Nuclear Spectroscopy & Reactions (D)

# IX.E ONE- AND TWO-NUCLEON TRANSFER REACTIONS

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## I. Introduction

Most experiments on particle transfer reactions are analyzed by means of the distorted wave Born approximation (DWBA) treatment of the reaction mechanism. The way in which this analysis is carried out, in practice, to obtain information on the structure of nuclei is discussed by Macfarlane and Schiffer in Chapter IV.B.2. This chapter is intended to describe the physical content as well

as the assumptions underlying this treatment of the reaction. It is not intended to be a review of the theory, nor does it provide derivations except when needed to support its purpose.

In recent years, alternative treatments of the reaction mechanism have been proposed. They are intended to deal with one of the shortcomings of the DWBA, and some applications of these theories have been made (Pearson and Coz, 1966; Butler *et al.*, 1967). However, it is probably fair to say that further evaluation of these alternatives is needed (Nagarajan, 1972).

The plan of this article is to describe first the simplest form of the DWBA theory for (d,p) reactions together with a discussion of the basic assumptions. The DWBA itself is undergoing constant refinements, the precise merit of which in some cases is yet to be decided. Improvements to the simple DWBA theory consist of two kinds. There are certain improvements which can or have been incorporated straightforwardly into existing computer programs and thus are readily accessible to the experimenter interested in analyzing his data for its nuclear structure information. These are discussed in Sections III and IV. The other type of improvements are those that are so intimately bound up with the nuclear structure and reaction mechanism (Sections V and VI) that a large theoretical and computational effort is first required before one can proceed to an analysis of experimental data. This class of improvements is therefore beyond the scope of routine analysis at the present time.

The two-nucleon transfer reaction is treated in Section VII. The discussion is brief since in many cases it would otherwise parallel that given for the (d,p) reaction.

## II. Simplest Form of DWBA for (d,p) Reactions

### A. Basic Assumptions Underlying DWBA

For definiteness we shall consider the (d,p) reaction, since it is the most extensively studied of the one-nucleon transfer reactions. An exact expression for the transition amplitude for the reaction A(d,p)B may be written down as the starting point for our discussion

$$T = \int \psi_p^{(-)*}(\mathbf{r}_p)\langle \Phi_B(A+1)|\, V_{pn} + V_{pA} - U_p \,|\Psi_{dA}^{(+)}(A, \mathbf{r}_n, \mathbf{r}_p)\rangle d\mathbf{r}_n\, d\mathbf{r}_p \quad (1)$$

The matrix element denotes integration over the coordinates of the $A$ nucleons of the target. In this expression, $\Psi_{dA}$ denotes an exact solution to the problem involving an incident deuteron on the target nucleus $A$, and as such, it contains information on all the processes that can be involved, such as elastic and inelastic scattering, breakup of the deuteron, the (d,p) reaction itself, as well as all other reactions and the distortion of the deuteron as it moves close to or in the nucleus. Of course, this ideal wave function is not available, but its presence in the exact amplitude gives us some insight into possible deficiencies in the approximations that are introduced to obtain a tractible expression.



The distorted wave Born approximation is obtained when the exact function $\Psi_{dA}(A, \mathbf{r}_n, \mathbf{r}_p)$ is replaced by a product of wave functions of the target ground state $\Phi_A(A)$, the deuteron ground state $\phi_0(r)$, and a function $\psi_d^{(+)}(R)$ describing the *elastic* scattering of the deuteron by an optical potential representing its interaction with the target nucleus

$$T = \int \psi_p^{(-)*}(\mathbf{r}_p)\langle \Phi_B(A+1)|\, V_{pn} + V_{pA} - U_p \,|\Phi_A(A)\phi_0(\mathbf{r})\rangle$$

$$\times \psi_d^{(+)}(\mathbf{R})\, d\mathbf{r}_n\, d\mathbf{r}_p \quad (2)$$

Here $\mathbf{r}$ and $\mathbf{R}$ are the relative and center-of-mass coordinates of the deuteron and are illustrated later. The other ingredients are $\psi_p^{(-)}$, a wave function describing the elastic scattering of the outgoing proton by an optical potential $U_p$, $\Phi_B(A+1)$, a wave function describing the residual nuclear state, and the interactions $V_{pn}$ between the proton and neutron and $V_{pA}$ between the proton and the remaining $A$ nucleons.

