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RELATION BETWEEN THE DEUTERON AND NUCLEON OPTICAL POTENTIALS

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Abstract: It is pointed out that in a generalization of the Watanabe model of the deuteron optical potential in which energy-independent, non-local nucleon optical potentials are averaged over the deuteron internal wave function, the resultant non-local deuteron potential is found to have a range of non-locality very close to one-half of the range of non-locality of the nucleon optical potential. This in turn implies that an equivalent energy-dependent, local deuteron optical potential may be obtained by averaging the local equivalent of the nucleon optical potential, evaluated at half of the incident deuteron kinetic energy, over the deuteron ground state. The relevance of these results to deuteron elastic scattering is discussed.

1. Non-local deuteron optical potential

We start by considering the dominant term in the real part of the deuteron optical potential [1-3]), the potential formed by folding the sum of the neutron and proton optical potentials into the deuteron wave function. This is the well-known Watanabe model [4]), but we shall here assume that the nucleon potentials are non-local and have the form †

$$\langle \boldsymbol{r}_\alpha | U_\alpha | \boldsymbol{r}'_\alpha \rangle = H(|\boldsymbol{r}_\alpha - \boldsymbol{r}'_\alpha|) U(\tfrac{1}{2}(\boldsymbol{r}_\alpha + \boldsymbol{r}'_\alpha)), \qquad \alpha = \mathrm{n, p}, \tag{1}$$

where $\boldsymbol{r}_\alpha$ is the coordinate of nucleon α relative to the target. Here, H is a peaked function having a range characterized by a length β, and normalized so that

$$\int \mathrm{d}\boldsymbol{s}\, H(s) = 1. \tag{2}$$

The non-local Watanabe potential is

$$\langle \boldsymbol{R} | U_\mathrm{d} | \boldsymbol{R}' \rangle = \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}'\, \phi(r) \langle \boldsymbol{R}\boldsymbol{r} | U_\mathrm{n} + U_\mathrm{p} | \boldsymbol{r}'\boldsymbol{R}' \rangle \phi(r'), \tag{3}$$

where $\phi(r)$ is the internal wave function of the deuteron, assumed to be pure S-state, and $\boldsymbol{r} = \boldsymbol{r}_\mathrm{n} - \boldsymbol{r}_\mathrm{p}$ and $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_\mathrm{n} + \boldsymbol{r}_\mathrm{p})$. Extracting the δ-functions from the potential matrix elements, e.g. for neutrons,

$$\langle \boldsymbol{R}\boldsymbol{r} | U_\mathrm{n} | \boldsymbol{r}'\boldsymbol{R}' \rangle = \delta(\boldsymbol{R} - \tfrac{1}{2}\boldsymbol{r} - \boldsymbol{R}' + \tfrac{1}{2}\boldsymbol{r}') \langle \boldsymbol{R} + \tfrac{1}{2}\boldsymbol{r} | U_\mathrm{n} | \boldsymbol{R}' + \tfrac{1}{2}\boldsymbol{r}' \rangle,$$

† For the moment we do not distinguish between the neutron and proton potentials.

and substituting eq. (1) into eq. (3), we find

$$\langle \boldsymbol{R}|U_{\rm d}|\boldsymbol{R}'\rangle = 8H(2|\boldsymbol{R}-\boldsymbol{R}'|)\int {\rm d}\boldsymbol{r}\,\phi(\boldsymbol{r}+(\boldsymbol{R}-\boldsymbol{R}'))\phi(\boldsymbol{r}-(\boldsymbol{R}-\boldsymbol{R}'))2U(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')+\tfrac{1}{2}\boldsymbol{r}). \tag{4}$$

