

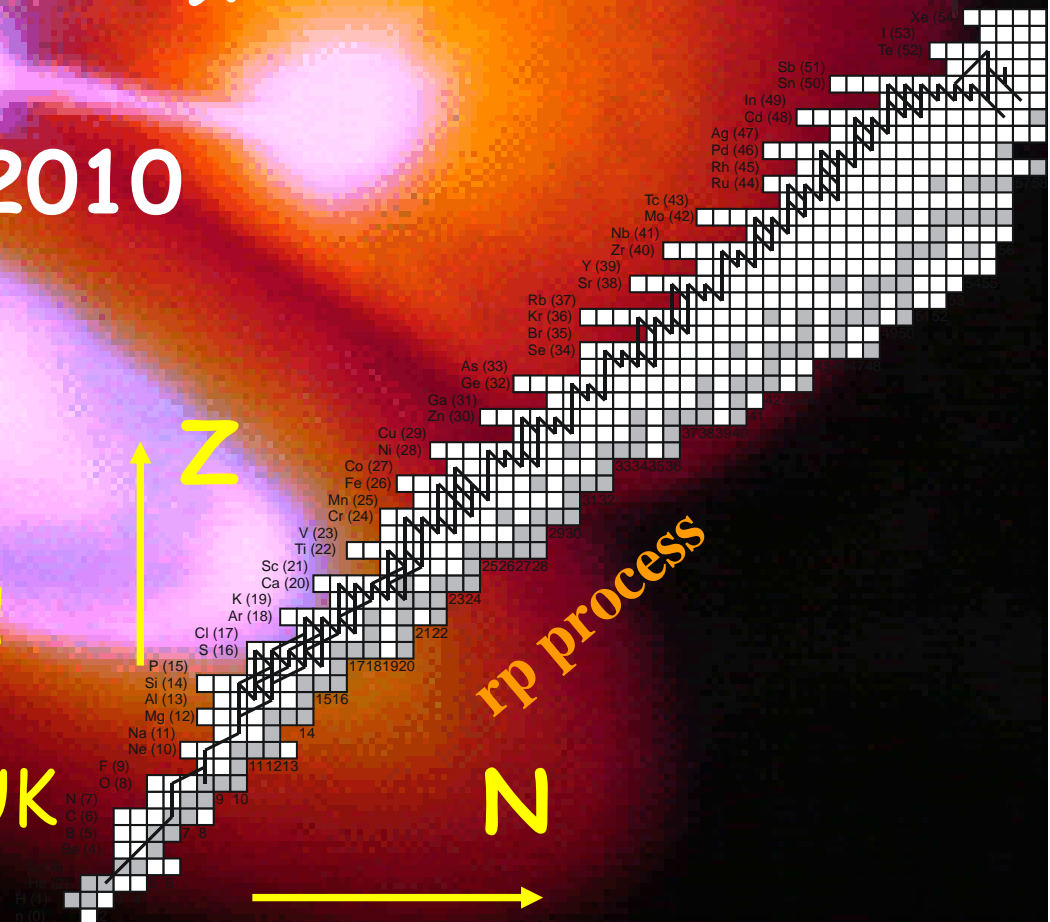
Reaction tools for the study of exotic nuclei: theory and applications - I

Hadrons and Nuclei under Extreme Conditions (HANE2010),

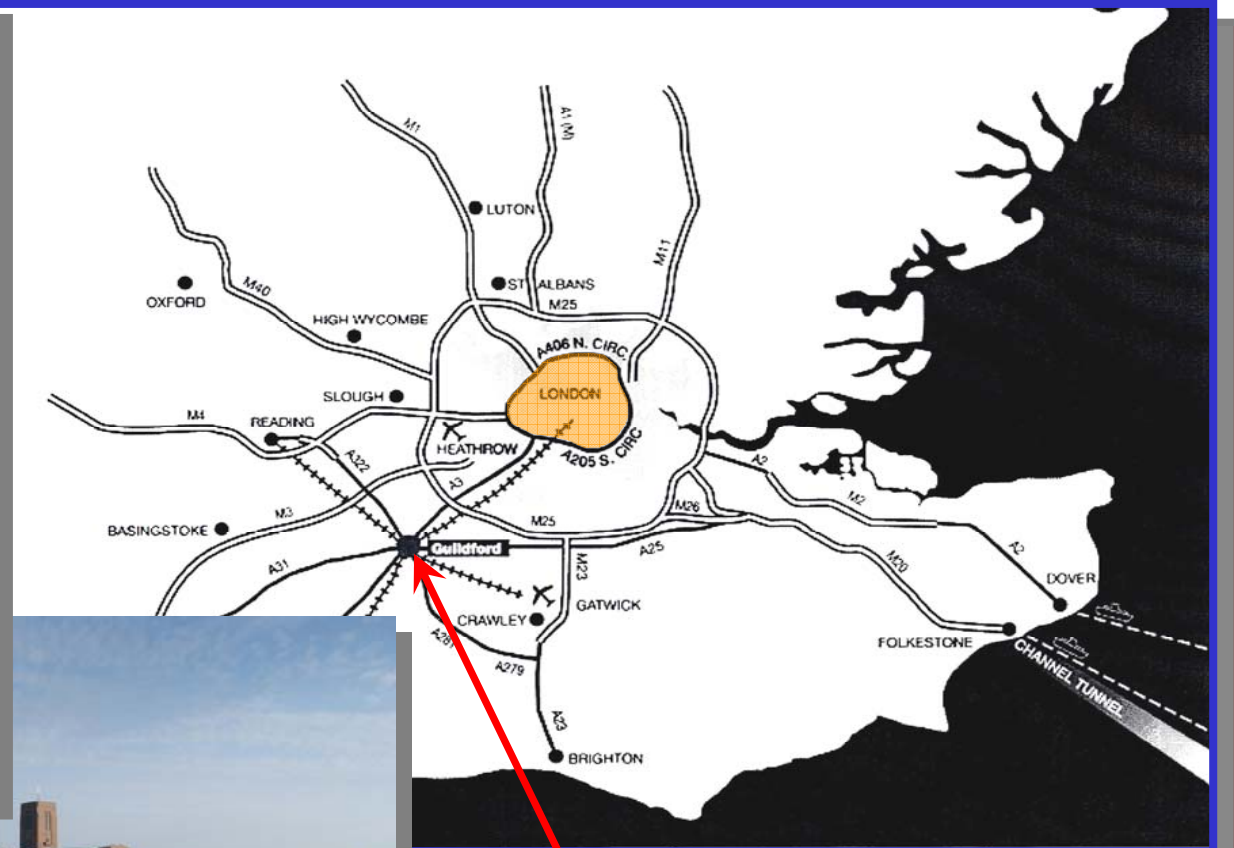
TIT, O-Okayama

16-17 September 2010

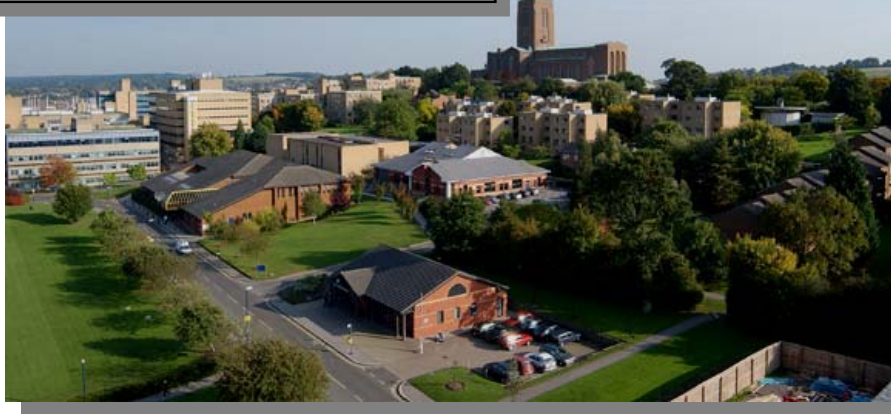
Jeff Tostevin, TIT and
Department of Physics
University of Surrey, UK



My part of the Cosmos - the University of Surrey



Guildford, Surrey



Exotic nuclei – fundamental questions

What are the limits of nuclear existence ... ?

How do the limits of weak binding and extreme proton-to-neutron asymmetries affect nuclear properties ... ?

How do the properties of nuclei evolve with changes in proton and neutron number ... ?

“Atomic Nuclei: Structure and Stability”, US 2002 Long-Range Plan for Nuclear Science

Some other slides/notes/resources can be found at:
http://www.nucleartheory.net/DTP_material/

There several good direct reactions texts:

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) [Norman Austern](#)

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages) [G R Satchler](#)

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) [L S Rodberg](#), [R M Thaler](#)

Direct Nuclear Reactions (World Scientific Publishing, 396 pages)
[Norman K. Glendenning](#)

Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages) [C A Bertulani](#), [P Danielewicz](#)

Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages) [Herman Feshbach](#)

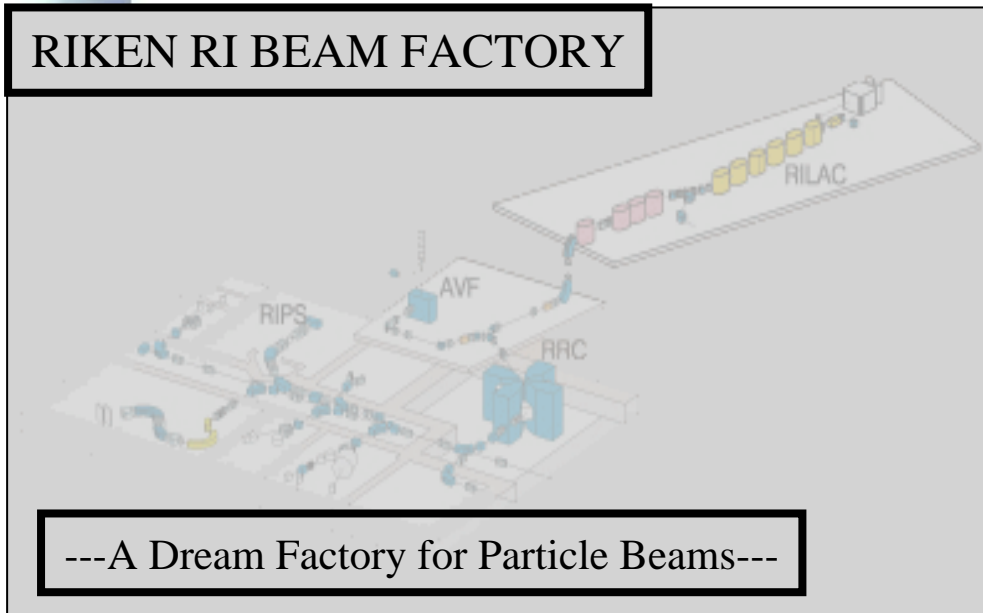
Introduction to Nuclear Reactions (Oxford University Press, 332 pages)
[G R Satchler](#)

Nuclear Reactions for Astrophysics (Cambridge University Press, 2010)
[Ian Thompson and Filomena Nunes](#)

Radioactive ion-beam facilities – and plans



RIKEN RI BEAM FACTORY

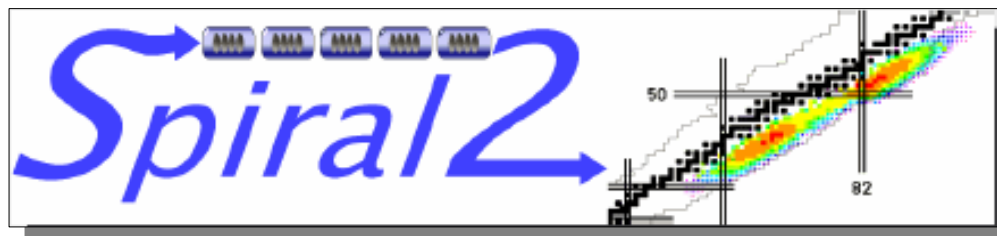


---A Dream Factory for Particle Beams---

RIBF operational
RIKEN

GSII

MSU -
FRIB
USA



GANIL, France

Lectures plan

Lecture 1: Introduction: history and key ideas

- nuclear shell structure, old and new
- Reactions at high-energy (removal)
- approximations, simplifications
- connection/interface with nuclear structures
- observables and what they tell us

Lecture 2: Illustrative and topical applications

- determining exotic structures
- testing of structure models and of shell model effective interactions
- nucleon and pair 'correlations'

Introduction: Motivations and context

Exotic nuclei – stability → structures → reactions

- How? history: have we got to present point?
- What ? are exotic nuclei – how are they different?
- Why ? are they of interest, and to whom?
- How ? (and where) are they produced
- Which ? can we/will we produce (in 2010-2015)?
- What ? are we now able to do with them?
- What ? are are they telling us?
- Where ? is this research area going/challenges

100 years (almost) of nuclear physics

[669]

LXXIX. *The Scattering of α and β Particles by Matter and the Structure of the Atom.* By Professor E. RUTHERFORD, F.R.S., University of Manchester *.

§ 1. **I**T is well known that the α and β particles suffer deflexions from their rectilinear paths by encounters with atoms of matter. This scattering is far more marked for the β than for the α particle on account of the much smaller momentum and energy of the former particle. There seems to be no doubt that such swiftly moving particles pass through the atoms in their path, and that the deflexions observed are due to the strong electric field traversed within the atomic system. It has generally been

Philosophical Magazine, volume **21** (1911), pages 669-688

Nuclear sizes: charge and mass distributions

REVIEWS OF MODERN PHYSICS

VOLUME 30, NUMBER 2

APRIL, 1958

International Congress on Nuclear Sizes and Density Distributions

Held at Stanford University, December 17–19, 1957

REVIEWS OF MODERN PHYSICS

VOLUME 30, NUMBER 2

APRIL, 1958

Nuclear Radii as Determined by Scattering of Neutrons

S. FERNBACH

University of California Radiation Laboratory, Livermore, California

REVIEWS OF MODERN PHYSICS

VOLUME 30, NUMBER 2

APRIL, 1958

Nuclear Density Distributions from Proton Scattering

A. E. GLASSGOLD

Department of Physics, University of California, Berkeley, California

REVIEWS OF MODERN PHYSICS

VOLUME 30, NUMBER 2

APRIL

Electron Scattering and Nuclear Charge Distributions

D. G. RAVENHALL

Department of Physics, University of Illinois



$$R = r_0 A^{1/3}$$

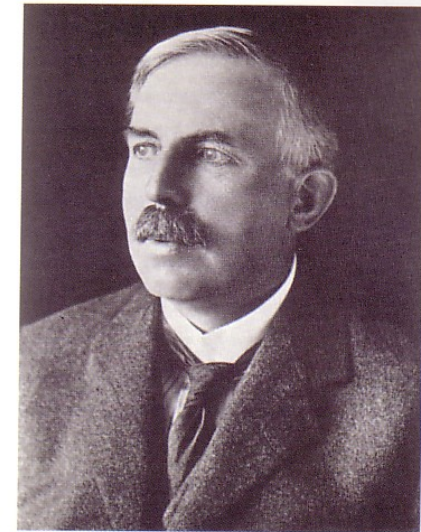
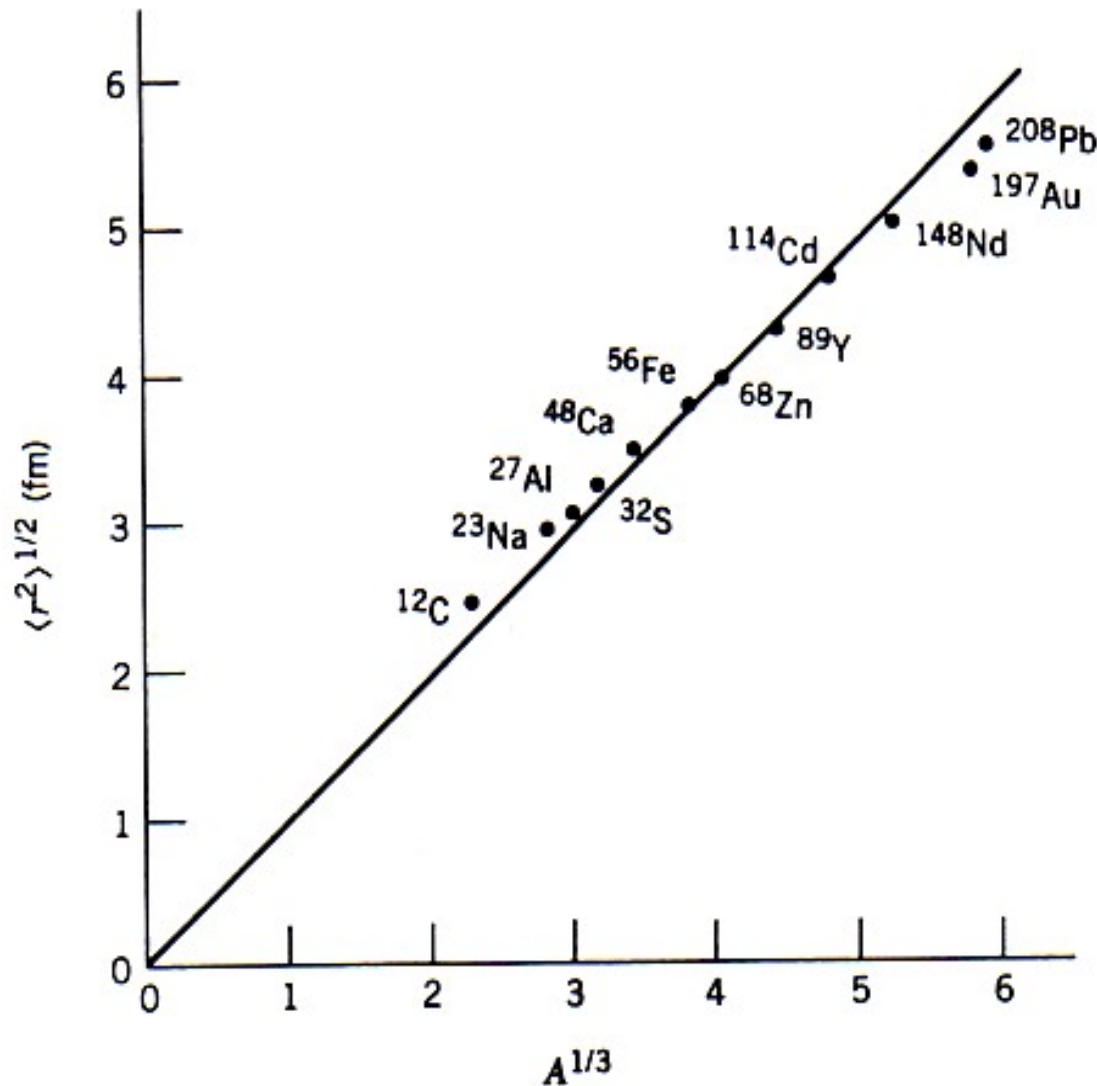
while for convenience the results on the other nuclei were analyzed in terms of the Fermi shape

$$\rho(r) = \rho_0 [1 + e^{(r-c)/t}]^{-1},$$

the only significant quantities determined were the radius and the surface thickness t . The radial parameter c , the distance to the point where ρ has dropped to half of its central value on this model, varies closely as $A^{1/3}$ for all of the nuclei, and t is effectively constant:

$$\begin{array}{ll} c = (1.07 \pm 0.02) A^{1/3} \times 10^{-13} \text{ cm} & \left. \begin{array}{l} \text{charge} \\ \text{distribution.} \end{array} \right\} \\ t = (2.4 \pm 0.3) \times 10^{-13} \text{ cm} & \end{array}$$

Nuclear sizes: charge and mass distributions



$$R = r_0 A^{1/3}$$

For convenience the results on the other nuclei analyzed in terms of the Fermi shape

$$\rho(r) = \rho_0 [1 + e^{(r-a)/a}]^{-1},$$

the significant quantities determined were the radius and the surface thickness t . The radial parameter a is the distance to the point where ρ has dropped to its central value on this model, varies closely for all of the nuclei, and t is effectively constant:

$$\begin{aligned} a &= (1.07 \pm 0.02) A^{1/3} \times 10^{-13} \text{ cm} \quad \left. \begin{array}{l} \text{charge} \\ \text{distribution.} \end{array} \right\} \\ t &= (2.4 \pm 0.3) \times 10^{-13} \text{ cm} \end{aligned}$$

Shell (single particle) structures in nuclei

On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER

Argonne National Laboratory and Department of Physics,
University of Chicago, Chicago, Illinois

February 4, 1949

Phys. Rev. **75**, 1969 (1949)

Cited 240 times

LETTERS TO THE EDITOR

1969

with spin $3/2$ in stead of the expected $d_{3/2}$, and $d_{5/2}$ Mn^{55} with $5/2$ instead of the expected $f_{7/2}$, are the only violations.

Table II lists the known spins and orbital assignments from magnetic moments when these are known and unambiguous, for the even-odd nuclei up to 83. Beyond 83 the data is limited and no exceptions to the assignment appear.

Up to Z or $N=20$, the assignment is the same as that of Feenberg and Nordheim. At the beginning of the next shell, $f_{7/2}$ levels occur at 21 and 23, as they should. At 28 the $f_{7/2}$ levels should be filled, and no spins of $7/2$ are encountered any more in this shell. This subshell may contribute to the stability of Ca^{48} . If the $g_{9/2}$ level did not cross the $p_{1/2}$ or $f_{7/2}$

TABLE I.

| Occ. No. | Square well | Spinet. term | Spin term | No. of states | Shells | Total No. |
|----------|-------------|--------------|--------------------|---------------|--------|-----------|
| 0 | 1s | 1s | 1s | 2 | 2 | 2 |
| 1 | 1p | 2p | 1p _{1/2} | 4 | 6 | 8 |
| 2 | 1d | 3d | 1d _{3/2} | 6 | 12 | 20 |
| 3 | 2s | 2s | 2s _{1/2} | 2 | 8 | 28 |
| 4 | 1f | 4f | 1f _{7/2} | 8 | 22 | 50 |
| 5 | 2p | 3p | 2p _{3/2} | 4 | 30 | 82 |
| 6 | 1g | 5g | 1g _{7/2} | 8 | 44 | 126 |
| 7 | 2d | 4d | 2d _{5/2} | 6 | 58 | 198 |
| 8 | 3s | 3s | 3s _{1/2} | 2 | 70 | 270 |
| 9 | 1h | 6h | 1h _{11/2} | 12 | 82 | 352 |
| 10 | 2f | 5f | 2f _{7/2} | 8 | 90 | 442 |
| 11 | 3p | 4p | 3p _{3/2} | 4 | 94 | 486 |
| 12 | 4s | 4s | 4s _{1/2} | 2 | 96 | 500 |

On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics,
University of Chicago, Chicago, Illinois
February 4, 1949

THE spins and magnetic moments of the even-odd nuclei have been used by Feenberg¹ and Nordheim² to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin-orbit coupling, increasing with angular momentum, is assumed, a level assignment different from either Feenberg or Nordheim is obtained. This assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

Table I contains in column two, in order of decreasing binding energy, the levels of the square well potential. The quantum number gives the number of radial nodes. Two levels of the same quantum number cannot cross for any type of potential well, except due to spin-orbit splitting. No evidence of any crossing is found. Column three contains the usual spectroscopic designation of the levels, as used by Nordheim and Feenberg. Column one groups together those levels which are degenerate for a three-dimensional isotropic oscillator potential. A well with rounded corners will have a behavior in between these two potentials. The shell grouping is given in column five, with the numbers of particles per shell and the total number of particles up to and including each shell in column six and seven, respectively.

Within each shell the levels may be expected to be close in energy, and not necessarily in the order of the table, although the order of levels of the same orbital angular momentum and different spin should be maintained. Two exceptions, $^{11}Na^{23}$

levels, the first spin of $9/2$ should occur at 41, which is indeed the case. Three nuclei with N or $Z=19$ have $g_{9/2}$ orbits. No s or d levels should occur in this shell and there is no evidence for any.

The only exception to the proposed assignment in this shell is the spin $5/2$ instead of $7/2$ for Mn^{55} , and the fact that the magnetic moment of $^{55}Co^{56}$ indicates a $g_{7/2}$ orbit instead of the expected $f_{7/2}$.

