CENTRE FOR NUCLEAR & RADIATION PHYSICS

Role of break-up in transfer reactions with deuterons

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Why study transfer reactions with deuterons?

d+A $\longrightarrow$ p+B B=A+n

(i) Angular distribution of p gives orbital angular momentum, $l_{n,}$, of neutron orbit in B. Polarization may give j_n.

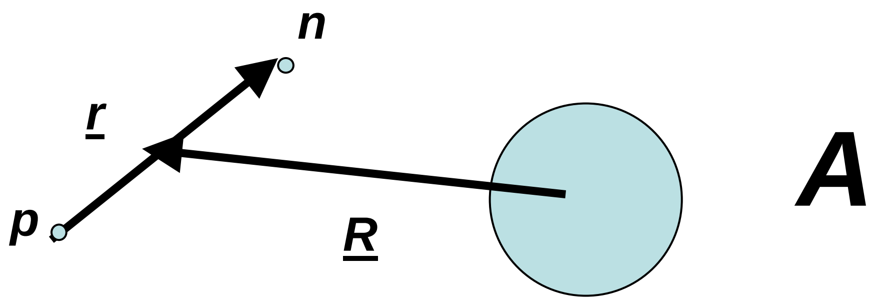
(ii) Cross-section magnitude reveals single-particle nature of neutron state in B.

(iii) Gives insight into the way single particle strength is distributed among states of a given nucleus and over the periodic table.

(iv) Modern studies seek to answer these questions for exotic nuclei near the drip lines. This may involve experiments in inverse kinematics.

(v) It is essential that we have credible reaction theories if reliable nuclear structure information is to result.

Three Body Model of d+A



$$H = K_R + H_{np} + V(\vec{R}, \vec{r})$$

$$H_{np} = K_r + V_{np}$$

$$V(\vec{R}, \vec{r}) = V_{nA}(\vec{R} + \vec{r}/2) + V_{pA}(\vec{R} - \vec{r}/2)$$

The Deuteron case

$$
\begin{aligned}
H_{np} &= K_r + V_{np}, \\
H_{np}\phi_0(\vec{r}) &= -\epsilon_0\phi_0(\vec{r}), \\
H_{np}\phi_{\vec{k}}^{(+)}(\vec{r}) &= \epsilon_k\phi_{\vec{k}}^{(+)}(\vec{r}).
\end{aligned}
$$

$$\epsilon_0 = 2.236\mathrm{MeV},$$

$$\phi_0(\vec{r}) \stackrel{r\to\infty}{\to} \mathcal{N}\exp(-\gamma r)/r,$$

–

=

$$\gamma = 0.236\mathrm{fm}^{-1}$$

Calculating Transfer and Elastic Break-up Amplitudes

The 3-body Schroedinger equation:

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \quad = \quad V_{np}\Psi_{\vec{K}_d}$$

Exact solution of inhomogeneous equation
($M_A \rightarrow$ infinity):

$$T_{d,p} \quad = \quad \langle \chi_p^{(-)} \phi_n \mid V_{np} \mid \Psi_{\vec{K}_d} \rangle,$$

$$T_{d,np} \quad = \quad \langle \chi_p^{(-)} \chi_n^{(-)} \mid V_{np} \mid \Psi_{\vec{K}_d} \rangle$$

NB The 3-body wavefunction is only needed for n and p within the range of V_{np}

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Adiabatic, "frozen-halo", approx.

$$\Psi^{trans} = \exp(-\frac{iH_{np}t}{\hbar})\Psi.$$

$$(T_R + V(\vec{R}, \vec{r}(t)))\Psi^{trans}(\vec{R}, \vec{r}, t) = i\hbar\frac{\partial\Psi^{trans}}{\partial t}.$$

$$\vec{r}(t) = \exp(\frac{iH_{np}t}{\hbar})\vec{r}\exp(-\frac{iH_{np}t}{\hbar}).$$

$$\left|\frac{< H_{np} > T}{\hbar}\right| \ll 1.$$

$$(T_R + V(\vec{R}, \vec{r}) - E_d)\Psi^{ad}(\vec{R}, \vec{r}) = 0$$

where $\vec{r}$ is now a parameter

Implementing the Adiabatic approximation: elastic scattering

$$(T_R + V(\vec{R},\vec{r}) - E_d)\Psi^{ad}(\vec{R},\vec{r}) = 0$$

$$\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r})$$

$$(T_R + V(\vec{R},\vec{r}) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = 0$$

$$\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) \stackrel{R\to\infty}{\to} \exp(i\vec{K}_d.\vec{R}) + f(\hat{R},\vec{r})\frac{\exp(iK_dR)}{R}.$$

$$f_{elastic}(\hat{R}) = \int d\vec{r}\phi^*_o(\vec{r})f(\hat{R},\vec{r})\phi_o(\vec{r})$$

Johnson and Soper PRC1(1970)976,
Amakawa, et al, PLB82B(1979)13

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Application to transfer and elastic break-up

***NB* Adiabatic wavefunction not valid for r large! Iteration needed!**

Johnson and Soper, PRC1(1970)976.

