TALENT Course 6: Theory for exploring nuclear reaction experiments
Exercises: Fusion of heavy-ion projectiles and coupled channels effects
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Analytical/Mathematical exercises

1. Show that the following pair of coupled radial equations, for a degenerate two-channel problem each with orbital angular momentum L, and where the channels are coupled by the interaction $F(R)$, can be decoupled by taking suitable linear combinations of the equations and/or solutions

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + V_L(R) - E\right]\begin{pmatrix} \phi_1(R) \\ \phi_2(R) \end{pmatrix} = -\begin{pmatrix} 0 & F(R) \\ F(R) & 0 \end{pmatrix}\begin{pmatrix} \phi_1(R) \\ \phi_2(R) \end{pmatrix}.$$

   If the incident particles are initially in state 1, and thus the incident scattering waves in $\phi_1(R)$, express the transmission probability $P_L(E)$ through the potential barrier (represented by $V_L(R)$) in terms of the S-matrix elements of both the uncoupled solutions (denoted ϕ^+ and ϕ^-) and the original coupled solutions ϕ_1 and ϕ_2.

Computational exercises on heavy-ion fusion

The program `ccfull` (of Hagino, Rowley and Kruppa) implements a quantum mechanical coupled-channels calculation of the barrier-passing fusion model. The home page for these theoretical tools is at http://www.nucl.phys.tohoku.ac.jp/∼hagino/ccfull.html

The executable `ccfull` provided here has been modified so as to also estimate the *distribution of barriers* function, $D(E) = d^2[E\sigma_{fus}(E)]/dE^2$, based on the calculated fusion cross sections. This is written to the file `barr_dist`. Input data for `ccfull` must be provided in a data set called `ccfull.inp`.

1. (a) Construct a data set for `ccfull` to calculate the fusion observables for the ^{16}O + ^{144}Sm (with $Z = 62$) system. Assume a real ion-ion nuclear potential geometry with radius and diffuseness parameters $r_0 = 1.10$ fm and $a_0 = 0.75$ fm. Assume, at first, that the projectile and target are inert and that there are no excitations included. Having established the potential well depth that reproduces the experimental cross section data (see `sigma.16O+144Sm.dat`) at the higher energies, calculate the fusion cross section (output to `cross.dat`) and the barrier distribution over a range of centre-of-mass energies from sub- to above-barrier energies, e.g. from 50 – 70 MeV.

2. Generalise your data set to allow dynamical excitation of the ^{144}Sm target in the collision, assuming the (single octupole phonon, $\lambda = 3$, vibrational) coupling from the 0^+ ground-state to the 3^-(1.81 MeV) excited state with an octupole strength

$\beta_3 = 0.20$. Study the effects of including this excitation on the fusion cross section and associated barrier distribution. You should assume a coupling interaction radius parameter for the target, `RT`, of $r_t = 1.06$ fm.

3. Compare this 3^-(1.81 MeV) excitation barrier distribution with that obtained if the 3^- state was assumed degenerate with the ground state (the adiabatic, small excitation energy limit) discussed in the lecture.

4. Compare the results from exercise 2 above, i.e. the more realistic vibrational excitation case, with what is obtained if one assumes a *rotational* spectrum and excitation of ^{144}Sm (as is the case for ^{154}Sm). It is suggested that you include excitations to the 2^+, 4^+ and 6^+ excited states, `NROTP=3`, assuming only a quadrupole strength, i.e. $\beta_2 = 0.3$, $\beta_4 = 0$, and a $E(2^+) \approx 0.1$ MeV. It follows therefore that we assume that the $J^\pi = 4^+, 6^+$ state energies are $E(J^+) = J(J+1)E(2^+)/6$.

Experimental data for the ^{16}O + ^{144}Sm and ^{16}O + ^{154}Sm reactions (courtesy M. Dasgupta, ANU) can be found in the files:
for the cross sections, `sigma.16O+144Sm.dat`, `sigma.16O+154Sm.dat`
for barrier distributions, `dist.16O+144Sm.dat`, `dist.16O+154Sm.dat`.