where the square bracket denotes vector coupling.[16] The factor $\binom{A+2}{2}$ has to be understood as symbolic in the following sense: In case the isospin formalism is not used in constructing the wave functions, then

$$\binom{A+2}{2}\to\binom{N+\nu}{\nu}\binom{Z+\pi}{\pi}, \qquad (3.2b)$$

where $\nu$ and $\pi$ are the number of neutrons and protons transferred ($\nu+\pi=2$). *In any case*, if, as is usual, the overlap is computed with wave functions that refer only to a certain antisymmetrized subgroup of the total number of nucleons, then $A$ (or $N$ and $Z$) stands only for the number in the group to which the pair is added.

If the wave functions of $(A)$ and $(A+2)$ are known, say from a shell-model calculation, then $\beta$ can be computed. As a simple example, consider a nucleus $(A)$ that has closed shells. Some states of the nucleus $(A+2)$ might therefore have the structure

$$\Psi_{J_2T_2}(A+2)=\Psi_0(A)\sum_{j_1j_2} C_{(j_1j_2)J_2T_2}\phi_{(j_1j_2)J_2T_2}(r_1,r_2), \quad (3.3)$$

where the $C$'s are the mixture coefficients for the levels above the closed shells of $(A)$. To calculate $\beta_{\gamma LSJT}$ we want to transform $\phi_{(j_1j_2)J}$ from the $j$-$j$ scheme to the $L$-$S$ scheme; this is achieved with the coefficients

$$\begin{bmatrix} l_1 & \frac{1}{2} & j_1\\ l_2 & \frac{1}{2} & j_2\\ L & S & J\end{bmatrix}=\{[L][S][j_1][j_2]\}^{1/2}\begin{Bmatrix} l_1 & \frac{1}{2} & j_1\\ l_2 & \frac{1}{2} & j_2\\ L & S & J\end{Bmatrix}, \quad (3.4)$$

where $[j]=2j+1$ and $\{\ \}$ is a 9-$j$ coefficient.[17] Upon doing this and inserting the resulting expression for $\Psi_{J_2}(A+2)$ into Eq. (3.2), we can perform the integrations immediately, obtaining

$$\beta_{\gamma LSJT}(0,J_2)=C_{(j_1j_2)J_2T_2}\begin{bmatrix} l_1 & \frac{1}{2} & j_1\\ l_2 & \frac{1}{2} & j_2\\ L & S & J\end{bmatrix}\delta_{JJ_2}\delta_{TT_2}, \quad (3.5)$$

which is the parentage factor connecting the ground state of $(A)$ and the state $J_2T_2$ of $(A+2)$.

It is very important to notice from Eqs. (2.1) and (2.3) that the configuration mixture coefficients $C$ in the wave function contribute *coherently* to the structure factors. Thus, the two-nucleon stripping reaction is sensitive to the phases as well as the magnitudes of the mixture coefficients. The single-nucleon stripping reaction by contrast depends only on the absolute values of these coefficients. It should be evident however that, starting with experimental results, it is in general impossible to deduce the wave function. Even supposing that the experiment uniquely determined the $G$'s, there is an infinity of ways in which the product of the three factors on the right side of Eq. (2.3) could be arranged to yield them. However, if we have a wave function obtained from a shell-model calculation, say, we can compute from it the structure factors, and thus test whether the wave function is compatible with the experimental results. In the next section this procedure is illustrated in detail for the $N^{14}$ wave functions.

The parentage factors can be easily obtained when a pair of like nucleons is added or taken out of a given shell $j$. In particular, when $n$ is even, the ground state is (assuming a pure configuration):

$$|(j^n);0\rangle=\sum_{v,j}((j^{n-2})vJ,(j^2)J]\!](j^n)0)\,|(j^{n-2})vJ,(j^2)J;0\rangle, \quad (3.6)$$

where $v$ is the seniority, and the bracket $]\!]$ is a coefficient of fractional parentage.[18] Again expanding the $(j^2)J$ configuration on an $L$-$S$ basis, and inserting Eq. (3.6) into Eq. (3.2), we obtain immediately

$$\beta_{LSJ}[(j^{n-2})vJ\leftrightarrow(j^n)0]=\left[\frac{n(n-1)}{2}\right]^{1/2}((j^{n-2})vJ,(j^2)J]\!](j^n)0)\begin{bmatrix} l & \frac{1}{2} & j\\ l & \frac{1}{2} & j\\ L & S & J\end{bmatrix}. \quad (3.7)$$

Similarly, the wave function for an excited state $|(j^n)v=2,J\rangle$ can be expanded and one finds

$$\beta_{LSJ}[(j^{n-2})v_1J_1\leftarrow(j^n)v_2J_2]=\left[\frac{n(n-1)}{2}\right]^{1/2}((j^{n-2})v_1J_1,(j^2)J]\!](j^n)v_2J_2)\begin{bmatrix} l & \frac{1}{2} & j\\ l & \frac{1}{2} & j\\ L & S & J\end{bmatrix}. \quad (3.8)$$

Explicit formulas for coefficients of fractional parentage can be obtained for states of low seniority by methods

[16] In our earlier work (Ref. 1) the factor coming from antisymmetrization was left as a multiplying factor in front of the cross section. We now incorporate it into the definition of $\beta$ in the same way that a similar factor is incorporated in the definition ofthe spectroscopic factor in single-nucleon stripping. That symbol denotes $\binom{m}{n}\equiv m!/[(m-n)!n!]$.