3-body Schroedinger equation:

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \quad = \quad V_{np}\Psi_{\vec{K}_d}$$

1st iteration:

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \quad = \quad V_{np}\Psi^{ad}_{\vec{K}_d}$$

Exact solution of inhomogeneous equation:

ADW

$$T_{d,p} \quad = \quad \langle \chi_p^{(-)}\phi_n \mid V_{np} \mid \Psi^{ad}_{\vec{K}_d}\rangle,$$

$$T_{d,np} \quad = \quad \langle \chi_p^{(-)}\chi_n^{(-)} \mid V_{np} \mid \Psi^{ad}_{\vec{K}_d}\rangle$$

(see Timofeyuk and Johnson, PRC59(1999)1545 for finite A corrections)

Application to (d,p)

Johnson and Soper PRC1(1970)976); Harvey and Johnson, PRC3(1971)636; Wales and Johnson, NPA274(1976))168.

$$V_{np}\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$$

$$T_{d,p} = \int d\vec{R}\chi_p^{(-)*}(\vec{R})\phi_n^*(\vec{R})[V_{np}\phi_0(\vec{r})]\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$$

$$(T_R + V(\vec{R},0) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0) = 0$$

$$V(R,0) = V_{nA}(R) + V_{pA}(R)$$

N.B. $V(R,0)$ not designed to fit elastic deuteron scattering. Designed to generate amplitude for finding n and p at R with r less than range of V_{np} and not necessarily in the form of a deuteron in its ground state.

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Comparison with DWBA

(i) Ignore all components of the 3-body wave function except the elastic A(d, d)A components.
(ii) Find deuteron and proton optical potentials which correctly describe elastic data.
(iii) Deuteron excitation (break-up) and excitation of A taken into account only in so far as these channels influence elastic d channel.

$$\Psi^{el}_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(\vec{r})\chi^{el(+)}_{\vec{K}_d}(\vec{R}),$$

$$(T_R + V^{opt}(\vec{R}) - E_d)\chi^{el(+)}_{\vec{K}_d}(\vec{R}) = 0,$$

DWBA:

$$\chi^{el(+)}_{\vec{K}_d}(\vec{R}) \stackrel{R\to\infty}{\to} \exp(\imath\vec{K}_d.\vec{R}) + f^{el}(\hat{R})\frac{\exp(\imath K_d R)}{R}.$$

D
D
D
D
W

D
W
B
A

$$\begin{aligned} T_{dp} &= \langle\chi^{(-)}_p\phi_n \mid V_{np} \mid \Psi^{el}_{\vec{K}_d}\rangle, \\ &= \langle\chi^{(-)}_p\phi_n \mid V_{np} \mid \phi_0\ \chi^{el(+)}_{\vec{K}_d}\rangle, \end{aligned}$$

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Some applications of the Johnson-Soper ADW model to (d,p) and (p,d) reactions.

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1 Cadmus and Haeberli, Nucl Phys A327(1979)419; A349(1980)103. E_d=12.9Mev, Sn target. DWBA fails. ADW works.

2. Liu, et al, Phys.Rev.C69(2004)064313 E_d=12-60MeV, $^{12}C(d,p)^{13}C$ and $^{13}C(p,d)^{12}C$. Consistent spectroscopic factors with ADW.

3.Stephenson, et al, Phys. Rev. C42(1990)2562.
E_d=79MeV, $^{116}Sn(d,p)^{117}Sn$, l_n=0;
E_d=88.2MeV, $^{66}Zn(d,p)$,$^{67}Zn(p,d)Zn^{66}$.

4. Catford, et al, J.Phys.G.31(2005)S1655. Nucleon Transfer via (d,p) using TIARA with a ^{24}Ne radioactive beam.

5. Guo, et al, J.Phys.G.34(2007)103. $^{11}B(d,p)^{12}B$, E_d=12MeV. Asymptotic Normalization Coefficients; Astrophysical S-factor.

