

AN APPROXIMATE THREE-BODY THEORY OF DEUTERON STRIPPING

R. C. JOHNSON and P. C. TANDY †

Department of Physics, University of Surrey, Guildford, Surrey, England

Received 1 April 1974

(Revised 24 July 1974)

Abstract: A treatment of deuteron stripping is developed in which the three-body effects associated with deuteron break-up in the nuclear field are included explicitly. The essence of the method is the choice of a convenient discrete set of n-p eigenfunctions as a representation of the three-body continuum effects. This approach leads to a distorted wave stripping matrix element similar to that of the DWBA, except that the elastic deuteron wave is replaced by a three-body wave function given as the solution of a set of coupled two-body Schrödinger equations. The adiabatic theory of Johnson and Soper appears as the solution in a suitable first approximation. This new formalism should prove useful in the evaluation of corrections to three-body models of the deuteron-nucleus system, in particular those models in which the nucleon-target interaction is represented by a complex local optical potential.

1. Introduction

The weakly bound structure of the deuteron strongly suggests the relevance of three-body effects in the mechanism of deuteron stripping. However the question of deuteron break-up in the nuclear field is usually ignored, and cannot be explicitly treated by the conventional DWBA approach, which employs a deuteron-target elastic scattering wave function¹⁾. Nevertheless this prescription has been very successful in accounting for much of the angular distribution and polarization data from stripping reactions¹⁾. The accuracy and validity of the DWBA description has recently been the subject of several studies of three-body models of stripping.

A practical prescription for including break-up effects in the analysis of stripping data has been given by the adiabatic theory of Johnson and Soper²⁾. This theory gives rise to a distorted wave calculation similar to the DWBA, except that the elastic deuteron wavefunction is replaced by a three-body wave function of similar form in which the low energy relative S-wave n-p break-up effects are treated in an effective range theory manner. This prescription has been used to analyse (p, d) and (d, p) reactions on a wide variety of targets and for several bombarding energies, and in most cases results in a systematic improvement of the fit to experimental angular distributions, and reasonable absolute spectroscopic factors are obtained without the use of arbitrary parameters such as a cut-off radius^{3, 4)}.

A key feature of the adiabatic theory is the derivation of an effective potential – the

† Present address: Dept. of Physics, University of Maryland, College Park, Maryland 20742, USA.

adiabatic potential – which generates the appropriate three-body wave function by an effective two-body Schrödinger equation. The adiabatic potential is derived from neutron and proton optical potentials obtained from fits to nucleon elastic scattering. The inclusion of break-up effects gives to this potential a geometry differing significantly from that of conventional deuteron phenomenological optical potentials. This feature is largely responsible for eliminating the need for arbitrary suppression of the interior contributions to stripping integrals when the adiabatic wave function is used³⁾.

Although the adiabatic theory has provided a systematic improvement over the conventional DWBA, a number of stripping and pick-up transitions are not well fitted by either theory^{3,4)}. It is natural to ask whether an improved treatment of the three-body aspects can help in this respect before greater attention is paid to the many-body nature of the reaction. An understanding of the nature and magnitude of any significant corrections to this approach is also necessary to facilitate extensions to similar reactions, e.g. (${}^6\text{Li}$, α).

In this paper, which addresses itself to processes dominated by a direct reaction mechanism only¹⁾, we attempt to go some way towards this goal by developing an approximate three-body theory of stripping as an extension of the ideas and methods of the adiabatic approach. Our method is based upon a representation of the n-p continuum in terms of a complete set of discrete eigenfunctions. A feature of this new approach is that the adiabatic prescription appears as the solution of lowest order[†]. Close connections with distorted wave methods are retained, and the method leads to a stripping matrix element similar to that of the DWBA, except that the elastic deuteron wave is replaced by the solution of a set of coupled channel two-body Schrödinger equations. The n-p system is treated in an approximate way, while the nucleon-nucleus systems are described by an effective interaction which in the first approximation is taken to be appropriate optical potentials.

Other approaches^{5,6)} to the three-body aspects of stripping reactions have been formulated in terms of the exact three-particle scattering techniques that have enjoyed so much success in describing the three-nucleon problem. In particular the work of Boudin and Levin⁶⁾ has close connections with several aspects of the present approach. However with a view to fitting data, the relevance of a direct carry-over of the present three-nucleon techniques into the realm of direct reactions upon a heavy target is not obvious. For practical necessity, the interactions must be described by separable potentials of low rank, and the number of partial waves in each two-body system must be severely limited. Since a substantial portion of the physics of a (d, p) reaction resides in the complexity of the optical potential of the two-body channels, and the importance of surface partial waves, it seems desirable to develop an approximate three-body theory to accommodate these features. As far as deuteron stripping is concerned, the adiabatic theory, and its extension in this paper may contribute towards this goal.

[†] The sense in which the term “lowest order” is to be interpreted will become clear later.

In sect. 2 we describe the three-body model of the reaction, and outline the three-body effects in which we are interested. The basic approximation of the theory is introduced by developing an appropriate expansion for the wave function. The resulting coupled equations are reduced to convenient form in sect. 3, and the properties of the coupling potentials and the solutions are illustrated. A simple estimate of the convergence of the method is presented. In sect. 4 our approximate theory is discussed in the light of related work.

2. The approximate three-body method

We are interested in the many-body problem appropriate to a deuteron incident upon a heavy nucleus in its ground state, and in particular we require the (d, p) transition amplitude to the ground state of the residual nucleus. In the limit of weak coupling between the ground and excited states for both target and residual nuclei, we will treat the target as an inert core, and the residual as a neutron-core bound state. It will be convenient to discuss such a three-body model as if the nucleon-core interactions are static real potentials. The relevance of this model to the deuteron-nucleus problem with a realistic nuclear target will be discussed in subsect. 2.3.

2.1. THE THREE-BODY MODEL

The three-body wave function initiated by an incident deuteron beam of kinetic energy $E_d (= (\hbar^2/4m)K_d^2)$ and internal binding energy $+\varepsilon_d$, satisfies ($E = E_d - \varepsilon_d$)

$$[E + i\varepsilon - T_r - T_R - V_n(\mathbf{R} - \frac{1}{2}\mathbf{r}) - V_p(\mathbf{R} + \frac{1}{2}\mathbf{r}) - V_{np}(\mathbf{r})]\psi^{(+)}(\mathbf{r}, \mathbf{R}) = i\varepsilon\phi_d(\mathbf{r})e^{i\mathbf{K}_d \cdot \mathbf{R}}, \quad (1)$$

where

$$T_r = -\frac{\hbar^2}{m}\nabla_r^2, \quad T_R = -\frac{\hbar^2}{4m}\nabla_R^2. \quad (2)$$

The coordinates \mathbf{r} , \mathbf{R} are respectively the relative and c.m. coordinates of the neutron and proton, $\mathbf{r} = \mathbf{r}_p - \mathbf{r}_n$, $\mathbf{R} = \frac{1}{2}(\mathbf{r}_n + \mathbf{r}_p)$, and the origin of coordinates has been taken at the c.m. of the target nucleus which we assume is infinitely heavy. The n-p interaction is V_{np} and V_n , V_p are the nucleon-core potentials which we have taken to be local, although a small non-locality would not hinder the following developments. The term on the r.h.s. of (1) specifies the incident boundary condition of a deuteron with internal wave function ϕ_d , and the physical total wave function is to be calculated in the limit $\varepsilon \rightarrow 0+$.