[17] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, New Jersey, 1957).

[18] G. Racah, Phys. Rev. **63**, 367 (1943).

discussed by Schwartz and de-Shalit.[19] One finds

$$((j^{n-2})vJ,(j^2)J]\!](j^n)0)=\left\{\frac{2(n-2)}{n-1}\frac{2J+1}{(2j-1)(2j+1)}\right\}^{1/2} \quad \text{for} \quad v=2,\ J\neq 0$$

$$=\left\{\frac{2j+3-n}{(n-1)(2j+1)}\right\}^{1/2} \quad \text{for} \quad v=0,\ J=0, \tag{3.9}$$

$$((j^{n-2})v_1J_1,(j^2)J]\!](j^n)v=2J_2)=\delta_{J_1J_2}\delta_{v_12}\left\{\frac{n-2}{n(n-1)}\frac{2j+1-n}{2j+1}\right\}^{1/2} \quad \text{for} \quad J=0$$

$$=\delta_{JJ_2}\delta_{v_10}\left\{\frac{2}{n(n-1)}\frac{(2j+1-n)(2j+3-n)}{(2j-1)(2j+1)}\right\}^{1/2} \quad \text{for} \quad J_1=0. \tag{3.10}$$

[See Eq. (36) of Ref. 19 for the case when $J\neq 0$, $J_1\neq 0$.]

Similarly to the above, when $n$ is odd we obtain

$$\beta_{LSJ}[(j^n)v=1j\leftrightarrow(j^{n-2})v=1j]=\left[\frac{n(n-1)}{2}\right]^{1/2}((j^{n-2})v=1j,(j^2)J]\!](j^n)v=1j)\begin{Bmatrix} l & \frac{1}{2} & j \\ l & \frac{1}{2} & j \\ L & S & J\end{Bmatrix}, \tag{3.11}$$

where

$$((j^{n-2})v=1j,(j^2)J]\!](j^n)v=1j)=-\frac{2}{2j-1}\left\{\frac{(2J+1)(2j+2-n)}{n(2j+1)}\right\}^{1/2} \quad \text{for} \quad J\neq 0$$

$$=\left\{\frac{2j+2-n}{n(2j+1)}\right\}^{1/2} \quad \text{for} \quad J=0. \tag{3.12}$$

We now consider the situation in which the nucleons are transferred to or from different shells. Then

$$\beta_{\gamma LSJ}[(j_a^{n_a})J_a,(j_b^{n_b})J_b;J_2\leftrightarrow(j_a^{n_a-1})J_a',(j_b^{n_b-1})J_b';J_1]$$
$$=(n_an_b)^{1/2}((j_a^{n_a-1})J_a',j_a]\!](j_a^{n_a})J_a)((j_b^{n_b-1})J_b',j_b]\!](j_b^{n_b})J_b)\begin{Bmatrix} J_a' & j_a & J_a \\ J_b' & j_b & J_b \\ J_1 & J & J_2\end{Bmatrix}\begin{Bmatrix} l_a & \frac{1}{2} & j_a \\ l_b & \frac{1}{2} & j_b \\ L & S & J\end{Bmatrix}. \tag{3.13}$$

The coefficients of fractional parentage are exactly those familiar from $(d,p)$ reactions,[6] and for states of lowest seniority can be written down [cf. Eq. (67) in Ref. 1].

The parentage factor for configuration mixed-wave functions based upon the above configurations can easily be found from those given for the pure configurations. Thus, for example, if

$$|J_2\rangle=\sum_{j_aj_bJ_aJ_b}C_{j_aj_bJ_aJ_b}^{(2)}|(j_a^{n_a})J_a,(j_b^{n_b})J_b;J_2\rangle,$$

$$|J_1\rangle=\sum_{j_aj_bJ_a'J_b'}C_{j_aj_bJ_a'J_b'}^{(1)}|(j_a^{n_a-1})J_a',(j_b^{n_a-1})J_b';J_1\rangle, \tag{3.14a}$$

then

$$\beta_{\gamma LSJ}[J_2\leftrightarrow J_1]=\sum_{J_aJ_bJ_a'J_b'}C^{(1)}C^{(2)}\beta_{\gamma LSJ}[J_aJ_b;J_2\leftrightarrow J_a'J_b';J_1]. \tag{3.14b}$$

For several other configurations that might rise in the conventional shell model, we have given the corresponding parentage factors elsewhere.[1]

In regions of the periodic table removed by more than several nucleons from closed shells, the conventional shell model becomes very cumbersome. In such situations, the Bardeen-Cooper-Schrieffer method has been applied to the nuclear-structure problem.[20,21] With some sacrifices, one can obtain a solution to the many-body problem. Using this nuclear model, Yoshida[22] has considered the two-nucleon stripping reaction and ob-

[19] C. Schwartz and A. de-Shalit, Phys. Rev. 94, 1257 (1954).

[20] S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 31, No. 11 (1959).

[21] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 32, No. 9 (1960).

[22] S. Yoshida, Nucl. Phys. 33, 693 (1962).