Cadmus and Haeberli, Nucl.Phys.A327(1979)419

$^{116}Sn(d,p)^{117}Sn$,   4.11MeV/A

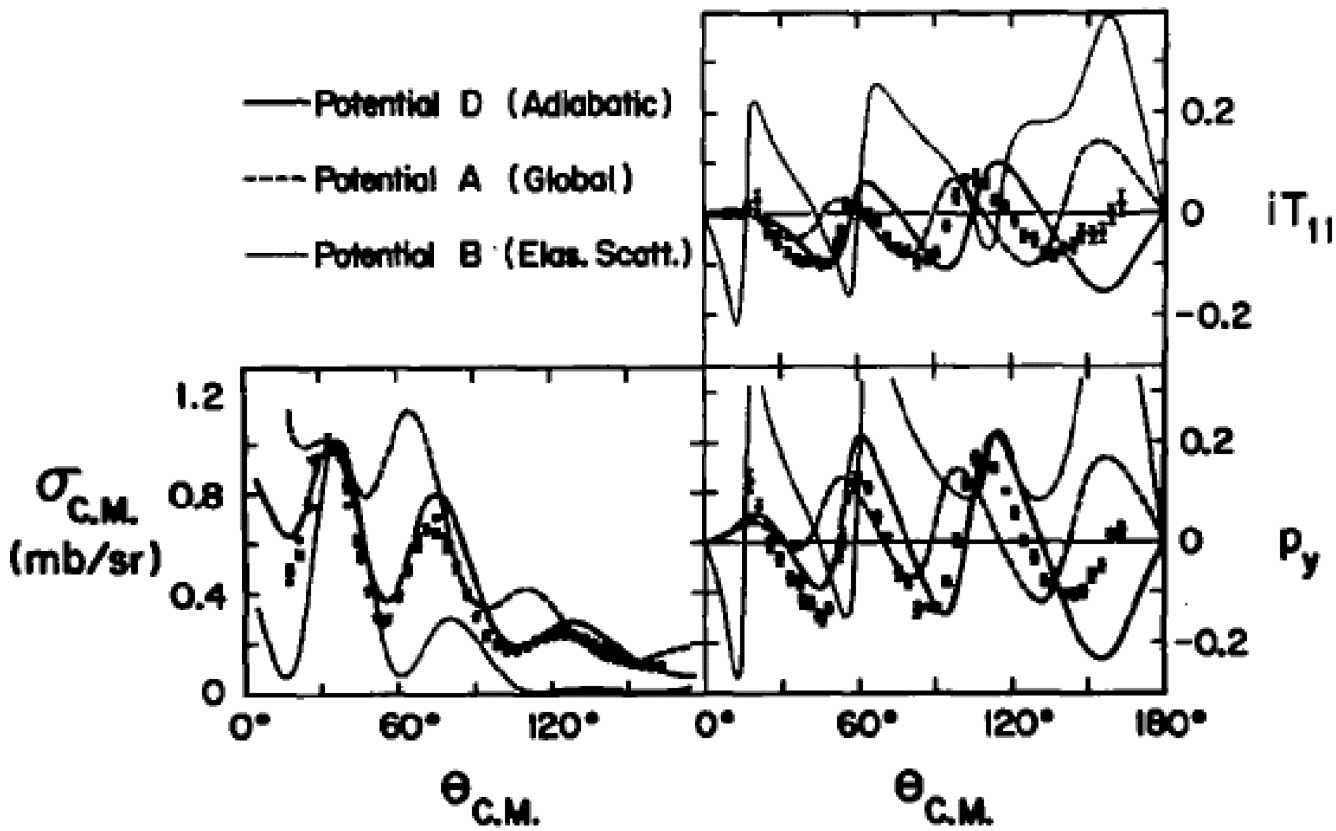

Fig. 2. Cross section, vector analyzing power, and proton polarization data for the ^{116}Sn(d, p)^{117}Sn(g.s.) reaction. The cross section data represented by the closed symbols were obtained directly from observations of the (d, p) reaction, while those represented by the open symbols were obtained from the (p, d) measurements and renormalized as explained in the text. The curves are the results of distorted-wave calculations using different deuteron optical potentials.

Spectroscopic Factors

(Cadmus and Haeberli, 4.11 MeV/A)

TABLE 2

Spectroscopic factors for states in ^{117}Sn

E_x (MeV)	Potential A (global analysis [25])	Potential B (elastic scattering)	Potential D (adiabatic)	Carson and McIntyre (ref. [45])
0.0	0.90	0.51	0.45	0.52
0.159	0.87	1.08	0.54	0.76
0.317	0.62	0.88	0.48	0.79
1.020	0.10	0.094	0.058	0.064
1.179	0.059	0.053	0.034	0.039
1.308	0.034	0.063	0.030	0.037

Carson and McIntyre, 2.5MeV/A

Application of Johnson-Soper method to ^{10}Be(d,p)^{11}Be(g.s.)

Data: Auton,Nucl.Phys. A157(1970)305

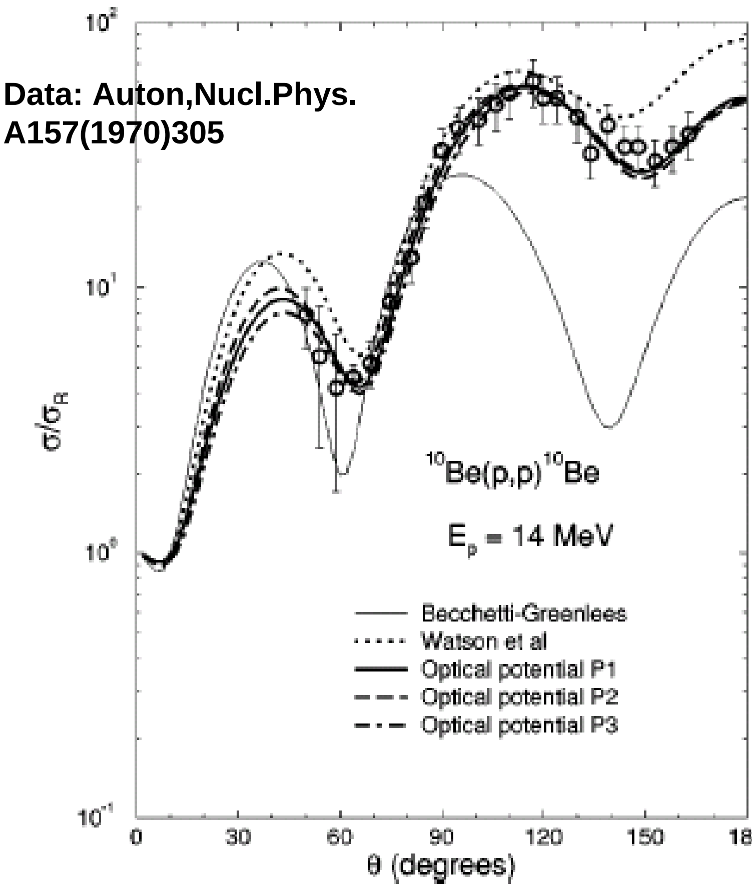

FIG. 3. Elastic scattering of the $p+{}^{10}$Be at $E_p=14$ MeV c culated with different proton optical potentials.

