Breakup and Knockout reactions

Mini-school on Nuclear Reaction Theories for Nuclear Astrophysics
Surrey, 9th January 2009

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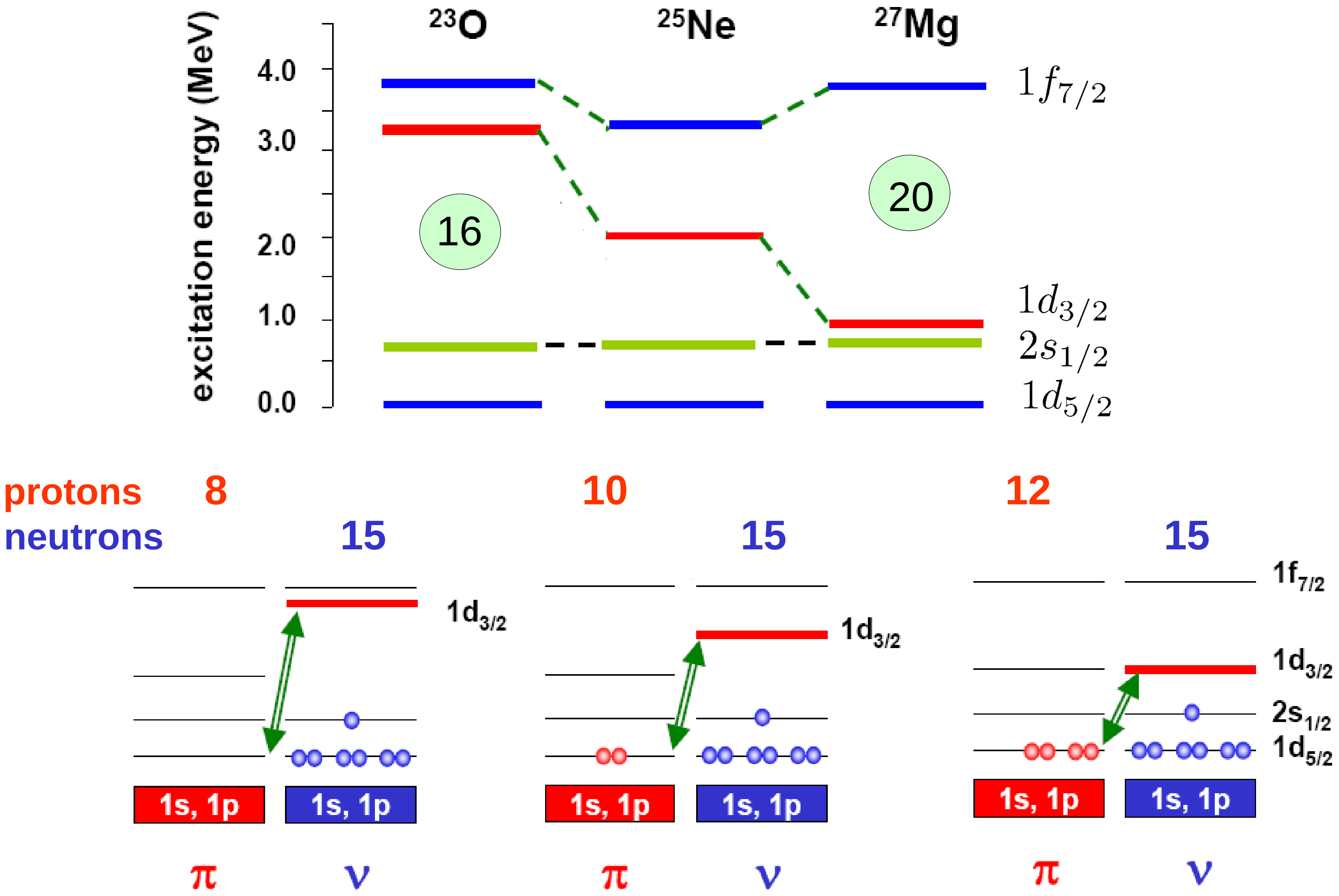
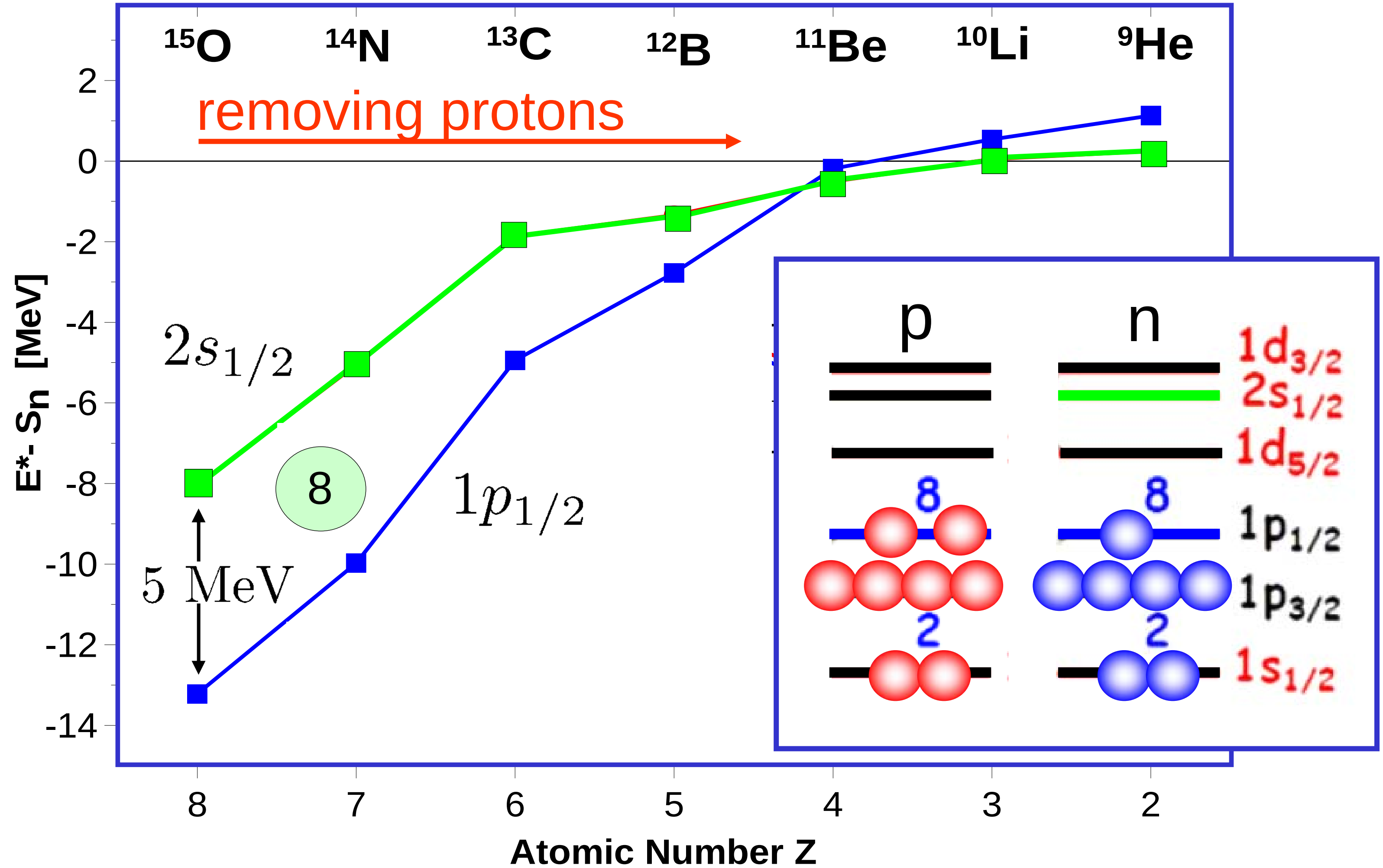
Magic numbers change with "neutron richness"

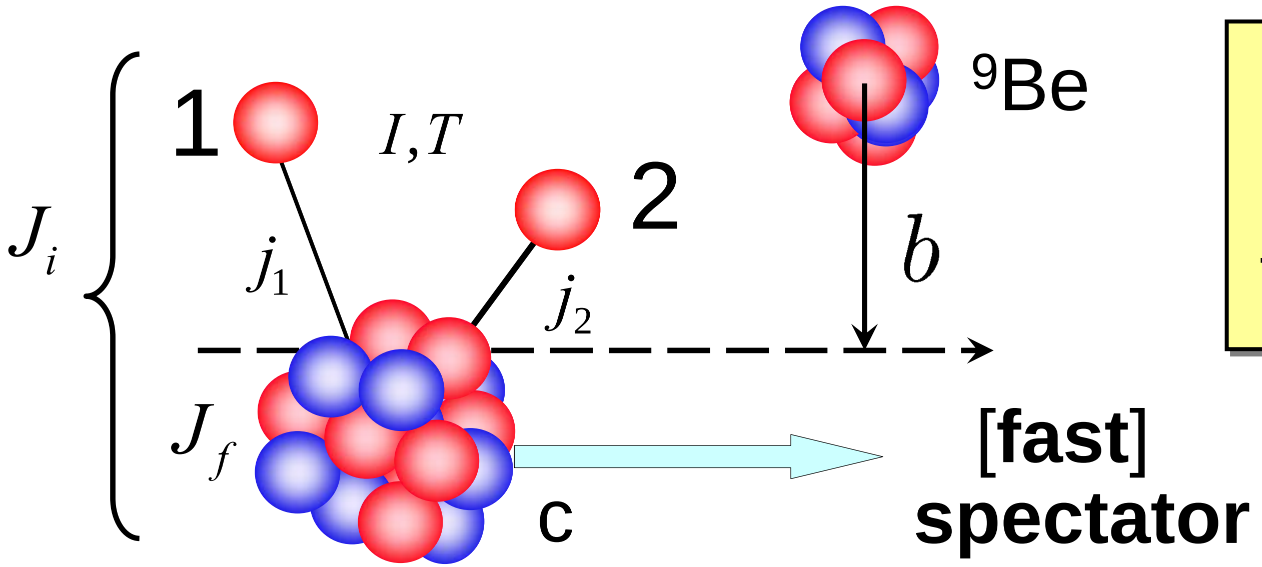


Migration of single particle levels for N=7



From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

One and two nucleon knockout, ~100 MeV/u



light
nuclear
target

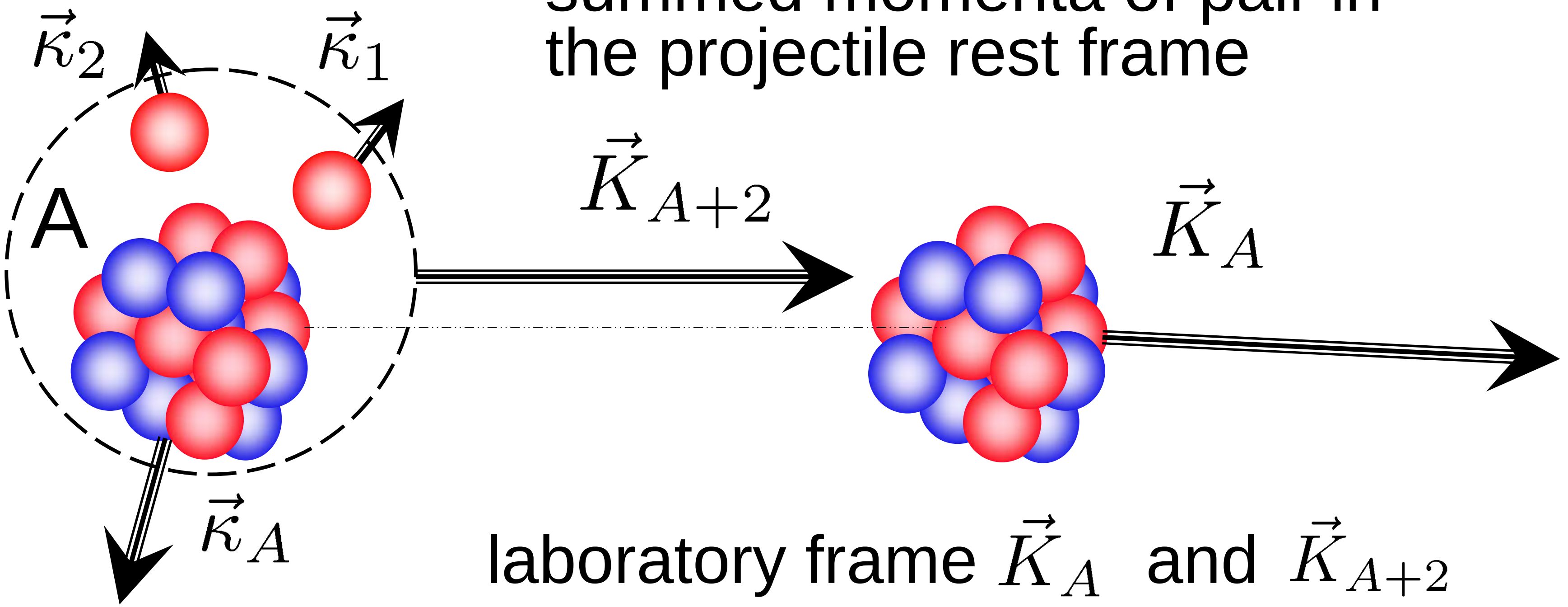
Experiments are inclusive (with respect to the target final states). Core final state measured – using gamma rays – whenever possible – and the momenta of the residues.

Cross sections are large and they include both: Break-up (elastic) and stripping (inelastic/absorptive) interactions of the removed nucleon(s) with the target

P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame



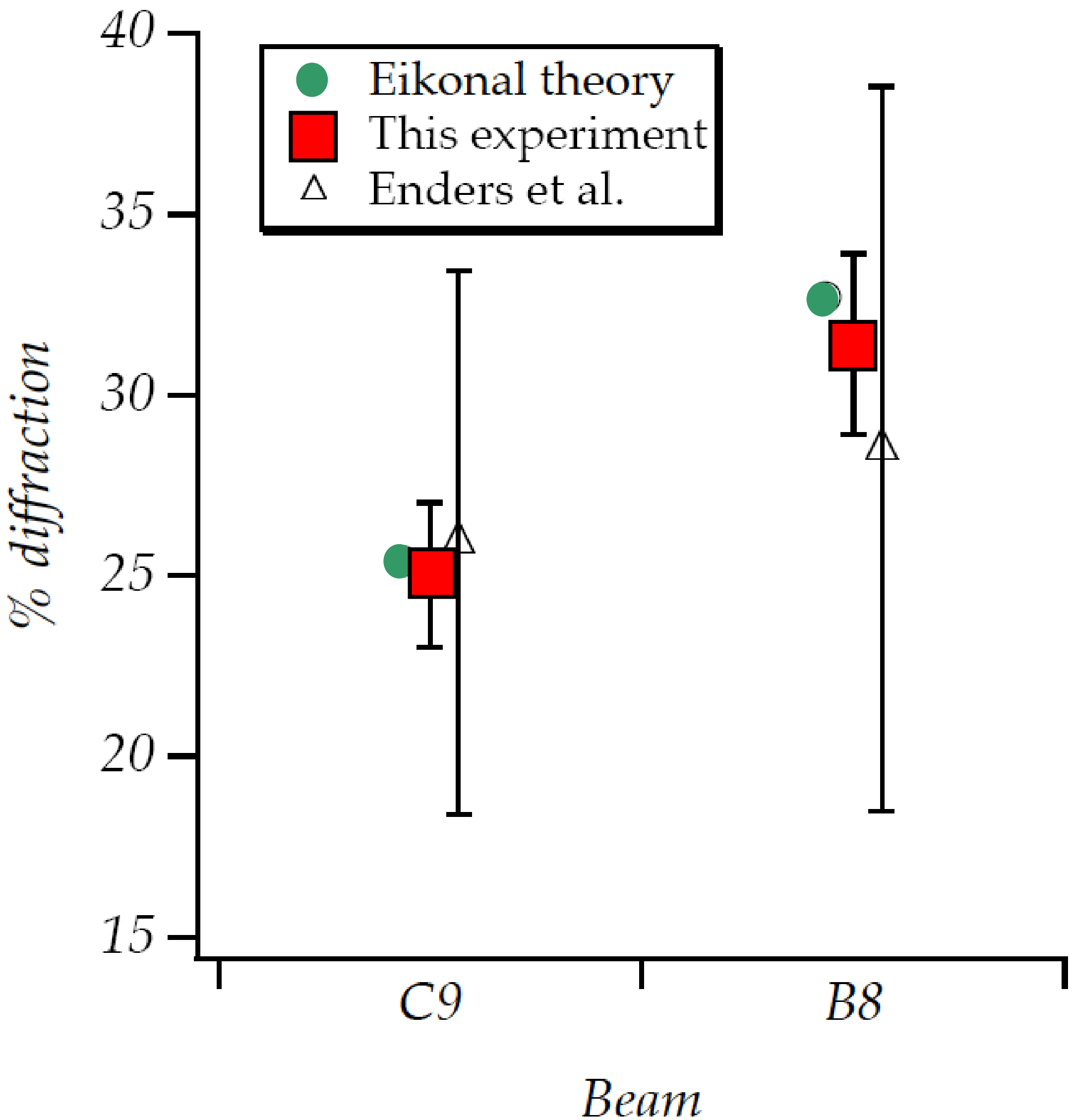
Projectile rest frame

laboratory frame $\vec{K}_A$ and $\vec{K}_{A+2}$

$$\vec{K}_A = \frac{A}{A+2}\vec{K}_{A+2} - [\vec{\kappa}_1 + \vec{\kappa}_2]$$

and component equations

Two mechanisms – stripping and diffraction



In those cases where the two, separate contributions have been measured, the Eikonal theory does a good job. Here for ^{9}C(-p) and ^{8}B(-p) Sum is measured – only the heavy residue is detected

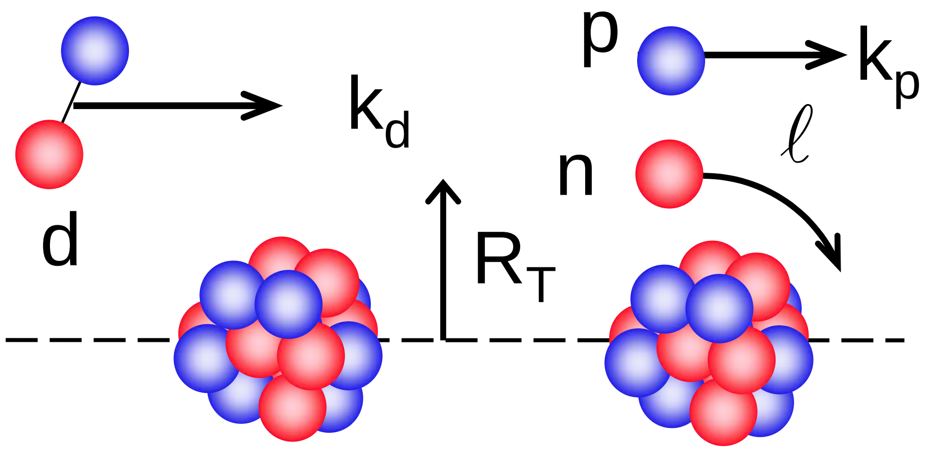
D. Bazin et al., to be published; Proc of International Nuclear Physics Conference(INPC07), (Tokyo, Japan 2007) Volume 2, pp. 406ed,.