It is customarily argued that $V_{pA}$ and $U_p$ approximately cancel each other in (2). The starting point for DWBA calculations is then

$$T = \int \psi_p^{(-)*}(\mathbf{r}_p)\langle \Phi_B(A+1)|\, V_{pn}(\mathbf{r})\,|\Phi_A(A)\phi_0(\mathbf{r})\rangle\, \psi_d^{(+)}(\mathbf{R})\, d\mathbf{r}_n\, d\mathbf{r}_p \quad (3)$$

The expression is intuitively pleasing because it has the form of a matrix element between unperturbed initial and final states of the system. However, there are three basic assumptions underlying this expression.

First, it assumes that the transfer takes place directly from the target state to the final state by the simple deposit of the neutron. Since $V_{pn}$ does not depend on the $A$ coordinates of the target, according to this expression the reaction can take place only to the extent that they are in the same state of motion in the final nucleus; otherwise the matrix element would vanish. Thus, processes in which some of the $A$ nucleons are excited through inelastic scattering of the deuteron or proton are excluded from (3) although as remarked earlier, they are present in the original expression (1). Intuitively one feels that such processes should not be too important when $\Phi_B(A+1)$ is a state in which the $A$ nucleons are predominantly in the same arrangement as in $\Phi_A(A)$, as one expects would be the case for low-lying states in $(A+1)$. For weakly populated states as well as higher lying states, such higher order processes are likely to be much more important. Second order effects are discussed in Chapter IV.B.2 and the theoretical treatment of such processes is discussed later in this chapter.

A second assumption underlying (3) concerns the distorted waves $\psi_p$ and $\psi_d$. In practice, they are chosen to be wave functions of optical potentials whose parameters are chosen to reproduce the elastic cross sections. (For a discussion of the optical potential refer to Perey, Chapter IV.B.1.) The essential point here is that the elastic scattering data determine the wave function in that channel *outside* the region of the nucleus. Any wave function which has the same phase shifts at large distance yields the same elastic cross section. Since there are ambiguities and uncertainties in the optical model parameters, there are corresponding uncertainties in the wave functions in the nuclear region, just where they are needed in (3). Aside from this point, the meaning of the wave function inside the nucleus is open to question, especially for composite particles which may be distorted or broken up by the nuclear field yet still contribute to the reaction (Pearson and Coz, 1966; Butler *et al.*, 1967; Johnson and Soper, 1970).

A third assumption is that the (d,p) reaction is weak so that it may be treated in first order. While this assumption may usually be valid, Rawitscher and Mukherjee (1969) have pointed out that some (d,p) cross sections are unusually large. This implies that after the neutron is stripped it can be picked up again to form a deuteron. Thus there is coupling between the deuteron channels and the (d,p) channel which may not always have negligible effect. Usually, however, the (d,p) cross sections are smaller than enhanced inelastic scattering cross sections. Thus, the inelastic processes, especially those involving collective states, are likely to be generally the more important of the higher order corrections to the DWBA.

## B. Zero-Range Approximation

The DWBA expression for the transition amplitude involves a 6-fold integration over $\mathbf{r}_n$ and $\mathbf{r}_p$ after the integration of the nuclear coordinates $A$. How the 6-fold integration can be handled has been discussed by Austern *et al.* (1964) and Sawaguri and Tobocman (1967). However, it is usually reduced to a 3-fold integration by assuming that the product

$$D(\mathbf{r}) = V_{pn}(\mathbf{r})\phi_0(\mathbf{r}) \tag{4}$$

has zero range. In almost all applications, the zero-range approximation is made, although in some few cases the 6-fold integration has been performed (Drisko and Satchler, 1964). Generally speaking, the cross section computed with a finite-range interaction is slightly smaller than that computed in zero range, but the angular distribution is little modified. An approximate way of accounting for the finite range of $D(r)$, which is a simple modification of the zero-range expression for $T$, has been found and is discussed in Section III.



So if $D(\mathbf{r})$ is chosen to have zero range,

$$D(\mathbf{r}) = V_{pn}(\mathbf{r})\phi_0(\mathbf{r}) = D_0\delta(\mathbf{r}) \tag{5}$$

An appropriate value of $D_0$ can be found by integrating this equation over $\mathbf{r}$ and using a Hulthén function for $\phi_0$ and the corresponding potential $V_{pn}$. This yields (see Lee *et al.*, 1964)

$$D_0 = \int d\mathbf{r}\, r^2 V_{pn}(\mathbf{r})\phi_0(\mathbf{r}) \cong (1.5 \times 10^4 \quad (\text{MeV}^2\ \text{fm}^3)^{1/2} \tag{6}$$

An alternate approach using effective range theory yields about the same value (Austern, 1962).

We may note at this point that the deuteron has been assumed to exist purely in the S state. The D state contribution is under investigation by several authors (Johnson and Santos, 1967; Delic and Robson, 1970) and probably does not alter the cross section greatly, but it does appear to be more important for stripping into states of higher angular momentum.