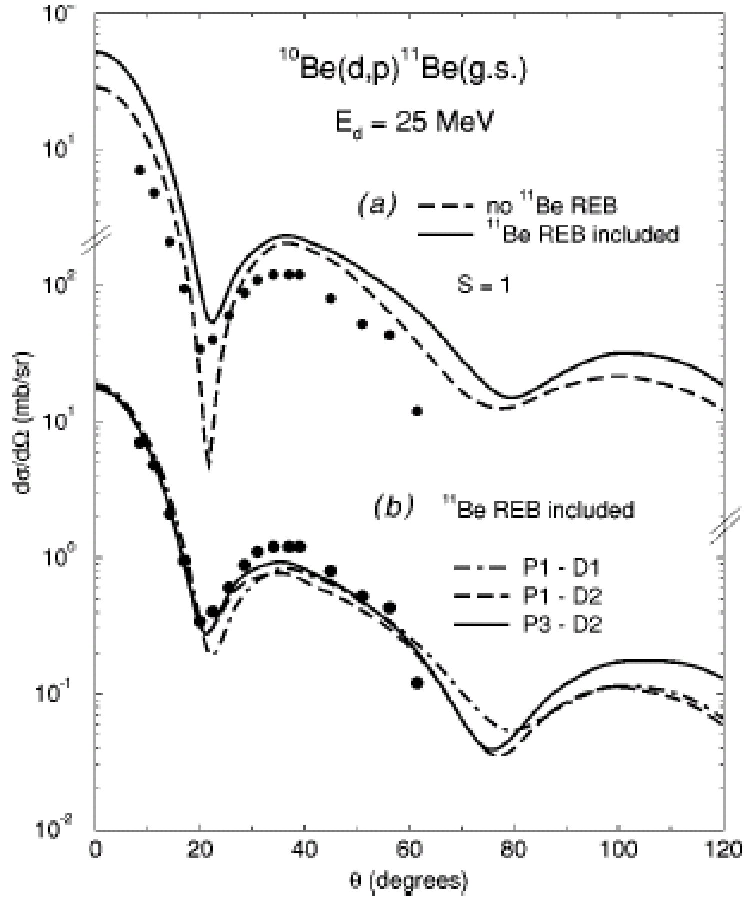

FIG. 4. Cross sections of the ^{10}Be$(d,p)^{11}$Be reaction at $E_d=25$ MeV calculated with adiabatic deuteron wave function. (a) Calculations have been done with proton optical potential P1 and deuteron adiabatic potential D1; spectroscopic factor $S=1$ was used. (b) Different sets of proton and deuteron optical potentials were used and theoretical curves are normalized to the experimental data.

Data: Zwieglinki,et al.Nucl.Phys.A315(1979)124

Timofeyuk and Johnson Phys.Rev. 59(1999)1545

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Application of Johnson-Soper method

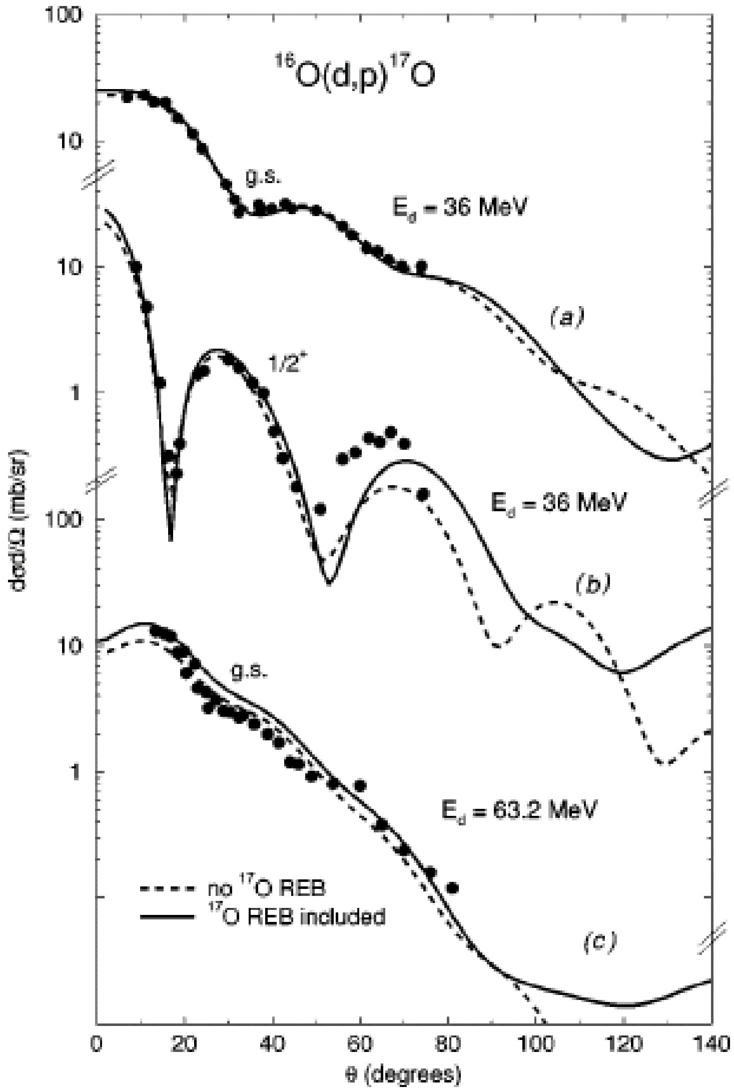

FIG. 2. (a) ^{16}O$(d,p)^{17}$O(g.s.) at E_d=36 MeV, (b) ^{16}O$(d,p)^{17}$O$(1/2^+)$ at E_d=36 MeV, and (c) ^{16}O$(d,p)^{17}$O(g.s.) at E_d=63.2 MeV calculated with deuteron breakup taken into account. Solid lines denote REB and dashed, no-REB calculations.

