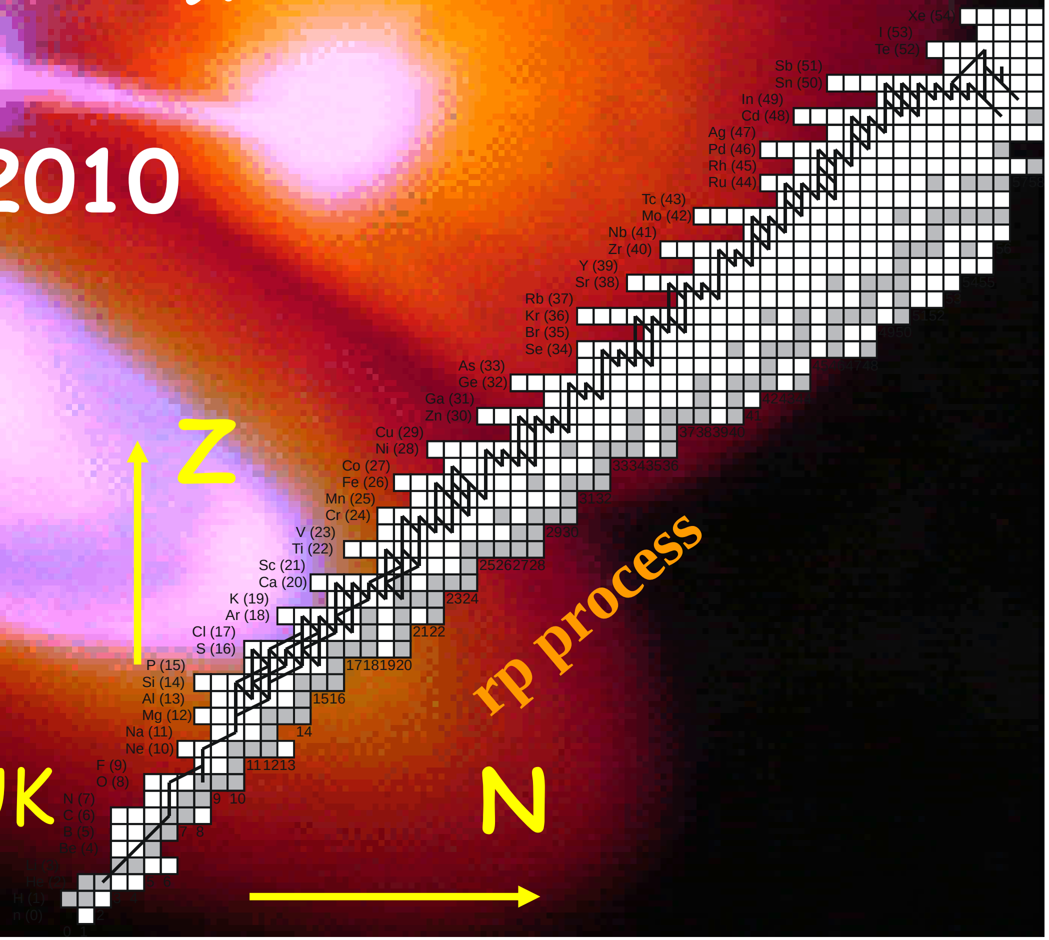
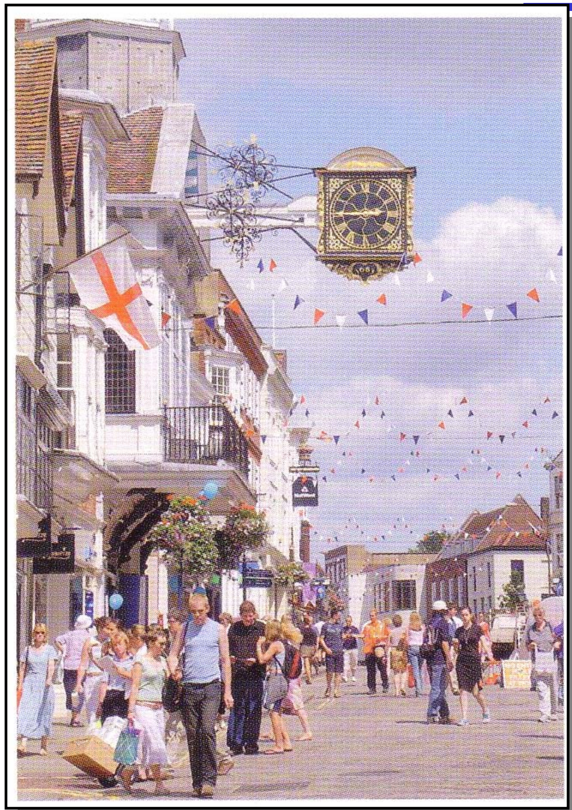
Reaction tools for the study of exotic nuclei: theory and applications - I

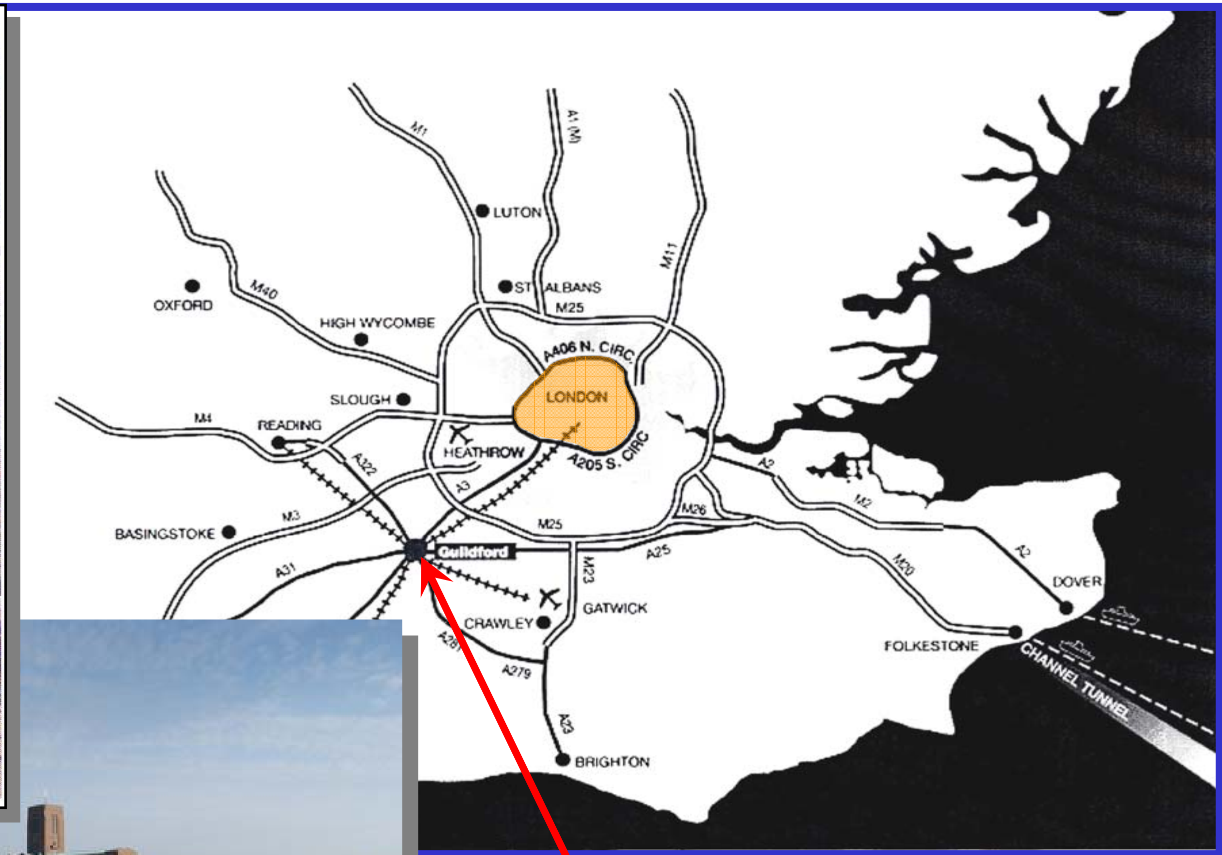
Hadrons and Nuclei under Extreme Conditions (HANEC2010), TIT, O-Okayama 16-17 September 2010

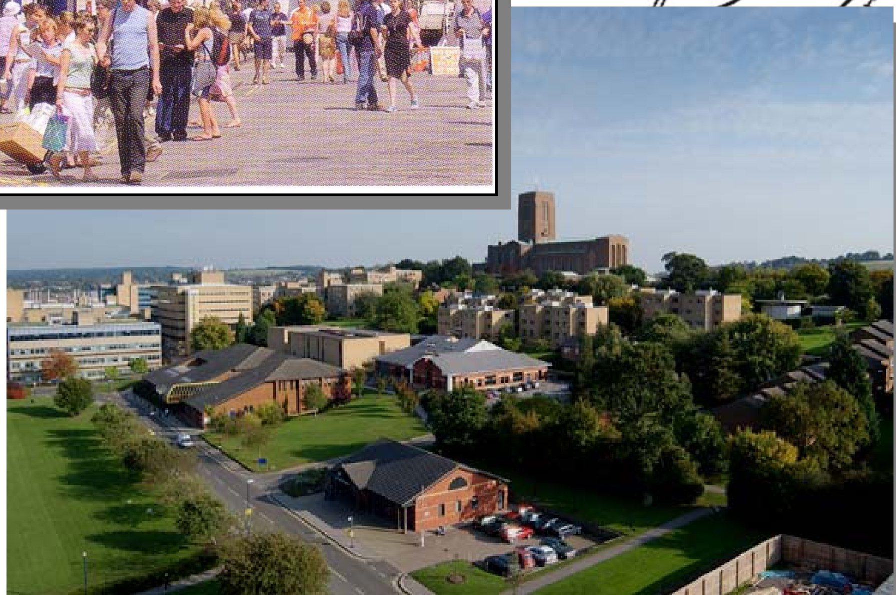


Jeff Tostevin, TIT and
Department of Physics
University of Surrey, UK

My part of the Cosmos - the University of Surrey







Guildford, Surrey

UNIVERSITY OF SURREY

Exotic nuclei – fundamental questions

What are the limits of nuclear existence … ?

How do the limits of weak binding and extreme proton-to-neutron asymmetries affect nuclear properties …?

How do the properties of nuclei evolve with changes in proton and neutron number … ?

"Atomic Nuclei: Structure and Stability", US *2002 Long-Range Plan for Nuclear Science*

Some other slides/notes/resources can be found at:

http://www.nucleartheory.net/DTP_material/

There several good direct reactions texts:

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) Norman Austern

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages) G R Satchler

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) L S Rodberg, R M Thaler

Direct Nuclear Reactions (World Scientific Publishing, 396 pages) Norman K. Glendenning

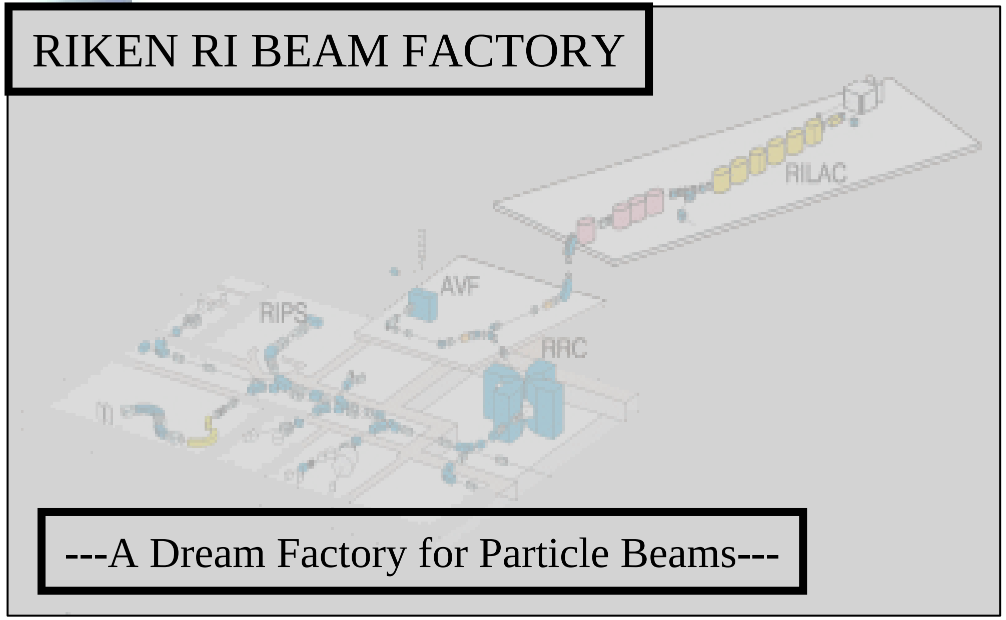
Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages) C A Bertulani, P Danielewicz

Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages) Herman Feshbach

Introduction to Nuclear Reactions (Oxford University Press, 332 pages) G R Satchler

Nuclear Reactions for Astrophysics (Cambridge University Press, 2010) Ian Thompson and Filomena Nunes

Radioactive ion-beam facilities – and plans



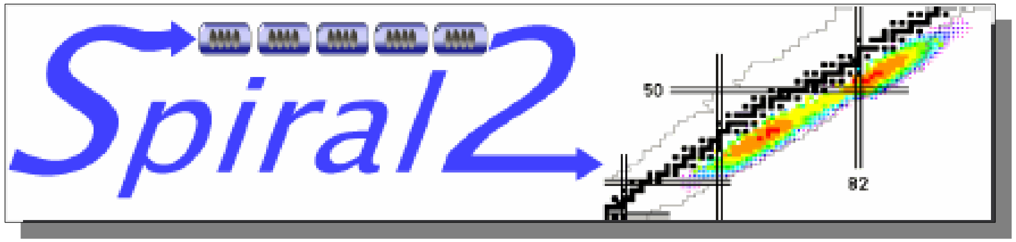
RIBF operational

RIKEN

MSU -

FRIB

USA



GANIL, France

Lectures plan

Lecture 1: Introduction: history and key ideas
- nuclear shell structure, old and new

Reactions at high-energy (removal)
- approximations, simplifications
- connection/interface with nuclear structures
- observables and what they tell us

Lecture 2: Illustrative and topical applications
- determining exotic structures
- testing of structure models and of shell model effective interactions
- nucleon and pair ‘correlations’

Introduction: Motivations and context

Exotic nuclei – stability → structures → reactions

How? history: have we got to present point?

What ? are exotic nuclei – how are they different?

Why ? are they of interest, and to whom?

How ? (and where) are they produced

Which ? can we/will we produce (in 2010-2015)?

What ? are we now able to do with them?

What ? are are they telling us?

Where ? is this research area going/challenges

100 years (almost) of nuclear physics

[669]

LXXIX. *The Scattering of α and β Particles by Matter and the Structure of the Atom.* *By* Professor E. RUTHERFORD, *F.R.S., University of Manchester* *.

§ 1. IT is well known that the α and β particles suffer deflexions from their rectilinear paths by encounters with atoms of matter. This scattering is far more marked for the β than for the α particle on account of the much smaller momentum and energy of the former particle. There seems to be no doubt that such swiftly moving particles pass through the atoms in their path, and that the deflexions observed are due to the strong electric field traversed within the atomic system. It has generally been

Philosophical Magazine, volume **21** (1911), pages 669-688

Nuclear sizes: charge and mass distributions

REVIEWS OF MODERN PHYSICS VOLUME 30, NUMBER 2 APRIL, 1958

International Congress on Nuclear Sizes and Density Distributions

Held at Stanford University, December 17–19, 1957

REVIEWS OF MODERN PHYSICS VOLUME 30, NUMBER 2 APRIL, 1958

Nuclear Radii as Determined by Scattering of Neutrons

S. FERNBACH

University of California Radiation Laboratory, Livermore, California

REVIEWS OF MODERN PHYSICS VOLUME 30, NUMBER 2 APRIL, 1958

Nuclear Density Distributions from Proton Scattering

A. E. GLASSGOLD

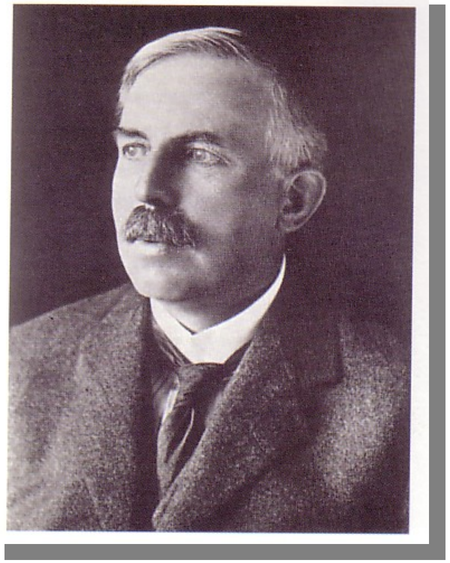
Department of Physics, University of California, Berkeley, California

REVIEWS OF MODERN PHYSICS VOLUME 30, NUMBER 2 APRI

Electron Scattering and Nuclear Charge Distributions

D. G. RAVENHALL

Department of Physics, University of Illinois



$$R = r_0\, A^{1/3}$$

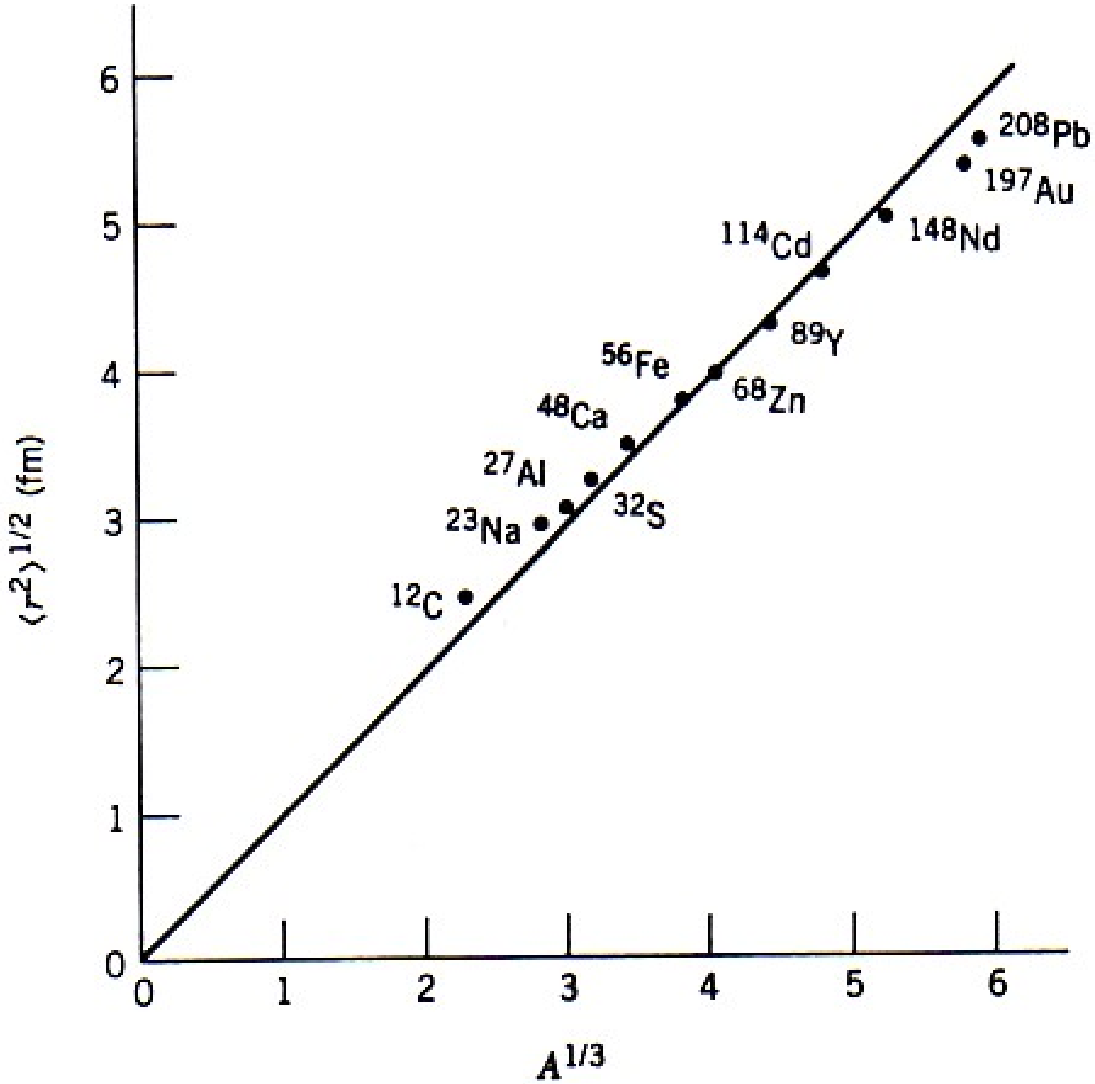
while for convenience the results on the other nucle were analyzed in terms of the Fermi shape

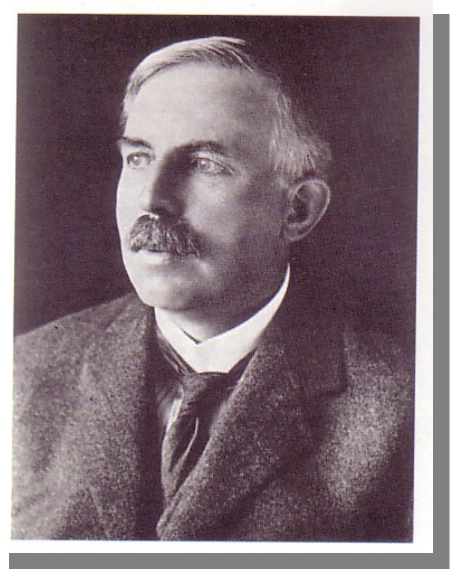
$$\rho(r)=\rho_0[1+e^{(r-c)/z_1}]^{-1},$$

the only significant quantities determined were the radius and the surface thickness t. The radial parameter c, the distance to the point where ρ has dropped to half of its central value on this model, varies closely as $A^{\frac{1}{3}}$ for all of the nuclei, and t is effectively constant:

$$\left.\begin{aligned} c&=(1.07\pm0.02)A^{\frac{1}{3}}\times10^{-13}\text{ cm}\\ t&=(2.4\pm0.3)\times10^{-13}\text{ cm}\end{aligned}\right\}\text{charge distribution.}$$

Nuclear sizes: charge and mass distributions





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$t = (2.4\pm0.3)\times10^{-13}$ cm } distribution.