In the next shell two exceptions to the assignment occur. The spin of $1/2$ for Mo^{98} with 43 would be a violation, but is experimentally doubtful. The magnetic moment of Eu^{153} indicates $f_{7/2}$ instead of the predicted $d_{5/2}$. No $h_{11/2}$ levels appear. It seems that these levels are filled in pairs only,

1970

LETTERS TO THE EDITOR

which does not seem a serious drawback of the theory as this tendency already shows up at the filling of the $g_{9/2}$ levels. Otherwise, the agreement is satisfactory. The shell begins with $s_{1/2}$, which has two isotopes with $d_{1/2}$ and $g_{7/2}$ levels, respectively, as it should. The thallium isotopes with 81 neutrons and a spin of $1/2$ indicate a crossing of the $h_{11/2}$ and $3s$ levels. This is not surprising, since the energies of these levels are close together in the square well. The assignment

TABLE II. Spins of even-odd nuclei.

| Occ. No. | Element | Odd protons | | Odd neutrons | | Even-odd nuclei | | Spin | Orbit | Levels |
|----------|---------|-------------|----------|--------------|----------|-----------------|----------|------|------------------|--------|
| | | Mean No. | Mean No. | Mean No. | Mean No. | Mean No. | Mean No. | | | |
| 1 | H | 1 | 1 | 1 | 1 | 1 | 1 | 1/2 | s _{1/2} | 1 |
| 2 | He | 2 | 2 | 2 | 2 | 2 | 2 | 1/2 | s _{1/2} | 2 |
| 3 | Li | 3 | 3 | 3 | 3 | 3 | 3 | 1/2 | s _{1/2} | 3 |
| 4 | Be | 4 | 4 | 4 | 4 | 4 | 4 | 1/2 | s _{1/2} | 4 |
| 5 | B | 5 | 5 | 5 | 5 | 5 | 5 | 1/2 | s _{1/2} | 5 |
| 6 | C | 6 | 6 | 6 | 6 | 6 | 6 | 1/2 | s _{1/2} | 6 |
| 7 | N | 7 | 7 | 7 | 7 | 7 | 7 | 1/2 | s _{1/2} | 7 |
| 8 | O | 8 | 8 | 8 | 8 | 8 | 8 | 1/2 | s _{1/2} | 8 |
| 9 | F | 9 | 9 | 9 | 9 | 9 | 9 | 1/2 | s _{1/2} | 9 |
| 10 | Ne | 10 | 10 | 10 | 10 | 10 | 10 | 1/2 | s _{1/2} | 10 |
| 11 | Na | 11 | 11 | 11 | 11 | 11 | 11 | 1/2 | s _{1/2} | 11 |
| 12 | Mg | 12 | 12 | 12 | 12 | 12 | 12 | 1/2 | s _{1/2} | 12 |
| 13 | Al | 13 | 13 | 13 | 13 | 13 | 13 | 1/2 | s _{1/2} | 13 |
| 14 | Si | 14 | 14 | 14 | 14 | 14 | 14 | 1/2 | s _{1/2} | 14 |
| 15 | P | 15 | 15 | 15 | 15 | 15 | 15 | 1/2 | s _{1/2} | 15 |
| 16 | S | 16 | 16 | 16 | 16 | 16 | 16 | 1/2 | s _{1/2} | 16 |
| 17 | Cl | 17 | 17 | 17 | 17 | 17 | 17 | 1/2 | s _{1/2} | 17 |
| 18 | Ar | 18 | 18 | 18 | 18 | 18 | 18 | 1/2 | s _{1/2} | 18 |
| 19 | K | 19 | 19 | 19 | 19 | 19 | 19 | 1/2 | s _{1/2} | 19 |
| 20 | Ca | 20 | 20 | 20 | 20 | 20 | 20 | 1/2 | s _{1/2} | 20 |
| 21 | Sc | 21 | 21 | 21 | 21 | 21 | 21 | 1/2 | s _{1/2} | 21 |
| 22 | Ti | 22 | 22 | 22 | 22 | 22 | 22 | 1/2 | s _{1/2} | 22 |
| 23 | V | 23 | 23 | 23 | 23 | 23 | 23 | 1/2 | s _{1/2} | 23 |
| 24 | Cr | 24 | 24 | 24 | 24 | 24 | 24 | 1/2 | s _{1/2} | 24 |
| 25 | Mn | 25 | 25 | 25 | 25 | 25 | 25 | 1/2 | s _{1/2} | 25 |
| 26 | Fe | 26 | 26 | 26 | 26 | 26 | 26 | 1/2 | s _{1/2} | 26 |
| 27 | Co | 27 | 27 | 27 | 27 | 27 | 27 | 1/2 | s _{1/2} | 27 |
| 28 | Ni | 28 | 28 | 28 | 28 | 28 | 28 | 1/2 | s _{1/2} | 28 |
| 29 | Cu | 29 | 29 | 29 | 29 | 29 | 29 | 1/2 | s _{1/2} | 29 |
| 30 | Zn | 30 | 30 | 30 | 30 | 30 | 30 | 1/2 | s _{1/2} | 30 |
| 31 | Ga | 31 | 31 | 31 | 31 | 31 | 31 | 1/2 | s _{1/2} | 31 |
| 32 | Ge | 32 | 32 | 32 | 32 | 32 | 32 | 1/2 | s _{1/2} | 32 |
| 33 | As | 33 | 33 | 33 | 33 | 33 | 33 | 1/2 | s _{1/2} | 33 |
| 34 | Se | 34 | 34 | 34 | 34 | 34 | 34 | 1/2 | s _{1/2} | 34 |
| 35 | Br | 35 | 35 | 35 | 35 | 35 | 35 | 1/2 | s _{1/2} | 35 |
| 36 | Kr | 36 | 36 | 36 | 36 | 36 | 36 | 1/2 | s _{1/2} | 36 |
| 37 | Rb | 37 | 37 | 37 | 37 | 37 | 37 | 1/2 | s _{1/2} | 37 |
| 38 | Sr | 38 | 38 | 38 | 38 | 38 | 38 | 1/2 | s _{1/2} | 38 |
| 39 | Y | 39 | 39 | 39 | 39 | 39 | 39 | 1/2 | s _{1/2} | 39 |
| 40 | Zr | 40 | 40 | 40 | 40 | 40 | 40 | 1/2 | s _{1/2} | 40 |
| 41 | Nb | 41 | 41 | 41 | 41 | 41 | 41 | 1/2 | s _{1/2} | 41 |
| 42 | Mo | 42 | 42 | 42 | 42 | 42 | 42 | 1/2 | s _{1/2} | 42 |
| 43 | Tc | 43 | 43 | 43 | 43 | 43 | 43 | 1/2 | s _{1/2} | 43 |
| 44 | Ru | 44 | 44 | 44 | 44 | 44 | 44 | 1/2 | s _{1/2} | 44 |
| 45 | Rh | 45 | 45 | 45 | 45 | 45 | 45 | 1/2 | s _{1/2} | 45 |
| 46 | Pd | 46 | 46 | 46 | 46 | 46 | 46 | 1/2 | s _{1/2} | 46 |
| 47 | Ag | 47 | 47 | 47 | 47 | 47 | 47 | 1/2 | s _{1/2} | 47 |
| 48 | Cd | 48 | 48 | 48 | 48 | 48 | 48 | 1/2 | s _{1/2} | 48 |
| 49 | In | 49 | 49 | 49 | 49 | 49 | 49 | 1/2 | s _{1/2} | 49 |
| 50 | Sn | 50 | 50 | 50 | 50 | 50 | 50 | 1/2 | s _{1/2} | 50 |
| 51 | Pb | 51 | 51 | 51 | 51 | 51 | 51 | 1/2 | s _{1/2} | 51 |
| 52 | Bi | 52 | 52 | 52 | 52 | 52 | 52 | 1/2 | s _{1/2} | 52 |
| 53 | Po | 53 | 53 | 53 | 53 | 53 | 53 | 1/2 | s _{1/2} | 53 |
| 54 | At | 54 | 54 | 54 | 54 | 54 | 54 | 1/2 | s _{1/2} | 54 |
| 55 | Rn | 55 | 55 | 55 | 55 | 55 | 55 | 1/2 | s _{1/2} | 55 |
| 56 | Fr | 56 | 56 | 56 | 56 | 56 | 56 | 1/2 | s _{1/2} | 56 |
| 57 | Ac | 57 | 57 | 57 | 57 | 57 | 57 | 1/2 | s _{1/2} | 57 |
| 58 | Th | 58 | 58 | 58 | 58 | 58 | 58 | 1/2 | s _{1/2} | 58 |
| 59 | Pa | 59 | 59 | 59 | 59 | 59 | 59 | 1/2 | s _{1/2} | 59 |
| 60 | U | 60 | 60 | 60 | 60 | 60 | 60 | 1/2 | s _{1/2} | 60 |
| 61 | Np | 61 | 61 | 61 | 61 | 61 | 61 | 1/2 | s _{1/2} | 61 |
| 62 | Pu | 62 | 62 | 62 | 62 | 62 | 62 | 1/2 | s _{1/2} | 62 |
| 63 | Am | 63 | 63 | 63 | 63 | 63 | 63 | 1/2 | s _{1/2} | 63 |
| 64 | Cm | 64 | 64 | 64 | 64 | 64 | 64 | 1/2 | s _{1/2} | 64 |
| 65 | Bk | 65 | 65 | 65 | 65 | 65 | 65 | 1/2 | s _{1/2} | 65 |
| 66 | Cf | 66 | 66 | 66 | 66 | 66 | 66 | 1/2 | s _{1/2} | 66 |
| 67 | Es | 67 | 67 | 67 | 67 | 67 | 67 | 1/2 | s _{1/2} | 67 |
| 68 | Fm | 68 | 68 | 68 | 68 | 68 | 68 | 1/2 | s _{1/2} | 68 |
| 69 | Md | 69 | 69 | 69 | 69 | 69 | 69 | 1/2 | s _{1/2} | 69 |
| 70 | No | 70 | 70 | 70 | 70 | 70 | 70 | 1/2 | s _{1/2} | 70 |
| 71 | Lr | 71 | 71 | 71 | 71 | 71 | 71 | 1/2 | s _{1/2} | 71 |
| 72 | Be | 72 | 72 | 72 | 72 | 72 | 72 | 1/2 | s _{1/2} | 72 |
| 73 | B | 73 | 73 | 73 | 73 | 73 | 73 | 1/2 | s _{1/2} | 73 |
| 74 | C | 74 | 74 | 74 | 74 | 74 | 74 | 1/2 | s _{1/2} | 74 |
| 75 | N | 75 | 75 | 75 | 75 | 75 | 75 | 1/2 | s _{1/2} | 75 |
| 76 | O | 76 | 76 | 76 | 76 | 76 | 76 | 1/2 | s _{1/2} | 76 |
| 77 | F | 77 | 77 | 77 | 77 | 77 | 77 | 1/2 | s _{1/2} | 77 |
| 78 | Ne | 78 | 78 | 78 | 78 | 78 | 78 | 1/2 | s _{1/2} | 78 |
| 79 | Na | 79 | 79 | 79 | 79 | 79 | 79 | 1/2 | s _{1/2} | 79 |
| 80 | Mg | 80 | 80 | 80 | 80 | 80 | 80 | 1/2 | s _{1/2} | 80 |
| 81 | Al | 81 | 81 | 81 | 81 | 81 | 81 | 1/2 | s _{1/2} | 81 |
| 82 | Si | 82 | 82 | 82 | 82 | 82 | 82 | 1/2 | s _{1/2} | 82 |
| 83 | P | 83 | 83 | 83 | 83 | 83 | 83 | 1/2 | s _{1/2} | 83 |

demands that there be no spins of $9/2$ in this shell, and none have been found. No f or p levels should occur and, except for Eu^{153} , there is no indication of any.

The spin and magnetic moment of $s_{1/2}$, indicating an $h_{1/2}$ state, is a beautiful confirmation of the correct beginning of the next shell. Here information begins to be scarce. The spin and magnetic moment of Pb^{207} with 125 neutrons interpret as $p_{1/2}$. This is the expected end of the shell since $7s$ and $4p$ have practically the same energy in the square well model. No spins of $11/2$ and no s , d , or g orbits should occur in this shell, and the data indicates none.

The prevalence of isomerism towards the end of a shell, noticed by Feenberg and Nordheim, is easily understood by this assignment. These are the regions where levels with very different spins are adjacent. These ground and isomeric

states are due to spin-orbit coupling. The origin of this paper.

Thanks are due to Enrico Fermi for the remark, "Is there any indication of spin-orbit coupling?" which was the origin of this paper.

Eugene Feenberg, Phys. Rev. **75**, 320 (1949).
Eugene Feenberg and K. C. Littman, Phys. Rev. **75**, 1877 (1949).
Ludwig Nordheim, Phys. Rev. **75**, 1896 (1949). The author is indebted to these authors for having obtained copies of both references 2 and 3.

H. H. Goldstein and D. R. Inglis, *The Properties of Atomic Nuclei*, 2nd ed. (Information Division, Brookhaven National Laboratory).



Enrico Fermi (1901–1954)

Thanks are due to Enrico Fermi for the remark, "Is there any indication of spin-orbit coupling?" which was the origin of this paper.

Shell (single particle) structures in nuclei

On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER

Argonne National Laboratory and Department of Physics,
University of Chicago, Chicago, Illinois

February 4, 1949



1969
of the expected $d_{5/2}$ and $d_{3/2}$ and Mn^{55} with $5/2$ $f_{7/2}$ are the only violations.
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and no spins of $7/2$ are encountered
. This subshell may contribute to the
e $g_{7/2}$ level did not cross the $p_{1/2}$ or $f_{7/2}$

TABLE I.

| Spin term | No. of states | Shells | Total No. |
|--------------|------------------|--------|--------------|
| $1p_{1/2}$ | 2 | 2 | 2 |
| $1p_{3/2}$ | 4 | | |
| $1d_{3/2}$ | 2 | 6 | 8 |
| $1d_{5/2}$ | 6 | | |
| $1d_{7/2}$ | 4 | 12 | |
| $2d_{3/2}$ | 2 | | 20 |
| $1f_{7/2}$ | 8 | 87 | 287 |
| $1f_{5/2}$ | 6 | | |
| $2f_{7/2}$ | 4 | 22 | |
| $2f_{5/2}$ | 2 | | |
| $1g_{7/2}$ | 10 | | 50 |
| $1g_{9/2}$ | 8 | | |
| $2d_{5/2}$ | 6 | | |
| $2d_{3/2}$ | 4 | 32 | |
| $3p_{1/2}$ | 2 | | |
| $1h_{11/2}$ | 12 | | 82 |
| $1h_{9/2}$ | 10 | | |
| $2f_{7/2}$ | 8 | | |
| $2f_{5/2}$ | 6 | | |
| $3p_{3/2}$ | 4 | 44 | |
| $3p_{1/2}$ | 2 | | |
| $1i_{13/2}$ | 14 | | 126 |
| $1i_{11/2}$ | | | |

levels, the first spin of $9/2$ should occur at 41, which is indeed the case. Three nuclei with N or $Z=49$ have $g_{9/2}$ orbits. No s or d levels should occur in this shell and there is no evidence for any.

The only exception to the proposed assignment in this shell is the spin $5/2$ instead of $7/2$ for Mn^{55} , and the fact that the magnetic moment of $^{55}Co^{56}$ indicates a $g_{7/2}$ orbit instead of the expected $f_{7/2}$.

In the next shell two exceptions to the assignment occur. The spin of $1/2$ for Mo^{98} with $S3$ would be a violation, but is experimentally doubtful. The magnetic moment of Eu^{149} indicates $f_{5/2}$ instead of the predicted $d_{5/2}$. No $h_{11/2}$ levels appear. It seems that these levels are filled in pairs only,

1970

LETTERS TO THE EDITOR

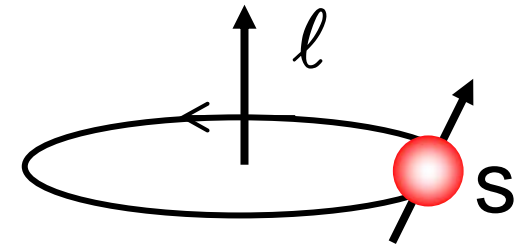
which does not seem a serious drawback of the theory as this tendency already shows up at the filling of the $g_{7/2}$ levels. Otherwise, the agreement is satisfactory. The shell begins with ^{83}Sb , which has two isotopes with $d_{5/2}$ and $g_{7/2}$ levels, respectively, as it should. The thallium isotopes with 81 neutrons and a spin of $1/2$ indicate a crossing of the $h_{11/2}$ and $3s$ levels. This is not surprising, since the energies of these levels are close together in the square well. The assignment

TABLE II. Spins of even-odd nuclei.

| No. of neutrons or protons | Element | Odd protons | | Odd neutrons | | Even neutrons | | Spin | Levels |
|-------------------------------|---------|-------------|------|--------------|------|---------------|------|------|--------|
| | | Val. No. | Unit | Val. No. | Unit | Val. No. | Unit | | |
| 1 | H | 1 s | | 1 s | | 1 s | | 1/2 | 1s |
| 3 | Li | 7 p | | 3 s | | 3 p | | 3/2 | 3p |
| 5 | B | 11 p | | 5 p | | 5 p | | 5/2 | 5p |
| 7 | N | 15 p | | 7 p | | 7 p | | 7/2 | 7p |
| 9 | F | 19 p | | 9 p | | 9 p | | 9/2 | 9p |
| 11 | Na | 23 p | | 11 p | | 11 p | | 11/2 | 11p |
| 13 | Al | 27 p | | 13 p | | 13 p | | 13/2 | 13p |
| 15 | P | 31 p | | 15 p | | 15 p | | 15/2 | 15p |
| 17 | Cl | 35 p | | 17 p | | 17 p | | 17/2 | 17p |
| 19 | K | 39 p | | 19 p | | 19 p | | 19/2 | 19p |
| 21 | Sc | 45 p | | 21 p | | 21 p | | 21/2 | 21p |
| 23 | Y | 51 p | | 23 p | | 23 p | | 23/2 | 23p |
| 25 | La | 55 p | | 25 p | | 25 p | | 25/2 | 25p |
| 27 | Co | 59 p | | 27 p | | 27 p | | 27/2 | 27p |
| 29 | Cu | 63 p | | 29 p | | 29 p | | 29/2 | 29p |
| 31 | Ga | 67 p | | 31 p | | 31 p | | 31/2 | 31p |
| 33 | As | 75 p | | 33 p | | 33 p | | 33/2 | 33p |
| 35 | Br | 79 p | | 35 p | | 35 p | | 35/2 | 35p |
| 37 | I | 85 p | | 37 p | | 37 p | | 37/2 | 37p |
| 39 | Tl | 89 p | | 39 p | | 39 p | | 39/2 | 39p |
| 41 | Bi | 93 p | | 41 p | | 41 p | | 41/2 | 41p |
| 43 | Fr | 101 p | | 43 p | | 43 p | | 43/2 | 43p |
| 45 | At | 105 p | | 45 p | | 45 p | | 45/2 | 45p |
| 47 | Ac | 107 p | | 47 p | | 47 p | | 47/2 | 47p |
| 49 | Th | 113 p | | 49 p | | 49 p | | 49/2 | 49p |
| 51 | Pa | 121 p | | 51 p | | 51 p | | 51/2 | 51p |
| 53 | Np | 127 p | | 53 p | | 53 p | | 53/2 | 53p |
| 55 | Pu | 133 p | | 55 p | | 55 p | | 55/2 | 55p |
| 57 | Am | 139 p | | 57 p | | 57 p | | 57/2 | 57p |
| 59 | Cm | 145 p | | 59 p | | 59 p | | 59/2 | 59p |
| 61 | Bk | 151 p | | 61 p | | 61 p | | 61/2 | 61p |
| 63 | Cf | 157 p | | 63 p | | 63 p | | 63/2 | 63p |
| 65 | Es | 163 p | | 65 p | | 65 p | | 65/2 | 65p |
| 67 | Fm | 169 p | | 67 p | | 67 p | | 67/2 | 67p |
| 69 | M | 175 p | | 69 p | | 69 p | | 69/2 | 69p |
| 71 | La | 181 p | | 71 p | | 71 p | | 71/2 | 71p |
| 73 | Pr | 187 p | | 73 p | | 73 p | | 73/2 | 73p |
| 75 | Sr | 193 p | | 75 p | | 75 p | | 75/2 | 75p |
| 77 | Zr | 199 p | | 77 p | | 77 p | | 77/2 | 77p |
| 79 | Hf | 205 p | | 79 p | | 79 p | | 79/2 | 79p |
| 81 | Ti | 211 p | | 81 p | | 81 p | | 81/2 | 81p |
| 83 | Bi | 217 p | | 83 p | | 83 p | | 83/2 | 83p |

demands that there be no spins of $9/2$ in this shell, and none have been found. No f or p levels should occur and, except for Eu^{149} , there is no indication of any.

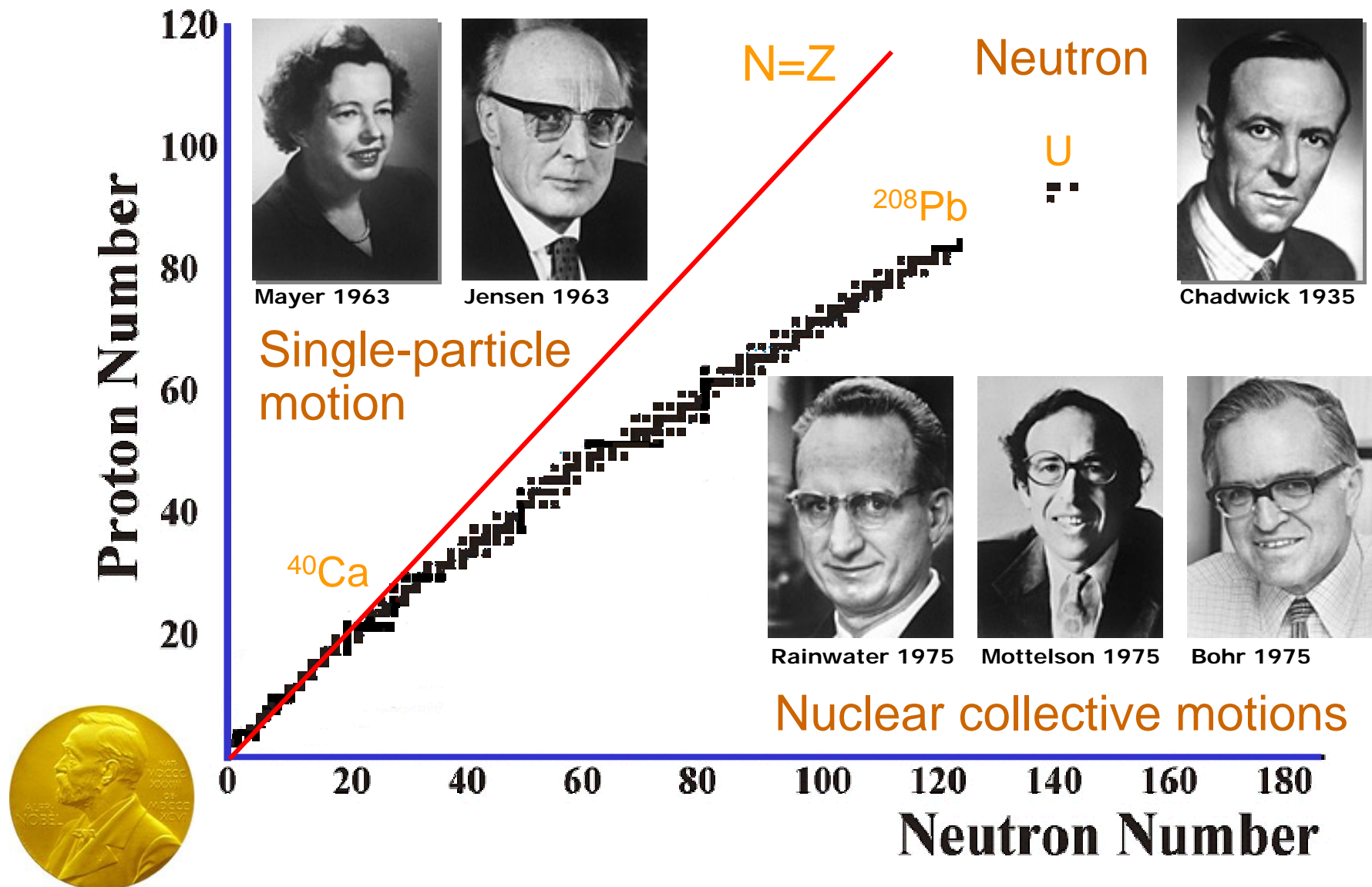
$$\ell, s = 1/2 \quad \begin{cases} j_{<} = \ell - 1/2 \\ j_{>} = \ell + 1/2 \end{cases}$$



$$V_{\ell s}(r) \vec{\ell} \cdot \vec{s}$$

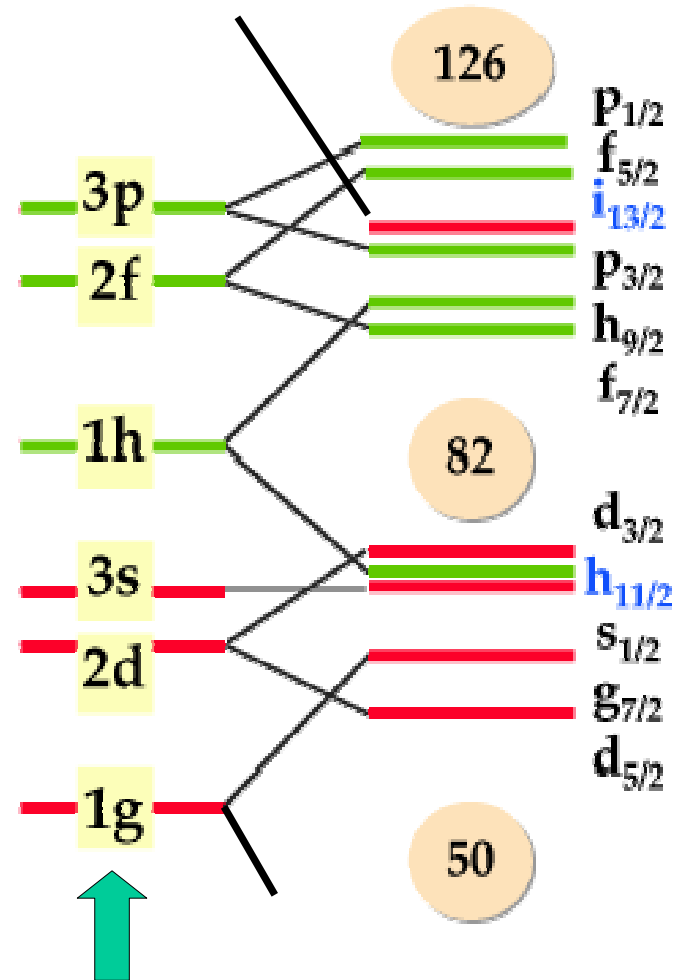
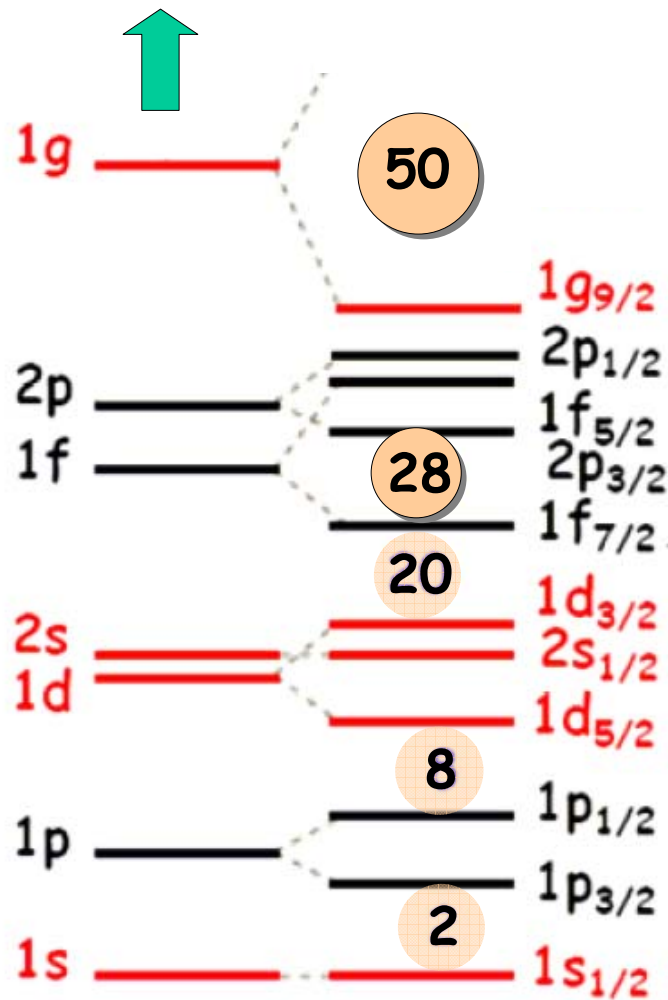
Thanks are due to Enrico Fermi for the remark, "Is there any indication of spin-orbit coupling?" which was the origin of this paper.