For other single-nucleon transfer reactions, such as ($^3$He,d), a formulation analogous to the above is used. The normalization, $D_0$, has been calculated by Bassel (1966) but it is sometimes determined from experiment by comparison of this reaction with the (d,p) reaction, or by studying levels whose spectroscopic factors (discussed later) one feels confident are known.

Figure 1 shows the coordinates that occur naturally in the problem. In particular, the proton distorted wave coordinate $\mathbf{r}_p$ is measured from the center

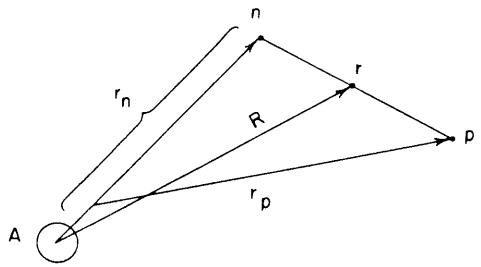


**Fig. 1.** Natural coordinates for (d,p) reaction.

of mass of A + n, in the nucleus B. When the zero-range approximation is made one sees that

$$\mathbf{r}_n \to \mathbf{R}, \qquad \mathbf{r}_p \to A\mathbf{r}_n/(A+1) \tag{7}$$

Recalling that the matrix element in (3) denotes integration over the coordinates of the $A$ nucleons of the target, it remains a function of the neutron

coordinate which we denote by

$$\Phi(\mathbf{r}_n) = \langle \Phi_B(A+1) | \Phi_A(A) \rangle \equiv \int \Phi_B^*(A, \mathbf{r}_n) \Phi_A(A) \, dA \tag{8}$$

Thus (3) becomes, after use of the zero-range approximation,

$$T = D_0 \int \psi_p^{(-)*}\left(\frac{A\mathbf{R}}{(A+1)}\right) \Phi(\mathbf{R}) \psi_d^{(+)}(\mathbf{R}) \, d\mathbf{R} \tag{9}$$

This is the form of the integral evaluated in computer programs that are used in calculating direct reaction cross sections in the DWBA. The same general form, containing a distorted wave for the incident and outgoing particle and a *form factor* $\Phi(\mathbf{R})$ depending on the reaction and nuclear structure, is encountered in all single-step direct reactions.

Although (9) is a three-dimensional integral, two of the integrations, the angles of R, are trivially done when the distorted waves are expanded in partial waves. Then the amplitude (9) is a linear combination of one-dimensional radial integrals.

In applications of this theory, it is often found that better agreement with experimental angular distributions is obtained if the integration on R is started not at the origin but at a "cutoff" radius, so that contributions to the integral from the nuclear interior are rejected. This cutoff radius is, of course, an arbitrary quantity having no definite physical meaning. Fortunately there are several physical effects which tend to reduce the contribution from the nuclear interior as seems to be often required, and these are discussed in Sections III and IV.

### C. Spectroscopic Factor

In case the target nucleus is a closed shell nucleus, it is customarily assumed that the overlap of the $A$ nucleons in (8) is perfect and that the neutron is deposited into a single-particle state $nlj$, with wave function $\psi_{nlj}$, say a product of space and spin functions. Then the nuclear overlap integral is trivial and yields

$$\Phi^*(\mathbf{r}_n) = \psi_{nlj}^m(\mathbf{r}_n) = R_{nlj}(r_n)[Y_l(\theta, \phi) X_{1/2}(\sigma)]_j^m \tag{10}$$

with $j = J_B$. The correct asymptotic behavior for the radial function $R$ in (10) should be $e^{-\alpha r}$ where $\alpha$ is determined from the separation energy of the neutron. Although nuclear structure calculations are often carried out with harmonic oscillator radial functions – because of their convenience when it comes to evaluating the matrix elements that occur in shell model calculations – their asymptotic behavior, which is $\sim e^{-\nu r^2/2}$, is incorrect. While this is of little importance in shell-model calculations, since the main contribution to the matrix elements comes from the interior region, it is of great importance for the (d,p) reaction because a large, if not the main, contribution to (9) comes from the surface region. The weight which is given to (9) from various radii is quite different in the two cases.



There are two ways in which the correct asymptotic behavior of Eq. (10) can be guaranteed. One is to use an oscillator function in the interior which is matched to a Hankel function tail which decays according to the separation energy of the neutron. The other, and more common method nowadays, is to use a wave function corresponding to a Woods–Saxon potential whose parameters are adjusted so that the binding energy in the well is equal to the separation energy.