The (d, p) stripping amplitude in this model is given by

$$T_{d,p} = \langle e^{i\mathbf{k}_p \cdot \mathbf{r}_p}, \Phi_n(\mathbf{r}_n) | V_p + V_{np} | \psi^{(+)} \rangle \quad (3)$$

$$= \langle \chi_p^{(-)}(\mathbf{r}_p), \Phi_n(\mathbf{r}_n) | V_{np} | \psi^{(+)} \rangle, \quad (4)$$

where Φ_n is the neutron-core bound state wave function with binding energy $-\varepsilon_n$, and $\chi_p^{(-)}$ is the proton distorted wave generated by V_p and describing proton-core

elastic scattering with incoming boundary conditions at the proton energy $E - \varepsilon_n = (\hbar^2/2m)k_p^2$. The existence of the exact expression (4) is crucial to the development of our approximate three-body treatment. To compute (4) we only require $\psi^{(+)}(r, \mathbf{R})$ for $r < r_0$ and $R < R_0 + \frac{1}{2}r_0$, where r_0, R_0 are measures of the ranges of V_{np} and Φ_n respectively. In the limit of a zero-range approximation¹⁾ for V_{np} , only $\psi^{(+)}(0, \mathbf{R})$ is required, and we expect that this wave function which satisfies much simpler boundary conditions than the full three-body wave function, would be amenable to an effective two-body treatment⁷⁾. In the present work we develop an effective two-body method for the projected wave function $V_{np}|\psi^{(+)}\rangle$, where V_{np} is of short range.

To consider the details of the exact wave function $\psi^{(+)}$ we write the integral equation equivalent to (1), in the form

$$|\psi^{(+)}\rangle = |\phi_d\rangle|\mathbf{K}_d\rangle + G_{np}(E^+)[V_n + V_p]|\psi^{(+)}\rangle, \quad (5)$$

where

$$G_{np}(E^+) = (E + i\varepsilon - T_r - T_R - V_{np})^{-1}. \quad (6)$$

The break-up components of $\psi^{(+)}$ that we are concerned with, can be seen explicitly by introducing the spectral representation of the three-body Green function G_{np} in terms of the complete orthonormal set of eigenstates of the n-p Hamiltonian $H_{np} = T_r + V_{np}$,

$$G_{np}(E^+) = \frac{|\phi_d\rangle\langle\phi_d|}{E + i\varepsilon + \varepsilon_d - T_R} + \int d\mathbf{k} \frac{|\phi_k^{(+)}\rangle\langle\phi_k^{(+)}|}{E + i\varepsilon - \varepsilon_k - T_R}, \quad (7)$$

where $\varepsilon_k = (\hbar^2/m)k^2$ and the eigenstates are defined by

$$H_{np}|\phi_d\rangle = -\varepsilon_d|\phi_d\rangle, \quad H_{np}|\phi_k^{(+)}\rangle = \varepsilon_k|\phi_k^{(+)}\rangle. \quad (8)$$

The use of (7) in (5) yields the following formal representation of the wave function

$$|\psi^{(+)}\rangle = |\phi_d\rangle|\chi_d^{(+)}\rangle + \int d\mathbf{k} |\phi_k^{(+)}\rangle|\chi_k^{(+)}\rangle, \quad (9)$$

where $\chi_d^{(+)}(\mathbf{R})$ is the elastic deuteron c.m. scattering wave function given by

$$|\chi_d^{(+)}\rangle = |\mathbf{K}_d\rangle + \frac{1}{E_d + i\varepsilon - T_R} \langle\phi_d|V_n + V_p|\psi^{(+)}\rangle. \quad (10)$$

The continuum components $\chi_k^{(+)}(\mathbf{R})$ describe the motion of the c.m. of an n-p pair scattering with relative energy ε_k , and are formally given by

$$|\chi_k^{(+)}\rangle = \frac{1}{E + i\varepsilon - \varepsilon_k - T_R} \langle\phi_k^{(+)}|V_n + V_p|\psi^{(+)}\rangle. \quad (11)$$

The exact wave function has outgoing waves corresponding to elastic deuterons given by the first term of (9), plus "outgoing" waves in which the neutron and proton are not bound together and which are represented by the second term of (9). The two stripping components which leave one nucleon bound to the core are implicitly

included in this second term. The conventional DWBA ignores break-up effects in the sense that the second term of (9) is neglected, and $\chi_d^{(+)}(\mathbf{R})$ is approximated by using a suitable optical potential and is substituted into the amplitude (4). We wish to include effects due to both terms of (9), but not necessarily in that form.

We will restrict our attention to the ^3S states of the n-p system. This restriction is not expected to be unduly severe, since the zero-range approximation ¹⁾ has proved quite successful in stripping calculations. In this limit other spin triplet states only contribute to the product $V_{np}|\psi^{(+)}\rangle$ through second and higher order terms, and although it is by no means obvious that such effects are negligible ⁸⁾, it is reasonable to ignore them in the first approximation. Contributions from singlet spin states arise from the relatively small spin dependence of V_n and V_p . There are no effects due to ^1S states unless the nucleon optical potentials have isovector spin-orbit components. Such effects have been investigated by Harvey and Johnson ⁹⁾ and found to be small.

2.2. THE WEINBERG EXPANSION OF THE WAVE FUNCTION

Since any approximate wave function, say $|\psi_A^{(+)}\rangle$, which satisfies $V_{np}|\psi_A^{(+)}\rangle = V_{np}|\psi^{(+)}\rangle$, will give the correct amplitude (4), we will not attempt to preserve the correct form of the wave function for large n-p separations such as the asymptotic form in the stripping channels. From (9) we have for the projected wave function,

$$V_{np}|\psi^{(+)}\rangle = V_{np}|\phi_d\rangle|\chi_d^{(+)}\rangle + \int d\mathbf{k} V_{np}|\phi_k^{(+)}\rangle|\chi_k^{(+)}\rangle. \quad (12)$$

If the dominant break-up effects are associated with low n-p relative energies, then application of effective range theory yields

$$V_{np}(\mathbf{r})\phi_k^{(+)}(\mathbf{r}) \approx g(\mathbf{k})V_{np}(\mathbf{r})\phi_d(\mathbf{r}), \quad (13)$$

and the projected wave function takes on the simple form

$$V_{np}(\mathbf{r})\psi^{(+)}(\mathbf{r}, \mathbf{R}) \approx V_{np}(\mathbf{r})\phi_d(\mathbf{r})\bar{\chi}^{(+)}(\mathbf{R}), \quad (14)$$

where

$$|\bar{\chi}^{(+)}\rangle \approx |\chi_d^{(+)}\rangle + \int d\mathbf{k} g(\mathbf{k})|\chi_k^{(+)}\rangle. \quad (15)$$

The utility of the above low-energy assumption lies in the demonstration that a product form (14) of the three-body wave function can be used in the stripping amplitude. The calculation of the amplitude parallels that of the DWBA, except that the low-energy break-up effects can be included in $\bar{\chi}^{(+)}(\mathbf{R})$ which replaces the elastic $\chi_d^{(+)}(\mathbf{R})$. The existence of the product form (14) provided the motivation for the adiabatic theory ²⁾.