300 stable isotopes exist on Earth – Segré chart

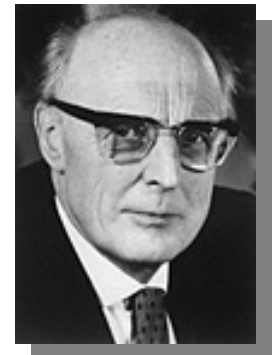


Magic numbers – a framework for nuclear stability

Nuclear mean-field levels



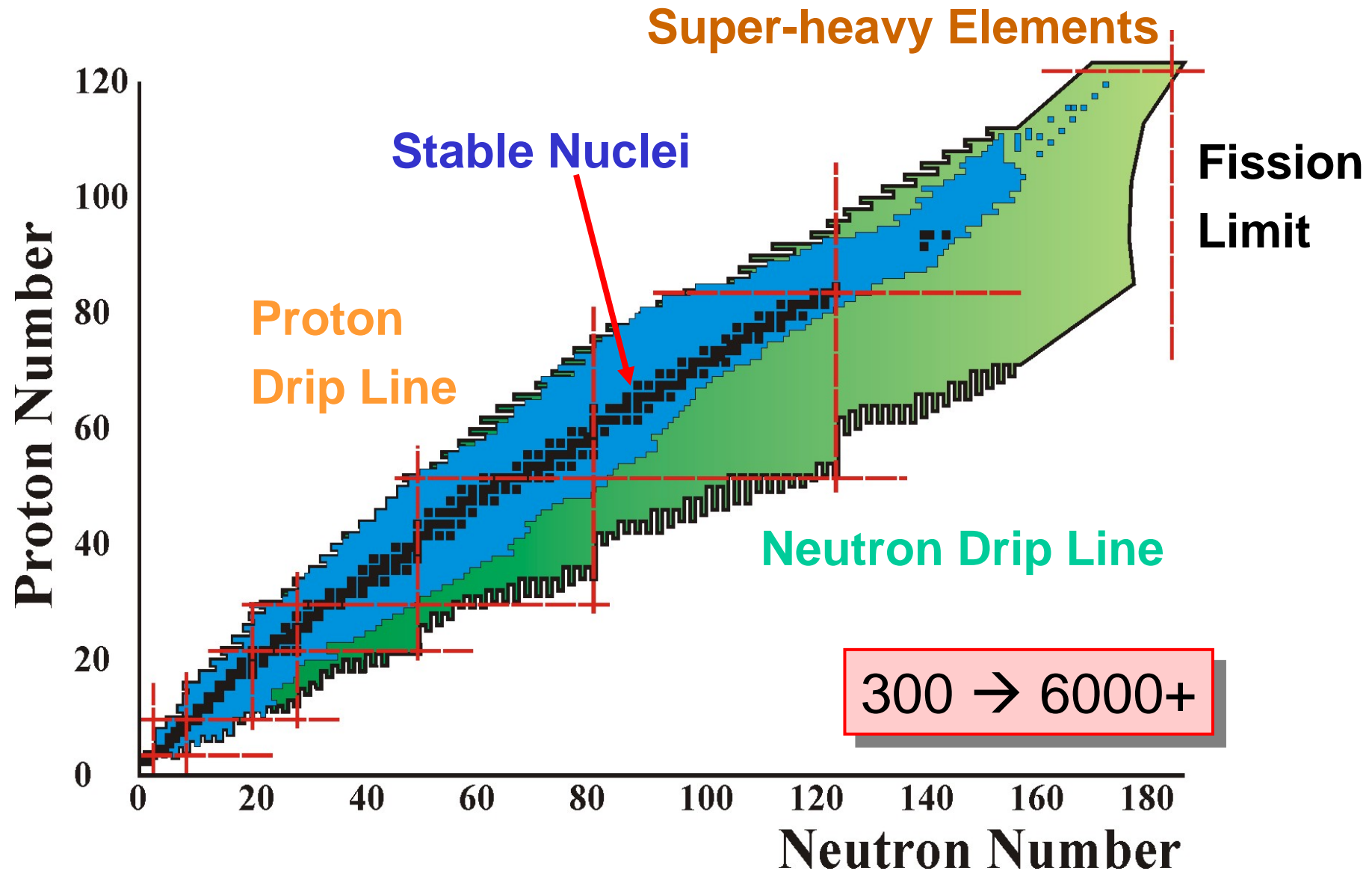
Mayer



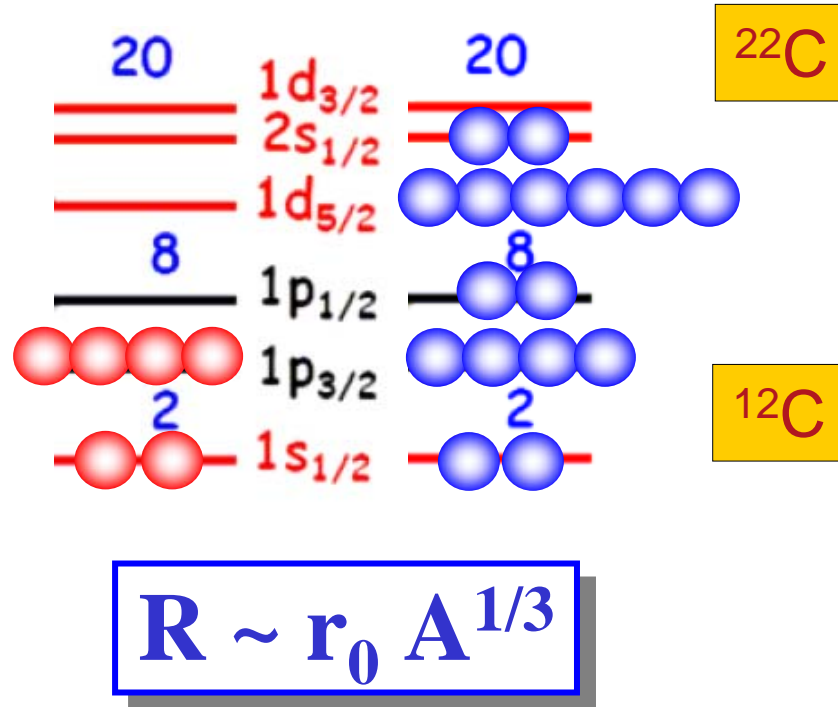
Jensen

$$V_{Cent}(r) + V_{\ell s}(r)\vec{\ell} \cdot \vec{s}$$

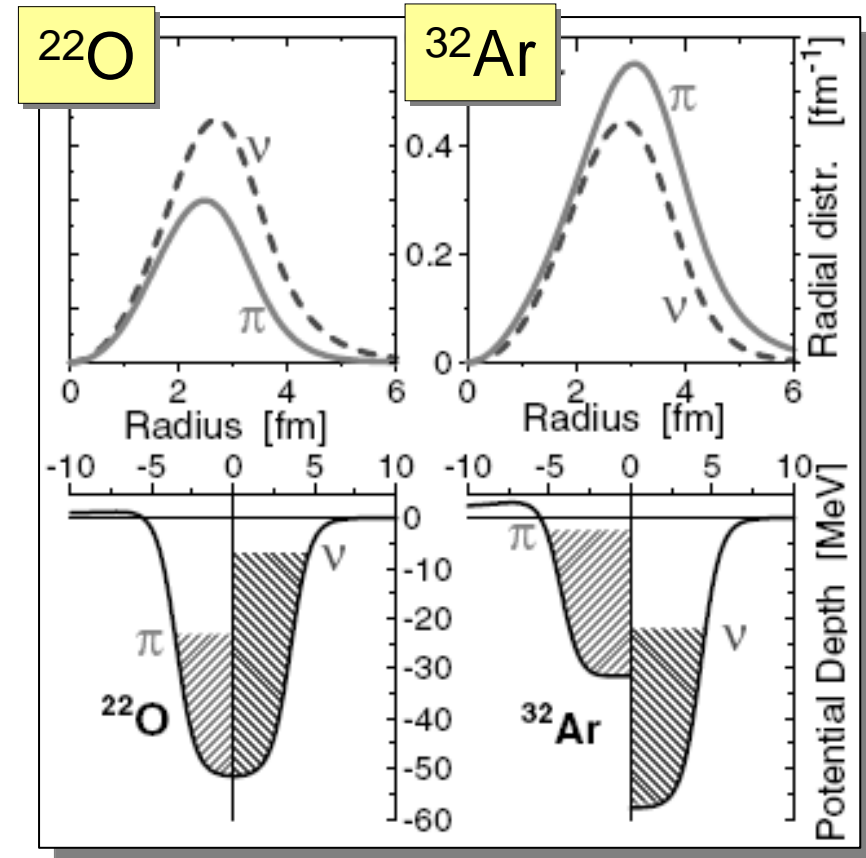
So, the limits to nuclear existence are ...?



Exotic neutron + proton combinations - expectations



Very weak binding of last nucleons as the driplines are approached (1.4 MeV in ^{22}C , 18.7 MeV for neutron in ^{12}C)



Z=8 N=14

$S_n = 6.8 \text{ MeV}$

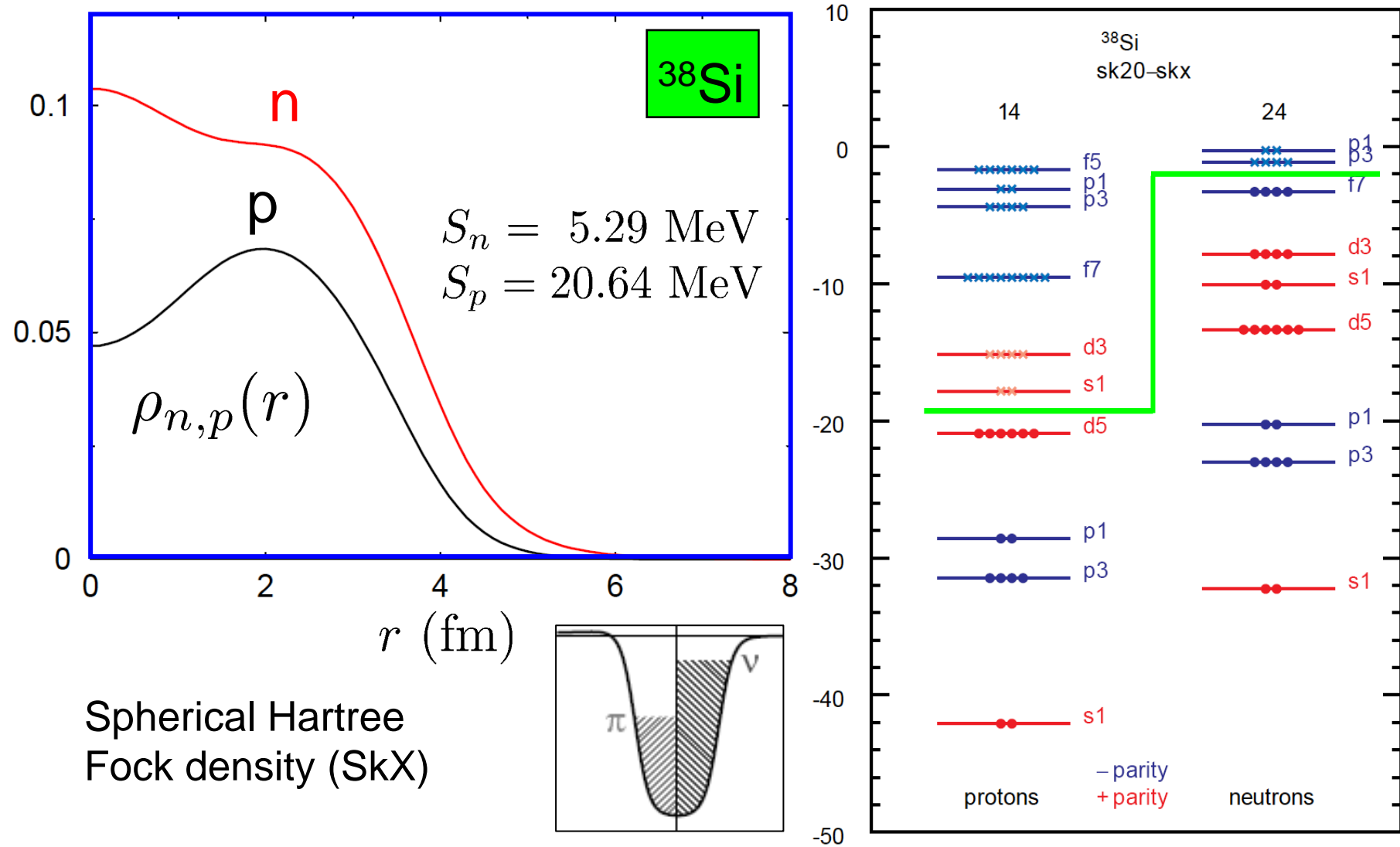
$S_p = 23 \text{ MeV}$

Z=18 N=14

$S_n = 22 \text{ MeV}$

$S_p = 2.4 \text{ MeV}$

Make use of available tools – locate Fermi surfaces



Bound states – the mean field helps our intuition

```

*****
*
* IA,IZ =    24    8 *
*
*****

INPUT VALUES

----- Neutron bound state results -----

k n l j      e  IE  OCC
1 1 s 1/2 -26.757 1  2.00  36.70  35.28
2 1 p 3/2 -16.883 1  4.00  36.70  35.80
3 1 p 1/2 -12.396 1  2.00  36.70  36.04
4 1 d 5/2 -6.166 1  6.00  36.70  36.37
5 1 d 3/2 -0.109 1  0.00  36.70  36.69
6 2 s 1/2 -3.360 1  2.00  36.70  36.52
7 1 f 7/2 -0.200 3  0.00  46.02  46.01
8 1 f 5/2 -0.200 3  0.00  60.56  60.55
9 2 p 3/2 -0.200 3  0.00  48.10  48.09

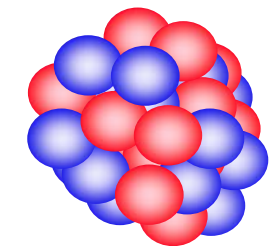
----- Neutron single-particle radii -----

```

| | | | | R(2) | R(4) | OCC | rho(8.9) | rho(9.9) | rho(10.9) |
|---|---|---|-----|-------|-------|-------|-----------|-----------|-----------|
| 1 | 1 | s | 1/2 | 2.274 | 2.575 | 2.000 | 0.848E-09 | 0.706E-10 | 0.600E-11 |
| 2 | 1 | p | 3/2 | 2.863 | 3.133 | 4.000 | 0.188E-07 | 0.244E-08 | 0.325E-09 |
| 3 | 1 | p | 1/2 | 2.954 | 3.268 | 2.000 | 0.727E-07 | 0.122E-07 | 0.210E-08 |
| 4 | 1 | d | 5/2 | 3.434 | 3.757 | 6.000 | 0.524E-06 | 0.129E-06 | 0.327E-07 |
| 5 | 1 | d | 3/2 | 4.662 | 6.063 | 0.000 | 0.131E-04 | 0.675E-05 | 0.371E-05 |
| 6 | 2 | s | 1/2 | 4.172 | 4.895 | 2.000 | 0.769E-05 | 0.278E-05 | 0.102E-05 |
| 7 | 1 | f | 7/2 | 3.865 | 4.440 | 0.000 | 0.324E-05 | 0.134E-05 | 0.600E-06 |
| 8 | 1 | f | 5/2 | 3.890 | 4.477 | 0.000 | 0.341E-05 | 0.141E-05 | 0.631E-06 |
| 9 | 2 | p | 3/2 | 6.815 | 8.635 | 0.000 | 0.451E-04 | 0.270E-04 | 0.167E-04 |

The mean field – e.g. spherical HF - gives an excellent estimate and guide

$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



$^{24}\text{O}(g.s.)$

And more realistic (complex) structure models?

MARIA GOEPPERT MAYER *Nobel Lecture, December 12, 1963*

The shell model → more properly: independent particle model

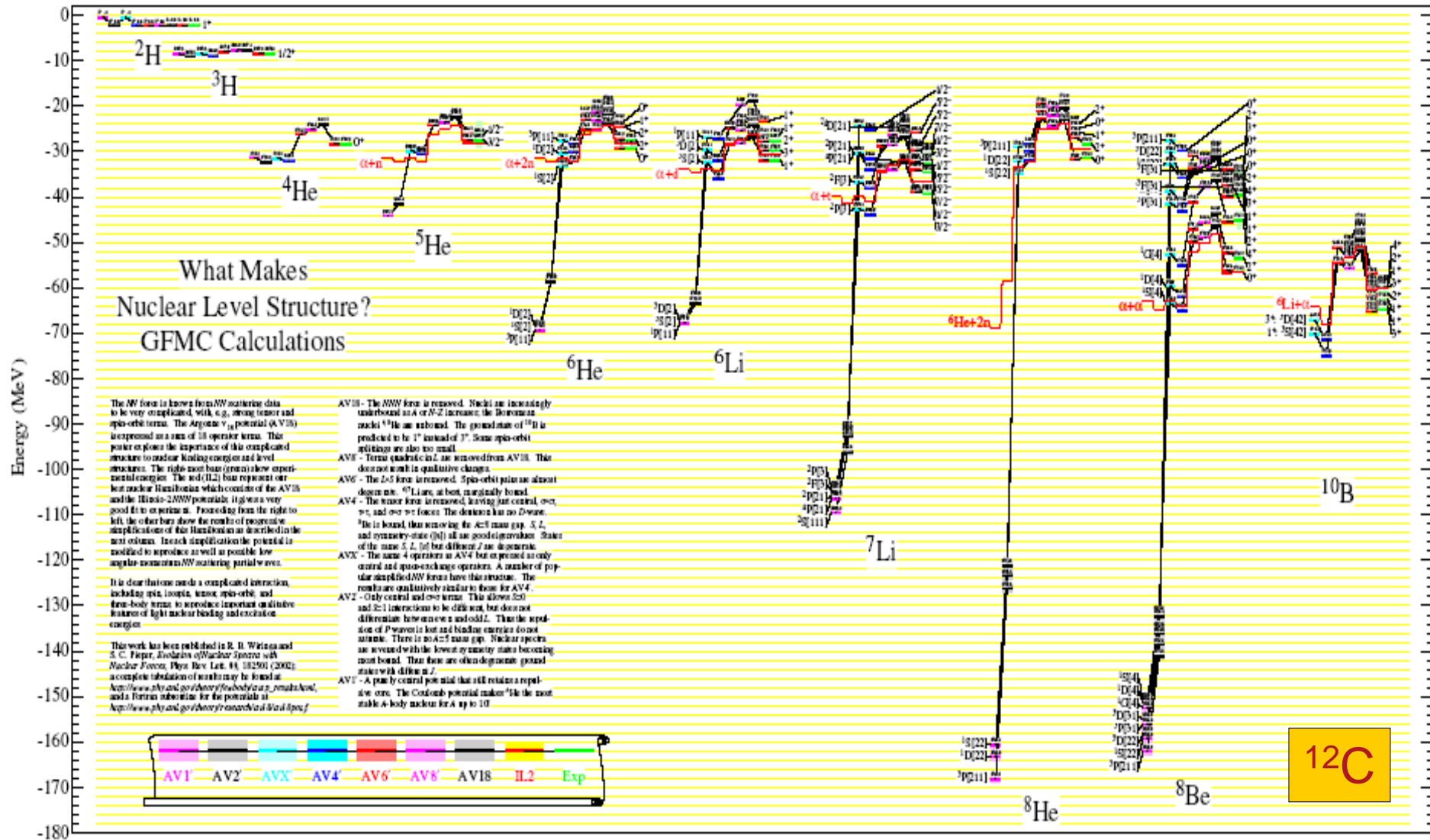
.....

If the forces are known, one should in principle be able to calculate deductively the properties of individual complex nuclei. Only after this has been accomplished can one say that one completely understands nuclear structures.

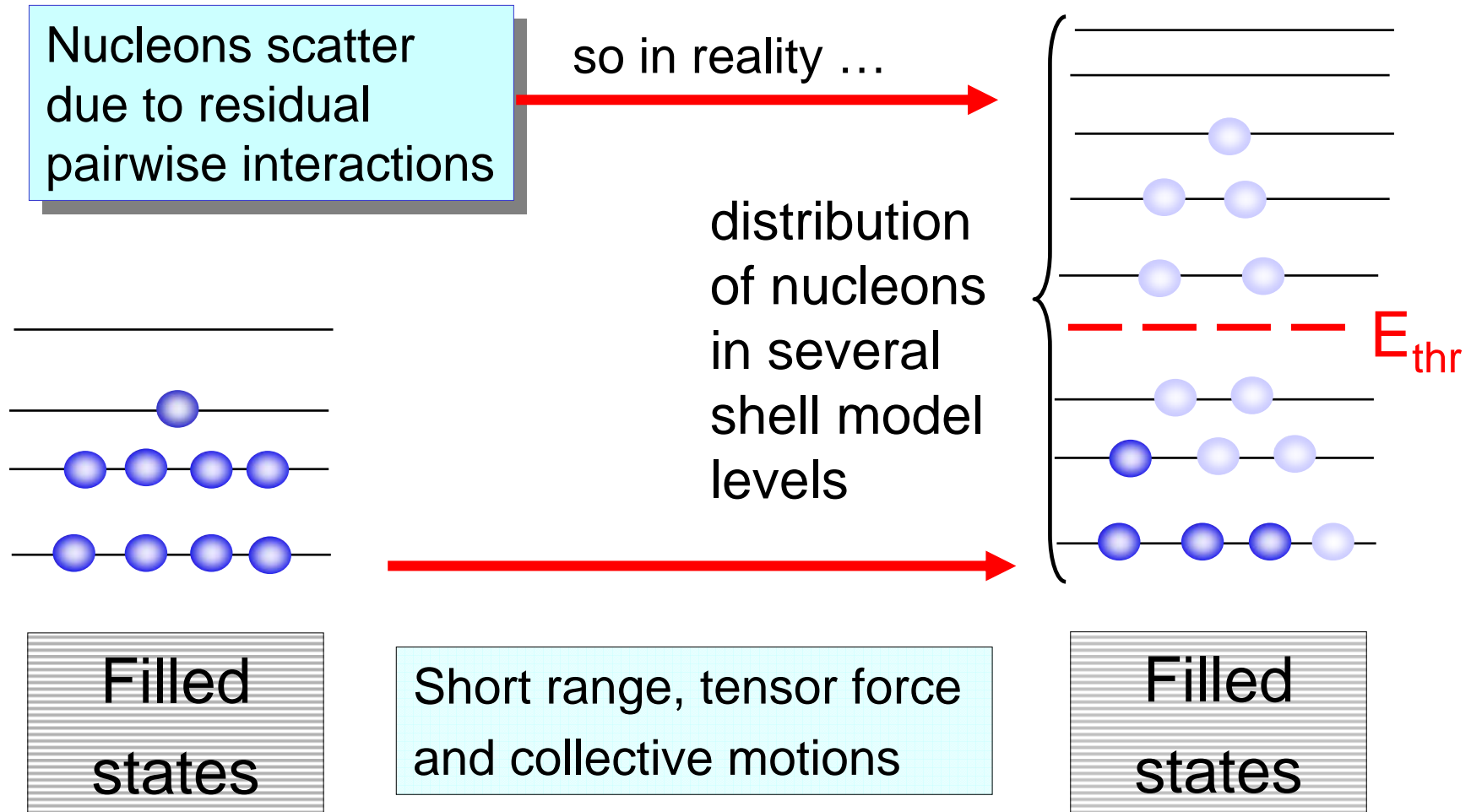
Ground state of ^{12}C 75,000 CPU-hours (27 Teraflop hours)

The shell model has initiated a large field of research. It has served as the starting point for more refined calculations. There are enough nuclei to investigate so that the shell modellists will not soon be unemployed.

