Adiabatic approximation for nucleus-nucleus scattering

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Abstract. Adiabatic approximations to few-body models of nuclear scattering are described with emphasis on reactions with deuterons and halo nuclei (frozen halo approximation) as projectiles. The different ways the approximation should be implemented in a consistent theory of elastic scattering, stripping and break-up are explained and the conditions for the theory's validity are briefly discussed. A formalism which links few-body models and the underlying many-body system is outlined and the connection between the adiabatic and CDCC methods is reviewed .

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1. ADIABATIC APROXIMATIONS

The use of adiabatic approximations[1] in nuclear reaction theories has a long history. The key idea is to separate the relevant degrees of freedom into 'slow' and 'fast' categories. The 'slow' variables are treated as fixed during the collision and the associated scattering problem for the 'fast' variables is treated quantum mechanically or semiclassically depending on the masses and energies involved in the reaction. This is the analogue of the Bohr-Oppenheimer approximation for bound states of molecules, where the electronic motion for fixed nuclear positions is calculated quantum mechanically.

An early example is Barrett's treatment of neutron scattering by a nucleus regarded as a rigid rotor[1]. He calculated the scattering amplitude for the scattering of a neutron by a deformed potential as a function of the orientation of the nuclear body-fixed axes. Scattering amplitudes were then calculated by taking matrix elements of this amplitude between the nuclear states of interest. Here the 'slow' motion is obviously the nuclear collective rotation and the incident neutron motion is regarded as 'fast'. Other applications of the adiabatic approximation to theories of elastic and inelastic scattering in nuclear physics are reviewed in ref.[2], pages 83-84 and elsewhere in that book.

The approximation in the energy domain which complements this time picture, is that the energy associated with relevant rotational excitations is assumed to be small compared to the translational energy of the neutron. In the adiabatic limit all excited rotational states are assumed to be effectively degenerate on a scale determined by the incident energy. We will see how this comes about formally below. This way of thinking tends to give too conservative an idea of the usefulness of the adiabatic approximation because it fails to take into account the crucial role played by absorption in nuclear reactions. In addition, we now have a better understanding of the spatial regions in which adiabatic solutions of the few-body Schrodinger equation are expected to be most valid and how to exploit this knowledge in applications to particular reaction channels.

Historically an important feature of the adiabatic approximation was that it is a cheap way of doing a complicated coupled channels calculation. For example, in Barrett's calculation, channels with a fast neutron and the rotor in any one of its excited states are taken into account coherently and non-perturbatively. With modern computing power this may not be a big advantage, but when the projectile is loosely bound and the relevant excitation spectrum is in the continuum the adiabatic approximation can be a powerful tool as well as frequently providing important insight and checks of more complete calculations. CDCC calculations, which were pioneered and developed in [3],[4],[5],[6], discretise the continuum and are in principle an improvement over the adiabatic approximation, but CDCC codes are generally available only for 2-body projectiles. See [7] and [8] for recent reviews. The first CDCC calculations for a 3-body projectile have only just been published[9]. An adiabatic code which treats 3-body projectiles has been available for some time[10].

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[1] in "Reaction Mechanims for Rare Isiotope Beams", 2nd Argonne/MSU/JINA/INT RIA Workshop, MSU, East Lansing, 9-12 March. 2005 (ed. B.A.Brown, AIP Conf Procs. Vol 79) pp128-139.

In this historical context we note that the adiabatic approximation in the sense we use here is the basis of Glauber's theory[11] of high energy composite particle scattering which has been widely used in the analysis of reaction experiments with halo and other light nuclei[7],[12]. In these calculations selected co-ordinates are treated adiabatically and the eikonal approximation is used to describe the scattering of the frozen object. In his development of a microscopic theory of the nucleon optical potential, Glauber goes further and treats *all* the internal co-ordinates of the target nucleus as frozen during the scattering. The resulting 2-body problem for a set of frozen internal nuclear co-ordinates is then solved using the eikonal approximation. It is one of the purposes of the present paper to advertise the fact that the adiabatic approximation can be useful even when the eikonal approximation breaks down.

2. DEUTERON-NUCLEUS COLLISIONS

This section discusses a 3-body model of the system $n+p+A$, where A is a heavy nucleus in its ground state. The theory we describe can, and has been, applied to other systems and we shall mention several in passing. Some cases, for which the adiabatic approximation is potentially useful, have special problems, e.g., Coulomb and antisymmetrisation effects. These are best discussed separately and will be ignored here.

In a 3-body model of deuteron-nucleus collisions, for example, channels corresponding to elastic deuteron scattering and elastic deuteron break-up in which the target is left in its ground state are all included in a unified way. Excited states of the target A do not appear explicitly. The relation between this model and the underlying many-body problem will be discussed below.

In a widely used notation we use $\vec{r}$ for the position of the neutron relative to the proton and $\vec{R}$ for the position of the centre-of-mass of n and p relative to A. The Hamiltonian of the model is

$$H = T_R + H_{np} + V(\vec{R},\vec{r}),\ V(\vec{R},\vec{r}) \equiv V_{nA}(\vec{R}+\vec{r}/2) + V_{pA}(\vec{R}-\vec{r}/2) \tag{1}$$

where $H_{np} = T_r + V_{np}$ is the Hamitonian for relative motion of the $n-p$ system. The T's are kinetic energy operators. For the purpose of this talk we assume that all Coulomb interactions are screened at large distances. We use $\phi_0(\vec{r})$ for the ground state of the deuteron with energy $-\varepsilon_0 < 0$, and $\phi_{\vec{k}}^{(+)}(\vec{r})$ for the continuum of $n-p$ scattering states which are eigenstates of H_{np} with energy $\varepsilon_k > 0$ and satisfy outgoing wave boundary conditions. It is the purpose of the adiabatic approximation to treat the coupling between the deuteron and the scattering state continuum in as accurate and transparent a way as possible. In the 3-body model this coupling comes from the tidal forces generated by the fact that, over the volume of the deuteron, $V_{nA}(\vec{R}+\vec{r}/2)$ and $V_{pA}(\vec{R}-\vec{r}/2)$ generate forces on the nucleons which differ in magnitude and direction.