The notion that the neutron is deposited into a pure single-particle state is an idealization which is approached in only a few cases in real nuclei. Usually only a part of the wave function can be so described, the remaining part having some of the $A$ nucleons in excited states. This could be expressed by writing a parentage expansion

$$\Phi_B(A+1) = \sum_{A'} \sum_{nlj} \beta_{nlj}(A', B)[\Phi_{A'}(A)\psi_{nlj}(\mathbf{r}_n)]_{J_B} \tag{11}$$

where $A'$ is used to denote the various states of the target nucleus. The integration over the $A$ coordinates in (8) picks out only the terms with $A' = A$, yielding, for even targets $A$,

$$\Phi^*(\mathbf{r}_n) = \beta_{nlj}(A, B)\psi_{nlj}(\mathbf{r}_n)\delta_{jJ_B} \tag{12}$$

Again, there is an assumption here that the neutron goes into a single-particle state $nlj$. While the parity of $B$ limits $l$ to one value when $A$ is even, there could in principle be a sum on the radial quantum number $n$ in (12). In case $\Phi$ has the value of Eq. (12), Eq. (9) becomes

$$T = S_{nlj}^{1/2}(A, B) \; D_0 \int \psi_p^{(-)*}\left(\frac{A\mathbf{R}}{A+1}\right) \psi_{nlj}^*(\mathbf{R}) \psi_d^{(+)}(\mathbf{R}) \, d\mathbf{R} \tag{13}$$

where we have written

$$S_{nlj}(A, B) = \beta_{nlj}^2(A, B) \tag{14}$$

This is called the spectroscopic factor and its interpretation is clear from the foregoing. It measures the probability that the nuclear state $\Phi_B$ produced in the (d,p) reaction has its parentage based on the ground state of the target, with a single particle in the shell-model state $nlj$ (Macfarlane and French, 1960).

Of course, it is the interaction between the stripped neutron and the other nucleons in the nucleus which destroys the simple single-particle picture first discussed above and causes the single-particle strength to be distributed over

several, or many, levels. When $\beta$ in Eq. (12) is less than unity it implies that there are a whole group of states of the form (11), which lie in the vicinity of where the single-particle state would lie if the odd neutron did not interact with the others. From the orthogonality properties of the coefficients in (10) a sum rule exists for the fragments of this idealized single-particle state for a closed-shell target

$$\sum_B S_{nlj}(A, B) = 1 \tag{15}$$

As described in Chapter IV.B.2, the integral in (13) is computed typically for a pure single-particle state, and the resulting computed cross section is compared with experiment. The discrepancy is ascribed to the value of $S$. There, also, applications of sum rules are discussed. In more typical cases, $A$ is not a closed-shell nucleus. The general definition of the spectroscopic factor and its calculation for various situations, as well as sum rules, are covered by Macfarlane and French (1960). The main purpose of (d,p) experiments is spectroscopic: the determination of the position, spin and parity of nuclear levels, their spectroscopic factors, and the spreading of the single-particle strength.

## III. Nonlocal and Finite-Range Effects

### A. Effect of Nonlocality of the Optical Potential

In most analyses of elastic scattering a phenomenological optical potential is sought which yields agreement with the data (see Chapter IV.B.1). This optical potential, $U_L(\mathbf{r})$, is usually taken to have a simple local form. By local it is meant that at the point $\mathbf{r}$ the particle feels the potential only at that same point. The Shroedinger equation then reads

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + U_L(\mathbf{r}) - E\right)\psi(\mathbf{r}) = 0 \tag{16}$$

The situation in a real scattering problem is always more complicated than encompassed by this equation. For example, the incident particle can excite the nucleus. The true state vector for the system therefore has many components describing the many things that can happen, and these are coupled to each other (see the coupled equations for scattering) by virtue of the mutual interactions that can connect the various components or channels. Nonetheless, fundamental theory shows that the complicated problem involving many channels can be reduced to a simpler one containing few, or even only the elastic channel, provided that the interaction between the scattered particle and the nucleus is suitably modified. This modified, or effective, interaction is, however, a very complicated object. No really satisfactory calculation of it can be made, and



certainly not an exact one. However, its formal structure is instructive since from it we can learn what properties the optical potential ought to have. Besides, being complex valued (the assumption always carried over to phenomenological optical potentials), the optical potential should depend explicitly on the energy of the system and it should also be nonlocal. The nonlocality means that the term $U_L(\mathbf{r})\psi(\mathbf{r})$ above must be replaced by

$$\int U(\mathbf{r}, \mathbf{r}')\psi(\mathbf{r}')\, d\mathbf{r}' \tag{17}$$

where $U(\mathbf{r}, \mathbf{r}')$ is the nonlocal potential. Thus the wave function at point $\mathbf{r}$ depends on conditions at all other points in the range of the nonlocal potential.