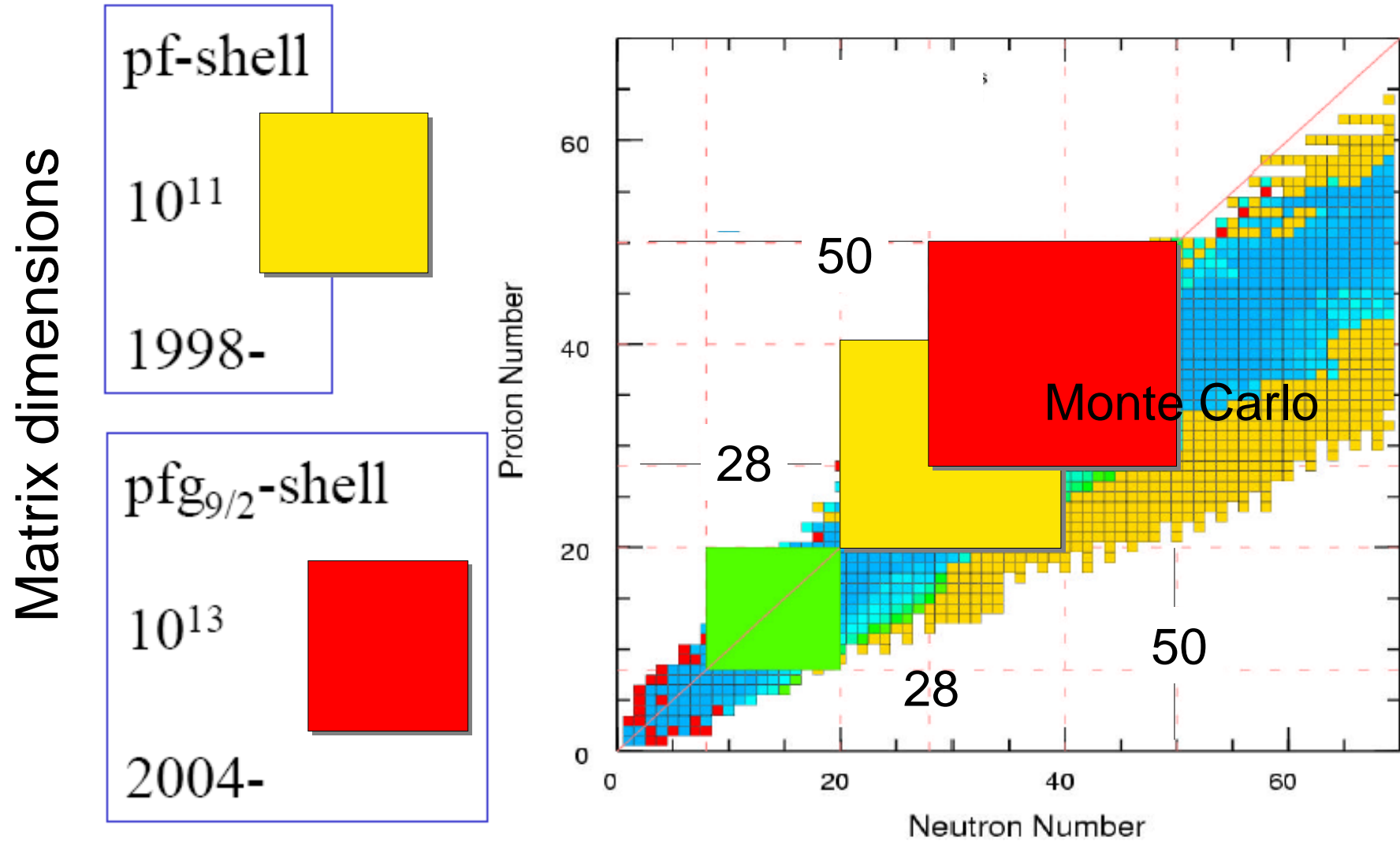
The state-of-the art – A=12 and less – really hard!



Modern 'shell model' calculations do much more

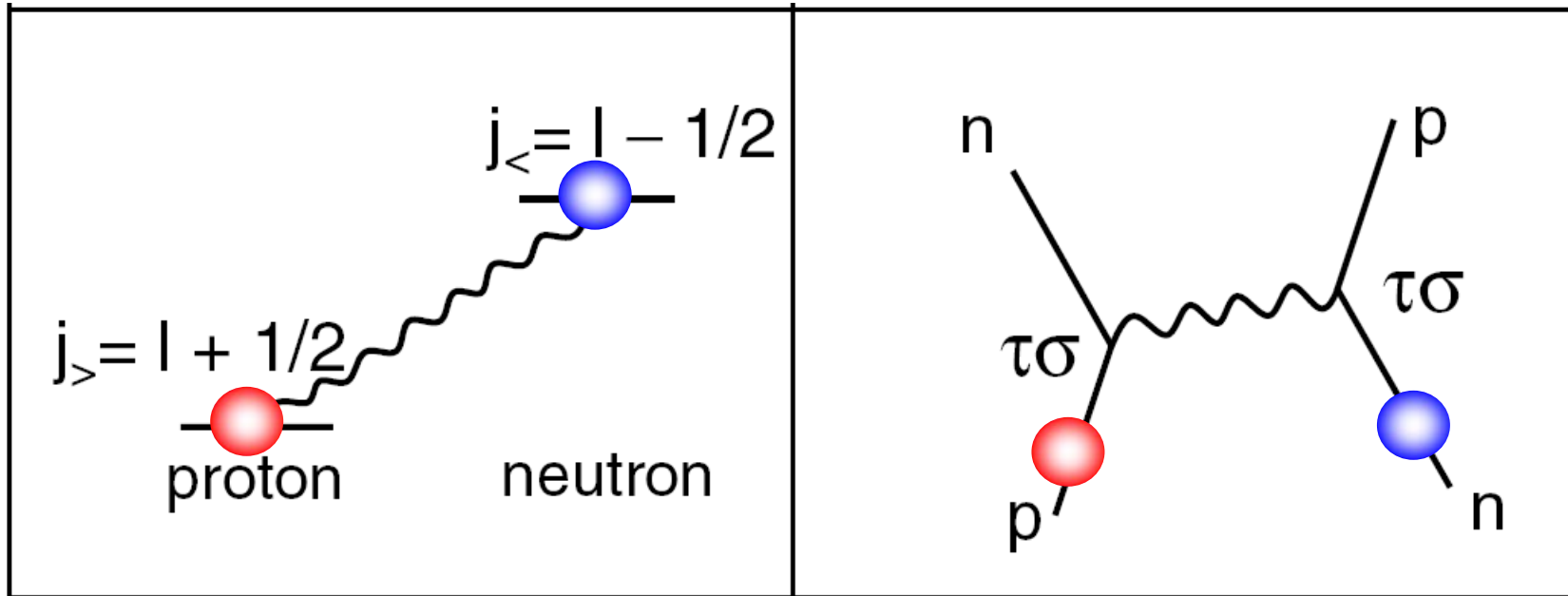


Calculations are big - computational challenge



From Alex Brown

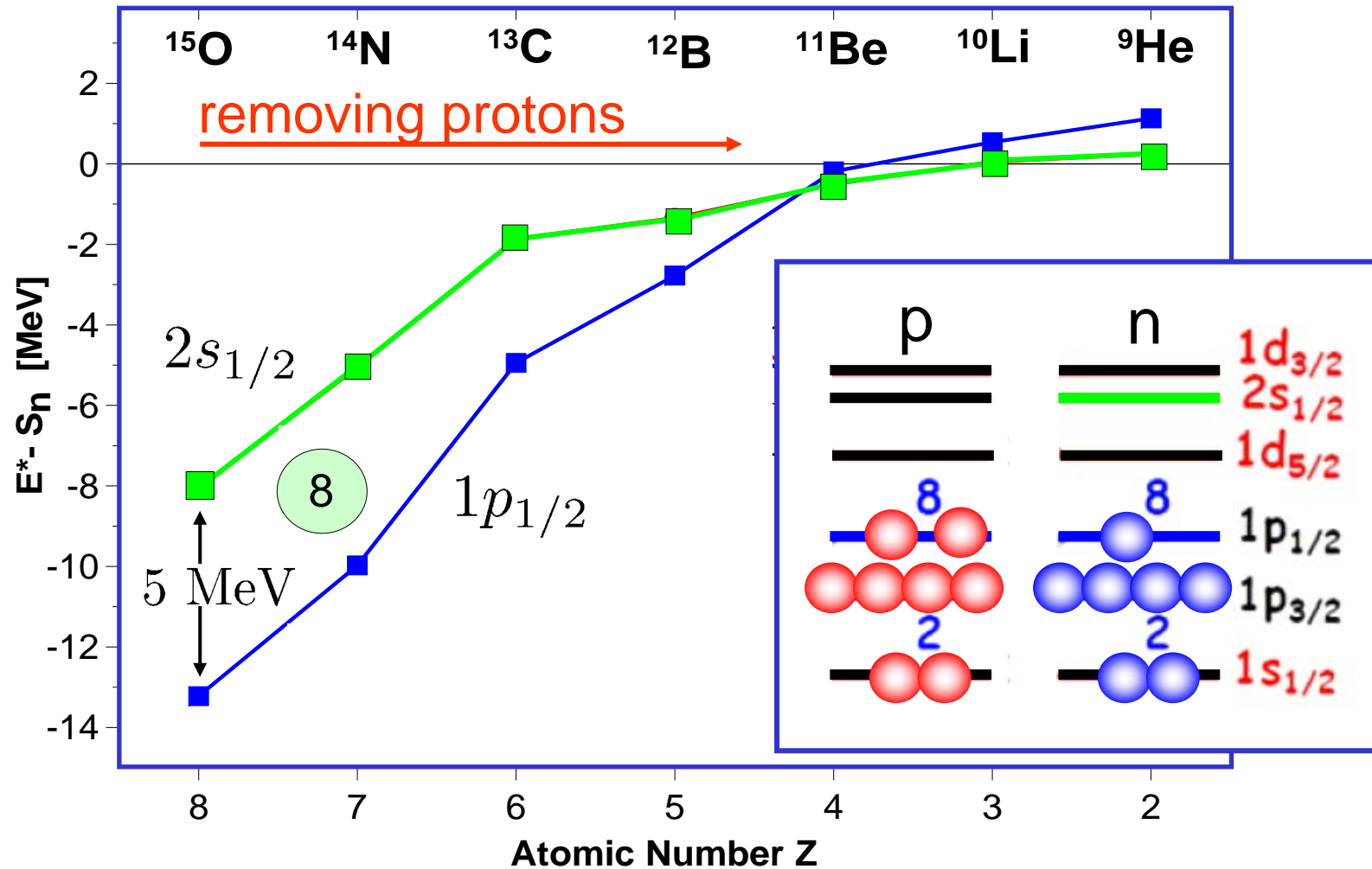
Implications for shell structure: asymmetric nuclei



Attractive interaction between neutrons and protons occupying $j_>$ and $j_<$ levels, repulsive $j_>$ and $j_>$ levels – from several sources, but mainly the tensor force

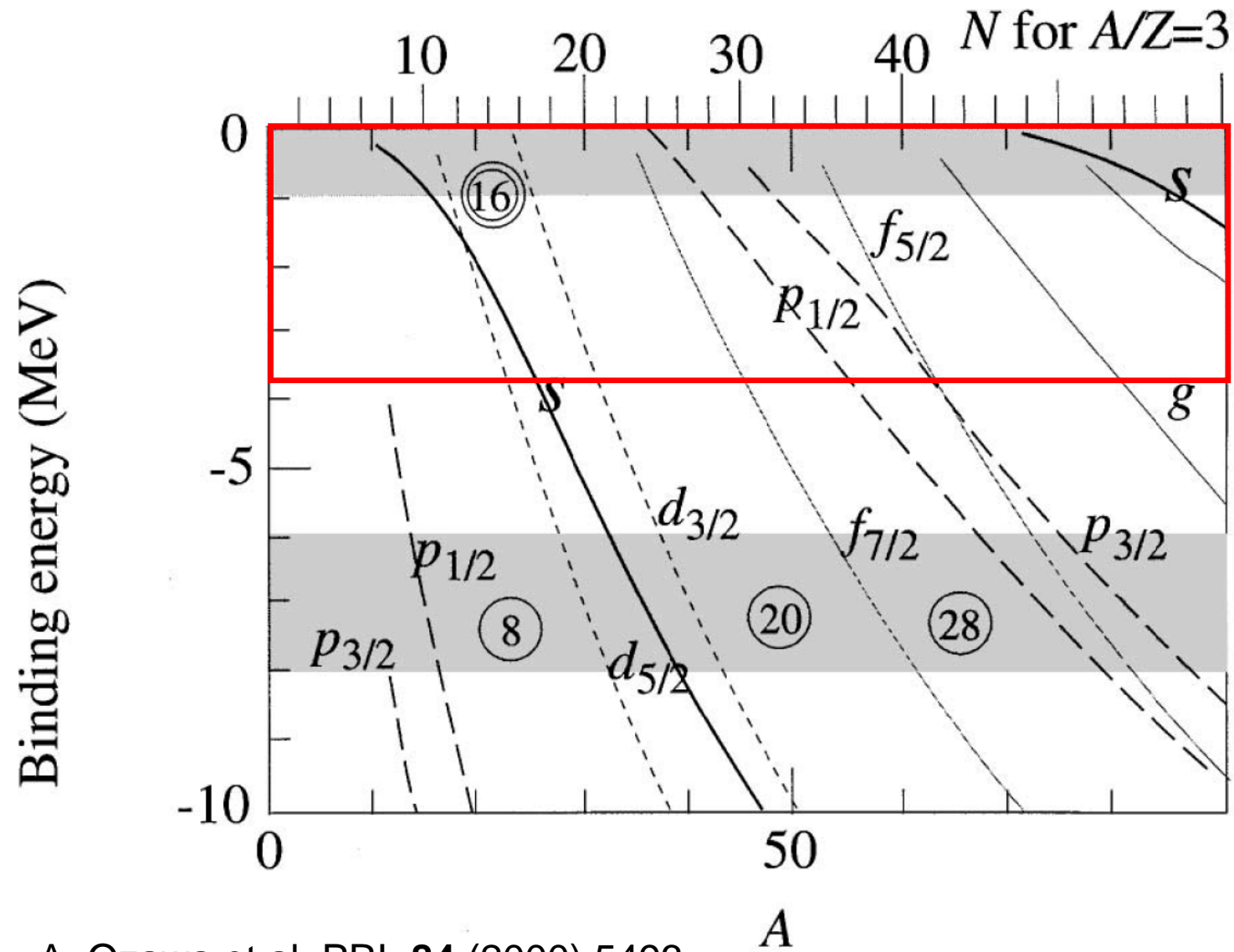
Takaharu Otsuka et al, Phys. Rev. Lett. **87**, 082502 (2001), **95**, 232502 (2005), **105**, 032501 (2010)

Migration of levels for N=7 – loss of the N=8 magic



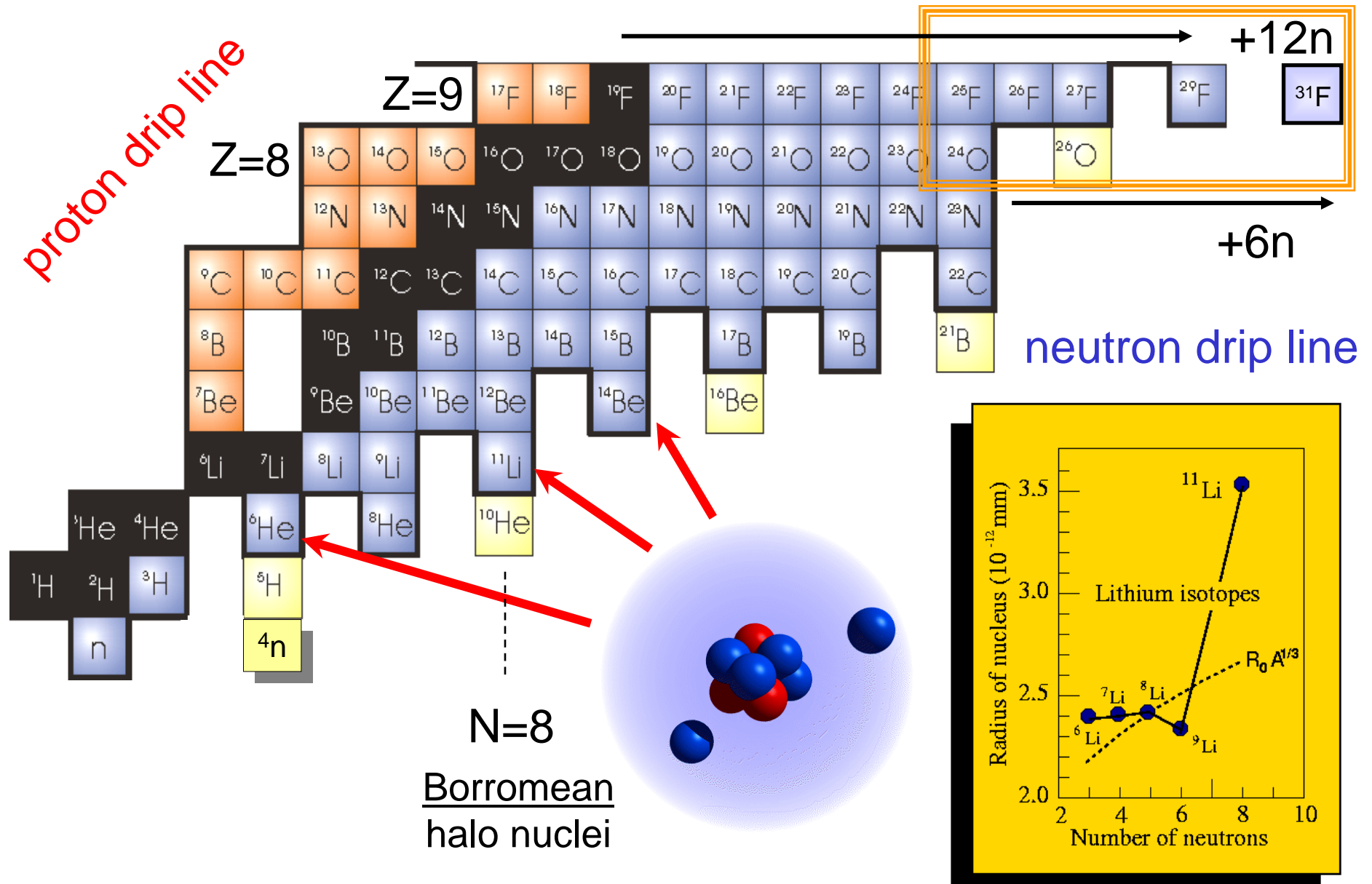
From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Low angular momentum states see well diffuseness

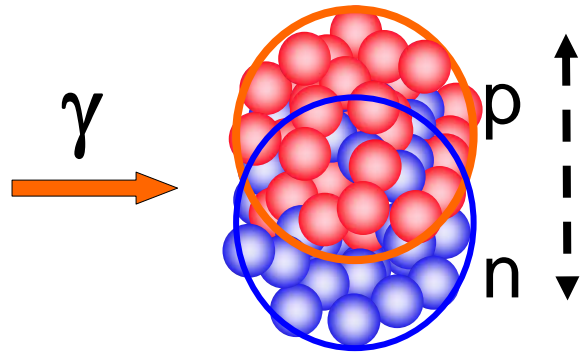


A. Ozawa et al, PRL **84** (2000) 5493

We have only reached the driplines in the light nuclei

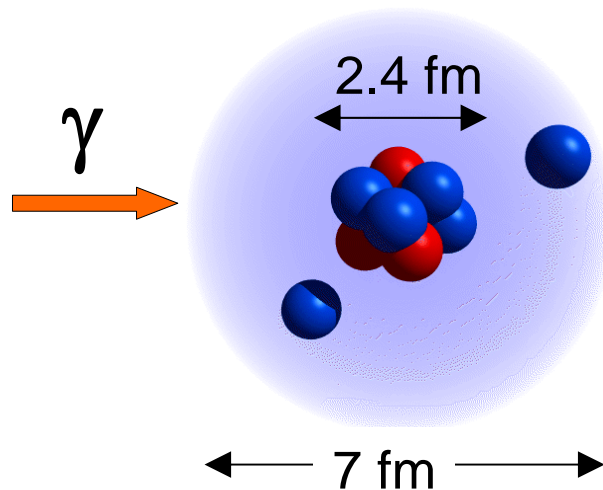


Halo nuclei –extremes of weak binding - responses

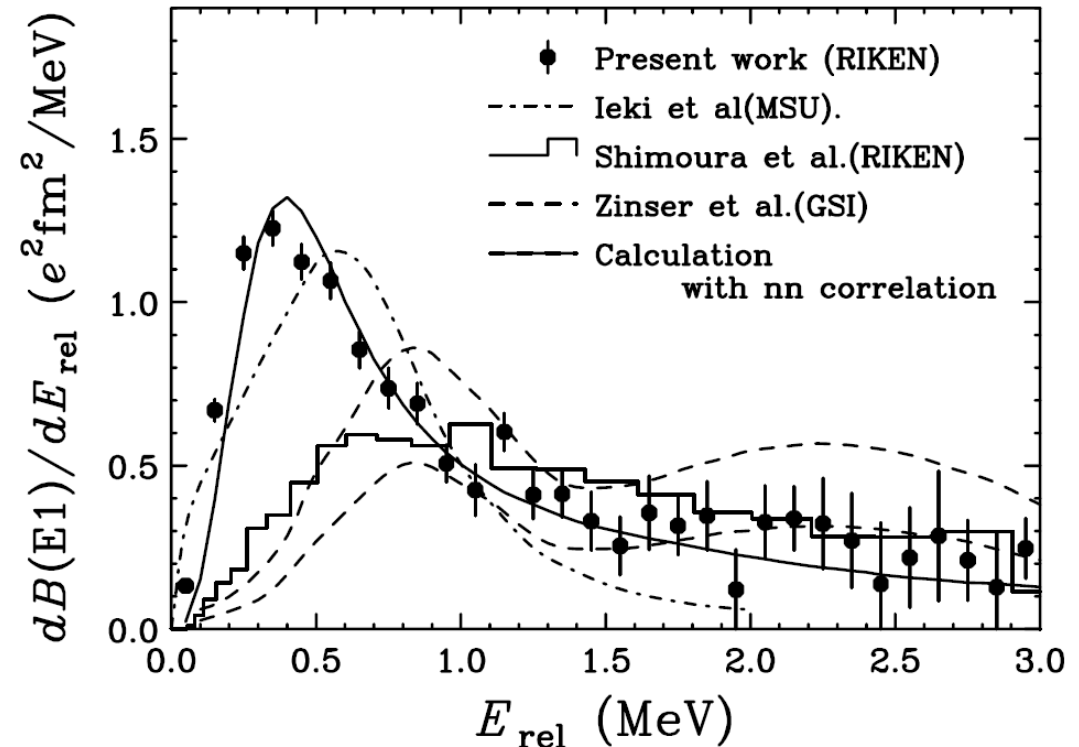


${}^6\text{He}$ (2n, 1 MeV) ${}^{11}\text{Be}$ (1n, 0.5 MeV)
 ${}^{11}\text{Li}$ (2n, 0.5 MeV) ${}^{14}\text{Be}$ (2n, ~1 MeV)

