

Q1.

$$P_L(E) = \frac{1}{2}(1 - |S^+|^2) + \frac{1}{2}(1 - |S^-|^2)$$

$$= \frac{1 - \sum_{i=1,2} |S_i|^2}{2}$$

$$\Phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2) \Rightarrow \frac{1}{\sqrt{2}}(H^- - S_1 H^+ - S_2 H^+)$$

potential $V_L + F$

$$= \frac{1}{\sqrt{2}}(H^- - S^+ H^+)$$

$$\Phi^- = \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) \Rightarrow \frac{1}{\sqrt{2}}(H^- - S_1 H^+ + S_2 H^+)$$

potential $V_L - F$

$$= \frac{1}{\sqrt{2}}(H^- - S^- H^+)$$

$$\frac{1}{2}(1 - |S^+|^2) + \frac{1}{2}(1 - |S^-|^2)$$

$$= \frac{1}{2} \{ 1 - (S_1 + S_2)(S_1^* + S_2^*) \} + \frac{1}{2} \{ 1 - (S_1 - S_2)(S_1^* - S_2^*) \}$$

$$= 1 - \{ S_1 S_1^* + S_2 S_2^* + \cancel{S_2 S_1^*} + \cancel{S_1 S_2^*} + S_1 S_1^* + S_2 S_2^* - \cancel{S_2 S_1^*} - \cancel{S_1 S_2^*} \} / 2$$

$$= 1 - |S_1|^2 - |S_2|^2 = \frac{1 - \sum_{i=1,2} |S_i|^2}{2}$$

Coupling matrix (diagonalize)

$$\left\{ \begin{array}{l} F \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} \Rightarrow \lambda = \pm 1, \\ \text{eigenvalues} \end{array} \right. \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

eigenvectors

$$\left\{ \begin{array}{l} \hookrightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} F \end{array} \right.$$