TALENT Course 6: Theory for exploring nuclear reaction experiments
Exercises: One- and two-particle overlaps
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Analytical/Mathematical exercises

1. Assuming the coefficients of fractional parentage for the lowest seniority states, that neutrons occupy the $\nu 1f_{7/2}$ orbital, that we have closed $N, Z = 20$ shells of neutrons and protons, i.e. the neutron-rich calcium isotopes, calculate the expected *spectroscopic factors* for the (p, d) neutron transfer or nucleon knockout reactions from the $^{48,47,46,\ldots 41}$Ca systems based on the independent particle model (IPM).

   The coefficients of fractional parentage are, for a seniority = 1 final state

   $$((j^{n-1})j, j; 0|\}(j^n)\, 0) = 1, \quad n = \text{even},$$

   for a seniority = 0 final state

   $$((j^{n-1})0, j; j|\}(j^n)\, j) = \left(\frac{2j + 2 - n}{n(2j + 1)}\right)^{\frac{1}{2}}, \; n = \text{odd},$$

   and for a seniority = 2 final state

   $$((j^{n-1})J, j; j|\}(j^n)\, j) = -\left(\frac{2(n-1)(2J + 1)}{n(2j - 1)(2j + 1)}\right)^{\frac{1}{2}}, \; n = \text{odd}, \; J \neq 0 \text{ and even}.$$

2. The very neutron-rich carbon isotope ^{20}C has $N = 14$ and is assumed to have a closed core of neutrons, with $N = 8$, and of protons with $Z = 6$. We will assume that the configurations of the six least bound sd-shell neutrons are either:

   (a) $|\ldots \nu(1d_{5/2})^6; 0^+\rangle$,

   (b) $|\ldots \nu(1d_{5/2})^4, (2s_{1/2})^2; 0^+\rangle$,

   and that the $1d_{5/2}$ and $2s_{1/2}$ levels are essentially degenerate, $E(1d_{5/2}) \approx E(2s_{1/2})$.

   Calculate the relative cross sections expected for the population of the 0^+ ground state of ^{18}C in the (p, t) two-neutron pick-up reaction. You should use the independent particle model and the two-particle coefficients of fractional parentage for the nuclear structure amplitudes.

   You should calculate these relative cross sections for:
   (i) case (a) above, with the removal of a $(1d_{5/2})^2$ neutron pair,
   (ii) case (b) above, and the removal of a $(1d_{5/2})^2$ neutron pair,
   (iii) case (b) above, and the removal of a $(2s_{1/2})^2$ neutron pair,
   assuming the nucleon single-particle wave functions are described by 3D harmonic oscillator states.

The two-particle coefficients of fractional parentage for the lowest seniority states, for removal of two neutrons from a single j shell are:

$$((j^{n-2})v = 0\,0, (j^2)0|(j^n)0) = \left[\frac{2j+3-n}{(n-1)(2j+1)}\right]^{1/2},$$

and for $J \neq 0$ (and even),

$$((j^{n-2})v = 2\ J, (j^2)J|(j^n)0) = \left[\frac{2(n-2)}{(n-1)} \times \frac{(2J+1)}{(2j-1)(2j+1)}\right]^{1/2},$$

where n is even and v is the seniority of the final state.

You can compute 9-j angular momentum coefficients (if needed) using the tool at http://personal.ph.surrey.ac.uk/~phs3ps/cleb.html