(Giant) electric Dipole excitation \rightarrow 10-20 MeV

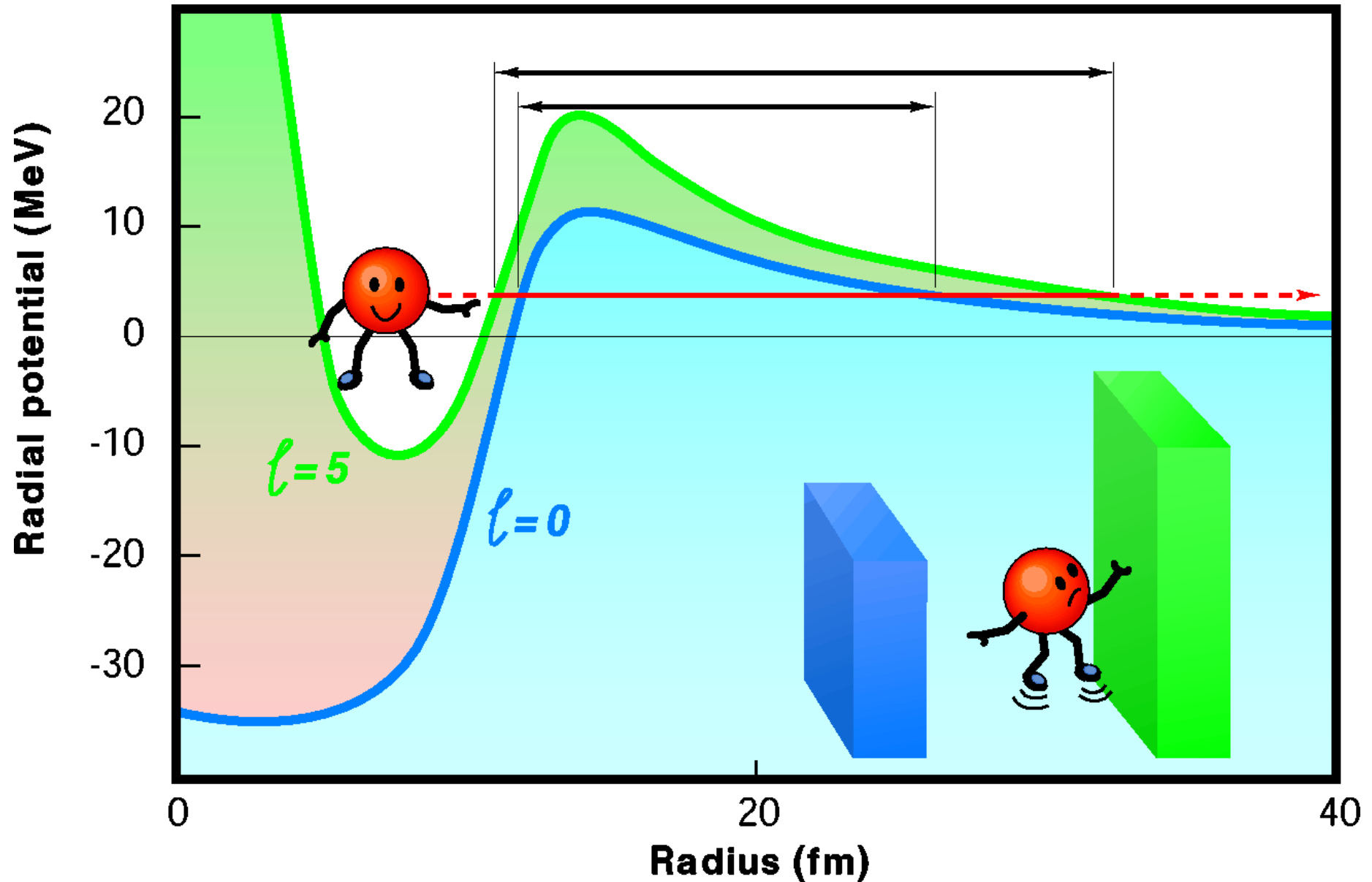


Electric dipole response – ${}^{11}\text{Li}$

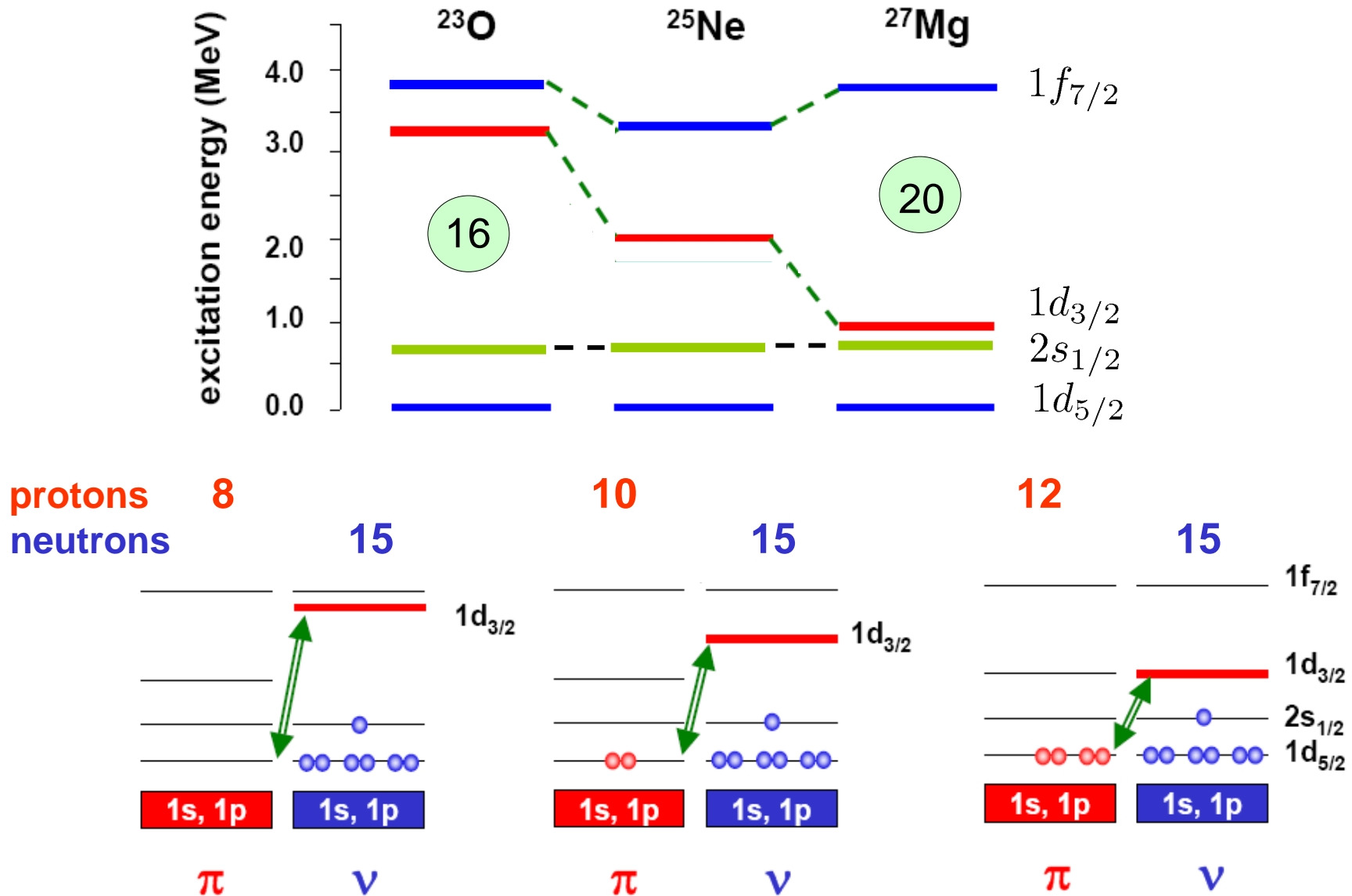


T. Nakamura et al., PRL (2006) in press

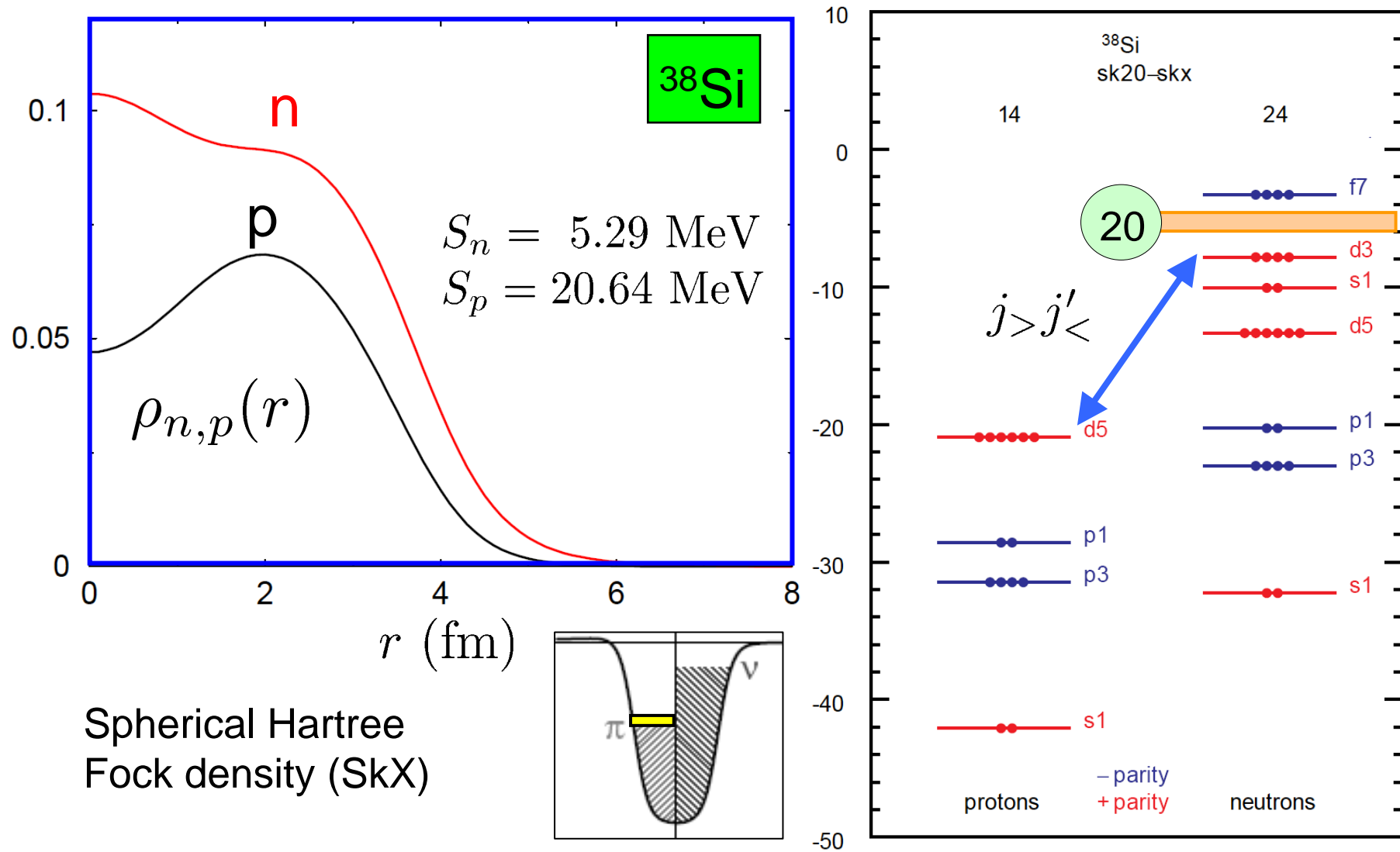
Proton decay – a probe of proton-dripline structures



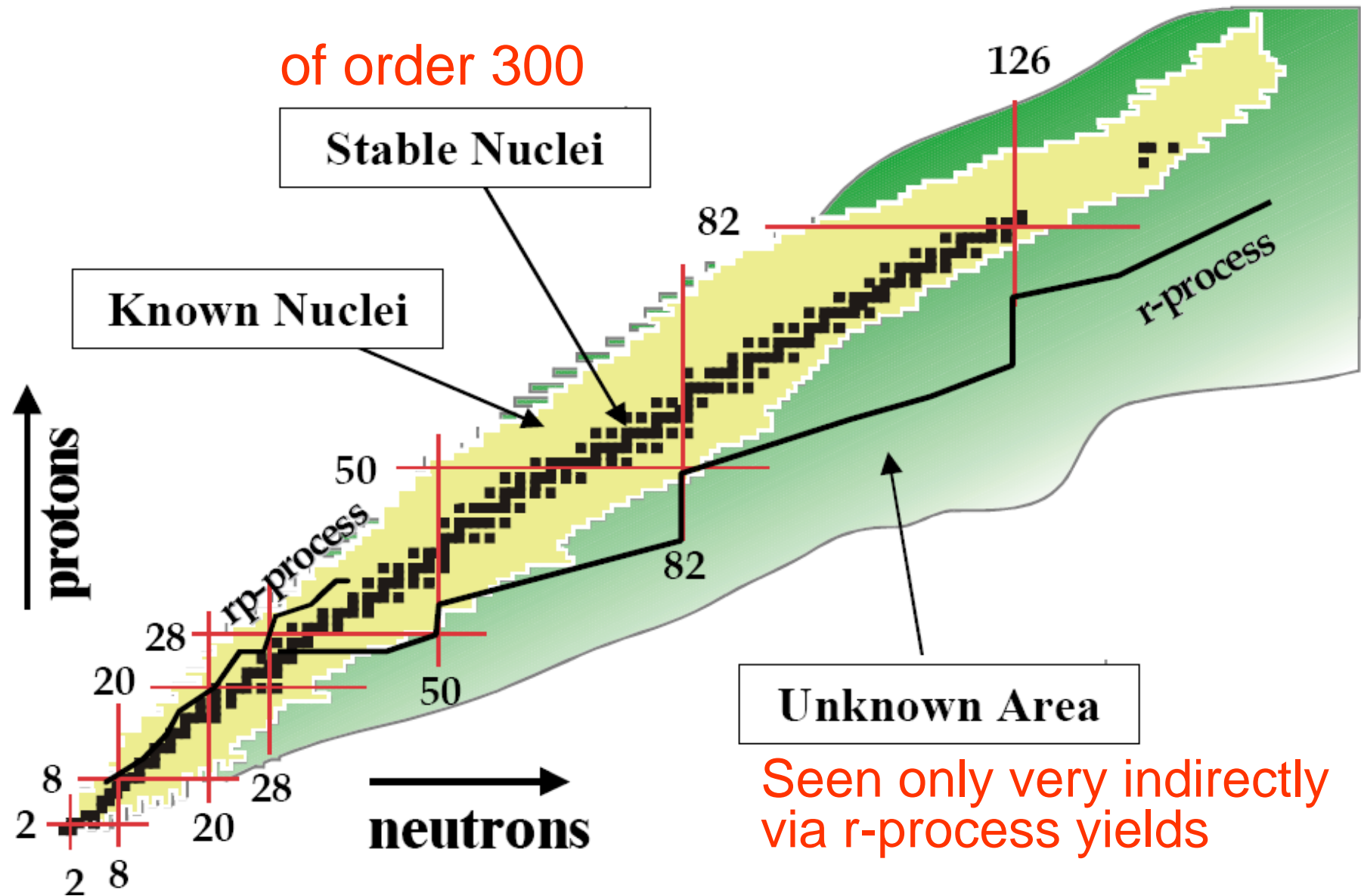
Magic numbers change with “neutron richness”



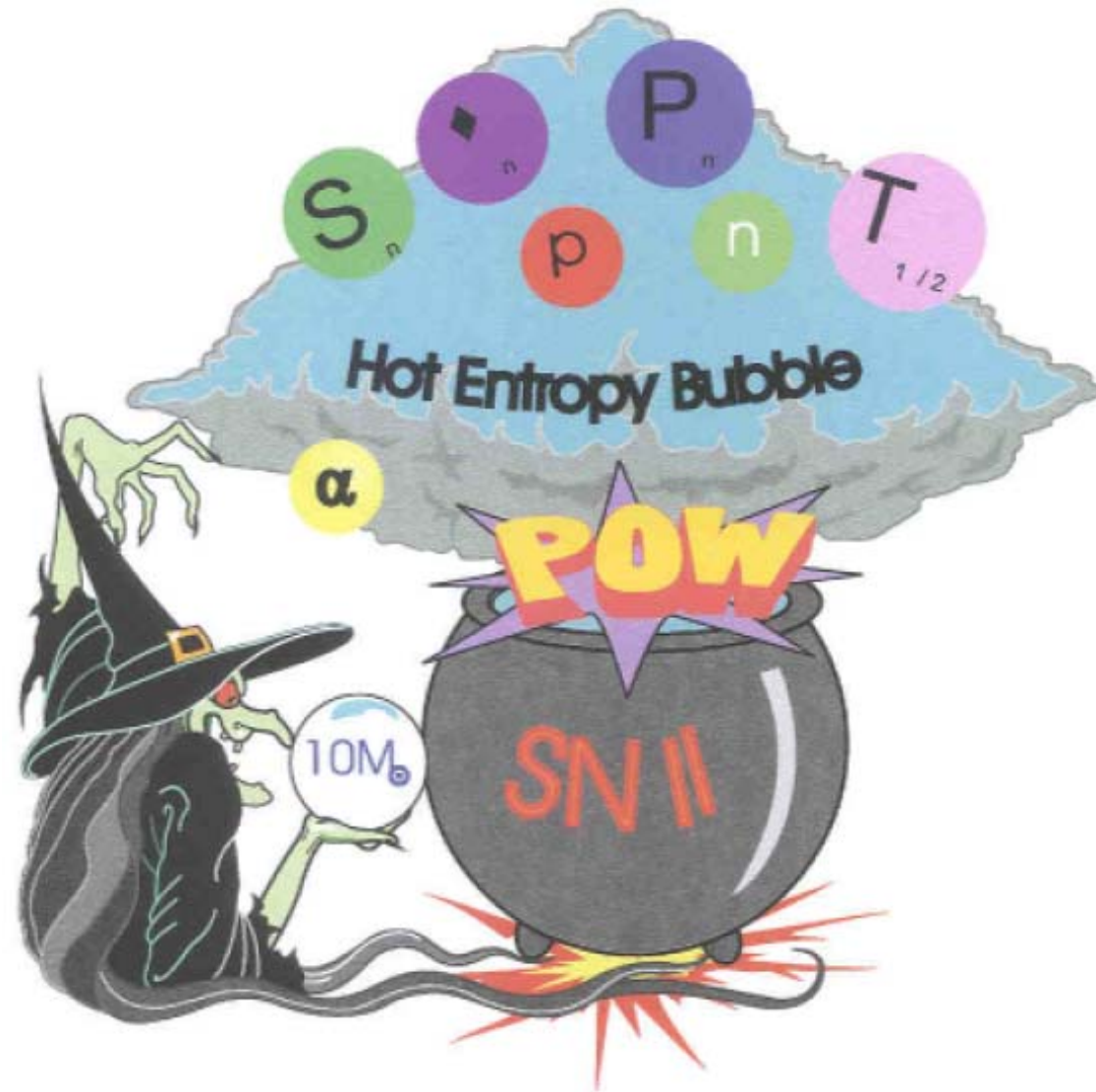
Otsuka: the np interaction tensor correlation



Nuclear landscape (circa 2010)



$\frac{1}{2}$ of all --- is produced in r-process synthesis





The Joint Institute for Nuclear Astrophysics



<http://groups.nsl.msui.edu/nero/Web/materials.html>

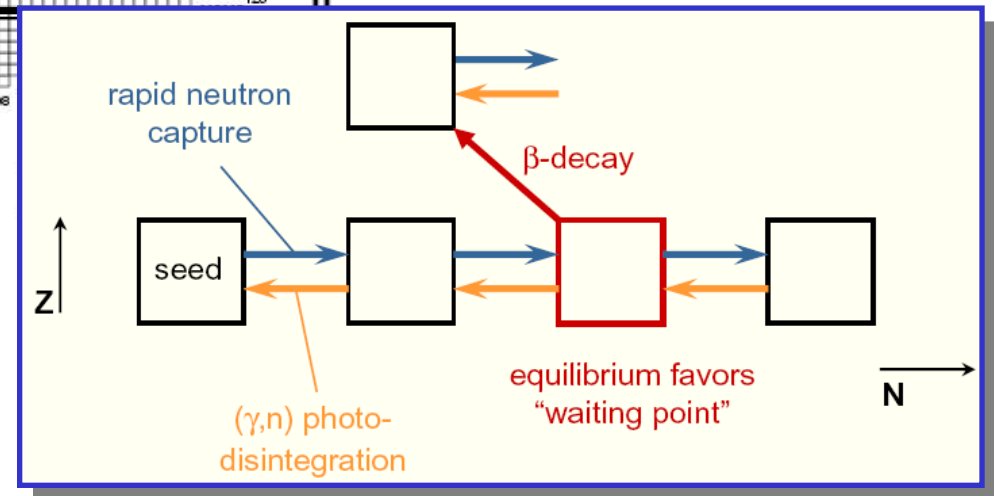
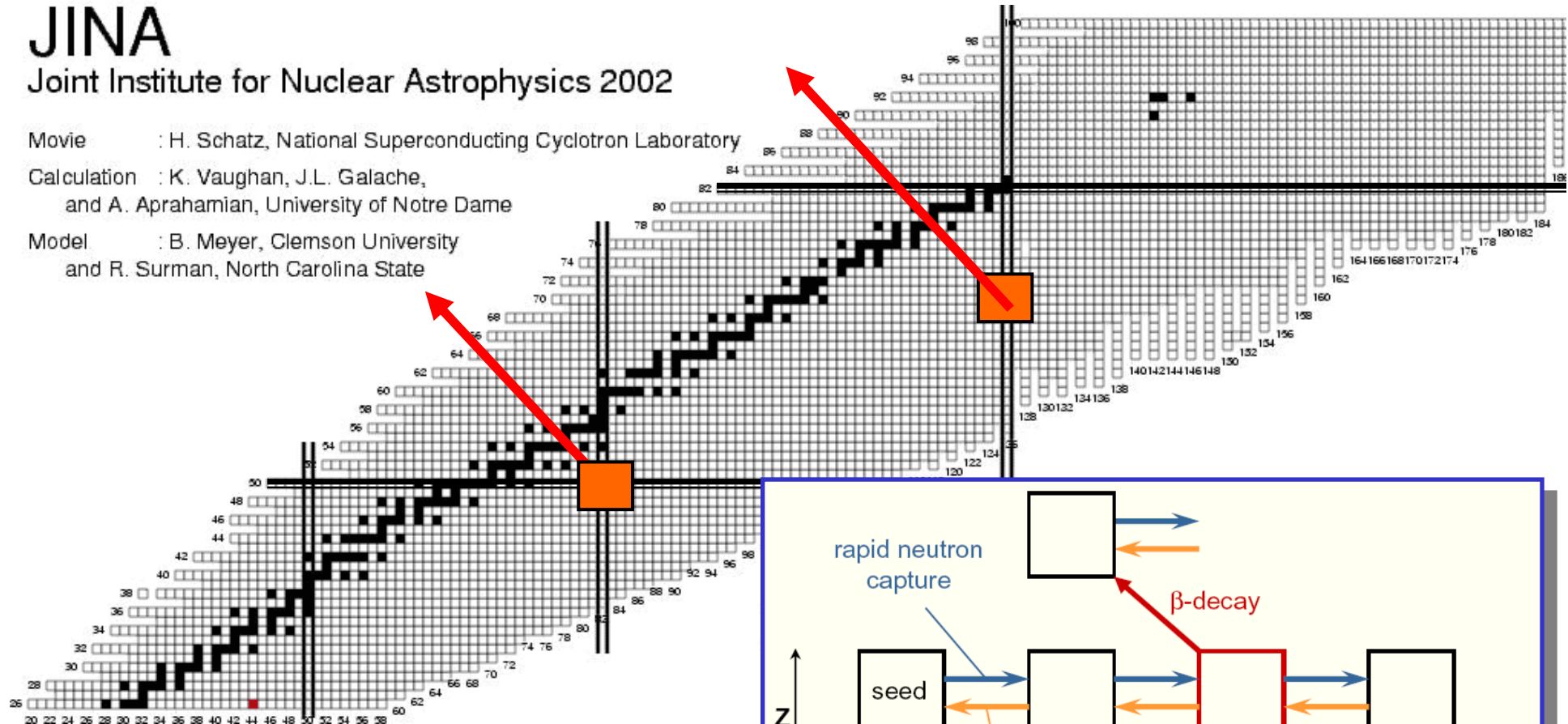
JINA

Joint Institute for Nuclear Astrophysics 2002

Movie : H. Schatz, National Superconducting Cyclotron Laboratory

Calculation : K. Vaughan, J.L. Galache,
and A. Aprahamian, University of Notre Dame

Model : B. Meyer, Clemson University
and R. Surman, North Carolina State

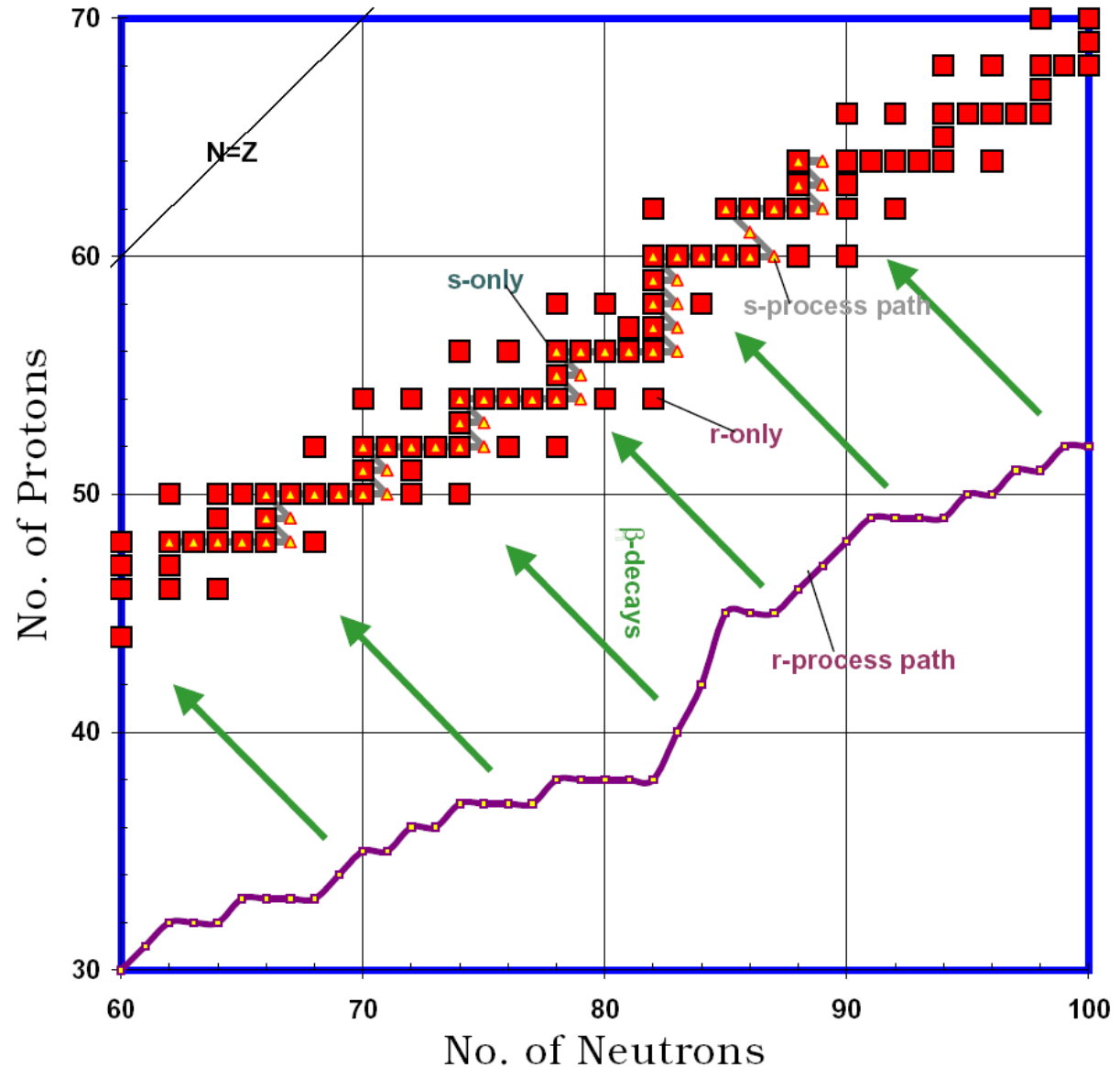
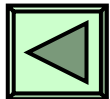


r-process synthesis modelling – inputs needed?

