^\sigma \right\| \Psi_{l_i I}^{J_i \pi_i}(E_i) \right\rangle \right|^2$$

- There is no interference between different l_i in the integrated cross sections.
- If there is no resonance in the $l_i = l_{min}$ partial wave but there is a resonance at $l_i = l_R \neq l_{min}$ then

$$\sigma(E) \approx \sigma_{l_{\min}}(E) + \sigma_{l_R}(E)$$

Microscopic models

$$\left(\sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V_{ij} \right) \Psi = E\Psi$$

Resonating Group Method (RGM), Generator coordinate method (GCM), Microscopic cluster model (MCM)

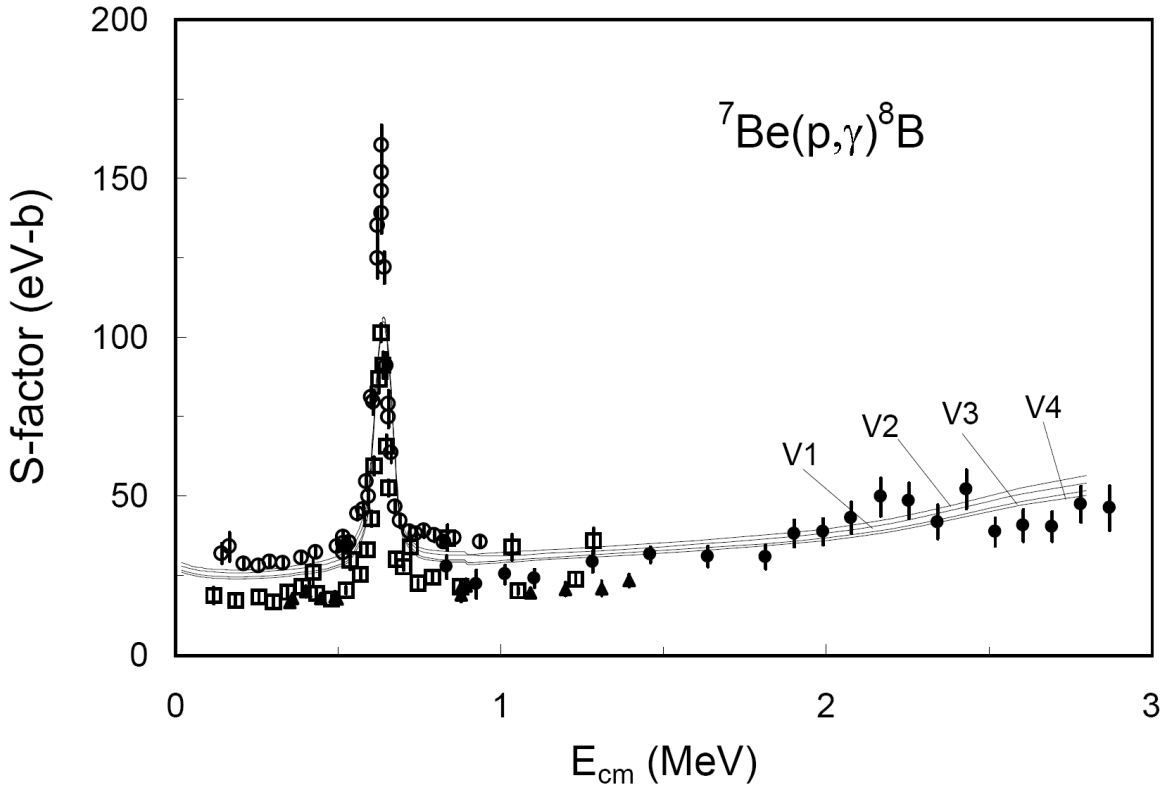
$$\Psi^{J_f M_f \pi_f} = \sum_{l_f I} \mathcal{A} \varphi^{J_f}_{l_f I}(r) \left[\left[\Psi^{J_1 \pi_1} \otimes \Psi^{J_2 \pi_2} \right]_I \otimes Y_{l_f}(\hat{r}) \right]^{J_f}_{M_f}$$

$\mathcal{A}$ is the antisymmetrization operator, it permutes nucleons between A_1 and A_2

The internal wave functions for A_1 and A_2 are made from single-particle wave functions of the oscillator shell model and $\varphi^{J_f}_{l_f I}(r)$ is found by solving the Schrödinger equation with proper boundary conditions.