2.1. Time dependent picture

Under the transformation

$$\Psi^{trans} = \exp(-\frac{iH_{np}t}{\hbar})\Psi. \tag{2}$$

the time dependent Schrodinger equation for $\Psi(\vec{R},\vec{r},t)$ becomes

$$(T_R + V(\vec{R},\vec{r}(t)))\Psi^{trans}(\vec{R},\vec{r},t) = i\hbar\frac{\partial\Psi^{trans}}{\partial t}. \tag{3}$$

In eq.(3) the $n-p$ relative co-ordinate $\vec{r}(t)$ has acquired a time dependence through the relation

$$\vec{r}(t) = \exp(\frac{iH_{np}t}{\hbar})\vec{r}\exp(-\frac{iH_{np}t}{\hbar}). \tag{4}$$

The adiabatic approximation assumes that the collision time T is so short that we can replace $\vec{r}(t)$ by $\vec{r}(0) = \vec{r}$. A sufficient condition for this step to be valid is that T satisfy

$$\left|\frac{< H_{np} > T}{\hbar}\right| \ll 1. \tag{5}$$

where $< H_{np} >$ is the maximum eigenvalue of H_{np} excited in the collision through the tidal forces. For the strong interaction this maximum is related to the shape of the nuclear surface and is insensitive to the incident deuteron energy. Hence for sufficiently high energy the collision time will always become small enough that the condition (5) is satisfied.

Implementation of the adiabatic approximation for a stationary state requires the solution of the adiabatic equation[13]

$$(T_R + V(\vec{R},\vec{r}) - E_d)\Psi^{ad}(\vec{R},\vec{r}) = 0, \tag{6}$$

which for fixed $\vec{r}$ is a 2-body problem. Note that even for central V_{nA} and V_{pA} the potential in eq. (6) is not central when considered as a function of $\vec{R}$ for fixed $\vec{r}$ so coupled equations still have to be solved in general.

2.2. Solution of the adiabatic equation

The solution of the adiabatic equation can be reduced to a manageable set of coupled equations by either of two methods which are based on different truncation schemes. Both methods assume the nucleon potentials V_{nA} and V_{pA} are central.

(i) In this method[3],[4],[14][15] the adiabatic wavefunction is expanded in the basis $[Y_l(\hat{r}) \times Y_L(\hat{R})]_{JM}$ and uses the fact that $V_{nA} + V_{pA}$ is diagonal in $J, M,$ although not in l and L. Truncation in these angular momenta is required.

(ii) The method used by Barrett[1] uses the fact that $V_{nA} + V_{pA}$ is diagonal in $\vec{L}.\hat{r}$ and proceeds by making a truncated multipole expansion of the potentials.

Method (ii) is well adapted to the case of scattering by a deformed nucleus when a natural truncation of the multipole expansion occurs. In the present context, convergence in l is found to be very rapid and truncation is linked with the adiabatic assumption of low excitations in the $\vec{r}$ co-ordinate.

The adiabatic wavefunction corresponding to a deuteron incident with momentum $\vec{K}_d$ has the structure

$$\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}), \tag{7}$$

where $\chi^{ad(+)}_{\vec{K}_d}$ satisfies

$$(T_R + V(\vec{R},\vec{r}) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = 0, \tag{8}$$

with the boundary condition ($\vec{r}$ fixed)

$$\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) \overset{R\to\infty}{\to} \exp(i\vec{K}_d.\vec{R}) + f(\hat{R},\vec{r})\frac{\exp(iK_dR)}{R}. \tag{9}$$

The function $\chi^{ad(+)}_{\vec{K}_d}$ and the scattering amplitude $f(\hat{R},\vec{r})$ both depend on $\vec{r}$. The physical meaning of $f(\hat{R},\vec{r})$ is that it describes the elastic scattering in the direction $\hat{R}$ of an $n-p$ pair with fixed separation $\vec{r}$ by the potential $V(\vec{R},\vec{r})$. The factor ϕ_0 in (7) ensures that the coefficient of the plane wave in eq.(7) and the exact 3-body wave function $\Psi^{(+)}_{\vec{K}_d}(\vec{R},\vec{r})$ coincide. The formula (9) can be derived by taking the adiabatic limit of formal expressions for the exact three-body wavefunction[16].

The adiabatic equation must be solved for as many values of $\vec{r}$ as is required for the application. For example, for elastic deuteron scattering we have to evaluate the scattering amplitude $f(\hat{R},\vec{r})$ for as many values of $\vec{r}$ as is required for the accurate evaluation of the integral

$$f_{elastic}(\hat{R}) = \int d\vec{r}\,\phi^*_o(\vec{r})\,f(\hat{R},\vec{r})\,\phi_o(\vec{r}), \tag{10}$$

i.e., for $0 < r < r_d$, where r_d is a measure of the size of the deuteron.