Shell (single particle) structures in nuclei

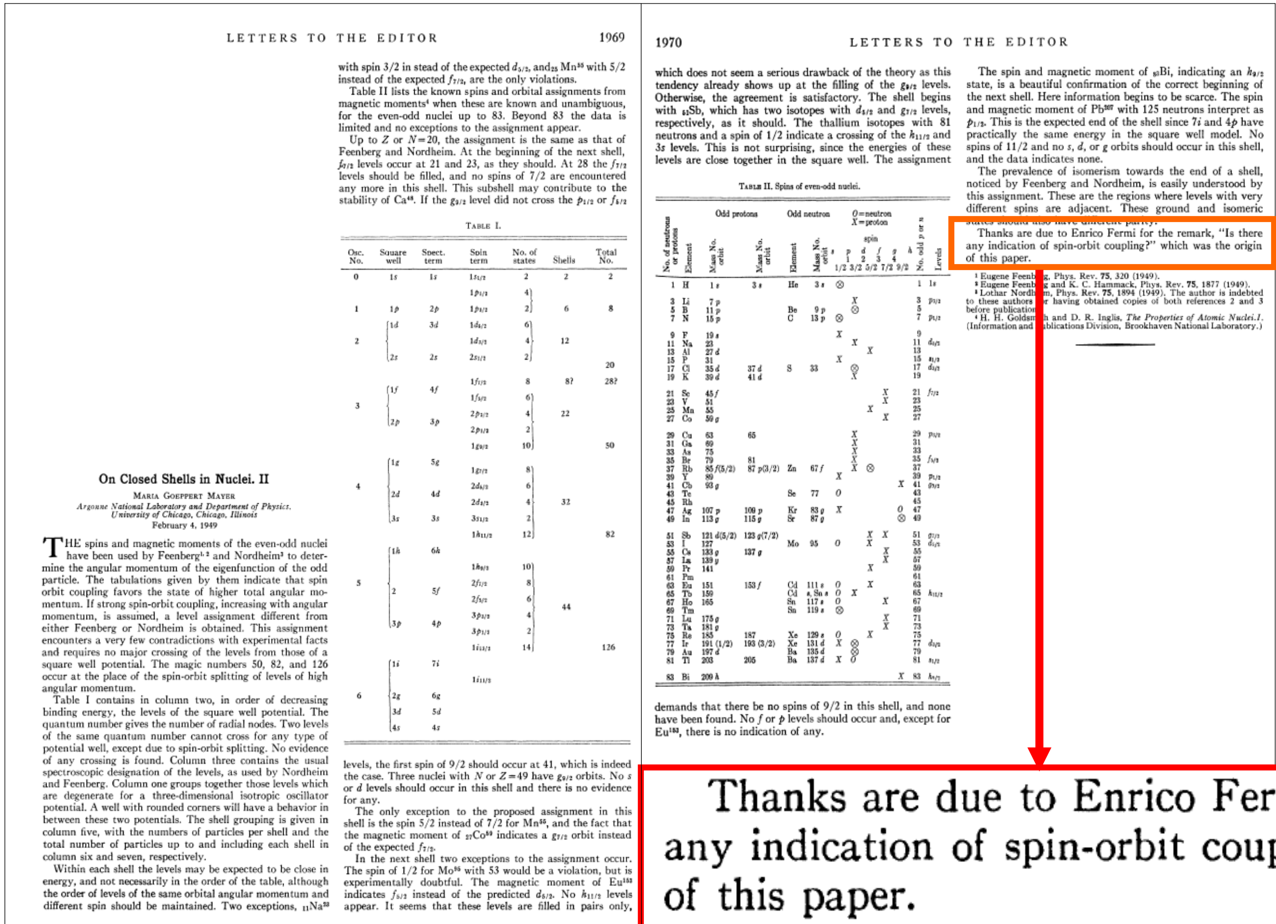
On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics, University of Chicago, Chicago, Illinois

February 4, 1949

Phys. Rev. **75**, 1969 (1949)

Cited 240 times


LETTERS TO THE EDITOR 1969

On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics, University of Chicago, Chicago, Illinois
February 4, 1949

TABLE I.

1970 LETTERS TO THE EDITOR

TABLE II. Spins of even-odd nuclei.

Thanks are due to Enrico Fermi for the remark, "Is there any indication of spin-orbit coupling?" which was the origin of this paper.

Enrico Fermi (1901–1954)

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Shell (single particle) structures in nuclei

On Closed Shells in Nuclei. II

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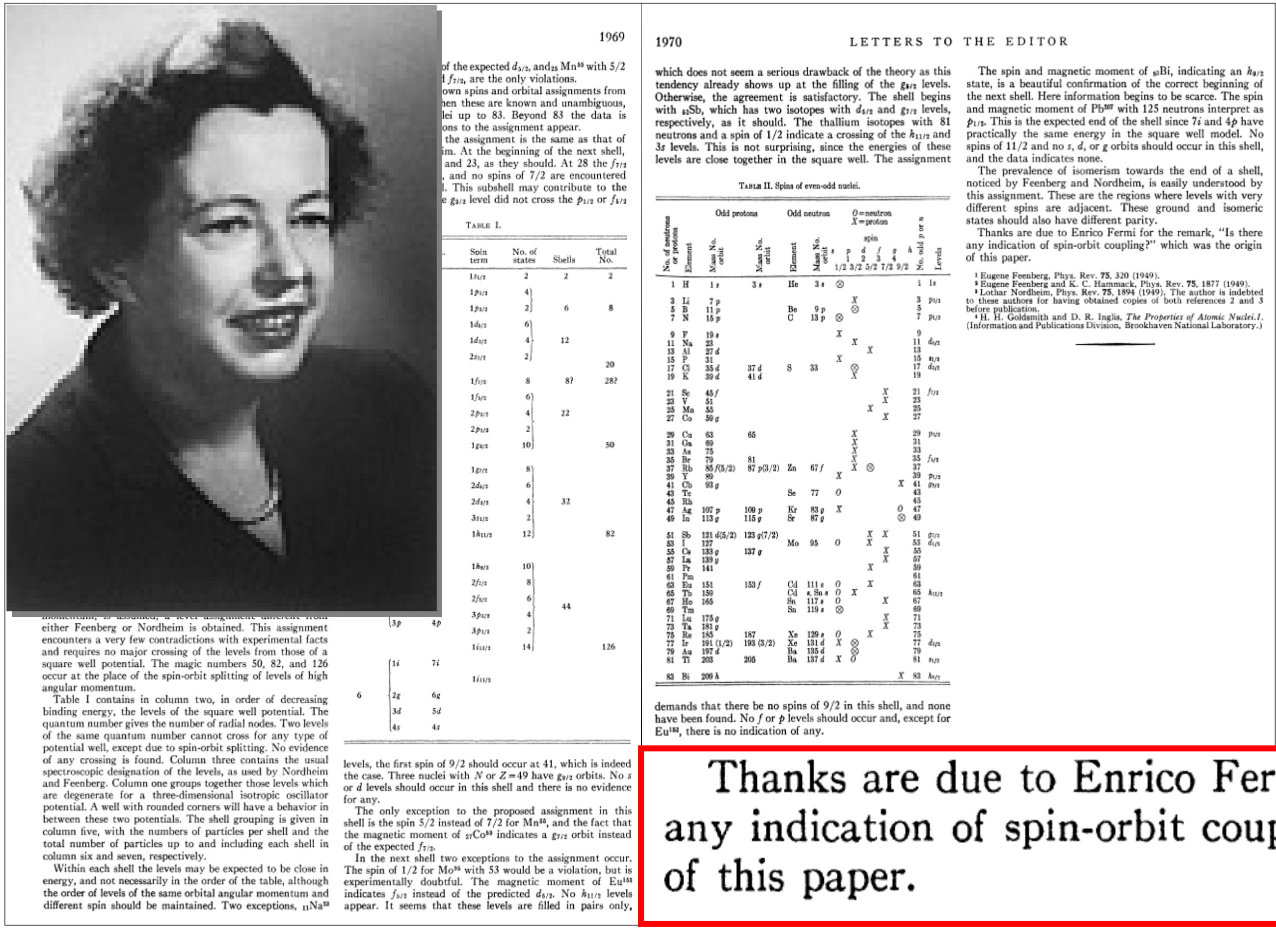
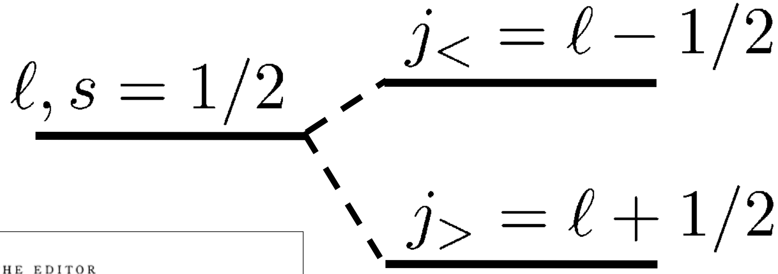
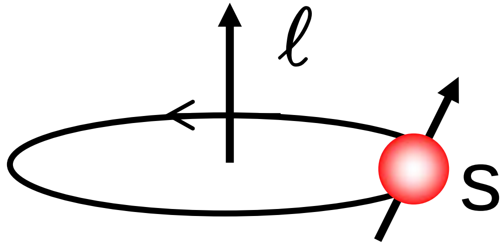


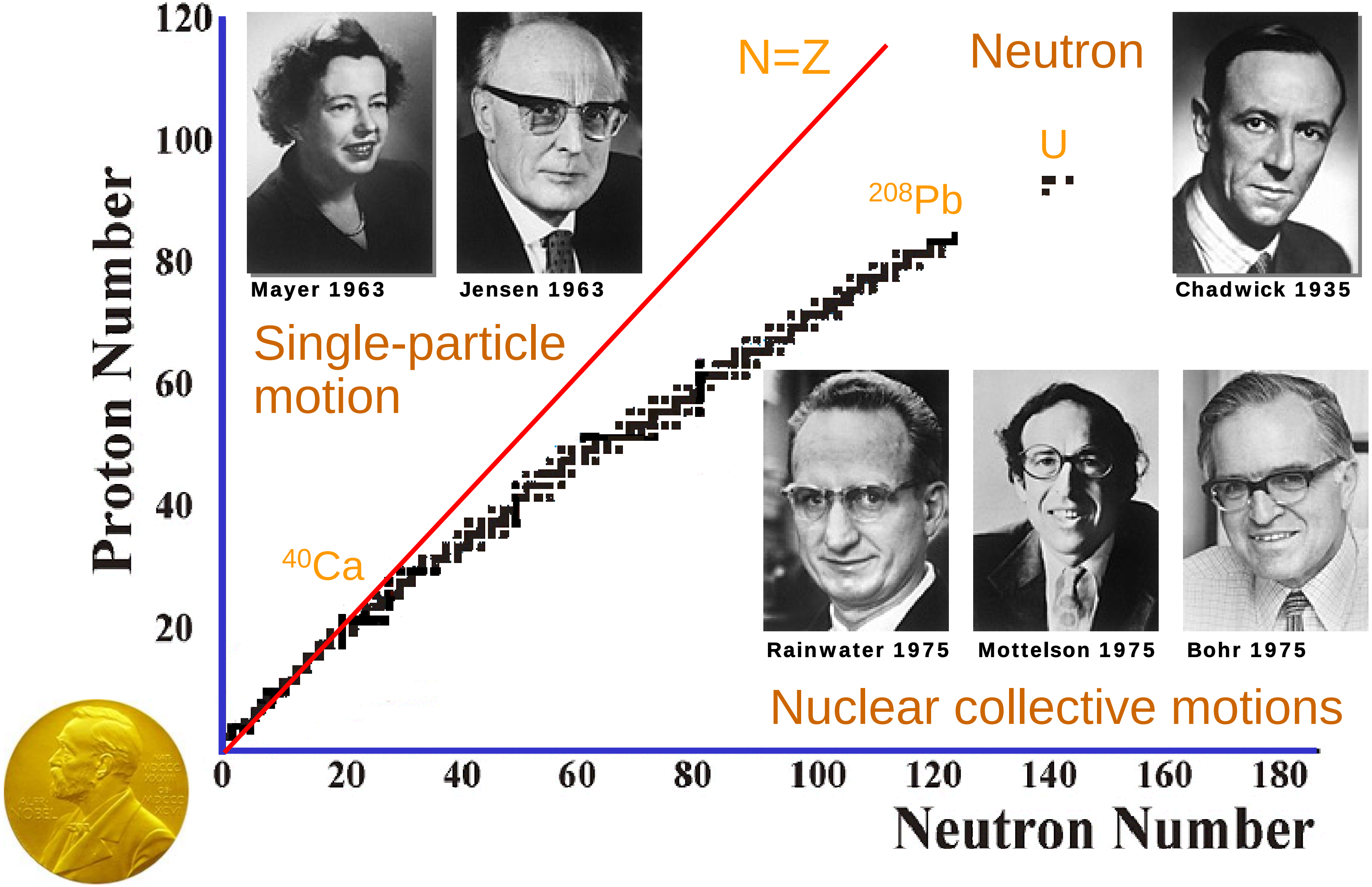




$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

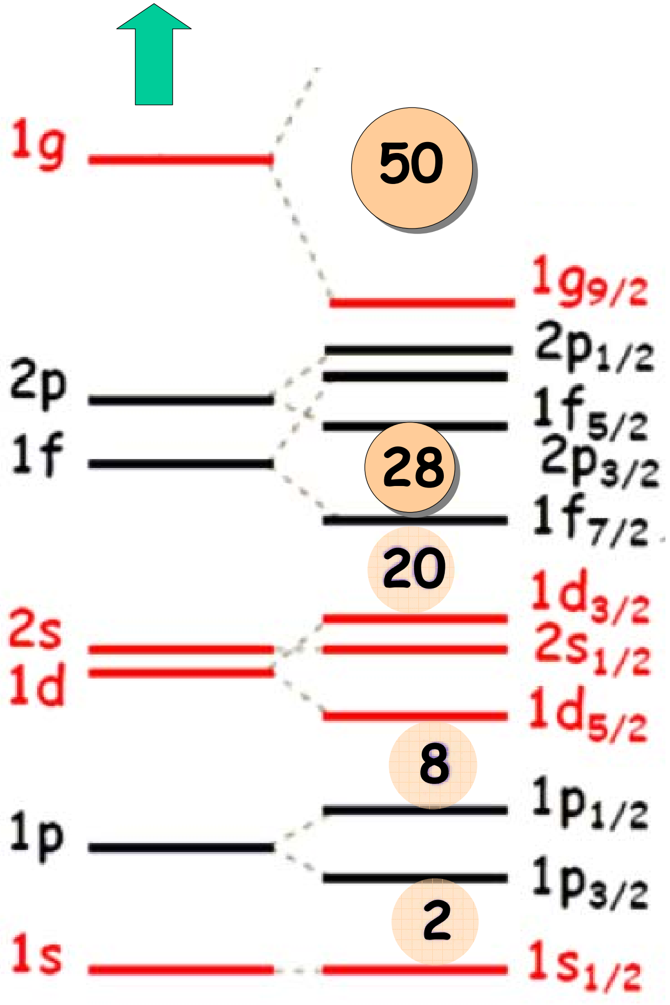
Thanks are due to Enrico Fermi for the remark, "Is there any indication of spin-orbit coupling?" which was the origin of this paper.

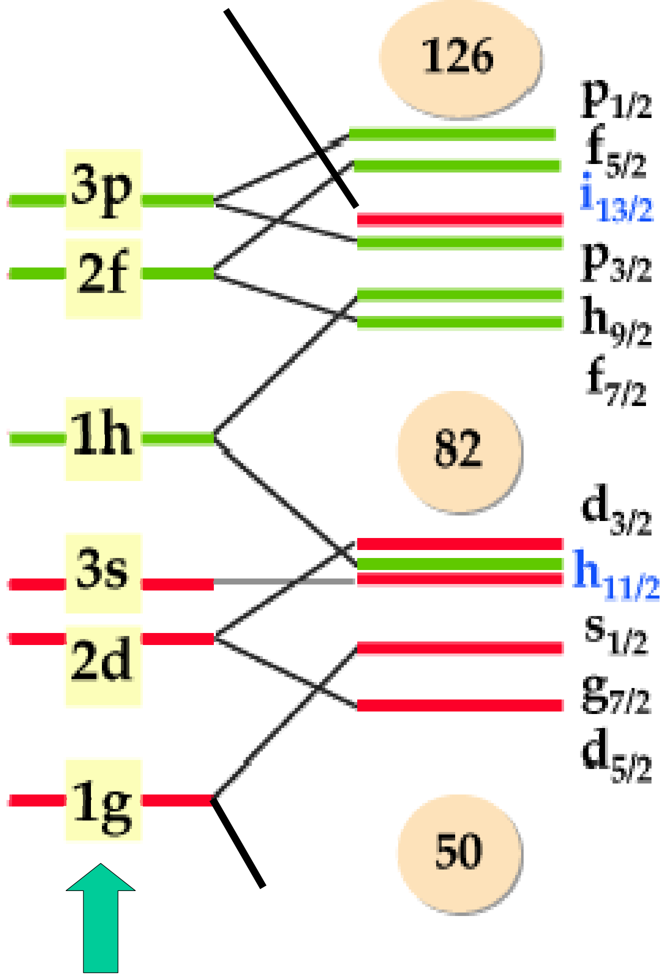
300 stable isotopes exist on Earth – Segré chart