The consequences of a *particular* separable form of nonlocality have been explored by Perey and Buck (1962). Their result is

$$U(r, r') = U_0(1/2 \mid \mathbf{r} + \mathbf{r}' \mid) H(\mathbf{r} - \mathbf{r}') \tag{18}$$

where, in numerical applications, the function $H$ is taken to be Gaussian. While it should be stressed that this simple form does not follow from the formal theory, the general effects and conclusions based on its use are probably sound. After the scattering problem was solved by means of nonlocal interaction [Eq. (18)], the computed cross section was fitted with a conventional local potential $U_L$. Two interesting facts were noted: first, that the equivalent local potential that yielded the same scattering as the nonlocal potential was weaker, $\mid U_L \mid < \mid U_0 \mid$; and second, that in the interior region the wave function of the nonlocal potential, $\psi_{NL}(\mathbf{r})$, was smaller than that of the local potential $\psi(\mathbf{r})$ (the so-called "Perey effect"). In fact, when the form of the nonlocality $H(\mathbf{r} - \mathbf{r}')$ in (18) is taken to be a Gaussian of range $\beta$, then a relationship between these two wave functions can be found such that

$$\psi_{NL}(\mathbf{r}) = F(r)\psi(\mathbf{r}) \tag{19}$$

where $F(r)$ goes to unity in the exterior region (both functions reproduce the same scattering cross sections) but in the internal region $F$ is less than unity and is (see Austern, general references)

$$F(r) = \left\{1 - (\mu\beta^2/2\hbar^2)U_L(r)\right\}^{-1/2} \tag{20}$$

where $\mu$ is the reduced mass. The value of $\beta$ which Perey and Buck found for nucleons to yield a best fit to the data over the energy range studied (up to 24 MeV) and $\beta \simeq 0.85$ fm. With this, and $U_L \sim -40$ MeV, one finds $F(0) \sim 0.75$. Thus the conventional local optical potential yields a wave function which should be decreased by about 25% in the nuclear interior. [Generally only the real part of $U_L$ is used in (19).] Together with a similar factor for the deuteron function, with $\beta_d \simeq 0.54$ fm, the contribution of the interior to the stripping amplitude is reduced by about 40%.

The bound neutron's wave function is usually taken from a local potential also, but in principle it should be the solution to a Hartree–Fock problem which again has a nonlocal potential. Since the overall wave function in the second case must retain its unit normalization, the interior damping must be compensated for by an increase in the magnitude of the wave function in the tail. This could be compensated for by an adjustment in radius of the bound state potential; however, and in general, the correction is not applied to the neutron.

We recognize that the suppression of the interior contribution due to the nonlocality of the optical potential alleviates the problems that are often handled by introducing the cutoff radius mentioned in Section II.B. However for light nuclei the suppression is usually still not sufficient (McAllen *et al.*, 1971). We must keep in mind however that the actual nonlocality of the effective interaction is more complicated than the one [Eq. (18)] on which these results are based. Indeed in another investigation (Schenter, 1967) a much greater damping of the interior wave function was found for nucleon–alpha scattering in which the nature of the nonlocality was closer to what we believe to be implicit in the formal theory. However in this case a simple connection between the ordinary optical model wave function and the nonlocal one, as expressed in Eqs. (19) and (20), does not exist.

At the present stage it appears most attractive to use the damping that can be incorporated in such a simple way into the DWBA integrals by introduction of a damping factor [Eq. (20)] for each of the particle wave functions. It is unfortunate but not surprising, as discussed above, that it may be necessary to introduce a stronger damping than would be implied by use of the Perey–Buck values of the nonlocal range parameter $\beta$ in (20). This is sometimes achieved by using an *ad hoc* greater range for the nonlocality. However, this undesirable arbitrary procedure seems to have been solved by the recent development discussed in Section IV.

Inasmuch as the effect of the nonlocality is to reduce the contribution to stripping that comes from the nuclear interior, the main change produced in the cross section is to reduce the large angle scattering while leaving the forward or peak cross section little changed (Philpott *et al.*, 1968) (grazing or distant collisions lead mostly to forward scattering while close ones, to large angle scattering).

