# Potential models, bound and continuum states

**Mini-school on Nuclear Reaction Theories for Nuclear Astrophysics**
**Surrey, 7th January 2009**

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# This session (learning) aims:

Solutions of the Schrodinger equation for states of two bodies, with specific quantum numbers, over a wide range of energies – the need for bound, resonant, continuum and continuum bin states for reactions.

Reminder of the form of these two-body solutions at large separations and their relationships to absorption, reaction and scattering studies.

Constraints on two-body potentials and their parameters. Parameter conventions. The need to cross reference to known nuclear structure, resonances, nuclear sizes and experiment (whenever possible) in constraining all parameter choices.

# This session (learning) outcomes:

To (re)gain familiarity with solutions of the Schrodinger equation over a range of energies: properties of bound, resonant, continuum and bin states. These enter reaction models and codes (such as FRESCO). To appreciate the need to understand sensitivities to input parameters that enter into final calculations.

To recall the role of the S-matrix in determining scattering and absorptive properties of a two-body interaction.

To question the use of ‘standard’ potential parameter sets and understand the need to explore the quality of these by reference to experimental data and to systematics from theoretical models – e.g. from the shell model and Hartree-Fock. To see examples in each energy regime.

# There is a lot of literature to refer to:

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) Norman Austern

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages ) G R Satchler

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) L S Rodberg, R M Thaler

Direct Nuclear Reactions (World Scientific Publishing, 396 pages) Norman K. Glendenning

Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages ) C A Bertulani, P Danielewicz

Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages ) Herman Feshbach

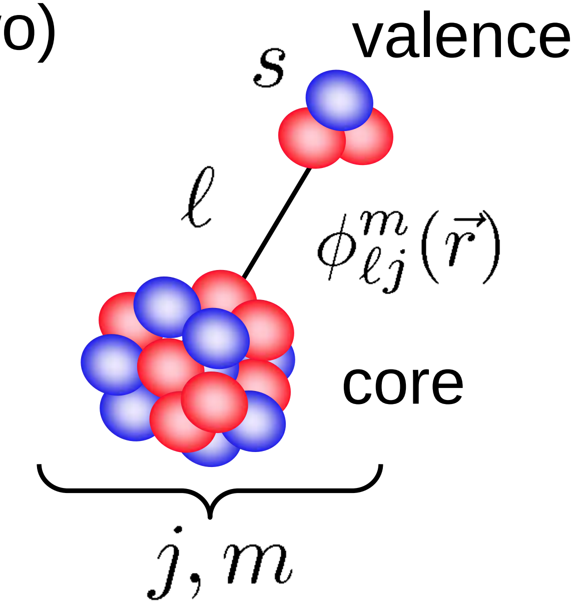
Introduction to Nuclear Reactions (Oxford University Press, 332 pages) G R Satchler

Nuclear Reactions for Astrophysics (Cambridge University Press, almost in press) Ian Thompson and Filomena Nunes

# Introduction to "The tools of the trade…"

Solution of Schrodinger's equation for (two) bodies interacting via a potential energy function of the form*

$$U(r) = \underbrace{V_C(r)}_{\text{Coulomb}} + \underbrace{V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}}_{\text{Nuclear}}$$




Need descriptions of wave functions of:

(1) Bound states of nucleons or clusters (valence particles)
. to a core (that is assumed for now to have spin zero).

(2) Unbound scattering or resonant states at low energy

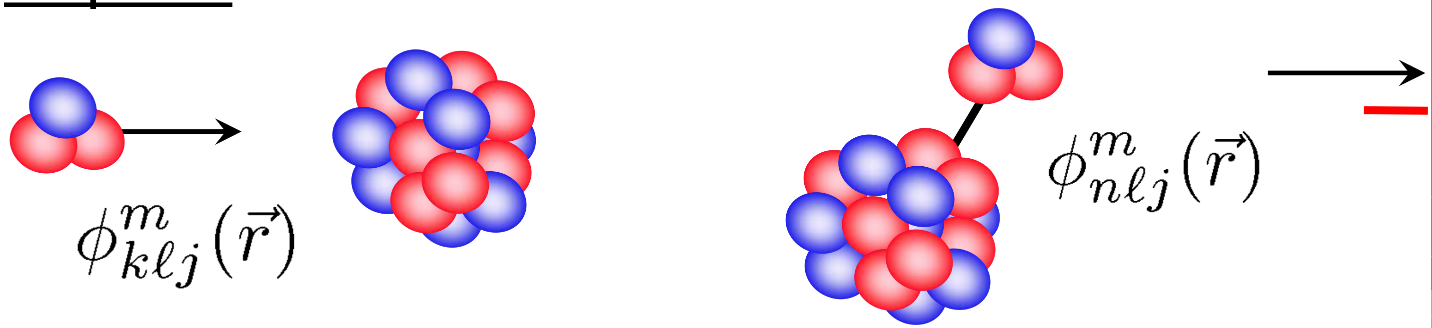
(3) Distorted waves of such bodies in complex potentials

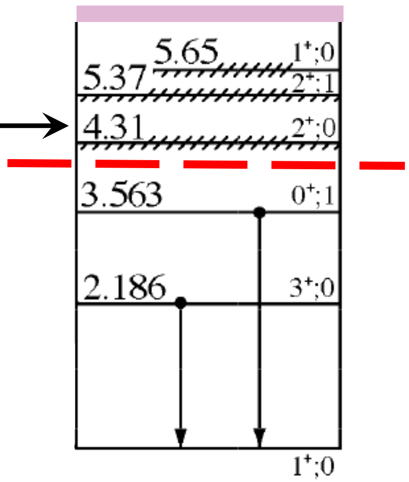
$$U(r) = V_C(r) + V(r) + \boxed{iW(r)} + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

*Additional, e.g. tensor terms, when s=1 or greater neglected

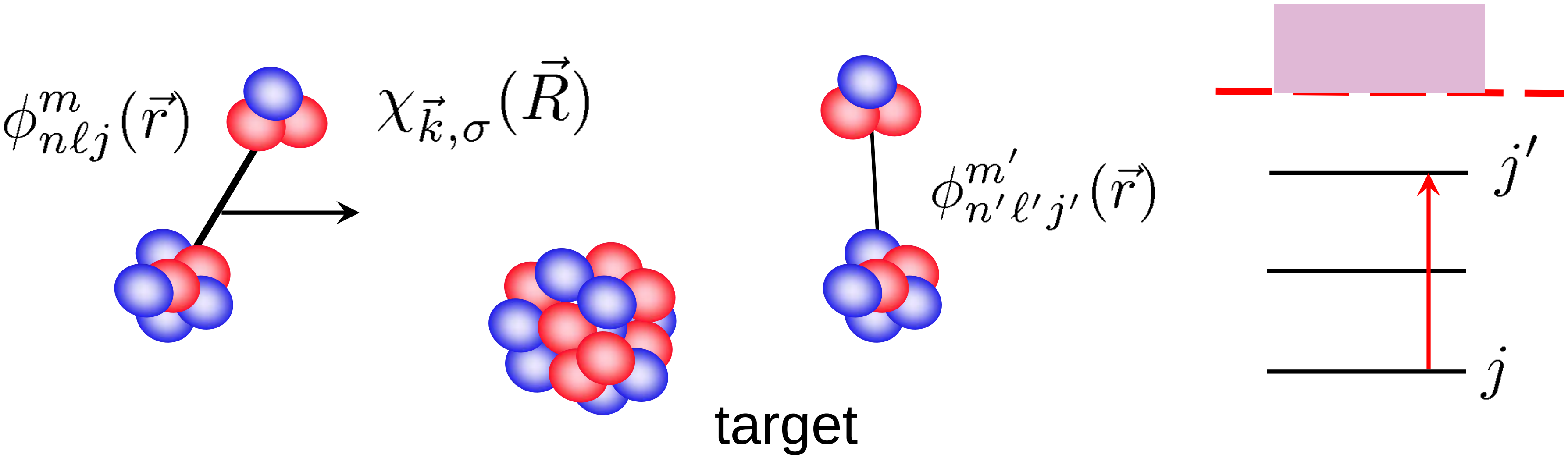
# Direct reactions – types and characteristics

## Capture



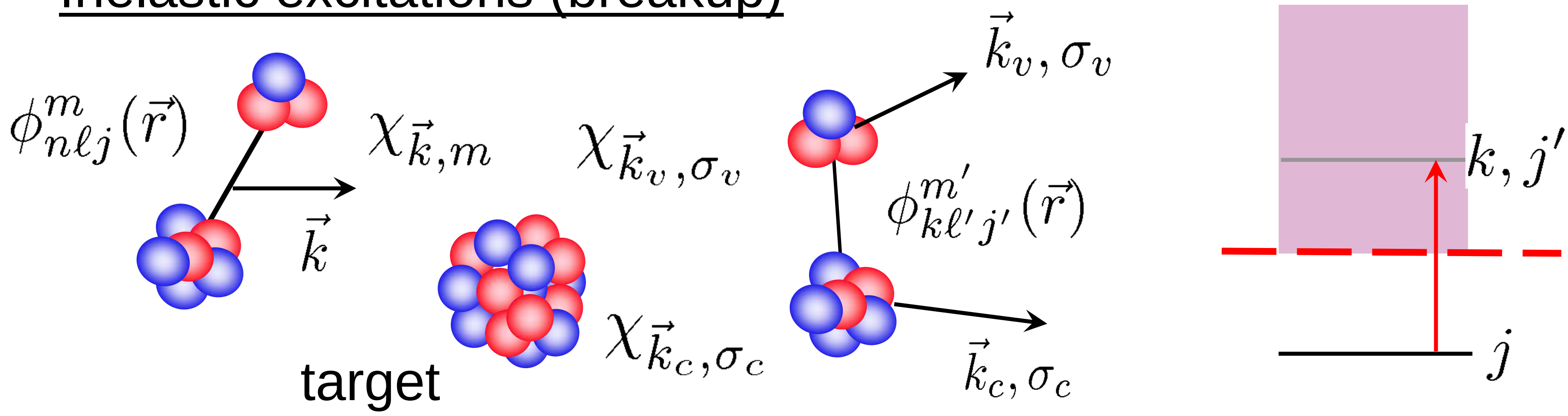




## Inelastic excitations (bound to bound states) DWBA

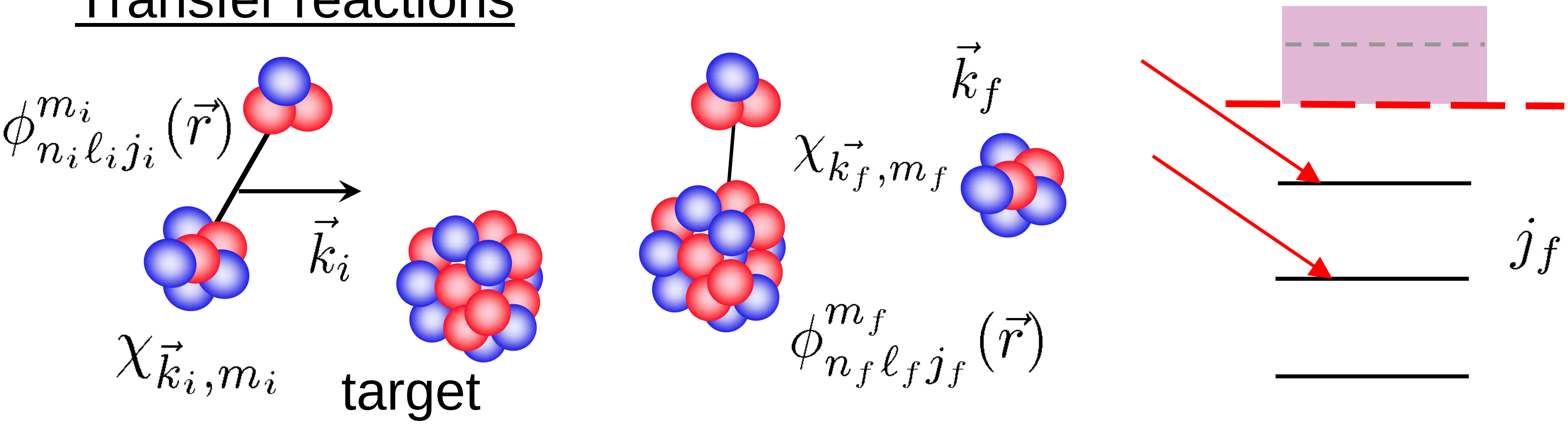


# Direct reactions – types and characteristics

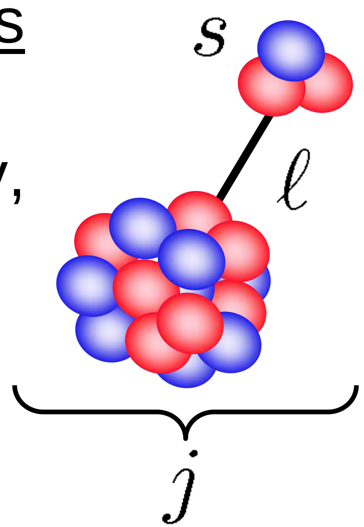
## Inelastic excitations (breakup)




## Transfer reactions



# Direct reactions – requirements (1)




Description of wave functions of **bound** systems (both nucleons or clusters) – (a) can take from structure theory, if available or, (b) more usually, use a real potential model to bind system with the required experimental separation energy.