Magic numbers – a framework for nuclear stability

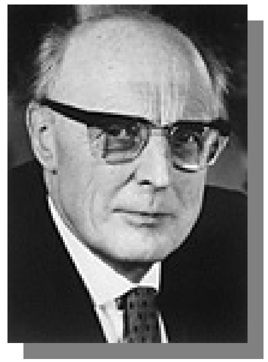
Nuclear mean-field levels





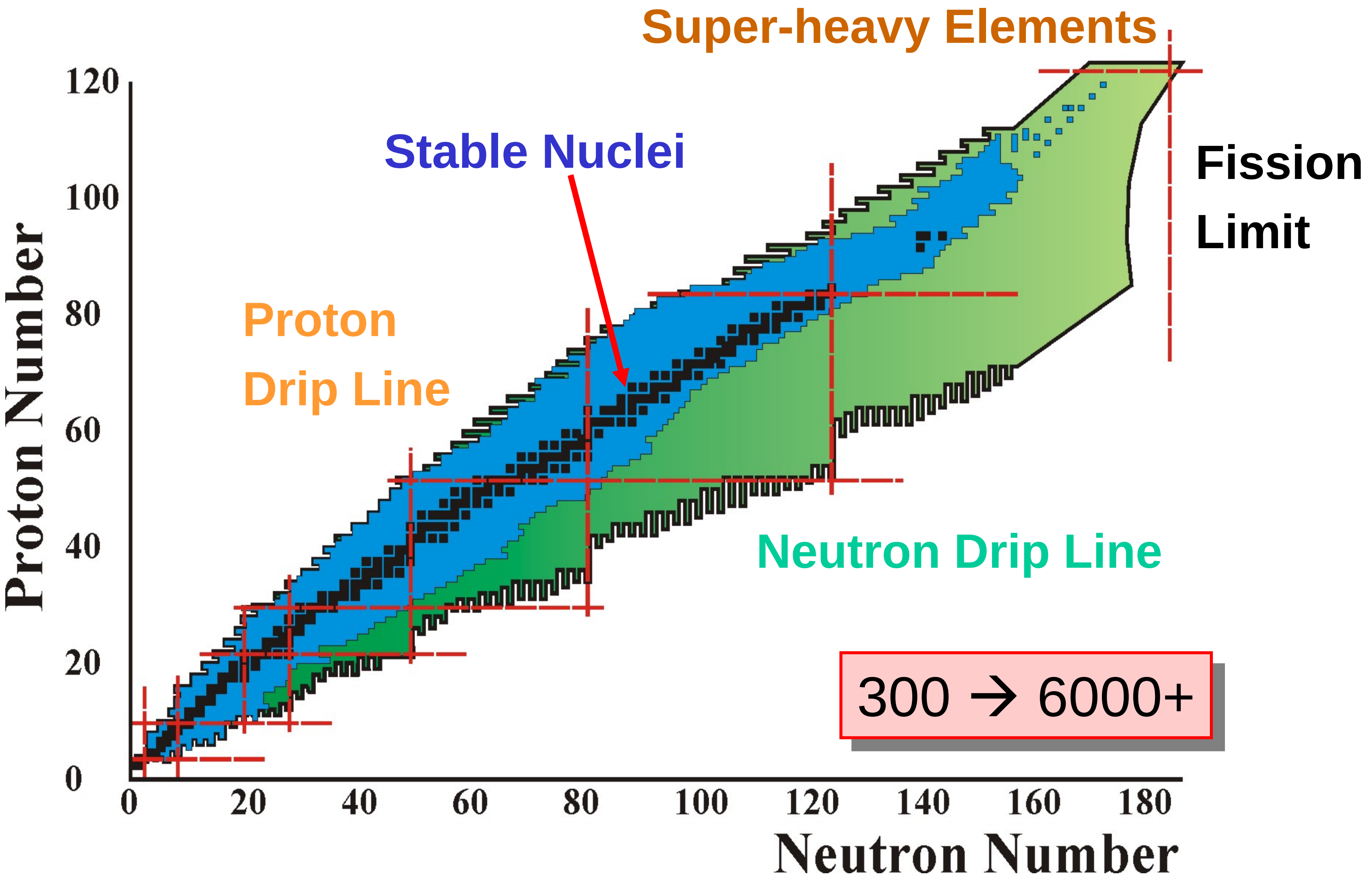
$$V_{Cent}(r) + V_{\ell s}(r)\vec{\ell} \cdot \vec{s}$$

Mayer

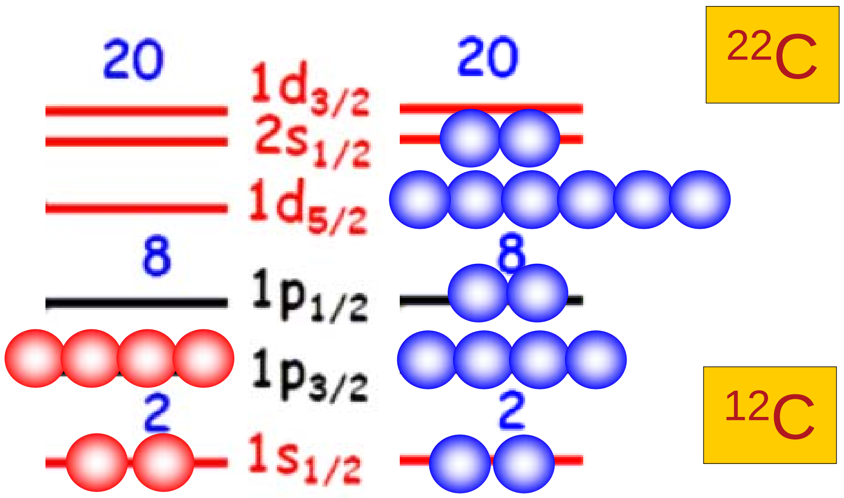


Jensen

So, the limits to nuclear existence are …?

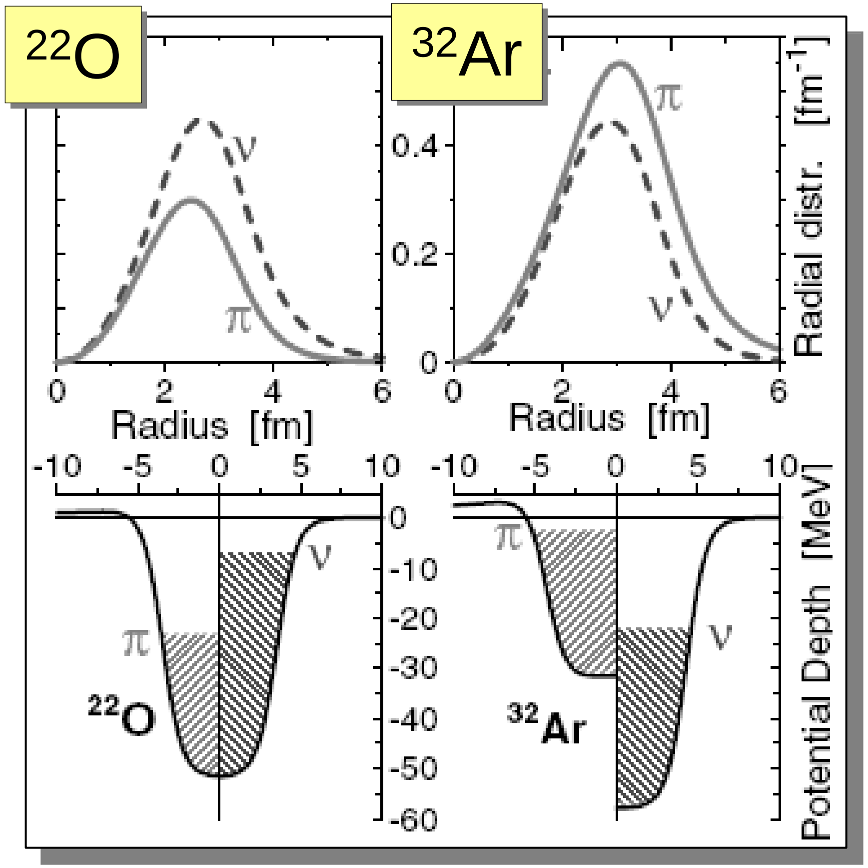


Exotic neutron + proton combinations - expectations



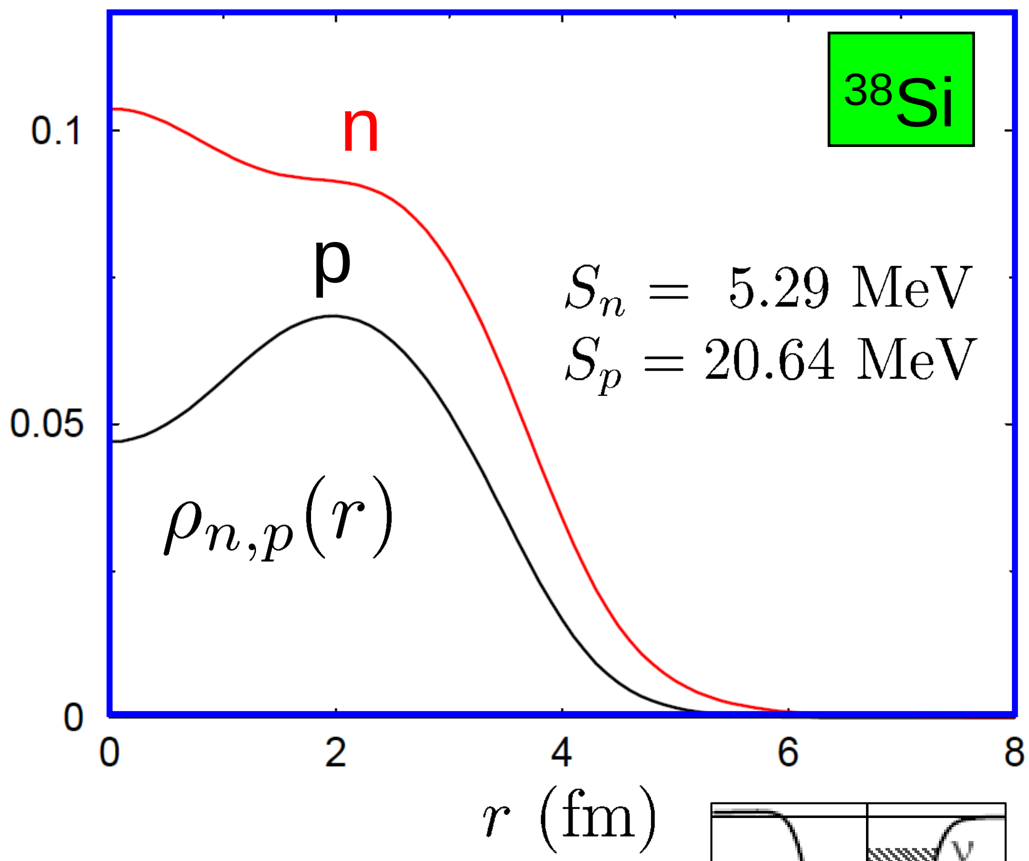
$$R \sim r_0 A^{1/3}$$

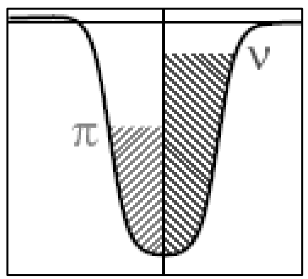
Very weak binding of last nucleons as the driplines are approached (1.4 MeV in ^{22}C, 18.7 MeV for neutron in ^{12}C)

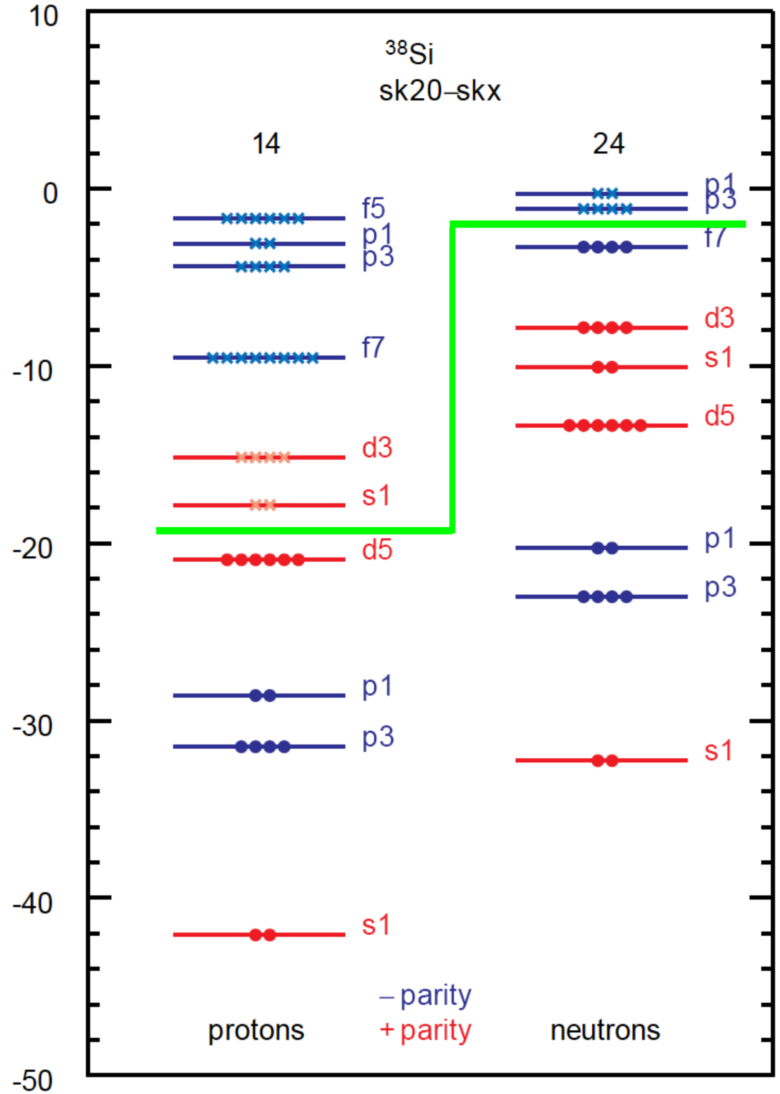


^{22}O	^{32}Ar
Z=8 **N=14**	Z=18 **N=14**
S_n=6.8 MeV	S_n=22 MeV
S_p=23 MeV	S_p=2.4 MeV

Make use of available tools – locate Fermi surfaces







Spherical Hartree
Fock density (SkX)

Bound states – the mean field helps our intuition

```
                          **********************
                          *                    *
INPUT VALUES              * IA,IZ =    24    8 *
                          *                    *
                          **********************
```

----- Neutron bound state results -----

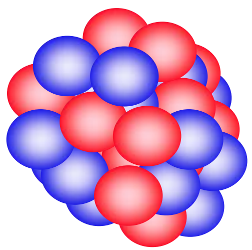
k	n	l	j	e	IE	OCC		
1	1	s	1/2	-26.757	1	2.00	36.70	35.28
2	1	p	3/2	-16.883	1	4.00	36.70	35.80
3	1	p	1/2	-12.396	1	2.00	36.70	36.04
4	1	d	5/2	-6.166	1	6.00	36.70	36.37
5	1	d	3/2	-0.109	1	0.00	36.70	36.69
6	2	s	1/2	-3.360	1	2.00	36.70	36.52
7	1	f	7/2	-0.200	3	0.00	46.02	46.01
8	1	f	5/2	-0.200	3	0.00	60.56	60.55
9	2	p	3/2	-0.200	3	0.00	48.10	48.09

----- Neutron single-particle radii -----

				R(2)	R(4)	OCC	rho(8.9)	rho(9.9)	rho(10.9)
1	1	s	1/2	2.274	2.575	2.000	0.848E-09	0.706E-10	0.600E-11
2	1	p	3/2	2.863	3.133	4.000	0.188E-07	0.244E-08	0.325E-09
3	1	p	1/2	2.954	3.268	2.000	0.727E-07	0.122E-07	0.210E-08
4	1	d	5/2	3.434	3.757	6.000	0.524E-06	0.129E-06	0.327E-07
5	1	d	3/2	4.662	6.063	0.000	0.131E-04	0.675E-05	0.371E-05
6	2	s	1/2	4.172	4.895	2.000	0.769E-05	0.278E-05	0.102E-05
7	1	f	7/2	3.865	4.440	0.000	0.324E-05	0.134E-05	0.600E-06
8	1	f	5/2	3.890	4.477	0.000	0.341E-05	0.141E-05	0.631E-06
9	2	p	3/2	6.815	8.635	0.000	0.451E-04	0.270E-04	0.167E-04

The mean field – e.g. spherical HF - gives an excellent estimate and guide

$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



$^{24}\mathrm{O}(g.s.)$

And more realistic (complex) structure models?

MARIA GOEPPERT MAYER *Nobel Lecture, December 12,* ***1963***

The shell model → more properly: independent particle model

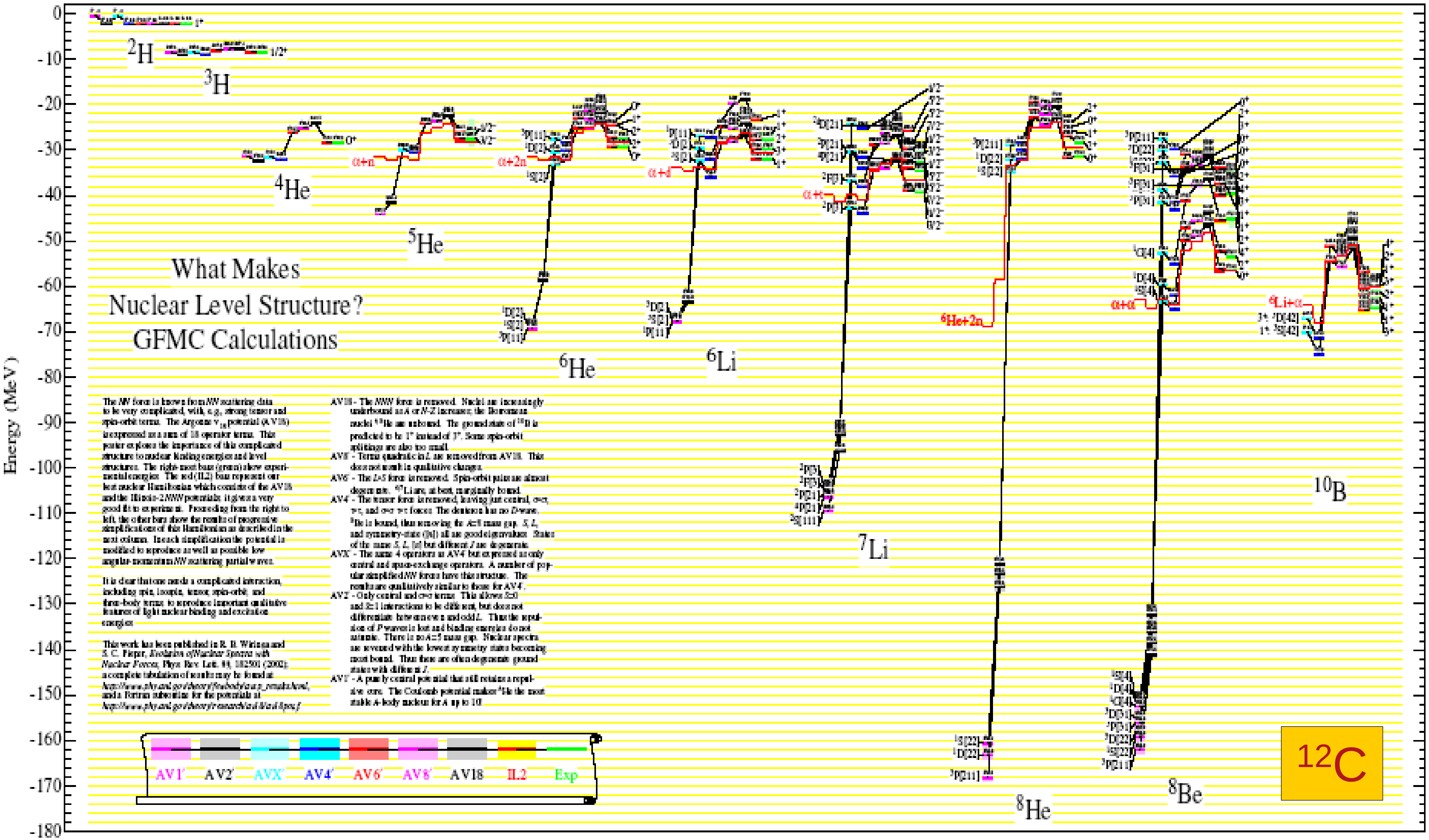
……

If the forces are known, one should in principle be able to calculate deductively the properties of individual complex nuclei. Only after this has been accomplished can one say that one completely understands nuclear structures.