### B. Finite Range

As discussed in Section II, almost all transfer reaction calculations are based on the use of a zero-range interaction which very much simplifies the numerical work in evaluating the cross section. Two schemes for evaluating the amplitude for a finite-range interaction have been devised (Austern *et al.*, 1964; Sawaguri and Tobocman, 1967). A comparison in several instances with the zero-range approximation has been made (Drisko and Satchler, 1964). The zero-range



approximation was found generally to overemphasize contributions coming from the nuclear interior and thus, to overestimate the cross section. The finite-range calculations are much more costly in computer time, and a means has been found of approximating the finite-range theory so that it reduces to the form of the zero-range theory, except that a new radially dependent factor appears in the zero-range integral (9) just as for the nonlocal effects discussed above (Buttle and Goldfarb, 1964). This factor is

$$\Lambda(R) = 1 - (\alpha/\beta)^2 (1/B_d)[U_d(R) - V_n(R) - U_p(R) - B_d] \tag{21}$$

corresponding to a Hulthén wave function for the deuteron

$$\phi_0(r) \propto (1/r)(e^{-\alpha r} - e^{-\beta r}) \tag{22}$$

In these equations $B_d$ is the deuteron binding energy, $U_d$ and $U_p$ are the deuteron and proton optical potentials, and $V_n$ is the potential that binds the neutron. The last potential is real but the first two are complex so that the correction factor $\Lambda(\mathbf{R})$ is complex.

Since it is now generally believed that the deuteron optical potential should be about equal in depth to twice the nucleon optical potential, the square bracket should be small near the origin. The departure of $\Lambda(R)$ from unity then comes mainly in the surface because the potentials for different particles tend to have different geometrical parameters. Unfortunately, however, because of the well known ambiguities in optical model parameters, a corresponding ambiguity in $\Lambda(\mathbf{R})$ exists. Whether this ambiguity is compensated for in the cross section by the distorted waves and bound neutron wave function has not been investigated.

The above approximate method of incorporating finite-range effects has been tested in several cases and found to agree very well with the full calculation when a deep deuteron potential is used (Dickens *et al.*, 1965). Generally this correction factor, along with those accounting for nonlocal effects of the optical potential, are used in modern analyses of (d,p) reactions.

## IV. Adiabatic Model for the Deuteron Optical Potential

It is always possible to find an optical potential that reproduces an elastic cross section, as needed in the original formulation of the DWBA approach to transfer reactions. This fact is most critical for composite particles. In the (d,p) reaction, for example, the effect of distortion and break up of the deuteron in the field of the nucleus is neglected in the usual treatment. Several attempts to remedy this defect have been made (Pearson and Coz, 1966; Butler *et al.*, 1967). The one which fits into the old framework most readily is the adiabatic model for the deuteron optical potential introduced by Johnson and Soper (1970).

They define the deuteron optical potential to be

$$\bar{U}_d(\mathbf{r}) = (1/D_0) \int [U_n(\mathbf{r} + \tfrac{1}{2}\mathbf{s}) + U_p(\mathbf{r} - \tfrac{1}{2}\mathbf{s})] V_{pn}(\mathbf{s})\phi_0(\mathbf{s})\, d\mathbf{s} \tag{23}$$

where $U_n$ and $U_p$ are the nucleon optical potentials corresponding to energies of half the deuteron bombarding energy and $\phi_0$ is the deuteron wave function. They show that the distorted waves generated by this potential contain, in an approximate way, outgoing waves associated with the breakup of the deuteron into low-energy relative $^3S$ states as well as elastic scattering.

This model has been applied so far to selected light, medium, and heavy nuclei and apparently has a distinct advantage over the elastic deuteron optical potential (Harvey and Johnson, 1971; McAllen *et al.*, 1971; Satchler, 1971). The effect produced on the stripping or pickup cross section is to cause a faster fall off with angle and to create stronger oscillations. In each of the cases cited, these results were achieved in earlier analyses by the introduction of a sharp cutoff. Indeed for the heavy nucleus (lead), the cutoff was larger than the nuclear radius, tentatively suggesting that the breakup process is most important for heavy nuclei. This can be understood in terms of the greater Coulomb field experienced by the proton.

The question comes immediately to mind as to how the potential of (23) differs from the usual elastic deuteron optical potential. Since the nucleon potentials are averaged over the short-range function

$$D(\mathbf{s}) = V_{pn}(\mathbf{s})\phi_0(\mathbf{s}) \tag{24}$$

the adiabatic potential $U_d$ differs from the sum $U_n + U_p$ mainly in having a greater diffuseness (assuming the same geometry for $U_n$ and $U_p$). Its radius is about the same as $U_n$ and $U_p$. Perhaps this is the key point, because the radius of the imaginary part of the elastic deuteron potential is usually considerably larger (sometimes 30–40%) than that of the real part, whereas for nucleon potentials they are close to each other. As emphasized by Harvey and Johnson (1971), the adiabatic potential produces a stronger localization in $l$ space than the conventional potential. That is to say, the main contribution to the stripping comes from a narrow band of partial waves. Convenient formulae for approximating (23) have been given in the literature quoted above. The validity of the approximations on which the adiabatic theory is founded improves at high deuteron energies, and this should be kept in mind. The model appears to be very promising and attractive in that the artificial radial cutoff problem appears to be solved. It deserves a great deal of further investigation.