Refer to core and valence particles

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r}Y^\lambda_\ell(\hat{r})\chi^\sigma_s, \quad \int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$

Usually just one or a few such states are needed.

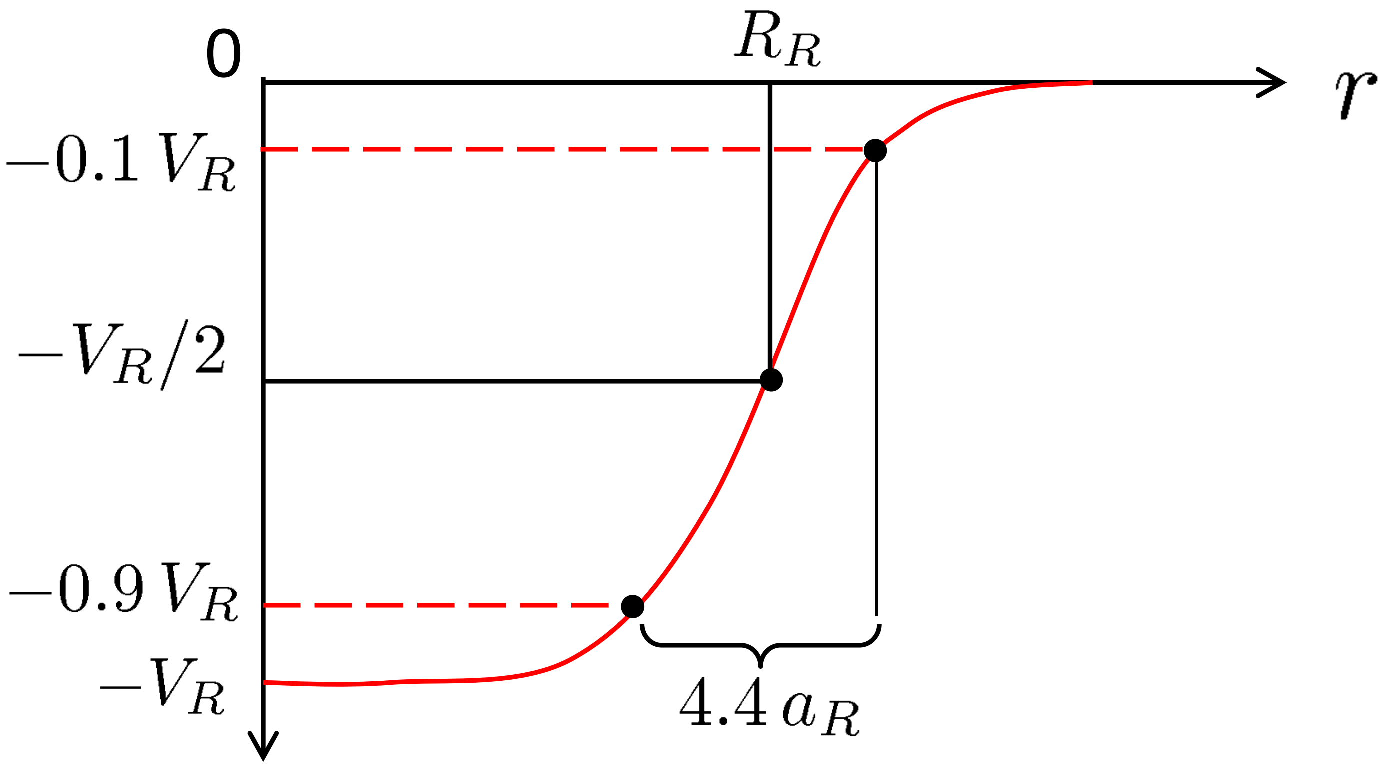
Separation energies/Q-values: many sites, e.g.

http://ie.lbl.gov/toi2003/MassSearch.asp

# Bound states – real potentials

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_R = \frac{r - R_R}{a_R}$$



# Bound states – potential conventions

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

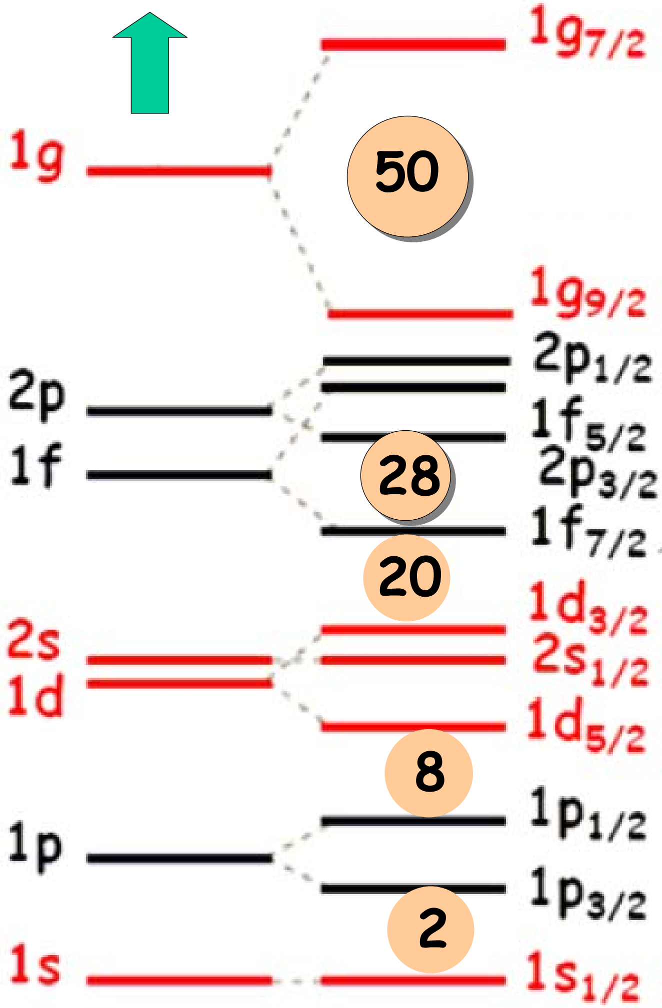
$$V_{so}(r) = -\frac{2\,V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

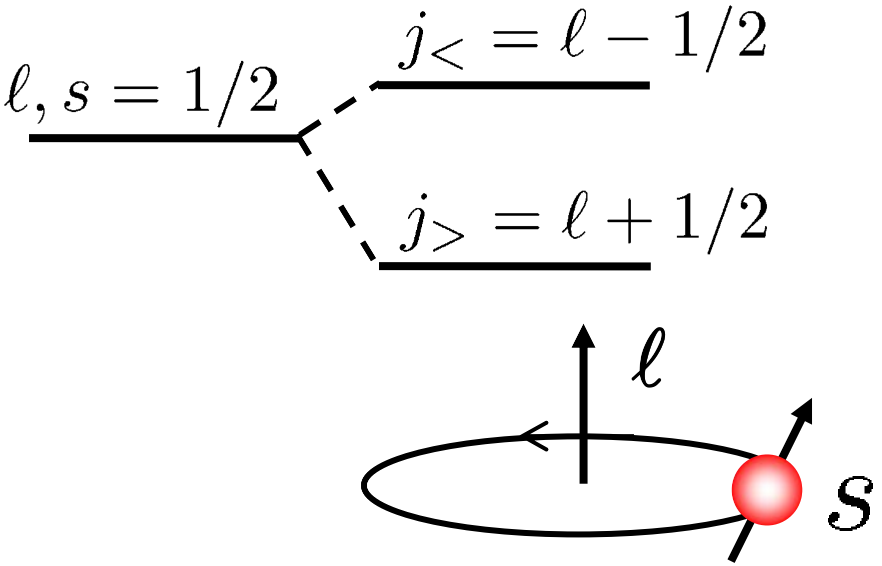
Conventions

$$R_i = r_i A_2^{1/3} \qquad \text{or} \qquad R_i = r_i\left[A_1^{1/3} + A_2^{1/3}\right]$$

$$\begin{aligned} V_C(r) &= Z_1 Z_2 e^2/r\ , r > R_C \\ &= \frac{Z_1 Z_2 e^2}{2R_C}\left[3 - \left(\frac{r}{R_C}\right)^2\right]\ , r \leq R_C \end{aligned}$$

# Bound states – nuclear single particle structures



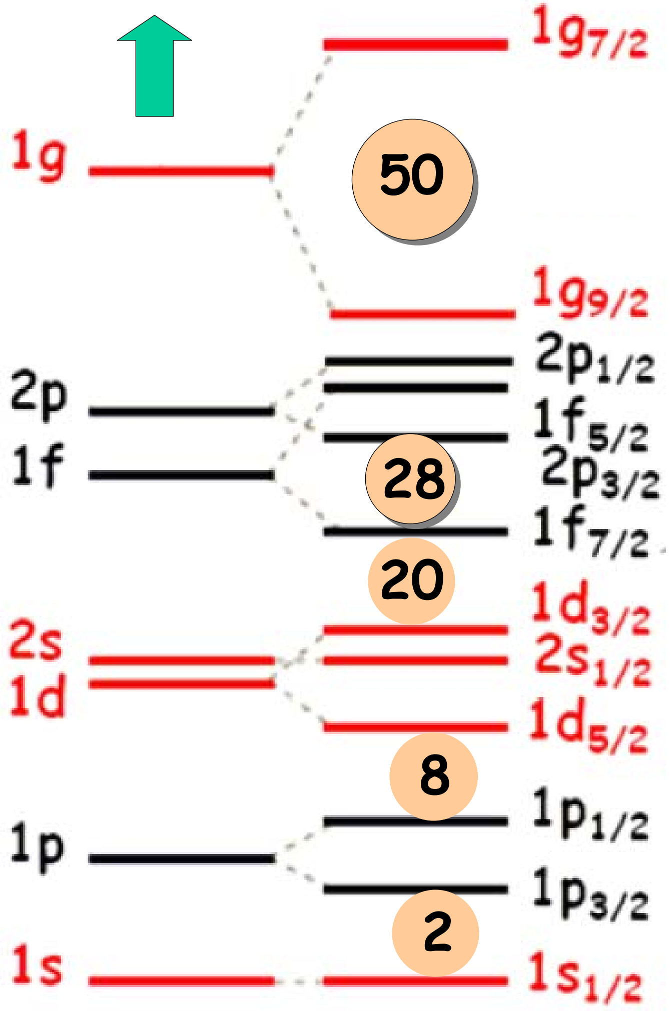




$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V_{so}(r) < 0$$

# Bound states – for nucleons - conventions




Conventions $\phi^{m}_{n\ell j}(\vec{r})$

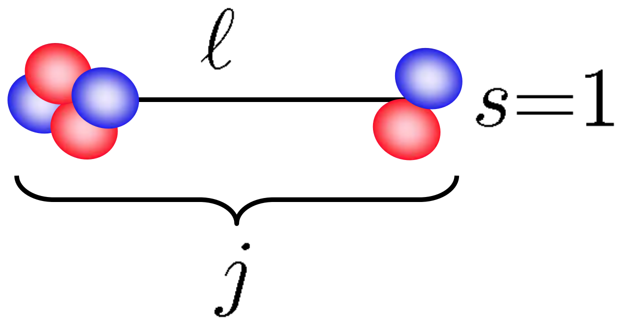
With this potential, and using sensible parameters, we will obtain the independent-particle shell model level orderings, shell closures with spin-orbit splitting.