Ground state of ^{12}C 75,000 CPU-hours (27 Teraflop hours)

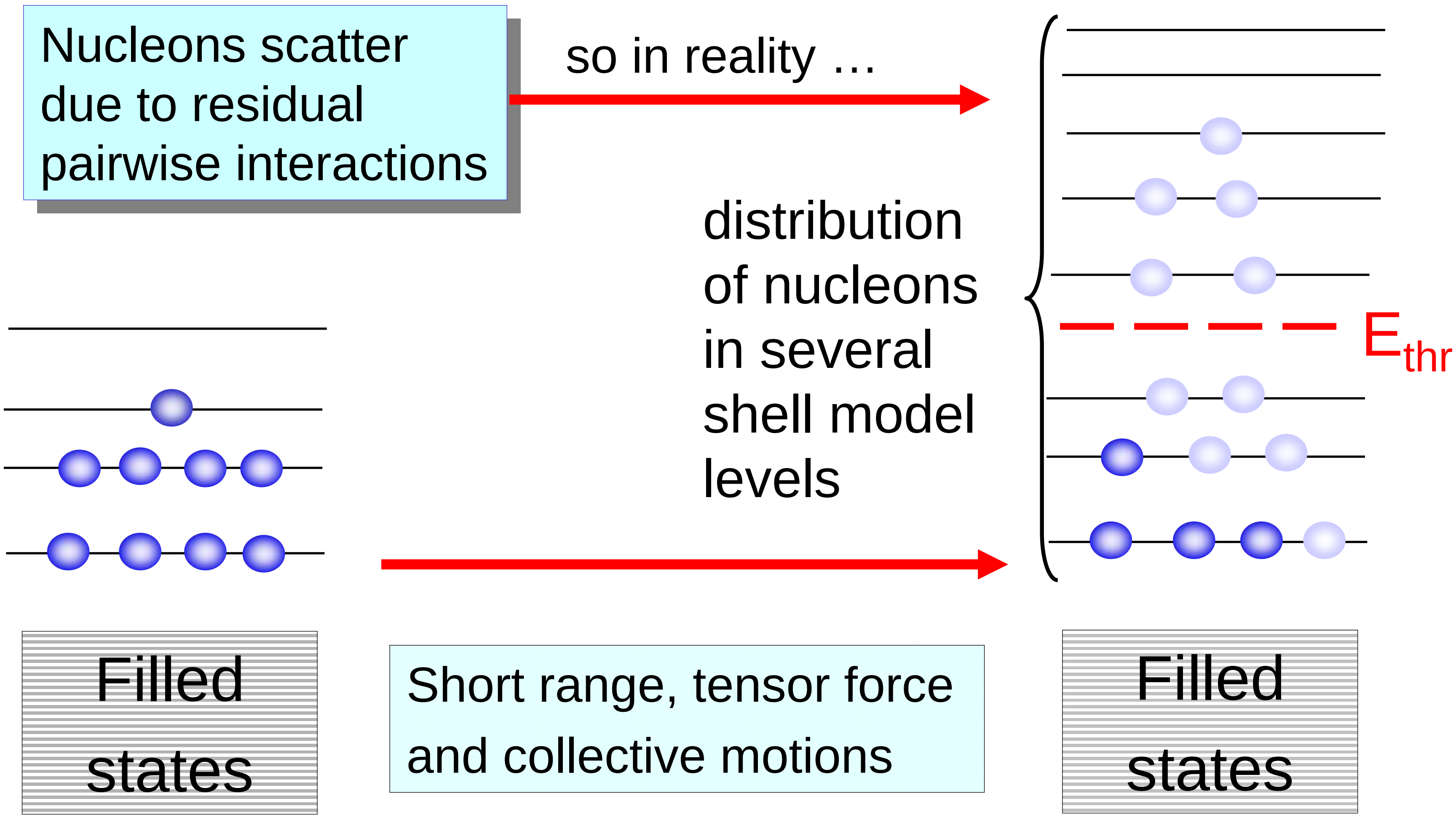
The shell model has initiated a large field of research. It has served as the starting point for more refined calculations. There are enough nuclei to investigate so that the shell modellists will not soon be unemployed.

The state-of-the art – A=12 and less – really hard!



http://www.phy.anl.gov/theory/poster.pdf

Modern 'shell model' calculations do much more ….



Calculations are big - computational challenge

Matrix dimensions

pf-shell

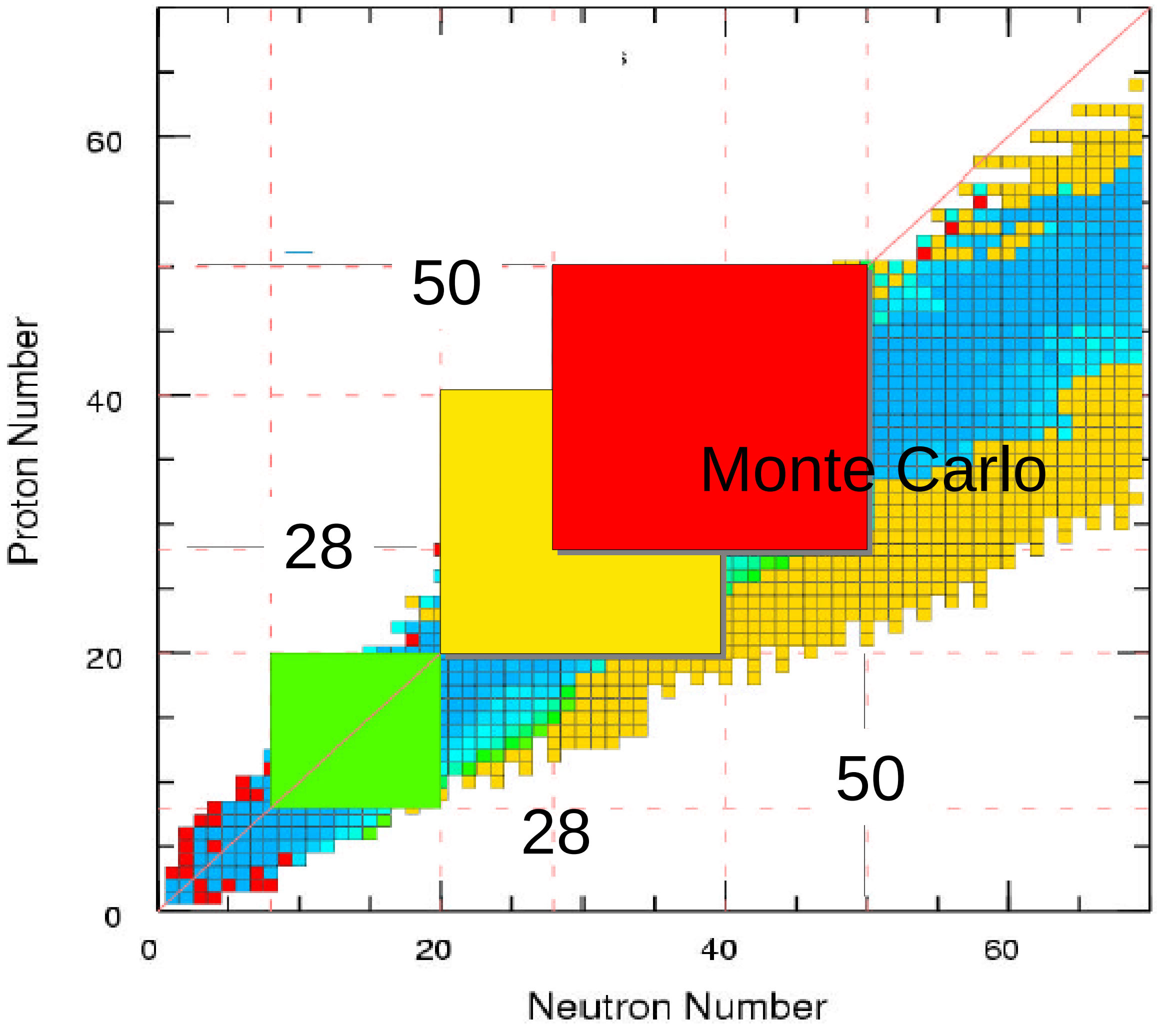
10^{11}

1998-

$pfg_{9/2}$-shell

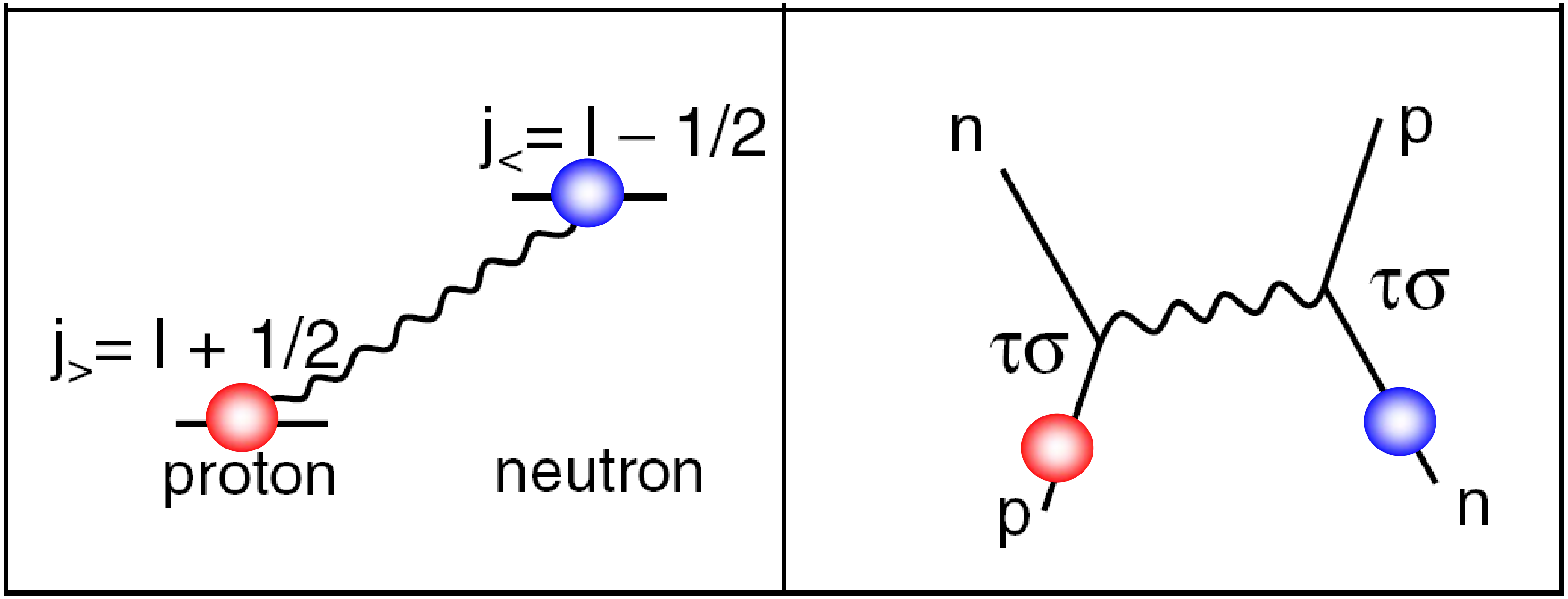
10^{13}

2004-



From Alex Brown

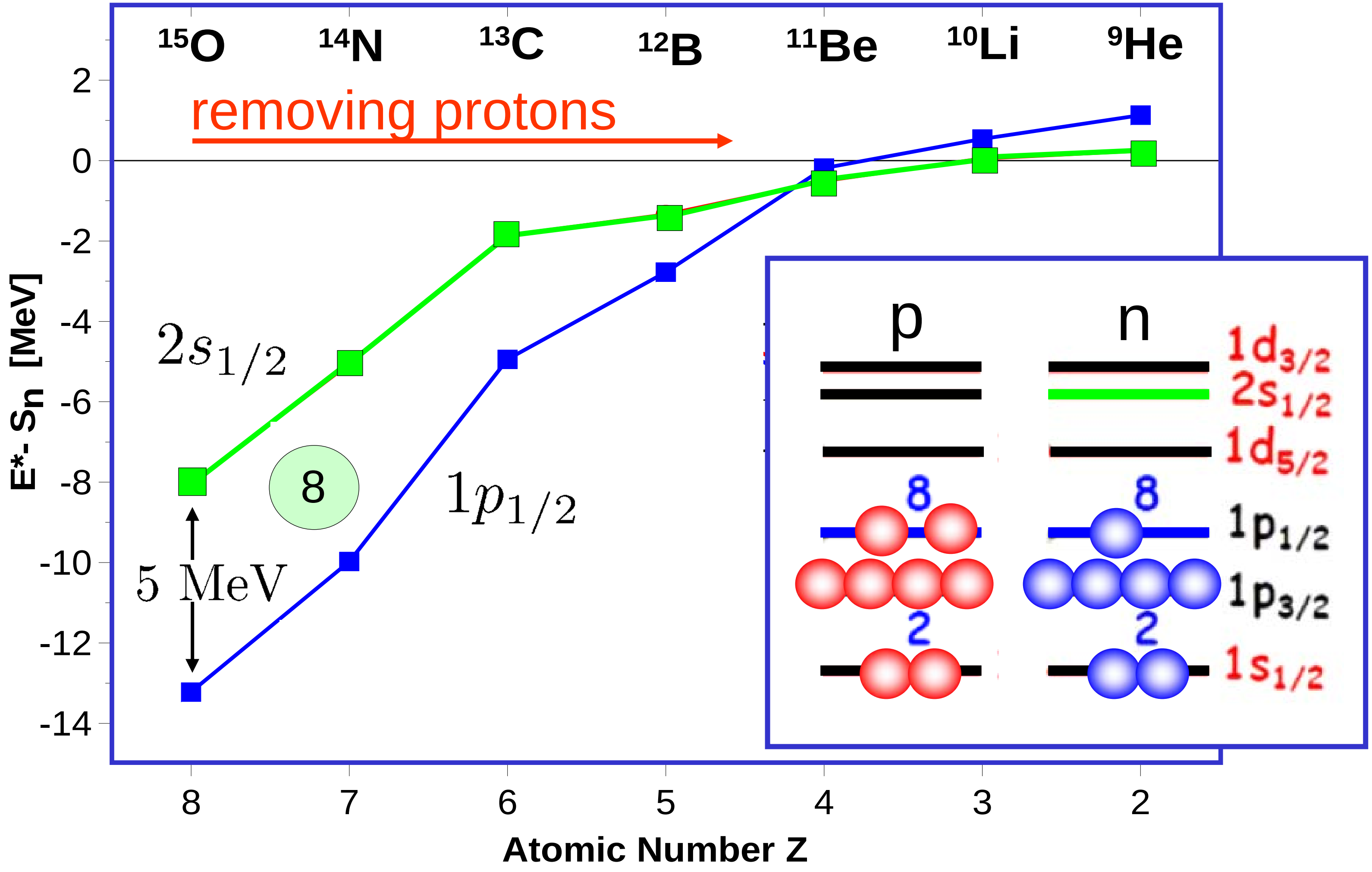
Implications for shell structure: asymmetric nuclei



Attractive interaction between neutrons and protons occupying $j_>$ and $j_<$ levels, repulsive j> and j> levels – from several sources, but mainly the tensor force

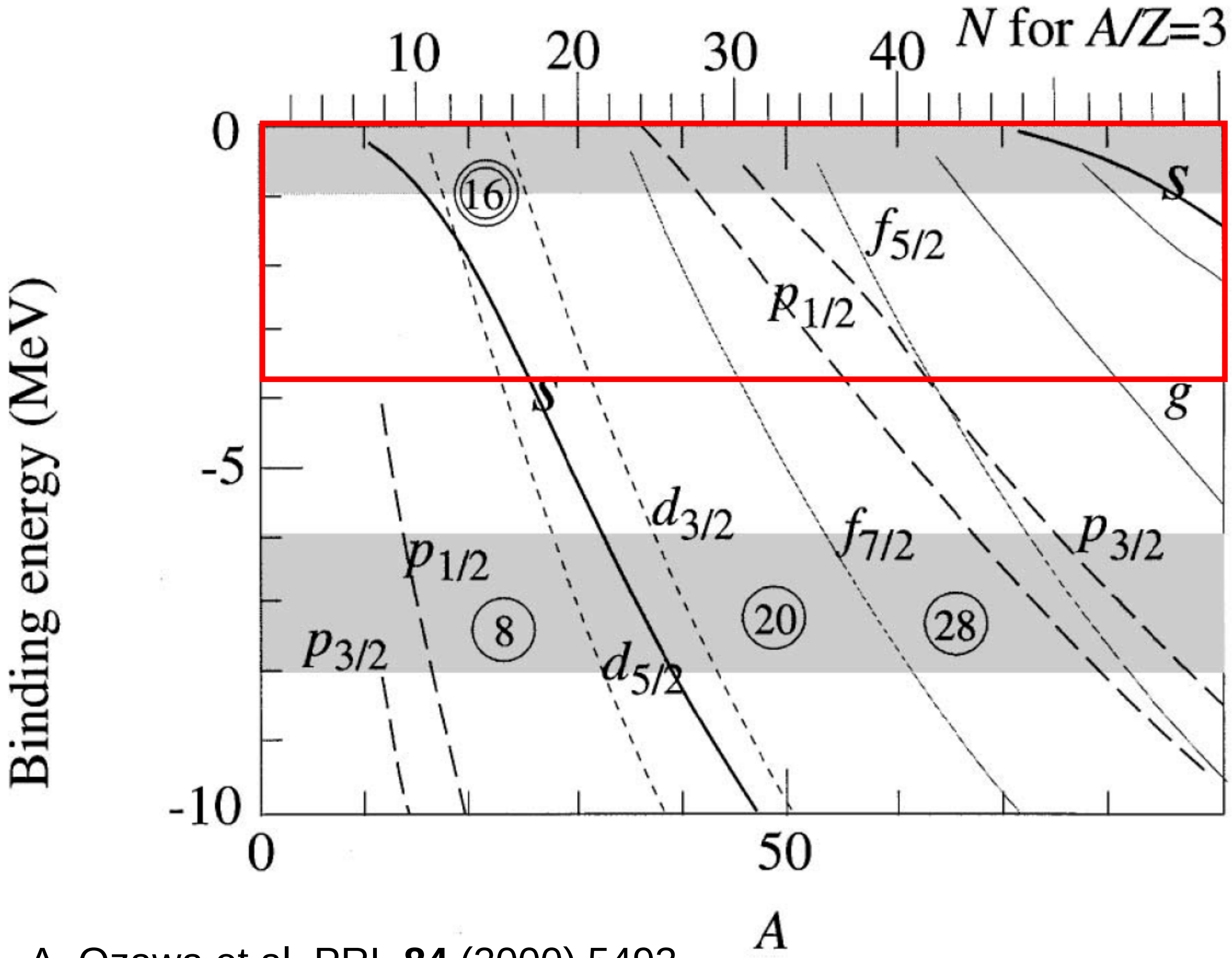
Takaharu Otsuka et al, Phys. Rev. Lett. **87**, 082502 (2001), **95**, 232502 (2005), **105**, 032501 (2010)

Migration of levels for N=7 – loss of the N=8 magic



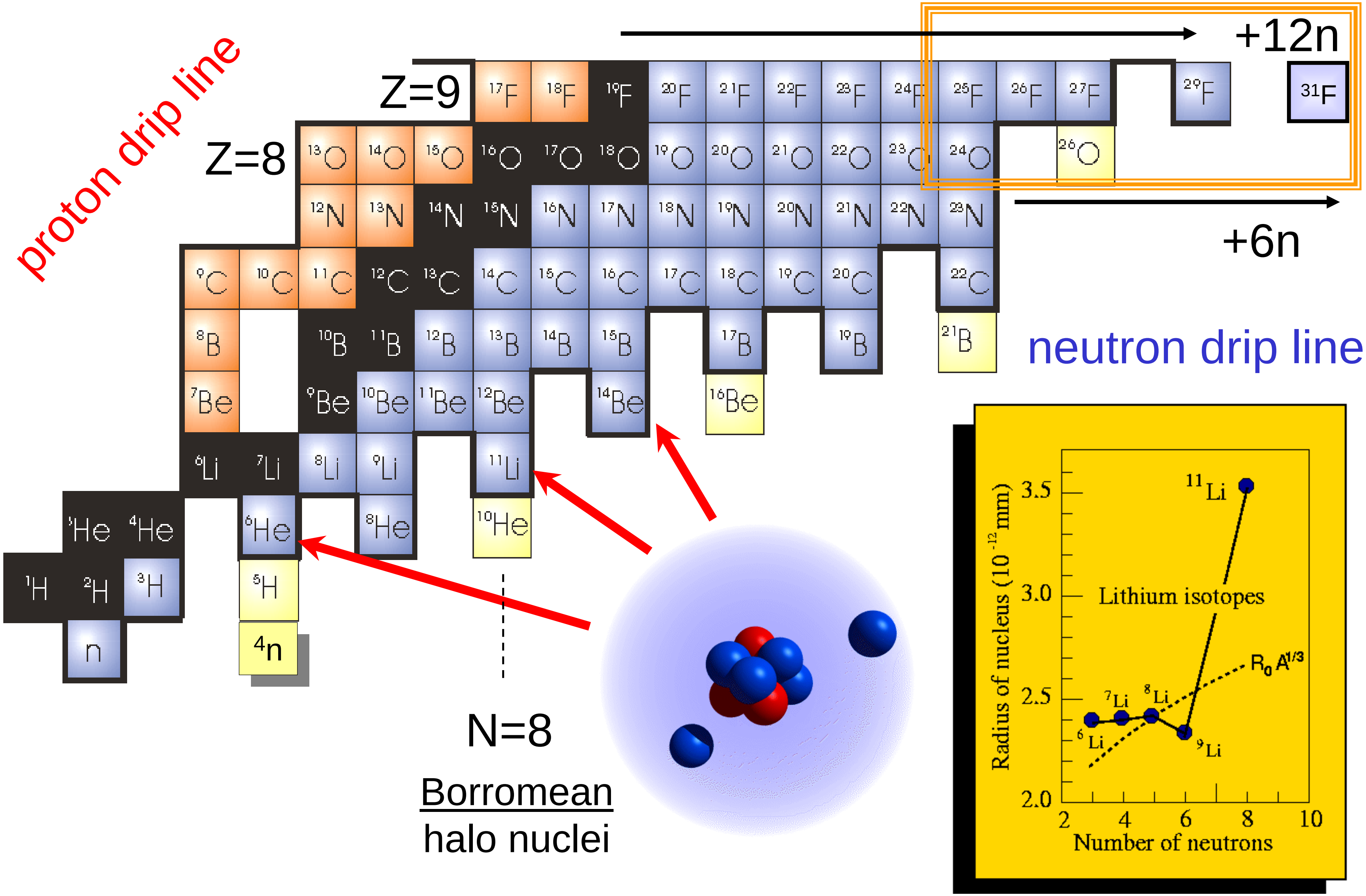
From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Low angular momentum states see well diffuseness