## V. Form Factors Related to Nuclear Structure

In most analyses of single-nucleon transfer reactions it is assumed that the nucleon is picked up or deposited into a shell-model state. The corresponding



wave function is usually taken to be an eigenfunction of a Woods–Saxon potential whose depth is adjusted so that the eigenvalue equals the separation energy. This assumption permits DWBA theory to be applied to the analysis of data in a routine manner. Having made the assumption, one then regards the data and its analysis through the DWBA as a means of picking apart the single-particle structure of the nucleus and determining the spectroscopic amplitudes with which the single-particle strength occurs in the various nuclear states. Without it, the practitioner is caught up immediately in rather difficult and detailed questions concerning the structure of the nuclei involved that go beyond the scope of routine analysis. Indeed, in principle, one needs to know the detailed nuclear wave functions in order to calculate the appropriate form factor. From this angle the emphasis is shifted to testing a model of the nucleus.

We recall that the transition amplitude contains the overlap

$$\Phi(x) \equiv \int \Psi_B(A, x)\Psi_A(A)\, dA \tag{25}$$

If $A$ is a closed shell nucleus one feels intuitively that the lowest few levels in $B$ should correspond very closely to single-particle states. What we mean by this is that the $A$ nucleons occupy the closed shells and the odd neutron occupies one or another of the shell-model levels above the closed shells. This cannot be so in general simply because the states of an odd nucleus are eigenstates of a many-body system and are not in general the states of a potential well. In other words, $\Phi(x)$ is not the eigenfunction of a potential well but satisfies a much more complicated differential equation (Pinkston and Satchler, 1965; Berggren, 1965). It is true, however, that the tail of $\Phi(x)$ is determined by the separation energy, provided this energy is not too big. If the separation energy is so large that the tail decays more rapidly than the nuclear forces, then not even this is true. In any case, the magnitude of the tail, of $\Phi(x)$, as compared to a Woods–Saxon eigenfunction can be different, which would lead to an incorrect spectroscopic factor in the latter case. It has been argued that the difference is greatest for light nuclei. Further, one feels intuitively that it would be greater for weakly populated levels than for strong ones.

An accurate calculation of $\Phi(x)$ is a very difficult problem, as in its entirety it is tantamount to solving the many-body problem. Various approximate methods and schematic models have been tried, sometimes yielding differences with cross sections computed in the usual way by a factor of 2 or 3 (Philpott *et al.*, 1968; Pinkston *et al.*, 1969; McAllen *et al.*, 1971). However the angular distributions are usually very similar. It is difficult to judge therefore whether the large difference in magnitude is real, coming from the many-body effects, or whether it is a result of the approximations made in trying to incorporate them. The situation at present is quite unsatisfactory and a great deal of work has yet to be done on this difficult problem.

At this point the reader may wonder how it was made so plausible in Section

II that the form factor reduces to a single-particle wave function. The point is that the parentage expansion (11) is formally correct, but is really unknown in any practical situation. Even if conventional shell-model calculations were performed for nuclei $A$ and $B$ they would not provide an accurate enough representation of the wave function for the computation of $\Phi(x)$ except for those few states that have a strong single-particle character. Recall that the starting point of a shell-model calculation is the choice of a set of single-particle wave functions which are used as the basis on which the nuclear wave functions are expanded. Since this choice is made in the beginning, the asymptotic region of the wave functions is never correct. This can be seen already for a simple $j^2$ configuration. The residual interaction splits the various states of different angular momentum $j^2(J), J = 0, 2, \ldots$ yet the radial forms of the wave functions are identical.

Finally we add that, if the radial shape of $\Phi(x)$ is strongly altered from the single-particle wave function, then the concept of a spectroscopic factor loses its meaning.

## VI. Higher Order Processes

Recall from our discussion in Section II that the DWBA amplitude is based on the assumption that the only important process is the simple direct deposit of a particle which does not disturb the other nucleons in the target ground state. In general, the states of the nucleus $A + 1$ will each have several or many states of $A$ as parent, but the DWBA takes into account only transitions to the components of the wave function of $A + 1$ which have the ground state of $A$ as parent. It had been believed for a long time that even if the parentage of a state were not pure, the direct single step process leading to the component having the target ground state as parent would account for the cross section. Recent investigations indicate that this assumption is not always true (Glendenning and Mackintosh, 1971). For example, some states in odd nuclei can be described approximately as a single-particle state coupled to a core-vibrational state. Since these latter states have enhanced inelastic cross sections, they are prime examples of that kind of state which may be produced in a several step process. This is illustrated in Fig. 2 for the idealized case in which two states have pure parentage based on the ground state, and are produced in the normal single step process, while one has parentage based on an excited state and can be produced only by means of an intermediate step. In practical situations in which nuclear states have mixed parentage, whether indirect modes compete with the direct will depend both on the fraction of parentage based on excited states of the core, and on the strength with which these states are produced in inelastic collisions. Generally, we expect that low-lying states of $A + 1$ will have a larger portion of their parentage based on the ground state of $A$ than higher-lying ones.