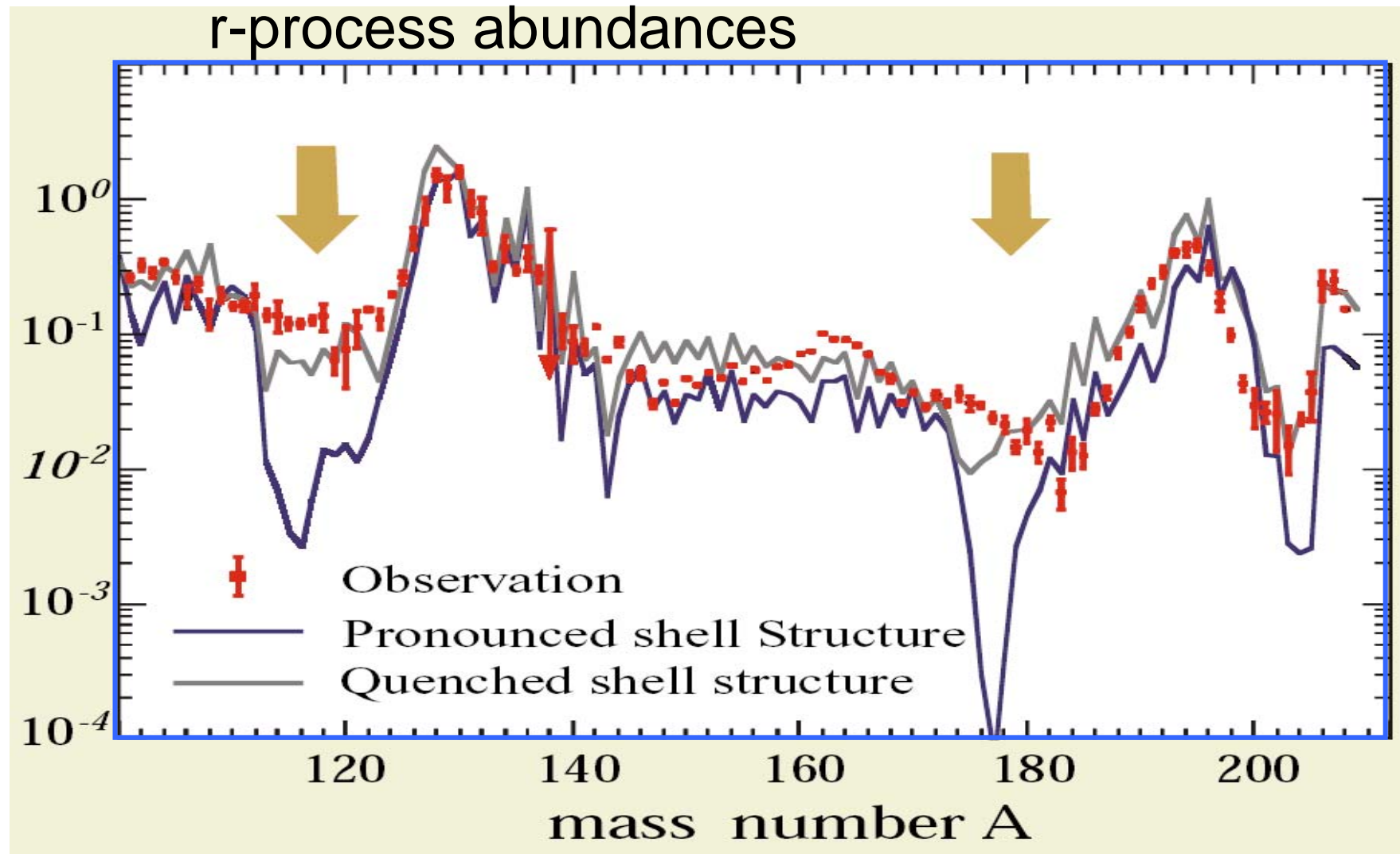
Need is for:

Masses – stability -
determine reaction rates

beta-decay half-lives – masses,
but also structure effects – need to
test shell model predictions – too
many systems to measure sensibly

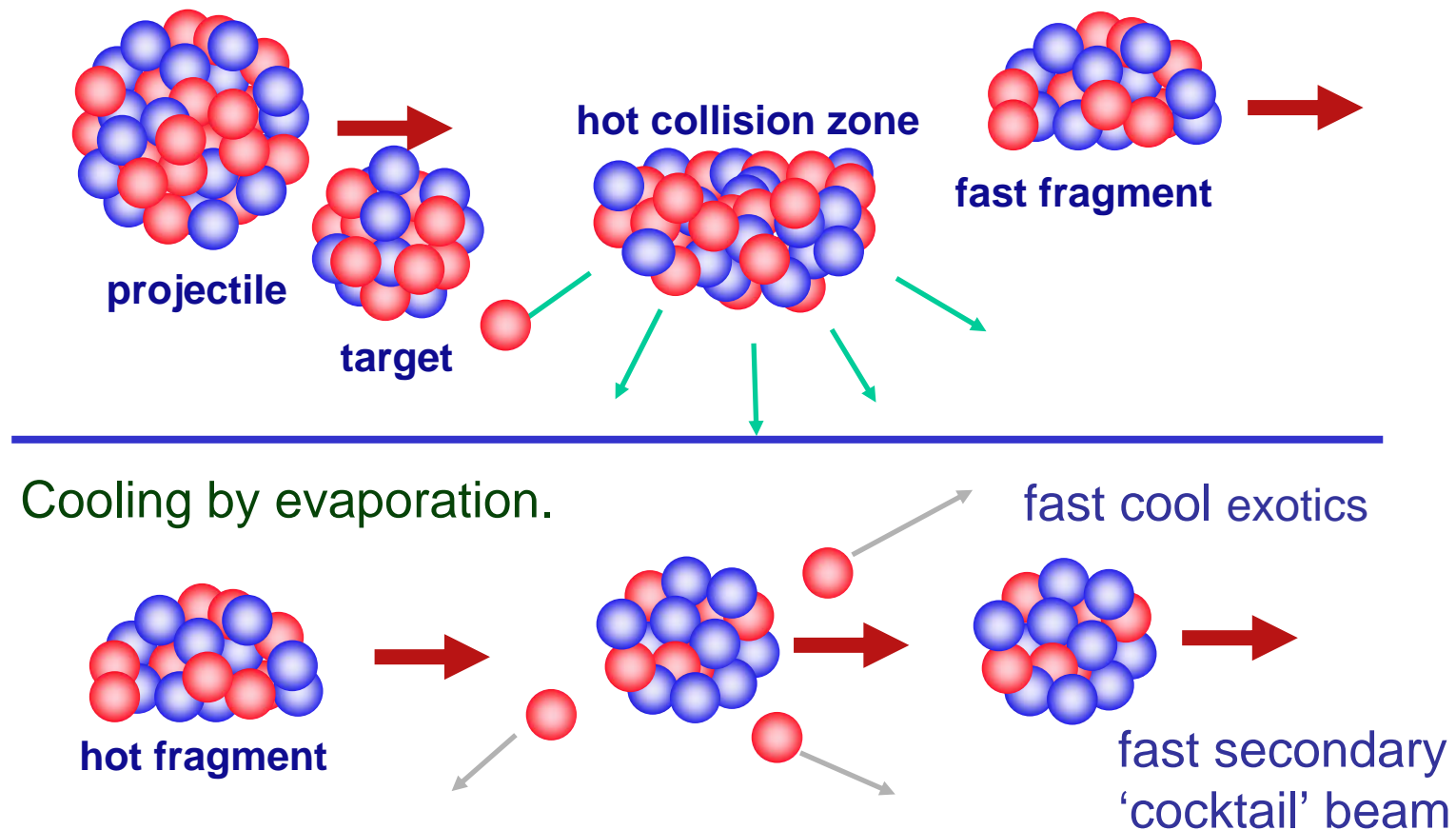


'Less magic' in heavier systems – evidence?



Exotic nuclei production - projectile fragmentation

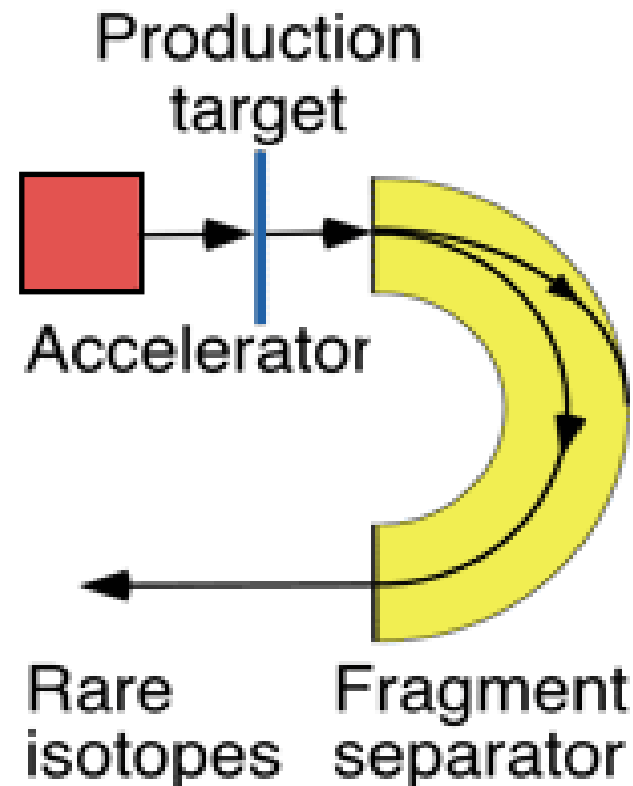
Random removal of protons and neutrons from heavy projectile in peripheral collisions at high energy - 100 MeV per nucleon or more



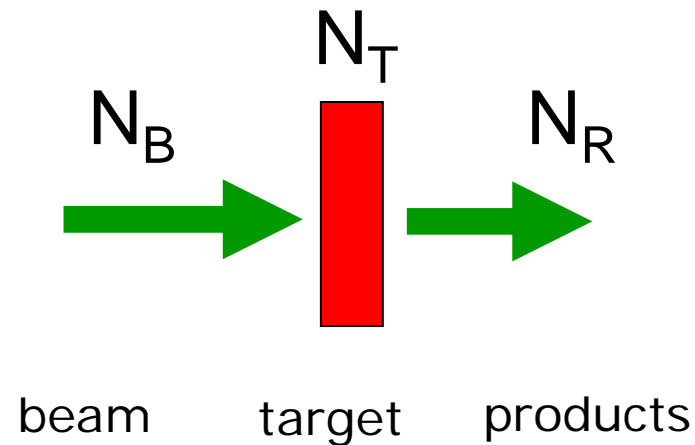
Schematic of a Projectile Fragmentation Facility

GANIL (France), GSI (Germany), NSCL (USA), RIKEN (Japan)

- High-energy beams ($E/A > 50$ MeV) of only modest beam quality
- Fast, physical method of separation, no chemistry
- Suitable for short-lived isotopes (lifetimes $> 10^{-6}$ s)
- But, low-energy beams are (very) difficult
- Modest beam quality



Use of rarest beams a 'few' atomic nuclei/second



- Fast exotic beams allow for
 - thick secondary targets
 - event-by-event identification
 - clean product selection
 - nevertheless

- $N_R = s \times N_T \times N_B$
 - s cross section
 - N_T atoms in target
 - N_B beam rate
 - N_R reaction rate

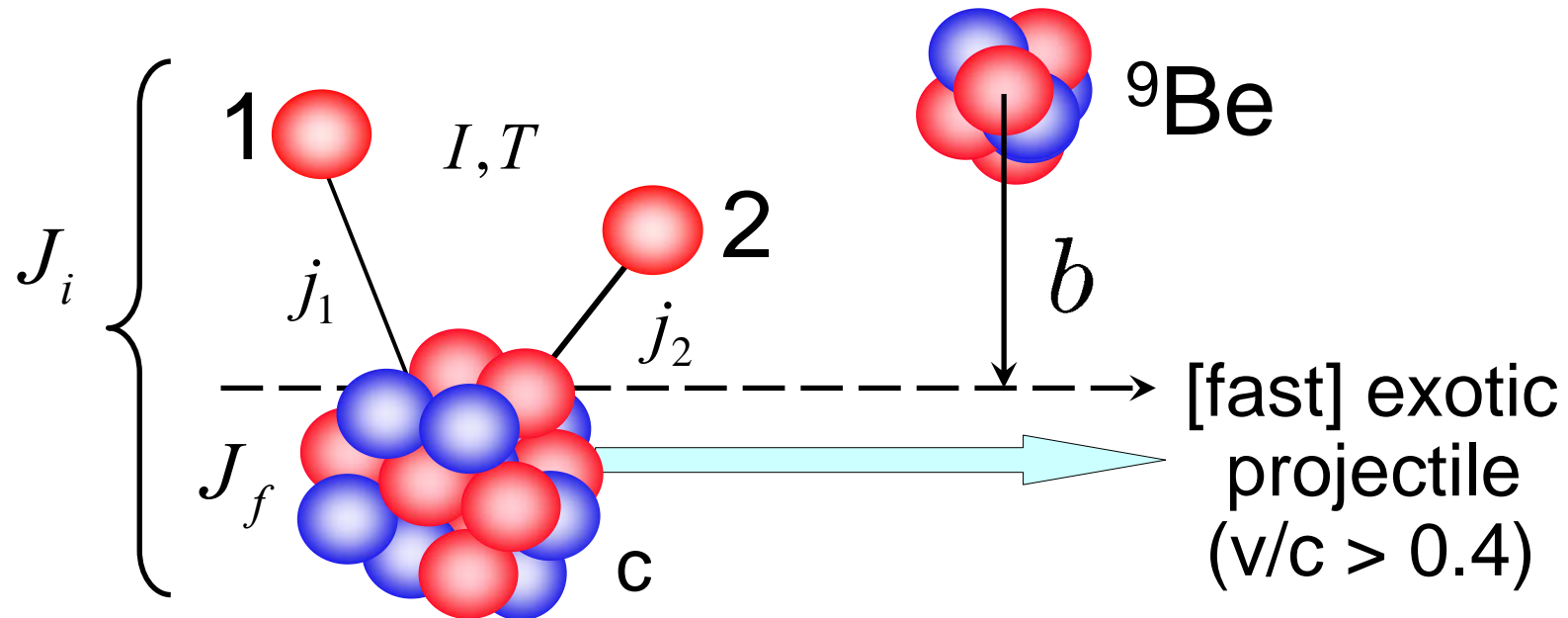
- Example
 - $s = 100$ millibarn
 - $N_T = 10^{21}$
 - $N_B = 3$ Hz
 - $N_R = 26/\text{day}$
 $= 3 \times 10^{-4}$ Hz

So, learned so far

- 1: Must rethink 'textbook' nuclear shell structure when treating asymmetric N:Z systems – see breakdown of N=8, 20, (28 ...?) shell gaps.
- 2: Asymmetry allows the observation and study of novel NN interaction effects – and new structure
- 3: An effective means of production is using high energy fragmentation – that produce the exotic nuclei as fast secondary beams: 100 MeV/u
4. Experiments for the rarest cases, with intensities of a few particles/s are hard – reactions with high cross sections/efficient detection are essential
- 5: Structure calculations for $A > 12$ are hard, they use effective interactions that need to be tested

Probing single particle (shell model) states

One such experimental option is one or two-nucleon removal – at ~ 100 MeV/nucleon

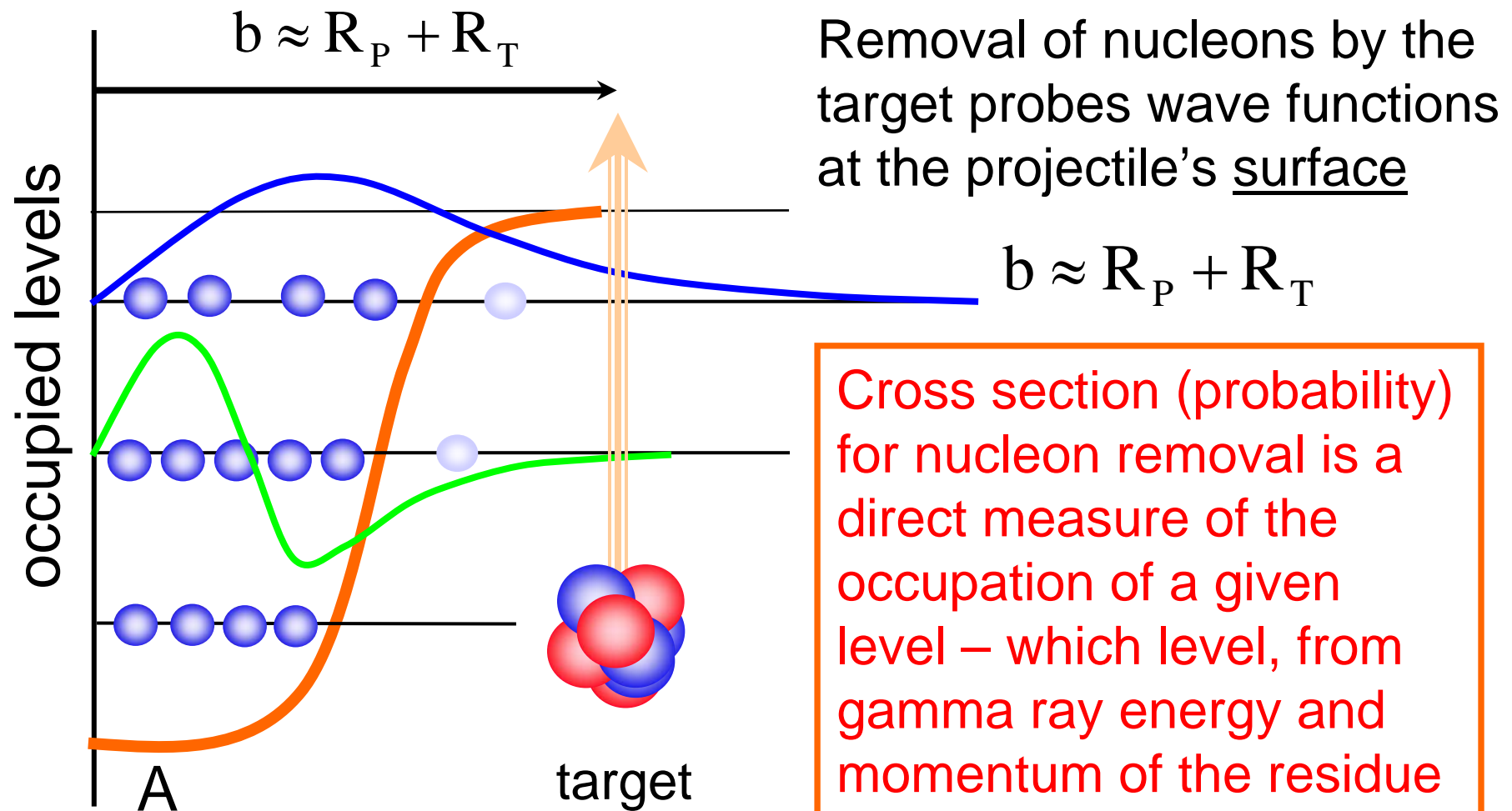


Experiments do not measure target final states. Final state of core c measured – using decay gamma rays.

How can we describe and what can we learn from these?

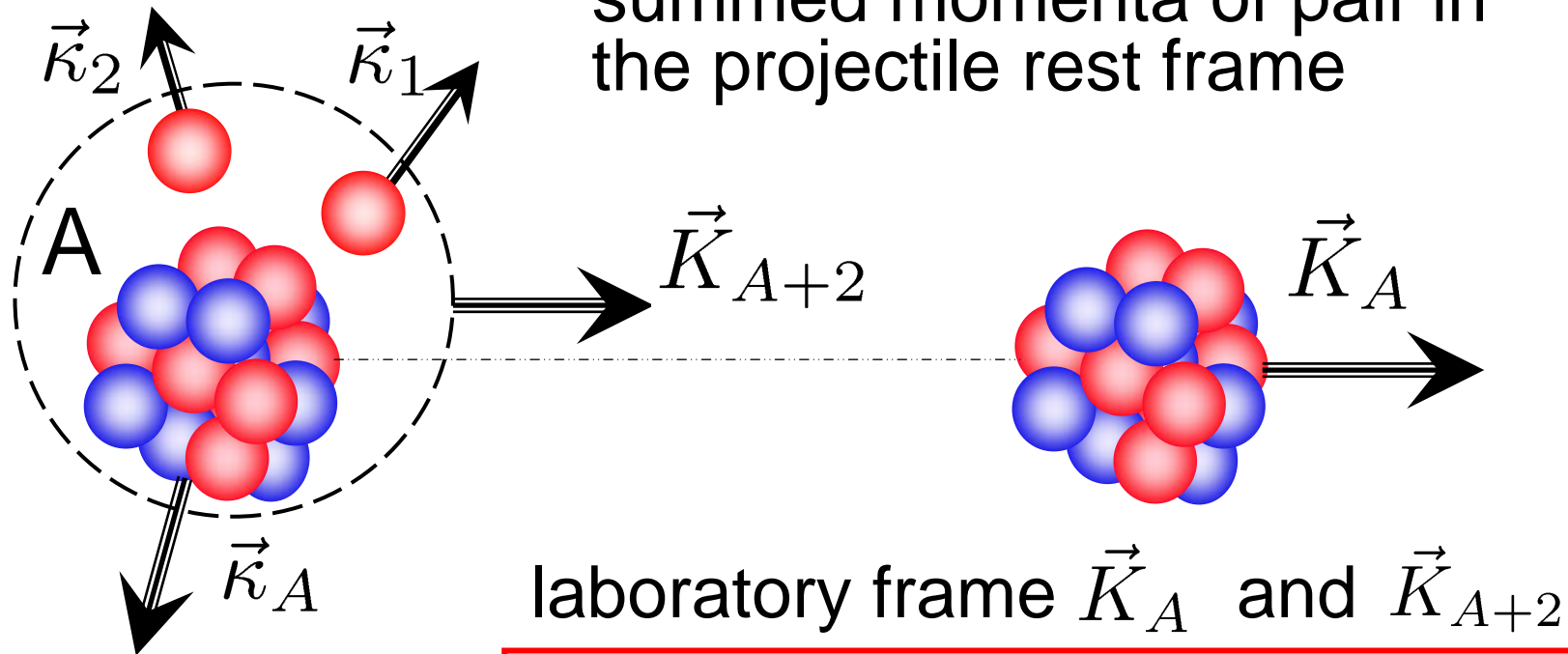
Viewed from the rest frame of the projectile

Projectile-target interaction absorptive when come close



Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame

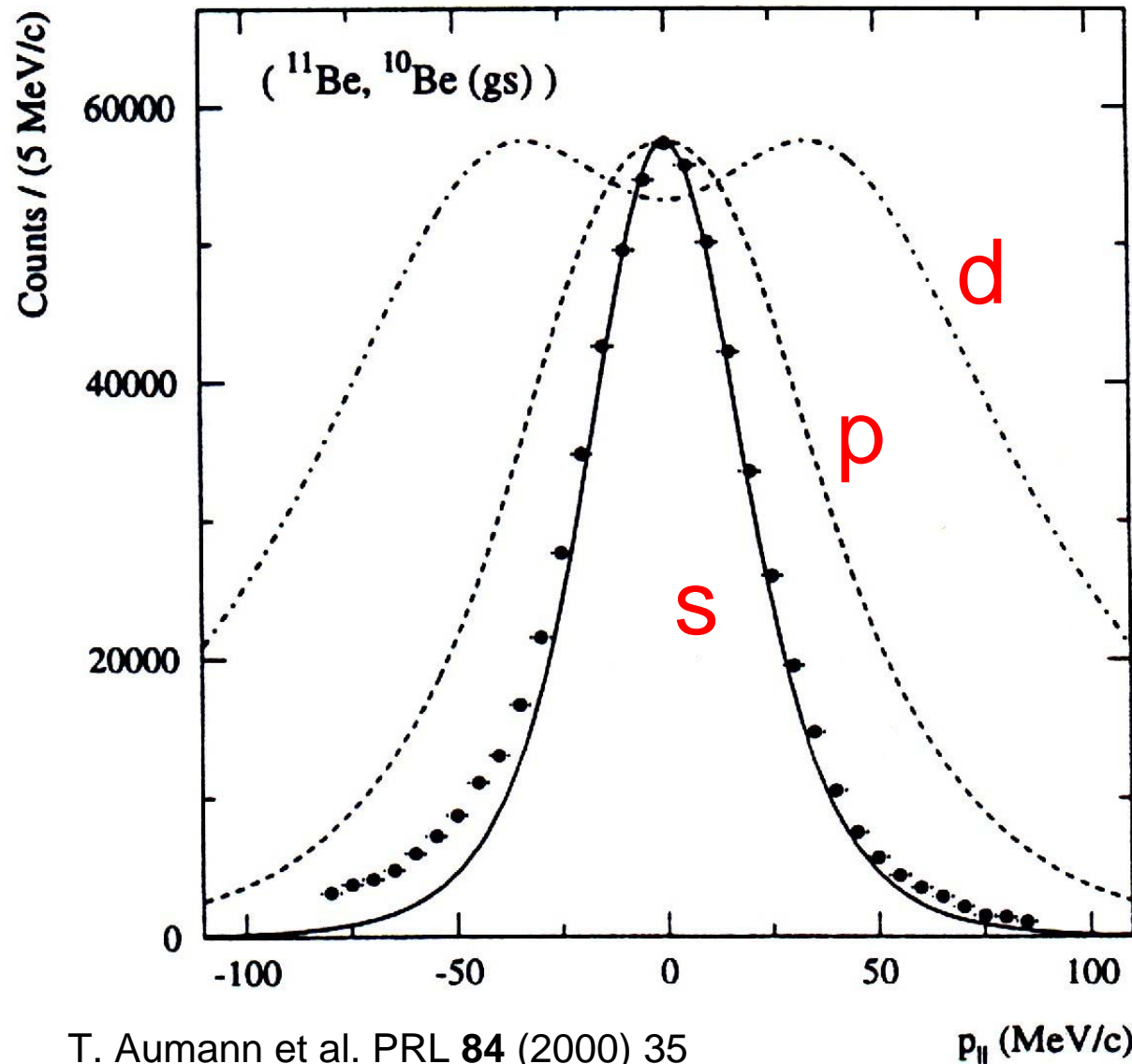


Projectile rest frame

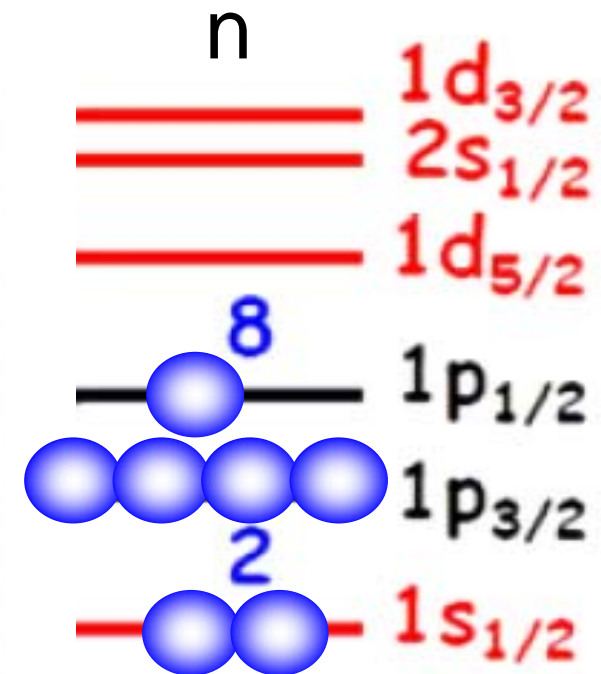
$$\vec{K}_A = \frac{A}{A+2} \vec{K}_{A+2} - [\vec{k}_1 + \vec{k}_2]$$

and component equations

Residue momentum $^{11}\text{Be} \rightarrow ^{10}\text{Be}$ – halo case



$$Z = 4, N = 7$$



^{11}Be

Need description of high-energy nuclear collisions

Need composite particle (nucleons and core), but proceed in steps:

1. Point particle scattering – summary only
 - S-matrix and the eikonal (high energy, forward-focussed reaction) approximation
2. Composite few-particle system
 - adiabatic/sudden approximation (reaction fast)
 - adiabatic plus eikonal scattering solution
3. Equations for one and two-nucleon removal cross sections and other observables
4. Link of reactions & many-body nuclear structure calculations (and assumptions made/needed)

Point particles: phase shift and partial wave S-matrix

Scattering states

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2} U_{\ell j}(r) + k^2 \right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2 \right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr)$, $G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\begin{aligned} u_{k\ell j}(r) &\rightarrow e^{i\delta_{\ell j}} [\cos \delta_{\ell j} F_\ell(\eta, kr) + \sin \delta_{\ell j} G_\ell(\eta, kr)] \\ &\rightarrow (i/2) [H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)] \end{aligned}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm iF_\ell(\eta, kr)$$

[illegible]

Eikonal approximation: for point particles (1)

Approximate (semi-classical) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu} \nabla_r^2 + U(r) - E_{cm} \right) \chi_{\vec{k}}^+(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2} U(r) + k^2 \right) \chi_{\vec{k}}^+(\vec{r}) = 0$$

valid when $|U|/E \ll 1, \quad ka \gg 1$ small wavelength
→ high energy

Key steps are: (1) the distorted wave function is written

$$\chi_{\vec{k}}^+(\vec{r}) = \exp(i\vec{k} \cdot \vec{r}) \omega(\vec{r})$$

all effects due to $U(r)$,
modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k} \cdot \nabla \omega(\vec{r}) - \frac{2\mu}{\hbar^2} U(r) \omega(\vec{r}) + \nabla^2 \omega(\vec{r}) \right] \exp(i\vec{k} \cdot \vec{r}) = 0$$

Eikonal approximation: point neutral particles (2)

$$\left[2i\vec{k} \cdot \nabla \omega(\vec{r}) - \frac{2\mu}{\hbar^2} U(r) \omega(\vec{r}) + \cancel{\nabla^2 \omega(\vec{r})} \right] \exp(i\vec{k} \cdot \vec{r}) = 0$$

The conditions $|U|/E \ll 1$, $ka \gg 1 \rightarrow$ imply that

$$2\vec{k} \cdot \nabla \omega(\vec{r}) \gg \nabla^2 \omega(\vec{r}) \quad \text{Slow spatial variation cf. } k$$

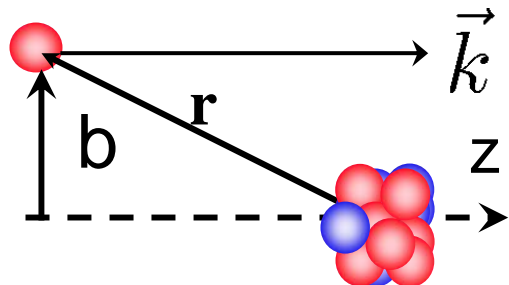
and choosing the z-axis in the beam direction \vec{k}

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k} U(r) \omega(\vec{r})$$

with solution

phase that develops with z

$$\omega(\vec{r}) = \exp \left[-\frac{i\mu}{\hbar^2 k} \int_{-\infty}^z U(r) dz' \right]$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point neutral particles (3)

$$\chi_{\vec{k}}^+(\vec{r}) = \exp(i\vec{k} \cdot \vec{r}) \quad \omega(\vec{r}) \approx \exp(i\vec{k} \cdot \vec{r}) \exp \left[-\frac{i\mu}{\hbar^2 k} \int_{-\infty}^z U(r) dz' \right]$$

So, after the interaction and as $z \rightarrow \infty$

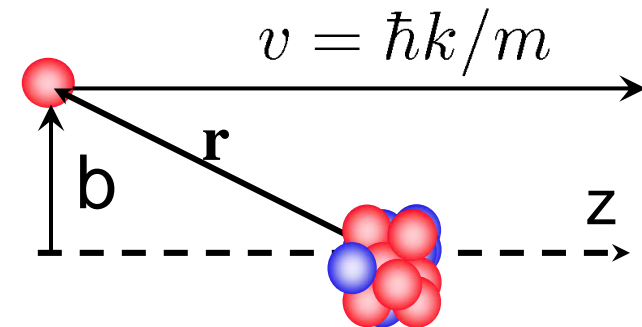
$$\chi_{\vec{k}}^+(\vec{r}) \rightarrow \exp(i\vec{k} \cdot \vec{r}) \exp \left[-\frac{i\mu}{\hbar^2 k} \int_{-\infty}^{\infty} U(r) dz' \right] = S(b) \exp(i\vec{k} \cdot \vec{r})$$

$$\chi_{\vec{k}}^+(\vec{r}) \rightarrow S(b) \exp(i\vec{k} \cdot \vec{r})$$

Eikonal approximation to the S-matrix $S(b)$

$$S(b) = \exp \left[-\frac{i}{\hbar v} \int_{-\infty}^{\infty} U(r) dz' \right]$$

$S(b)$ is amplitude of the forward going outgoing waves from the scattering at impact parameter b

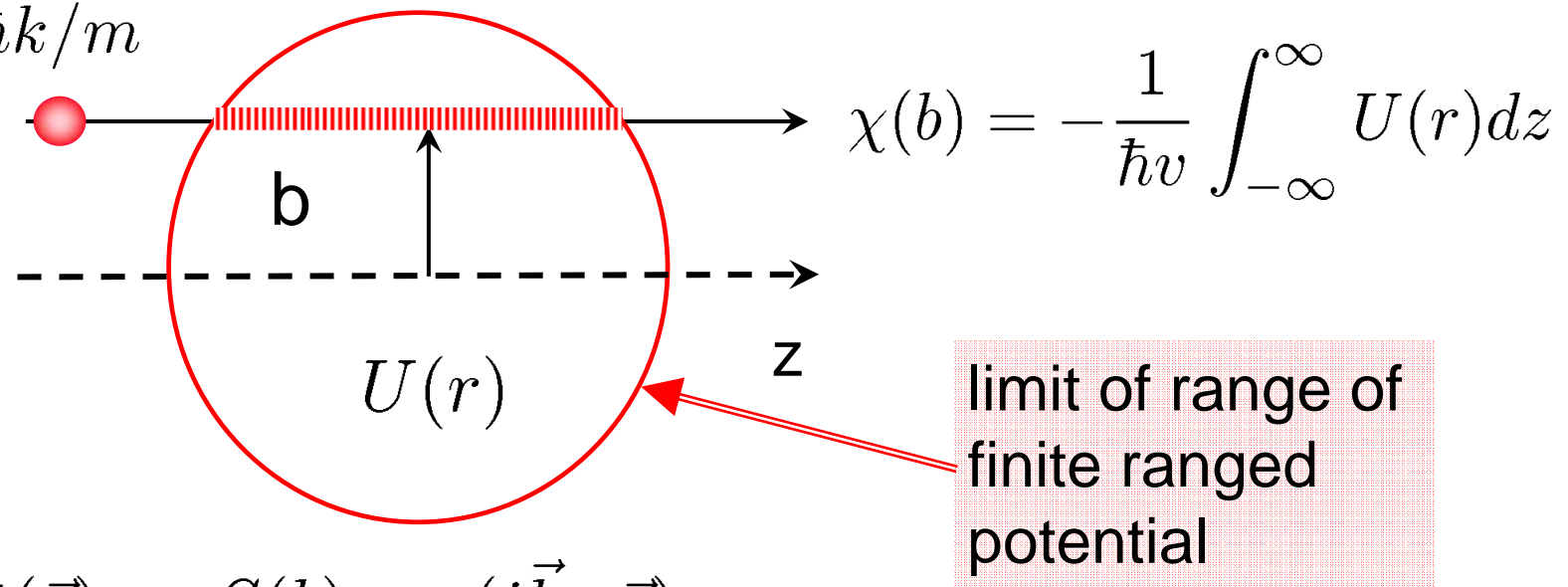


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal approximation: point particles - summary

$$\chi_{\vec{k}}^+(\vec{r}) = \exp(i\vec{k} \cdot \vec{r}) \exp \left[-\frac{i\mu}{\hbar^2 k} \int_{-\infty}^z U(r) dz' \right]$$

$$v = \hbar k / m$$

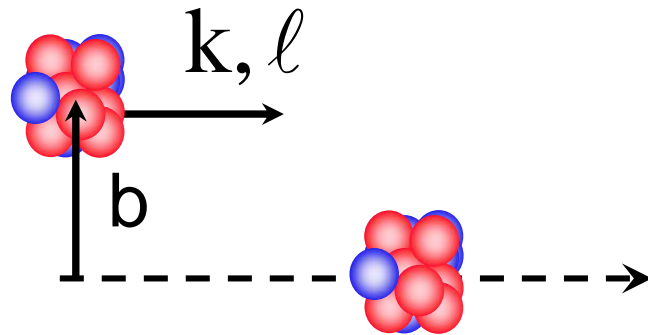


$$\chi_{\vec{k}}^+(\vec{r}) \rightarrow S(b) \exp(i\vec{k} \cdot \vec{r})$$

$$S(b) = \exp[i\chi(b)] = \exp \left[-\frac{i}{\hbar v} \int_{-\infty}^{\infty} U(r) dz' \right]$$

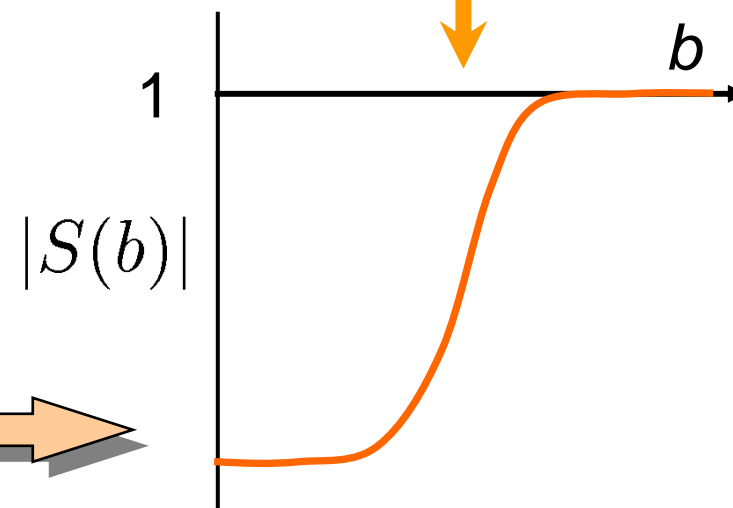
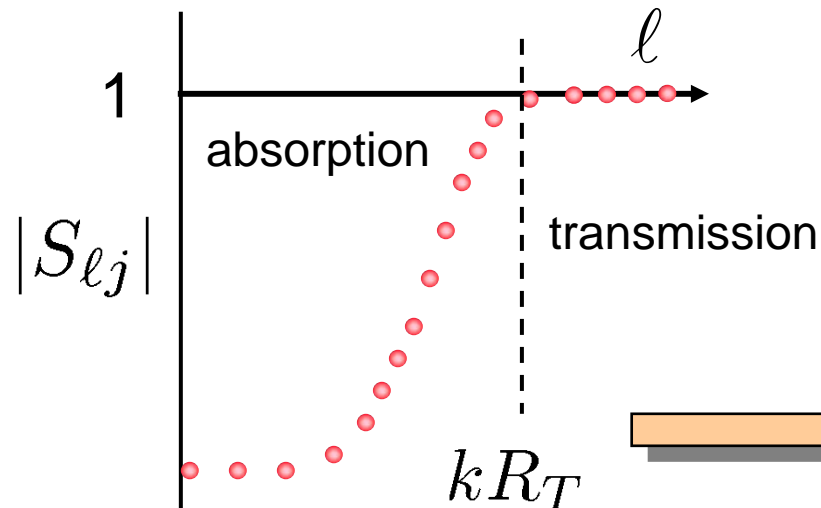
Semi-classical models for the S-matrix - $S(b)$

b =impact parameter



for high energy/or large mass,
semi-classical ideas are good

$$kb \cong \ell, \text{ actually } \Rightarrow \ell + 1/2$$



$$u_{k\ell j}(r) \rightarrow (i/2)[H_{\ell}^{(-)} - S_{\ell j}H_{\ell}^{(+)}]$$

$$S(b) = \exp \left[-\frac{i}{\hbar v} \int_{-\infty}^{\infty} U(r) dz' \right]$$

Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the eikonal (impact parameter) representation, for example (spinless case):

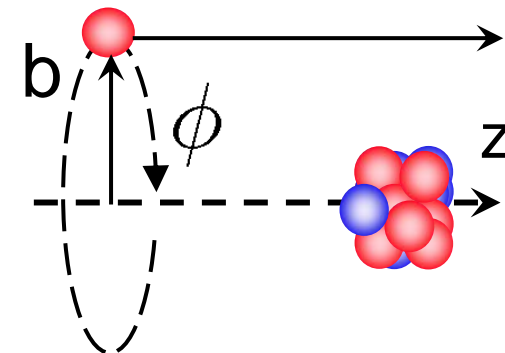
$$\sigma_{el} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) |1 - S_{\ell}|^2 \approx \int d^2\vec{b} |1 - S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) (1 - |S_{\ell}|^2) \approx \int d^2\vec{b} (1 - |S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el} + \sigma_R = 2 \int d^2\vec{b} [1 - \text{Re}.S(b)] \quad \text{etc.}$$

and where (cylindrical coordinates)

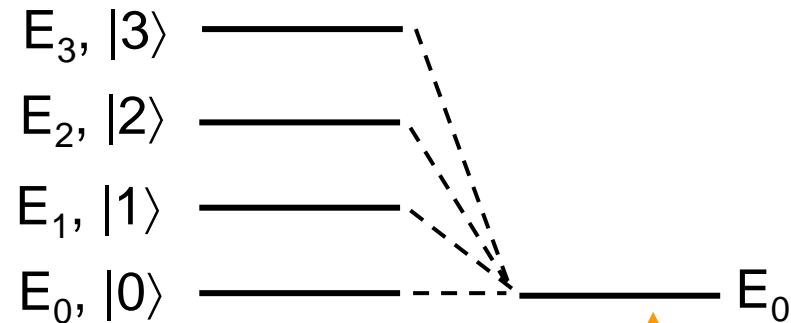
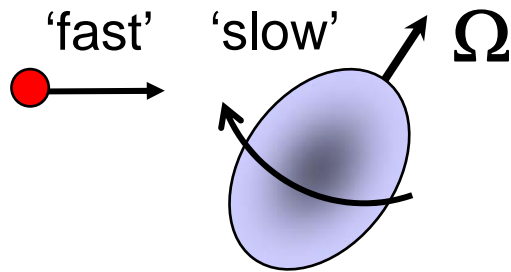
$$\int d^2\vec{b} \equiv \int_0^{\infty} b db \int_0^{2\pi} d\phi = 2\pi \int_0^{\infty} b db$$



Adiabatic (sudden) approximations in physics

Identify high energy/fast and low energy/slow degrees of freedom

Fast neutron scattering
from a rotational nucleus

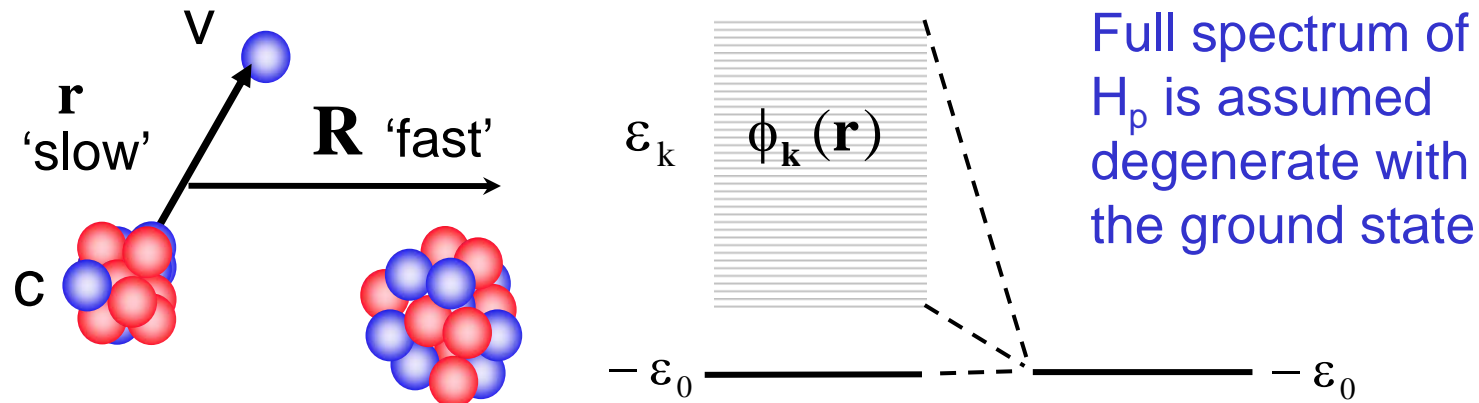


Fix Ω , calculate scattering
amplitude $f(\theta, \Omega)$ for each
(fixed) Ω .

moment of inertia $\rightarrow \infty$
and rotational spectrum
is assumed degenerate

Transition amplitudes $f_{\alpha\beta}(\theta) = \langle \beta | f(\theta, \Omega) | \alpha \rangle_{\Omega}$

Few-body projectiles – the adiabatic model



Freeze internal co-ordinate \mathbf{r} then scatter $c+v$ from target and compute $f(\theta, \mathbf{r})$ for all required fixed values of \mathbf{r}

Physical amplitude for breakup to state $\phi_k(\mathbf{r})$ is then,

$$f_k(\theta) = \langle \phi_k | f(\theta, \mathbf{r}) | \phi_0 \rangle_{\mathbf{r}}$$

Achieved by replacing $H_p \rightarrow -\epsilon_0$ in Schrödinger equation

Adiabatic approximation - time perspective

The time-dependent equation is

$$H\Psi(\mathbf{r}, \mathbf{R}, t) = i\hbar \frac{\partial \Psi}{\partial t}$$

and can be written

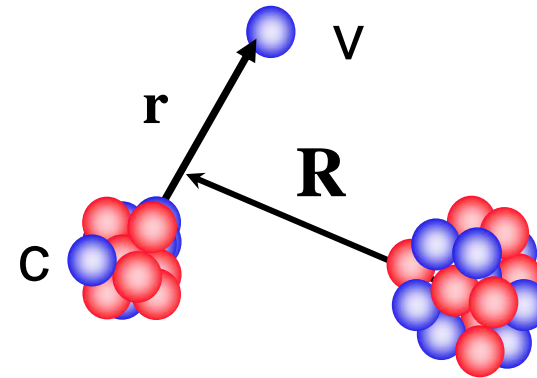
$$\Psi(\mathbf{r}, \mathbf{R}, t) = \Lambda \Phi(\mathbf{r}(t), \mathbf{R}), \quad \mathbf{r}(t) = \Lambda^+ \mathbf{r} \Lambda$$

$$\Lambda = \exp\{-i(H_p + \varepsilon_0)t/\hbar\} \quad \text{and where}$$

$$[T_R + U(\mathbf{r}(t), \mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t), \mathbf{R}) = i\hbar \frac{\partial \Phi}{\partial t}$$

Adiabatic
equation

$$[T_R + U(\mathbf{r}, \mathbf{R})]\Phi(\mathbf{r}, \mathbf{R}) = (E + \varepsilon_0)\Phi(\mathbf{r}, \mathbf{R})$$



Adiabatic step
assumes

$\mathbf{r}(t) \approx \mathbf{r}(0) = \mathbf{r} = \text{fixed}$
or $\Lambda = 1$ for the
collision time t_{coll}

requires

$$(H_p + \varepsilon_0)t_{\text{coll}}/\hbar \ll 1$$

Reaction timescales – surface grazing collisions

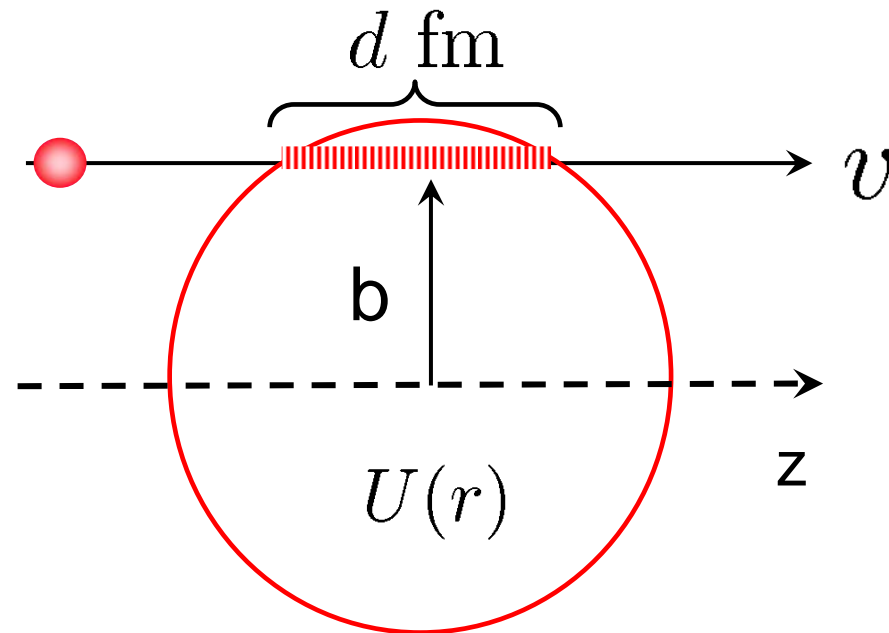
For 100 and 250 MeV/u incident energy:

$$\gamma = 1.1, \quad v/c = 0.42,$$

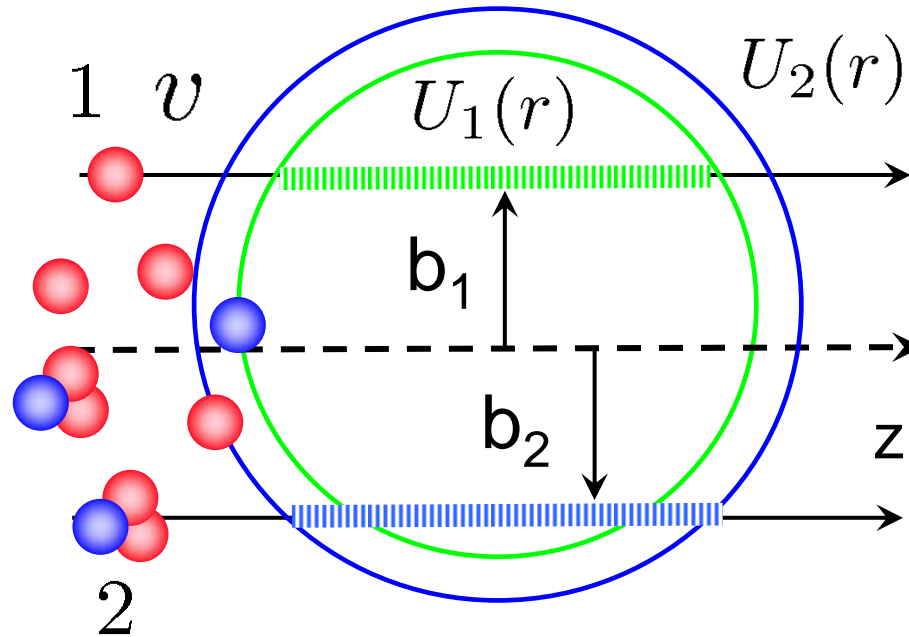
$$\gamma = 1.25, \quad v/c = 0.6,$$

$$\Delta t = 7.9 \times d \times 10^{-24} s,$$

$$\Delta t = 5.6 \times d \times 10^{-24} s$$



Adiabatic approximation: composite projectile



$$\chi_i(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} U_i(r) dz$$

Total interaction energy

$$U(r_1, \dots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp[i\chi_i(b_i)] = \exp\left[-\frac{i}{\hbar v} \int_{-\infty}^{\infty} U_i(r_i) dz'\right]$$

$$\chi(b_1, \dots) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} \sum_i U_i(r_i) dz$$

with composite systems: get products of the S-matrices

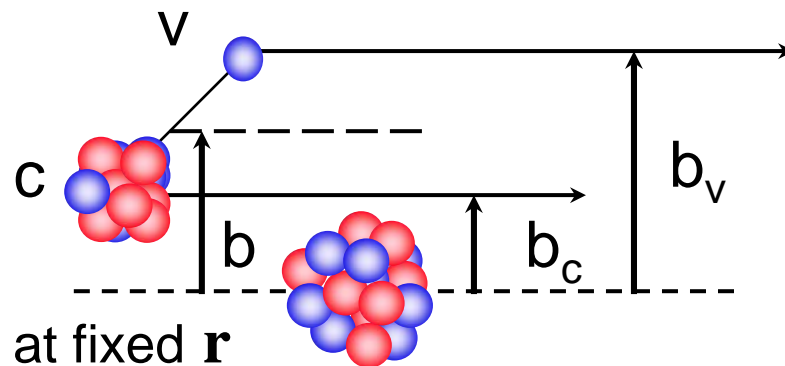
$$\exp[i\chi(b_1, \dots)] = \prod_i S_i(b_i)$$