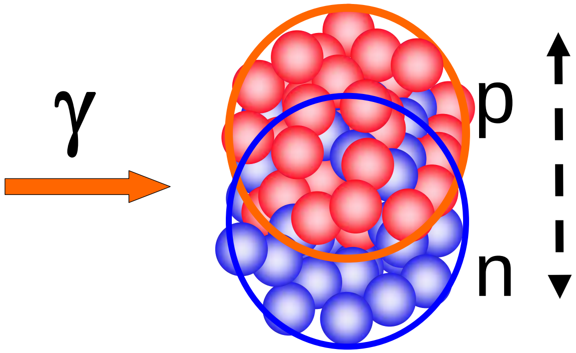


A. Ozawa et al, PRL **84** (2000) 5493

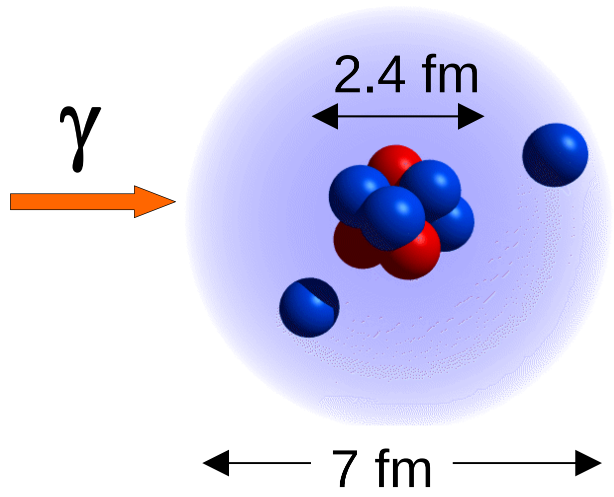
We have only reached the driplines in the light nuclei



Halo nuclei –extremes of weak binding - responses

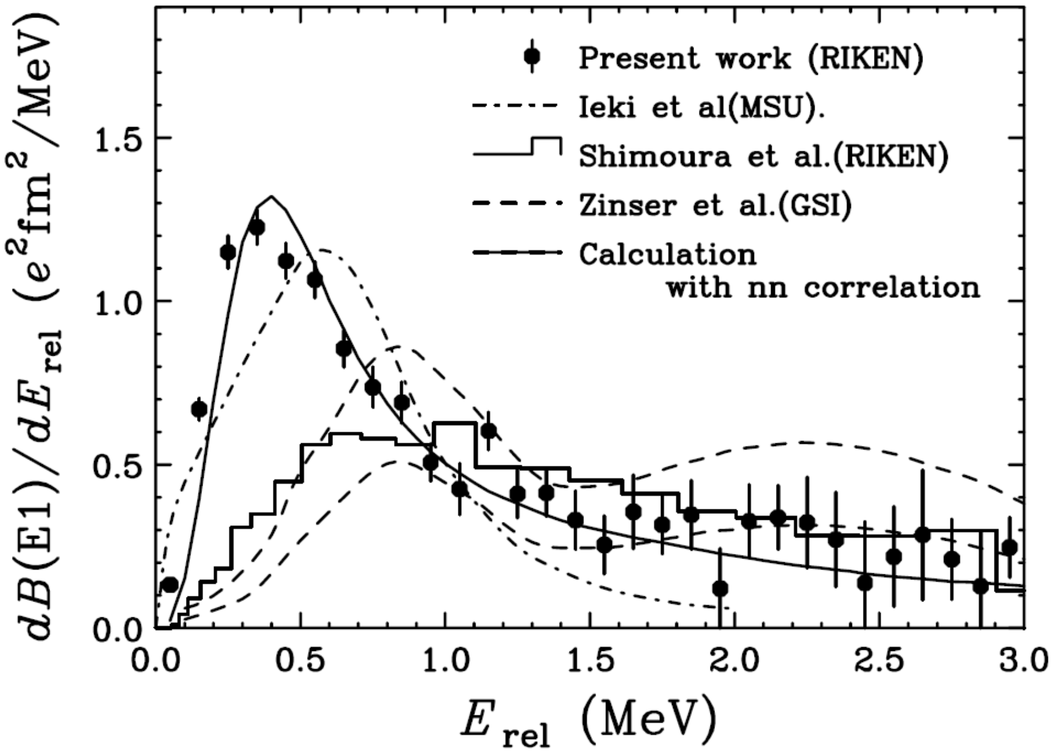


(Giant) electric Dipole excitation → 10-20 MeV



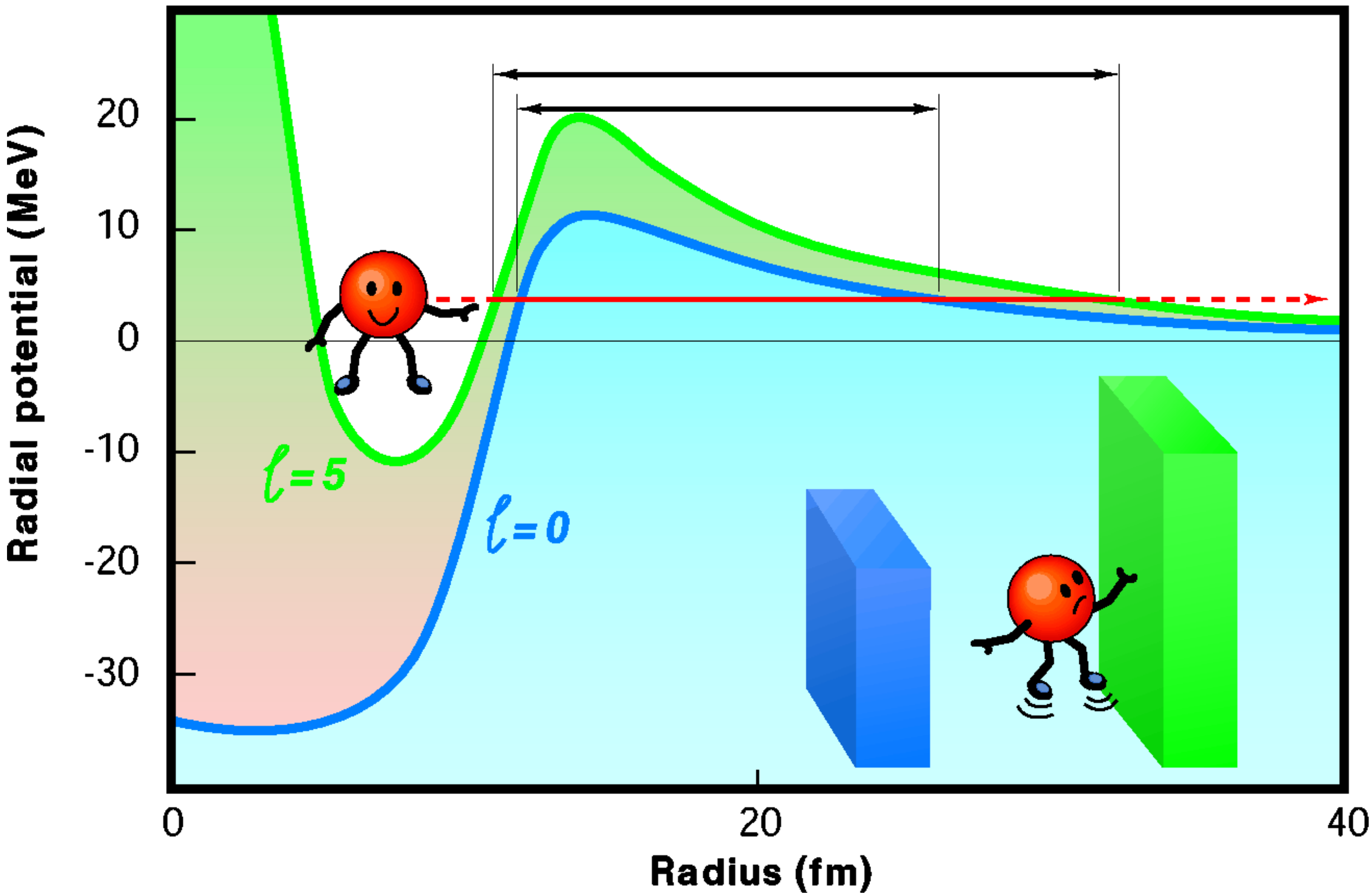
^{6}He (2n,1 MeV) ^{11}Be (1n,0.5 MeV)
^{11}Li (2n,0.5 MeV) ^{14}Be (2n,~1 MeV)

Electric dipole response – ^{11}Li

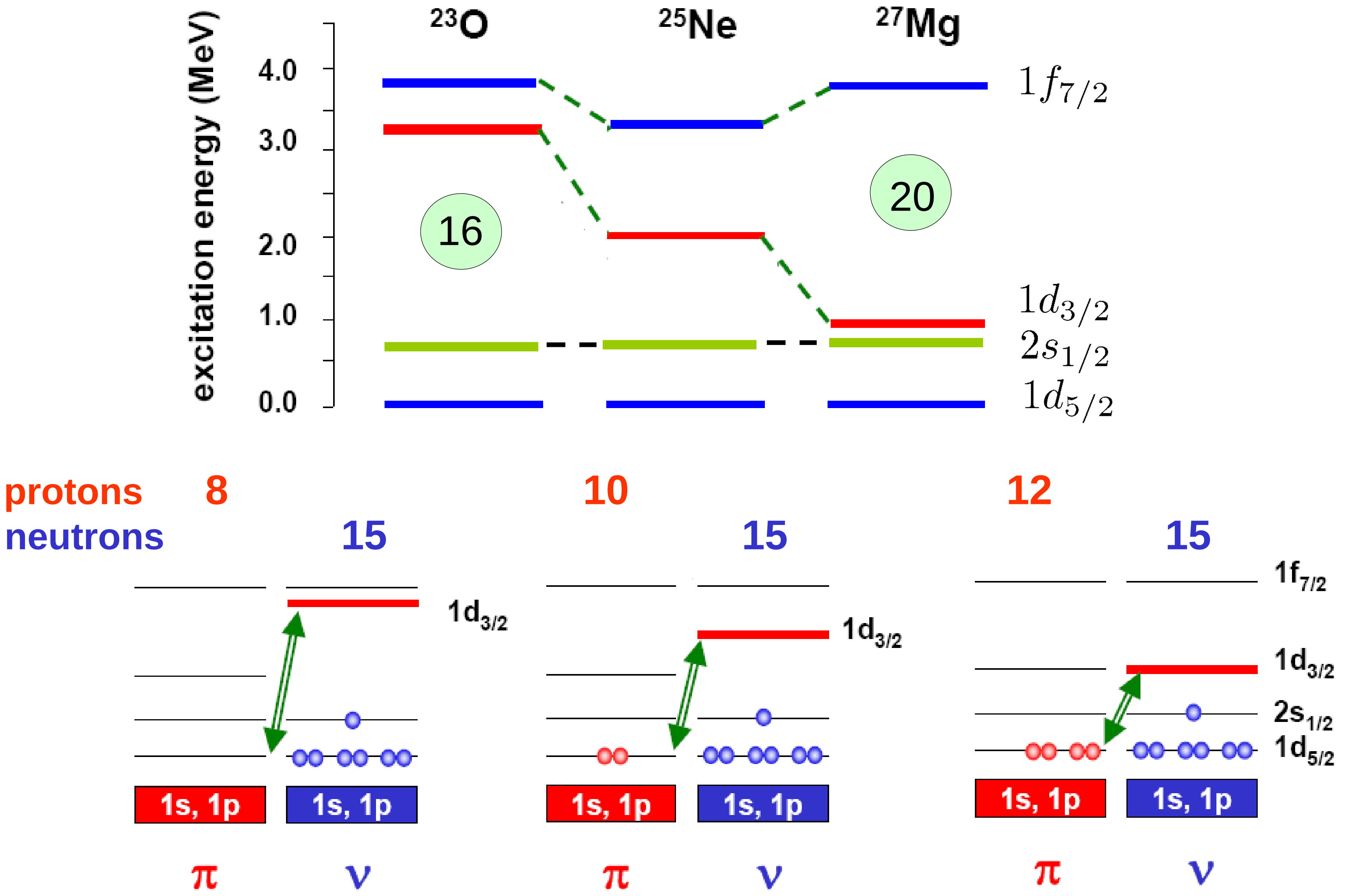


T. Nakamura et al., PRL (2006) in press

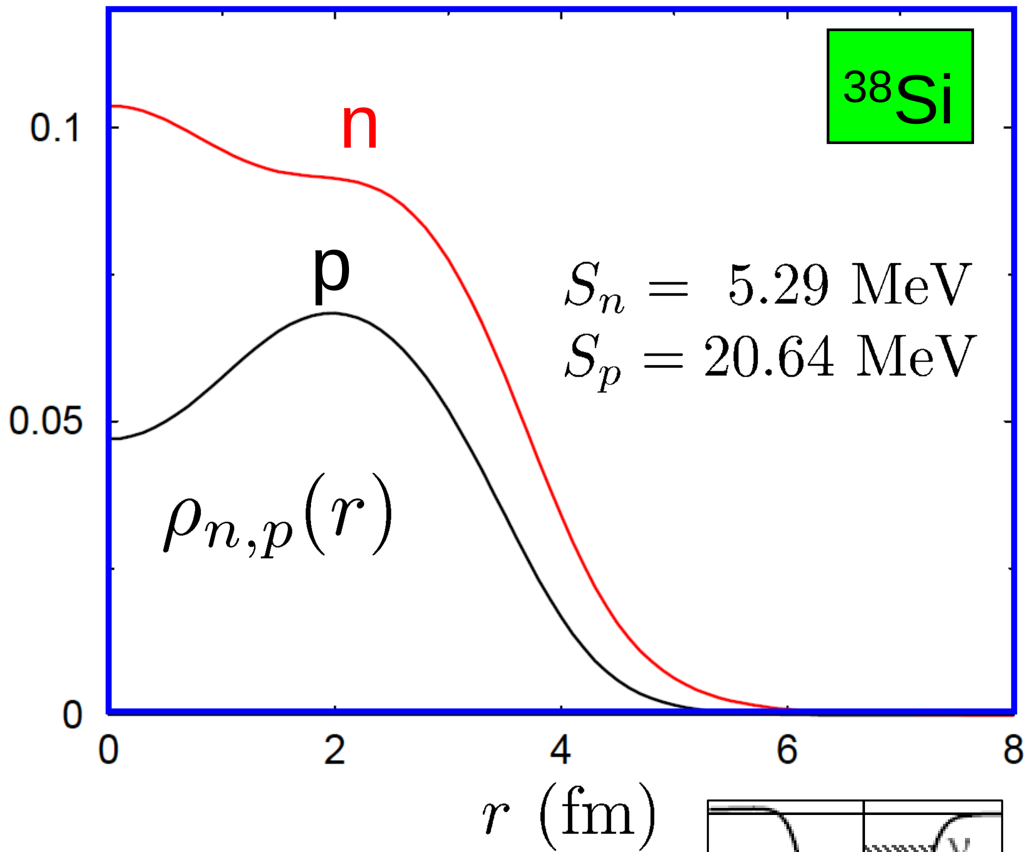
Proton decay – a probe of proton-dripline structures



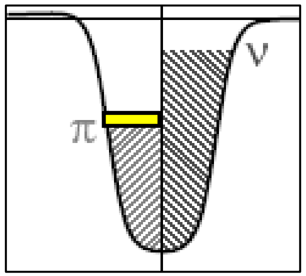
Magic numbers change with ''neutron richness''

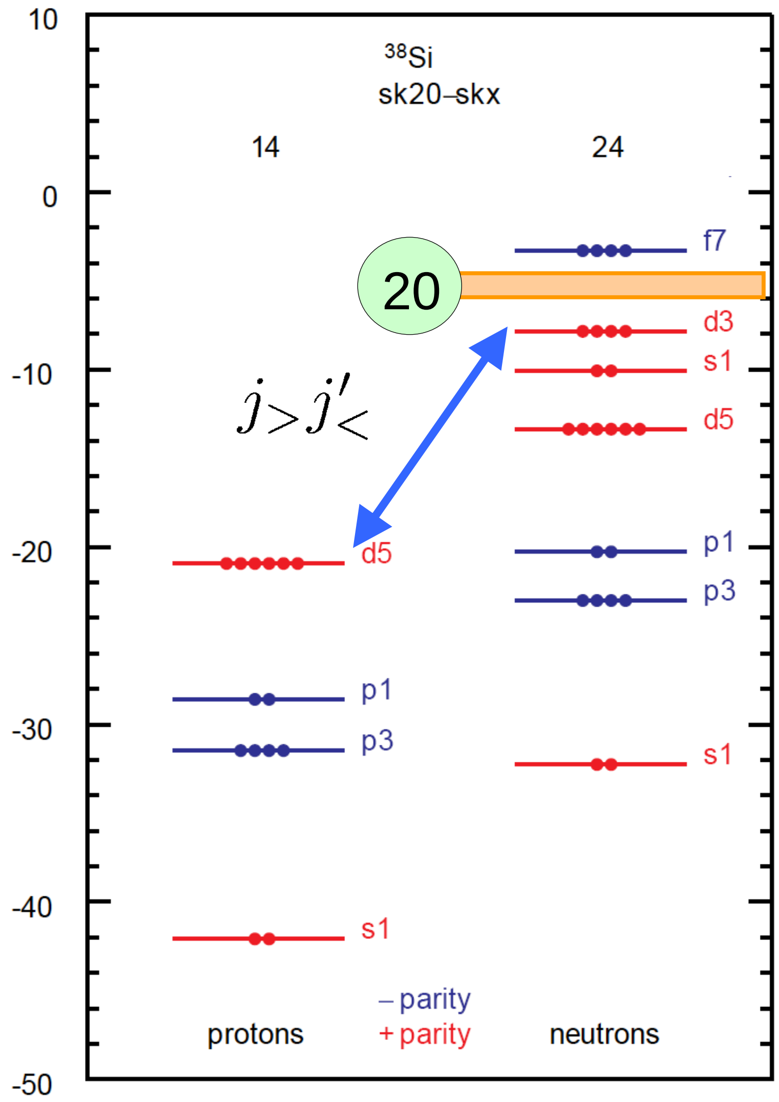


Otsuka: the np interaction tensor correlation

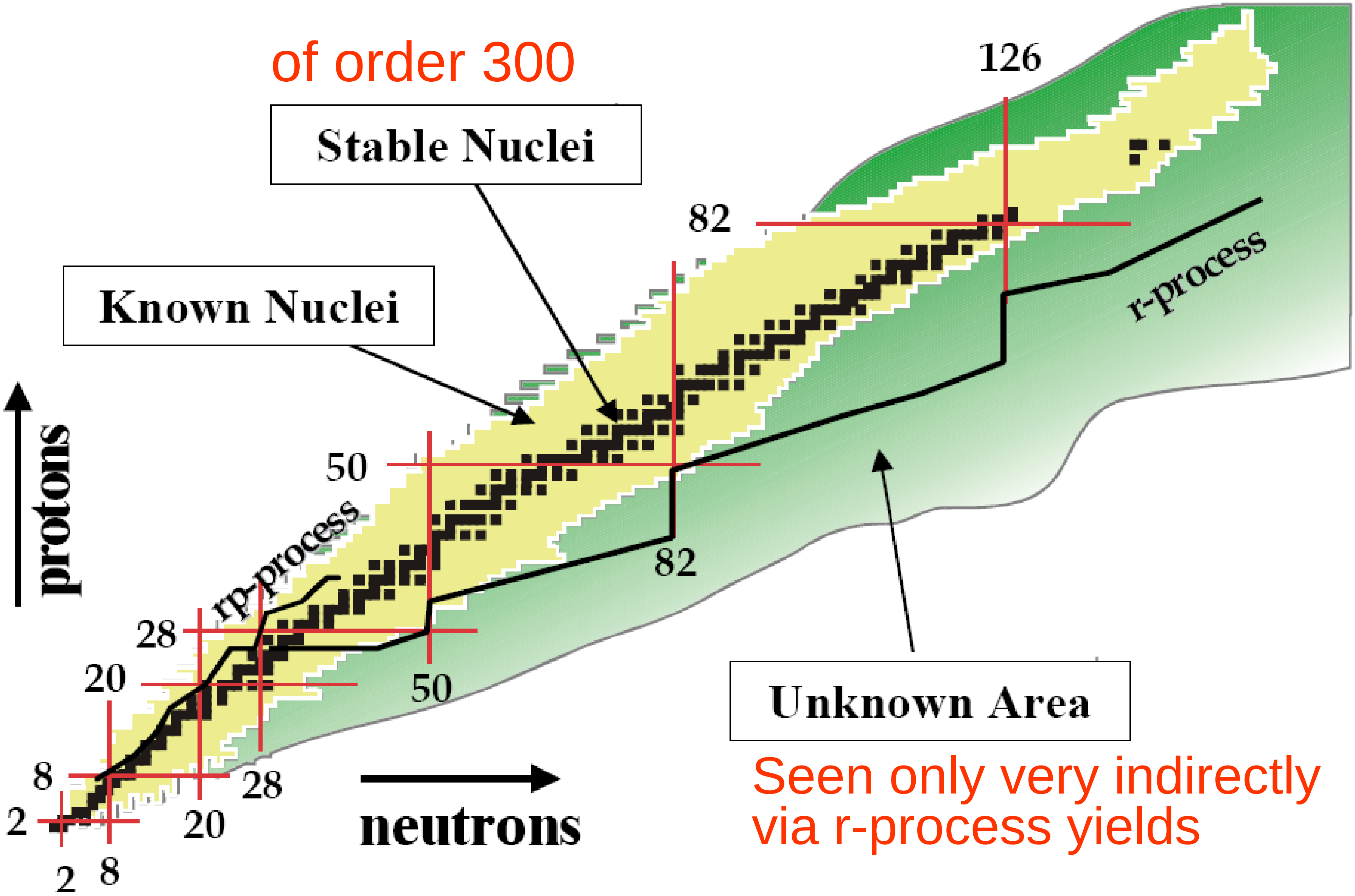


Spherical Hartree Fock density (SkX)