Specific limitations and positives

Have only the (ground and isomeric?) structure of the state presented by the incident beam – limitation? –

but

the removal reaction mechanism finesses some sensitivities of a transfer reaction – where linear and angular momentum matching/mismatch can result in greater sensitivity to optical and bound states potential parameters - especially for large Q.

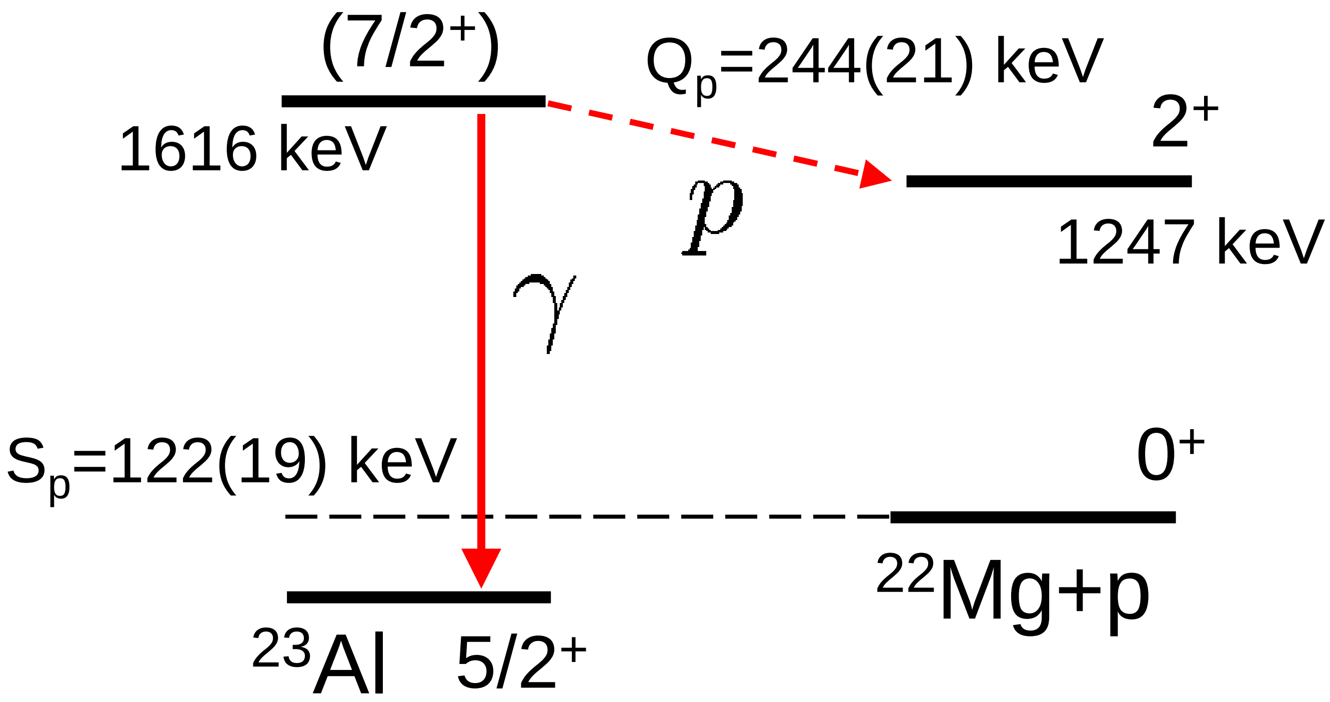


An example of p-pickup – $^{22}Mg + {}^{9}Be \rightarrow {}^{23}Al + X$

proposed structure
$[2^+ \otimes d_{5/2}]_{7/2^+}$

100 MeV/nucleon

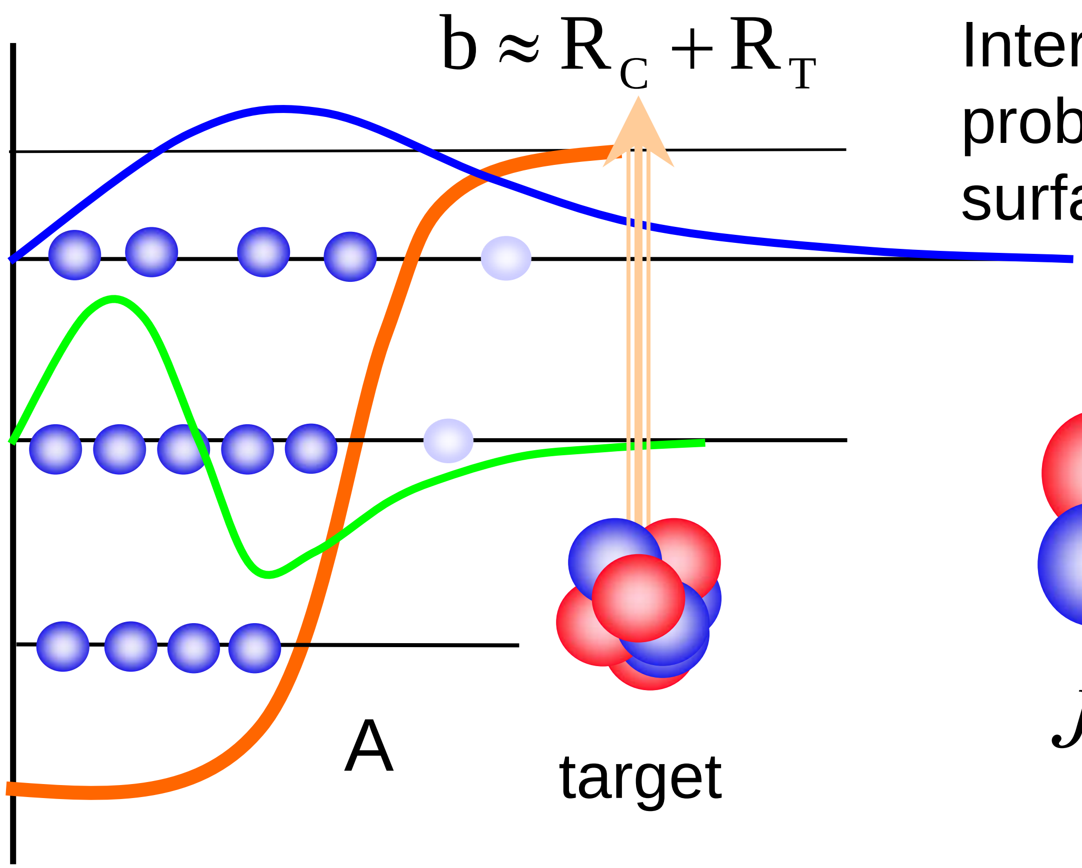
[0.54(5) mb]

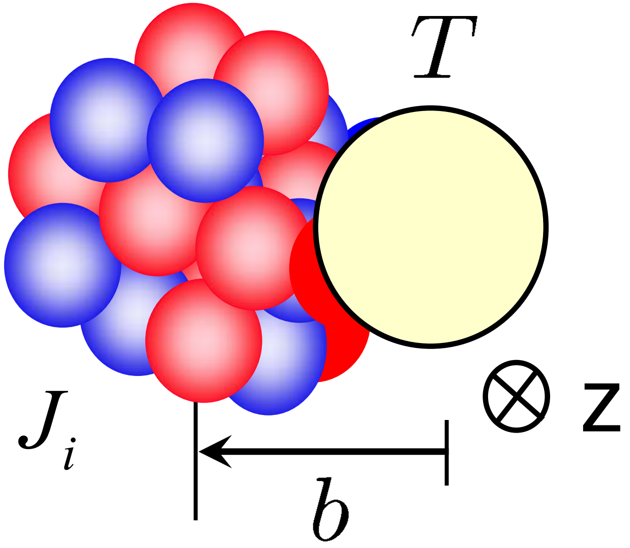


A. Gade at al., Phys. Lett. B 666, (2008) 218

Sampling the single-nucleon wave function

Interaction with the target probes wave functions at surface



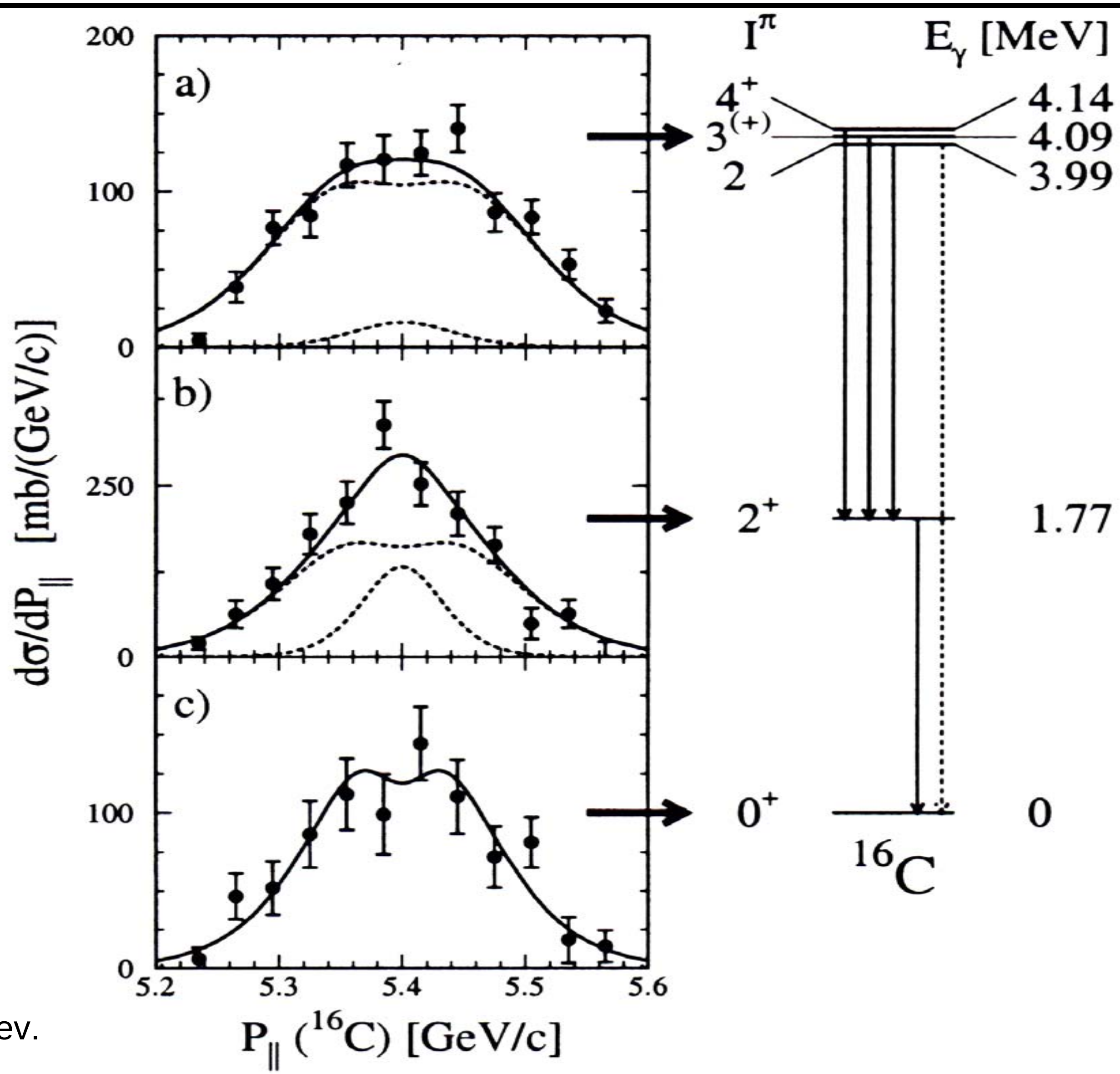


Single-neutron knockout – momentum distributions

ℓ=0,2 admixture

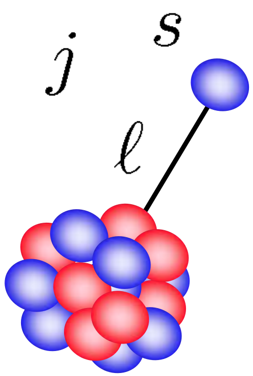
ℓ=0,2 admixture

pure ℓ=2



V. Maddalena et al. Phys. Rev. C **63** (2001) 024613

Large r: The Asymptotic Normalisation Coefficient



Bound states

$$E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2} U_{\ell j}(r) - \kappa_b^2 \right) u_{n\ell j}(r) = 0$$

but beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta_b \kappa_b}{r} - \kappa_b^2 \right) u_{n\ell j}(r) = 0, \quad \eta_b = \frac{\mu Z_c Z_v e^2}{\hbar \kappa_b}$$

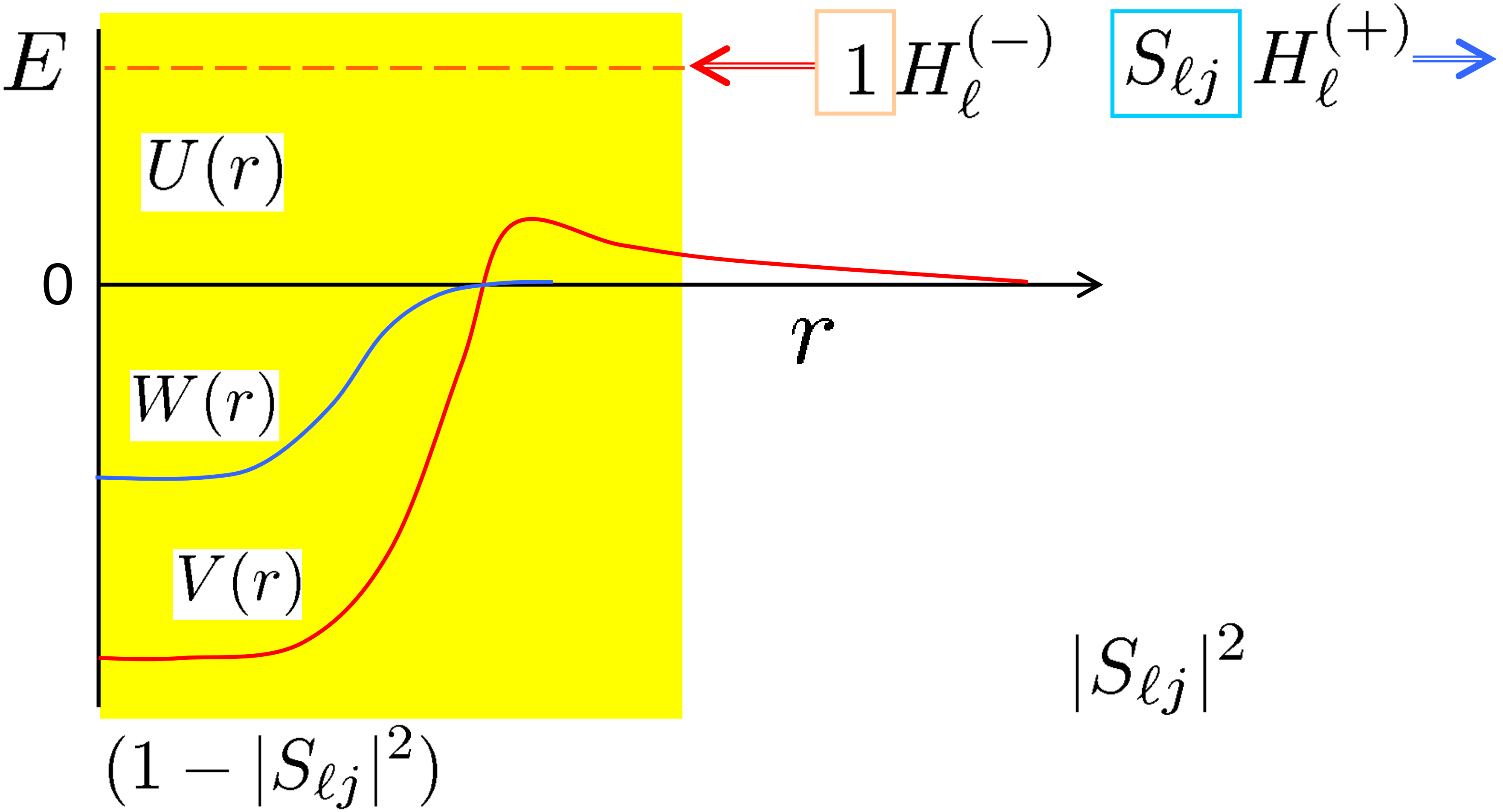
$$u_{n\ell j}(r) \rightarrow C_{\ell j} W_{-\eta_b, \ell+1/2}(2\kappa_b r) \underset{r \rightarrow \infty}{\longrightarrow} C_{\ell j} \exp(-\kappa_b r)$$

