1. Consider a simple two-body model for a one-neutron halo nucleus. For E1 transitions, the electric transition probability is given by

$$\frac{dB(E\lambda)}{dE_c} = \left(\frac{4\pi}{k}\right)^2 \frac{\mu k}{(2\pi)^3\hbar^2}\frac{3}{4\pi}(e^{E1}_{eff})^2 \left|\langle \ell_i 010|\ell_f 0\rangle \int dr\, r\phi_f(k,r)\,\mathcal{A}\phi_i(r)\right|^2 \tag{1}$$

where μ is the reduced mass o the two-body system and e^{E1}_{eff} is the effective charge which, for E1 transitions in a neutron+core system is given by $e^{E1}_{eff} = Ze/A$. The function $\phi_i(r)$ represents the radial part of the ground-state wavefunction, normalized to unity according to:

$$\int_0^\infty |\phi_i(r)|^2 dr = 1 \tag{2}$$

and $\phi_f(k,r)$ is the radial part of the positive-energy state with energy $E_c = \hbar^2k^2/2\mu$ and asymptotic behaviour

$$\phi_f(k,r) \xrightarrow{r\gg} F_\ell(k,r) \tag{3}$$

The constant factor $\mathcal{A}$ can be interpreted as a spectroscopic amplitude multiplying the ground state single-particle wavefunction.

(a) Show that, in the special case in which the final states are approximated by plane-waves, the $B(E1)$ distribution is related to the Fourier transform of the ground state wavefunction. Give arguments to justify that, in the case of weakly bound systems, the $B(E1)$ distribution is mostly sensitive to g.s. wavefunction at large neutron-core separations.

(b) In the situation above, the ground state wavefunction can be approximated by its asymptotic form which, for a s-wave configuration, can be written as

$$\phi_i(r) \simeq \sqrt{2k_0}e^{-k_0 r}. \tag{4}$$

Recalling the general asymptotic form of a bound-state wavefunction, give the expression for k_0 in terms of the neutron separation energy (S_n) and the reduced mass of the neutron-core system. Compute it numerically for the ^{11}Be (^{10}Be+n) case.

(c) Show that, under these approximations, the $B(E1)$ distribution is given by

$$\frac{dB(E1)}{dE_c} = \mathcal{A}^2\frac{3\hbar^2}{\pi^2\mu}\left(\frac{Ze}{A}\right)^2 \frac{\sqrt{S_n}E_c^{3/2}}{(E_c+S_n)^4}. \tag{5}$$

Hint: Recall the asymptotic behaviour of the regular Coulomb function $F_\ell(k,r)$ in the no-Coulomb case ($\eta = 0$) and use $\int_0^\infty dr\, r^2 j_1(br)e^{-ar} = 2b/(a^2+b^2)^2$

(d) Show that the maximum of this $B(E1)$ distribution is located at $E_c = \frac{3}{5}S_n$.

(e) In the work of Fukuda *et al*, PRC70, 054606, 2004 (available at the TALENT website) the total $B(E1)$, integrated along the full continuum spectrum, is estimated to be $B(E1) = 1.17$ e^{2}fm^2. From this value, determine the constant $\mathcal{A}$.

(f) Using the extracted parameters, compute the values of dB/dE_c in a uniform energy grid, from $E_c = 0.01$ MeV to 5 MeV, and store these values in tabular form in a text file (you can use XMGRACE for this purpose). Using the code EPM, calculate the differential cross section, $d\sigma/d\Omega$. Compare with Fig. 3 of the work of Fukuda *et al.* Calculate also the differential cross section $d\sigma/dE_c$ integrated for $\theta < 6°$ and $\theta < 1.3°$. Compare with Fig. 2a of Fukuda *et al.*

N.b.: For a better comparison with the data, you can smear the energy differential cross section with the experimental energy resolution, which is done using Gaussian functions with $\sigma = 0.19\sqrt{E_c}$. In the EPM code this achieved defining `foldtype=1`, `resfact=0.19` in the `egrid` namelist.

(g) Using the EPM code, calculate the number of equivalent photons as a function of the continuum energy (dn/dE_c). Evaluate this number at the maximum of the $B(E1)$ distribution for incident energies $E_{\rm lab}$=69 MeV/u and $E_{\rm lab}$=35 MeV/u. In view of the obtained results, which energy would be more appropriate for a Coulomb dissociation experiment?

Note: The data from PRC70, 054606, 2004 can be also obtained from `http://www.nndc.bnl.gov/exfor/servlet/X4sGetSubent?reqx=60546&subID=141915002&plus=1`

2. Consider the capture reaction ^{14}C(n,γ)^{15}C and its inverse, the photo-dissociation ^{15}C(γ,n)^{14}C. Using the $B(E1)$ distribution quoted in Fig. 2 of Phys. Rev. C 79, 035805 (2009), estimate the ratio of cross sections of these processes for $E_{\rm c.m} = 0.5$ and 1.0 MeV. Comment on the results.