**NB**: In diagram $2d_{5/2}$ means the second $d_{5/2}$ state. Defined this way, $n>0$ and $n-1$ is the number of nodes in the radial wave function. This is the convention used in e.g. FRESCO but many codes ask for the actual number of nodes. Care is needed.

# Direct reactions – requirements (2)

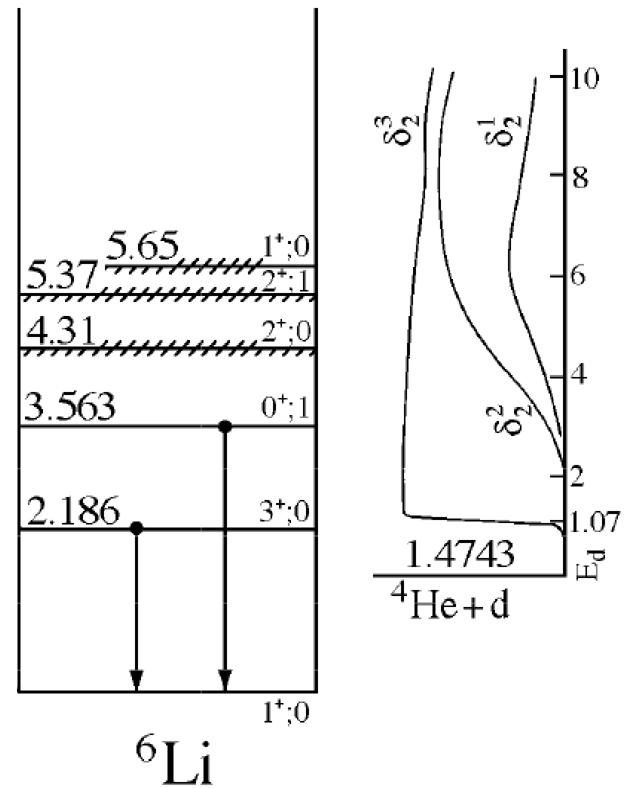
Description of wave functions for unbound (often light) systems (nucleons or clusters) with low relative energy: Usually have low nuclear level density of isolated resonances. Use the same real potential model as binds the system → scattering wave functions in this potential. Also 'bin' wave functions.

$^{6}$Li (α+d)



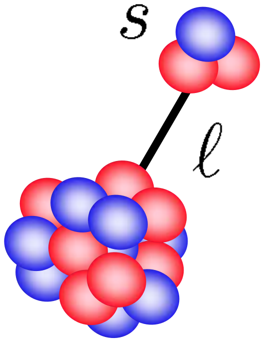

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^{m}_{k\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{k\ell j}(r)}{kr}Y^{\lambda}_{\ell}(\hat{r})\chi^{\sigma}_{s}$$



# Completeness and orthogonality

Given a fixed two-body Hamiltonian




$$H = T + U(r) = T + V_C(r) + V(r) + V_{so}(r)\vec{\ell} \cdot \vec{s}$$

the set of all of these bound and unbound wave functions

$$\{\phi^m_{n\ell j}(\vec{r}),\ \phi^m_{k\ell j}(\vec{r})\}$$

form a complete and orthogonal set, and specifically

$$\langle \phi^m_{n\ell j}(\vec{r}) | \, \phi^m_{k\ell j}(\vec{r}) \rangle = 0$$

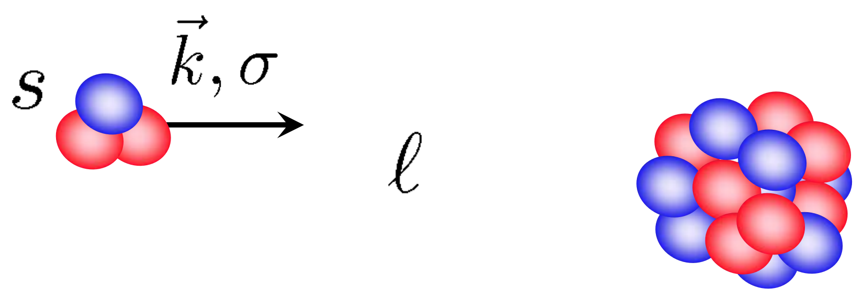
When including coupling of bound to unbound states it is essential we use a fixed Hamiltonian for both the bound and unbound states (in each $\ell\, j$ channel) else we lose the orthogonality and the states will couple even without a perturbation or interaction with the target.

# Direct reactions – requirements (3)

Description of wave functions for scattering of nucleons or clusters from a heavier target and/or at higher energies: (a) high nuclear level density and broad overlapping resonances, (b) many open reaction channels, inelasticity and absorption. Use a complex (absorptive) optical model potential – from theory or simply fitted to elastic scattering data for the system and the energy of interest.

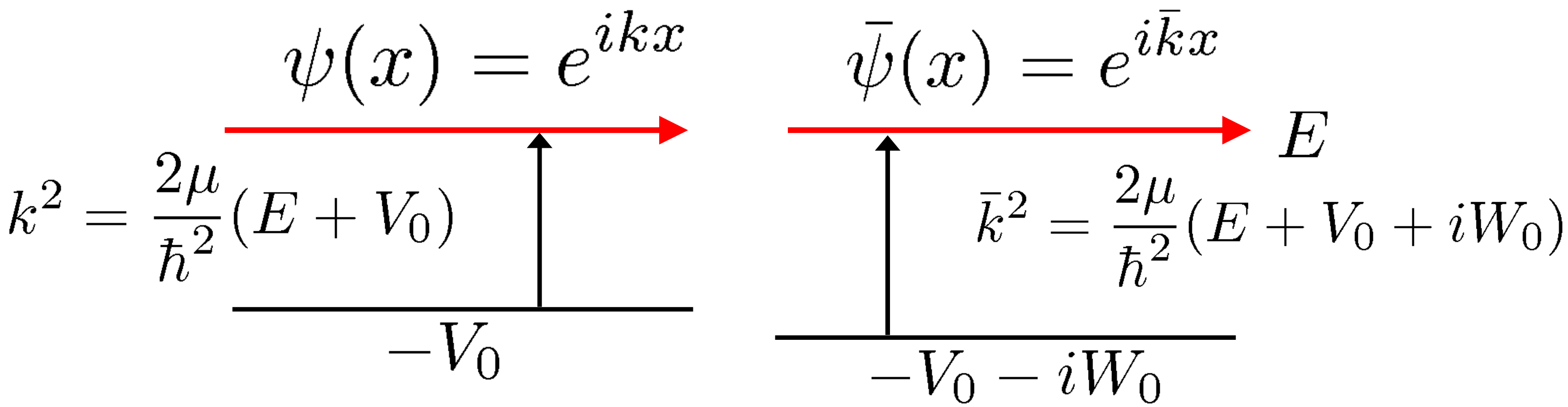
**Distorted waves**:

$$\chi_{\vec{k},\sigma}(\vec{r})$$




$$U(r) = V_C(r) + V(R) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

# Optical potentials – the role of the imaginary part




$$\bar{k}^2 = \frac{2\mu}{\hbar^2}(E + V_0 + iW_0) = \frac{2\mu}{\hbar^2}(E + V_0)\left[1 + \frac{iW_0}{E + V_0}\right]$$

$$\bar{k} = k\left[1 + \frac{iW_0}{E + V_0}\right]^{1/2} \approx k\left[1 + \frac{iW_0}{2(E + V_0)}\right], \quad W_0 \ll E, V_0$$

So, for $W_0 > 0$, $\bar{k} = k + ik_i/2$, $k_i = kW_0/(E + V_0) > 0$,

$$\bar{\psi}(x) = e^{i\bar{k}x} = e^{ikx}e^{-\frac{1}{2}k_i x}, \qquad |\bar{\psi}(x)|^2 = e^{-k_i x}$$

# Optical potentials - parameter conventions

$$U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

$$V_{so}(r) = -\frac{2V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

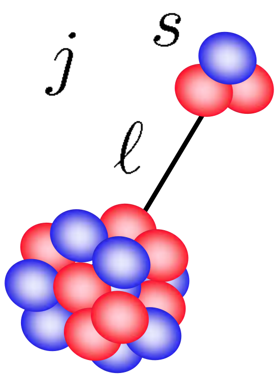
FRESCO conventions

$$W(r) = -\frac{W_V}{[1+\exp(X_V)]} - \frac{4W_S\exp(X_S)}{[1+\exp(X_S)]^2}\ ,$$

$$R_i = r_i\,A_2^{1/3} \quad \text{or} \quad R_i = r_i\,\left[A_1^{1/3} + A_2^{1/3}\right]$$

# The Schrodinger equation (1)

So, using usual notation




$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\phi_{\ell j}^m(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

and defining $\phi_{\ell j}^m(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{\ell j}(r)}{r}Y_\ell^\lambda(\hat{r})\chi_s^\sigma$

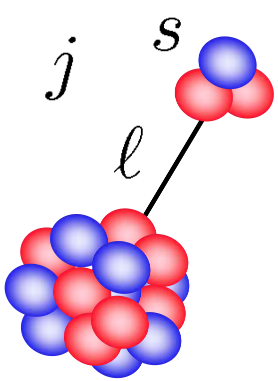
$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right)u_{\ell j}(r) = 0$$

bound states $E_{cm} < 0$ scattering states $E_{cm} > 0$

With $U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$

$$\begin{aligned} U_{\ell j}(r) &= V_C(r) + V(r) + iW(r) \\ &+ V_{so}(r)[j(j+1) - \ell(\ell+1) - s(s+1)]/2 \end{aligned}$$

# The Schrodinger equation (2)




Must solve

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right) u_{\ell j}(r) = 0$$

<u>bound states</u> $E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) \boxed{u_{n\ell j}(r)} = 0$$

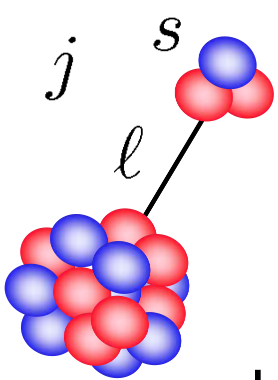
Discrete spectrum

<u>scattering states</u> $E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) \boxed{u_{k\ell j}(r)} = 0$$