Whittaker function

$C_{\ell j}$ ANC completely determines the wave function outside of the range of the nuclear potential – only requirement if a reaction probes only these radii

Ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[1\, H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$



Eikonal approximation: neutral point particles (1)

Approximate (semi-classical) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0, \qquad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2}U(r) + k^2\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0$$

valid when $|U|/E \ll 1, \quad ka \gg 1$ → small wavelength, high energy

Key steps are: (1) the distorted wave function is written

$$\chi_{\vec{k}}^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r})$$

all effects due to *U(r)*, modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

Eikonal approximation: point neutral particles (2)

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

The conditions $|U|/E \ll 1, \quad ka \gg 1$ → imply that

$$2\vec{k}\cdot\nabla\omega(\vec{r}) \gg \nabla^2\omega(\vec{r})$$

Slow spatial variation cf. k

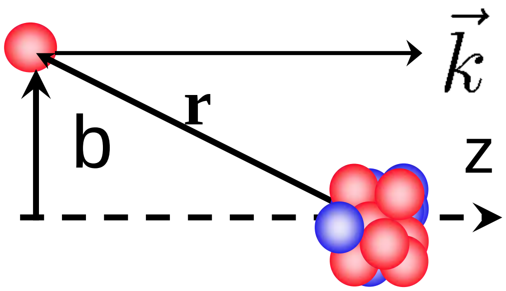
and choosing the z-axis in the beam direction $\vec{k}$

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r})$$

with solution

phase that develops with z

$$\omega(\vec{r}) = \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point neutral particles (3)

$$\chi_{\vec{k}}^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r}) \approx \exp(i\vec{k}\cdot\vec{r}) \boxed{\exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]}$$

So, after the interaction and as $z\rightarrow\infty$

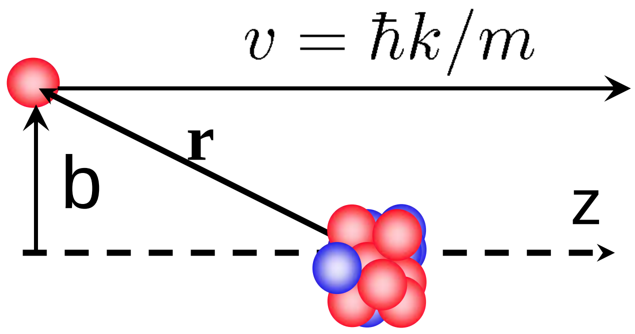
$$\chi_{\vec{k}}^{+}(\vec{r}) \rightarrow \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{\infty} U(r)dz'\right] = S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{\chi_{\vec{k}}^{+}(\vec{r}) \rightarrow S(b)\exp(i\vec{k}\cdot\vec{r})}$$

S(b) is amplitude of the forward going outgoing waves from the scattering at impact parameter b

Eikonal approximation to the S-matrix S(b)

$$\boxed{S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]}$$

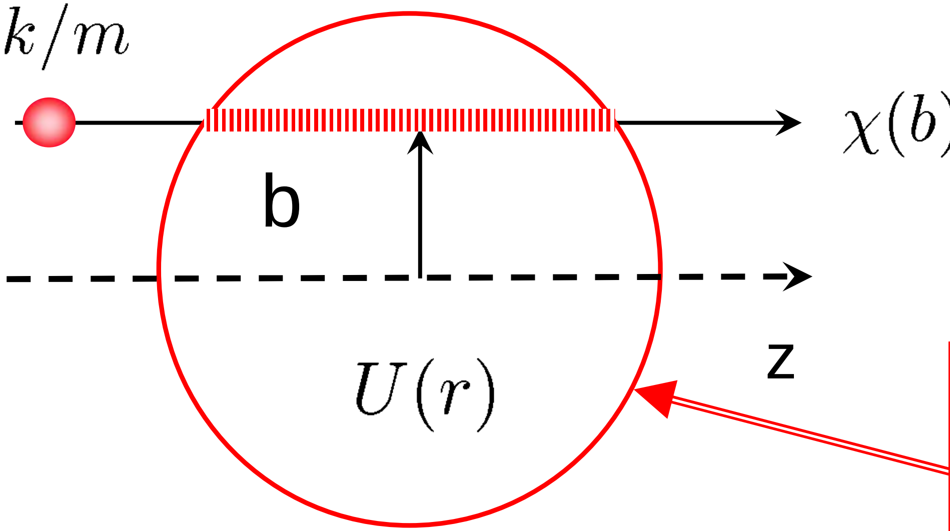


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal approximation: point particles

$$\chi^{+}_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r}) \ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$

$$v = \hbar k/m$$



$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U(r)dz$$

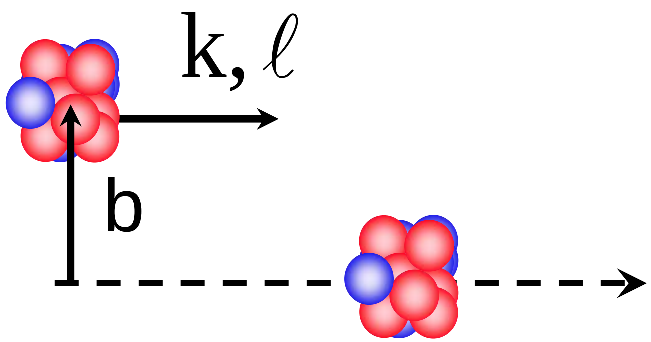
limit of range of finite ranged potential

$$\chi^{+}_{\vec{k}}(\vec{r}) \rightarrow S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]$$

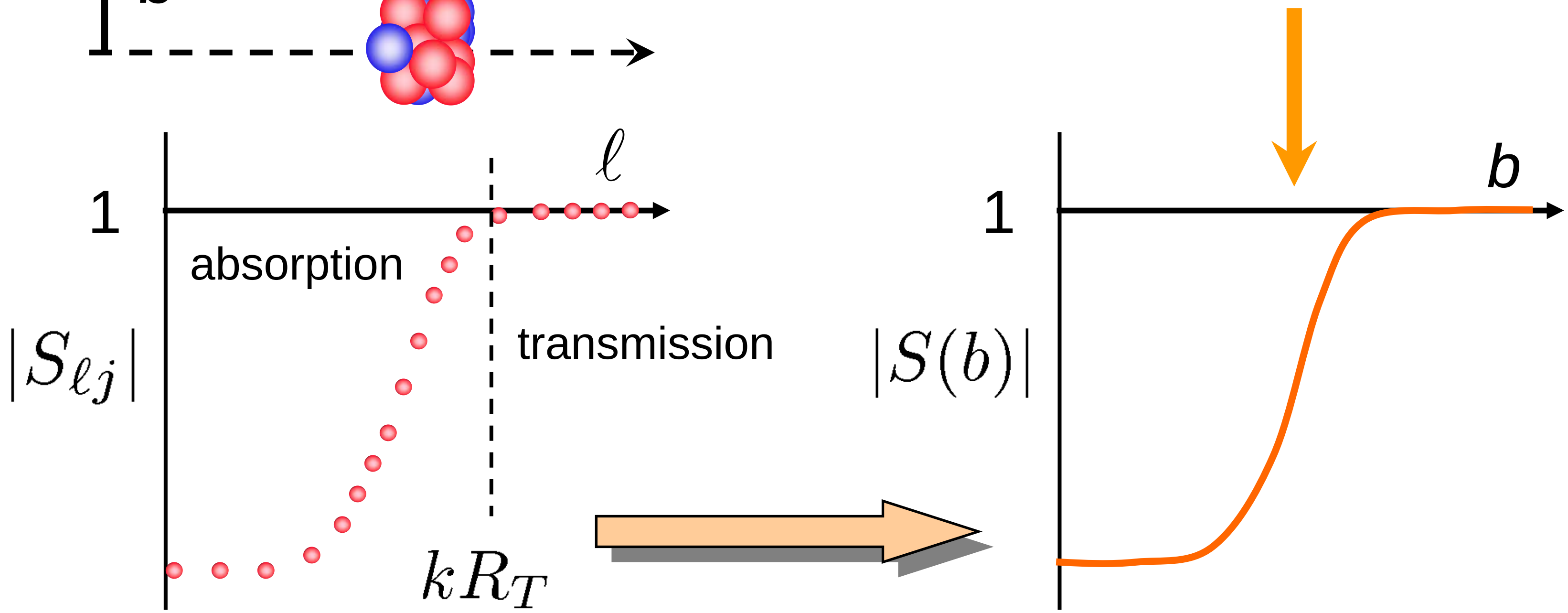
Semi-classical models for the S-matrix - S(b)

b=impact parameter



for high energy/or large mass, semi-classical ideas are good

$kb \cong \ell$, actually $\Rightarrow \ell + 1/2$



$$u_{k\ell j}(r) \to (i/2)[H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}] \qquad S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)\,dz'\right]$$

Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the eikonal (impact parameter) representation, for example (spinless case):

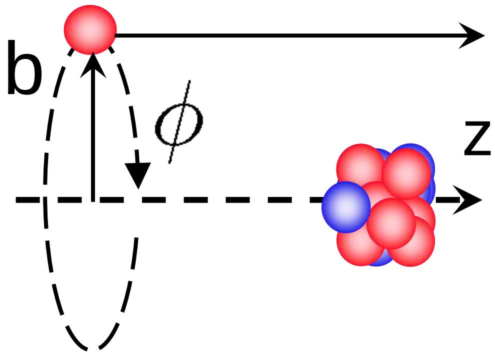
$$\sigma_{el} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\; |1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\; (1-|S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el} + \sigma_R = 2\int d^2\vec{b}\; [1 - \mathrm{Re}.S(b)]$$ etc.

and where (cylindrical coordinates)

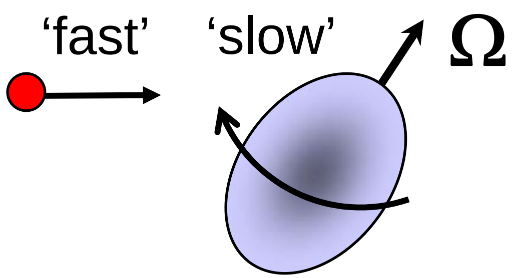
$$\int d^2\vec{b} \equiv \int_0^\infty bdb \int_0^{2\pi} d\phi = 2\pi \int_0^\infty bdb$$

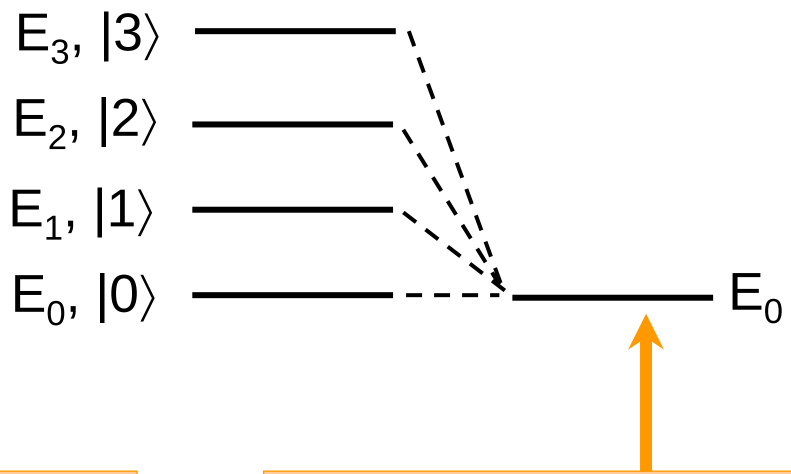


Adiabatic (sudden) approximations in physics

Identify high energy/fast and low energy/slow degrees of freedom

Fast neutron scattering from a rotational nucleus



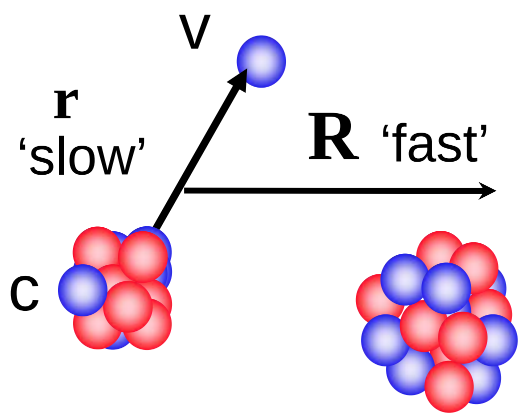


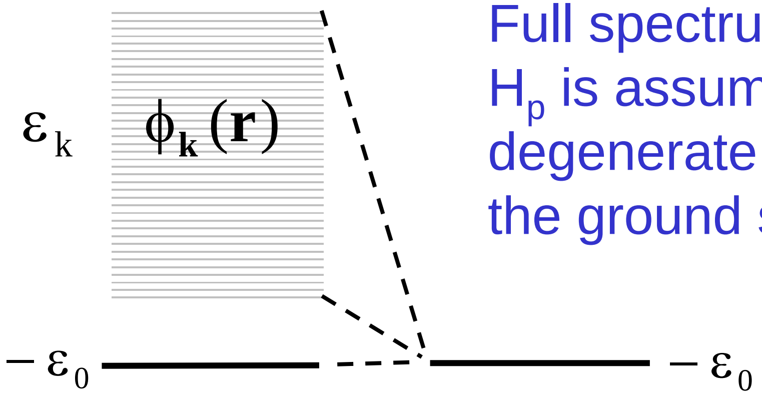
Fix Ω, calculate scattering amplitude $f(\theta, \Omega)$ for each (fixed) Ω.

moment of inertia $\rightarrow \infty$ and rotational spectrum is assumed degenerate

Transition amplitudes $f_{\alpha\beta}(\theta) = \langle \beta \,|\, f(\theta, \Omega) \,|\, \alpha \rangle_\Omega$

Few-body projectiles – the adiabatic model





Full spectrum of H_p is assumed degenerate with the ground state

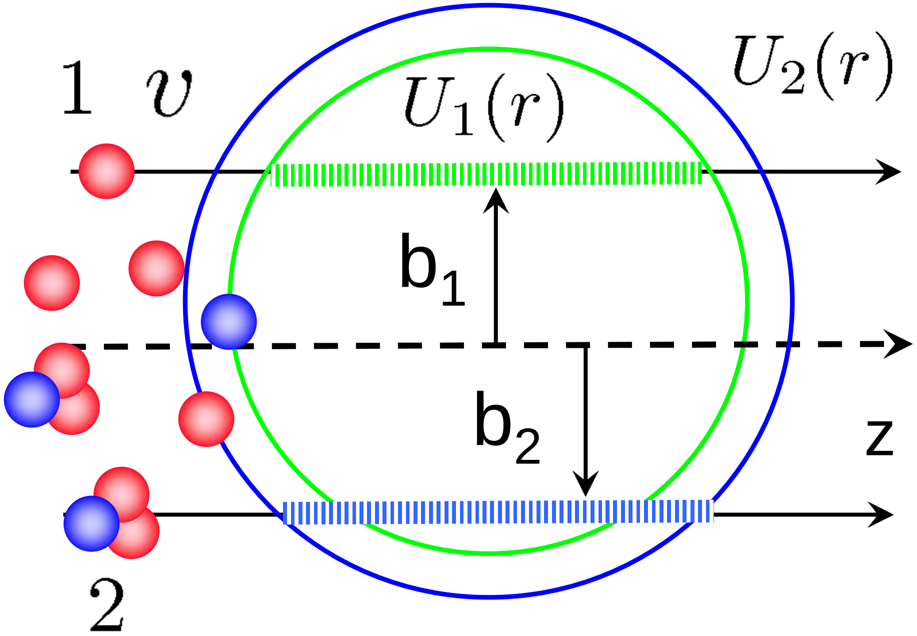
Freeze internal co-ordinate $\mathbf{r}$ then scatter c+v from target and compute $f(\theta,\mathbf{r})$ for all required fixed values of $\mathbf{r}$