This formalism can be simplified considerably in several interesting cases. When the conditions are such that eq. (6) can be solved in the eikonal approximation an explicit formula for $f(\hat{R},\vec{r})$ in terms of a path integral of $V_{nA} + V_{pA}$ can

be obtained. This is Glauber's theory of deuteron-nucleus scattering. The method has been extensively applied to the calculation of elastic scattering and reaction cross-sections of halo nuclei with 2 or more clusters[7],[17]. The integral in (10) is carried out numerically without further approximation of the expression using the best available models for the ground state wave function ϕ_0.

There are also considerable simplifications in the zero range limit of deuteron stripping or when one of the potentials V_{nA} and V_{pA} vanishes. These cases will also be discussed below.

The non-eikonal adiabatic method has been applied to the elastic scattering of ^{6}Li in an $\alpha + d$ model[18], ^{7}Li in an $\alpha + t$ model[19] and to the scattering of ^{11}Li in a $n + n$+^{9}Li 3-body model[10].

It is shown in[16] that estimates based on eq.(5) are too conservative. For short range forces the collision time decreases as the impact parameter increases. Hence for a given range of excitation energies the worst violations of the inequality (5) tend to occur at low impact parameters. Under conditions of strong absorption these are just the impact parameters whose contributions to the excitation are suppressed. A revised adiabatic condition which includes this effect is given in ref.[16]. By comparison with CDCC calculations in a special case the revised condition is shown to give an excellent idea of the accuracy of the adiabatic approximation for a model of ^{11}Be scattering.

2.3. An instructive special case

We show the result for deuteron scattering as an example. When one of the interactions V_{nA}, V_{pA} vanishes (or is a constant) the adiabatic equation can be solved exactly in a very simple way. We take $V_{nA} = 0$ for definiteness. This is obviously not a very realistic model of elastic deuteron scattering in general, but it is very relevant to Coulomb break-up of the deuteron[23]. Eq.(6) becomes

$$(T_R + V_{pA}(\vec{R} - \vec{r}/2) - E_d)\Psi^{ad}(\vec{R}, \vec{r}) = 0. \tag{11}$$

For a deuteron incident with momentum $\vec{K}_d$ this has the exact solution[20][21][22]

$$\Psi^{ad}_{\vec{K}_d}(\vec{R}, \vec{r}) = \phi_0(\vec{r}) \exp(i\vec{K}_d.\vec{r}/2)\chi^{(+)}_{\vec{K}_d}(\vec{R} - \vec{r}/2), \tag{12}$$

where $\chi^{(+)}_{\vec{K}_d}$ is the distorted wave for a particle with the mass of the deuteron by the potential V_{pA} and evaluated at the argument $\vec{R} - \vec{r}/2$ i.e., at the $p - A$ relative co-ordinate.

In this limit the elastic deuteron cross-section is simply expressed in terms of the deuteron ground state form factor and the deuteron elastic cross-section generated by V_{pA}. In the generalisation to the case of a projectile with unequal mass clusters the factors of $1/2$ are replaced by ratios involving the masses of the clusters [20]. The generalisation gives a good account of some features of ^{11}Be scattering[20].

The explicit form (12) also makes a deficiency of the adiabatic wavefunction very clear. It predicts that for any $\vec{R}$, $\Psi^{ad}_{\vec{K}_d} \to 0$ exponentially for $r \to \infty$, i.e, in regions of space where we look for outgoing waves in the stripping and break-up channels. We therefore cannot expect the adiabatic wavefunction to be accurate for large r even though it may be perfectly adequate for finite values of r. The reason for this shortcoming can be traced to the treatment of the break-up continuum as degenerate. It is then not possible for the 3-body wavefunction to carry the phase relations between the R and r dependence which are essential to generate the correct asymptotic form in rearrangement and break-up channels. We must use the adiabatic wavefunction in ways which respect this observation. We do this by using it as the basis for an iterative solution to the Schrödinger equation.

2.4. Iteration of the adiabatic solution

We re-write the 3-body Schrödinger equation in the form

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \quad = \quad V_{np}\Psi_{\vec{K}_d}, \tag{13}$$

where we have transferred the V_{np} term to the right-hand-side. This term requires $\Psi_{\vec{K}_d}$ only within the range of V_{np}, and for which we can therefore use the adiabatic wavefunction if the adiabatic conditions are satisfied[13].

The method proceeds by treating the equation

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \quad = \quad V_{np}\Psi^{ad}_{\vec{K}_d}, \tag{14}$$

as an inhomogeneous equation for $\Psi_{\vec{K}_d}$ with the right-hand-side given. In particular, by examining the outgoing Green's function for the operator $E - T_R - T_{np} - V_{pA} - V_{nA}$ it is found that the iterated solution has the correct asymptotic form in the stripping and break-up channels with outgoing waves with momenta correctly given by the conservation of energy, i.e., without the assumption of degenerate break-up channels used in the adiabatic wavefunction. In the exact solution of the inhomogeneous equation the coefficients of the outgoing waves in the stripping and break-up channels (transition amplitudes) are given by

$$T_{d,p} \quad = \quad \langle \chi^{(-)}_p \phi_n \mid V_{np} \mid \Psi^{ad}_{\vec{K}_d} \rangle, \tag{15}$$

$$T_{d,np} \quad = \quad \langle \chi^{(-)}_p \chi^{(-)}_n \mid V_{np} \mid \Psi^{ad}_{\vec{K}_d} \rangle, \tag{16}$$

where the $\chi^{(-)}_p$ and $\chi^{(-)}_n$ are distorted waves generated by V_{pA} and V_{nA}, respectively, and ϕ_n is a neutron bound state, all evaluated at the correct energies predicted by energy conservation.