Continuous spectrum

# Large r: The Asymptotic Normalisation Coefficient




## Bound states

$$E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

but beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta_b\kappa_b}{r} - \kappa_b^2\right) u_{n\ell j}(r) = 0, \quad \eta_b = \frac{\mu Z_c Z_v e^2}{\hbar\kappa_b}$$

$$u_{n\ell j}(r) \to C_{\ell j} W_{-\eta_b,\ell+1/2}(2\kappa_b r) \xrightarrow[r \to \infty]{} C_{\ell j} \exp(-\kappa_b r)$$

Whittaker function

$C_{\ell j}$ ANC completely determines the wave function outside of the range of the nuclear potential – only requirement if a reaction probes only these radii

# Large r: The phase shift and partial wave S-matrix

Scattering states

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr), \; G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\begin{aligned} u_{k\ell j}(r) &\rightarrow e^{i\delta_{\ell j}}[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)] \\ &\rightarrow (i/2)[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)] \end{aligned}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm i F_\ell(\eta, kr)$$

# Phase shift and partial wave S-matrix: Recall

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}} \underbrace{[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)]}$$

If *U(r)* is real, the phase shifts $\delta_{\ell j}$ are real, and […] also

$$u_{k\ell j}(r) \quad \rightarrow \quad (i/2)[\underline{H_\ell^{(-)}}(\eta, kr) - S_{\ell j} \underline{H_\ell^{(+)}}(\eta, kr)]$$

Ingoing waves

outgoing waves

$$S_{\ell j} = e^{2i\delta_{\ell j}}$$

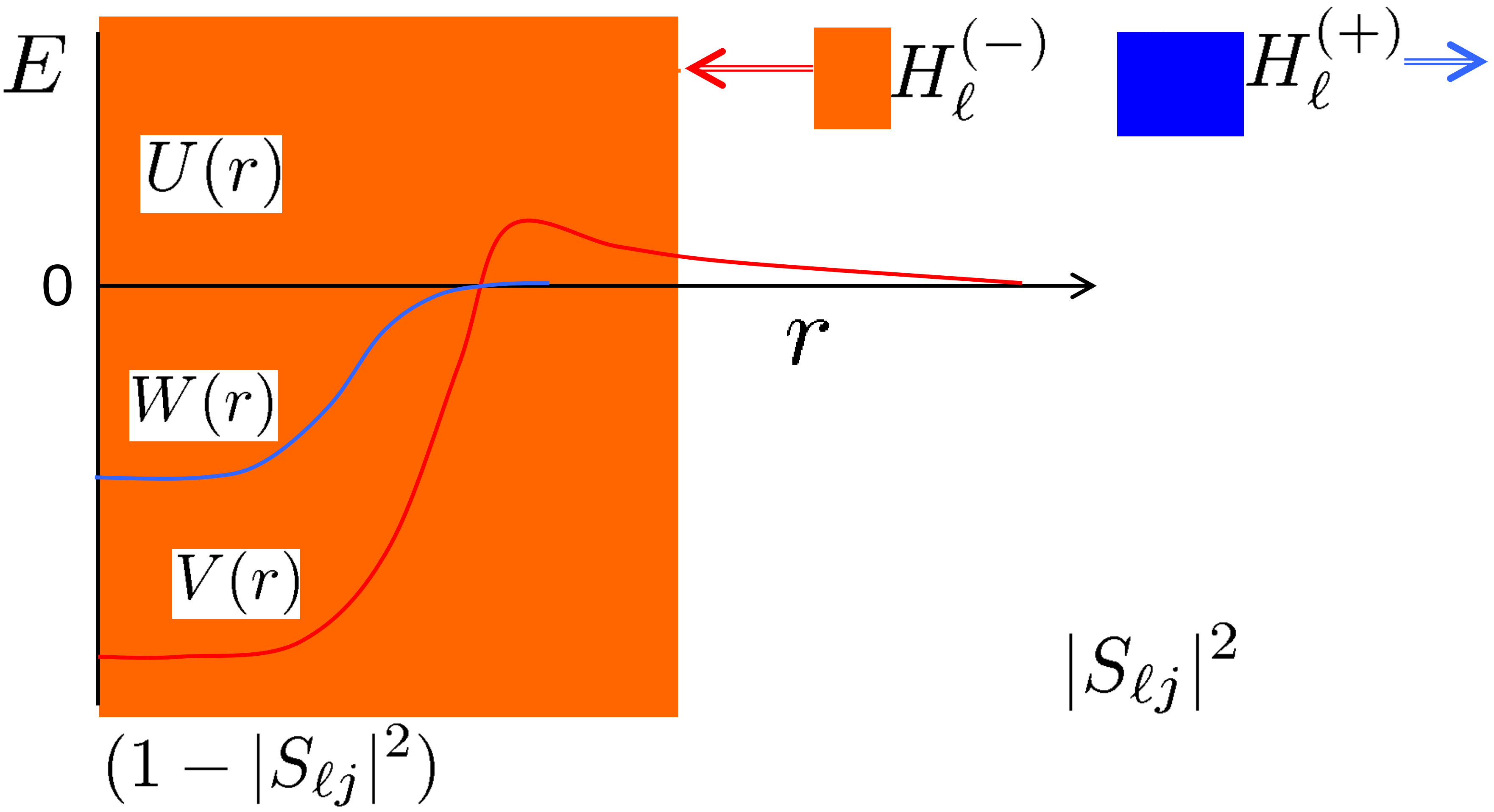
$|S_{\ell j}|^2$ survival probability in the scattering

$(1 - |S_{\ell j}|^2)$ absorption probability in the scattering

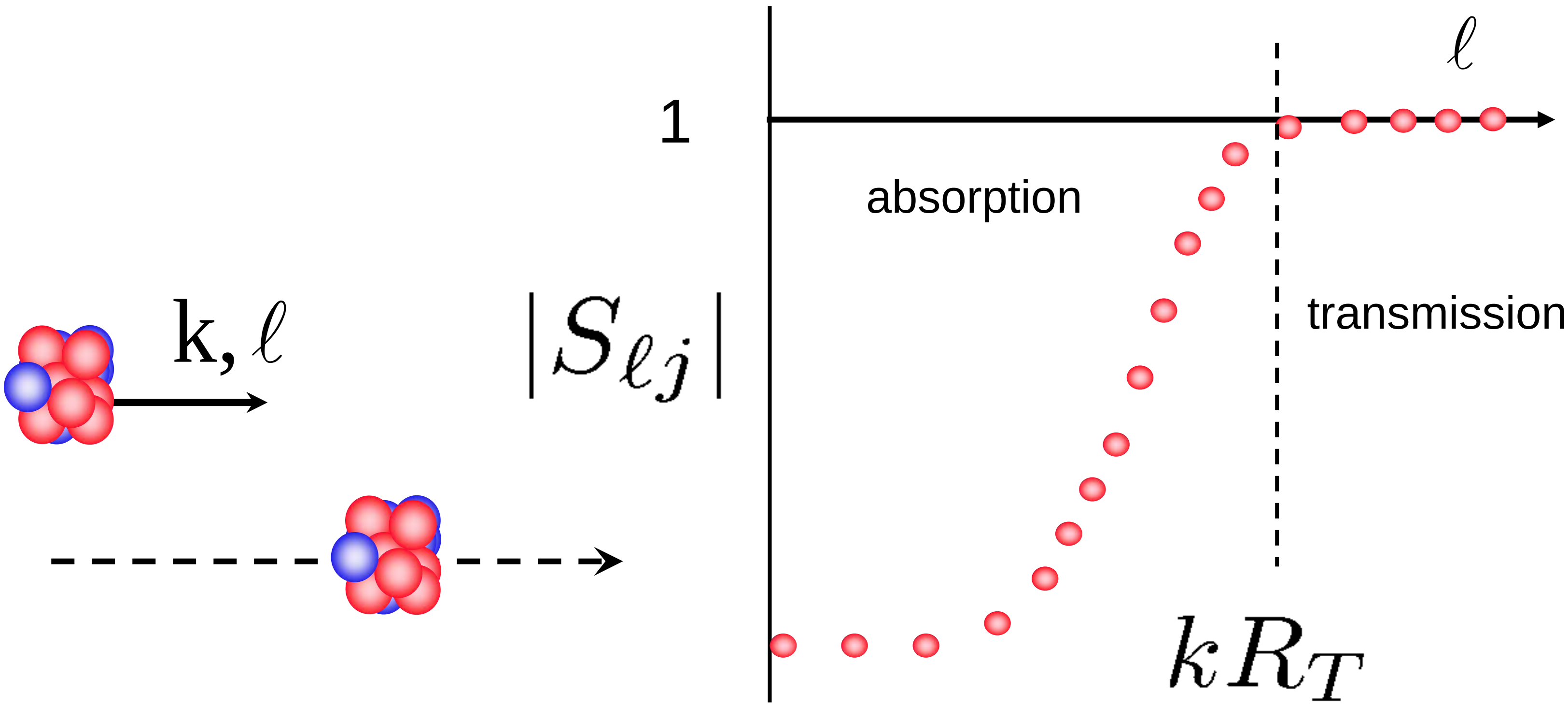
Having calculate the phase shifts and the partial wave S-matrix elements we can then compute all scattering observables for this energy and potential (but later).

# Ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[\;\; H_\ell^{(-)} - \;\; H_\ell^{(+)}]$$



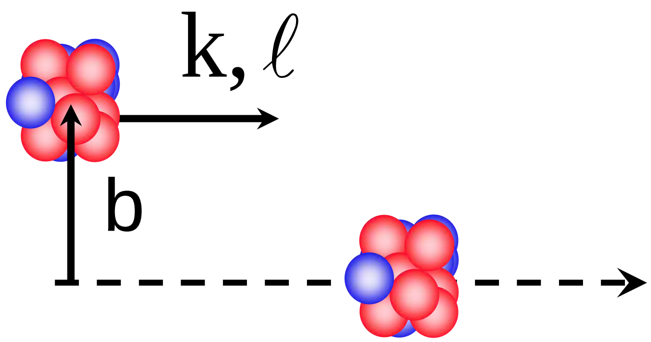
# S-matrix with absorption




$$u_{k\ell j}(r) \rightarrow (i/2)[H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$

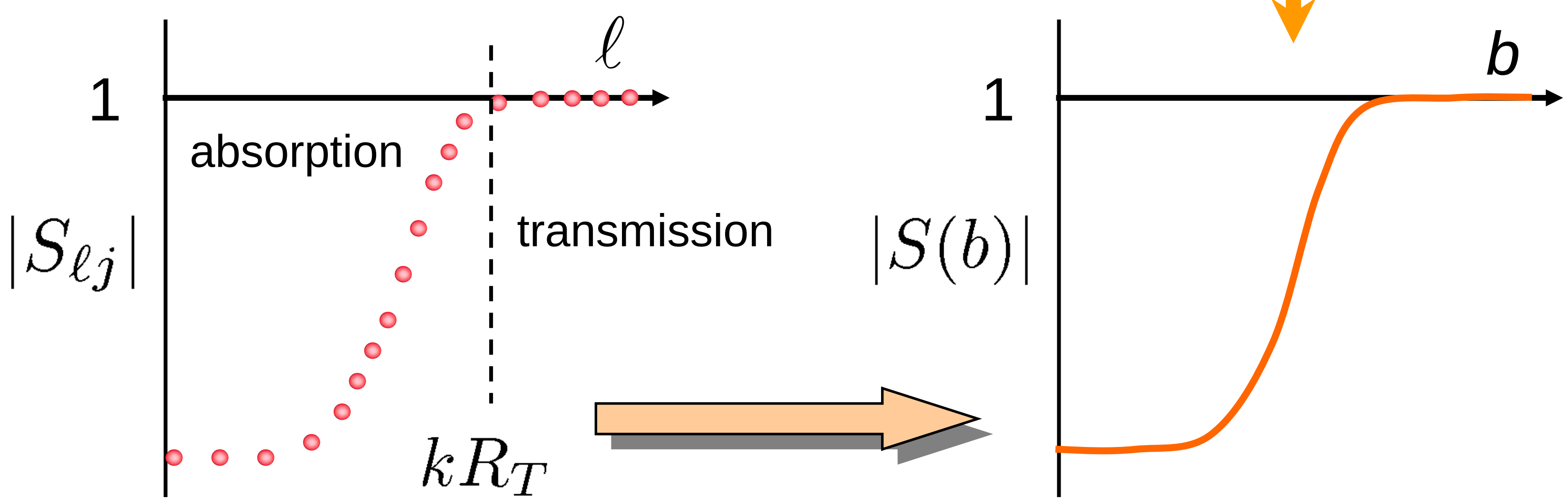
# Semi-classical models for the S-matrix - S(b)

b=impact parameter




for high energy/or large mass, semi-classical ideas are good

kb $\cong \ell$, actually $\Rightarrow \ell$+1/2



# Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the semi-classical (impact parameter) representation, for example (spinless case):