Physical amplitude for breakup to state $\phi_\mathbf{k}(\mathbf{r})$ is then,

$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_\mathbf{r}$$

Achieved by replacing $H_p \to -\varepsilon_0$ in Schrödinger equation

Adiabatic approximation: composite projectile



$$\chi_i(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_i(r)dz$$

Total interaction energy

$$U(r_1,\ldots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp\left[i\chi_i(b_i)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_i(r_i)dz'\right]$$

$$\chi(b_1,\ldots) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}\sum_i U_i(r_i)dz$$

with composite systems: get products of the S-matrices

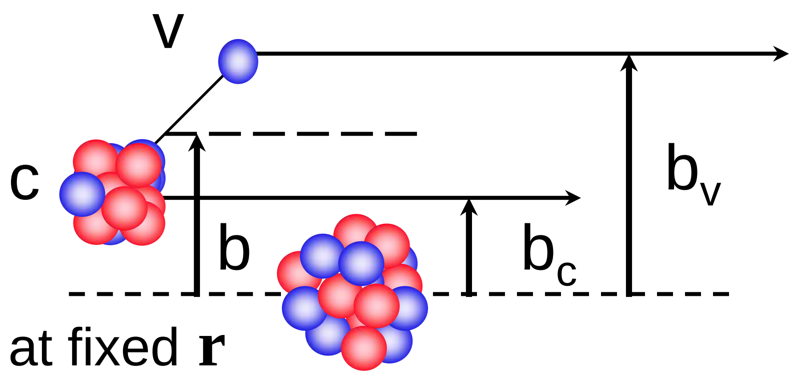
$$\exp[i\chi(b_1,\ldots)] = \prod_i S_i(b_i)$$

Few-body eikonal model amplitudes

So, after the collision, as $Z \rightarrow \infty$ $\quad \omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\, S_v(b_v)$

$$\Psi^{\mathrm{Eik}}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}}\, S_c(b_c)\, S_v(b_v)\, \phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

at fixed $\mathbf{r}$

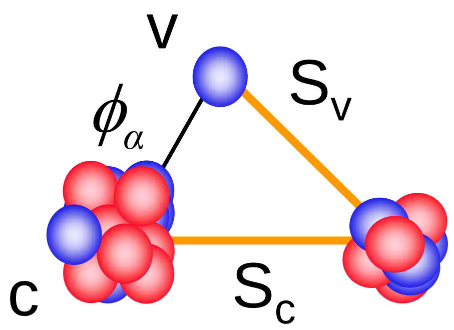
adiabatic

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)} \,|\, \phi_0 \rangle_{\mathbf{r}}$$

averaged over position probabilities of c and v ← amplitude that c,v survive interaction with b_c and b_v

Eikonal theory - dynamics and structure

Independent scattering information of c and v from target



dynamics

$$S_{\alpha\beta}(b) = \langle \phi_\beta \,|\, \overbrace{S_c(b_c)\, S_v(b_v)}\, |\, \phi_\alpha \rangle$$

structure

Use the best available few- or many-body wave functions

More generally,

$$S_{\alpha\beta}(b) = \langle \varphi_\beta \,|\, S_1(b_1)\, S_2(b_2) \ldots\ldots\; S_n(b_n) \,|\, \varphi_\alpha \rangle$$

for any choice of 1,2 ,3, ….. n clusters for which a most realistic wave function φ is available

Absorptive cross sections - target excitation

Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels → $|S(b)|^2 \leq 1$

$$\sigma_{abs} = \sigma_R - \sigma_{diff} = \int d\mathbf{b}\ \langle \phi_0 | \underbrace{1 - |S_c S_v|^2}\, | \phi_0 \rangle$$

$$\left\{ \begin{array}{l} |S_v|^2 (1 - |S_c|^2) + \\ |S_c|^2 (1 - |S_v|^2) + \\ (1 - |S_c|^2)(1 - |S_v|^2) \end{array} \right.$$

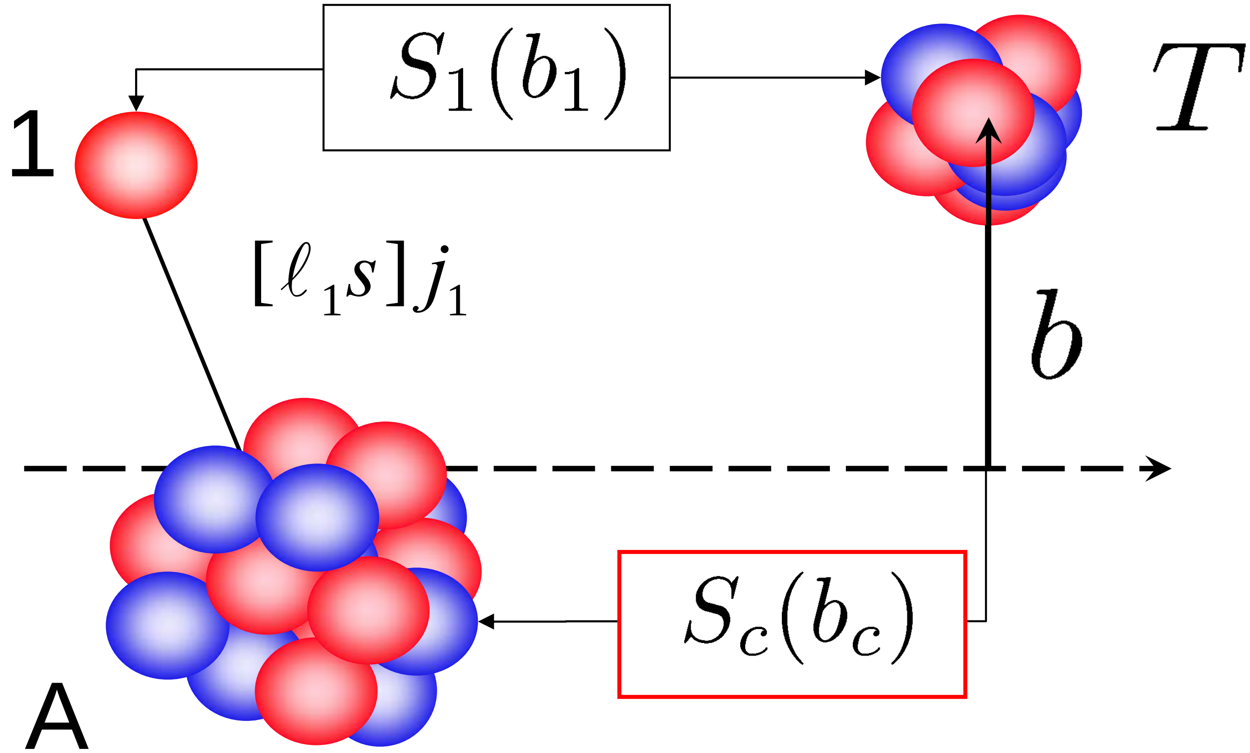
- $|S_v|^2 (1 - |S_c|^2)$: v survives, c absorbed
- $|S_c|^2 (1 - |S_v|^2)$: v absorbed, c survives
- $(1 - |S_c|^2)(1 - |S_v|^2)$: v absorbed, c absorbed

stripping of v from projectile exciting the target. c scatters at most elastically with the target

$$\sigma_{strip} = \int d\mathbf{b}\ \langle \phi_0 | |S_c|^2 (1 - |S_v|^2) | \phi_0 \rangle$$

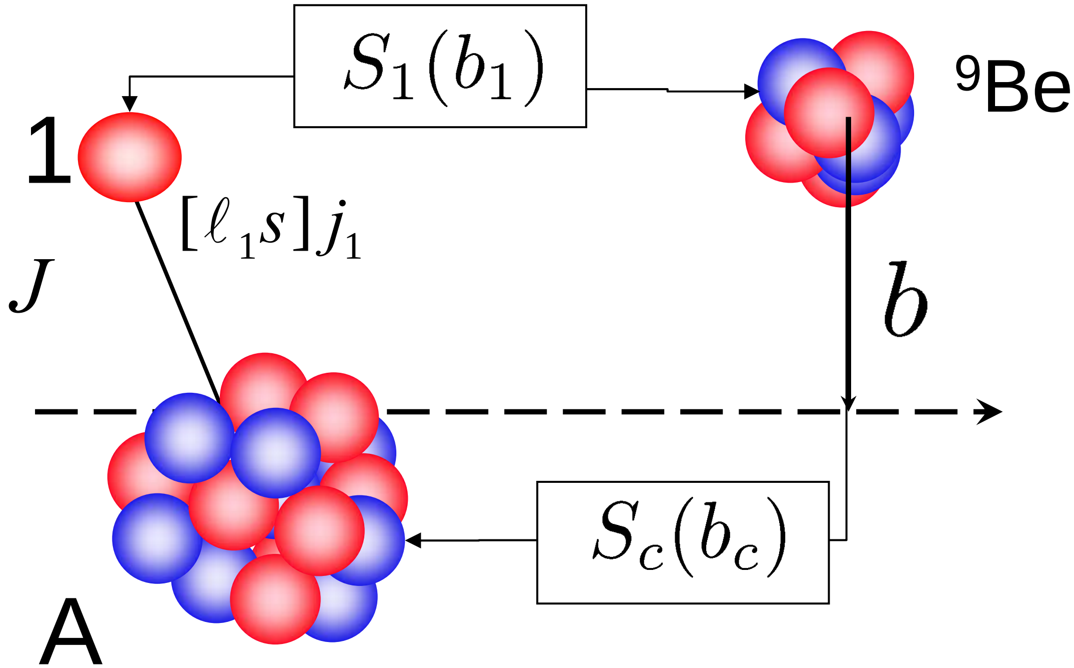
Related equations exist for the differential cross sections, etc.

Stripping of a nucleon



$$\sigma_{\text{strip}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_C|^2 \left(1 - |S_1|^2\right) | \phi_0 \rangle$$

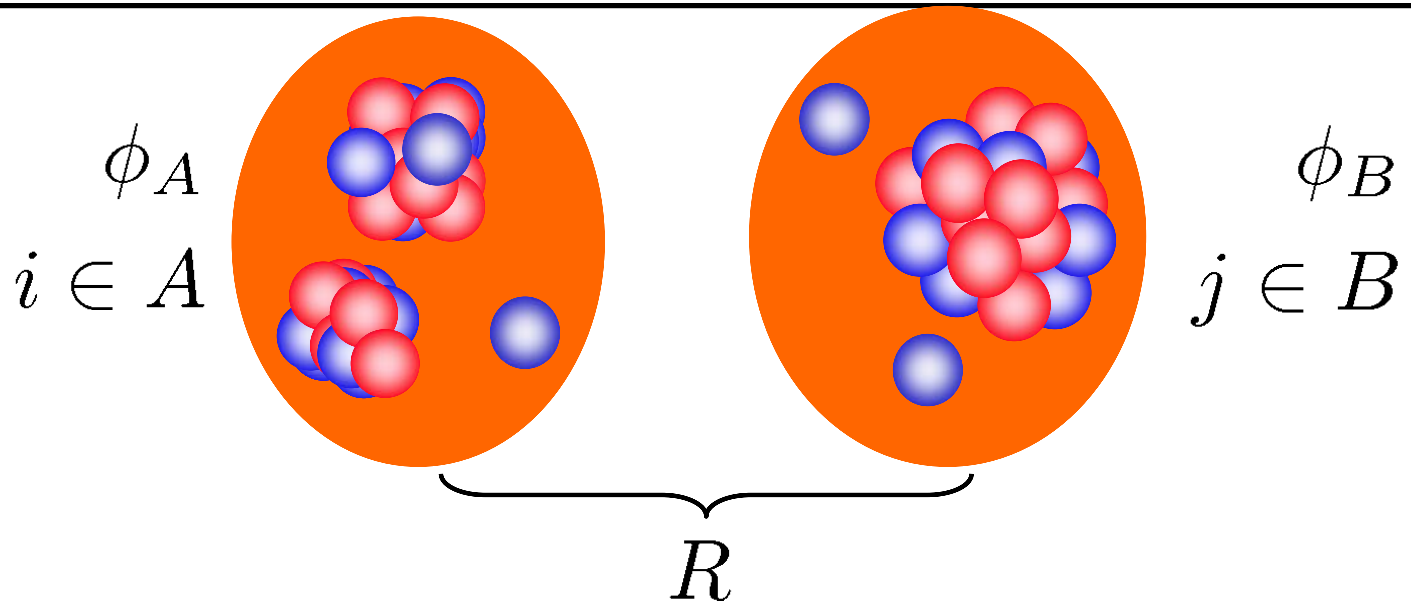
Sudden removal – eikonal model cross sections



At any given facility, and a programme of measurements (with an essentially fixed energy per nucleon) and given target then only two things change for different exotic beams (1) the core target interaction (2) the nuclear structure ***

J.A. Tostevin, G. Podolyák *et al.*, PRC **70** (2004) 064602.

Folding models are a general procedure



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} V_{ij}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

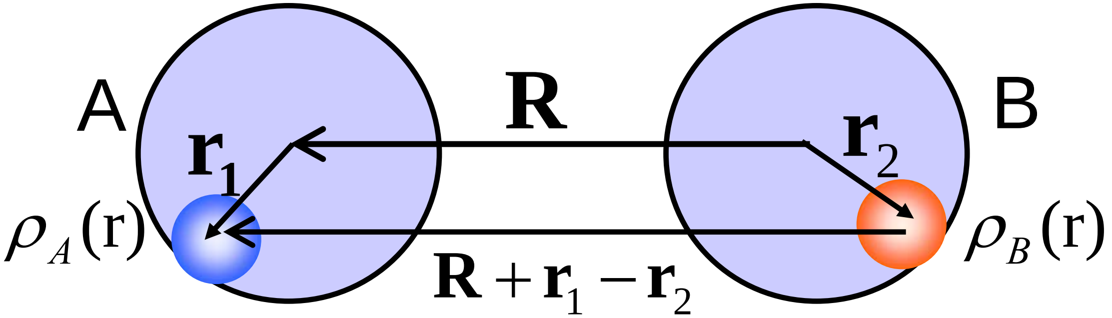
Pair-wise interactions integrated (averaged) over the internal motions of the two composites

Effective interactions – Folding models

Double folding

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$

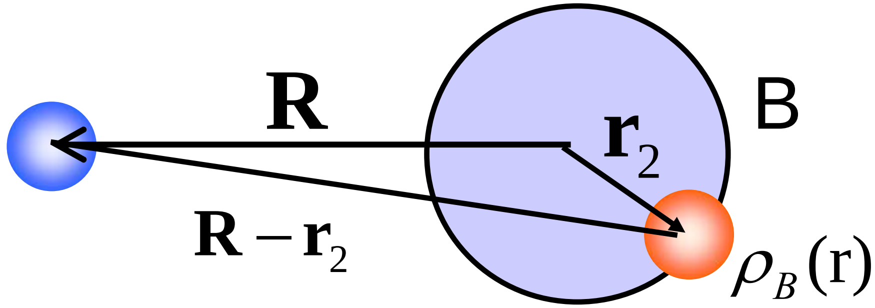
V_{AB}



Single folding

$$V_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} - \mathbf{r}_2)$$

V_B

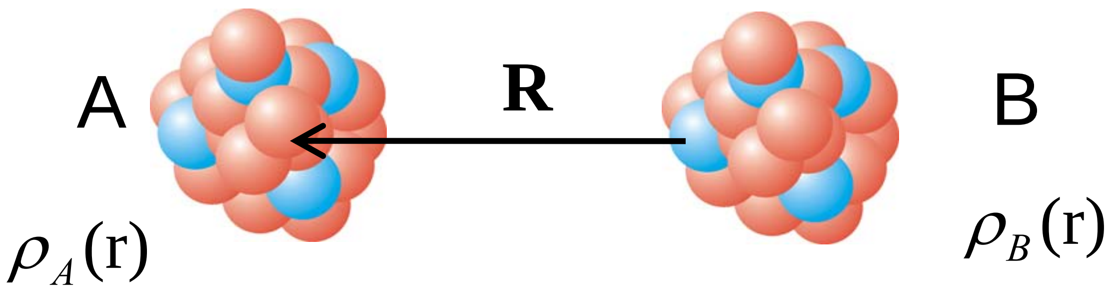


Core-target effective interactions

Double folding

$$V_{AB}(\mathrm{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(\mathrm{r}_1)\, \rho_B(\mathrm{r}_2)\, t_{\mathrm{NN}}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

$$V_{AB}$$



At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2}\sigma_{NN}(i + \alpha_{NN})\right] f(r), \quad \int d\vec{r} f(r) = 1$$

e.g. $f(r) = \delta(r)$

nucleon-nucleon cross section

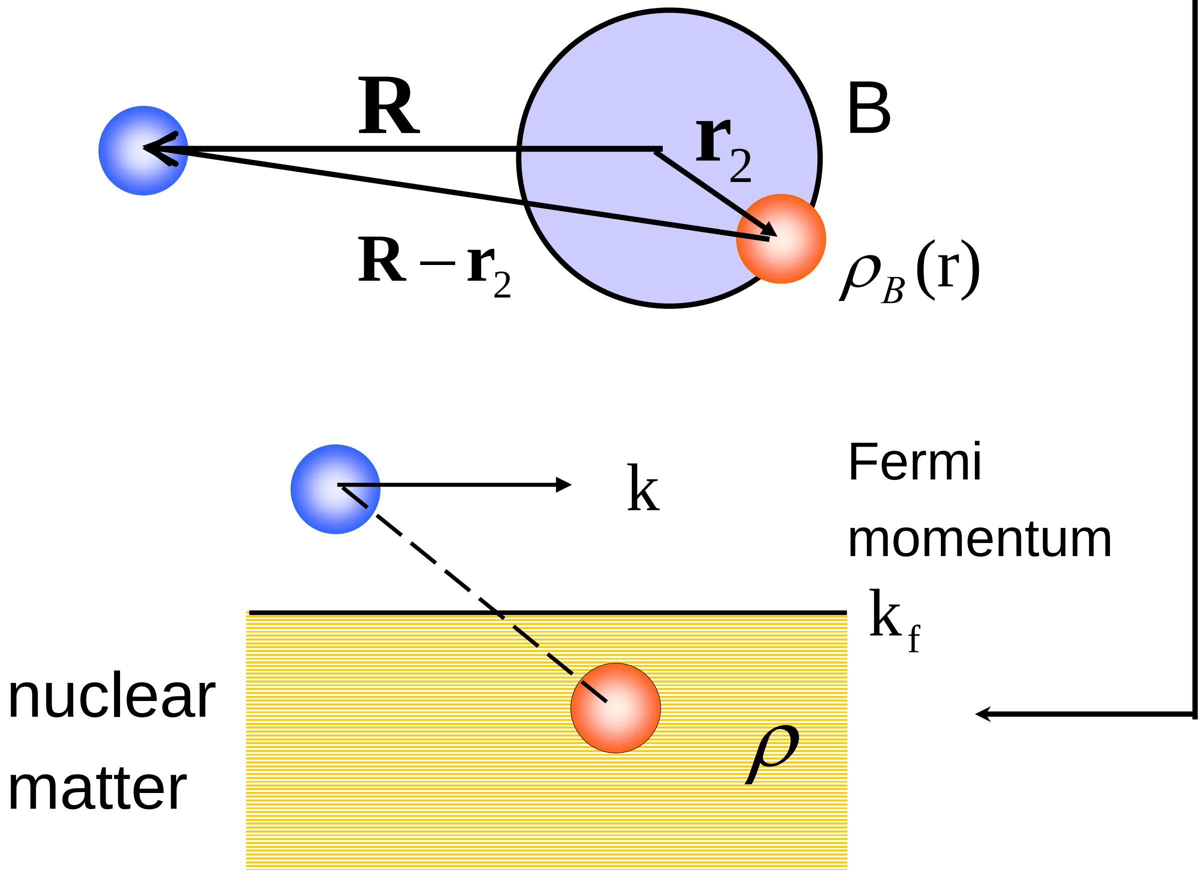
$$f(r) = (\sqrt{\pi}t)^{-3} \exp(-r^2/t^2)$$

resulting in a COMPLEX nucleus-nucleus potential

M.E. Brandan and G.R. Satchler, The Interaction between Light Heavy-ions and what it tells us, Phys. Rep. **285** (1997) 143-243.

