Hands-on session on capture cross sections

Excel program talent_cap.xlsm

The excel programme contains two sheets

1. *Coul*: computes Coulomb and Whittaker functions
2. *Potential*: computes (numerically, with the Numerov algorithm) the wave functions from the Schrödinger equation
$$-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}\right)u_\ell+(V_C(r)+V_N(r))u_\ell=Eu_\ell$$
The nuclear potential is parametrized by a Gaussian form factor
$$V_N(r)=V_0\exp(-(r/r_0)^2)).$$
The capture cross sections are determined from the wave functions of bound and of scattering states.

Inputs:

- masses, charges (B1, D1, B2, D2).
- rmax and step (G1 and G2) are used to compute the wave functions by the Numerov algorithm.
  Typical values: rmax=20 fm, step=0.1 fm.
- Angular momentum ℓ (row 5), range r_0 (row 7) and depth V_0 (row 6).
- For bound states: the number of radial nodes (B8).
  For scattering states: energies (G8,H8,I8, etc). Several energies can be considered simultaneously (row 8, from column G).
- For radiative capture: order of the multipole λ (J1).

Outputs

For bound states:

- Energy (negative) in B10.
- Total potential (nuclear + Coulomb + centrifugal) and wave functions $u_\ell(r)$ from B15 and C15.

For scattering states

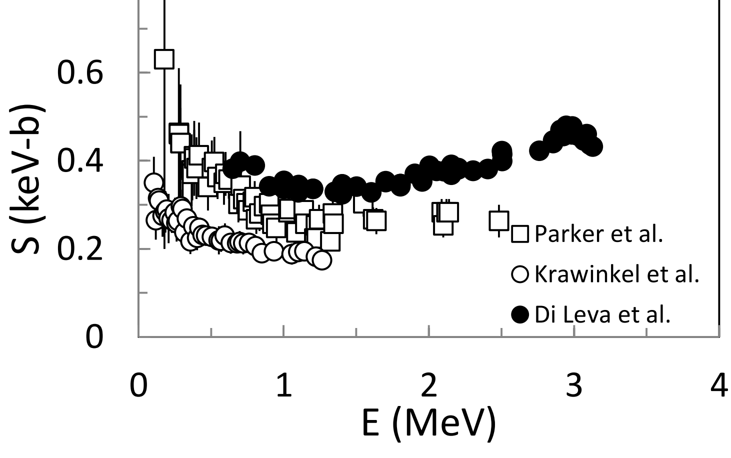
- Phase shifts in radians (G10, H10, etc).
- Capture cross section in fm^2(G11,H11, etc).
- S factor in MeV-b (G12, H12, etc).
- Potential and wave functions $u_\ell(r)$ (from F15 and G15).