We note that the projected wave function reduces exactly to the product form (14) in the special case in which V_{np} is taken to be a rank one separable potential. If we write

$$V_{np} = -|f\rangle\langle f|, \quad (16)$$

and require that this potential supports a bound state ϕ_d , then

$$|f\rangle = -V_{np}|\phi_d\rangle/\langle f|\phi_d\rangle. \quad (17)$$

In this case it is easily seen that the relations (13)–(15) are exact, and

$$\begin{aligned} g(\mathbf{k}) &= \langle f|\phi_k^{(+)}\rangle/\langle f|\phi_d\rangle, \\ |\bar{\chi}^{(+)}\rangle &= \langle f|\psi^{(+)}\rangle/\langle f|\phi_d\rangle. \end{aligned} \quad (18)$$

The definition (18) of $|\bar{\chi}^{(+)}\rangle$ is seen to be identical to that of the spectator wave function studied by Bouldin and Levin ⁶). The calculation of $|\bar{\chi}^{(+)}\rangle$ however still remains a three-body problem. Bouldin and Levin took V_n , V_p to be also of rank one separable form, and exploited the solubility of such a three-body model.

If the deuteron binding energy was so large that the low energy n-p scattering was not dominated by the bound state pole, we could replace (13) by

$$V_{np}(\mathbf{r})\phi_k^{(+)}(\mathbf{r}) \approx g_1(\mathbf{k})V_{np}(\mathbf{r})\phi_{k_1}^{(+)}(\mathbf{r}), \quad (19)$$

where $\phi_{k_1}^{(+)}(\mathbf{r})$ is a typical low-energy continuum state. The approximation (14) would then read

$$V_{np}(\mathbf{r})\psi^{(+)}(\mathbf{r}, \mathbf{R}) \approx V_{np}(\mathbf{r})[\phi_d(\mathbf{r})\chi_d^{(+)}(\mathbf{R}) + \phi_{k_1}^{(+)}(\mathbf{r})\chi_1^{(+)}(\mathbf{R})]. \quad (20)$$

A technique analogous to this is used by Harvey and Johnson ⁹) to investigate ¹S break-up effects. We seek to develop a general method for the representation and calculation of the projected wave function $V_{np}|\psi^{(+)}\rangle$.

The essence of our method [†] is to discretize the n-p spectrum by obtaining an expansion for $\psi^{(+)}$ in terms of a discrete set of n-p states. The exact n-p spectrum given by (8) is orthogonal with respect to integration over all n-p separations. However a restriction in the size of the domain to that of the projector V_{np} , will allow the choice of a discrete set of eigenfunctions. A convenient choice for this purpose is the set of Weinberg ¹⁰) eigenstates defined by

$$[-\varepsilon_d - T_r - \alpha_i V_{np}(\mathbf{r})]\phi_i(\mathbf{r}) = 0, \quad i = 1, 2, \dots, \quad (21)$$

for which the orthonormality relation is

$$\langle \phi_i | V_{np} | \phi_j \rangle = -\delta_{ij}. \quad (22)$$

The normalisation is of course arbitrary, but we have chosen unity for convenience, and the negative sign is due to the assumption of a purely attractive V_{np} , also for convenience. The first member of this set, apart from normalisation, is the deuteron bound state, such that $\phi_1 \approx \phi_d$ and $\alpha_1 = 1$. The Weinberg states so defined have the desirable property that the weight function for the orthogonality relation (22) is precisely the weighting that we wish to give to $\psi^{(+)}$ in the stripping matrix element. The member ϕ_i has i nodes within the range of V_{np} , and tends to zero exponentially

[†] A preliminary account of our method has been published in Proc. of the Int. Conf. on nuclear physics, Munich, ed. J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1974) p. 422.

after a turning point r_i given by

$$\alpha_i V_{np}(r_i) = -\varepsilon_d. \quad (23)$$

The eigenvalues α_i are real, and monotonically increasing with i , such that for large i , $\alpha_i \approx i^2$ for any finite range V_{np} .

Since the energy in (21) is fixed negative, these Weinberg states form a complete set of square integrable functions^{11,12}). Our expansion for the wave function is therefore,

$$\psi^{(+)}(r, R) = \sum_{i=1}^{\infty} \phi_i(r) \chi_i^{(+)}(R), \quad (24)$$

where, because of (22)

$$\chi_i^{(+)}(R) = -\langle \phi_i | V_{np} | \psi^{(+)} \rangle. \quad (25)$$

We expect that unless very high n-p continuum energies make a significant contribution to $V_{np}|\psi^{(+)}\rangle$, either directly or indirectly through coupling in the three-body Schrödinger equation, we may truncate the sum (24) to N terms. Substituting into the Schrödinger equation (1), we obtain ($i = 1, 2, \dots, N$),

$$[E + i\varepsilon + \varepsilon_d - T_R]|\chi_i^{(+)}\rangle = i\varepsilon\delta_{i1} N_d|\mathbf{K}_d\rangle - \langle \phi_i | V_{np}(V_n + V_p) | \psi^{(+)} \rangle - \langle \phi_i | V_{np}(H_{np} + \varepsilon_d) | \psi^{(+)} \rangle, \quad (26)$$

where $N_d = -\langle \phi_1 | V_{np} | \phi_d \rangle$. The second term on the r.h.s. of (26) will require $\psi^{(+)}$ only in the region where the truncated expansion (24) is valid, provided V_n and V_p are local or have only a small non-locality. Assuming for the present that in the third term of (26) we may also use the truncated expansion, we obtain the following set of N coupled-channel two-body Schrödinger equations,

$$[E_d + i\varepsilon - T_R - U_{ii}(\mathbf{R})]|\chi_i^{(+)}\rangle = i\varepsilon\delta_{i1} N_d|\mathbf{K}_d\rangle + \sum_{j \neq i}^N U_{ij}(\mathbf{R})|\chi_j^{(+)}\rangle. \quad (27)$$

In deriving this result we have employed the following definitions

$$U_{ij}(\mathbf{R}) = V_{ij}(\mathbf{R}) + \beta_{ij}(\alpha_j - 1), \quad (28)$$

$$V_{ij}(\mathbf{R}) = -\langle \phi_i | V_{np}(V_n + V_p) | \phi_j \rangle, \quad (29)$$

$$\beta_{ij}(\alpha_j - 1) = -\langle \phi_i | V_{np}(H_{np} + \varepsilon_d) | \phi_j \rangle \quad (30)$$

$$= \langle \phi_i | V_{np}^2 | \phi_j \rangle (\alpha_j - 1). \quad (31)$$

The coupling potentials V_{ij} describe the interaction of an n-p pair with the core while undergoing a transition from the relative motion state ϕ_j to ϕ_i . The real constant coupling terms in (28) and (30) arise because the Weinberg states are not eigenfunctions of the true n-p Hamiltonian H_{np} . Thus H_{np} itself can cause transitions between Weinberg states, and its representation in this discrete basis is not diagonal. However we will see in the next section that this situation is not a drawback of the method. The third term on the r.h.s. of (26) has been evaluated by using the Weinberg expansion of the wave func-

tion, and allowing H_{np} to operate to the right term by term. Such a procedure is valid in the present context because of the exponential fall-off of the Weinberg states at large r , so there can be no surface terms due to the kinetic energy operator in H_{np} . This situation is to be contrasted with that arising from the use of R -matrix type states to describe the n-p spectrum, wherein surface contributions would arise from the matching radius¹³). Integration of the third term of (26) by parts confirms the above procedure provided that we restrict $V_{np}(\mathbf{r})$ to be less singular than r^{-2} at the origin.