According to refs. [5, 6]), the range of non-locality of the nucleon optical potential is about 1 fm. Since $H(2|\boldsymbol{R}-\boldsymbol{R}'|)$ limits the values of $|\boldsymbol{R}-\boldsymbol{R}'|$ to values $\lessapprox \frac{1}{2}\beta$, and over this range the deuteron wave function is very smooth, an excellent approximation to eq. (4) is given by

$$\langle \boldsymbol{R}|U_{\rm d}|\boldsymbol{R}'\rangle \approx 8H(2|\boldsymbol{R}-\boldsymbol{R}'|)2\overline{U}(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')), \tag{5}$$

where

$$\overline{U}(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')) = \int {\rm d}\boldsymbol{r}\,\phi(r)^2 U(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')+\tfrac{1}{2}\boldsymbol{r}), \tag{6}$$

is simply the nucleon potential form factor folded into the deuteron wave function. We also note, from eq. (2), that the non-locality factor is still normalized,

$$8\int {\rm d}\boldsymbol{s}\,H(2s) = 1.$$

The corrections to eq. (5) are considered below and found to be rather small. Therefore eq. (5) stands as an important result showing that the range of non-locality of the deuteron optical potential $\beta_{\rm d}$ in the Watanabe model is approximately half the range of non-locality of the nucleon optical potentials:

$$\beta_{\rm d} \approx \tfrac{1}{2}\beta. \tag{7}$$

We now consider the corrections to eq. (5) by assuming Gaussian forms for the non-locality,

$$H(s) = (\pi^{\frac{1}{2}}\beta)^{-3}\exp\left\{-\left(\frac{s}{\beta}\right)^2\right\}, \tag{8}$$

and for the deuteron wave function;

$$\phi(r) = (\pi^{\frac{1}{2}}R_{\rm d})^{-3}\exp\left\{-\left(\frac{r}{R_{\rm d}}\right)^2\right\}. \tag{9}$$

Substituting eqs. (8) and (9) into eq. (4) leads exactly to the result,

$$\langle \boldsymbol{R}|U_{\rm d}|\boldsymbol{R}'\rangle = (\pi^{\frac{1}{2}}\beta_{\rm d})^{-3}\exp\left\{-\left(\frac{\boldsymbol{R}-\boldsymbol{R}'}{\beta_{\rm d}}\right)^2\right\}\left(\frac{\beta_{\rm d}}{\frac{1}{2}\beta}\right)^3 2\overline{U}(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')), \tag{10}$$

where the new non-locality is

$$\beta_{\rm d} = \tfrac{1}{2}\beta\left[1+2\left(\frac{\frac{1}{2}\beta}{R_{\rm d}}\right)^2\right]^{-\frac{1}{2}}, \tag{11}$$

and $\overline{U}$ is given by eq. (6) with ϕ as in eq. (9). Setting $\beta = 0.85$ fm, as found for phenomenological potentials [5, 6]) of the form of eqs. (1) and (8), and $R_{\rm d} = 4.32$ fm, we find a 1 % correction to approximation (7) and a consequent 3 % reduction in the depth of the potential form factor.

It might be objected that a Gaussian choice for the deuteron wave function is rather poor, and we have therefore considered the correction following from the Hulthén wave function. To simplify the calculation, we consider the region $\boldsymbol{R} \approx \boldsymbol{R}' \approx 0$ for which it is reasonable to pull out the slowly varying function $U(\frac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')+\frac{1}{2}\boldsymbol{r})$ in eq. (4) from under the integral sign. This gives

$$\langle \boldsymbol{R}|U_{\mathrm{d}}|\boldsymbol{R}'\rangle \approx 8H(2|\boldsymbol{R}-\boldsymbol{R}'|)F(\boldsymbol{R}-\boldsymbol{R}')2U(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')), \tag{12}$$

where the correction to the non-locality factor is now,

$$F(x) = \int \mathrm{d}\boldsymbol{r}\, \phi(\boldsymbol{r}+\boldsymbol{x})\phi(\boldsymbol{r}-\boldsymbol{x}). \tag{13}$$

We have compared $F(x)$ for three cases:

(a) With Gaussian form of $\phi(r)$ as in eq. (9).