Strictly speaking, to deduce (15) and (16) from eq.(14) requires the additional assumption that the target has infinite mass, $A \to \infty$. It is only then that the kinetic energy separates into n and p terms and solutions of the homogeneous equation have a product form. Recoil terms of order $1/A$ can mix in terms in the final state in which the neutron is excited out of the state ϕ_n. These corrections (Recoil Excitation and Break-up (REB) effects) can be very significant for light nuclei[24].

All the quantities in (15) and (16) are solutions of 2-body problems and are readily calculated. The evaluation of the amplitudes requires techniques similar to those used in the evaluation of DWBA amplitudes. We emphasise that this iterated theory goes far beyond DWBA. No Born approximation is involved. Couplings between 3-body channels are included to all orders in $\Psi^{ad}_{\vec{K}_d}$. It should be clear that if $\Psi^{ad}_{\vec{K}_d}$ were replaced in equations (15) and (16) by the exact three-body wavefunction $\Psi_{\vec{K}_d}$ then these expressions would give the exact reaction amplitudes. The approximations (15) and (16) assume that all coupling effects in the wavefunction for r less than the range of V_{np} can be adequately accounted for by the adiabatic wavefunction. In principle this whole procedure could be iterated by calculating the complete solution of the inhomgeneous equation (13) (not just the asymptotic form as explained above) and then using the solution to re-calculate an improved inhomogeneous term. As far as I know this has never been done.

In the zero range limit for V_{np} the evaluation of equations (15) and (16) becomes particularly simple because then we can use[13]

$$V_{np}\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)..\tag{17}$$

From eq.(8) we see that $\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$ satisfies

$$(T_R + V(\vec{R},0) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0) = 0, \tag{18}$$

and is, therefore, simply a distorted wave generated by the central potential $V_{nA}(R) + V_{pA}(R)$.

We see that in this limit the adiabatic theory of stripping looks even more like a DWBA theory, but this is misleading because the function $\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$ includes outgoing waves in break-up channels and the potential $V_{nA}(R) + V_{pA}(R)$ may have very little to do with elastic deuteron scattering.

When s-wave break-up dominates at small r the formalism can be modified to correct for a finite range V_{np}[13]. We expand the three-body wavefunction in terms of the complete set of $n-p$ relative motion states $\{\phi_0, \phi^{(+)}_{\vec{k}}\}$ introduced earlier. Provided the continuum states which contribute do not have very high energy we can safely assume that only s-wave states will overlap V_{np} and we can write

$$\Psi_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(r)\chi_o(\vec{R}) + \int d\vec{k}\phi^{(+)}_k(r)\chi_k(\vec{R}),\ r < r_{np}, \tag{19}$$

where r_{np} is the range of V_{np}. For continuum energies less than roughly 10 MeV the shape of the s-wave state $\phi^{(+)}_k(r)$ does not depend strongly on energy for $r < r_{np}$ and we can write

$$\phi_k(r) \simeq g(k)\phi_0(r),\ r < r_{np}, \tag{20}$$

where $g(k)$ is independent of r. Inserting this approximation into (19) gives

$$V_{np}\Psi_{\vec{K}_d}(\vec{R},\vec{r}) \simeq V_{np}\phi_0(r)\bar{\chi}_{\vec{K}_d}(\vec{R}), \tag{21}$$

where

$$\bar{\chi}_{\vec{K}_d}(\vec{R}) = \chi_o(\vec{R}) + \int d\vec{k} g(k)\chi_k(\vec{R}). \tag{22}$$

We emphasise that the form implied by (21) for the 3-body wavefunction is generally only valid inside the range of V_{np}. The r and R dependence of the adiabatic wavefunction does not factorise in this way in general (see, e.g., eq.(12)).

In the next subsection we will show how these qualitative ideas can be systematically exploited to give an equation for $\bar{\chi}_{\vec{K}_d}(\vec{R})$.

2.5. The method of Johnson and Tandy

A more general approach to obtaining $V_{np}\Psi$, the projection of the 3-body wavefunction which is most relevant to the transition amplitude for stripping, and break-up according to (15) and (16), is to expand in terms of a set of functions which are complete within the range of V_{np}.

A convenient set for this purpose is the set of Weinberg states, or Sturmians, $\bar{\phi}_i(r),\ i = 0\ldots\infty$, used by Johnson and Tandy[25]. They satisfy

$$(T_r + \alpha_i V_{np})\bar{\phi}_i = -\varepsilon_0\bar{\phi}_i \tag{23}$$

where the α_i's are Sturmian eigenvalues. For $i = 0$, $\alpha_0 = 1$ and $\bar{\phi}_0$ is proportional to the deuteron ground state ϕ_0. These states all look like the deuteron asymptotically, but as i and α_i increase they oscillate more and more rapidly at short distances.

An expansion in terms of this set converges rapidly if the dependence on r of the three-body wave function inside the range of V_{np} is similar to ϕ_0. Coupled equations for the coefficients are readily derived using the orthogonality property

$$\langle\bar{\phi}_i \mid V_{np} \mid \bar{\phi}_j\rangle = -\delta_{i,j}. \tag{24}$$

The first term in the expansion has the form $\phi_0\bar{\chi}$ with $\bar{\chi}$ defined by

$$(E_d - T_R - \bar{V}(R))\bar{\chi}_{\vec{K}_d}(\vec{R}) = 0, \tag{25}$$

where the potential $\bar{V}$ is given by

$$\bar{V}(R) = \frac{\langle\phi_0 \mid V_{np}(V_{nA}+V_{pA}) \mid \phi_0\rangle}{\langle\phi_0 \mid V_{np} \mid \phi_0\rangle}. \tag{26}$$

The bra and ket in this equation imply an integration over $\vec{r}$ with fixed $\vec{R}$. $\bar{V}$ reduces to the zero-range result $V(R,0)$ of eq.(18) if the variation of the nucleon optical potentials over a distance of the order of r_{np} can be neglected. For nucleon potentials with a Wood-Saxon shape the effect of the finite range correction in eq.(26) is to increase their diffuseness slightly. A simple way of incorporating these modifications, which can be important for light nuclei, can be found in refs.[26],[27]. Results that take into account terms in the expansion in $\bar{\phi}_i$'s beyond $i = 0$ are given in refs.[25],[28] in specific cases. They show that this is a promising approach to the calculation of break-up effects in stripping which go beyond the adiabatic approximation.

