Q2. $\tan\delta = \Gamma/2 / (E_r - E_{cm})$

e.g. $\cos^2\delta + \sin^2\delta = 1 \quad (\div \sin^2\delta)$

$$\cot^2\delta + 1 = 1/\sin^2\delta$$

$$\frac{(E_r - E_{cm})^2}{(\Gamma/2)^2} + 1 = \frac{(E_r - E_{cm})^2 + (\Gamma/2)^2}{(\Gamma/2)^2} = \frac{1}{\sin^2\delta}.$$

$$|T_\ell|^2 = |e^{i\delta}\sin\delta|^2 = \sin^2\delta$$

$$\therefore |T_\ell|^2 = \frac{(\Gamma/2)^2}{(E_{cm} - E_r)^2 + (\Gamma/2)^2}.$$

---

e.g. $$S_\ell = e^{2i\delta_\ell} = e^{i\delta_\ell}/e^{-i\delta_\ell}$$

$$= \frac{\cos\delta + i\sin\delta}{\cos\delta - i\sin\delta} = \frac{1 + i\tan\delta}{1 - i\tan\delta} = \frac{1 + i(\Gamma/2)/(E_r - E_{cm})}{1 - i(\Gamma/2)/(E_r - E_{cm})}$$

$$S_\ell = \frac{E_{cm} - E_r - i\Gamma/2}{E_{cm} - E_r + i\Gamma/2}$$

Denominator vanishes when $E_{cm} = E_r - i\Gamma/2$