Effective NN interactions $V_{ij}(|r_i - r_j|)$ is used.

Calculations of $^{7}Be(p,\gamma)^{8}B$ within Microscopic Cluster model



Energy dependence from theoretical calculations are used to extrapolate the experimental cross sections to stellar energies (E < 30 keV)

Continuum Shell Model

$$\sum_{i<j}^{A} V_{ij} = \sum_{i=1}^{A} \bar{V}_i + \sum_{i<j}^{A} V_{ij}^{res}$$

Nucleons move in a mean field and interact with each other via residual interactions

Nuclear wave function is made of single-particle wave functions, nucleons can occupy not only bound states but also continuum states

$^{17}F(p,\gamma)^{18}Ne$ (important for nucleosynthesis in novae)

$J^{\pi(^{17}F)} = 5/2^+$

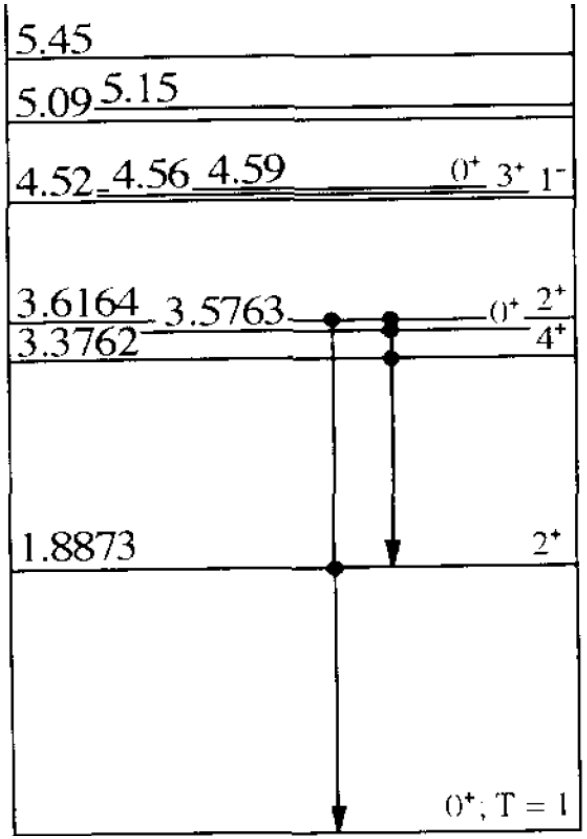
$I = 2^+$ and 3^+

$\pi(J_f) = 1$

Allowed transitions:

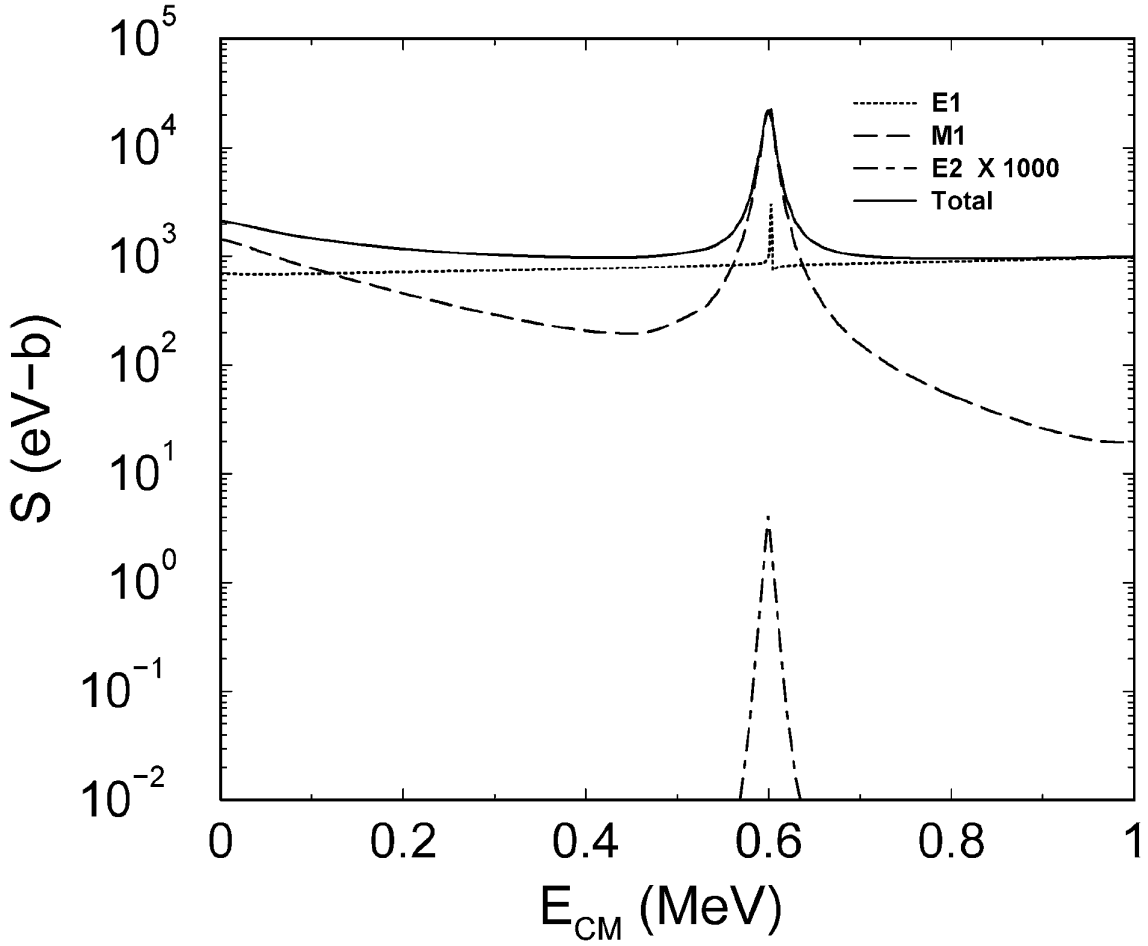
M1 from $l_i = 0$

E1 from $l_i = 1$



Calculations of $^{17}F(p,\gamma)^{18}Ne$ within Continuum Shell Model

R.Chatterjee, J.Okolowicz, M.Ploszajczak Nucl.Phys. A764, 528 (2006)



M1 transition dominates, which cannot occur in potential model

Ab-initio calculations of radiative capture

d(p,γ)^{3}He (*M.Viviani et al, PRC **61**, 064001 (2000)*)

J^π(d) = 1^+

I = $1/2^+$ and $3/2^+$

J^π (^{3}He) = $1/2^+$

$\pi(J_f) = 1$

Allowed transitions:

M1 from l_i =0

E1 from l_i = 1

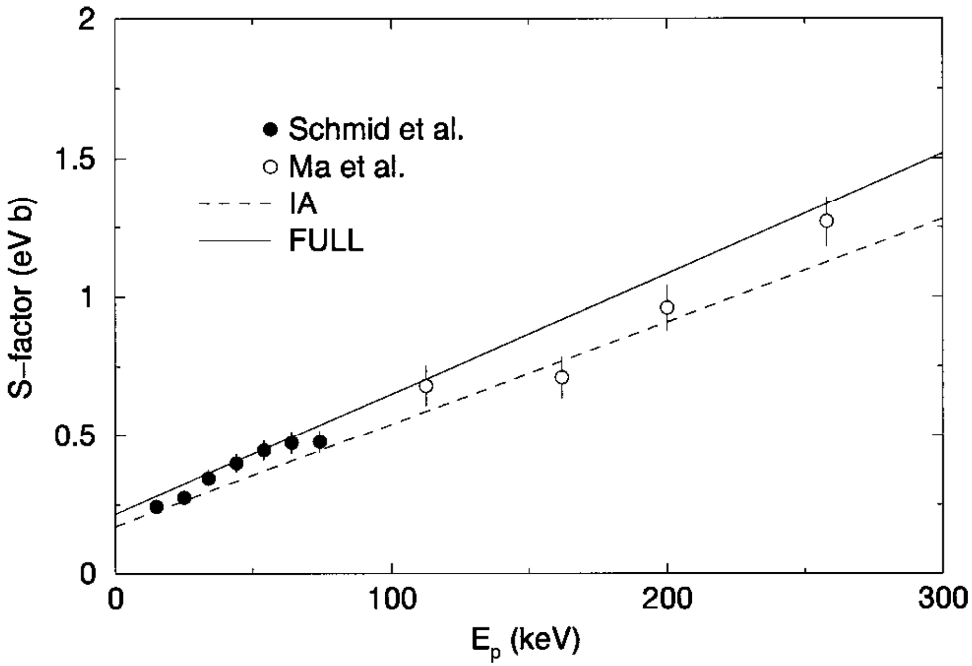


FIG. 1. The S factor for the $^2\mathrm{H}(p,\gamma)^3\mathrm{He}$ reaction, obtained with the AV18/UIX Hamiltonian model and one-body only (dashed line) or both one- and two-body (solid line) currents, is compared with the experimental values of Refs. [7,8].