An interesting feature of this derivation is that it makes no reference to the incident energy but only the assumption that the break-up states excited have low enough excitation that the 3-body wave function is well approximated by the form $\phi_0(r)\bar{\chi}(\vec{R})$ inside the range of V_{np}. This suggests that a stripping theory which takes into account break-up effects can be based on the use of $\bar{\chi}$ as a distorted wave even at low energies where the adiabatic condition is not well satisfied.

The situation for elastic deuteron scattering is quite different because there the adiabatic wave function is needed out to distances of the order of the size of the deuteron where the form $\phi_0(r)\bar{\chi}(\vec{R})$ has no justification. The use of Sturmians is not then appropriate.

There have been many comparisons between theory based on eqs. (15) with $V_{np}\Psi^{ad}$ given by (21), (25), (26) and stripping experiments. We call this the Adiabatic Distorted Wave Approximation (ADWA). Over a wide range of energies the ADWA has gives angular distributions for differential crosssections and polarization observables which agree with stripping and pick-up experiments more convincingly and consistently than the DWBA and without the extra ambiguities associated with the use of a deuteron optical potential in the DWBA. Everything in an ADWA calculation is determined by *nucleon* optical potentials for the appropriate energy and target. Some early examples are given in ref.[2], pages 732-734 but there have been many others since, *e.g.*[45]. The method has also been successfully used for (p,d*) [29] and (d,^{2}He)[30] charge exchange reactions.

The study of Cadmus and Haeberli[31] is particularly noteworthy. These authors measured a large number of deuteron and proton elastic scattering observables to pin down optical model parameters so that the DWBA could be applied unambiguously. It was found to fail badly. They were able to use their measurements of deuteron and proton polarization parameters to identify the source of the failures and how these were remedied by the ADWA.

A more recent example of how the ADWA can be used to give an improved account of the systematics of a particular (d,p) transition as function of energy is ref.[32].

Although the ADWA goes well beyond the DWBA and includes effects due to coupling between the elastic deuteron channel and other 3-body channels to all orders it is nevertheless an approximate theory which is expected to need correction at some level. An important example of a clear indication from experiment of the need to go beyond the ADWA theory and the nature of those corrections is the work of the Indiana-Surrey collaboration[33].

One way of going beyond the ADWA for stripping and pick-up is to use the Sturmian expansion method of refs.[25],[28]. An alternative is to use the CDCC wavefunction in (15). For deuteron stripping this is done in refs.[44],[46]. Ref.[47] reports a surprisingly large discrepancy between measured proton polarisation and CDCC predictions for ^{208}Pb(d,p)^{209}Pb at 20 MeV incident energy. Other observables are well reproduced.

2.6. Elastic Coulomb break-up

An interesting application of the expression (16) is the case of Coulomb break-up of a 2-body projectile where one body is uncharged and we can neglect its interaction with the target. In the deuteron case, for example, we can then use eq.(12) and the matrix element factorises[23] as

$$T^{ADWA}_{d,np} = \left(\int d\vec{r} \exp(i(\vec{k}_n - \frac{1}{2}\vec{K}_d).\vec{r}) V_{np}\phi_0(\vec{r}) \right) \int d\vec{r}_p \chi^{(-)*}_{\vec{k}_p}(\vec{r}_p) \exp(-i\vec{k}_n.\vec{r}_p) \chi^{(+)}_{\vec{K}_d}(\vec{r}_p). \tag{27}$$

where $\chi^{(+)}_{\vec{K}_d}$ and $\chi^{(-)}_{\vec{k}_p}$ are distorted waves describing the scattering of a point deuteron and a proton by the Coulomb field of the target. For a very large screening radius the second factor has the form of an unobservable phase factor which goes to infinity with the screening radius, multiplied by an integral which is similar to that which occurs in the theory of Bremsstrahlung and can be evaluated analytically for a point target. The first factor is easily evaluated for any V_{np}. The restriction to $A \to \infty$ is easily lifted[23].

This theory has been applied successfully to Coulomb break-up of the deuteron[23], ^{11}Be[34], ^{6}He [35] and ^{19}C[34],[36]

We emphasise that the theory which leads to (27) is not perturbation theory. Terms of all orders in V_{pA} and V_{np} are included. The effects of coupling between Coulomb break-up channels are included in all orders with the 2 assumptions that the adiabatic approximation is valid and that the nuclear interaction between the neutron and the target can be neglected.

A DWBA theory which is often used for Coulomb break-up starts from the expression

$$T^{DWBA}_{d,np} \quad = \quad \langle \chi^{(-)}_p \vec{k}_n \mid V_{np} \mid \Psi^{elast}_{\vec{K}_d} \rangle, \tag{28}$$

where the elastic deuteron wavefunction $\Psi^{elast}_{\vec{K}_d}$ has the form

$$\Psi^{elast}_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_o(\vec{r}) \chi^{(+)}_{\vec{K}_d}(\vec{R}). \tag{29}$$

For the case of Coulomb break-up $\chi^{(+)}_{\vec{K}_d}(\vec{R})$ is a Coulomb wavefunction describing the scattering of a point deuteron in the Coulomb field of the target.