We can immediately discuss several general properties of the effective two-body coupled channel formulation (27). It can be seen that only the first channel component $|\chi_1^{(+)}\rangle$ contains the elastic deuteron component of $|\psi^{(+)}\rangle$. This follows immediately from the definition (25) and the property (22) of the Weinberg states. However every channel wave function contains break-up effects. This situation can be summarised by applying the projection (25) to eq. (9) to obtain formally,

$$|\chi_i^{(+)}\rangle = \delta_{i1} N_d |\chi_d^{(+)}\rangle - \int d\mathbf{k} \langle \phi_i | V_{np} | \phi_{\mathbf{k}}^{(+)} \rangle |\chi_{\mathbf{k}}^{(+)}\rangle. \quad (32)$$

In the coupled equations (27), the diagonal elements of the constant coupling matrix $\beta_{ii}(\alpha_i - 1)$ have the effect of redefining the channel energy. The energy of the first channel ($i = 1$, $\alpha_1 = 1$) however remains unchanged at the elastic deuteron value E_d . If we assume that only the first term of the expansion (24) is a sufficiently accurate representation, then the set of coupled equations truncate to the single equation

$$[E_d + i\varepsilon - T_R - V_{11}(\mathbf{R})] |\chi_1^{(+)}\rangle = i\varepsilon N_d |\chi_d\rangle. \quad (33)$$

A comparison with the adiabatic theory²) shows that (33) is almost precisely the prescription for the adiabatic wave function, the difference being that $V_{11}(\mathbf{R})$ defined in (29) is not identical to the adiabatic potential²) defined by

$$V_{ad}(\mathbf{R}) = \frac{\langle \mathbf{k} = 0 | V_{np} (V_n + V_p) | \phi_d \rangle}{\langle \mathbf{k} = 0 | V_{np} | \phi_d \rangle}. \quad (34)$$

The numerical difference between these two potentials is however negligible for the same V_n and V_p . The important feature of both is the folding of the nucleon-core potentials over the shape of the short-ranged V_{np} . The resulting geometry in the c.m. coordinate \mathbf{R} has important differences from that of phenomenological elastic deuteron optical potentials, and it is these differences which are responsible for the improved description of stripping provided by the adiabatic theory^{2,3}). The type of break-up effects included in the approximation (33) are those that can be represented by taking a single Weinberg state (the deuteron) in the expansion of the wave function, and we have seen this form to be consistent with an effective-range theory description of the n-p system. An expansion extended to several Weinberg states can simulate those high n-p relative momentum components that introduce successively more radial nodes within the range of V_{np} . A solution of the coupled equations (27) will then, in principle, provide an improvement upon the adiabatic theory in the sense that, firstly

$|\chi_1^{(+)}\rangle$ will be modified by the coupling to the higher order channels, and secondly these channels themselves will be available to compute the stripping element if necessary. The number of Weinberg states necessary to obtain $V_{np}|\psi^{(+)}\rangle$ and so the stripping matrix element accurately, can only be finally settled by a numerical calculation, as indeed can the question of whether the method will converge as N is increased. In the next section we attempt to lay the foundation for such investigations by demonstrating a technique for reducing the coupled equations to a more readily soluble form. The nature of both the solutions and the coupling potentials will be discussed.

2.3. OPTICAL POTENTIALS IN THE THREE-BODY MODEL

The relevance of a purely three-body model of the deuteron-nucleus system, to the true many-body problem has been discussed by several authors¹⁴⁻¹⁶). An important result is that such a model can be made formally equivalent to the many-body system, if suitable effective interactions are introduced by systematically eliminating the internal coordinates of the target nucleus. It was shown in ref.¹⁴) that in the limit of weak coupling to excited states of the target, the three-body wave function $|\psi^{(+)}\rangle$ can be taken to be the projection of the full many-body wave function onto the target ground state. In such a situation it has been shown¹⁴) that the formula (4) is still exact if we interpret $\chi_p^{(-)}$ as the proton-target elastic wave generated by the proton-target optical potential, and ϕ_n as the target-residual nucleus overlap function. The wave function $|\psi^{(+)}\rangle$ is to be calculated from equation (1) with $(V_n + V_p)$ replaced by an energy-dependent non-local interaction $V_{\text{eff}}(n, p)$.

It is shown in refs.¹⁴⁻¹⁶) that an important class of contributions to V_{eff} can be interpreted as the sum of the neutron and proton optical potentials evaluated off the energy shell in a specified way. These contributions are referred to as one-body terms. The nature of corrections to these contributions is discussed in ref.¹⁴). Even in lowest order the Pauli principle introduces two-body corrections peculiar to the effective interaction for composite projectiles, and estimates have been made of these contributions to elastic deuteron scattering²) and break-up²⁵). In higher order, terms arise which require both incident nucleons to excite the nucleus. Attempts¹⁵) have been made to take these into account in elastic deuteron scattering in a semi-phenomenological manner but a detailed calculation of their magnitude does not appear to be available. It is known²⁶) on theoretical grounds that these terms must play a rôle at very low energies because the off-shell prescription in the one-body terms proposed in¹⁴) does not give a correct amount of the rôle of open channels involving excited states of the target. However, it has been already pointed out in ref.²) (sect. IV.C) that at 20 MeV by far the major part (within 3 %) of the observed deuteron absorption cross sections for medium mass targets can be accounted for in terms of the imaginary parts of the nucleon optical potentials evaluated at $\frac{1}{2}E_d$.

For the purposes of illustrating the treatment of the three-body problem given in earlier subsections it will be assumed that

$$V_{\text{eff}}(E_n, E_p; r_n, r_p) \approx V_n(\frac{1}{2}E_d, r_n) + V_p(\frac{1}{2}E_d, r_p) + V_C(R), \quad (35)$$

where the first two terms are the local phenomenological optical potentials describing nucleon-target elastic scattering evaluated at half the incident deuteron kinetic energy, and V_C is the Coulomb field of the target. In terms of the discussion of ref. ¹⁴), this ignores all two-body terms and Pauli corrections. As far as the off-shell prescription for the one-body terms is concerned it is assumed that the parts of the three-body wave functions that dominate the stripping amplitude, eq. (4), have nucleons with kinetic energy close to their values in the incident deuteron. This has been shown ²⁷) to give a good account of the Hartree-Fock contribution to V_n and V_p when deuteron break-up is neglected and should be a reasonable first approximation for break-up components involving low relative momentum. An obvious inadequacy of the right-hand side of eq. (35) is the fixed energy dependence resulting in fixed imaginary components ¹⁶). Thus those parts of the total three-body wave function involving nucleon-target bound states cannot be generated correctly by this form for V_{eff} . However, only the projection $V_{np}|\psi^{(+)}\rangle$ is required accurately in the matrix element (4). The interaction (35) may indeed generate inaccuracies in $|\psi^{(+)}\rangle$ in regions where the neutron and proton are widely separated, as for example, the asymptotic stripping regions. However these components of $|\psi^{(+)}\rangle$ are not directly relevant to the matrix element (4). Information concerning the asymptotic stripping regions is supplied by the independent calculation of $\langle\chi_p^{(-)}\phi_n|$ using neutron and proton potentials corresponding to the correct final state energies.