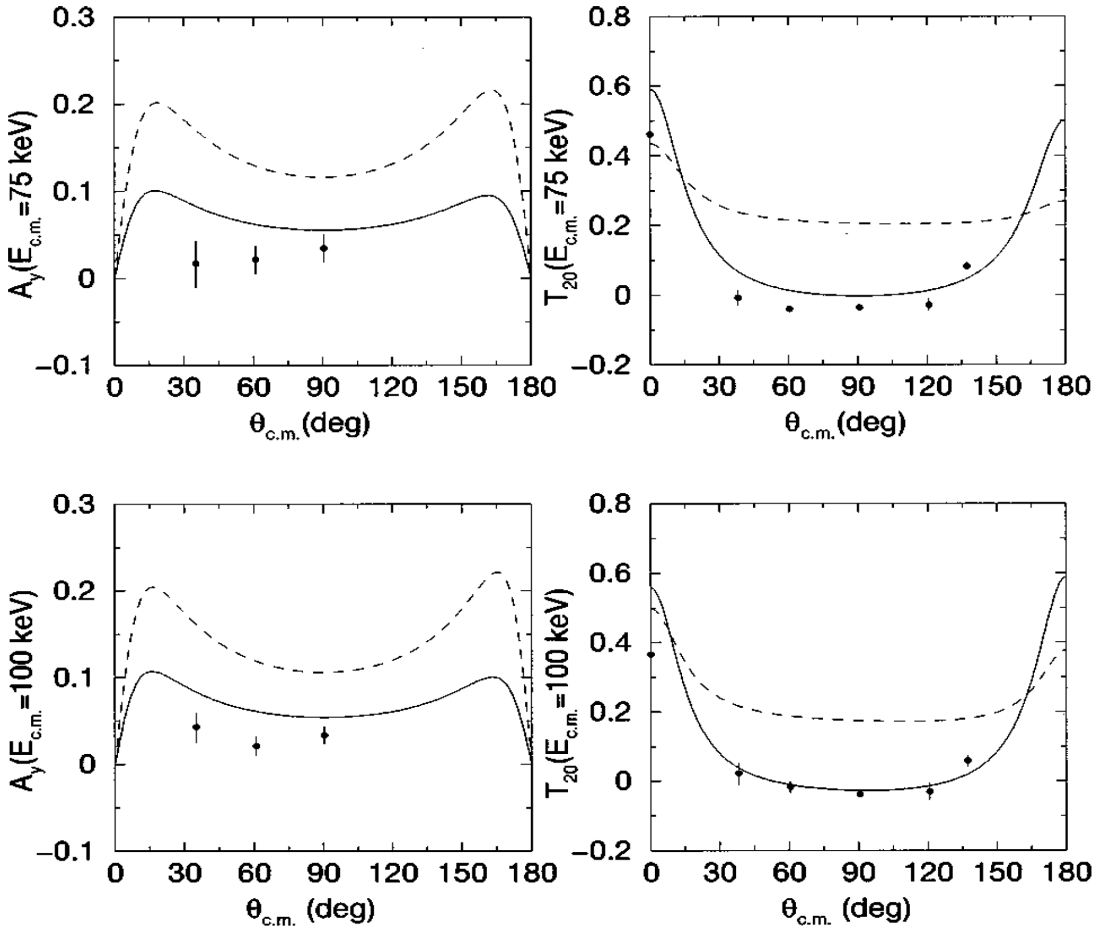


FIG. 4. Proton vector analyzing power A_y and deuteron tensor analyzing power T_{20} for pd capture at $E_{\mathrm{c.m.}}=75$ and 100 keV, obtained with the AV18/UIX Hamiltonian model and one-body only (dashed lines) or both one- and two-body currents (solid lines). The experimental values are from Ref. [8].

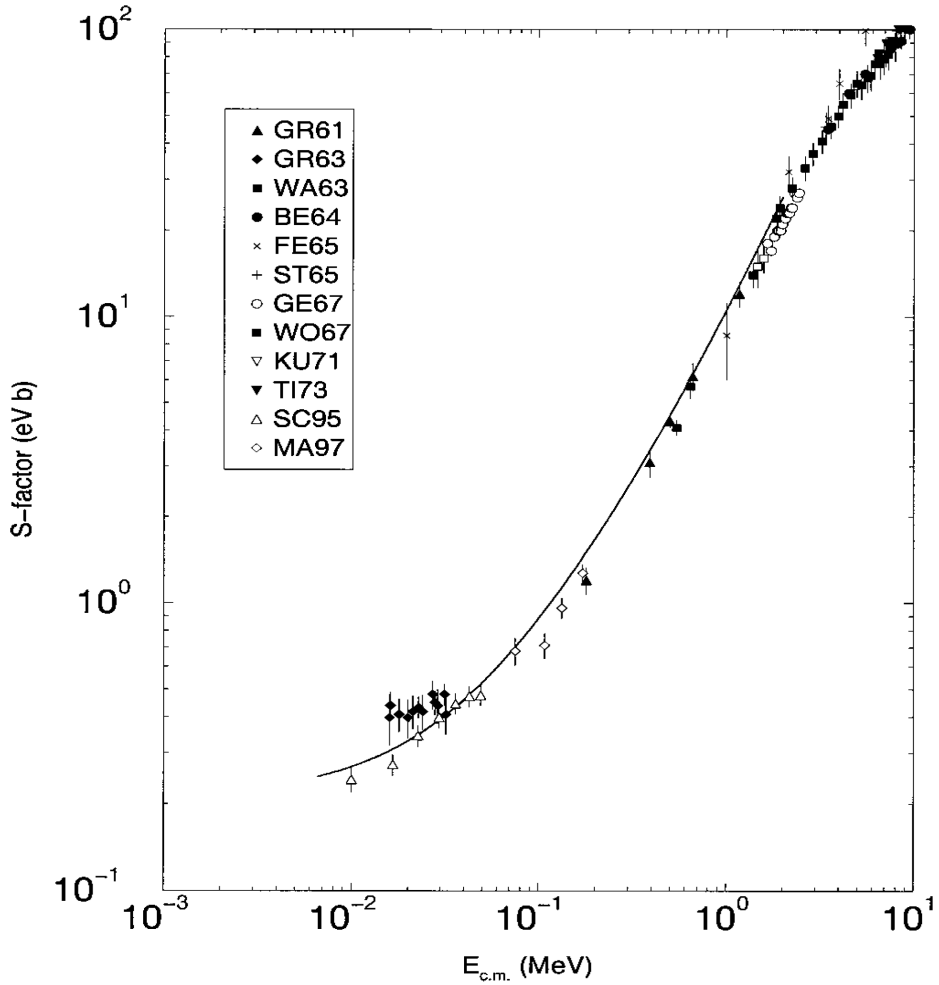


FIG. 6. The S factor for the $^2\mathrm{H}(p,\gamma)^3\mathrm{He}$ reaction, in the c.m. energy range 0–2 MeV, obtained with the AV18/UIX Hamiltonian model and one- and two-body currents (solid line) is compared with the experimental values listed in the web site http://pntpm.ulb.ac.be/nacre.htm.