Effective NN interactions – not free interactions

$$V_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(|\mathbf{R}-\mathbf{r}_2|)$$



include the effect of NN interaction in the “nuclear medium” – Pauli blocking of pair scattering into occupied states

$$\rightarrow v_{NN}(\rho,\mathbf{r})$$

But as E → high

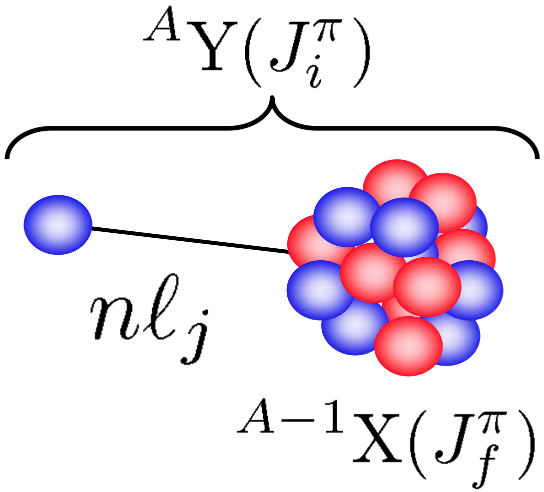
$$v_{NN} \rightarrow v_{NN}^{free}$$

Bound states – spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.

$$\phi_{n\ell j}^{m}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm) \frac{u_{n\ell j}(r)}{r} Y_{\ell}^{\lambda}(\hat{r}) \chi_{s}^{\sigma},$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$



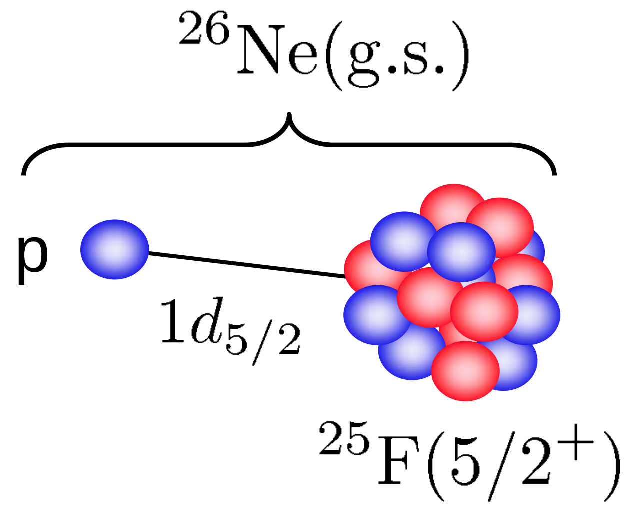
The potential model wave function approximates the overlap function of the A and A–1 body wave functions (A and A–n in the case of an n-body cluster) i.e. the overlap

$$\langle \ell j, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^{A}\mathrm{Y}(J_i^\pi) \rangle \rightarrow I_{\ell j}(r), \quad \int_0^\infty [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

S(…) is the spectroscopic factor ← a structure calculation

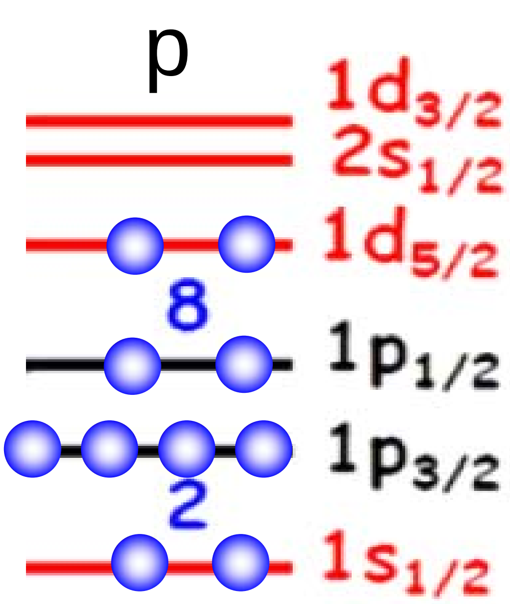
Bound states – shell model overlaps

$$\langle \vec{r}, {}^{25}\mathrm{Ne}(5/2^+, E^*) | {}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.}) \rangle$$



USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides spectroscopic factors but **not** the bound state radial wave function.

```
-- core state --- - overlap state -   (1 2  5)(
2j2t p   n      e  2j2t p    n      e     s
              -59.414             -81.625
 5 7 +   1    0.000 0 6 +    1    0.000  1.79039
 5 7 +   2    3.756 0 6 +    1    0.000  0.02316
 5 7 +   3    4.799 0 6 +    1    0.000  0.01084
 5 7 +   4    5.631 0 6 +    1    0.000  0.00012
 5 7 +   5    6.022 0 6 +    1    0.000  0.00589
 5 7 +   6    6.504 0 6 +    1    0.000  0.00044
 5 7 +   7    6.796 0 6 +    1    0.000  0.00002
 5 7 +   8    8.034 0 6 +    1    0.000  0.00006
 5 7 +   9    8.186 0 6 +    1    0.000  0.00097
 5 7 +  10    8.398 0 6 +    1    0.000  0.00006
total =                                  1.83196
centroid =    0.102     centroids =      0.000
centroid*= -22.313     centroids =    -22.211
```



Bound states – use mean field information

```
                        **********************
                        *                    *
INPUT VALUES            * IA,IZ =    24    8 *
                        *                    *
                        **********************
```

----- Neutron bound state results -----

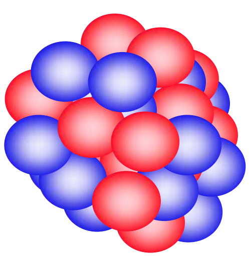
k	n	l	j	e	IE	OCC		
1	1	s	1/2	-26.757	1	2.00	36.70	35.28
2	1	p	3/2	-16.883	1	4.00	36.70	35.80
3	1	p	1/2	-12.396	1	2.00	36.70	36.04
4	1	d	5/2	-6.166	1	6.00	36.70	36.37
5	1	d	3/2	-0.109	1	0.00	36.70	36.69
6	2	s	1/2	-3.360	1	2.00	36.70	36.52
7	1	f	7/2	-0.200	3	0.00	46.02	46.01
8	1	f	5/2	-0.200	3	0.00	60.56	60.55
9	2	p	3/2	-0.200	3	0.00	48.10	48.09

----- Neutron single-particle radii -----

				R(2)	R(4)	OCC	rho(8.9)	rho(9.9)	rho(10.9)
1	1	s	1/2	2.274	2.575	2.000	0.848E-09	0.706E-10	0.600E-11
2	1	p	3/2	2.863	3.133	4.000	0.188E-07	0.244E-08	0.325E-09
3	1	p	1/2	2.954	3.268	2.000	0.727E-07	0.122E-07	0.210E-08
4	1	d	5/2	3.434	3.757	6.000	0.524E-06	0.129E-06	0.327E-07
5	1	d	3/2	4.662	6.063	0.000	0.131E-04	0.675E-05	0.371E-05
6	2	s	1/2	4.172	4.895	2.000	0.769E-05	0.278E-05	0.102E-05
7	1	f	7/2	3.865	4.440	0.000	0.324E-05	0.134E-05	0.600E-06
8	1	f	5/2	3.890	4.477	0.000	0.341E-05	0.141E-05	0.631E-06
9	2	p	3/2	6.815	8.635	0.000	0.451E-04	0.270E-04	0.167E-04

But must make small correction as HF is a fixed centre calculation

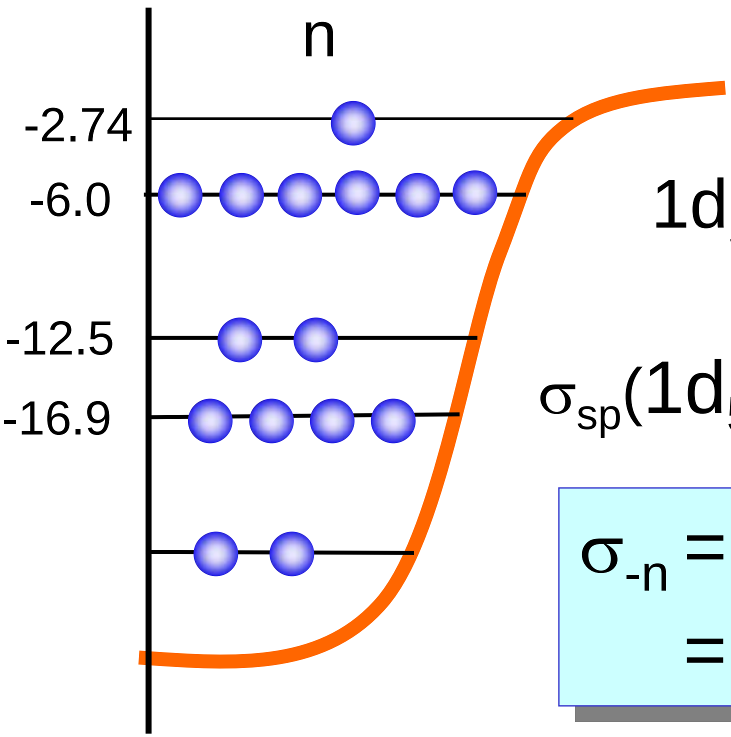
$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



$^{24}\mathrm{O}(g.s.)$

Orientation - extreme sp model – inclusive sigma

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$2s_{1/2}$ S_n=2.74 MeV

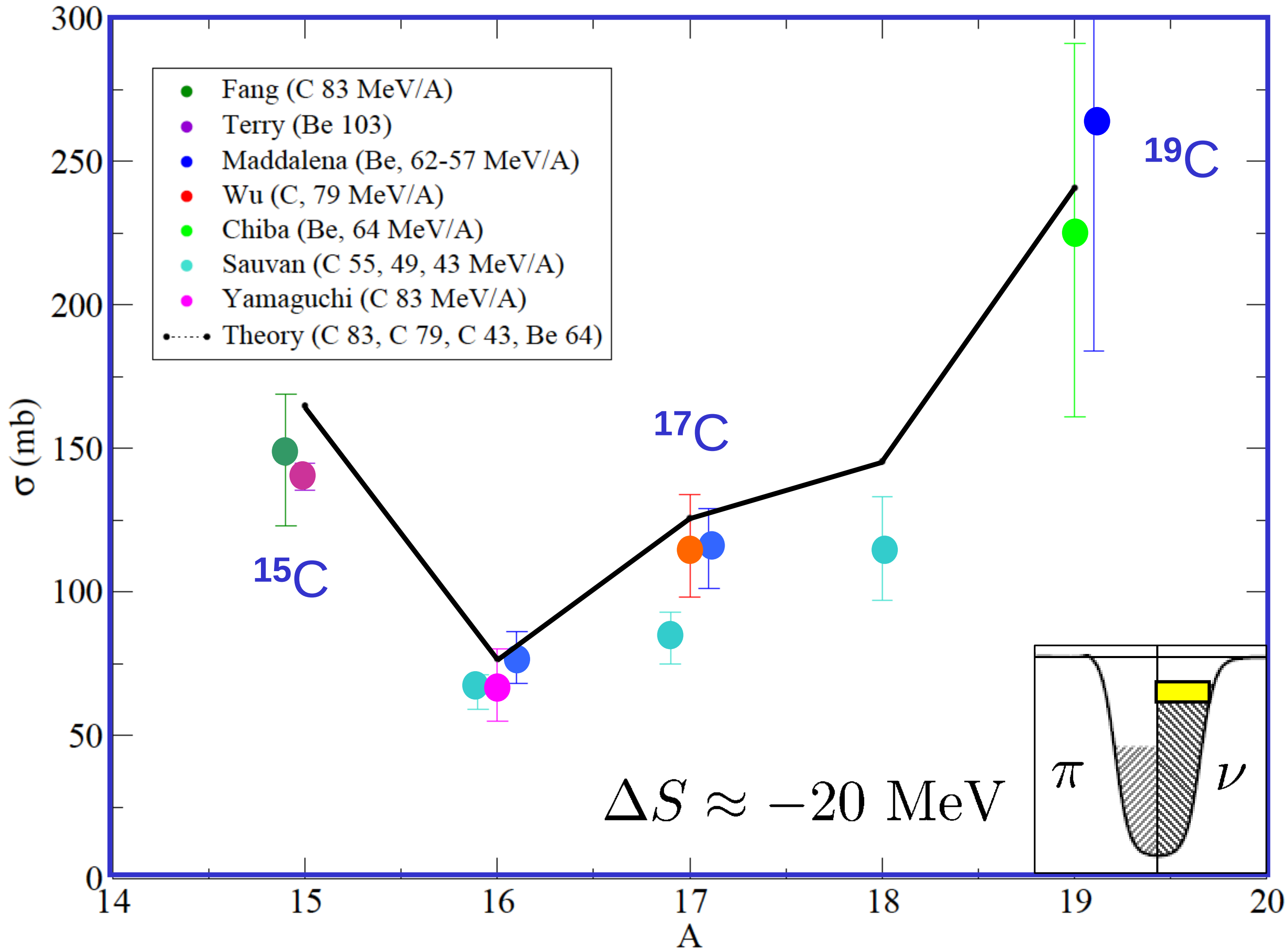
$1d_{5/2}$ S_n=6.0 MeV

$\sigma_{sp}(1d_{5/2})$=23 mb $\sigma_{sp}(2s_{1/2})$=64 mb

$$\sigma_{-n} = 6\,\sigma_{sp}(1d_{5/2}) + \sigma_{sp}(2s_{1/2})$$
$$= 202 \text{ mb}$$

Measurement at RIKEN [Kanungo *et al.* PRL **88** ('02) 142502]
at 72 MeV/nucleon on a ^{12}C target; σ_{-n} = 233(37)mb

Inclusive neutron removal – $^{15-19}$C isotopes



E.C. Simpson and J.A. Tostevin, submitted

Overlap function sensitivity: Hartree Fock ‘sizes’