These remarks are not meant to imply that corrections to the model (35) are negligible. Work aimed at elucidating this question is in progress ²⁸). The method of treating three-body effects generated by a given three-body Hamiltonian developed in this paper is well adapted to an exploration of these points. Thus in the present method only certain matrix elements of V_{eff} appear and these can be evaluated in principle for a V_{eff} of arbitrary complexity. In particular, since it is unlikely that an exact and tractable form for V_{eff} will ever be realised, it is important that the current method requires matrix elements of V_{eff} between a finite number of Weinberg states only.

It has been shown ²) that the effective interaction (35) can provide an understanding of elastic deuteron scattering at 20 MeV on medium mass nuclei, and has been used with some success in the adiabatic approach to stripping ²⁻⁴). The attitude taken here is that this does provide a basis from which a more accurate theory of stripping can be developed.

3. Details of the method

3.1. A TECHNIQUE FOR SOLUTION

To demonstrate that the approximate coupled-channel method can be solved in practice, it will prove convenient to use a matrix notation. We define the column of solutions

$$|\chi^{(+)}\rangle = \text{col} \{|\chi_1^{(+)}\rangle \dots |\chi_N^{(+)}\rangle\}, \quad (36)$$

and the matrix of constants C , whose elements are

$$C_{ij} = \beta_{ij}(\alpha_j - 1), \quad C_{i1} = 0. \quad (37)$$

The coupled equations (27) can then be written

$$[E_d + i\varepsilon - T_R]|\chi^{(+)}\rangle = C|\chi^{(+)}\rangle + V|\chi^{(+)}\rangle, \quad (38)$$

where the elements of V are the coupling potentials given in (29). The incoming deuteron boundary condition for the first channel has been omitted for convenience. The solutions can be properly defined by specifying the outgoing boundary conditions in each channel. This can be done once it is recognised that the matrix C can always be diagonalised. Defining

$$\alpha = \text{diagonal } \{1, (\alpha_2 - 1)^{\frac{1}{2}} \dots (\alpha_N - 1)^{\frac{1}{2}}\}, \quad (39)$$

we have

$$C = \alpha^{-1} B \alpha, \quad (40)$$

where

$$B = \begin{pmatrix} 0 & \mathbf{x} \\ 0 & \mathbf{b} \end{pmatrix} \quad (41)$$

and

$$\mathbf{x} = \text{row } \{\beta_{12}(\alpha_2 - 1)^{\frac{1}{2}}, \beta_{13}(\alpha_3 - 1)^{\frac{1}{2}} \dots \beta_{1N}(\alpha_N - 1)^{\frac{1}{2}}\}, \quad (42)$$

$$b_{ij} = (\alpha_i - 1)^{\frac{1}{2}} \beta_{ij} (\alpha_j - 1)^{\frac{1}{2}}, \quad i, j = 2, \dots, N. \quad (43)$$

The submatrix \mathbf{b} is real and symmetric, and therefore can always be diagonalised by a similarity transformation. Thus the matrix B is similar to a triangular matrix with diagonal elements $\{0, \lambda_2 \dots \lambda_N\}$ where the λ_i are the real eigenvalues of \mathbf{b} . A sufficient condition for a triangular matrix to be similar to a diagonal matrix is that its eigenvalues be distinct. However in the special case of B it is sufficient that none of the λ_i ($i = 2, \dots, N$) be zero. This condition is always satisfied due to the positive-definiteness of \mathbf{b} .

We can therefore write

$$C = A^{-1} \lambda A, \quad (44)$$

where

$$\lambda = \text{diagonal } \{0, \lambda_2 \dots \lambda_N\}, \quad (45)$$

and A is non-singular but not unitary. Therefore it is always possible to find a basis in which the coupled equations have no constant coupling terms. In this basis the equations become

$$[E_d + i\varepsilon - T_R]|F^{(+)}\rangle = \lambda|F^{(+)}\rangle + W|F^{(+)}\rangle, \quad (46)$$

where

$$|F^{(+)}\rangle = A|\chi^{(+)}\rangle, \quad (47)$$

$$W(R) = AV(R)A^{-1}. \quad (48)$$

The coupling potentials in the new basis are, explicitly,

$$W_{ij}(\mathbf{R}) = \sum_{l,n}^N A_{il} V_{ln}(\mathbf{R})(A^{-1})_{nj}, \quad (50)$$

$$= -\langle \Omega_i^N | V_{np} (V_n + V_p + V_c) | A_j^N \rangle, \quad (51)$$

where the new n-p basis states are

$$|A_j^N\rangle = \sum_{n=1}^N |\phi_n\rangle (A^{-1})_{nj}, \quad (52)$$

$$\langle \Omega_i^N | = \sum_{l=1}^N A_{il} \langle \phi_l | \neq \langle A_i^N |, \quad (53)$$

with orthonormality relation from (22)

$$\langle \Omega_i^N | V_{np} | A_j^N \rangle = -\delta_{ij}. \quad (54)$$

The representation of the wave function, using (24), (47) and (52), is now

$$|\psi^{(+)}\rangle = \sum_{i=1}^N |A_i^N\rangle |F_i^{(+)}\rangle, \quad (55)$$

where

$$|F_i^{(+)}\rangle = -\langle \Omega_i^N | V_{np} | \psi^{(+)} \rangle. \quad (56)$$

The nature of the approximation that we have now arrived at can be seen in a different light. The effect of using N Weinberg states to describe the n-p spectrum can be summarised by the replacement of H_{np} by the approximate Hamiltonian

$$H_{np}^N = \sum_{i,j=1}^N |\phi_i\rangle \langle \phi_i | V_{np} H_{np} |\phi_j\rangle \langle \phi_j | V_{np}, \quad (57)$$

such that in the N -dimensional basis of Weinberg states, H_{np}^N and H_{np} have the same matrix representation. The approximate Hamiltonian H_{np}^N is not an hermitian operator, but we have already seen that its matrix representation is similar to a diagonal matrix of its real eigenvalues. In the new basis (52) and (53), H_{np}^N has the representation

$$H_{np}^N = \sum_{i,j=1}^N |A_i^N\rangle \langle \Omega_i^N | V_{np} H_{np} | A_j^N \rangle \langle \Omega_j^N | V_{np}, \quad (58)$$

$$= -\sum_{i=1}^N |A_i^N\rangle [\lambda_i - \varepsilon_d] \langle \Omega_i^N | V_{np}. \quad (59)$$

A similar situation will occur if the Weinberg states are used to represent the n-p spectrum in a channel which does not contain a bound state. By suitable choice of normalisation of the Weinberg states the matrix representation of H_{np}^N can always be made hermitian. Thus the method can be extended to treat n-p channels other than ^3S . The only requirement necessary to preserve the property of similarity to an hermitian

representation of H_{np}^N , is that the weight function $V_{np}(r)$, of the Weinberg states be identical to the interaction part of H_{np} .