The input data required for the 2 expressions (27) and (28) are identical, i.e., V_{np}, and the Coulomb potential of the target, although they are based on very different physical assumptions. The DWBA expression assumes that the coupling between deuteron elastic and break-up channels is small and can be treated in first order. The adiabatic expression makes no such approximation but instead makes the assumption[2] that any break-up channels that are relevant for the 3-body scattering wavefunction inside the range of V_{np} have low energy compared with the incident deuteron energy.

The DWBA amplitude for Coulomb break-up involves a 6-dimensional integration. Various approximations, including the zero-range approximation for V_{np}, have invariably been made to simplify its evaluation. Recently Zadro[36],[37] has published a momentum-space technique for the exact evaluation of the DWBA amplitude with a finite range V_{np}. This has enabled a meaningful comparison to be made with the ADWA theory. He studied ^{11}Be$\rightarrow^{10}$Be$+n$ and ^{19}C$\rightarrow^{18}$C$+n$ elastic break-up on a ^{208}Pb target at energies near 70 MeV/nucleon. Both theories give very similar projectile-fragment relative energy distributions in quite good agreement with experiment[38] but the predicted DWBA crosssection magnitudes for a 2-body projectile model are up to 50% bigger. Crosssections for break-up into states of more than a few MeV are very small. The adiabatic approximation therefore ought to be excellent at 70MeV/nucleon[16]. This suggests that spectroscopic factors obtained by comparison of predicted DWBA crosssections with break-up data may be significantly underestimated.

3. THE UNDERLYING MANY-BODY THEORY

Our presentation so far is based on the 3-body Hamiltonian (1) in which V_{nA} and V_{pA} are optical potentials. These are usually taken at $\frac{1}{2}$ of the incident deuteron kinetic energy E_d. This is reasonable if any break-up components in the 3-body wave function have a small fraction of E_d and is certainly consistent with the adiabatic assumption. More generally the $\frac{1}{2}E_d$ prescription can be justified[39] by detailed calculation if the energy dependence arises purely from non-locality and break-up effects are negligible. A deeper question is why the effective interaction in the 3-body model should have anything to do with the nucleon optical potential.

To study this further we recall that the 3-body wavefunction in eq.(3) is the projection of the full many-body $A+2$ wavefunction onto the target ground state. In a standard fashion[40] we can show that the effective Hamiltonian which governs this component when the target is in its ground state in the incident channel is

$$H_{eff} = T_R + H_{np} + \langle \phi_A \mid U \mid \phi_A \rangle, \tag{30}$$

where the bra-ket notation implies integration over the target nucleus co-ordinates to leave an operator in n and p co-ordinates only. The complicated many-body operator U satisfies

$$U = (v_{nA} + v_{pA}) + (v_{nA} + v_{pA})\frac{Q_A}{e}U, \;\; v_{NA} = \sum_{i=1}^{A} v(N,i). \tag{31}$$

The v_{NA}'s, $N = p,n$, are the sums of the 2-body interactions between the incident p and n and the target nucleons $1...A$. The operator Q_A projects on to excited states of the target. U sums up all processes via excited states which begin and end on the target ground state.

U can be separated into its p and n contributions by using manipulations from multiple scattering theory. We obtain

$$U = (U_{nA} + U_{pA}) + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + ..., \tag{32}$$

where

$$U_{nA} = v_{nA} + v_{nA}\frac{Q_A}{e}U_{nA}, \;\; U_{pA} = v_{pA} + v_{pA}\frac{Q_A}{e}U_{pA}, \tag{33}$$

and the dots in (32) are terms of 3rd or higher order in U_{nA} and/or U_{pA}, always with an excited target (though not necessarily excited deuteron) as intermediate state.

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[2] Note that because we use the solution (12) in this case we have no need for the extra assumptions about the break-up spectrum which lead to (21).

The above expressions for U_{nA} and U_{pA} are strongly reminiscent of Feshbach's[40] expressions for the operator which gives the nucleon optical potential when sandwiched between the target ground state. Note however that the energy denominator e which appears everywhere is not the denominator one expects to see in the nucleon operator. It is given (for an infinitely massive target) by $e = E + i0 - T_n - T_p - V_{np} - H_A$ where H_A is the target Hamiltonian. It is plausible that if low energy weakly correlated break-up states dominate, then in U_{nA}, for example, we can neglect V_{np} and replace $E - T_p$ by $\frac{1}{2}E_d$ on the average. $\langle \phi_A \mid U_{nA} \mid \phi_A \rangle$ then reduces to a formal expression for the neutron optical potential at energy $\frac{1}{2}E_d$.

The higher order terms in (32) still have to be dealt with, however. The second order terms describe a process in which the neutron excites the nucleus and the proton subsequently de-excites it, and vice versa. The magnitude of such effects will be small for weakly correlated $n-p$ configurations such as in the deuteron or low energy break-up configurations. Their neglect is consistent with approximations already made.

We learn from this analysis that the validity of the 3-body model as usually assumed is intimately bound up with the assumption of dominance of low energy break-up configurations. However, all the arguments given above are very qualitative. Very little work has been done to give substance to them and estimate any corrections to the usual model. It would seem to be hardly worth while to go much beyond the adiabatic or CDCC treatments of three-body effects, both of which assume that break-up excitations can be truncated, without investigating many-body corrections to the 3-body Hamiltonian, eq.(1), more thoroughly.