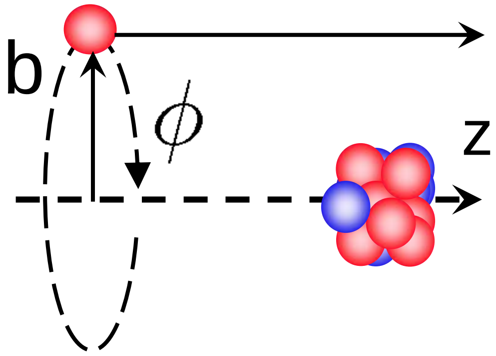
$$\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\ |1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\ (1-|S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el} + \sigma_R = 2\int d^2\vec{b}\ [1-\mathrm{Re.}S(b)] \quad \text{etc.}$$

and where (cylindrical coordinates)

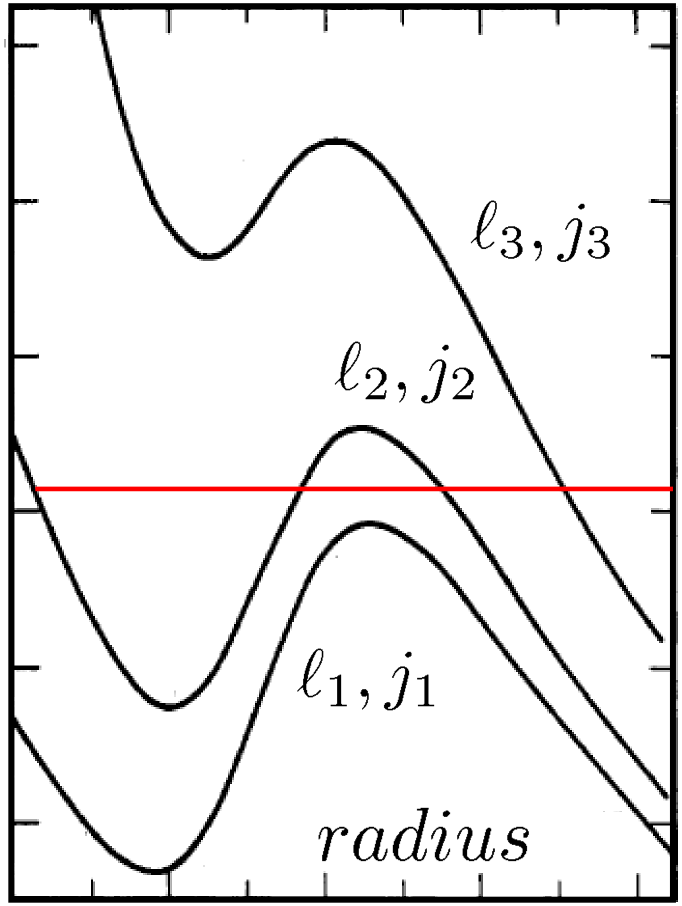
$$\int d^2\vec{b} \equiv \int_0^{\infty} bdb \int_0^{2\pi} d\phi = 2\pi \int_0^{\infty} bdb$$



# Phase shifts and S-matrix: Resonant behaviour

In real potentials, at low energies, the combination of an attractive nuclear, repulsive Coulomb and/or centrifugal terms can lead to potential pockets and resonant behaviour – the system being able to trapped in the pocket for some (life)time $\tau$.

$$\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+U_{\ell j}(r)$$

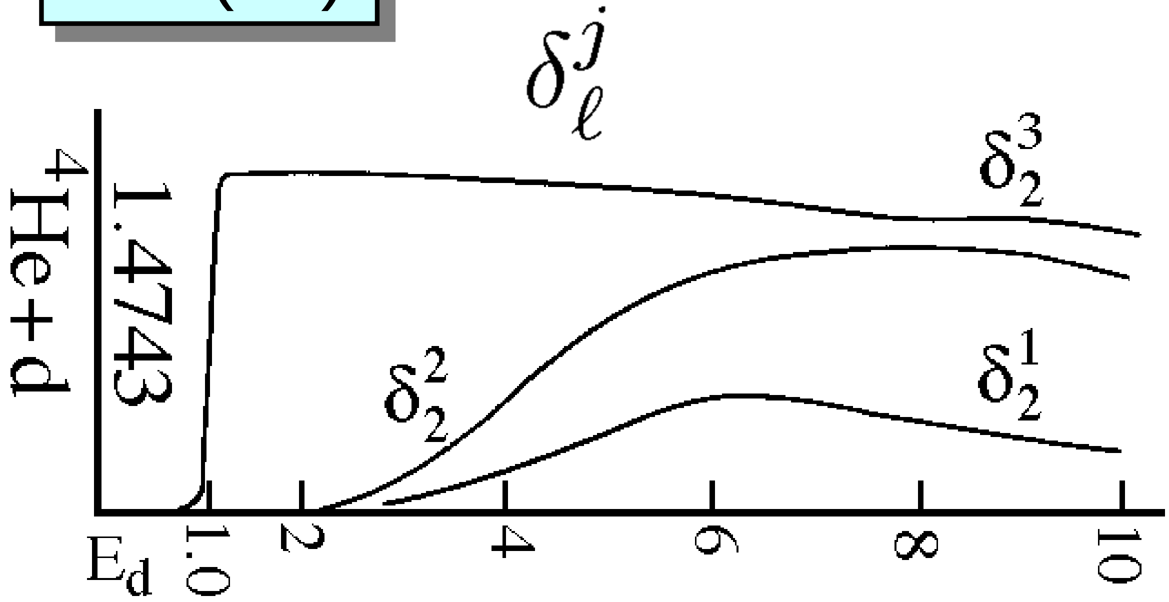


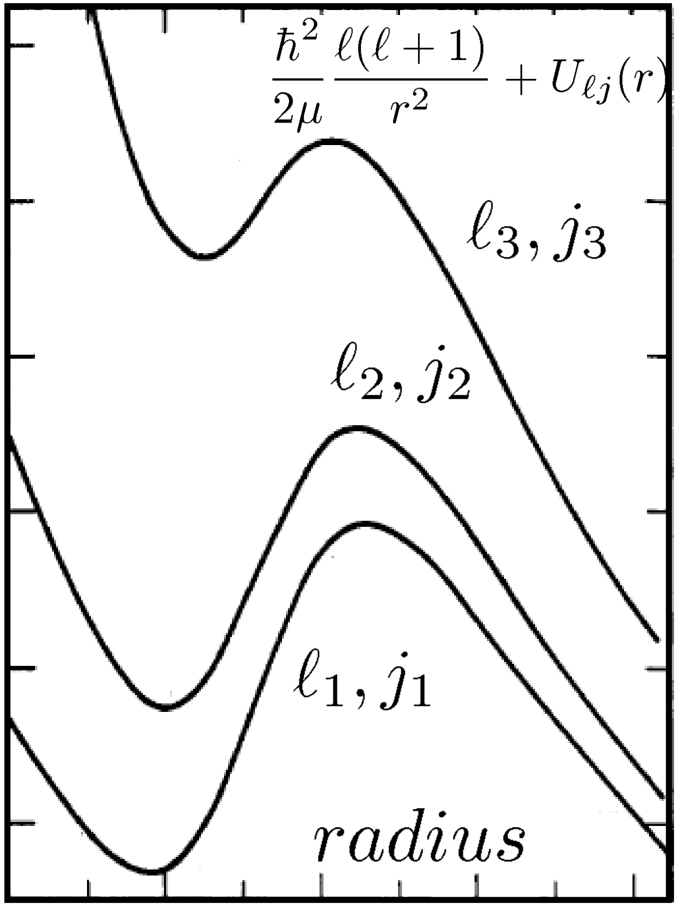
# Phase shifts and S-matrix: Resonant behaviour

Potential pockets can lead to resonant behaviour – the system being able to trapped in the pocket for some (life)time $\tau$.

**Clearest signal is the rise of the phase shift through 90 degrees.**

$\alpha$+d ($^6$Li)




Potential parameters should describe any known resonances

# Bound states potential parameters - nucleons

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

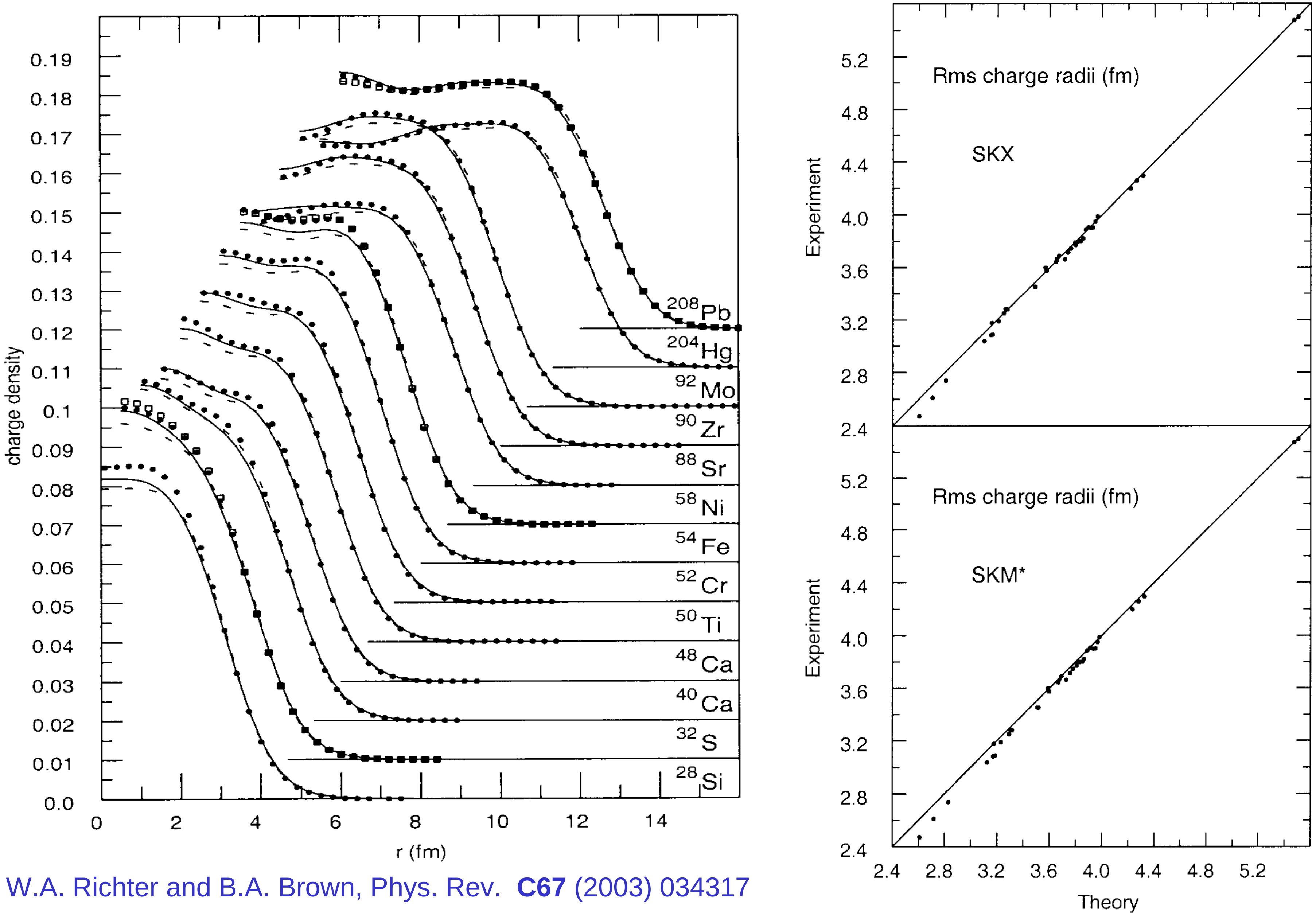
$$V_{so}(r) = -\frac{2\ V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