Timofeyuk and Johnson
Phys.Rev.C 59(1999)1545

Data:Cooper, Hornyak and Roos,
Nucl.Phys.A218(1974)249

The 3-body Wave function inside the range of V_{np}

Johnson and Soper,PRC1(1970)976,   Johnson, 2nd RIA Workshop 2005 (AIP Conf Proc 791)

JS Zero range

$$V_{np}\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0).$$

$$(T_R + V(\vec{R},0) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0) = 0,$$

Finite Range:

$$\Psi_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(r)\chi_o(\vec{R}) + \int d\vec{k}\,\phi^{(+)}_{\vec{k}}(r)\chi_k(\vec{R}),\ r < r_{np},$$

$$\phi_k(r) \simeq g(k)\phi_0(r),\ r < r_{np},$$

$$V_{np}\Psi_{\vec{K}_d}(\vec{R},\vec{r}) \simeq V_{np}\phi_0(r)\bar{\chi}_{\vec{K}_d}(\vec{R}),$$

$$\bar{\chi}_{\vec{K}_d}(\vec{R}) = \chi_o(\vec{R}) + \int d\vec{k}\,g(k)\chi_k(\vec{R}$$

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The Sturmian Method of Johnson and Tandy

Johnson and Tandy NuclPhys A235(1974)56

$$\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) = \sum_{0}^{\infty} \bar{\phi}_i(\vec{r})\bar{\chi}_i(\vec{R})$$

$$(T_r + \alpha_i V_{np})\bar{\phi}_i = -\epsilon_0 \bar{\phi}_i \qquad \langle \bar{\phi}_i \mid V_{np} \mid \bar{\phi}_j \rangle = -\delta_{i,j}$$

$$V_{np}\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) \approx V_{np}\bar{\phi}_0(\vec{r})\bar{\chi}_0(\vec{R})$$

$$(T_R + \bar{V}(\vec{R}) - E_d)\bar{\chi}_0(\vec{R}) = 0$$

$$\bar{V}(R) = \frac{\langle \phi_0 \mid V_{np}(V_{nA} + V_{pA}) \mid \phi_0 \rangle}{\langle \phi_0 \mid V_{np} \mid \phi_0 \rangle}$$