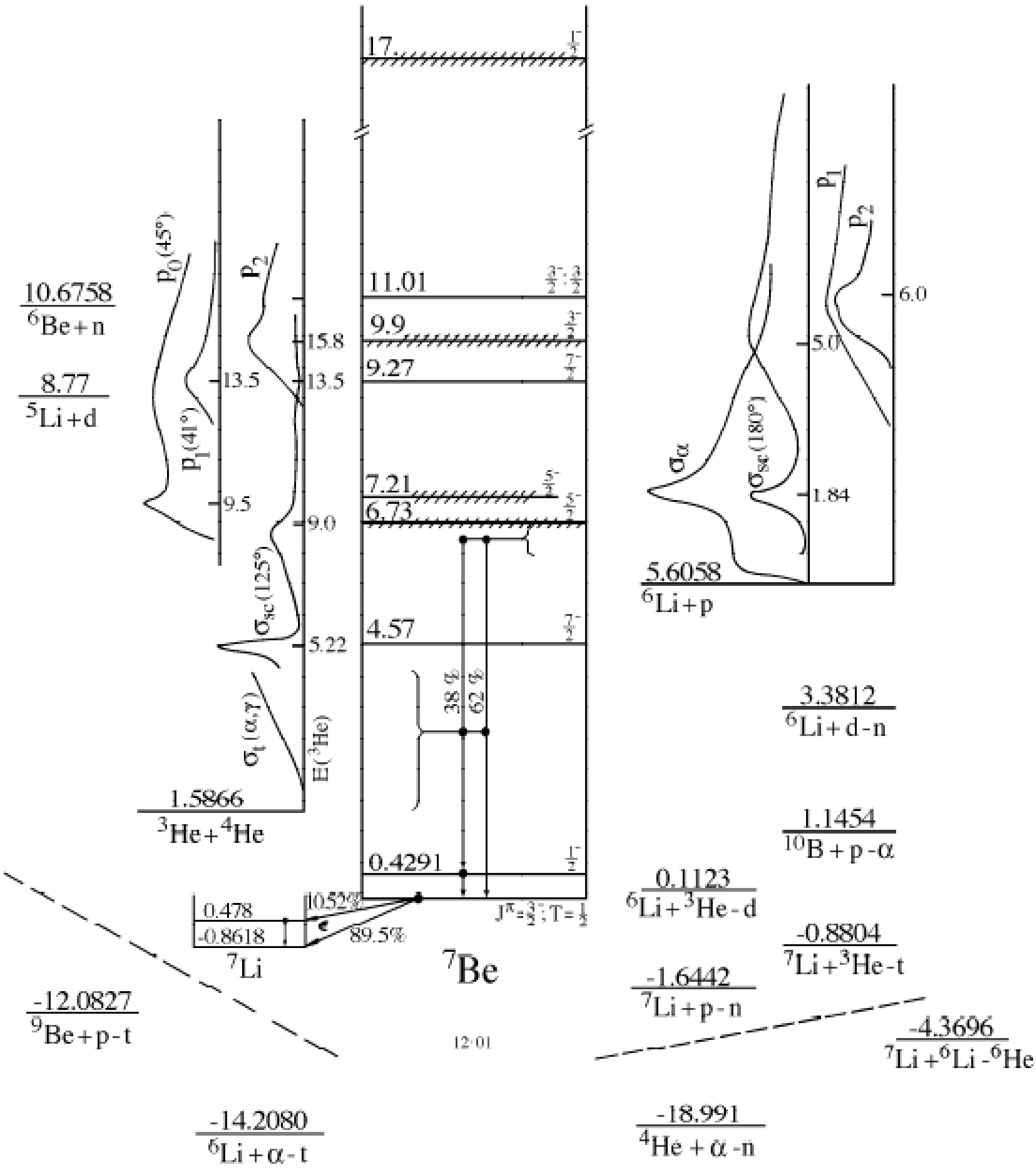
Questions

Consider the ^{3}He(α,γ)^{7}Be reaction. The ground state is $J_f = 3/2^-, \ell_f = 1$, and is bound by -1.59 MeV (see spectrum).

1. By using the sheet "Potential" and $r_0 = 2.5$ fm (Nradial=1), determine the depth V_0 to reproduce the experimental binding energy. Plot the corresponding wave function, and check (by a numerical integration) the normalization $\int_0^\infty u_\ell(r)^2 dr = 1$.
2. The Asymptotic Normalization Constant (ANC) of a bound state is defined by
$$u_f(r) \to C W_{-\eta_B,\ell+1/2}(2k_B r)$$
where $W_{-\eta_B,\ell+1/2}(x)$ is the Whittaker function, and where $k_B$and η_B refer to the bound state.

   With the exact wave function and the Whittaker function given by sheet "Coul", determine the ANC of the ^{7}Be ground state.
3. What is the expected dominant multipolarity λ? Which partial wave is expected to be the most important? Check that the electromagnetic selection rules are satisfied.
4. Using the same potential, determine the scattering wave function at $E = 0.1$ MeV (for $\ell = 0$), and plot the integrant $u_i(r)u_f(r)r^\lambda$. Give an estimate (in %) of the external contribution.
   Increase and decrease the binding energy (by modifying the depth of the potential), and see how this maximum moves.
5. Compute and plot the S factor up to 1 MeV. Compare with experiment (see figure below, the data are given in sheet he3ag). Use different discretizations for the initial and final wave functions, and check the stability of the S factor.




17.
11.01
9.9
9.27
7.21
6.73
4.57
0.4291
$J^{\pi}=\frac{3}{2}^-;T=\frac{1}{2}$
38 %
62 %
^{7}Be
12·01
10.6758
^{6}Be+n
8.77
^{5}Li+d
1.5866
^{3}He+^{4}He
$p_0(45°)$
$p_1(41°)$
p_2
$\sigma_{sc}(125°)$
$\sigma_t(\alpha,\gamma)$
E(^{3}He)
13.5
9.5
15.8
9.0
5.22
0.478
-0.8618
^{7}Li
10.52%
89.5%
-12.0827
^{9}Be+p-t
-14.2080
^{6}Li+α-t
5.6058
^{6}Li+p
σ_α
$\sigma_{sc}(180°)$
p_1
p_2
6.0
5.0
1.84
3.3812
^{6}Li+d-n
1.1454
^{10}B+p-α
0.1123
^{6}Li+^{3}He-d
-0.8804
^{7}Li+^{3}He-t
-1.6442
^{7}Li+p-n
-4.3696
^{7}Li+^{6}Li-^{6}He
-18.991
^{4}He+α-n