(b) With the Hulthén form,

$$\phi(r) = N(\mathrm{e}^{-\alpha r}-\mathrm{e}^{-6\alpha r}), \tag{14}$$

where N is the normalization and $\alpha = 0.232\ \mathrm{fm}^{-1}$.

(c) An approximate calculation of $F(x)$ exhibiting its dependence on the average kinetic energy of the nucleons in the deuteron,

$$F(x) \approx 1+\tfrac{2}{3}x^2 \int \mathrm{d}\boldsymbol{r}\, \phi(r)\nabla^2\phi(r), \tag{15}$$

evaluated for the Hulthén function.

In cases (b) and (c) we have calculated an effective β_{d} by fitting the resulting $F(x)$ to an equivalent Gaussian having the same volume integral and rms radius as $F(x)$. The results are displayed in table 1.

We conclude that the approximation used in eq. (5) is also rather accurate for the Hulthén wave function. Moreover we see that expansion (15) gives good agreement with the exact result, and so the magnitude of the correction is determined essentially by the average kinetic energy of the nucleons in the deuteron.

TABLE 1

The correction to the deuteron non-locality, $\delta\beta_{\mathrm{d}} = \beta_{\mathrm{d}}-\frac{1}{2}\beta$, and the consequent change in depth for various deuteron wave functions

Method		(a) Gaussian	(b) Hulthén	(c) Approx. Hulthén
decrease non-locality	$= -\delta\beta_{\mathrm{d}}/\frac{1}{2}\beta$	1.0 %	1.7 %	1.9 %
decrease depth	$= -\delta U/U$	2.8 %	5.1 %	5.8 %

2. Local equivalent potentials

We now discuss the implications of these results for calculations starting from local, energy-dependent nucleon optical potentials. One problem of the Watanabe model in this case is that the appropriate nucleon energies (E_n, E_p) used to specify the potentials are not known. We first note that this problem is by-passed altogether in the non-local formulation described above, provided, as we shall assume, the non-local potentials are energy independent. We find, however, by consideration of the equivalent local potentials derived from eqs. (1) and (5), called respectively V_α (α = n, p) and V_d, that the nucleon energies may be chosen in a simple way so that the local method yields the same result as the non-local.

Our discussion is based on Perey and Buck's [5]) transcendental equation of the general form

$$V(\mathcal{U}, \mathcal{E}) = \mathcal{U} \exp\{-x[\mathcal{E} - V(\mathcal{U}, \mathcal{E})]\}, \tag{16}$$

which is known to give a local potential $V(\mathcal{U}, \mathcal{E})$ accurately equivalent to a non-local potential of the form of eqs. (1) and (8). In eq. (16), $\mathcal{U}$ is the appropriate non-local form factor, $\mathcal{E}$ is the appropriate energy modified by any purely local parts of the interaction, and x is the *nucleon* non-locality factor, $x = m\beta^2/(2\hbar^2)$. From eq. (7) we have for deuterons,

$$x_d = \frac{m_d \beta_d^2}{2\hbar^2} \approx \frac{m\beta^2}{4\hbar^2} = \tfrac{1}{2}x. \tag{17}$$

Let us allow for the difference between the neutron and proton non-local form factors in eq. (1) by introducing a symmetry term U_s,

$$U(\text{neutron}) \rightarrow U - U_s, \qquad U(\text{proton}) \rightarrow U + U_s. \tag{18}$$