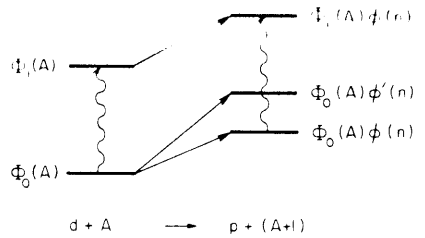


**Fig. 2.** An idealized situation showing states of pure parentage. Two are based on the ground state, which therefore can be reached by the direct transfer of a neutron, and one is based on an excited state, which can be reached, therefore, only in second order.

There are two available methods for incorporating higher order processes which pass through intermediate states produced in inelastic collisions. Both involve solving the coupled channel equations for inelastic scattering, and both are equivalent in their final outcome though they are different in their computational execution. One method is a generalization of the DWBA amplitude, Eq. (3), in which the elastic distorted waves, which refer to the elastic channel only, are replaced by wave functions which contain, in addition, components referring to excited states (Penny and Satchler 1964). Thus in Eq. (3) the following replacement is made

$$\psi_p^{(-)}\Phi_B(A+1) \rightarrow \sum_{B'} \psi_{pB'}^{(-)}\Phi_{B'}(A+1) \equiv \Psi_p \tag{26}$$

with a similar replacement for $\psi_d^{(+)}$. Whereas $\psi_p^{(-)}$ is the solution of an optical potential equation

$$(T + U_p - E_p)\psi_p^{(-)} = 0 \tag{27}$$

the $\psi_{pB'}{}^{(-)}$ satisfy coupled equations which account for the possibility of exciting the nucleus through inelastic collisions. We write the total Hamiltonian in terms of (a) a nuclear Hamiltonian $H_{A+1}$ of which the $\Phi_{B'}$ are eigenfunctions:

$$(H_{A+1} - E_{B'})\Phi_{B'}(A+1) = 0 \tag{28}$$

(b) the kinetic energy $T$ of relative motion p $-$ $(A + 1)$; and (c) an interaction $\mathcal{V}(p, A+1)$ between proton and nucleus

$$H = H_{A+1} + T + \mathcal{V} \tag{29}$$

Then inserting the expression for $\Psi_p$ of Eq. (26) into

$$(H - E)\Psi_p = 0 \tag{30}$$

steps may be important. The two-nucleon transfer reaction including such processes has been formulated (Ascuitto and Glendenning, 1970a). However, the calculation of such effects is not yet routine in the analysis of experiments.

The effect of higher order processes on the (p,t) reaction in spherical vibrational nuclei has been calculated to affect relative cross sections by up to a factor of 2 (Ascuitto and Glendenning, 1970b). However in the example studied, the angular distributions were unchanged. This, however, was not the case in deformed nuclei, where even angular distributions were in some cases strongly altered (Ascuitto *et al.*, 1971). In the deformed nuclei the effect of the higher order processes was corroborated by comparison with experiment.

## F. Other Two-Nucleon Transfer Reactions

Other reactions like ($^3$He,p) and ($\alpha$,d) and their inverses can be treated in a similar manner to the (p,t) reaction. The relative motion again is s state. The spin state of like nucleons is therefore singlet but for unlike nucleons it is a mixture of singlet and triplet. The state of the deuteron, however, allows only the triplet spin state to contribute in the ($\alpha$,d) reaction. The structure amplitudes defined before as $G_{NJ}$ need now to be generalized to include the orbital and spin transfer $L$ and $S$ in addition to the total angular momentum transfer $J$. Tables exist for these as well (Glendenning, 1968a). In the absence of spin–orbit interactions in the optical potentials there is no interference between different $L$'s or $S$'s, and never between different $J$'s.

In such a reaction as ($^3$He,p) where both $S = 0$ and 1 are allowed, there is evidence that the interaction $V$ depends on $S$ and this should be taken into account in calculations of the cross section (Fleming *et al.*, 1971).

The selection rules for these reactions as well as (p,t) have been detailed elsewhere (Chapter IV.B.2 and Glendenning, 1965).

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