The coupled Schrödinger equations (46) are explicitly ($i = 1, \dots, N$)

$$[(E_d - \lambda_i) + i\varepsilon - T_R + W_{ii}(\mathbf{R})]F_i^{(+)}(\mathbf{R}) = i\varepsilon\delta_{i1} N_d e^{i\mathbf{K}_d \cdot \mathbf{R}} + \sum_{j \neq i}^N W_{ij}(\mathbf{R})F_j^{(+)}(\mathbf{R}), \quad (60)$$

and it is seen that the incoming deuteron boundary condition is still confined to the first channel. This result follows from a special property of the transformation matrices A^{-1} , A for the change of basis. Because the deuteron bound state is the first Weinberg state, the matrices A^{-1} , A are easily seen to have a first column equal to $\text{col } \{1, 0, 0, \dots, 0\}$. Thus (52) gives $|A_1^N\rangle = |\phi_1\rangle = N_d^{-1}|\phi_d\rangle$, and from (56) the incoming components of channel functions $F_i^{(+)}$ are

$$-\langle \Omega_i^N | V_{np} | \phi_d \rangle | \mathbf{K}_d \rangle = \delta_{i1} N_d | \mathbf{K}_d \rangle. \quad (61)$$

Since the coupling potentials $^\dagger W_{ij}(\mathbf{R})$ have a finite range typically of the size of the target nucleus plus the range of $V_{np}(\mathbf{r})$, the form of the channel wave functions outside the range of the $W_{ij}(\mathbf{R})$ is given by

$$F_i^{(+)}(\mathbf{R}) \sim \delta_{i1} N_d e^{i\mathbf{K}_d \cdot \mathbf{R}} + a_i(\hat{\mathbf{K}}_i, \hat{\mathbf{R}}) \frac{e^{iK_i R}}{R}, \quad (62)$$

where

$$\frac{\hbar^2}{4m} K_i^2 = E_d - \lambda_i. \quad (63)$$

As discussed in the next subsection, only the diagonal coupling potentials have a Coulomb component which survives at large R , and the asymptotic form (62) is to be modified in the usual way by using the appropriate Coulomb functions.

The first channel has the elastic deuteron wave number since $\lambda_1 = 0$. The channels for which $(E_d - \lambda_i)$ is negative, decay exponentially outside the range of the potentials W_{ij} , and the channel wave numbers (63) in these cases are replaced by

$$K_i \rightarrow i\gamma_i, \quad \gamma_i = \left[\frac{4m}{\hbar^2} (\lambda_i - E_d) \right]^{\frac{1}{2}}. \quad (64)$$

In a given practical application the number of open and closed channels is determined by the incident deuteron kinetic energy E_d and the eigenvalues λ_i . The latter in turn are determined by the n-p interaction and the number N of states in the basis. As N is increased with E_d fixed, we expect that the eigenvalues λ_i will span an increasing range of positive energies (to simulate the n-p continuum) and the number of open channels will increase. The minimum value of N sufficient to obtain the stripping

[†] We note the non-symmetry of the coupling potentials in the channel indices, which, even in the case where the nucleon-core interactions are real, will produce absorption effects. We hope to include a discussion of this topic in a future article.

amplitude to a given accuracy must be determined by a numerical calculation. Note that eq. (23), with i replaced by N gives a qualitative relationship between N and the domain $0 \leq r \leq r_N$ over which we expect the truncated Weinberg representation of the wave function to be accurate. The above expected behaviour of the eigenvalues λ_i as N is increased, has been borne out in a preliminary calculation in which V_{np} was taken to be of Hulthén form and $N < 8$. This potential is convenient since the Weinberg states are known analytically.

The exact three-body wave function has break-up asymptotic outgoing components associated with n-p relative energies in the continuous range $[0, E]$. The approximate method we have developed for the projected wave function $V_{np}|\psi^{(+)}\rangle$ is capable only of selecting a discrete number of open break-up channels with n-p relative energies and eigenstates given by (59). We emphasise that the number (N) and therefore the nature of these discrete n-p eigenvalues and eigenstates is determined solely by the accuracy required for $V_{np}|\psi^{(+)}\rangle$ in the stripping amplitude. We are thus not concerned with obtaining an accurate representation of $|\psi^{(+)}\rangle$ in the asymptotic non-interacting region where the form (62) is clearly inadequate. The amplitude a_1 in (62) cannot be interpreted as the elastic deuteron amplitude since those break-up effects that have been included by the projector $\langle \Omega_1^N | V_{np}$ are present. Even if the channel $i = 1$ is the only open channel (a situation which is quite likely if only a few n-p eigenstates are used as a basis), $F_1^{(+)}(\mathbf{R})$ will differ from the elastic component $\langle \phi_d | \psi^{(+)} \rangle$.

3.2. THE COUPLING POTENTIALS

To complete this section we discuss some general properties of the coupling potentials and give an approximate formula for their computation. The considerations here are independent of whether the potentials are required in the basis of Weinberg states (29) or in the new basis (51). Ignoring the Coulomb component for the present, (29) becomes in the space of the n-p c.m. momenta

$$\langle \mathbf{K}' | V_{ij} | \mathbf{K} \rangle = -\langle \phi_i | V_{np}(r) \frac{\sin \frac{1}{2} Qr}{\frac{1}{2} Qr} | \phi_j \rangle [V_n(\mathbf{Q}) + V_p(\mathbf{Q})], \quad (65)$$

where $\mathbf{Q} = \mathbf{K}' - \mathbf{K}$, and we have used the locality of V_n , V_p in coordinate space to make this factorization. Now if the significant values of \mathbf{Q} required to compute V_{ij} accurately in coordinate space are such that in the integral over r we may expand the term $\sin x/x$ about $x = 0$, we obtain

$$V_{ij}(\mathbf{Q}) \approx [V_n(\mathbf{Q}) + V_p(\mathbf{Q})] [\delta_{ij} - \frac{1}{24} Q^2 \langle r^2 \rangle_{ij} + \frac{1}{1920} \langle Q^4 r^4 \rangle_{ij} + \dots]. \quad (66)$$

The parameters $\langle r^n \rangle_{ij}$ are dependent only upon properties of the n-p interaction and are given by

$$\langle r^n \rangle_{ij} = -\langle \phi_i | V_{np}(r) r^n | \phi_j \rangle. \quad (67)$$

Transforming into coordinate space and retaining only terms to second order, (65) yields the approximate formula

$$V_{ij}(\mathbf{R}) \approx [\delta_{ij} + \frac{1}{24} \langle r^2 \rangle_{ij} \nabla_{\mathbf{R}}^2] [V_n(\mathbf{R}) + V_p(\mathbf{R})]. \quad (68)$$

The accuracy of this approximation depends only upon the n-p interaction being of shorter range than the nucleon optical potentials. In the case that V_n , V_p are to have simple analytic forms, for example a Woods-Saxon shape for the real parts, and a derivative Woods-Saxon shape for the imaginary parts, the prescription (68) is quite accurate and represents a considerable saving in computational labour. The variation with the channel indices i, j is contained solely in the constants $\langle r^2 \rangle_{ij}$, whereas an exact calculation would require a different double folding integration for each set (i, j) . Two properties of the exact potentials, preserved by the approximate form (68), are the volume integrals and mean square radii, which are given by the relations

$$\int d\mathbf{R} V_{ij}(\mathbf{R}) = \delta_{ij} \int d\mathbf{R} [V_n(\mathbf{R}) + V_p(\mathbf{R})], \quad (69)$$

$$\int d\mathbf{R} R^2 V_{ij}(\mathbf{R}) = \int d\mathbf{R} [R^2 \delta_{ij} + \frac{1}{4} \langle r^2 \rangle_{ij}] [V_n(\mathbf{R}) + V_p(\mathbf{R})]. \quad (70)$$

It is easily verified that the above results are equally true for the coupling potentials $W_{ij}(\mathbf{R})$, defined in the transformed basis, with the parameters $\langle r^2 \rangle_{ij}$ replaced by their transformed values

$$\langle r^2 \rangle'_{ij} = -\langle \Omega_i^N | V_{np}(r) r^2 | \Delta_j^N \rangle. \quad (71)$$

Since the Coulomb component of our effective interaction (35) depends only on R , the Coulomb contribution to the coupling potentials is just $\delta_{ij} V_C(R)$. In fact, for $R >$ half of the range of V_{np} , this contribution is also obtained to high accuracy when the folding of the more obvious choice $V_C(r_p)$ is considered. The corrections to this treatment are associated with Coulomb break-up of the deuteron, and above the Coulomb barrier these effects are generally small compared to the nuclear effects¹⁷⁾.