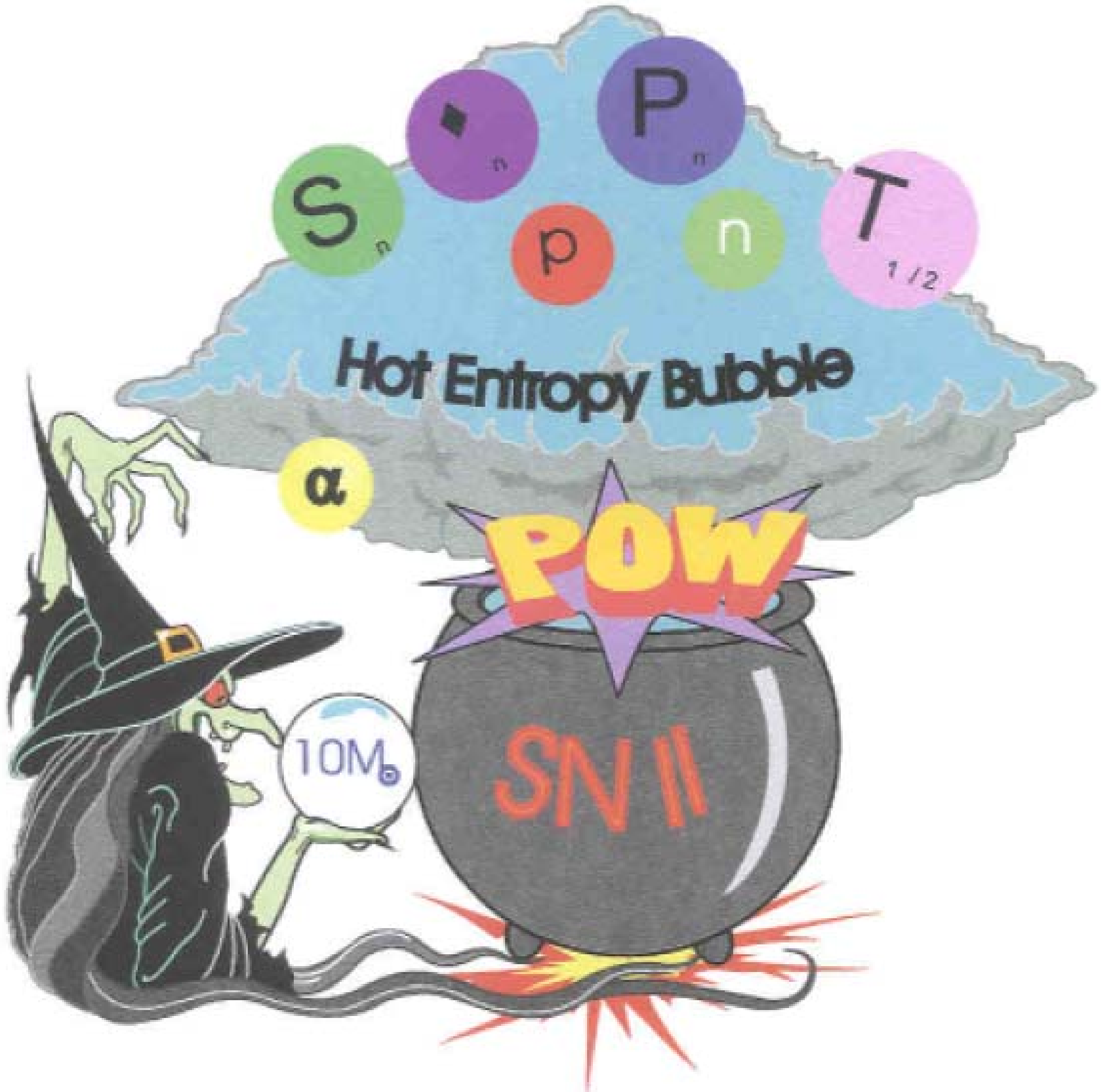




Nuclear landscape (circa 2010)



½ of all --- is produced in r-process synthesis

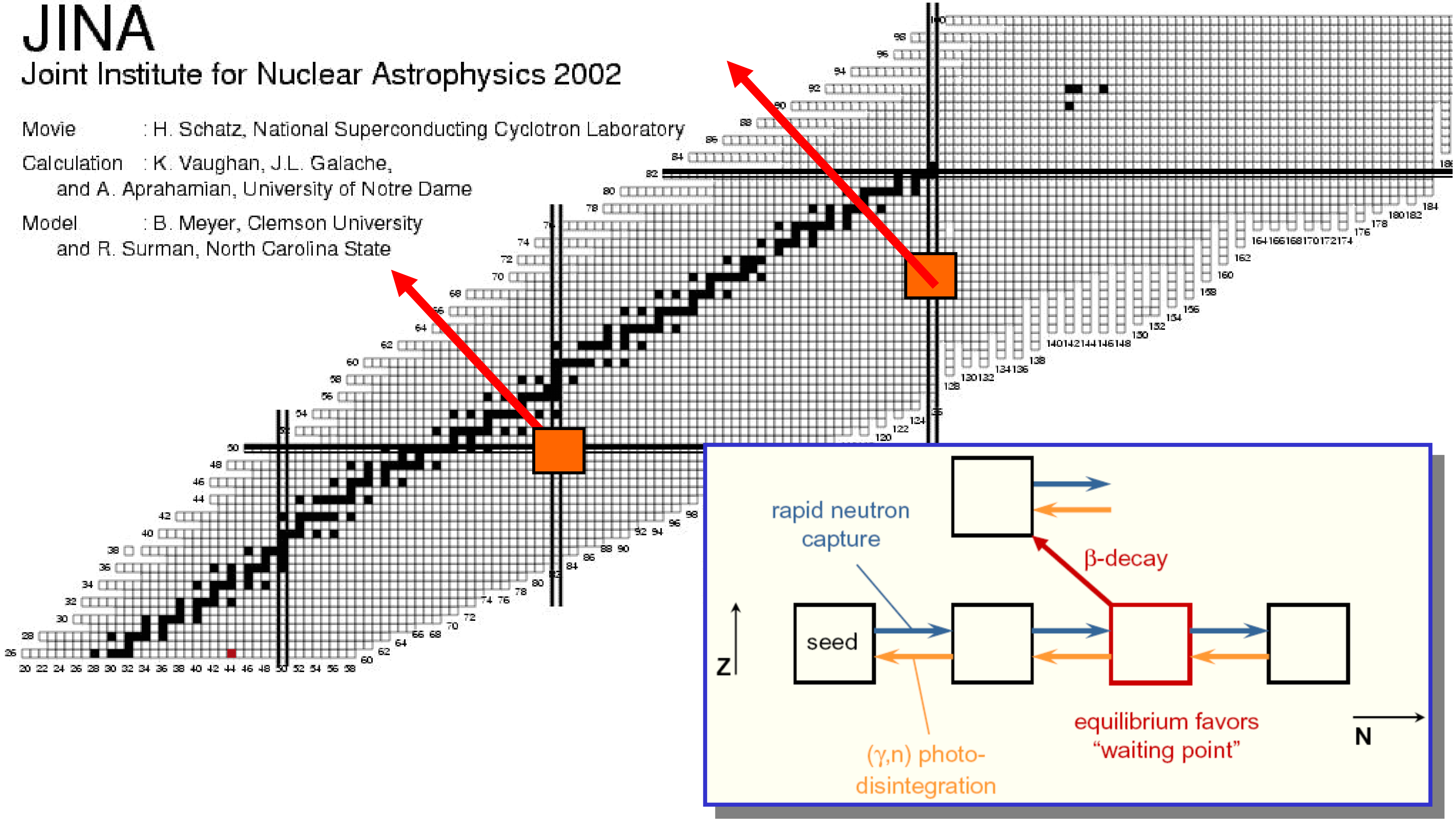


JINA

The Joint Institute for Nuclear Astrophysics

NSCL

http://groups.nscl.msu.edu/nero/Web/materials.html

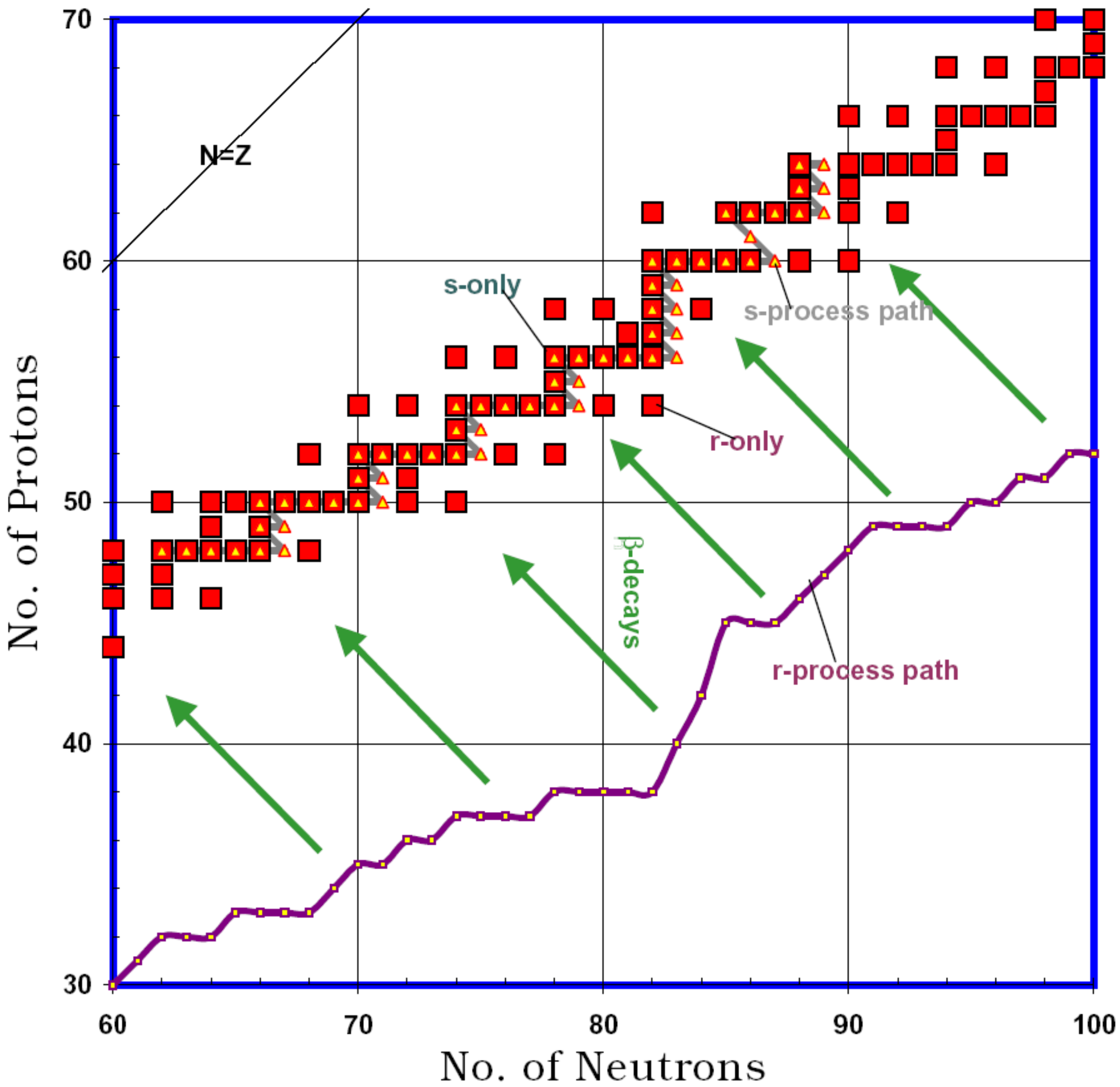


r-process synthesis modelling – inputs needed?

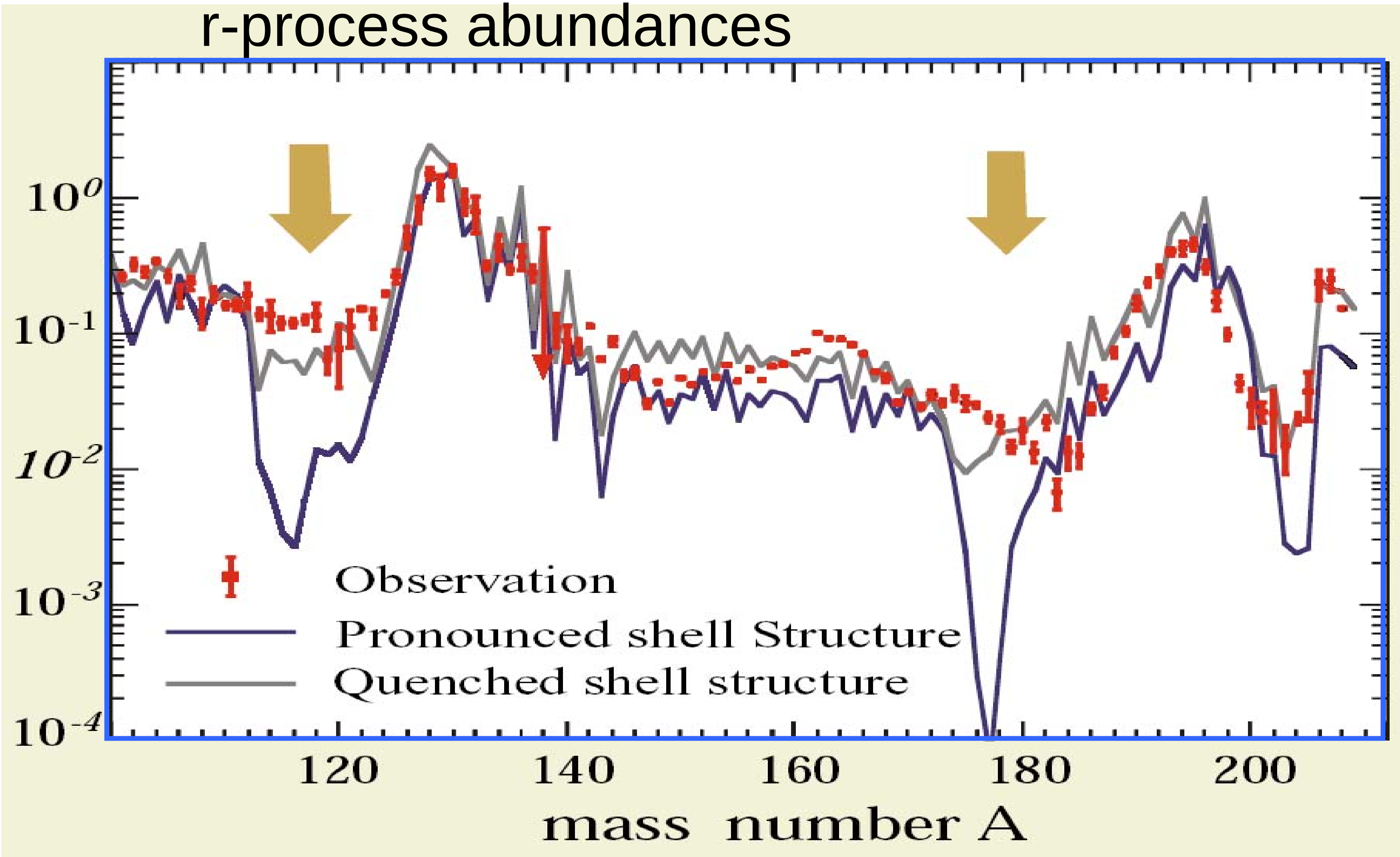
Need is for:

Masses – stability - determine reaction rates

beta-decay half-lives – masses, but also structure effects – need to test shell model predictions – too many systems to measure sensibly

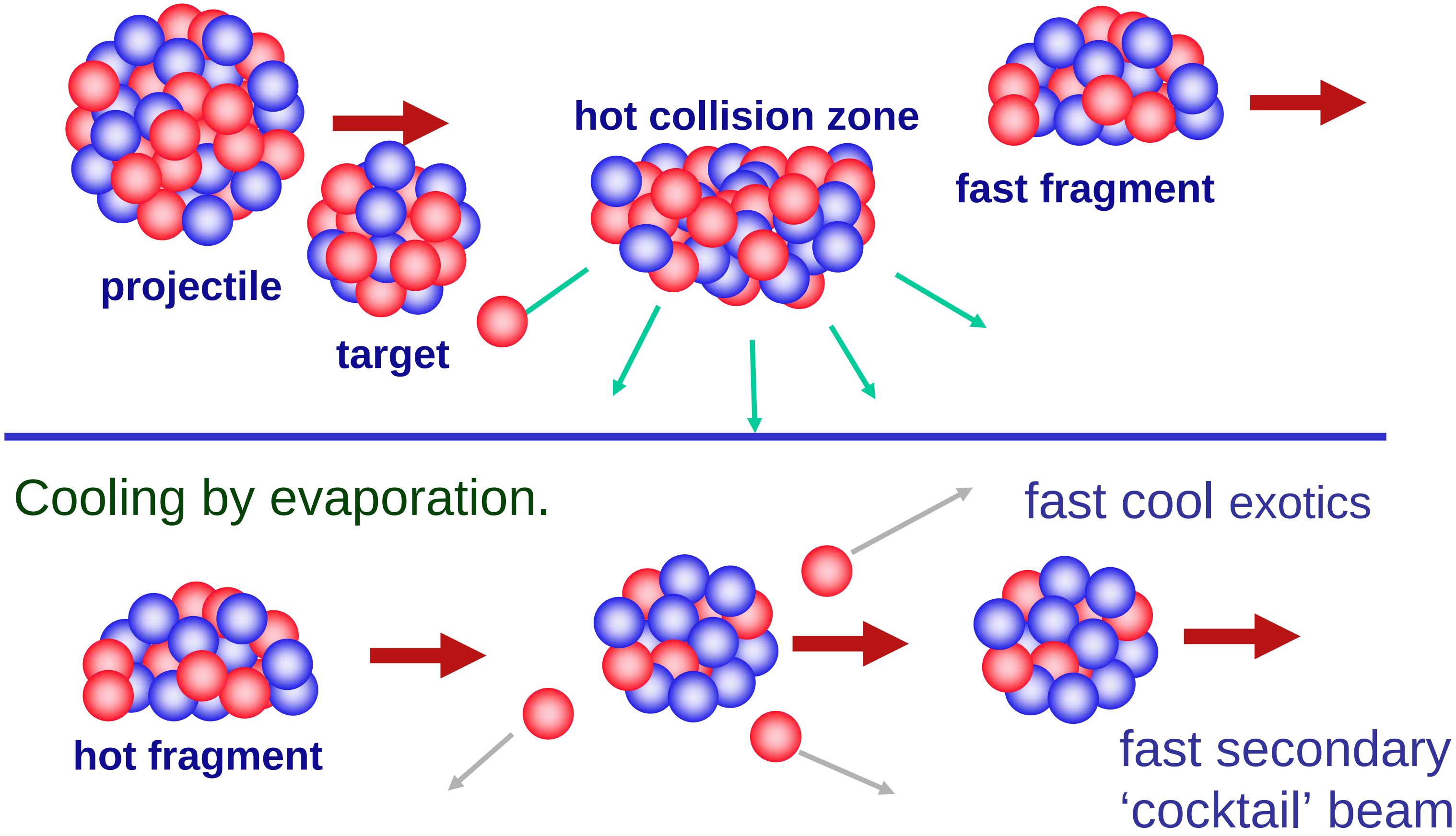


‘Less magic’ in heavier systems – evidence?