$$R_i = r_i\,A_2^{1/3}$$

$$r_R = r_C = r_{so} \approx 1.25\text{fm}$$

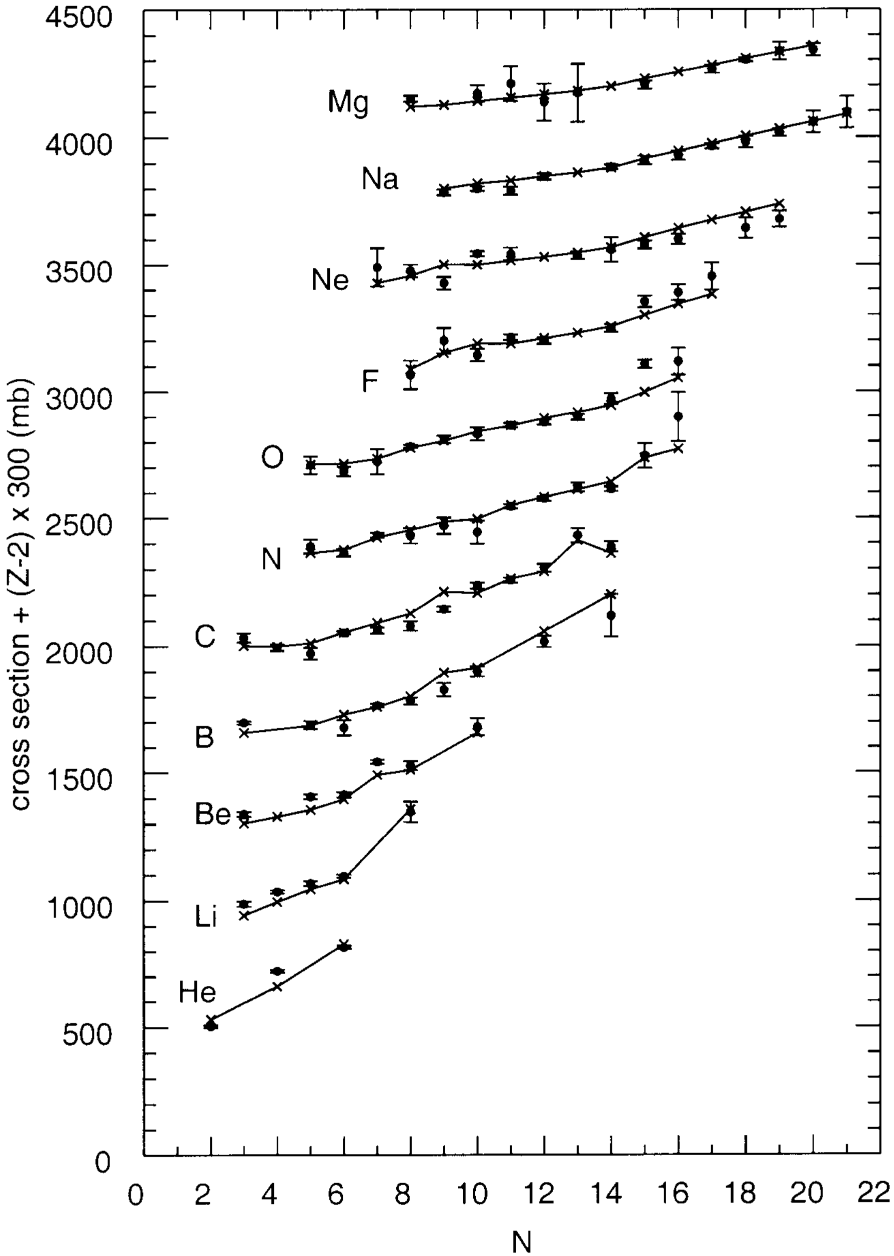
$$a_R = a_{so} \approx 0.65 - 0.7\text{fm}$$

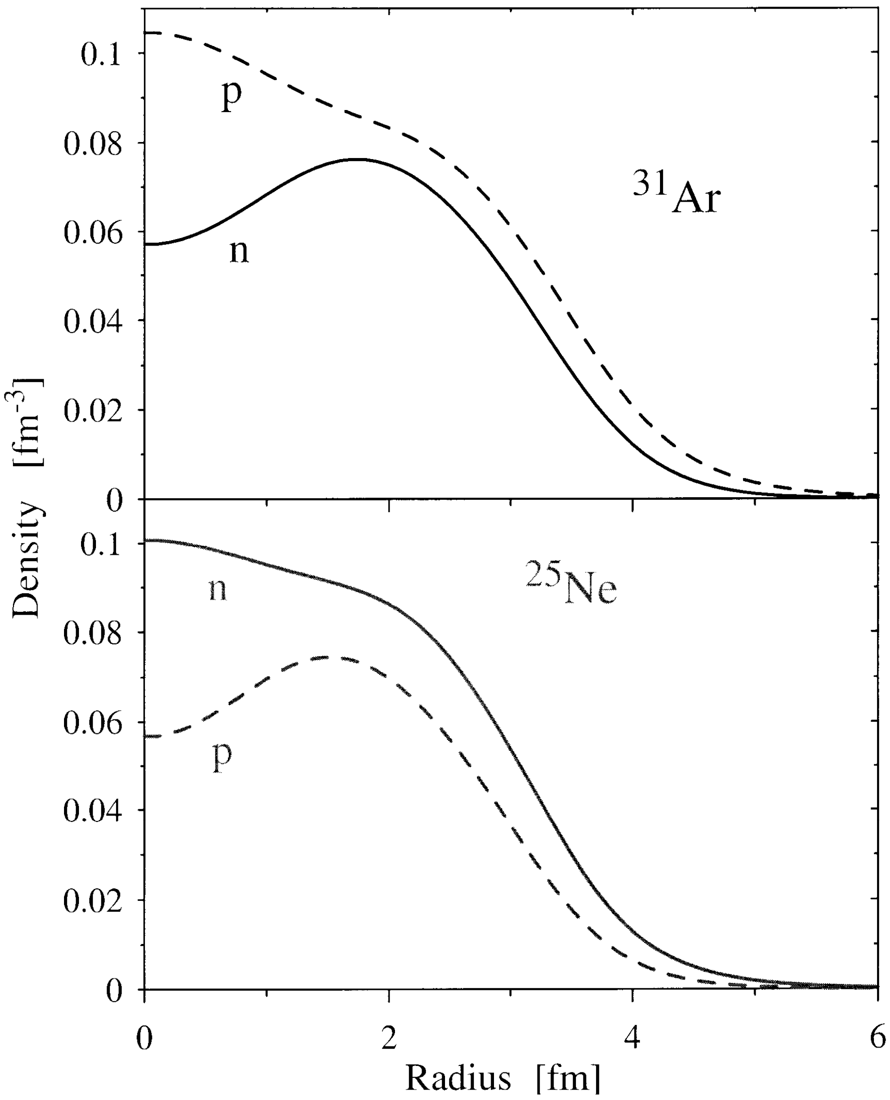
# Sizes - Skyrme Hartree-Fock radii and densities




W.A. Richter and B.A. Brown, Phys. Rev. **C67** (2003) 034317

# Sizes - Skyrme Hartree-Fock radii and densities




B.A. Brown, S. Typel, and W.A. Richter, Phys. Rev. **C65** (2002) 014612

# Bound states – use mean field information

```
                       **********************
                       *                    *
INPUT VALUES           * IA,IZ =    24    8 *
                       *                    *
                       **********************
```

----- Neutron bound state results -----

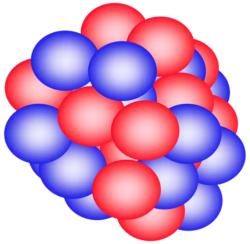
| k | n | l | j | e | IE | OCC | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | s | 1/2 | -26.757 | 1 | 2.00 | 36.70 | 35.28 |
| 2 | 1 | p | 3/2 | -16.883 | 1 | 4.00 | 36.70 | 35.80 |
| 3 | 1 | p | 1/2 | -12.396 | 1 | 2.00 | 36.70 | 36.04 |
| 4 | 1 | d | 5/2 | -6.166 | 1 | 6.00 | 36.70 | 36.37 |
| 5 | 1 | d | 3/2 | -0.109 | 1 | 0.00 | 36.70 | 36.69 |
| 6 | 2 | s | 1/2 | -3.360 | 1 | 2.00 | 36.70 | 36.52 |
| 7 | 1 | f | 7/2 | -0.200 | 3 | 0.00 | 46.02 | 46.01 |
| 8 | 1 | f | 5/2 | -0.200 | 3 | 0.00 | 60.56 | 60.55 |
| 9 | 2 | p | 3/2 | -0.200 | 3 | 0.00 | 48.10 | 48.09 |

----- Neutron single-particle radii -----

| | | | | R(2) | R(4) | OCC | rho( 8.9) | rho( 9.9) | rho(10.9) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | s | 1/2 | 2.274 | 2.575 | 2.000 | 0.848E-09 | 0.706E-10 | 0.600E-11 |
| 2 | 1 | p | 3/2 | 2.863 | 3.133 | 4.000 | 0.188E-07 | 0.244E-08 | 0.325E-09 |
| 3 | 1 | p | 1/2 | 2.954 | 3.268 | 2.000 | 0.727E-07 | 0.122E-07 | 0.210E-08 |
| 4 | 1 | d | 5/2 | 3.434 | 3.757 | 6.000 | 0.524E-06 | 0.129E-06 | 0.327E-07 |
| 5 | 1 | d | 3/2 | 4.662 | 6.063 | 0.000 | 0.131E-04 | 0.675E-05 | 0.371E-05 |
| 6 | 2 | s | 1/2 | 4.172 | 4.895 | 2.000 | 0.769E-05 | 0.278E-05 | 0.102E-05 |
| 7 | 1 | f | 7/2 | 3.865 | 4.440 | 0.000 | 0.324E-05 | 0.134E-05 | 0.600E-06 |
| 8 | 1 | f | 5/2 | 3.890 | 4.477 | 0.000 | 0.341E-05 | 0.141E-05 | 0.631E-06 |
| 9 | 2 | p | 3/2 | 6.815 | 8.635 | 0.000 | 0.451E-04 | 0.270E-04 | 0.167E-04 |