Zero range V_{np}: $\bar{V}(R) \to (V_{nA}(\vec{R}) + V_{pA}(\vec{R}))$

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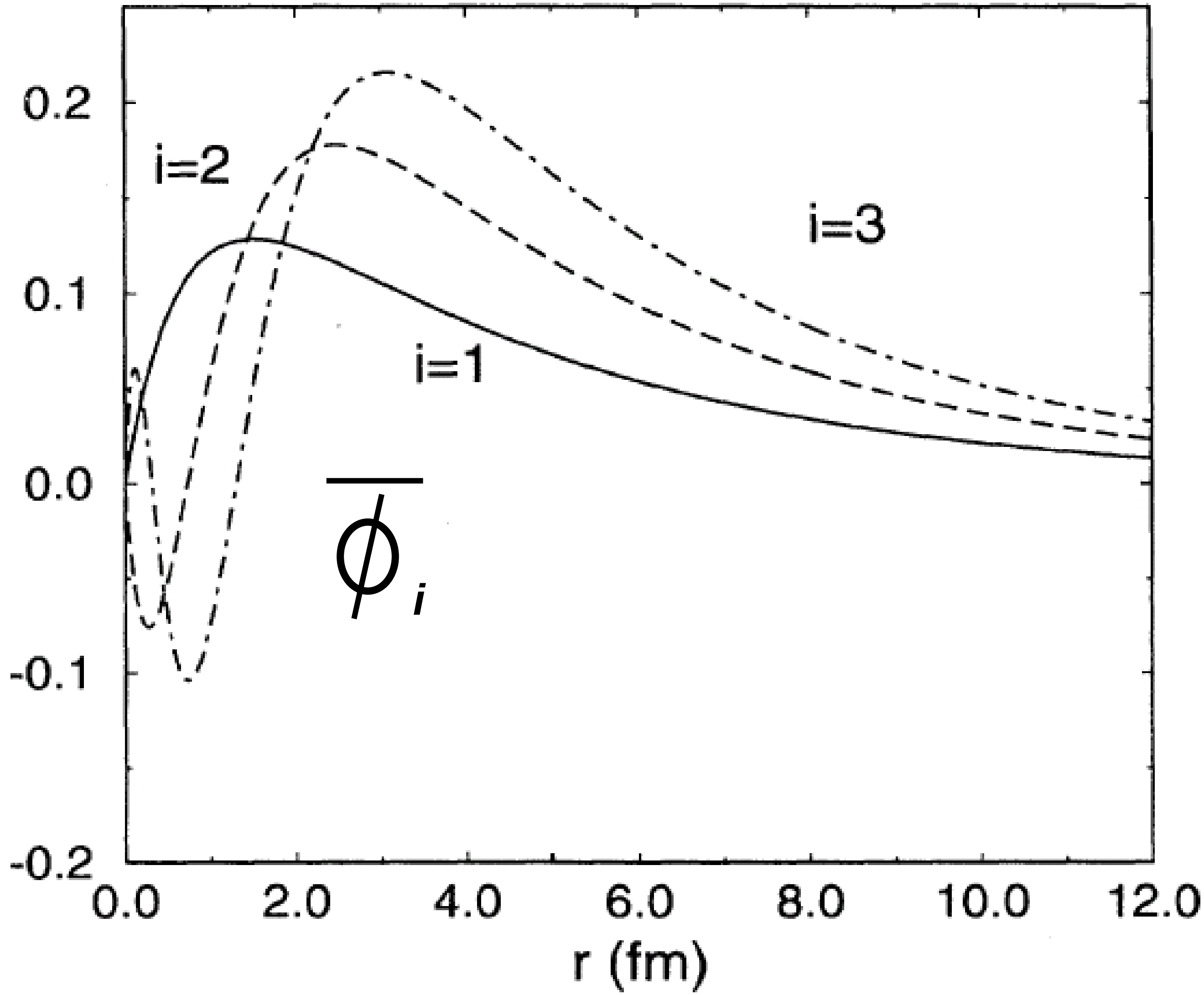
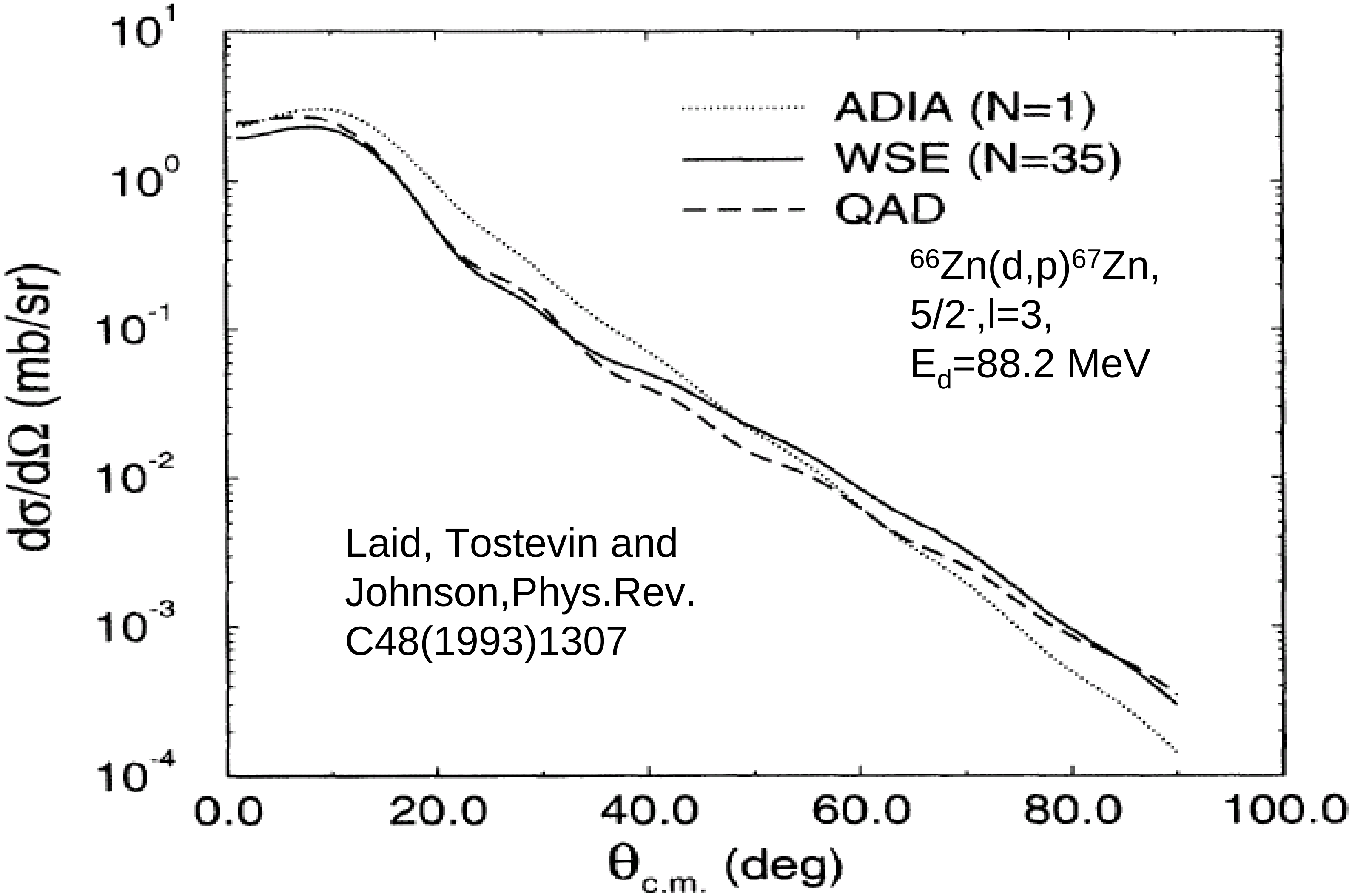
Sturmian States for V_{np} (Hulthen)



Laid, Tostevin and Johnson, PRC48(1993)1307

Corrections to the Adiabatic approximation at E_d=88.2 MeV



The CDCC Method

$$\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) = \sum_s \psi_s(\vec{r})\chi_s(\vec{R})$$

$$\langle \psi_s \mid H_{np} \mid \psi_{s'} \rangle = \epsilon_s \delta_{s,s'}$$

$$(E_d - \epsilon_s - T_R)\chi_s(\vec{R}) = \sum_{s'} \langle \psi_s \mid (V_{nA} + V_{pA}) \mid \psi_{s'} \rangle \chi_{s'}(\vec{R})$$

$$\Psi^{(+)}_{\vec{K}}(\vec{r},\vec{R}) \stackrel{\epsilon_s \to 0}{\longrightarrow} \Psi^{\rm ad}_{\vec{K}}(\vec{r},\vec{R})$$

Austern,et al, Phys.Rep.154(1987)125.