Few-body eikonal model amplitudes

So, after the collision, as $Z \rightarrow \infty$ $\omega(\mathbf{r}, \mathbf{R}) = S_c(b_c) S_v(b_v)$

$$\Psi_{\mathbf{K}}^{\text{Eik}}(\mathbf{r}, \mathbf{R}) \rightarrow e^{i\mathbf{K} \cdot \mathbf{R}} S_c(b_c) S_v(b_v) \phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



at fixed \mathbf{r}
adiabatic

So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

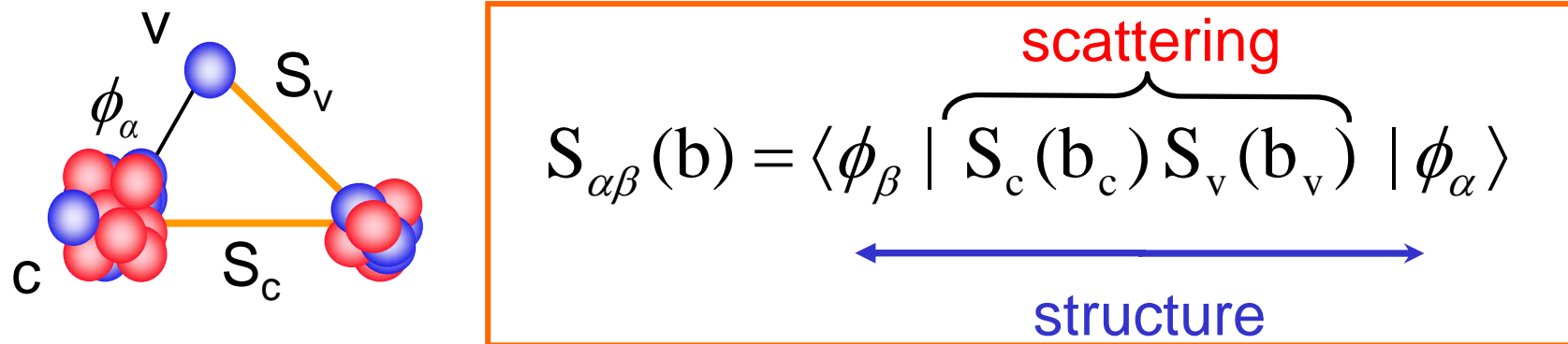
$$S_p(b) = \langle \phi_0 | \underbrace{S_c(b_c) S_v(b_v)} | \phi_0 \rangle_{\mathbf{r}}$$

averaged over position
probabilities of c and v

amplitude that c, v survive
interaction with b_c and b_v

Eikonal theory - dynamics and structure

Independent scattering information of c and v from target



Use the best available few- or many-body wave functions

More generally,

$$S_{\alpha\beta}(b) = \langle \varphi_{\beta} | S_1(b_1) S_2(b_2) \dots S_n(b_n) | \varphi_{\alpha} \rangle$$

for any choice of 1, 2, 3, n clusters for which a most realistic wave function φ is available

Absorptive cross sections - target excitation

Since our effective interactions are complex all our $S(b)$ include the effects of absorption due to inelastic channels

$$|S(b)|^2 \leq 1$$

$$\sigma_{\text{abs}} = \sigma_R - \sigma_{\text{diff}} = \int d\mathbf{b} \langle \phi_0 | 1 - |S_c S_v|^2 | \phi_0 \rangle$$

$$\left\{ \begin{array}{l} |S_v|^2 (1 - |S_c|^2) + \\ |S_c|^2 (1 - |S_v|^2) + \\ (1 - |S_c|^2)(1 - |S_v|^2) \end{array} \right.$$

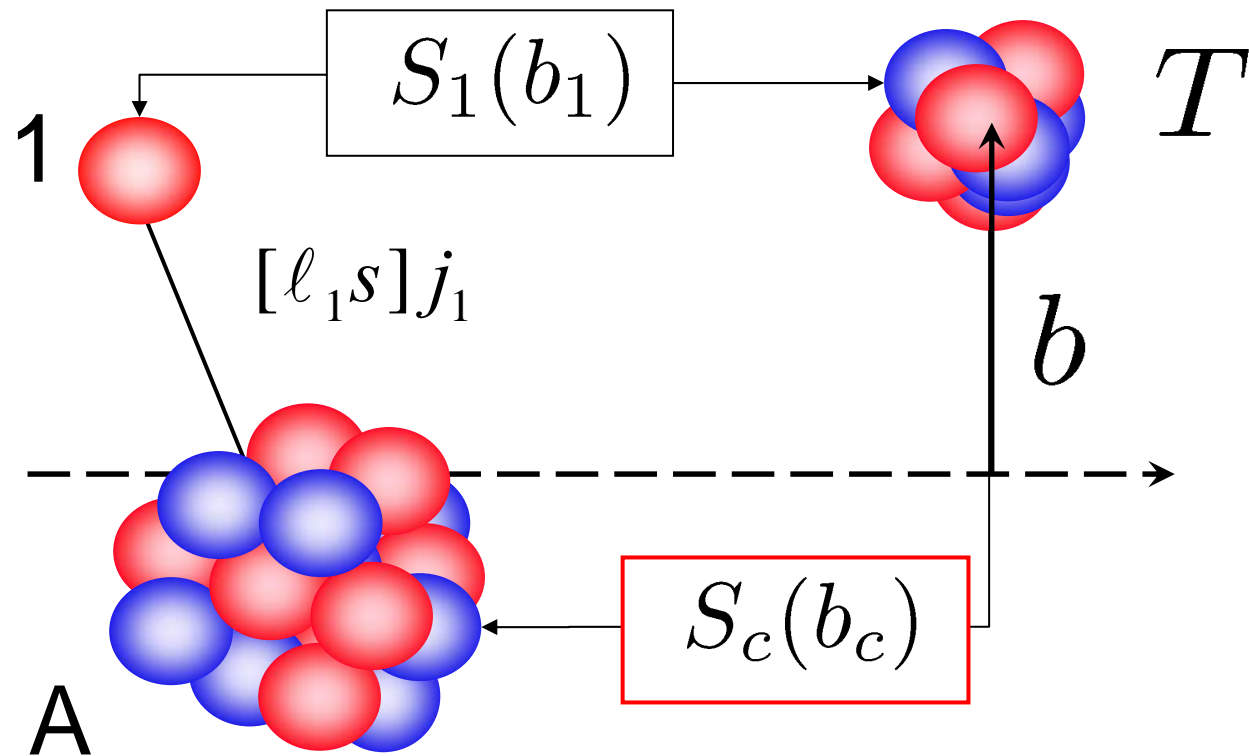
v survives, c absorbed
 v absorbed, c survives
 v absorbed, c absorbed

stripping of v from projectile exciting the target. c scatters at most elastically with the target

$$\sigma_{\text{strip}} = \int d\mathbf{b} \langle \phi_0 | |S_c|^2 (1 - |S_v|^2) | \phi_0 \rangle$$

Related equations exist for the differential cross sections, etc.

Stripping of a nucleon – nucleon ‘absorbed’



$$\sigma_{\text{strip}} = \int d\mathbf{b} \langle \phi_0 | |S_c|^2 (1 - |S_1|^2) | \phi_0 \rangle$$

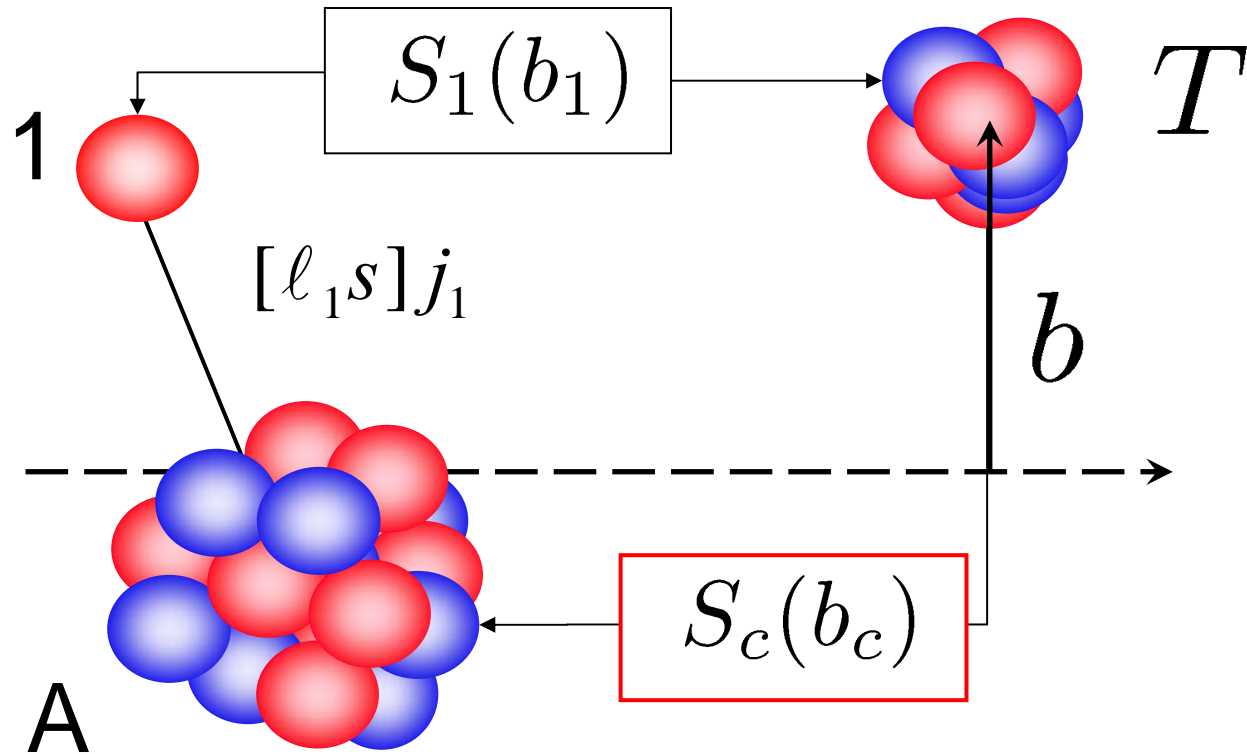
And for two-nucleon removal

$$\sigma_{abs} \rightarrow 1 - |S_c|^2 |S_1|^2 |S_2|^2$$

$$\begin{aligned}
 1 &= [|S_c|^2 + \cancel{(1 - |S_c|^2)}] \\
 &\times [|S_1|^2 + (1 - |S_1|^2)] \\
 &\times [|S_2|^2 + (1 - |S_2|^2)]
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1 &= \\ &\times \\ &\times \end{aligned}} \right\} \begin{array}{l} \text{core survival} \\ \text{and nucleon} \\ \text{"removal"} \end{array}$$

$$\begin{aligned}
 \sigma_{abs}^{KO} &\rightarrow |S_c|^2 (1 - |S_1|^2)(1 - |S_2|^2) \quad \text{2N stripping} \\
 &+ |S_c|^2 |S_1|^2 (1 - |S_2|^2) \\
 &+ |S_c|^2 (1 - |S_1|^2) |S_2|^2
 \end{aligned}
 \left. \vphantom{\begin{aligned} &+ \\ &+ \end{aligned}} \right\} \begin{array}{l} \text{1N stripped} \\ \text{1N diffracted} \end{array}$$

Diffractive (breakup) removal of a nucleon



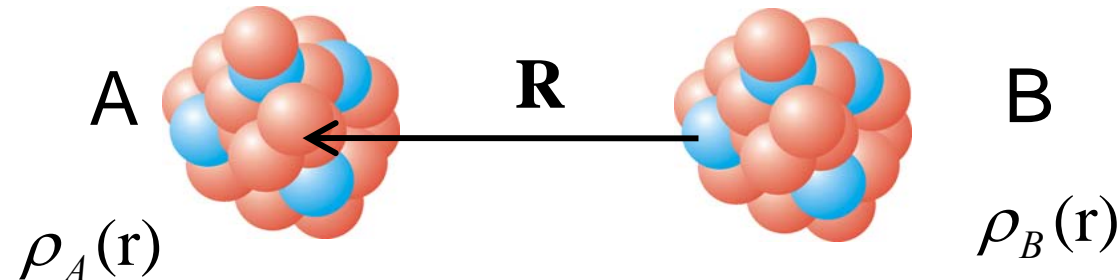
$$\sigma_{\text{diff}} = \int d\mathbf{b} \left\{ \langle \phi_0 | |S_c S_v|^2 | \phi_0 \rangle - |\langle \phi_0 | S_c S_v | \phi_0 \rangle|^2 \right\}$$

Core-target effective interactions – for $S_c(b_c)$

Double
folding

U_{AB}

$$U_{AB}(\mathbf{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_A(\mathbf{r}_1) \rho_B(\mathbf{r}_2) t_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$



At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2} \sigma_{NN} (i + \alpha_{NN}) \right] f(r), \quad \int d\vec{r} f(r) = 1$$

e.g. $f(r) = \delta(r)$

nucleon-nucleon cross section

$$f(r) = (\sqrt{\pi}t)^{-3} \exp(-r^2/t^2)$$

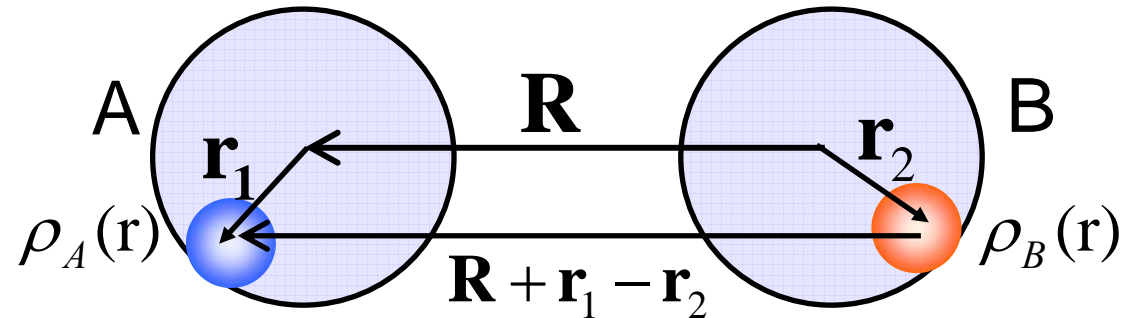
resulting in a **COMPLEX**
nucleus-nucleus potential

Effective interactions – Folding models

Double
folding

$$U_{AB}(\mathbf{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_A(\mathbf{r}_1) \rho_B(\mathbf{r}_2) v_{\text{NN}}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$

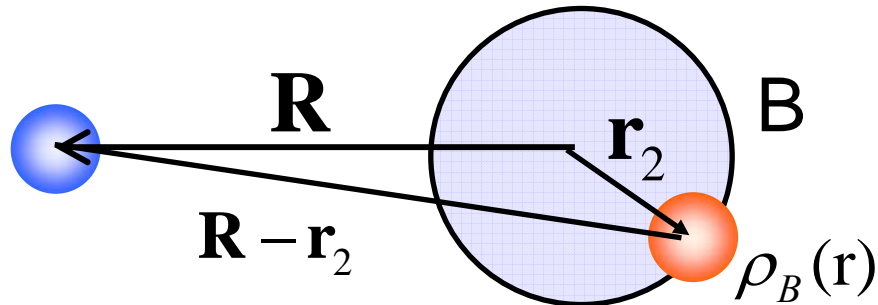
U_{AB}



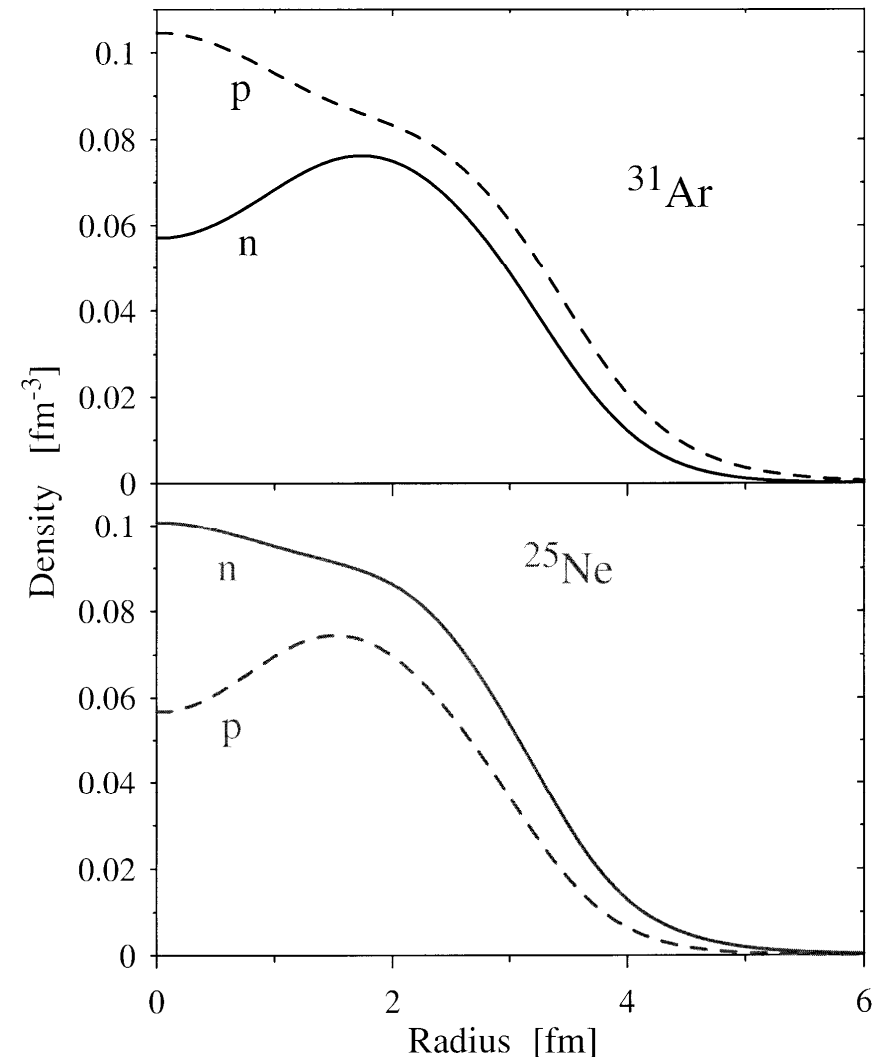
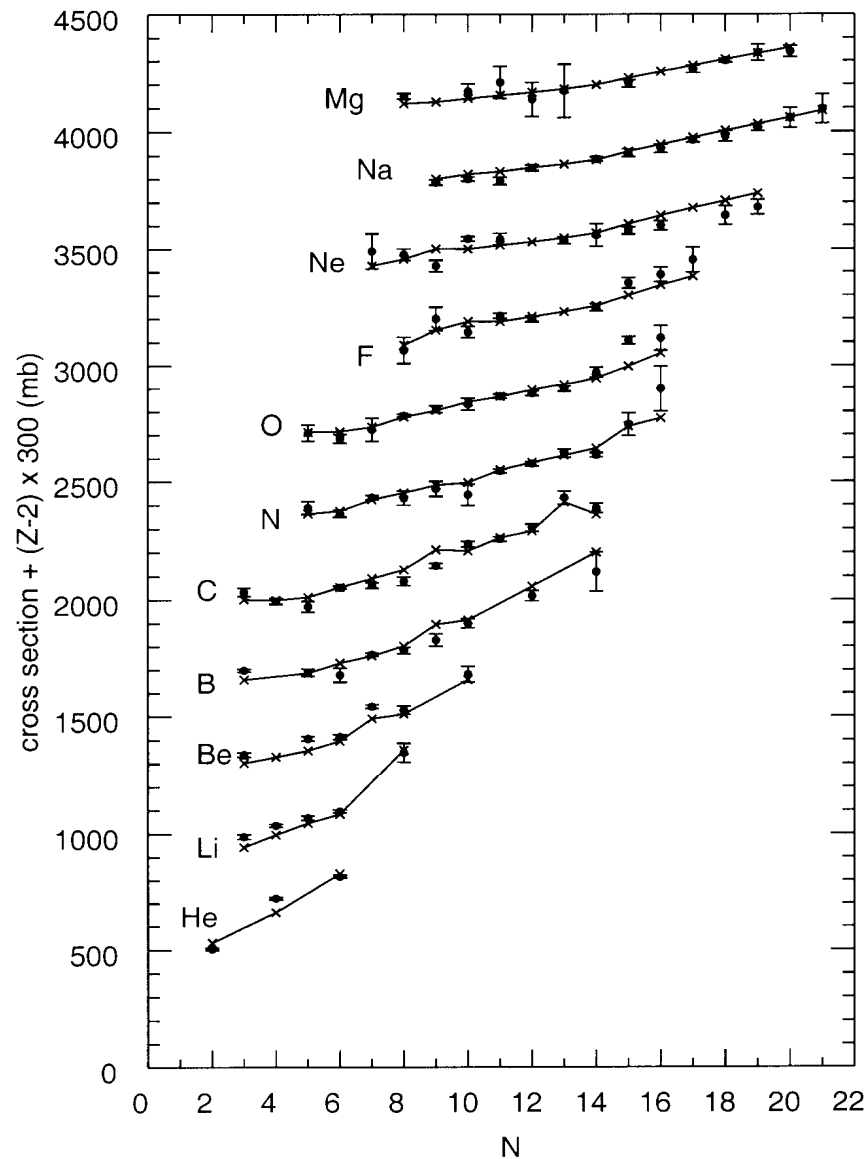
Single
folding

$$U_B(\mathbf{R}) = \int d\mathbf{r}_2 \rho_B(\mathbf{r}_2) v_{\text{NN}}(\mathbf{R} - \mathbf{r}_2)$$

U_B



Sizes - Skyrme Hartree-Fock radii and densities



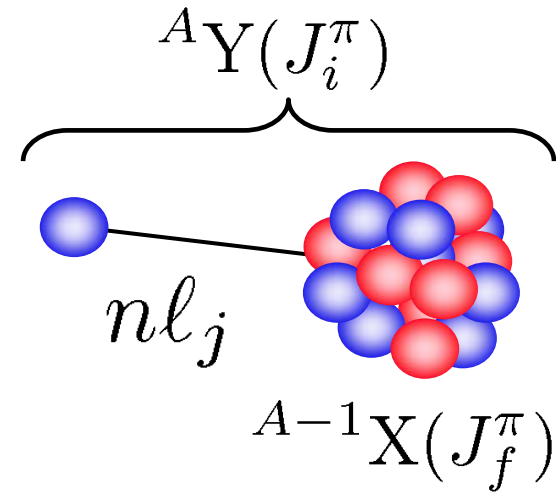
B.A. Brown, S. Typel, and W.A. Richter,
Phys. Rev. **C65** (2002) 014612

Connection to many-body – spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.

$$\phi_{n\ell j}^m(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma | jm) \frac{u_{n\ell j}(r)}{r} Y_{\ell}^{\lambda}(\hat{r}) \chi_s^{\sigma},$$

$$\int_0^{\infty} [u_{n\ell j}(r)]^2 dr = 1$$



The potential model wave function approximates the overlap function of the A and $A-1$ body wave functions (A and $A-n$ in the case of an n -body cluster) i.e. the overlap

$$\langle \ell j, \vec{r}, A-1 X(J_f^{\pi}) | A Y(J_i^{\pi}) \rangle \rightarrow I_{\ell j}(r), \quad \int_0^{\infty} [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

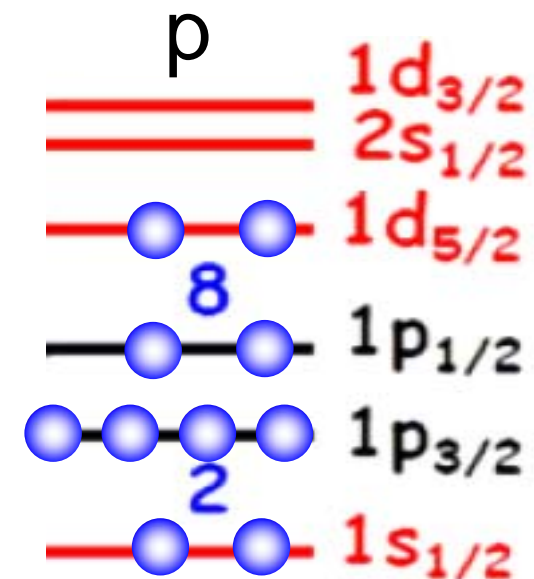
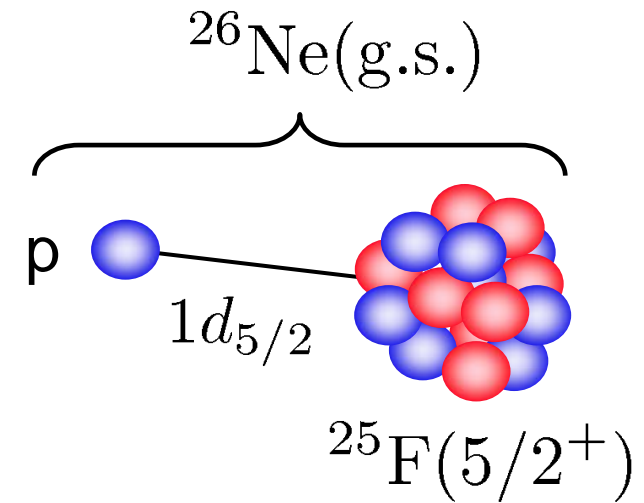
$S(\dots)$ is the spectroscopic factor ← a structure calculation

Many-body input – the shell model overlaps

$$\langle \vec{r}, {}^{25}\text{Ne}(5/2^+, E^*) | {}^{26}\text{Ne}(0^+, \text{g.s.}) \rangle$$

USDA sd-shell model overlap from
e.g. OXBASH (*Alex Brown et al.*).
Provides spectroscopic factors but
not the bound state radial wave
function.

```
-- core state --- - overlap state - (1 2 5) |
2j2t p  n      e  2j2t p  n      e      s
      -59.414
5 7 +  1  0.000 0 6 +  1  0.000 1.79039
5 7 +  2  3.756 0 6 +  1  0.000 0.02316
5 7 +  3  4.799 0 6 +  1  0.000 0.01084
5 7 +  4  5.631 0 6 +  1  0.000 0.00012
5 7 +  5  6.022 0 6 +  1  0.000 0.00589
5 7 +  6  6.504 0 6 +  1  0.000 0.00044
5 7 +  7  6.796 0 6 +  1  0.000 0.00002
5 7 +  8  8.034 0 6 +  1  0.000 0.00006
5 7 +  9  8.186 0 6 +  1  0.000 0.00097
5 7 + 10  8.398 0 6 +  1  0.000 0.00006
total =                                1.83196
centroid =    0.102      centroids =    0.000
centroid* = -22.313      centroids = -22.211
```



We have the background we need to apply this in a serious context and confront experiment: tomorrow we look at a number of applications.