3.3. A SIMPLE ESTIMATE OF CONVERGENCE

We have previously remarked that the number of basis states to be used in our representation of the three-body wave function is to be determined from the accuracy required of the stripping amplitude. This will be possible only if the coupled Schrödinger equations (60) do not admit significant contributions from coupling to channels of extremely high order. The fact that we have been guided the successful adiabatic theory, and have resolved the wave function into discrete components associated with increasingly higher n-p relative energies, is not in itself an immediate guarantee that the coupled equations (60) can be truncated without loss of accuracy. In this subsection we make use of an extremely idealized situation to obtain an estimate of the magnitude of the extremely high order components of the wave function. The basic requirements for this estimate is a knowledge of the behaviour of the ingredients of the coupled equations with increasing channel index. In particular we require simple estimates of the eigenvalues λ_i , and the strengths of the coupling potentials which we take from (68) to be given by the parameters $\langle r^2 \rangle_{ij}$. These quantities are properties of

the n-p interaction only. We will assume that the dominant physics of the problem will not be seriously altered, if for the purposes of the convergence estimate, we take the n-p interaction to be a square well with strength $-v_0$ and radius r_0 .

We note immediately that for this interaction the Weinberg states form a complete set only in the domain $0 \leq r \leq r_0$, and consequently surface contributions from the edge of the well will in principle arise in our derivation of the coupled channel method in subsect. 2.2. However we only wish to use the resulting forms for the parameters λ_i , $\langle r^2 \rangle_{ij}$ as a guideline to the behaviour expected for a more physical interaction. The square well Weinberg states are from (21)

$$\begin{aligned} \phi_i(r) &\sim \frac{1}{r} \sin k_i r, & r < r_0, \\ &\sim \frac{1}{r} e^{-\gamma_d r}, & r > r_0, \end{aligned} \quad (72)$$

where

$$\frac{\hbar^2}{m} k_i^2 = \alpha_i v_0 - \varepsilon_d, \quad \varepsilon_d = \frac{\hbar^2}{m} \gamma_d^2, \quad (73)$$

and the eigenvalues α_i are determined from the matching condition at r_0

$$k_i \cot k_i r_0 = -\gamma_d. \quad (74)$$

Since $\gamma_d/k_i \ll 1$, the approximate solution $k_i \approx (2i-1)\pi/2r_0$ yields the following form for the eigenvalues

$$\alpha_i \rightarrow \left(\frac{\hbar^2}{m} \frac{\pi^2}{v_0 r_0^2} \right) i^2, \quad i \gg 1. \quad (75)$$

The parameters $\langle r^2 \rangle_{ij}$ from (67) are given by the approximate formula

$$\langle r^2 \rangle_{ij} \approx 2 \left(\frac{r_0}{\pi} \right)^2 (-1)^{i-j} \left[\frac{1}{(i-j)^2} + \frac{1}{(i+j-1)^2} \right], \quad i \neq j, \quad (76)$$

$$\langle r^2 \rangle_{ii} \approx \frac{1}{3} r_0^2 + 2 \left(\frac{r_0}{\pi} \right)^2 \frac{1}{(2i-1)^2}. \quad (77)$$

The calculation of the eigenvalues λ_i is especially simple for the square well, since H_{np} is already diagonal in the basis of Weinberg states. The result is

$$\lambda_i = v_0(\alpha_i - 1), \quad (78)$$

and therefore for large i

$$\lambda_i \rightarrow \frac{\hbar^2}{m} \left(\frac{\pi}{r_0} \right)^2 i^2, \quad i \gg 1. \quad (79)$$

Because of (68) and (77), the strength of the diagonal potentials W_{ii} approaches a constant value as $i \rightarrow \infty$, and thus the only term on the l.h.s. of (60) of any consequence in this limit is λ_i . The magnitude of the channel wave functions for large i can therefore be estimated from (60), using (68), (76), (77) and (79) as

$$F_i^{(+)} \approx -\frac{m}{\hbar^2} \frac{1}{12} \left(\frac{r_0}{\pi}\right)^4 (\nabla_R^2(V_n + V_p)) \sum_{j \neq i}^{\infty} \frac{(-1)^{i-j}}{i^2} \left[\frac{1}{(i-j)^2} + \frac{1}{(i+j-1)^2} \right] F_j^{(+)}. \quad (80)$$

The slowest rate of fall-off of $F_i^{(+)}$ with i is provided by the coupling to the first channel $j = 1$. Taking only this term, we can make an estimate of the ratio of the sum total of all higher order channels to the first channel

$$\sum_{j=2}^{\infty} \frac{F_j^{(+)}}{F_1^{(+)}} \approx -\frac{m}{\hbar^2} \frac{1}{6} \left(\frac{r_0}{\pi}\right)^4 (\nabla_R^2(V_n + V_p)) \sum_{\mu=1}^{\infty} \frac{(-1)^{\mu}}{\mu^4}, \quad (81)$$

$$\approx \frac{m}{\hbar^2} \frac{1}{6} \left(\frac{r_0}{\pi}\right)^4 (\nabla_R^2(V_n + V_p)) \frac{7\pi^4}{720}. \quad (82)$$

For typical strengths of the optical potentials this ratio has the estimate $\frac{1}{40}$. It thus seems plausible to expect that detailed calculations with the truncated set of coupled equations can provide a meaningful solution.

4. Discussion

We have seen that the Weinberg eigenstates of the n-p system provide a convenient basis for the representation of the three-body wave function when the (d, p) stripping amplitude is required. A coupled channel method has been developed for the wave function calculation, and break-up effects are explicitly included. The method provides a convenient framework in which to numerically test the accuracy of the adiabatic theory of deuteron stripping, and to compute any significant corrections. The channel coupling potentials describing the interaction of a continuum n-p pair with the nuclear target are of finite range in the c.m. coordinate of the pair, and are readily calculable in terms of the nucleon optical potentials and the parameters $\langle r^2 \rangle_{ij}$ of the n-p system. These n-p parameters are dependent upon the details of the n-p interaction, e.g. the shape of $V_{np}(r)$. This shape dependence is to be expected since the adiabatic theory, which appears as a suitable lowest order solution to the present theory, employs only the shape-independent properties of the n-p system. We have treated only the ^3S n-p states, but have indicated that our basic method can be applied to the other states of the n-p system.

We note that the Weinberg states that we have employed as a basis are identical to the eigenstates used in the construction of the unitary pole expansion¹⁸⁾ of the two-body ^3S t -matrix into separable terms. This procedure is equivalent to a generalisation of (16) to several terms whose form factors are $|f_i\rangle = V_{np}|\phi_i\rangle$. A representation of the half-shell t -matrix by this method is equivalent to a representation of the projected two-body wave function $V_{np}|\phi_k^{(+)}\rangle$ in the basis of Weinberg states. The known

numerical convergence of these t -matrix expansions for n-p scattering ¹⁹⁾ demonstrates the ability of the Weinberg eigenstates to represent the n-p continuum effects. For the three-body problem however, these expansion techniques have been used chiefly within the context of the exactly soluble models of the Amado type ²⁰⁾. If the approximate method developed here is found successful, it would indicate that for stripping and similar rearrangement processes on a heavy nuclear target, a strict application of these exact models is not absolutely necessary to account for the major three-body physics of these reactions.