Exotic nuclei production - projectile fragmentation

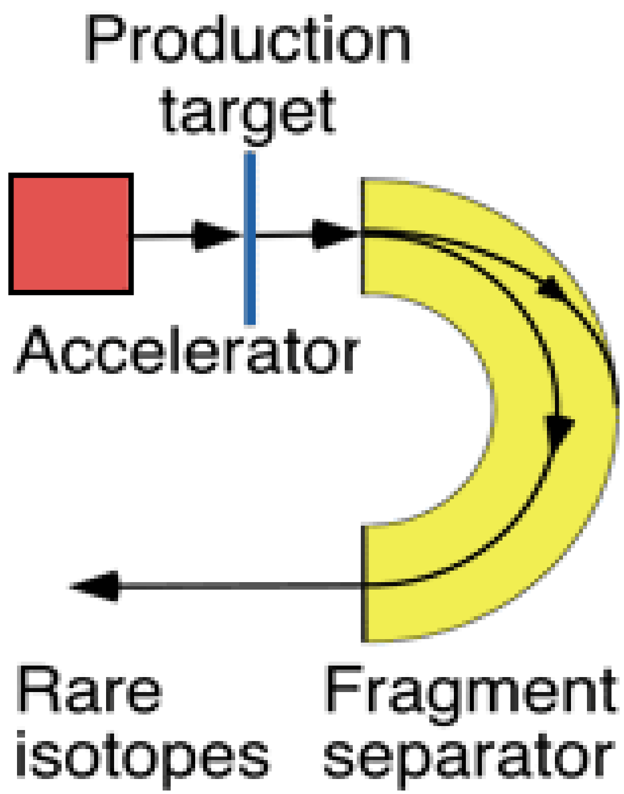
Random removal of protons and neutrons from heavy projectile in peripheral collisions at high energy - 100 MeV per nucleon or more



Schematic of a Projectile Fragmentation Facility

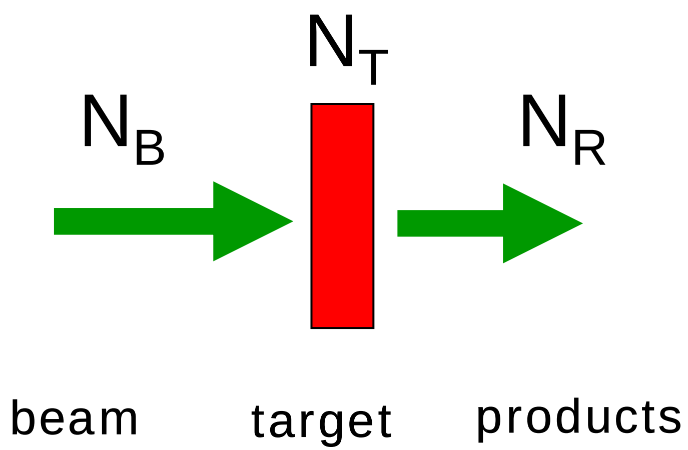
GANIL (France), GSI (Germany), NSCL (USA), RIKEN (Japan)

- High-energy beams (E/A > 50 MeV) of only modest beam quality
- Fast, physical method of separation, no chemistry
- Suitable for short-lived isotopes (lifetimes > 10^{-6} s)
- But, low-energy beams are (very) difficult
- Modest beam quality



From Thomas Glasmacher

Use of rarest beams a ‘few’ atomic nuclei/second



- Fast exotic beams allow for
  - thick secondary targets
  - event-by-event identification
  - clean product selection
  - nevertheless …..

- $N_R = s \times N_T \times N_B$
  - s cross section
  - N_T atoms in target
  - N_B beam rate
  - N_R reaction rate

- Example
  - s = 100 millibarn
  - $N_T = 10^{21}$
  - $N_B = 3$ Hz
  - $N_R = 26$/day
    $= 3 \times 10^{-4}$ Hz

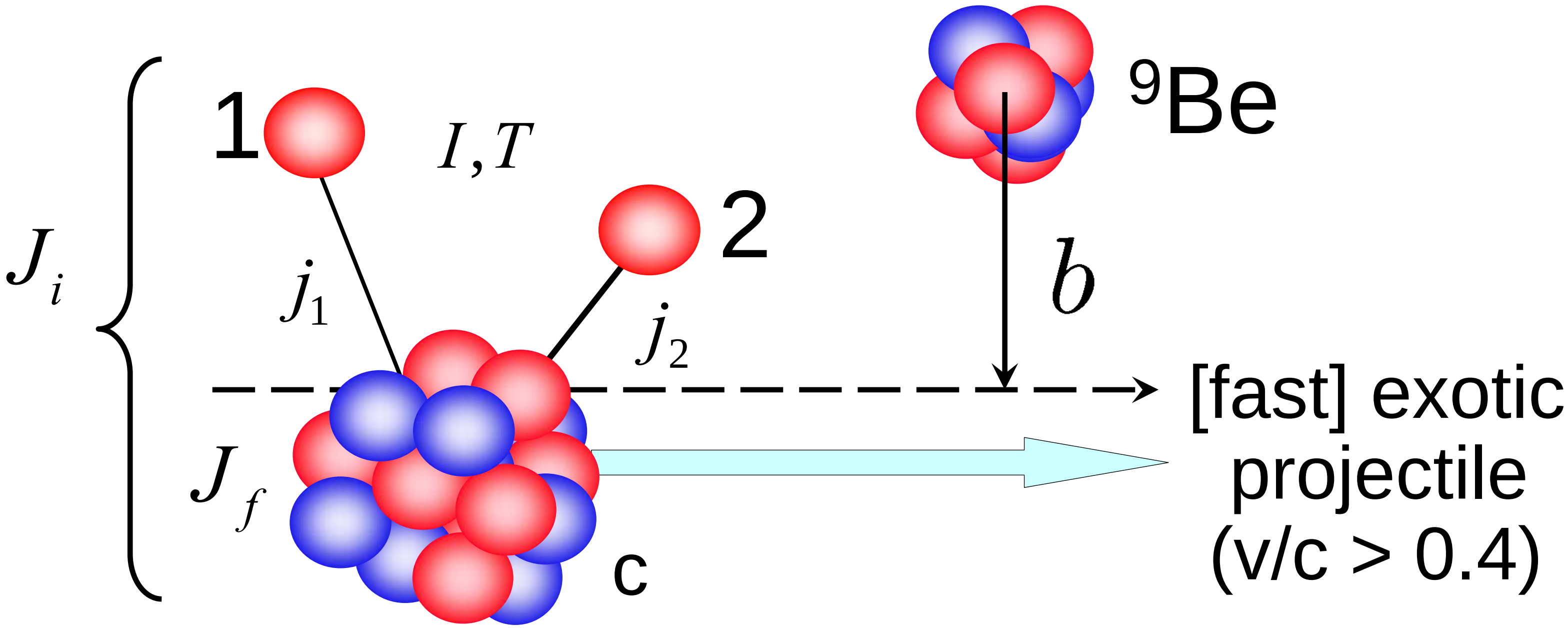
From Thomas Glasmacher

So, learned so far

1: Must rethink 'textbook' nuclear shell structure when treating asymmetric N:Z systems – see breakdown of N=8, 20, (28 …?) shell gaps.
2: Asymmetry allows the observation and study of novel NN interaction effects – and new structure
3: An effective means of production is using high energy fragmentation – that produce the exotic nuclei as fast secondary beams: 100 MeV/u
4. Experiments for the rarest cases, with intensities of a few particles/s are hard – reactions with high cross sections/efficient detection are essential
5: Structure calculations for A>12 are hard, they use effective interactions that need to be tested

Probing single particle (shell model) states

One such experimental option is one or two-nucleon removal – at ~100 MeV/nucleon



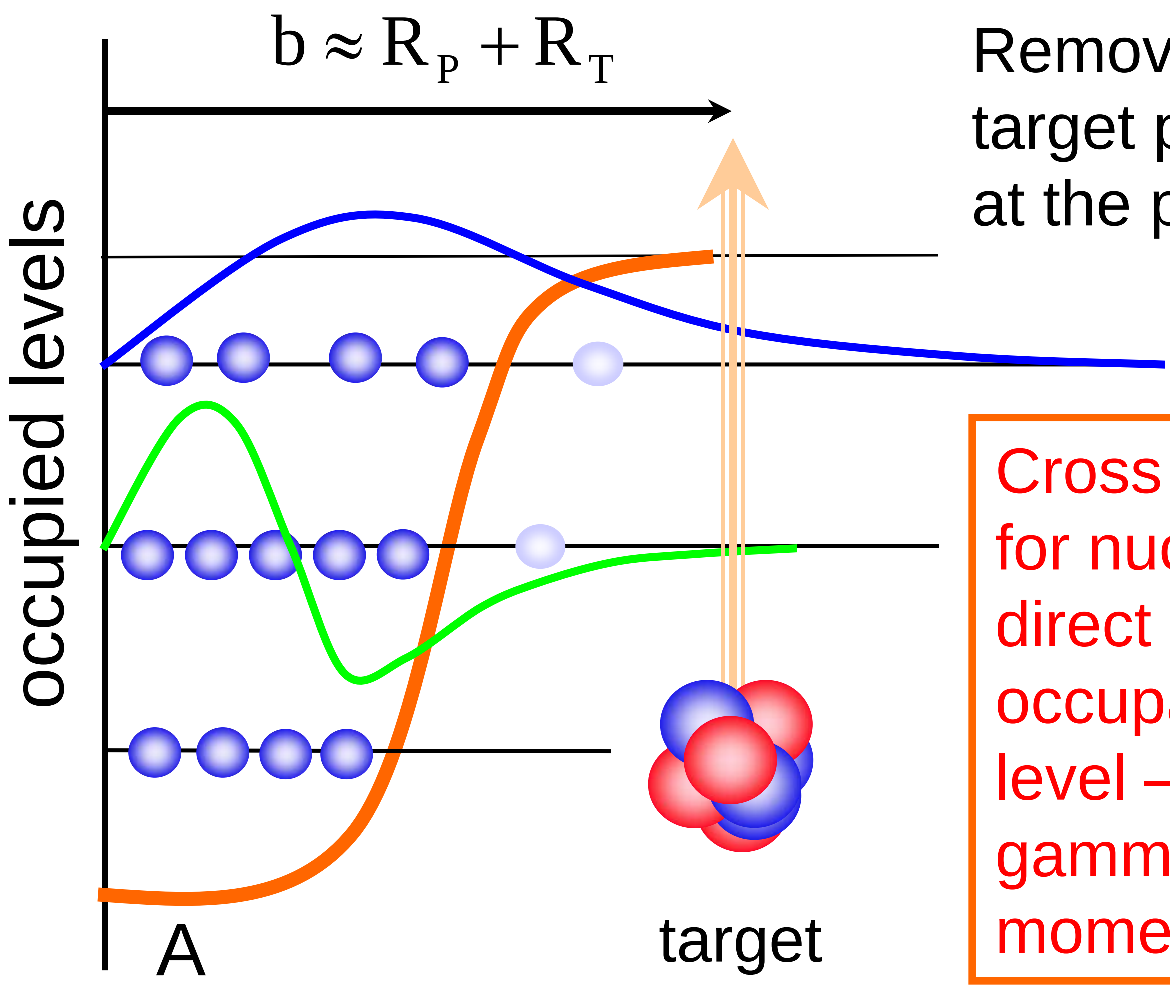
Experiments do not measure target final states. Final state of core c measured – using decay gamma rays.

How can we describe and what can we learn from these?

P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Viewed from the rest frame of the projectile

Projectile-target interaction absorptive when come close



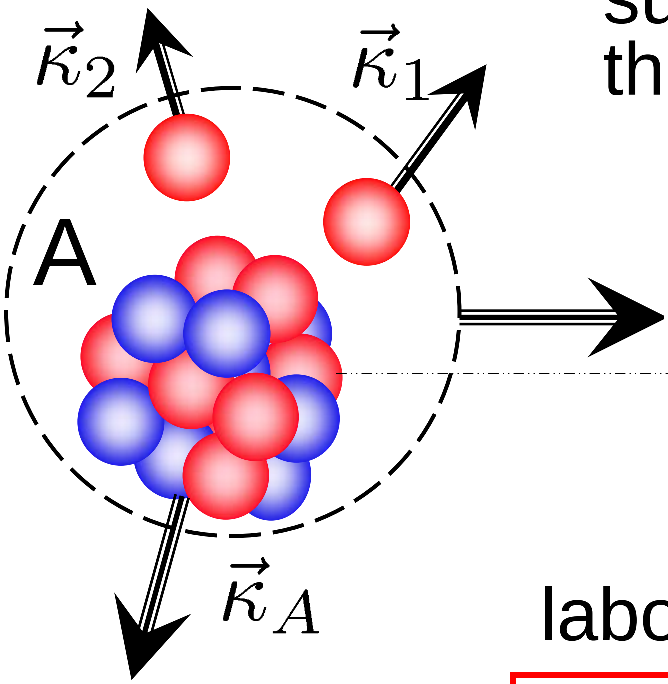
Removal of nucleons by the target probes wave functions at the projectile's surface

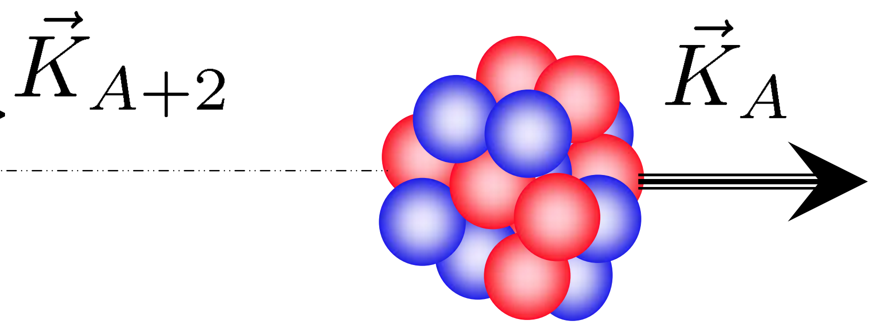
$b \approx R_P + R_T$

Cross section (probability) for nucleon removal is a direct measure of the occupation of a given level – which level, from gamma ray energy and momentum of the residue

Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame





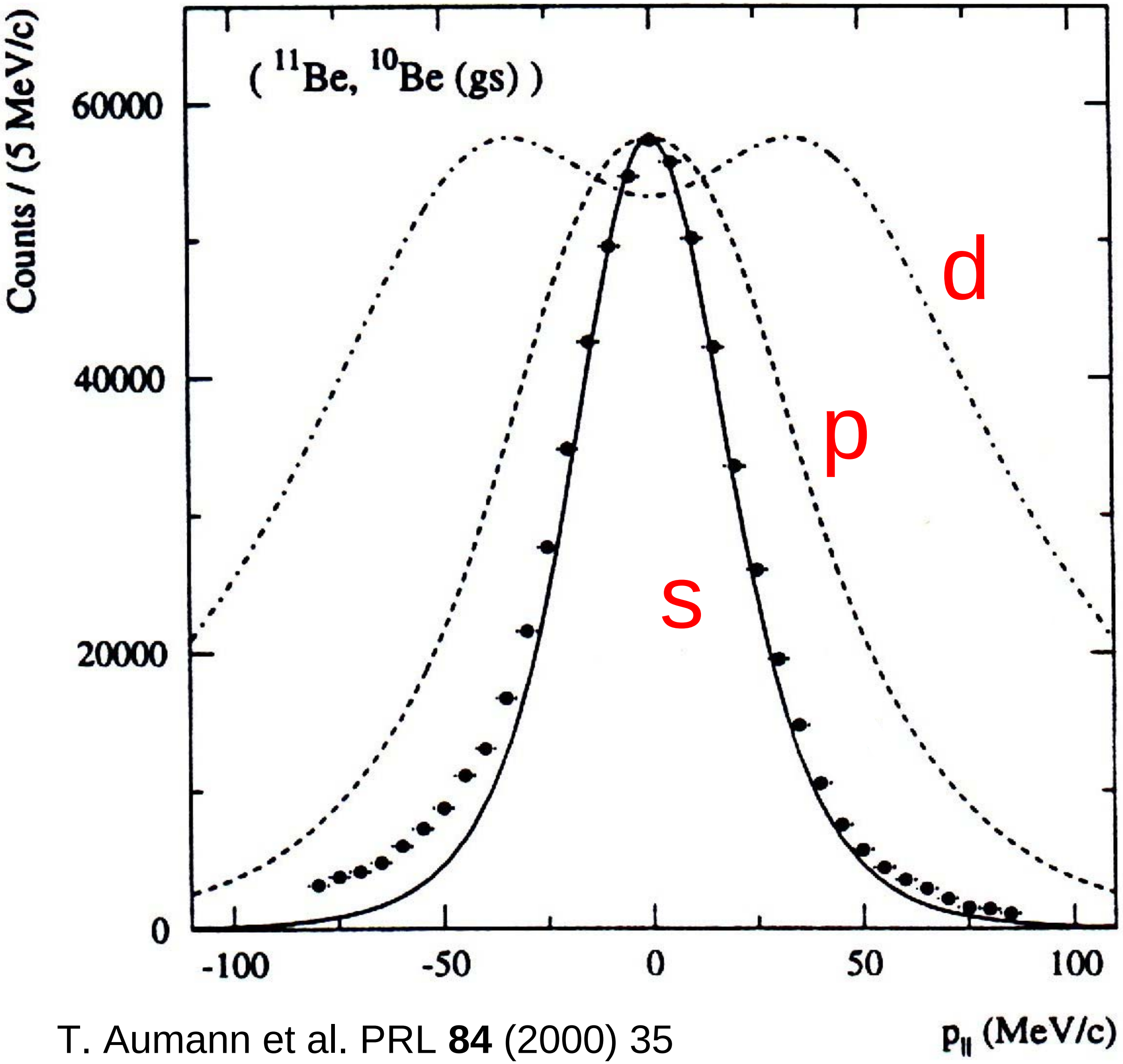
laboratory frame $\vec{K}_A$ and $\vec{K}_{A+2}$