With this generalization, we find that the equivalent local potentials V_n, V_p and V_d, derived from eqs. (1), (5), (17) and (18), are solutions of,

$$V_n(E_n) = (U - U_s) \exp\{-x[E_n - V_n(E_n)]\}, \tag{19}$$

$$V_p(E_p) = (U + U_s) \exp\{-x[E_p - V_C - V_p(E_p)]\}, \tag{20}$$

$$V_d(E_d) = 2\bar{U} \exp\{-x[\tfrac{1}{2}(E_d - V_C) - \tfrac{1}{2}V_d(E_d)]\}, \tag{21}$$

where V_C is the Coulomb potential. We see immediately that in the absence of U_s, V_C and the folding effect ($\bar{U} \rightarrow U$) the equations satisfied by $V_n(\frac{1}{2}E_d)$, $V_p(\frac{1}{2}E_d)$ and $\frac{1}{2}V_d(E_d)$ are identical. In other words the potential $V_d(E_d, R)$, depending only on the deuteron energy, can be constructed as the sum of the local nucleon potentials (without folding) provided that they are evaluated at $E_n = E_p = \frac{1}{2}E_d$. We shall see that this statement is not greatly disturbed by the presence of the various corrections.

It is clear that eqs. (19), (20) and (21) are forms of the general eq. (16) with specific values of the parameters $\mathcal{U}$ and $\mathcal{E}$. The deuteron potential is

$$V_d(E_d) = 2V(\bar{U}, \tfrac{1}{2}(E_d - V_C)), \tag{22}$$

while the sum of the nucleon potentials folded into the deuteron wave function (indicated by $\langle \dots \rangle$) is,

$$\langle V_n(E_n)+V_p(E_p)\rangle = \langle V(U-U_s, E_n)+V(U+U_s, E_p-V_C)\rangle. \tag{23}$$

If we expand the r.h.s. of eq. (23) about the values of the arguments on the right of eq. (22), we find, exhibiting terms up to first order,

$$\langle V_n(E_n)+V_p(E_p)\rangle = 2V(U, \tfrac{1}{2}(E_d-V_C))+(E_n+E_p-E_d)\left(\frac{\partial V}{\partial E}\right)_{E=\frac{1}{2}(E_d-V_C)}+\dots, \tag{24}$$

since the difference $\langle U-\bar{U}\rangle$ vanishes. So we see that if we choose $E_n+E_p = E_d$ we shall obtain the same result, to first order, by the non-local method eq. (22) and by the local method eq. (23). Furthermore this result is not dependent on the specific form of the Perey and Buck equation, although of course its accuracy does depend on the smoothness of the derivatives as a function of the parameters.

The decrease in depth of $V_d(E_d)$, as calculated from eq. (21), due to the corrections listed in table 1 is less than the correction to the non-local depth and typically less than 4 %. This estimate is in close agreement with that of Kunz [7]) who also estimates that there could be an additional 3 % decrease in depth due to a hard core contribution in the deuteron wave function, because, of course, the average kinetic energy in the deuteron is then increased (see eq. (15)).

We suggest that the above procedure for generating the deuteron optical potential is probably more useful than that proposed by Bauer and Bloch [8]). Following precisely the argument which led to eq. (5), they write the analogous equation in momentum space,

$$\langle \boldsymbol{K}|U_d|\boldsymbol{K}'\rangle \approx \tilde{\phi}^2(\tfrac{1}{2}(\boldsymbol{K}'-\boldsymbol{K}))[\langle\tfrac{1}{4}(3\boldsymbol{K}-\boldsymbol{K}')|U_n+U_p|\tfrac{1}{4}(3\boldsymbol{K}'-\boldsymbol{K})\rangle], \tag{25}$$

where $\tilde{\phi}^2$ is the Fourier transform of ϕ^2.

On the energy shell this equation is a relation between Born approximations: the Born approximation for elastic deuteron scattering at momentum transfer $\boldsymbol{Q} = \boldsymbol{K}-\boldsymbol{K}'$ and energy $E_d = \hbar^2K^2/4m = \hbar^2K'^2/4m$ is proportional to the sum of the Born approximations for nucleon elastic scattering at the same momentum transfer but at energies,

$$E_n = E_p = \tfrac{1}{4}(5-3\hat{\boldsymbol{K}}'\cdot\hat{\boldsymbol{K}})E_d = \tfrac{1}{2}E_d+\tfrac{1}{8}Q^2, \tag{26}$$

depending on the momentum transfer. If the Born approximation were valid, it would be correct to take nucleon optical potentials at these nucleon energies in the Watanabe model. In the relevant energy range, however, the Born approximation is not good, and off-energy-shell matrix elements are also important. In this situation a well tested method for calculating the equivalent local potential is Perey and Buck's [5]) procedure as outlined above.