But must make small correction as HF is a fixed centre calculation

$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



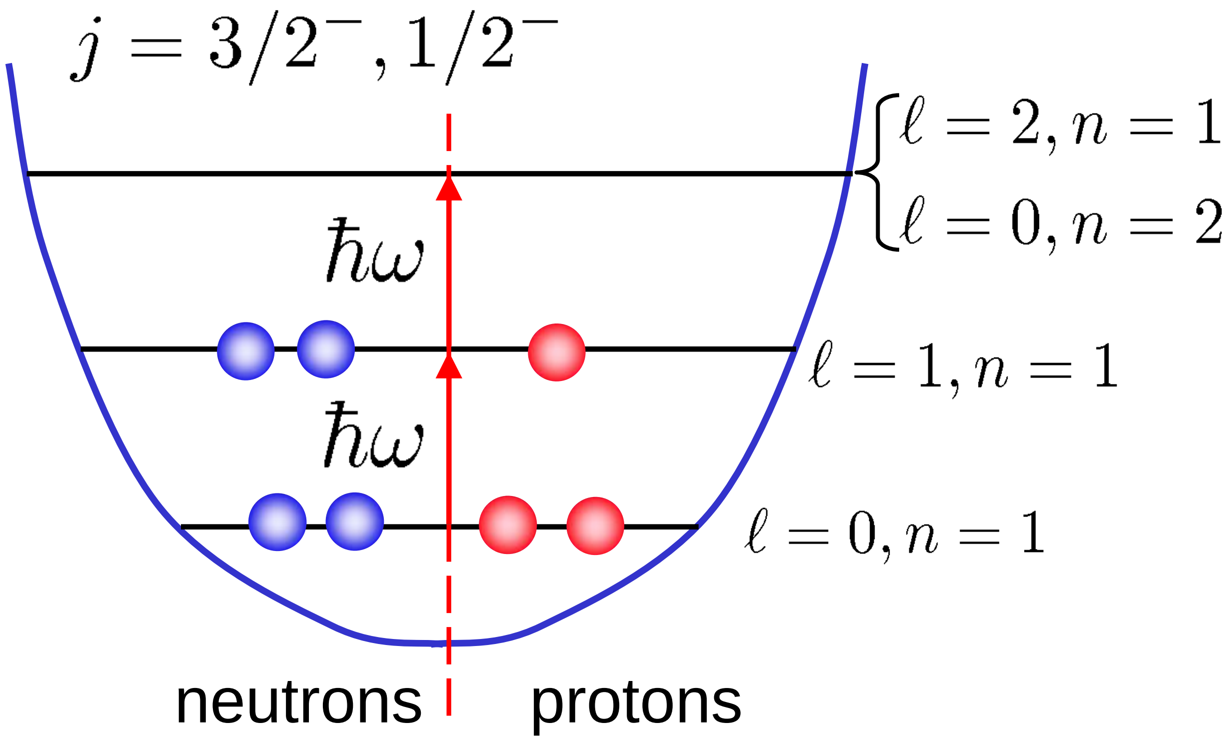
$^{24}O(g.s.)$

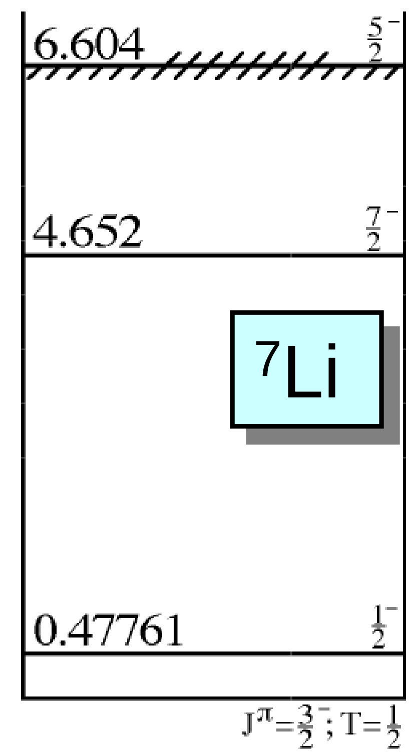
# Bound states – for clusters – conventions (1)

**How many nodes for cluster states ?** $\phi^{m}_{n\ell j}(\vec{r})$

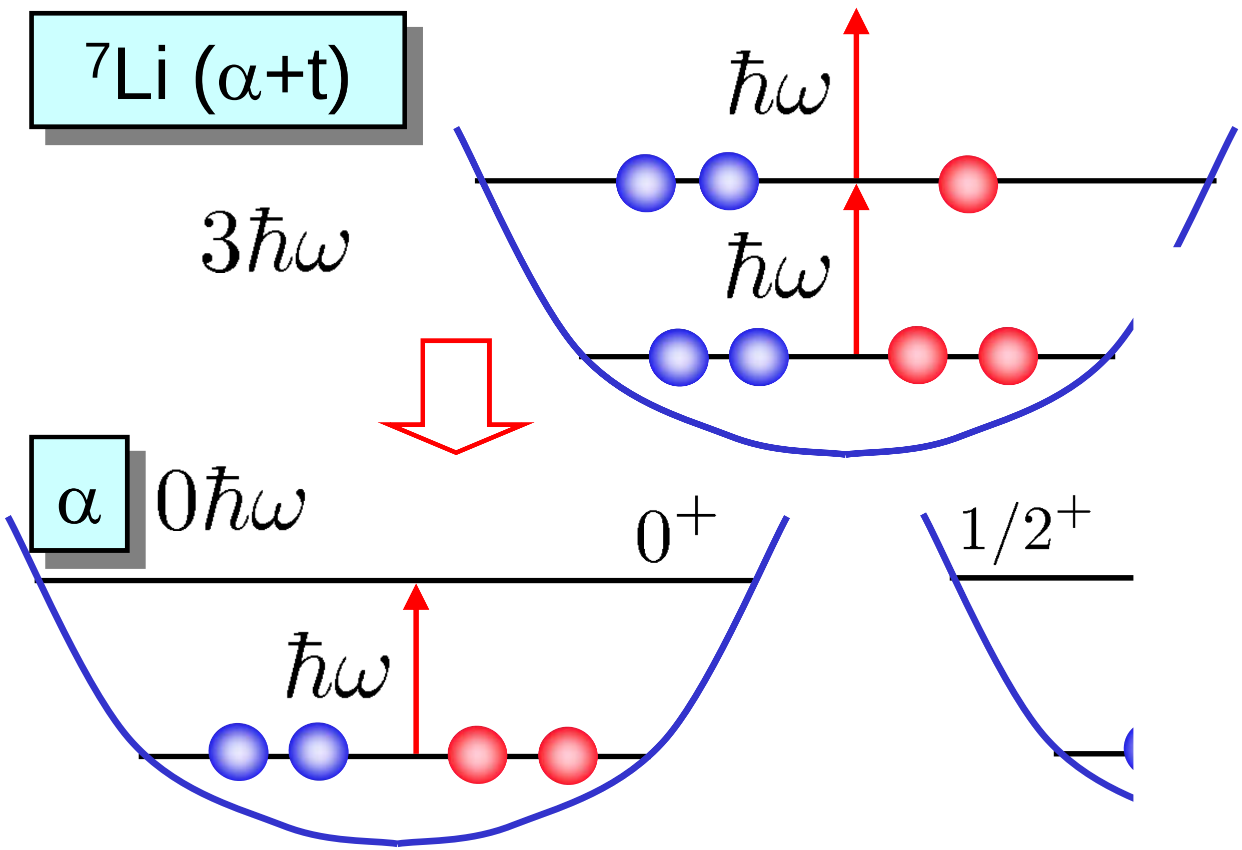
Usually guided by what the 3D harmonic oscillator potential requires - so as not to violate the Pauli Principle.

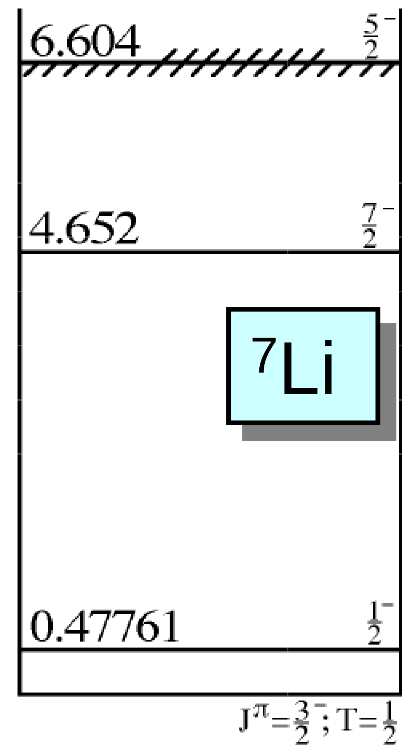
**$^{7}$Li (α+t)** $[2(n-1)+\ell\,]\hbar\omega$ { Excitation of a nucleon in this state



# Bound states – for clusters - conventions (2)



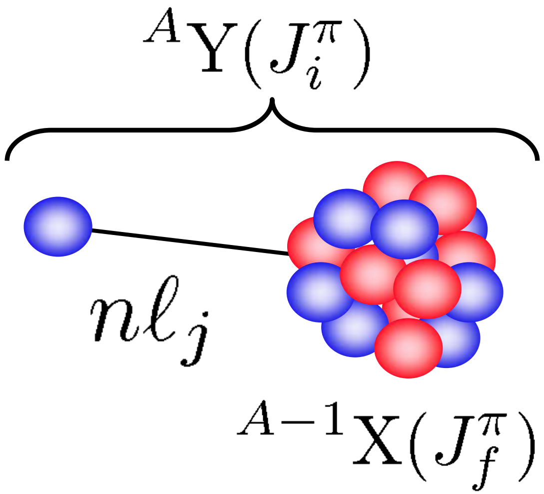

$$[2(N-1)+L\,]\hbar\omega = 3\hbar\omega$$

$L = 1, N = 2$     $L = 3, N = 1$

$j = 3/2^-, 1/2^-,$     $j = 7/2^-, 5/2^-$

# Bound states – spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.




$$\phi_{n\ell j}^{m}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm) \frac{u_{n\ell j}(r)}{r} Y_\ell^\lambda(\hat{r}) \chi_s^\sigma ,$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$

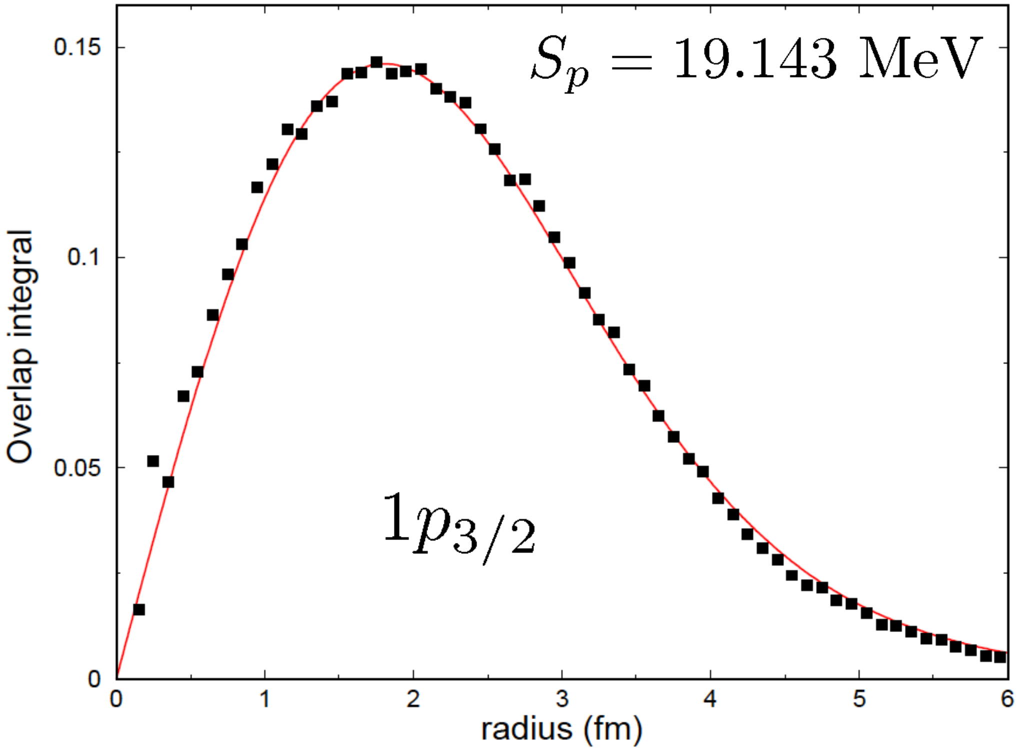
The potential model wave function approximates the overlap function of the A and A–1 body wave functions (A and A–n in the case of an n-body cluster) i.e. the overlap