Finally in this section, note that even in the lowest order version of the effective interaction one expects to see corrections arising from the identity of n and p and the target nucleons. There are essentially two distinct approaches to these antisymmetrization effects. The RGM methods starts from a many-body Hamiltonian with an assumed N-N interaction and puts in antisymmetrization right from the start, treating the nucleons in the deuteron and in the target on the same footing. On the other hand it is difficult in practice to treat all possible open channels and absorption has to be inserted by hand. RGM calculations have been published[41] which include deuteron break-up effects using discretisation methods similar to the CDCC (next section). Antisymmetrisation effects are very important in this approach[41].

The alternative approach of refs.[13],[42],[43] is based on the idea that because of the loosely bound extended spatial nature of the deuteron, the nucleons in the deuteron see the target nucleus much as if they were completely independent and so much of the effects of antisymmetry and coupling to excited target states are contained in the complex optical potentials of the 3-body model.In this way one automatically generates a total deuteron reaction cross section which is close to that observed even when deuteron elastic break-up is neglected. New effects arise for deuteron collisions only because the nucleons in the deuteron may scatter off each other into occupied target states (Pauli blocking). These effects are included through a generalisation of the Bethe-Goldstone equation. The role of break-up channels is to re-adjust the flow of flux into inelastic channels involving excited target states as well as transfering flux into break-up channels. For some impact parameters the effect of the break-up channel may even be to *decrease* the partial reaction crosssection because in the break-up configuration the nucleons may overlap spatially less well with the imaginary parts of the nucleon optical potentials. Tostevin, et al[43] found that absorption tends to suppress Pauli blocking and no new major qualitative effects were found. Aoki[44], using a formula for these effects due to Pong and Austern[42], reported that Pauli blocking effects gave a 10% repulsive correction to the deuteron optical potential and improved CDCC fits to elastic deuteron scattering on ^{208}Pb at 20 MeV. The effect on (d,p) crosssections was negligible.

3.1. Many-body theory of stripping

One advantage of a theory of stripping and elastic break-up based on the matrix elements (15) and (16), which we have here derived within a 3-body model, is that they generalise easily to include important many body effects. One starts from formulae analogous to these but with target nucleus co-ordinates still explicit. The wavefunction $\Psi^{ad}_{\vec{K}_d}$ is replaced by the full many-body scattering state corresponding to a deuteron incident on the target ground state. The final state wavefunction describes scattering of a proton incident on the residual nucleus and scattered by the *target* nucleons. There is no V_{np} involved in this state (see [49], pages 838-839, and [24]). The effect of the identity of the neutron in the deuteron and the N target neutrons is included exactly by multiplying the T-matrix by the factor $\sqrt{(N+1)}$ and using properly antisymmetrised wavefunctions for the target and residual nucleus (see [49], pages 836-838).

To obtain the connection with the 3-body theory we

(i) Ignore explicit contributions from channels in which the target is excited by the incident $n-p$ pair. [3]

(ii) Ignore Recoil Excitation and Break-up of the final nucleus by elastic scattering of the proton by the target nucleons in the final state [4].

Because the operator V_{np} in the matrix element is independent of the target internal co-ordinates these two steps automatically give a matrix element which involves the projection of the final nucleus state onto an un-excited target state.

(iii) Ignore the identity of the protons in the target and incident deuteron.

Very little is known about the validity of step (iii). The usual qualitative argument is that proton exchange terms in a (d,p) reaction involve the overlap of bound and continuum proton states instead of the continuum-continuum overlap which contributes to the direct term. The exchange term is therefore expected to be small.

The result of these steps is simply that in the expressions (15) ϕ_n is replaced by the *overlap function* ([2], page710)

$$\phi_n^{BA}(\vec{r}_n) = \sqrt{N+1}\int d\xi_A \phi_B^*(\xi_A,\vec{r}_n)\phi_A(\xi_A), \tag{34}$$

where $\phi_A,\ \phi_B$ are wavefunctions of the initial and final states in the stripping reaction and $\vec{r}_n$ is the coordinate of the neutron relative to the centre-of-mass of the target. By definition the spectroscopic factor S_{AB} is the norm of the overlap function.

If approximation (ii) is relaxed the (d,p) transition matrix will be a linear combination of terms involving the overlap function for different states B,A, but always with the target ground state. If approximation (i) is relaxed overlaps and spectroscopic factors for many different states A and B will enter, reflecting the many different paths between the initial and final states which then become possible. The corresponding generalisation of the DWBA is referred to as Coupled Channels Born Approximation (CCBA) but the theory described here is not Born approximation because of the way couplings in the n-p space are treated.

4. LINK WITH THE CDCC METHOD

In the CDCC method the 3-body wave function for deuteron-nucleus scattering, $\Psi(\vec{R},\vec{r})$, is expanded in a set of orthonormal functions $\phi_s(\vec{r}),\ s=0,1,2.....$, which diagonalise H_{np} with eigenvalues ε_s and discretise the $n-p$ continuum. The set are usually defined so that $\phi_{s=0}$ is the deuteron ground state. Coupled equations are then derived as a technique for solving the 3-body Schrödinger equation. See the talk by I J Thompson for further details of the CDCC method.

We can expand the adiabatic wavefunction $\Psi^{ad}(\vec{R},\vec{r})$ in a volume $\mathscr{V}$ in $\vec{r}$ space in terms of an orthonormal set $\psi_s(\vec{r}),\ s=0,1,2.....$ which is complete in $\mathscr{V}$:

$$\Psi^{ad}(\vec{R},\vec{r}) = \sum_{s=0}^{\infty}\psi_s(\vec{r})\chi_s(\vec{R}), \tag{35}$$

and derive coupled equations for the χ_s's of the form

$$(E_d - T_R)\chi_s(\vec{R}) = \sum_{s'}\langle\psi_s \mid V \mid \psi_{s'}\rangle\chi_{s'}(\vec{R}), \tag{36}$$

where V is defined in eq.(1) and the coupling matrix elements involve an integration over $\mathscr{V}$.