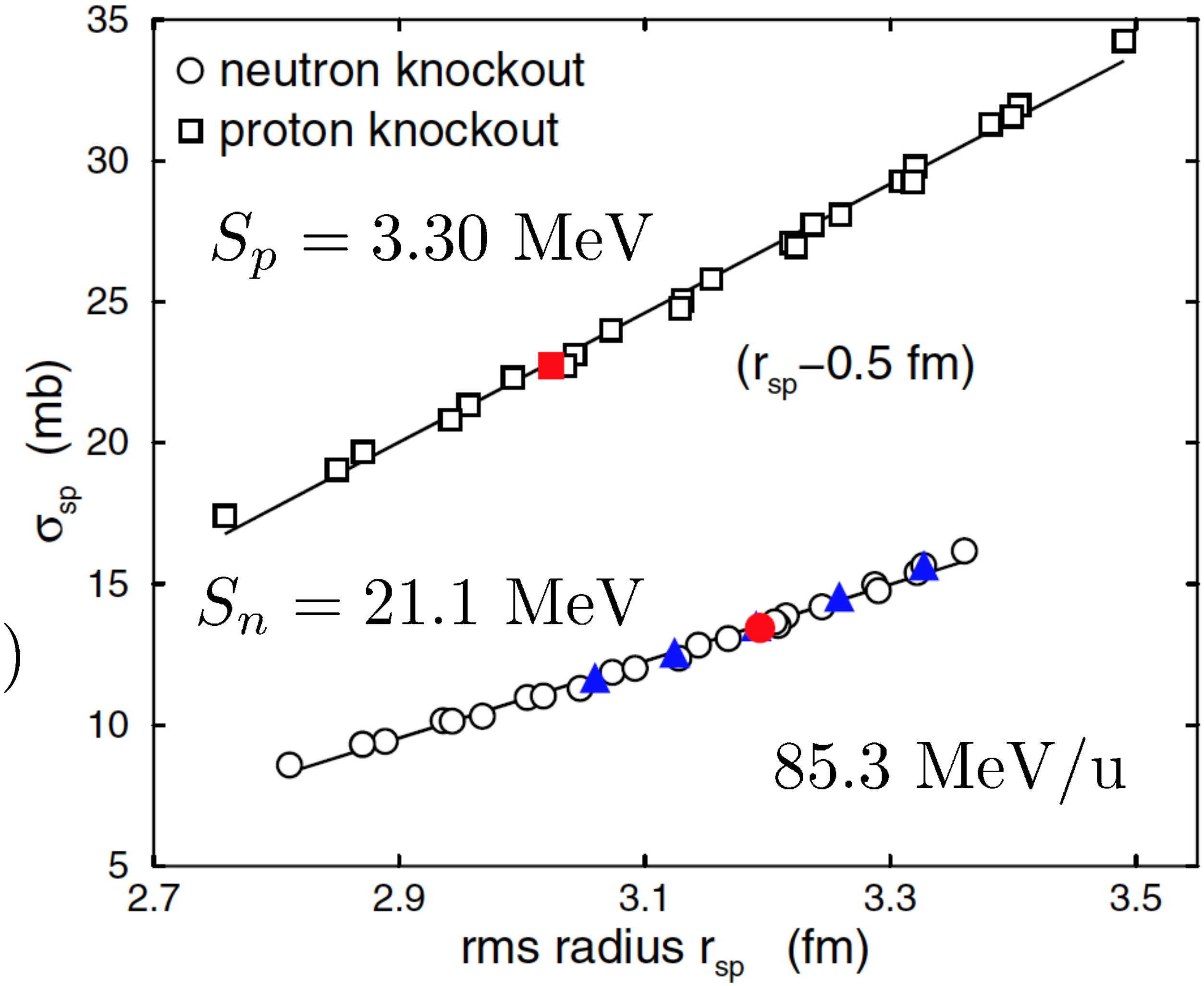
$^{24}\mathrm{Si}\ (-1\mathrm{N})$

$1d_{5/2}$

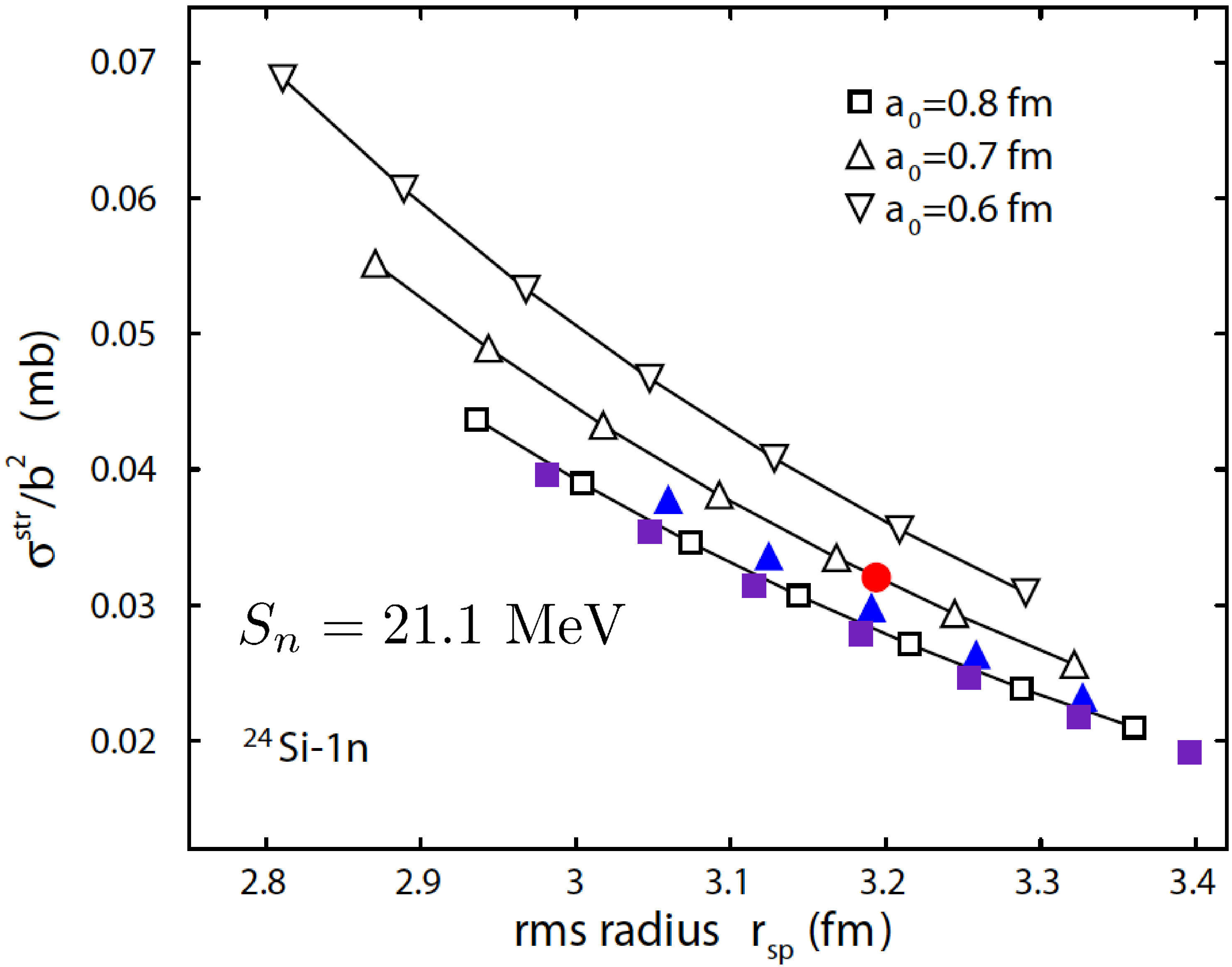
$\sigma(\langle r^2\rangle^{1/2})$

↑

$\sigma(r_0, a_0, V_{so}, \beta_{NL})$

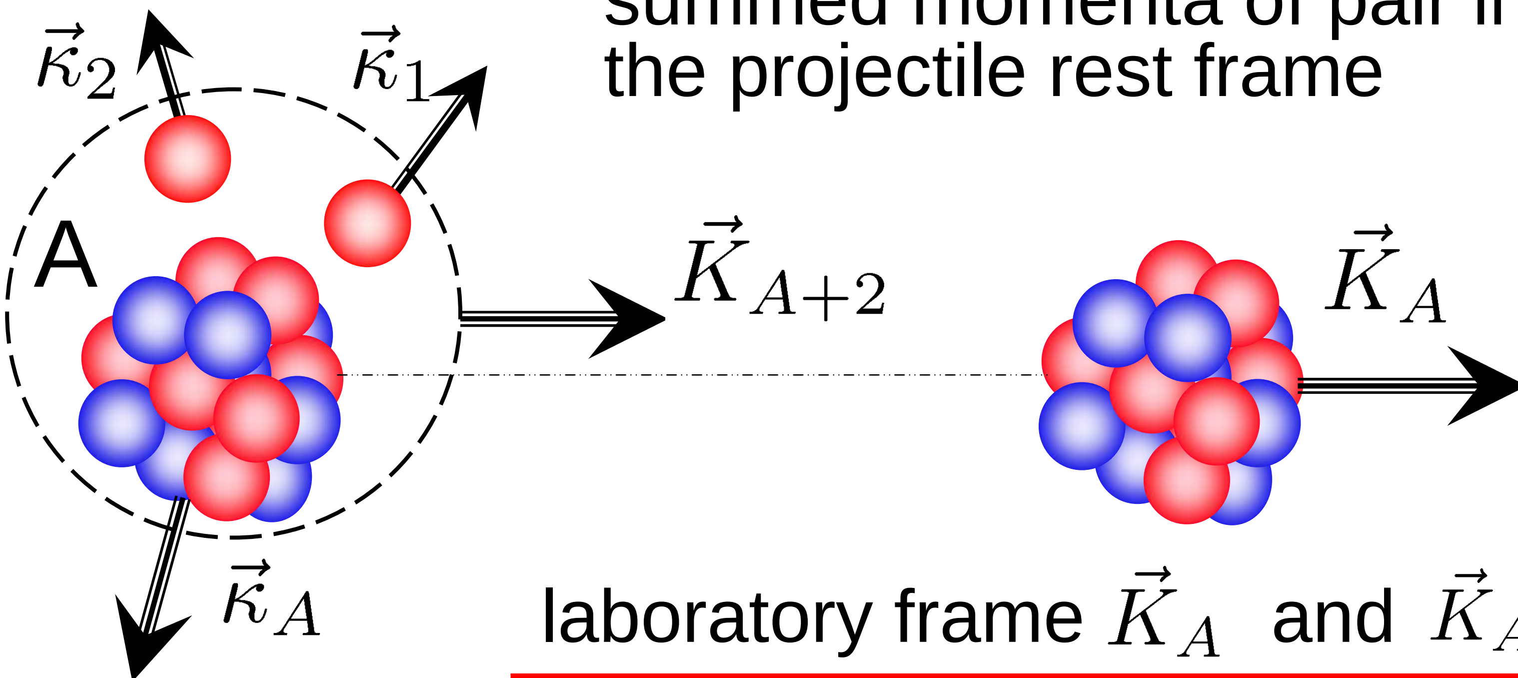


Sensitivity to ANC – or more?



Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame



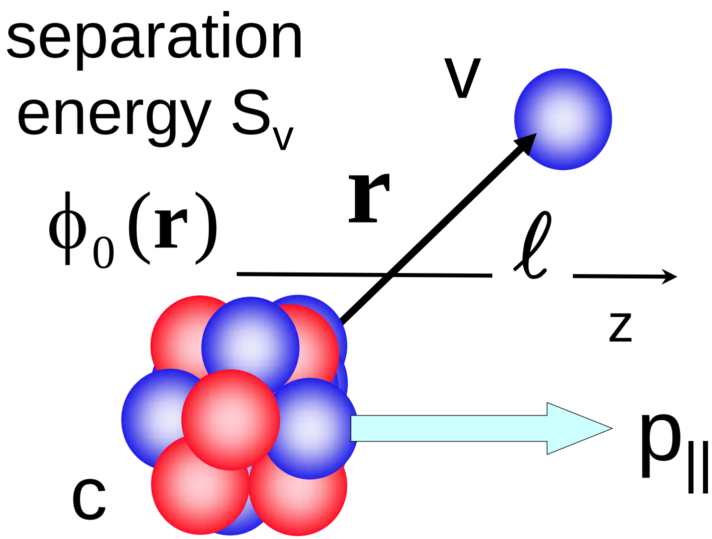
Projectile rest frame

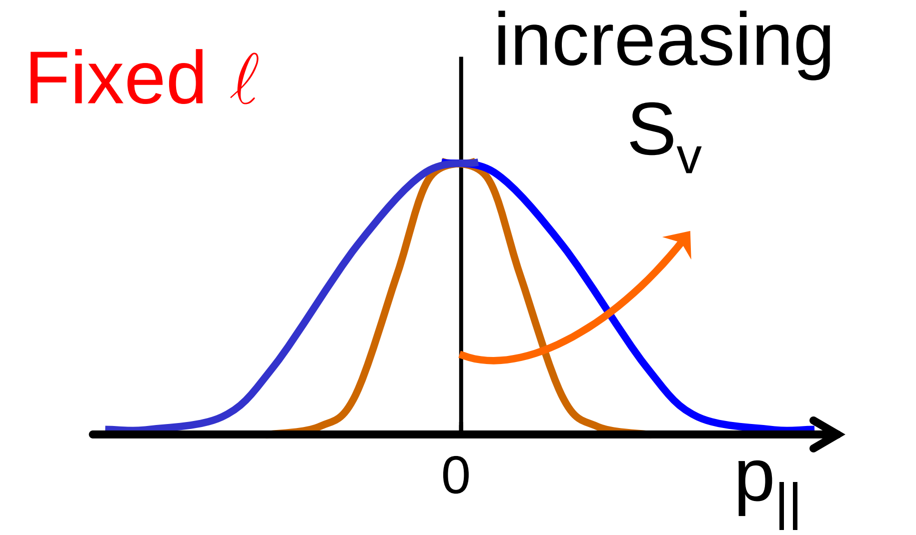
laboratory frame $\vec{K}_A$ and $\vec{K}_{A+2}$

$$\vec{K}_A = \frac{A}{A+2}\vec{K}_{A+2} - [\vec{\kappa}_1 + \vec{\kappa}_2]$$

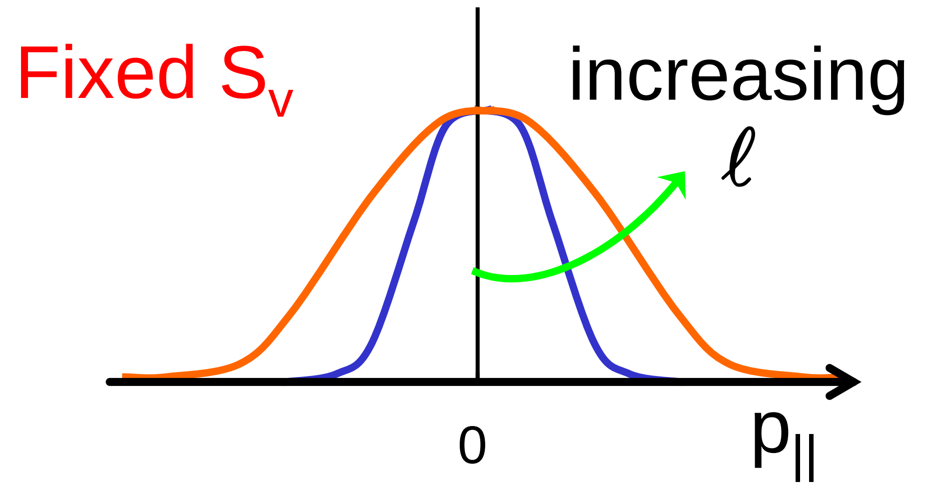
and component equations

Measurement of the residue's momentum





consider momentum components $p_{||}$ of the core parallel to the beam direction, in the projectile rest frame



$$\Delta p \Delta x > \hbar/2$$

Residue momentum distributions after knockout

$$\sigma_{str} = \frac{1}{2l+1}\sum_m \int d^2b\,\langle\psi_{lm}||S_c(b_c)|^2(1-|S_n(b_n)|^2)|\psi_{lm}\rangle$$

$$= \frac{1}{2l+1}\sum_m \int d^2b_n\,(1-|S_n(b_n)|^2)\langle\psi_{lm}|S_c^*\,\underbrace{}\,S_c|\psi_{lm}\rangle$$

$$\frac{1}{(2\pi)^3}\int d\vec{k}_c|\vec{k}_c\rangle\langle\vec{k}_c| = 1$$

In projectile rest frame:

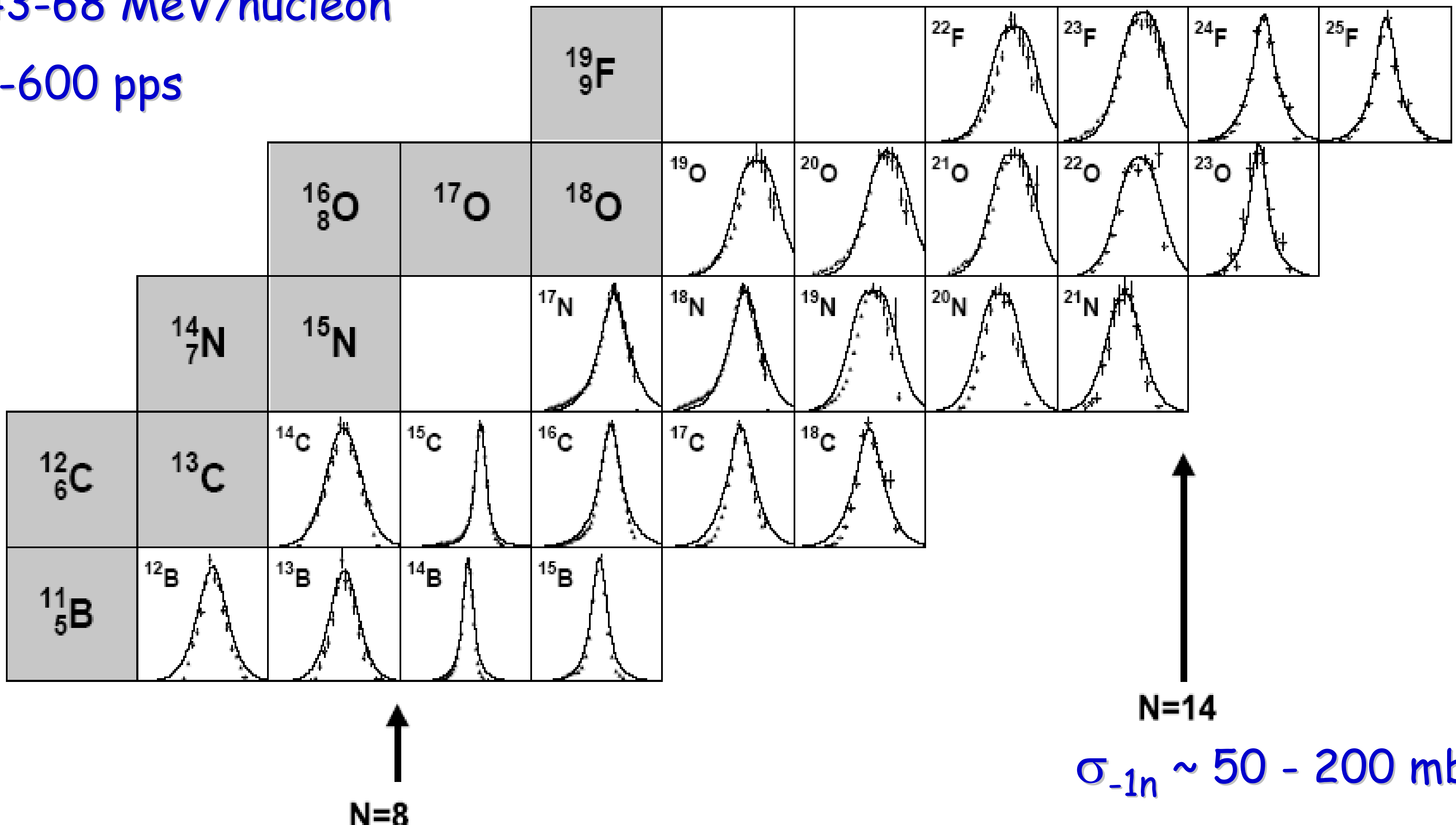
$$\frac{d\sigma_{str}}{d^3k_c} = \frac{1}{(2\pi)^3}\frac{1}{2l+1}\sum_m \int d^2b_n[1-|S_n(b_n)|^2]$$

$$\times\left|\int d^3r e^{-i\mathbf{k}_c\cdot\mathbf{r}}S_c(b_c)\psi_{lm}(\mathbf{r})\right|^2$$

Systematics show shell effects

43-68 MeV/nucleon

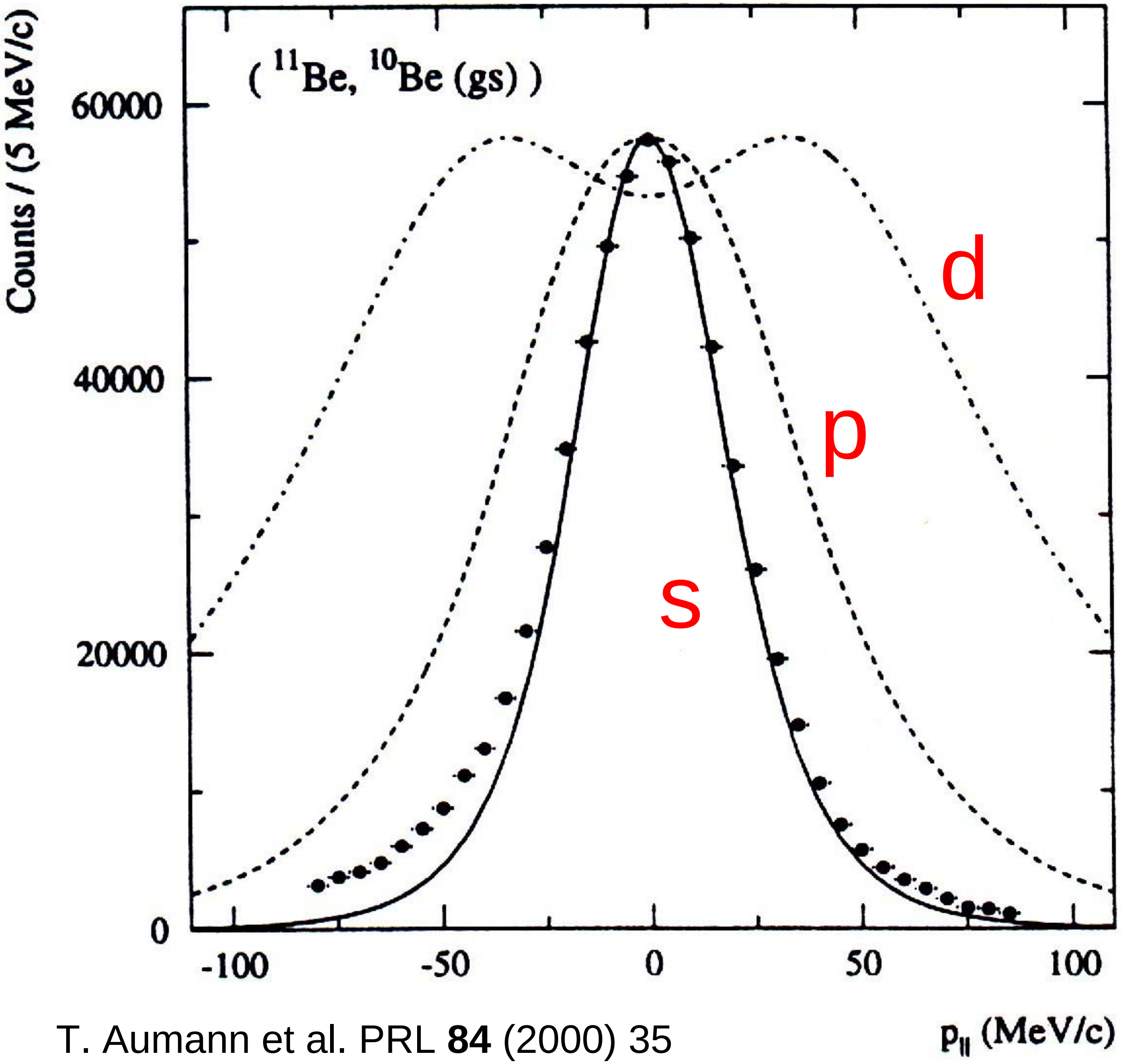
1-600 pps



σ_{-1n} ~ 50 - 200 mb

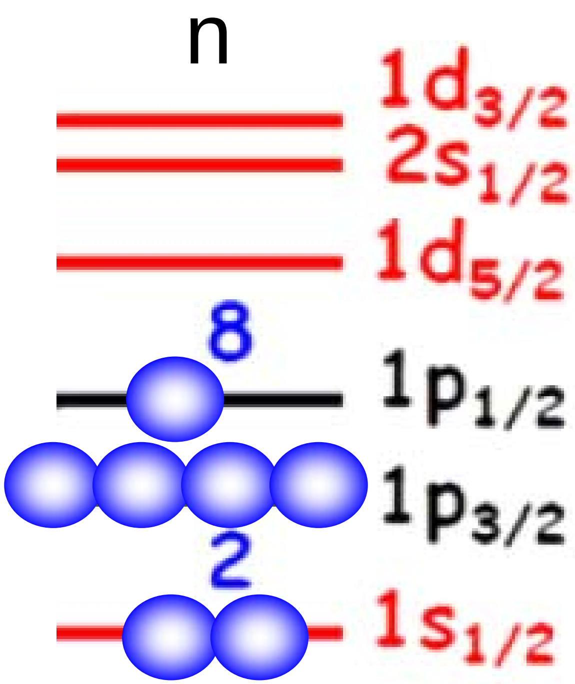
FWHM ~ 50 - 240 MeV/c

Residue momentum $^{11}Be \rightarrow ^{10}Be$ – 2s intruder



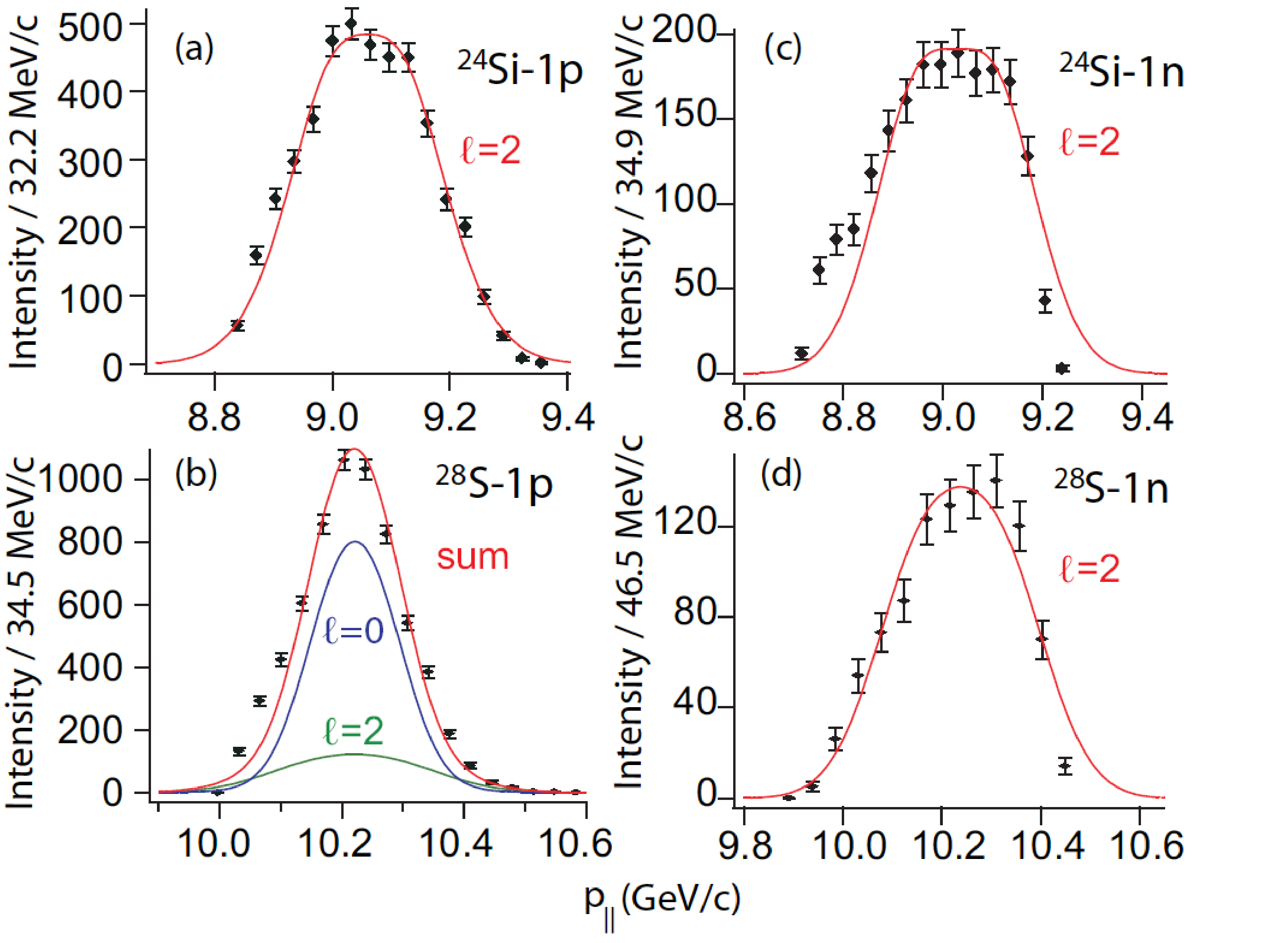
T. Aumann et al. PRL **84** (2000) 35

$Z = 4, N = 7$

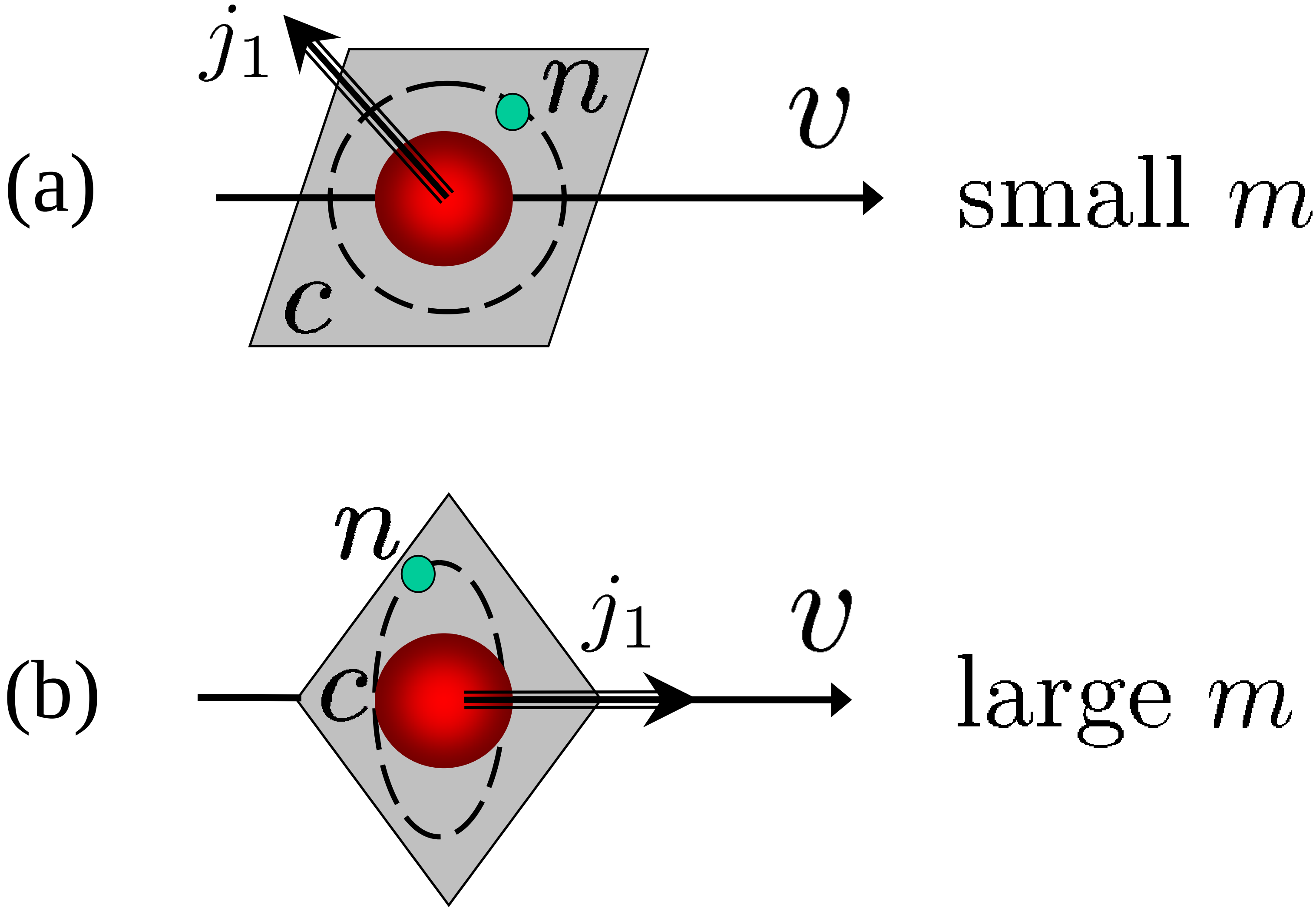


^{11}Be

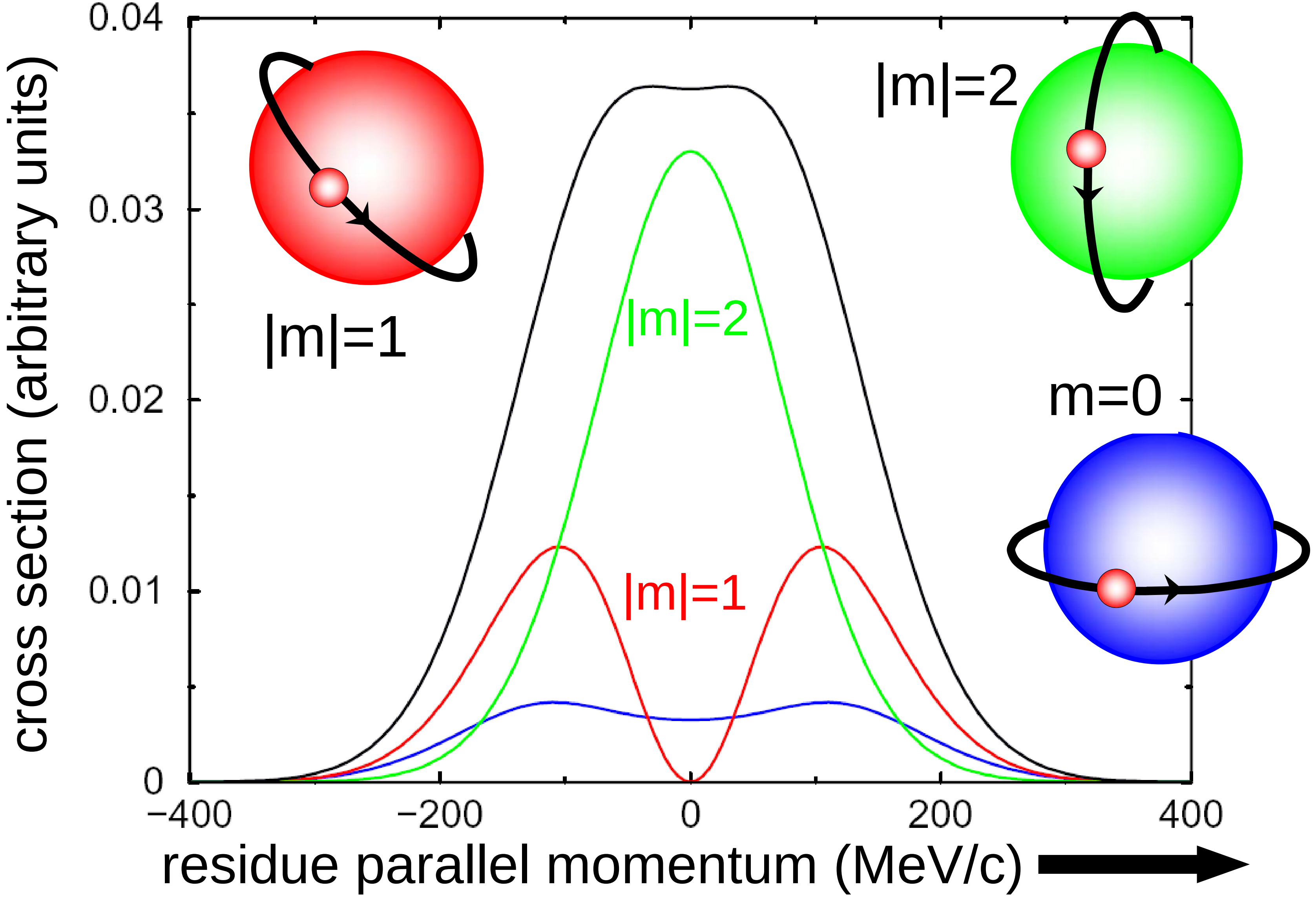
and – their momentum also distributions look OK