Within the context of deuteron stripping this situation can arise because the projected wave function $V_{np}|\psi^{(+)}\rangle$ that is required, satisfies much simpler boundary conditions than the full wave function $|\psi^{(+)}\rangle$. In particular the asymptotic stripping components are eliminated. The exact three-body models, by design, take full account of the three-body boundary conditions in all possible asymptotic channels by means of the primary singularity structure ²¹⁾. In contrast $V_{np}|\psi^{(+)}\rangle$ has only outgoing elastic deuteron waves which are preserved by our method, and outgoing continuum n-p components which are treated approximately in our method. Theoretical considerations ^{21, 22)} indicate that the spectator wave function $\langle\phi_d|V_{np}|\psi^{(+)}\rangle$ [see the discussion after eq. (18)] will have besides the asymptotic elastic deuteron component, an asymptotic continuum component whose slowest rate of fall-off is $R^{-\frac{5}{2}}h(R)$. The function $h(R)$ is oscillatory with a c.m. wave number $(4mE/\hbar^2)^{\frac{1}{2}}$. This continuum component is treated in an approximate fashion in our method for expanding the projected wave function. In a numerical study of deuteron stripping within an exact three-body model, by Bouldin and Levin ²³⁾, a comparison of the elastic and spectator wave functions in R -space indicated significant differences only inside the nucleus. The extent to which this result is model dependent however is not clear. In applications of the adiabatic theory, the elastic and adiabatic wave functions differ by a phase in the nuclear surface ³⁾. We expect that similar behaviour will result from use of our approximate three-body method.

Recently the technique of employing a discrete representation of the n-p continuum has been studied by Rawitscher ⁸⁾ and Austern and Farrell ²⁴⁾ in the related problem of deuteron elastic scattering. The exact scattering eigenstates of H_{np} , chosen at several discrete values of low relative n-p energy, are used to represent the continuum components, and a coupled channel formulation of the problem is obtained. The essential difference between elastic deuteron scattering and stripping is that different projections of the three-body wave functions are required in each case. For stripping we have been able to exploit a basis ideally suited to the required projection $V_{np}|\psi^{(+)}\rangle$. The elastic projection $\langle\phi_d|\psi^{(+)}\rangle$ requires information about $|\psi^{(+)}\rangle$ over a much more extended region, and could well be sensitive to three-body effects that lie outside of the scope of the present treatment. Rawitscher's calculation has indicated that coupling to continuum d-states of the n-p system has a significant effect in elastic scattering. The effects of such states in stripping could be investigated within the framework we have developed in this paper.

Both authors would like to thank Dr. P. J. R. Soper, and Professors G. H. Rawitscher and F. S. Levin for many useful discussions concerning both this work and their own work on related topics. We are also grateful for a very useful comment from Dr. D. M. Brink. The support for this research is provided by a Science Research Council contract.

References

- 1) N. Austern, Direct nuclear reaction theories (Wiley-Interscience, London, 1970)
- 2) R. C. Johnson and P. J. R. Soper, Phys. Rev. **C1** (1970) 976
- 3) J. D. Harvey and R. C. Johnson, Phys. Rev. **C3** (1971) 636
- 4) G. M. McAllen, W. T. Pinkston and G. R. Satchler, Particles and Nuclei **1** (1971) 412;
G. R. Satchler, Phys. Rev. **C4** (1971) 1485;
B. M. Preedom, Phys. Rev. **C5** (1972) 587;
M. Pignatelli, J. Gosset, F. Resmini, B. Mayer and J. L. Escudié, Phys. Rev. **C8** (1973) 2120;
M. D. Cooper, W. F. Hornyak and P. G. Roos, Nucl. Phys. **A218** (1974) 249;
B. Blum, W. Dünnweber and W. R. Hering, Nucl. Phys. **A206** (1973) 321;
C. F. Maguire, W. D. Callender and C. K. Bockleman, P433, and K. Yagi, Y. Aoki, M. Matoba and M. Hyakutake, P432, Proc. of the Int. Conf. on nuclear physics, Munich, ed. by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1974);
H. Ohnuma, T. Suehiro, M. Sekiguchi and S. Yamada, J. Phys. Soc. Japan **36** (1974) 1245
- 5) A. N. Mitra, Phys. Rev. **139** (1965) B1472;
A. S. Reiner and A. I. Jaffe, Phys. Rev. **161** (1967) 935;
P. E. Shanley and R. Aaron, Ann. of Phys. **44** (1967) 363;
J. V. Noble, Phys. Rev. **157** (1967) 939;
A. I. Baz, V. F. Demin and I. I. Kuz'min, Sov. J. Nucl. Phys. **4** (1967) 525, 815;
K. King and B. H. T. McKellar, Phys. Rev. Lett. **30** (1973) 562;
Gy. Bencze, University of Helsinki preprint
- 6) D. Bouldin and F. S. Levin, Phys. Lett. **37B** (1971) 145; Nucl. Phys. **A189** (1972) 449
- 7) A. I. Baz, Nucl. Phys. **51** (1964) 145
- 8) G. H. Rawitscher, Phys. Rev. **C9** (1974) 2210
- 9) J. D. Harvey and R. C. Johnson, J. of Phys. **A7** (1974) 2017
- 10) S. Weinberg, Phys. Rev. **131** (1963) 440
- 11) K. Meetz, J. Math. Phys. **3** (1962) 690
- 12) R. G. Newton, Scattering theory of waves and particles (McGraw-Hill, New York, 1966) p. 282
- 13) A. M. Lane and D. Robson, Phys. Rev. **151** (1966) 774; **185** (1969) 1403
- 14) W. F. Junkin and F. Villars, Ann. of Phys. **45** (1967) 93; **51** (1969) 68
- 15) S. Mukherjee, Nucl. Phys. **A118** (1968) 423
- 16) N. Austern and K. C. Richards, Ann. of Phys. **49** (1968) 309
- 17) C. F. Clement, Nucl. Phys. **66** (1965) 241;
F. P. Gibson and A. K. Kerman, Phys. Rev. **145** (1966) 758
- 18) E. Harms, Phys. Rev. **C1** (1970) 1667
- 19) I. R. Afnan and J. M. Read, Austral. J. Phys. **26** (1973) 449
- 20) R. Aarons, R. D. Amado and Y. Yam, Phys. Rev. **140** (1965) B1291
- 21) T. A. Osborn and D. Bollé, Phys. Rev. **C8** (1973) 1198
- 22) R. G. Newton, Ann. of Phys. **74** (1972) 324
- 23) D. Bouldin and F. S. Levin, Phys. Lett. **42B** (1972) 167
- 24) N. Austern and P. Farrell, private communication
- 25) B. L. Gambhir and J. J. Griffin, Phys. Rev. **C5** (1972) 1856
- 26) Gy. Bencze and P. Doleschall, Phys. Lett. **32B** (1970) 539
- 27) R. C. Johnson and P. J. R. Soper, Nucl. Phys. **A182** (1972) 619
- 28) E. F. Redish, private communication, and Saclay preprint D. Ph.T/74/34, to be published