3. Discussion

We first comment on the relevance of these results to deuteron elastic scattering. It was found in a previous study [9]) that the folded potentials generated by the two methods described above actually do give very similar elastic scattering. Perey and Satchler [1]) have noted, however, in an extensive study of the real part of the Watanabe model that it seems to be necessary, in order to fit the data, to introduce a 10–20 % correction to the real well depth. It appears that a correction of this type, a reduction of about 10 %, is still needed even when allowance is made for the contribution of break-up channels to the elastic scattering [3]). Some reduction in the real well depth is expected to come from a correct treatment of the Pauli exclusion principle [9, 10]), but it is also possible that a further significant contribution is made by the 4 % non-locality reduction discussed above and by Kunz [7]).

Finally we note that the relationship between the deuteron and nucleon optical potentials we have derived depends strongly on the assumption that the range of non-locality of the nucleon optical potential is small compared with a length characteristic of the deuteron internal wave function. In the opposite limit,

$$\beta \gg R_{\mathrm{d}}, \tag{27}$$

we obtain from eqs. (10) and (11),

$$\beta_{\mathrm{d}} \approx R_{\mathrm{d}}/\sqrt{2}, \tag{28}$$

$$\langle \boldsymbol{R}|U_{\mathrm{d}}|\boldsymbol{R}'\rangle \approx (\pi^{\frac{1}{2}}\beta_{\mathrm{d}})^{-3} \exp\left\{-\left(\frac{\boldsymbol{R}-\boldsymbol{R}'}{\beta_{\mathrm{d}}}\right)^2\right\}\left(\frac{\sqrt{2}R_{\mathrm{d}}}{\beta}\right)^3 2\bar{U}(\tfrac{1}{2}(\boldsymbol{R}+\boldsymbol{R}')). \tag{29}$$

A non-locality satisfying eq. (27) would be obtained if the Perey and Buck potential eq. (1) was modified by adding a suitable local, energy-independent potential to the non-local terms † and the parameters of U and H adjusted to fit the elastic nucleon scattering. According to eq. (29) the associated contribution to the deuteron optical potential would be very small and deuteron-nucleus scattering would not be simply related to nucleon-nucleus scattering at any particular energy. Since the non-locality of the nucleon optical potential is not well understood at present from a fundamental point of view [11]) it is not clear that the limit (27) is completely irrelevant.

† This is the form taken by the Hartree-Fock potential if the nucleon interaction is local.

References

1) F. G. Perey and G. R. Satchler, Nucl. Phys. **A97** (1967) 515
2) N. Austern and K. C. Richards, Ann. of Phys. **49** (1968) 309
3) R. C. Johnson and P. J. R. Soper, Phys. Rev. **C1** (1970) 976
4) S. Watanabe, Nucl. Phys. **8** (1958) 484
5) F. G. Perey and B. Buck, Nucl. Phys. **32** (1962) 353
6) H. Schulz and H. Wiebicke, Phys. Lett. **21** (1966) 190
7) P. D. Kunz, Phys. Lett. **35B** (1971) 16
8) M. Bauer and C. Bloch, Phys. Lett. **33B** (1970) 155
9) P. J. R. Soper, Ph.D. thesis, University of Surrey, 1968, unpublished
10) G. Baumgartner, Z. Phys. **204** (1967) 17
11) M. Coz, A. D. MacKellar and L. G. Arnold, Ann. of Phys. **58** (1970) 504