Al-Khalili and Tostevin, in "Scattering" Academic Press 2002, p1373.

Comparison with Faddeev calculations:

Deltuva, Moro, Cravo, Nunes and Fonseca, PRC76(2007)064602

Alt, Blokhintsev, Mukhamedzhanov and Sattarov, PRC75(02007)054003.

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Many-body theory of *A(d,p)B*

$$T_{d,p} = \langle \chi^{(-)}_{pB} \mid V_{np} \mid \Psi_{\vec{K}_{dA}} \rangle$$

(i) Ignore explicit $n + p + A'$ components in $\Psi_{\vec{K}_{dA}}$. See Delaunay,Nunes, Lynch and Tsang PRC72(2005)014610 for $^{12}\mathrm{C}(d,p)^{13}\mathrm{C}$ case.

(ii)Ignore explicit $p + B'$ components in $\chi^{(-)}_{pB}$. See Timofeyuk and Johnson PRC59(1999)1545 for $1/A$ REB effects.

(iii)Ignore identity of proton in deuteron with target protons.

$$T_{d,p} \quad = \quad \langle \chi^{(-)}_{p} \phi^{BA}_{n} \mid V_{np} \mid \Psi_{\vec{K}_{d}} \rangle$$

Nuclear Structure aspects. Overlap functions

Many-body theory of $d + A(N,Z) \rightarrow B(N+1,Z) + p$:

$$\phi_n(\vec{r}_n) \longrightarrow \phi_n^{AB}(\vec{r}_n)$$

where the Overlap Function ϕ_n^{AB} is given by

$$\phi_n^{BA}(\vec{r}_n) = \sqrt{N+1} \int d\xi_A \phi_B^*(\xi_A, \vec{r}_n) \phi_A(\xi_A)$$

The **Spectroscopic Factor** S^{AB} is defined by

$$S^{AB} = \int d\vec{r}_n \mid \phi_n^{AB}(\vec{r}_n) \mid^2$$

Effective 3-body potential (1): a challenge to theory.

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$$H_{eff} = T_R + H_{np} + \langle \phi_A \mid U \mid \phi_A \rangle$$

$$U = (v_{nA} + v_{pA}) + (v_{nA} + v_{pA}) \frac{Q_A}{e} U$$

$$v_{NA} = \sum_{i=1}^{A} v(N,i)$$

$$U = (U_{nA} + U_{pA}) + U_{nA} \frac{Q_A}{e} U_{pA} + U_{pA} \frac{Q_A}{e} U_{nA} + ..$$

$$U_{nA} = v_{nA} + v_{nA} \frac{Q_A}{e} U_{nA}, \quad U_{pA} = v_{pA} + v_{pA} \frac{Q_A}{e} U_{pA}$$

Effective 3-body potential (2)

$$U_{nA} = v_{nA} + v_{nA} \frac{Q_A}{e} U_{nA}$$

where

$$e = (E^+ - T_R - H_{np} - H_A)$$

Approximation:

$$e \approx (1/2E_d^+ - T_n - H_A)$$

$$U_n \approx \nu_{nA} + \nu_{nA} \frac{Q_A}{(1/2E_d^+ - T_n - H_A)} v_{nA} \cdots$$

$$= U_{nA}^{opt}(1/2E_d^+)$$

$$V_{nA}^{opt}(r_n, 1/2E_d) = \langle \phi_A \mid U_{nA}^{opt}(1/2E_d^+) \mid \phi_A \rangle$$

Effective 3-body Hamiltonian(3)

1.Multiple scattering corrections

$$
\begin{aligned}
H = T_R + T_r + V_{np}(r) \quad &+ \quad V_{nA}^{opt}(r_n, 1/2E_d) + V_{pA}^{opt}(r_p, 1/2E_d) \\
&+ \quad \langle \phi_A \mid U_{nA} \frac{Q_A}{e} U_{pA} \mid \phi_A \rangle + (n \leftrightarrow p) + \ldots
\end{aligned}
$$

High energy: corrections suppressed by the weak correlation of n and p in the deuteron

Effective 3-body Hamiltonian(4)

1.Multiple scattering corrections

$$H = T_R + T_r + V_{np}(r) \quad + \quad V_{nA}^{opt}(r_n, 1/2E_d) + V_{pA}^{opt}(r_p, 1/2E_d)$$
$$+ \quad \langle \phi_A \mid U_{nA} \frac{Q_A}{e} U_{pA} \mid \phi_A \rangle + (n \leftrightarrow p) + \ldots$$

2. Pauli blocking. The Bethe-Goldstone equation.

$$H \quad \rightarrow \quad T_R + T_r + Q_F V_{np}(r) + V_{nA}^{opt} + V_{pA}^{opt}$$
$$Q_F \mid n, p \rangle \quad = \quad 0, \quad \text{if } \epsilon_n \text{ or } \epsilon_p < \epsilon_F .$$

Tostevin, Lopes and Johnson, Nucl.Phys. A465(1987)83.

Ioannides and Johnson Phys. Rev. C17 (1978)1331.
Binding energy of a d propagating in nuclear matter.