If we identify the ψ_i's and ϕ_i's (to obtain the CDCC equations the ϕ_i's must diagonalise H_{np}) these equations are similar to the CDCC equations with all the channel energies set equal to $-\varepsilon_0$. We can put this another way. If the set $\psi_s(\vec{r}),\ s=0,1,2.....$, is complete in $\mathscr{V}$, and the functions $\chi_s(\vec{R})$ satisfy the coupled equations (36) then these equations show that, for $\vec{r}$ in $\mathscr{V}$, $\sum_s \psi_s(\vec{r})\chi_s(\vec{R})$ satisfies

$$\begin{aligned}(E_d - T_R)\sum_s \psi_s(\vec{r})\chi_s(\vec{R}) &= \sum_s \psi_s(\vec{r})\int d\vec{r}'\,\psi_s^*(\vec{r}')\sum_{s'}V(\vec{R},\vec{r}')\psi_{s'}(\vec{r}')\chi_{s'}(\vec{R}) \\ &= V(\vec{R},\vec{r})\sum_{s'}\psi_{s'}(\vec{r})\chi_{s'}(\vec{R}),\end{aligned} \tag{37}$$

[3] The implicit effects of target excitation are, of course included in the nucleon optical potentials.

[4] See [24] for an estimate of these effects for some light targets.

where the completeness of the ψ_s's has been used. Eq.(37) is just the adiabatic equation. Hence $\sum_s \psi_s(\vec{r})\chi_s(\vec{R})$ is the adiabatic solution $\Psi^{ad}(\vec{R},\vec{r})$.

We see that the Adiabatic approach can be regarded as an approximation to the CDCC method. Thompson[8] describes how this result can be used as a check of CDCC calculations by talking the limit when all the channel energies (the ε_s's) are set equal. In making these comparisons note that the adiabatic method does not take into account any restrictions imposed by the Pauli Principle on the states which should be included in the set ϕ_s. For example, in the case of ^{11}Be scattering the adiabatic calculations include transitions into a state in which the neutron is in a nodeless s-state with respect to the ^{10}Be core. Such contributions are easily excluded in the CDCC calculation or in the Johnson-Tandy approach[25], but it is not obvious how to do this in an adiabatic calculation without introducing non-local projection operators with the consequential loss of some of the characteristic simplicity of the adiabatic equation.

At first sight it is puzzling that the adiabatic calculation can take into account effects due to excited deuteron states when only the deuteron ground state wave function appears explicitly. In CDCC calculations the wave functions of all excited states deemed to be important must be inserted explicitly into the calculation of the coupling matrix elements. Our derivation above shows how this puzzle can be resolved but it is also helpful to note that the ground state wavefunction ϕ_0 determines the Hamiltonian through the identity (see the Appendix to [16])

$$H_{np} = -\varepsilon_0 - \frac{\hbar^2}{2\mu_{np}}\phi_0^{-1}\nabla_r\phi_0^2.\nabla_r\phi_0^{-1}, \tag{38}$$

where the ∇_r operators act on everything to the right of them.

We note that the CDCC method uses a basis which is complete in large volumes of $\vec{r}$ space. As we have seen the stripping and break-up matrix elements (15) and (16) explore a very restricted part of this space, i.e., within the range of V_{np}. For this purpose the complete set used by Johnson and Tandy[25] and its generalisations may be more efficient.

In their exploration of the adiabatic approximation Johnson and Soper[13] proposed an approximation to the CDCC method which replaced the deuteron continuum by a single pseudo state. In their method the component $V_{np}\Psi$ of the 3-body wave function is still governed by equations (21), (25) and (26), but the deuteron elastic scattering wavefunction and the pseudo break-up state satisfy a pair of coupled equations. This method was critically examined in great detail by Rawitscher[3],[4] within the CDCC framework. A more sophisticated version of the single pseudostate method was developed by Amakawa, Austern and Vincent[48] and is known as the quasi-adiabatic method.

5. CONCLUDING REMARKS

Some of the clearest evidence for the importance of deuteron break-up effects and the failure of the DWBA for (d,p) and (p,d) reactions has been obtained by using the adiabatic approximation as implemented in the ADWA. However, we have seen that the adiabatic approximation can be regarded as an approximation to the CDCC, so it might be argued that the adiabatic approximation no longer has a role. It is only recently, however, that the CDCC method has become available for projectiles with more than 2 clusters and, when coupled with the eikonal approximation where applicable, the adiabatic approximation is a powerful tool for the analysis of reactions with composite projectiles.

An attractive feature of the adiabatic approach which it shares with CDCC is that it provides a framework for inserting the systematics of the interaction of the constituents of the projectile with the target into reaction analyses. This means that the need for optical potentials for unstable projectiles can often be avoided, but it still requires reliable information about the constituents' optical potentials and hence good elastic scattering data for appropriate energies and targets.

An advantage of the adiabatic method over CDCC is that its implementation does not need detailed wavefunctions of strongly coupled excited bound and continuum states of the projectile. The construction of these states may introduce considerable uncertainties into a CDCC calculation. It is important therefore to understand the limitations of the adiabatic approximation.

Perhaps the most important feature of the adiabatic approximation is its ability to provide insights into the mechanism of complex reactions. It can be used to provide checks of CDCC and other theories as well as being a relatively easy and transparent way to take into account complicated effects of channel coupling in some important special cases.

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