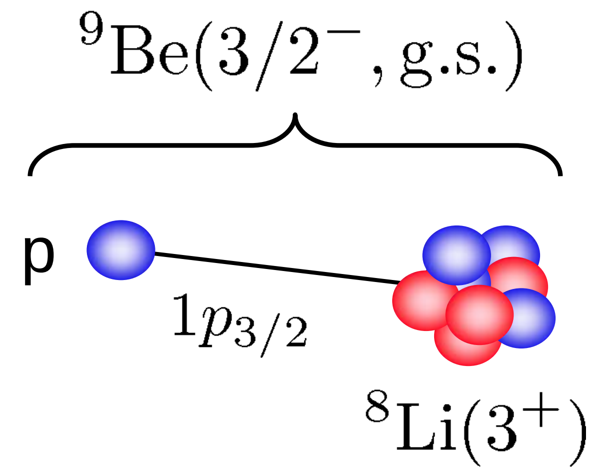
$$\langle \ell j, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^{A}\mathrm{Y}(J_i^\pi) \rangle \to I_{\ell j}(r), \quad \int_0^\infty [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

*S(…)* is the spectroscopic factor ← a structure calculation

# Bound states – microscopic overlaps

$$\langle \vec{r}, {}^8\mathrm{Li}(3^+) | {}^9\mathrm{Be}(3/2^-, \mathrm{g.s.}) \rangle$$




- Microscopic overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a few cases

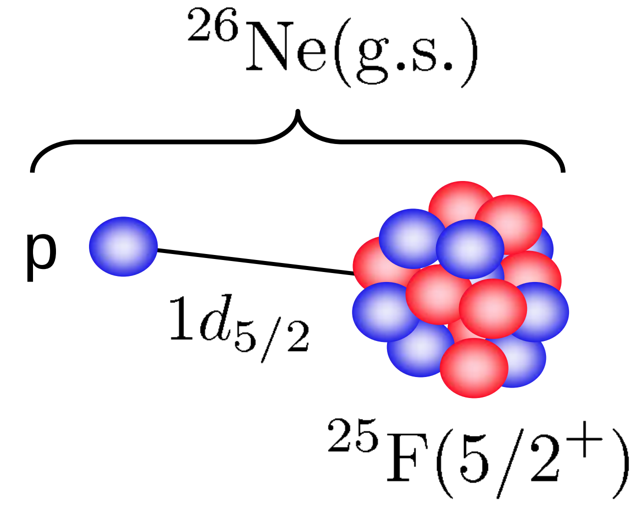
Normalised bound state in Woods-Saxon potential well x $(0.23)^{1/2}$ Spectroscopic factor

$$r_V = r_{so} = \text{fitted}, \; a_V = a_{so} = \text{fitted}, \; V_{so} = 6.0$$

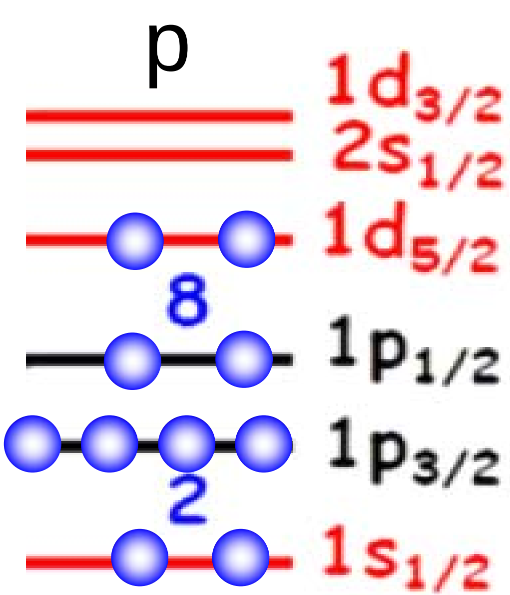
# Bound states – shell model overlaps

$$\langle \vec{r}, {}^{25}\mathrm{Ne}(5/2^+, E^*) | {}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.}) \rangle$$

USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides spectroscopic factors but **not** the bound state radial wave function.



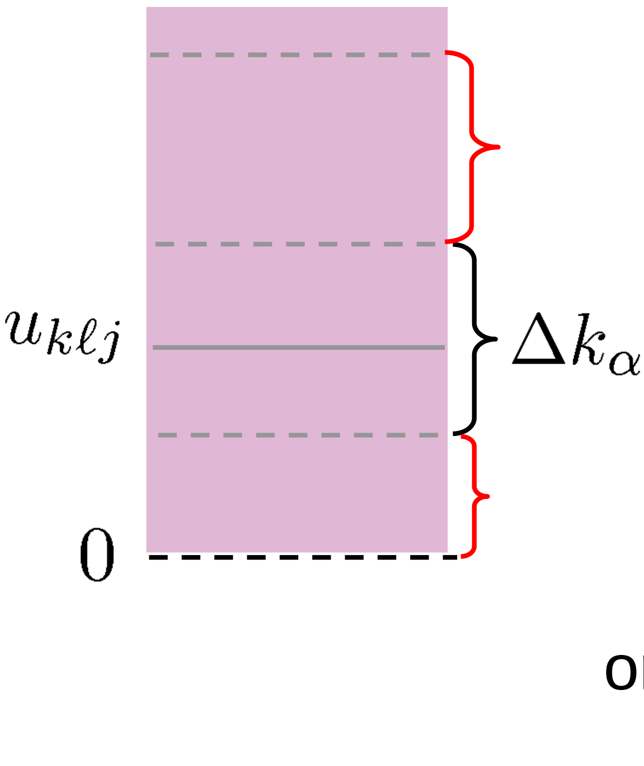

```
-- core state --- - overlap state -   (1 2   5)(
2j2t p    n      e  2j2t p    n      e     s
          -59.414              -81.625
 5 7 +    1    0.000 0 6 +    1    0.000  1.79039
 5 7 +    2    3.756 0 6 +    1    0.000  0.02316
 5 7 +    3    4.799 0 6 +    1    0.000  0.01084
 5 7 +    4    5.631 0 6 +    1    0.000  0.00012
 5 7 +    5    6.022 0 6 +    1    0.000  0.00589
 5 7 +    6    6.504 0 6 +    1    0.000  0.00044
 5 7 +    7    6.796 0 6 +    1    0.000  0.00002
 5 7 +    8    8.034 0 6 +    1    0.000  0.00006
 5 7 +    9    8.186 0 6 +    1    0.000  0.00097
 5 7 +   10    8.398 0 6 +    1    0.000  0.00006
total =                                   1.83196
centroid =     0.102     centroids =      0.000
centroid*= -22.313      centroids =   -22.211
```



# Neither bound nor scattering – continuum bins

## Scattering states

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right]$$




$$\int_0^\infty dr\, u_{k\ell j}(r)\, u^*_{k'\ell j}(r) = \frac{\pi}{2}\delta(k - k')$$

$$\hat{u}_{\alpha\ell j}(r) = \sqrt{\frac{2}{\pi N_\alpha}} \int_{\Delta k_\alpha} dk\, g(k)\, u_{k\ell j}(r)$$

$$N_\alpha = \int_{\Delta k_\alpha} dk\, [g(k)]^2$$

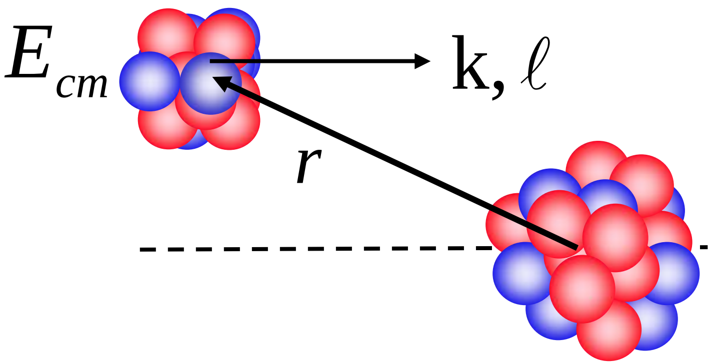
weight function

orthonormal set

$$\int_0^\infty dr\, \hat{u}^*_{\alpha\ell j}(r)\, \hat{u}_{\beta\ell j}(r) = \delta_{\alpha\beta}$$

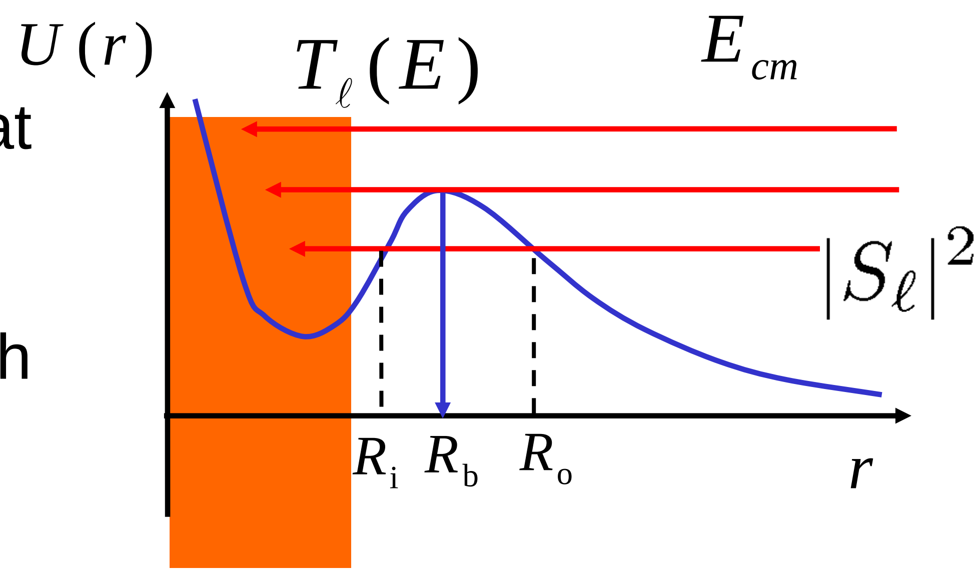
$$g(k) = 1 \qquad g(k) = \sin\delta_{\ell j}$$

# Barrier passing models of fusion




Gives basis also for simple (barrier passing) models of nucleus-nucleus fusion reactions

an imaginary part in *U(r)*, at short distances, can be included to absorb all flux that passes over or through the barrier – assumed to result in fusion




$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-|S_\ell|^2)$$

# Code is available - to practice these ideas

**`bound`** (bound states solver – see **`bound.outline`**)

**`scatter`** (spin-independent scattering solver, range of $\ell$ or energy - see **`scatter.outline`**)

**`scat`** (single $\ell$, j, scattering as a function of energy – see **`scat.outline`**)

**`scat_one`** (single $\ell$, j, and single energy scattering)

**`bins`** (single $\ell$, j, continuum bin state constructor – see **`bins.outline`** )

**`dens`** (Alex Brown Hartree Fock solver– for mean field wave functions and densities)