Binding energy of a 50MeV deuteron in ^{92}Zr in the local nuclear matter approximation (Ioannides and Johnson 1978)

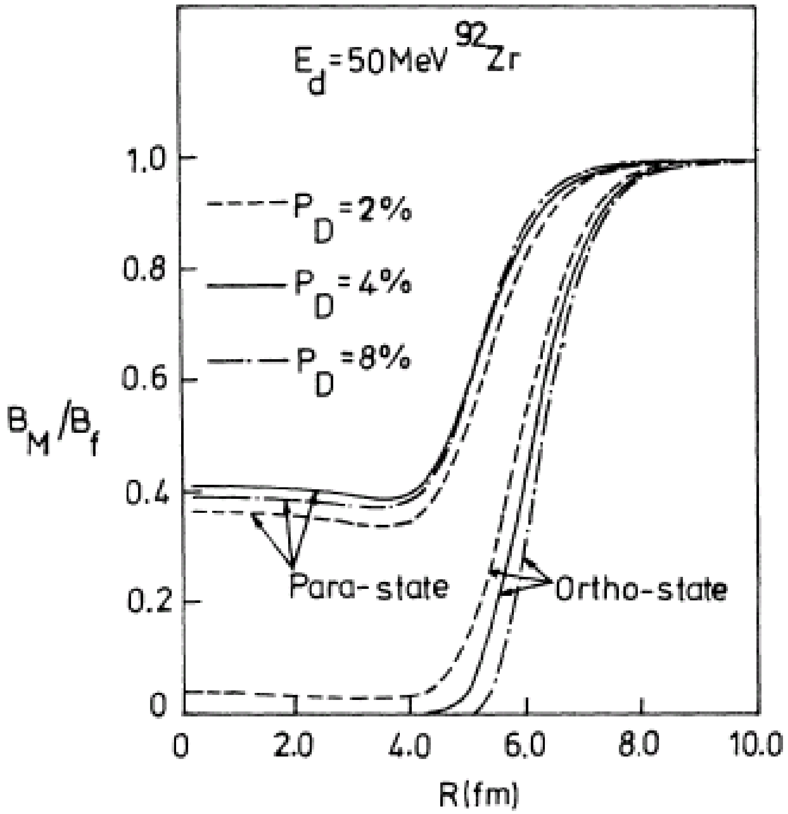

FIG. 9. The ratios of the binding energy of the deuteron in a finite nucleus to the free-space binding energy, for the ortho state and para state, for various D-state probabilities at E_d = 50 MeV.

Ortho: M=0

Para: |M|=1

Deuteron break-up effects

1. In 1970 Johnson-Soper and Harvey-Johnson gave a simple prescription for taking into account coherent , `entangled', effects of d break-up on (d,p) reactions. Need for Deuteron optical potentials disappears. Only need NUCLEON optical potentials at several energies. No change in the way nuclear structure parameters (overlap functions, spectroscopic factors, ANC's) appear in the theory.
2. H-J were able to explain some outstanding discrepancies between old DWBA calculations and experiment. New theory could use existing codes and was simple to implement.

3.J-S sought to justify their assumptions (`adiabatic') by applying the same ideas to 20 MeV elastic d scattering, with some success. (See Chau Huu-Tau, Nucl.Phys.A773(2006)56;A776(2006)80 for CDCC developments.)

4.Johnson and Tandy introduced a new approach which made clear that the adiabatic assumption was not a necessary condition for the validity of the J-S method for <u>transfer.</u> Implemented by Laid, Tostevin and Johnson.
5.For application to modern low energy experiments (GANIL, MSU) the big question is `What is the correct 3-body Hamiltonian?'
6.Validity of adiabatic approximation for elastic scattering investigated by comparison with CDCC calculations. Generalised to 3-body projectiles.

Concluding Remarks

1. “Adiabatic” approximation. Must distinguish between

(a) Stripping and pick-up.

Johnson and Soper. Johnson and Tandy;

Laid, Tostevin and Johnson, PRC(1993)1307.

(b) Elastic Scattering.

Summers, Al-Khalili and Johnson, PRC66(2002)014614.

Johnson and Soper:

2. Implementation needs optical potentials for constituents, not composite system.

3. CDCC is very complicated to implement:. It needs excited continuum state wavefunctions.

4-body CDCC: Matsimoto, et al, PRC70,(2004)061601, et al,

4-body adiabatic: Christley, et al, Nucl.Phys.A624(1997)275.

Concluding Remarks

1. 3-body aspects of (d,p).
    DWBA does not work.
    3-body wavefunction needed within V_{np} only.
    Adiabatic approximation is sufficient but not necessary condition for the J-S distorting potential to be valid for transfer.
    Evidence for validity at low energy(4 MeV/A).
    Need to extend work of Laid et al PRC48(1993)1307 on Johnson-Tandy expansion to low energy.
    Validity of adiabatic approximation for elastic d scattering is a separate question. (See Summers, et al, PRC66(2002)014614).

2. Overlap functions.
    What properties of this quantity are measured in any one (d,p) experiment?
    See Nunes,et al, PRCC72(2005)017602, C75(2005)024601.

3. What is the effective 3-body Hamiltonian at low energy?
    Multiple scattering effects?
    Effective V_{np} ?   Pauli blocking?

4. To deduce reliable nuclear structure information we need a co-ordinated programme of transfer and <u>relevant</u> nucleon elastic and inelastic scattering measurements, including polarization variables.