Astrophysical S-factor at E=0

Capture from

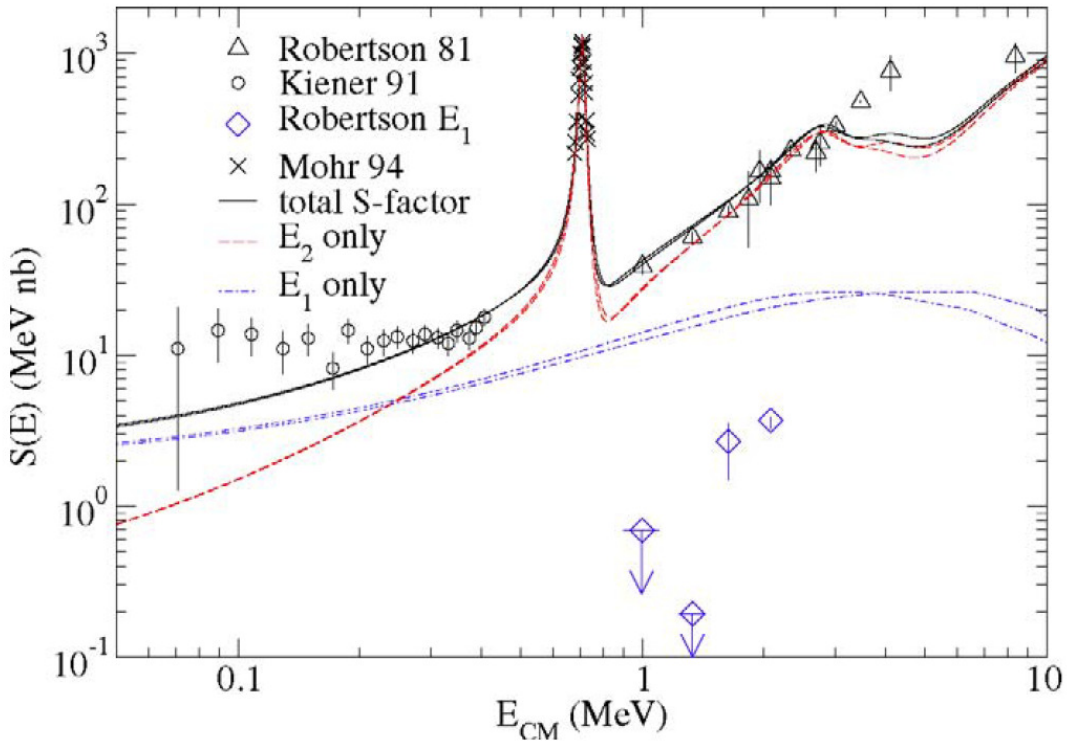
S-wave 0.110 eV b
P-wave 0.109 eV b

Total 0.219 eV b

LUNA data
S(0) = 0.216 ± 0.010 eV b

d(^{4}He,γ)^{6}Li *K.M. Nollett et al, PRC **63**, 024003 (2001)*

Variational Monte-Carlo calculations for ^{6}Li and ^{4}He but potential model wave function for d-^{4}He scattering



In potential model cross sections for E_1 equal to zero because Z_1A_2-Z_2A_1=0

$$\left\langle \Psi^{J_f\pi_f}_{l_f I_f} \left\| M_1^E(\boldsymbol{r}) \right\| \Psi^{Ji\pi_i}_{l_i I_i} \right\rangle \sim \left(Z_1 \frac{A_2}{A} - Z_2 \frac{A_1}{A} \right)$$

$^{3}He(^{4}He,\gamma)^{7}Be$ and $^{3}H(^{4}He,\gamma)^{7}Li$

*K.M. Nollett, PRC **63**, 054002 (2001)*

Variational Monte-Carlo calculations for ^{7}Li, ^{7}Be, ^{3}H, ^{3}He and ^{4}He

but potential model for ^{3}He-^{4}He and ^{3}H-^{4}He scattering wave function