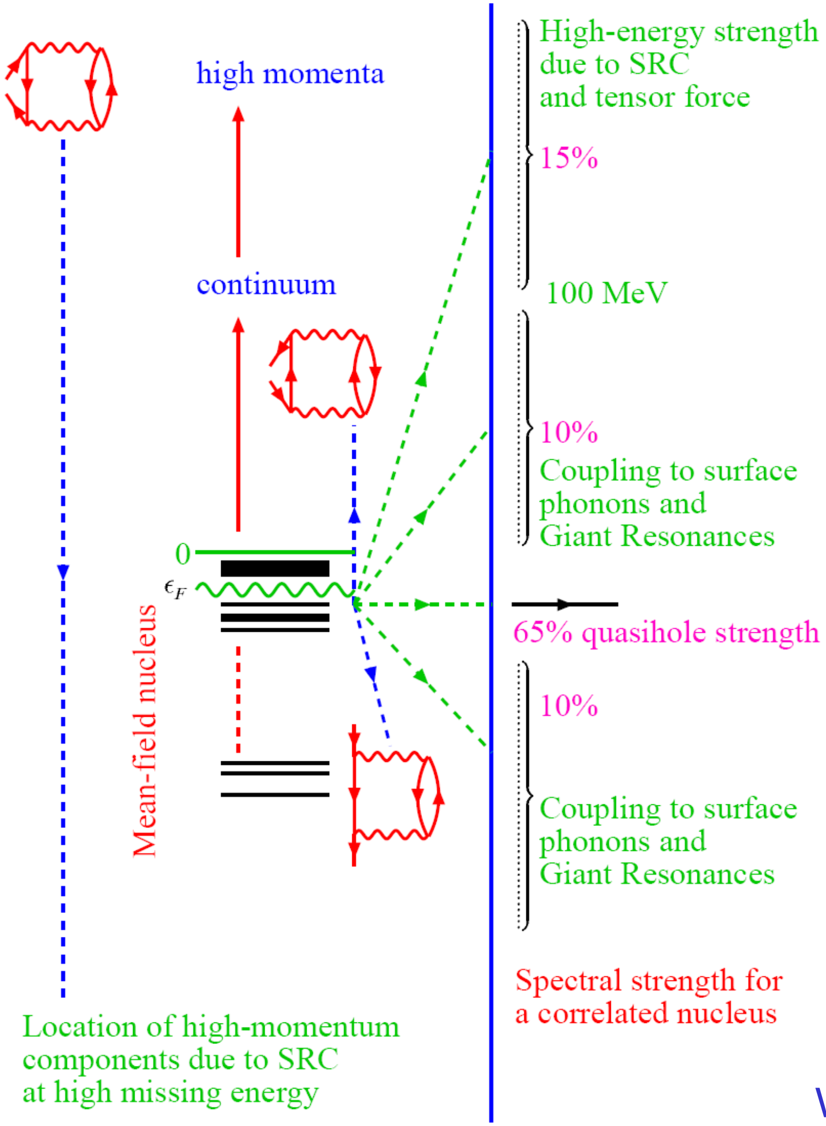
projection dependence … what do we expect?

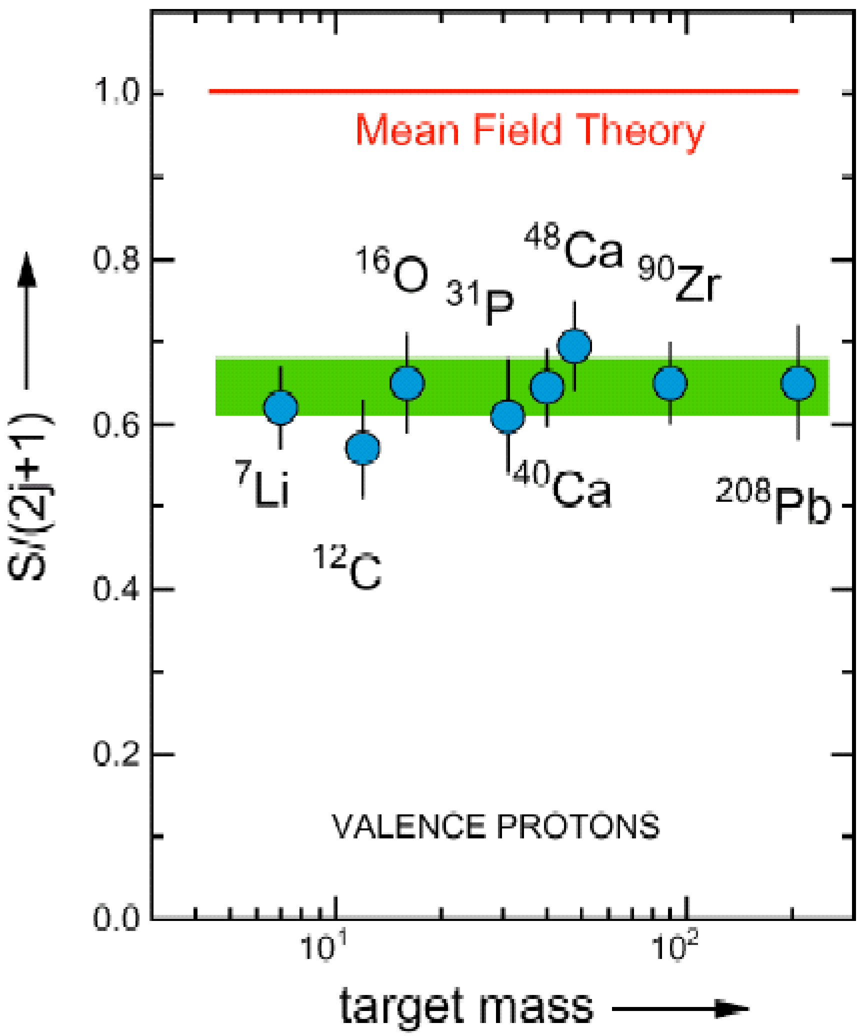


One nucleon knockout – ^{28}Mg (–p,ℓ=2) 82A MeV



Strength from e-induced knockout – stable nuclei





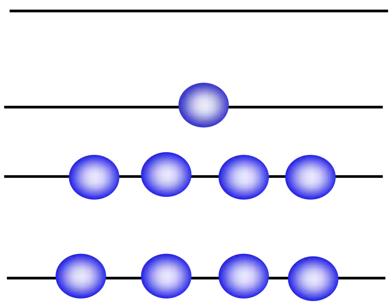
W. Dickhoff and C. Barbieri, Progress in Particle and Nuclear Physics **52** 377 (2004)

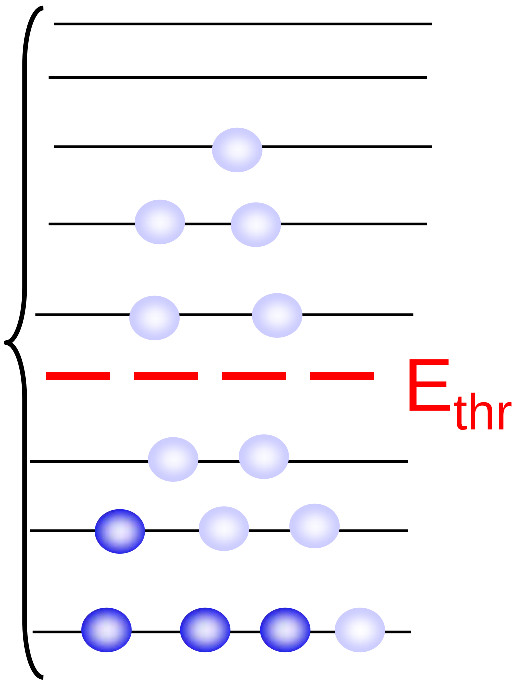
Modern ‘shell model’ calculations do much more ....

Nucleons scatter due to residual pairwise interactions

so in reality …

distribution of nucleons in several shell model levels



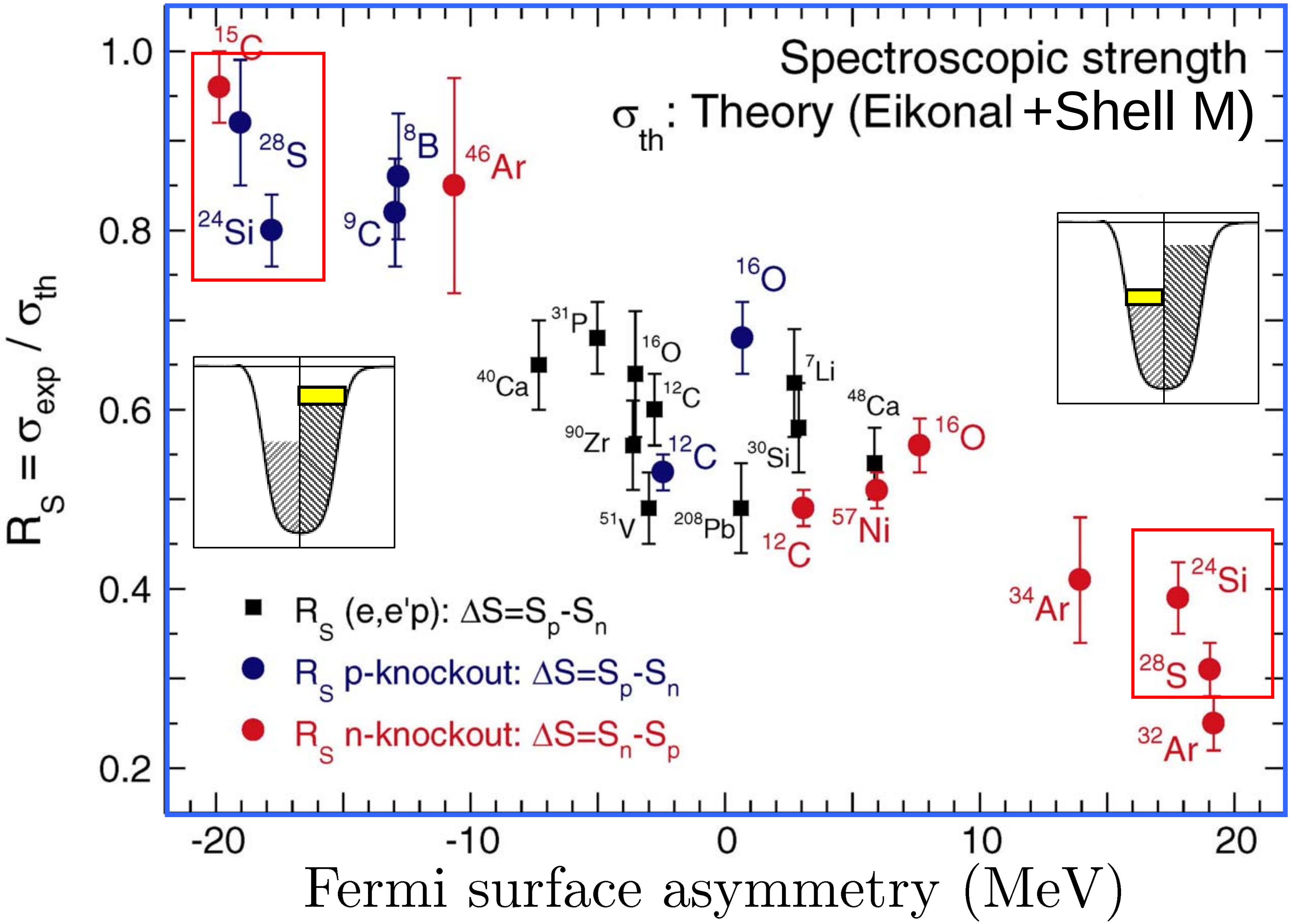


Filled states

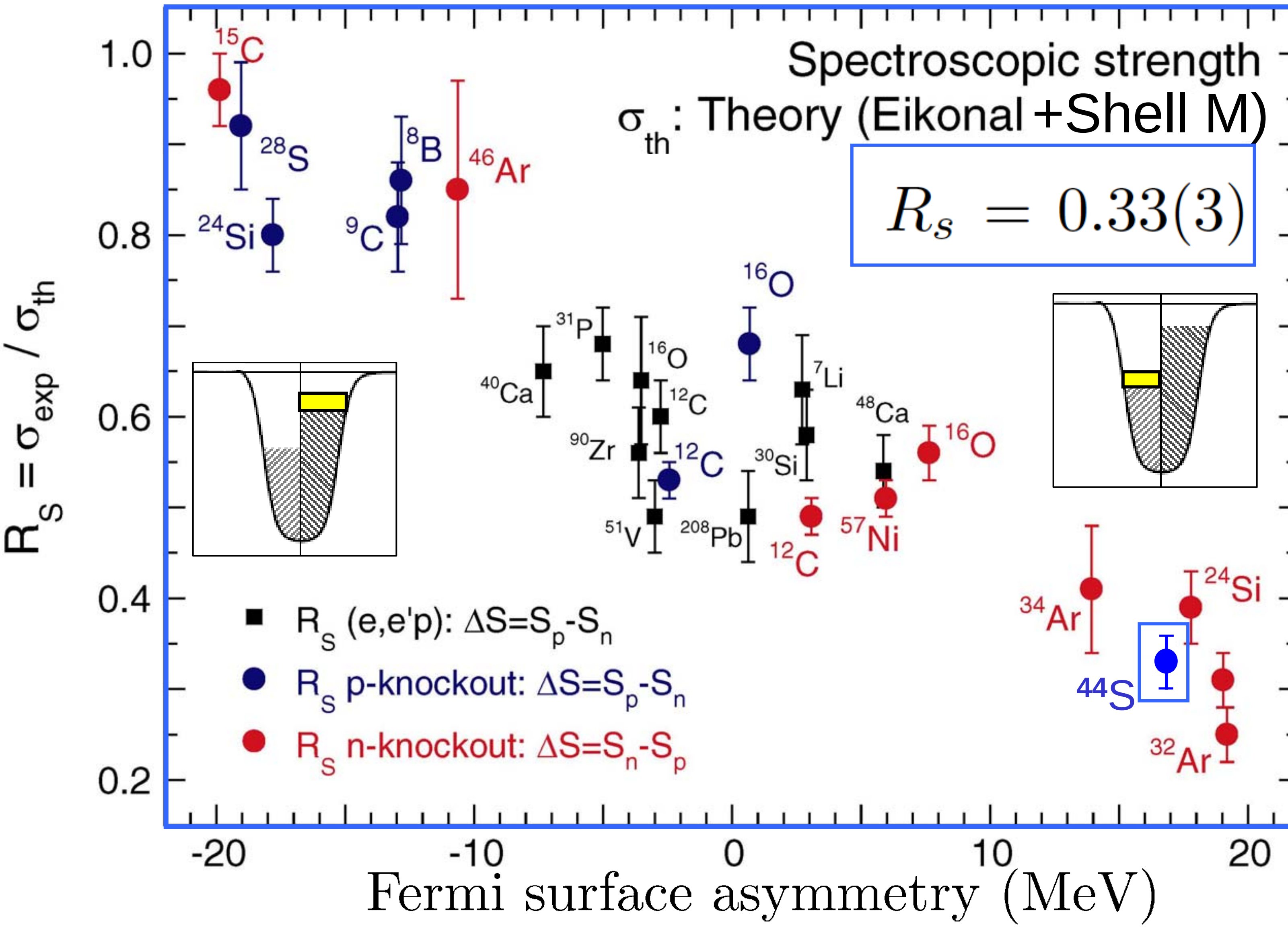
Short range, tensor force and collective motions

Filled states

Removal strengths at the Fermi surface (2008)



Removal strengths at the Fermi surfaces – ^{44}S



L.A. Riley et al., Phys Rev C 78, 011303(R) 2008