Projectile rest frame

$$\vec{K}_A = \frac{A}{A+2}\vec{K}_{A+2} - [\vec{\kappa}_1 + \vec{\kappa}_2]$$

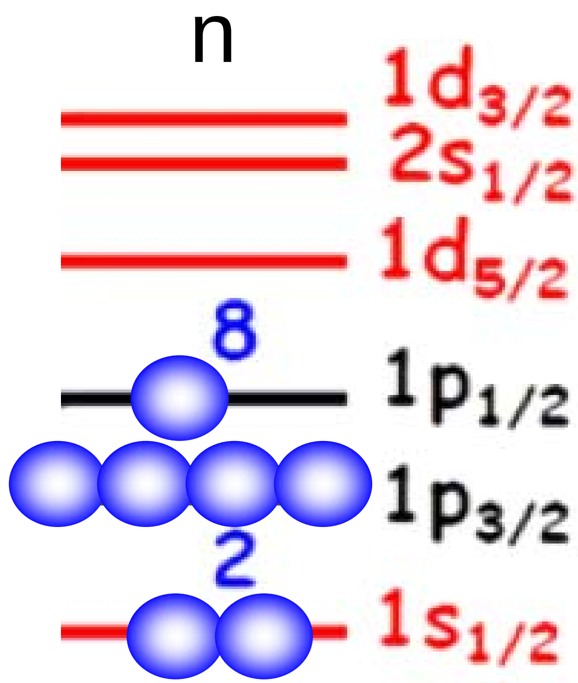
and component equations

Residue momentum $^{11}Be \rightarrow {}^{10}Be$ – halo case



T. Aumann et al. PRL **84** (2000) 35

Z = 4, N = 7



^{11}Be

Need description of high-energy nuclear collisions

Need composite particle (nucleons and core), but proceed in steps:

1. Point particle scattering – summary only
   - S-matrix and the eikonal (high energy, forward-focussed reaction) approximation
2. Composite few-particle system
   - adiabatic/sudden approximation (reaction fast)
   - adiabatic plus eikonal scattering solution
3. Equations for one and two-nucleon removal cross sections and other observables
4. Link of reactions & many-body nuclear structure calculations (and assumptions made/needed)

Point particles: phase shift and partial wave S-matrix

Scattering states

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

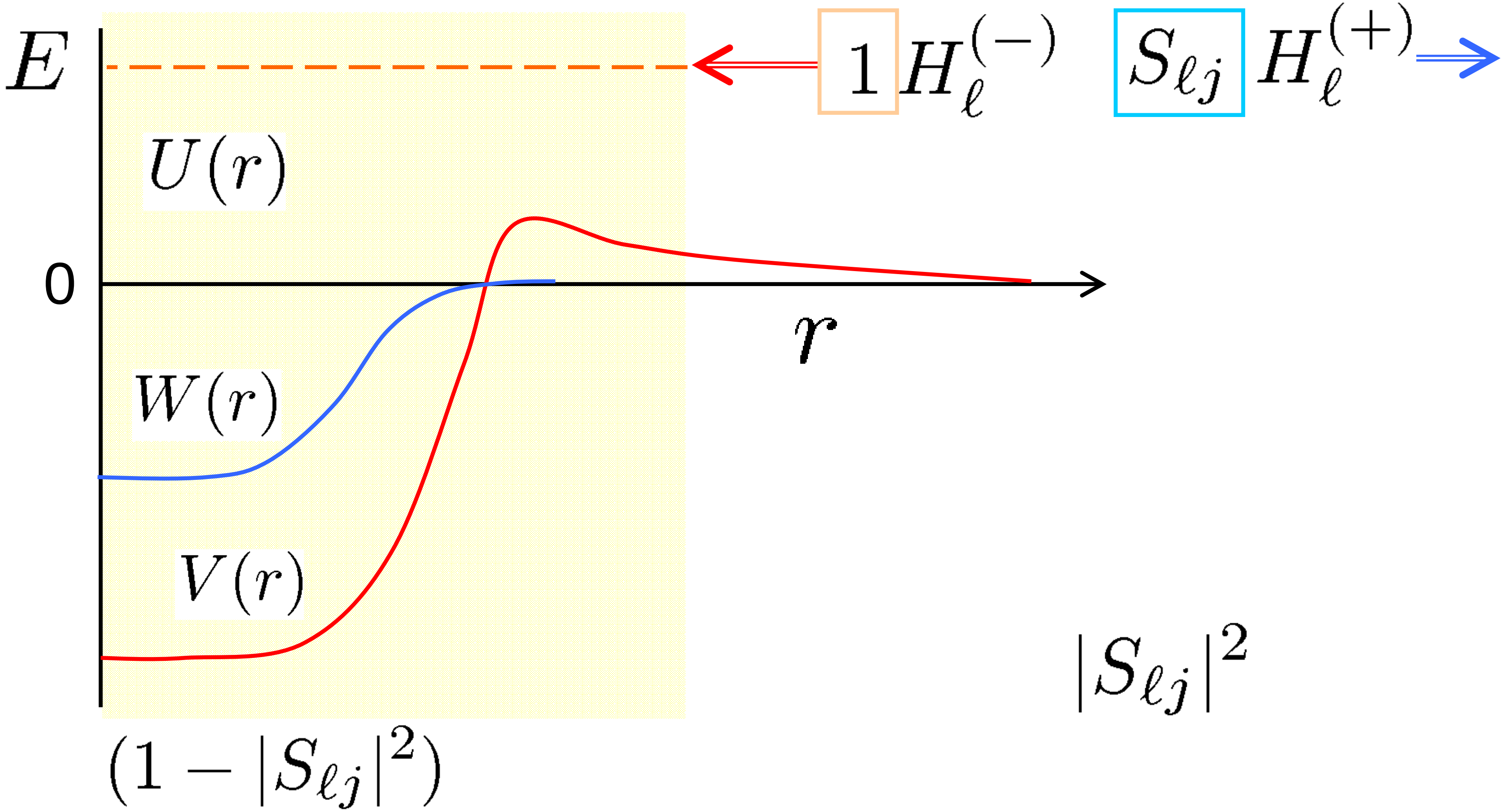
$F_\ell(\eta, kr),\ G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\begin{aligned} u_{k\ell j}(r) \quad &\rightarrow \quad e^{i\delta_{\ell j}}[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)] \\ &\rightarrow \quad (i/2)[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)] \end{aligned}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm iF_\ell(\eta, kr)$$

S-matrix - ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[1\, H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$



Eikonal approximation: for point particles (1)

Approximate (semi-classical) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0, \qquad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2}U(r) + k^2\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0$$

valid when $|U|/E \ll 1, \quad ka \gg 1$ → small wavelength, high energy

Key steps are: (1) the distorted wave function is written

$$\chi_{\vec{k}}^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r})$$

← all effects due to *U(r)*, modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

Eikonal approximation: point neutral particles (2)

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

The conditions $|U|/E \ll 1, \quad ka \gg 1$ → imply that

$$2\vec{k}\cdot\nabla\omega(\vec{r}) \gg \nabla^2\omega(\vec{r})$$

Slow spatial variation cf. k

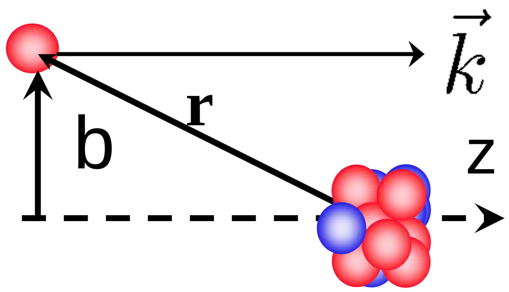
and choosing the z-axis in the beam direction $\vec{k}$

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r})$$

with solution

phase that develops with z

$$\omega(\vec{r}) = \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point neutral particles (3)

$$\chi^{+}_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r}) \approx \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$

So, after the interaction and as $z \rightarrow \infty$

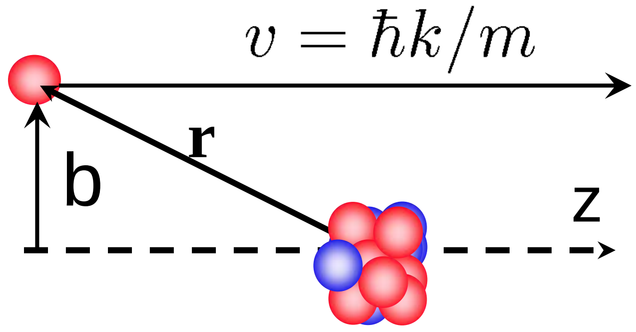
$$\chi^{+}_{\vec{k}}(\vec{r}) \rightarrow \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{\infty} U(r)dz'\right] = S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\chi^{+}_{\vec{k}}(\vec{r}) \rightarrow S(b)\exp(i\vec{k}\cdot\vec{r})$$

S(b) is amplitude of the forward going outgoing waves from the scattering at impact parameter b

Eikonal approximation to the S-matrix S(b)

$$S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]$$

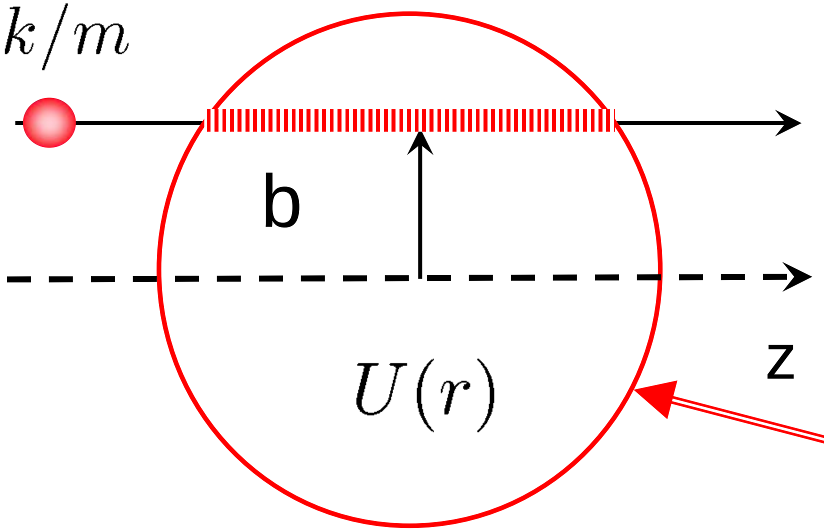


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal approximation: point particles - summary

$$\chi^+_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$

$$v = \hbar k/m$$



$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U(r)dz$$

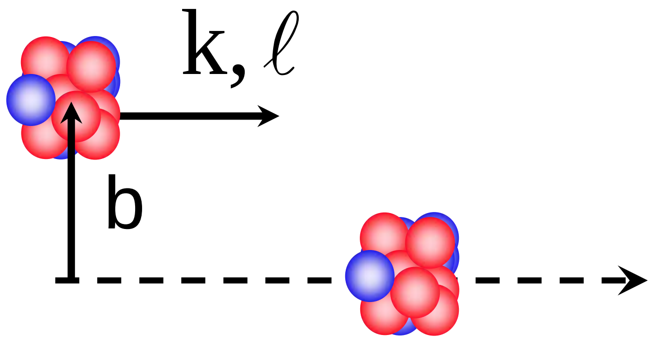
limit of range of finite ranged potential

$$\chi^+_{\vec{k}}(\vec{r}) \longrightarrow S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]$$

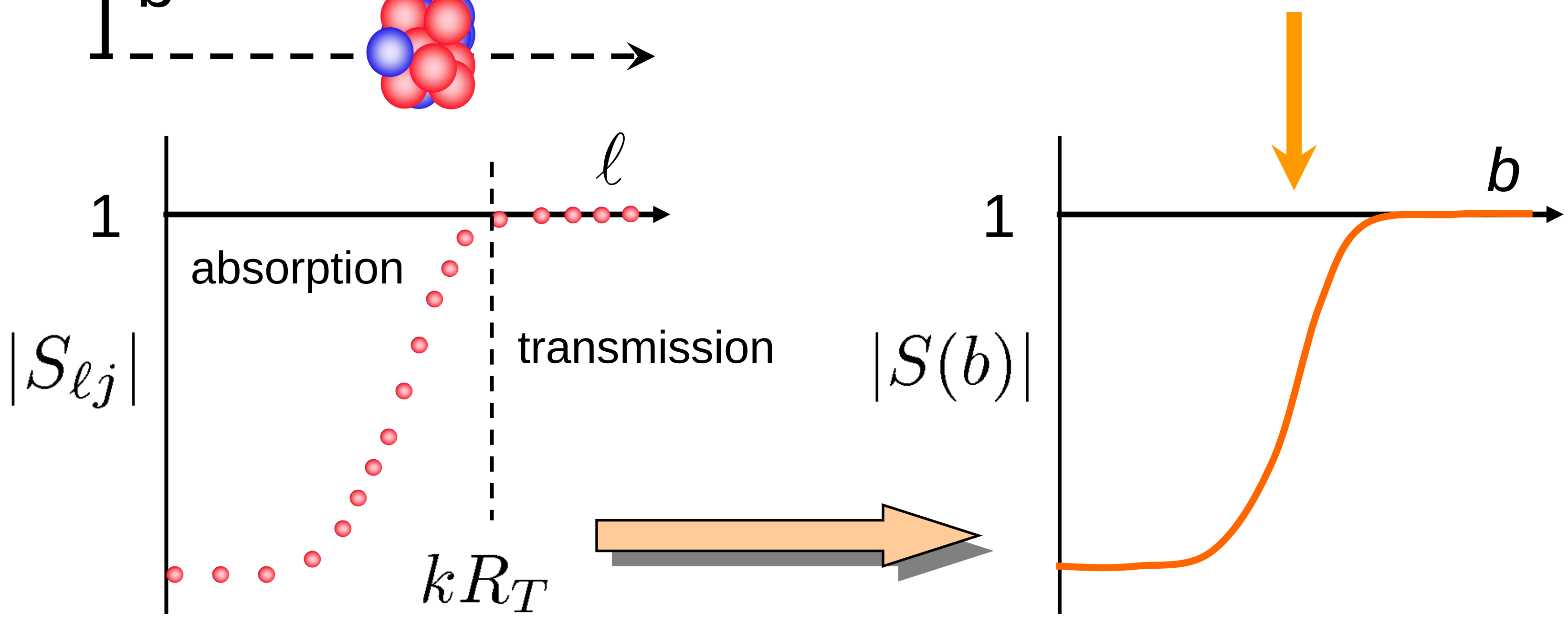
Semi-classical models for the S-matrix - S(b)

b=impact parameter



for high energy/or large mass, semi-classical ideas are good

$kb \cong \ell$, actually $\Rightarrow \ell+1/2$



$$u_{k\ell j}(r) \to (i/2)[H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}] \qquad S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]$$

Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the eikonal (impact parameter) representation, for example (spinless case):

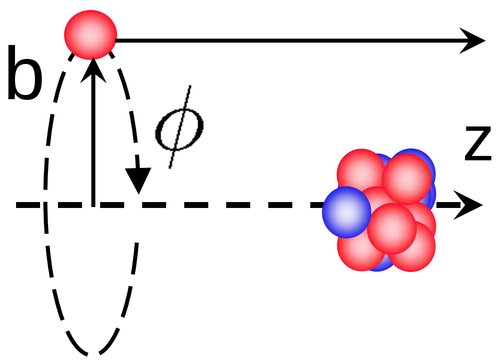
$$\sigma_{el} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\; |1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\; (1-|S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el} + \sigma_R = 2\int d^2\vec{b}\; [1-\mathrm{Re}.S(b)]$$ etc.

and where (cylindrical coordinates)

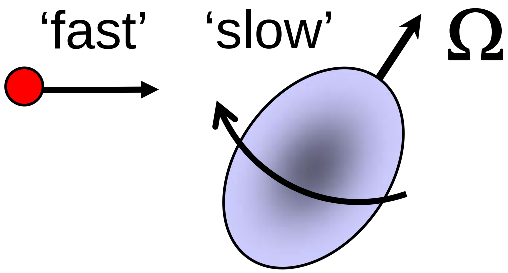
$$\int d^2\vec{b} \equiv \int_0^{\infty} bdb \int_0^{2\pi} d\phi = 2\pi \int_0^{\infty} bdb$$

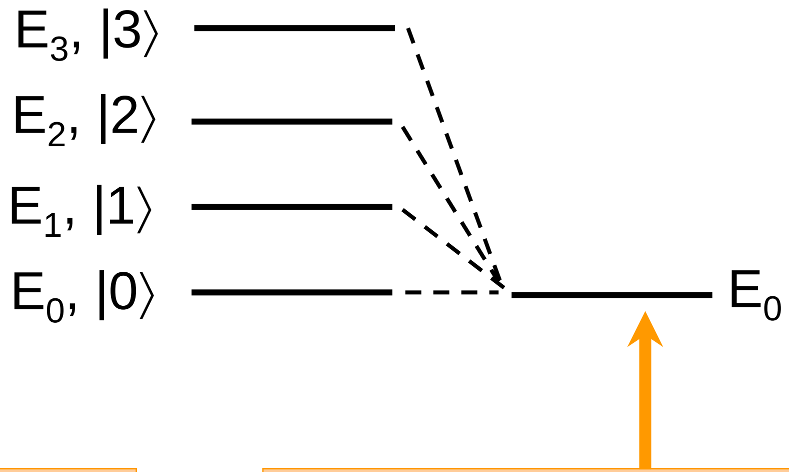


Adiabatic (sudden) approximations in physics

Identify high energy/fast and low energy/slow degrees of freedom

Fast neutron scattering from a rotational nucleus



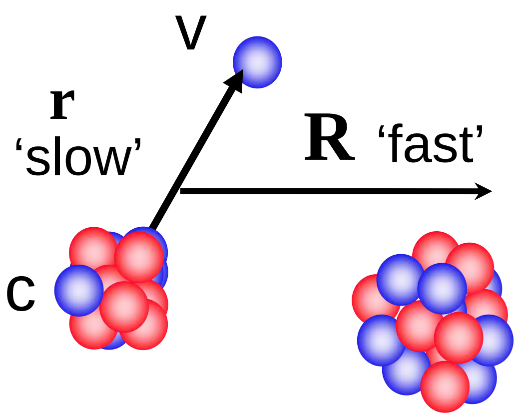


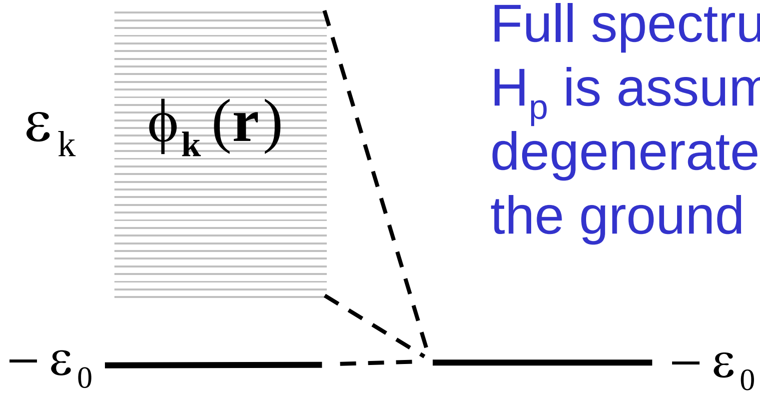
Fix Ω, calculate scattering amplitude $f(\theta, \Omega)$ for each (fixed) Ω.

moment of inertia $\to \infty$ and rotational spectrum is assumed degenerate

Transition amplitudes $f_{\alpha\beta}(\theta) = \langle \beta \,|\, f(\theta, \Omega) \,|\, \alpha \rangle_\Omega$

Few-body projectiles – the adiabatic model





Full spectrum of H_p is assumed degenerate with the ground state

Freeze internal co-ordinate $\mathbf{r}$ then scatter c+v from target and compute f(θ,$\mathbf{r}$) for all required fixed values of $\mathbf{r}$

Physical amplitude for breakup to state $\phi_{\mathbf{k}}(\mathbf{r})$ is then,

$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_{\mathbf{r}}$$

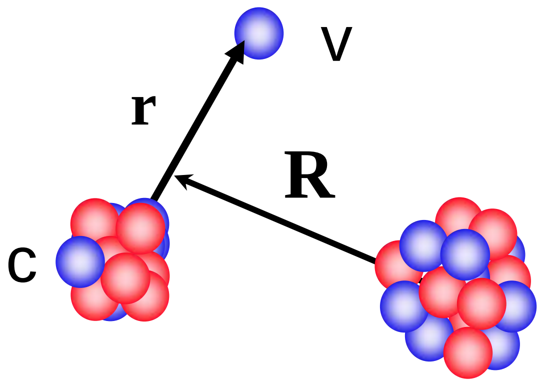
Achieved by replacing $H_p \rightarrow -\varepsilon_0$ in Schrödinger equation

Adiabatic approximation - time perspective

The time-dependent equation is

$$H\Psi(\mathbf{r},\mathbf{R},t) = i\hbar\frac{\partial\Psi}{\partial t}$$

and can be written



$$\Psi(\mathbf{r},\mathbf{R},t) = \Lambda\,\Phi(\mathbf{r}(t),\mathbf{R}), \quad \mathbf{r}(t) = \Lambda^{+}\,\mathbf{r}\,\Lambda$$

$\Lambda = \exp\{-i(H_p+\varepsilon_0)t/\hbar\}$ and where

$$[T_R + U(\mathbf{r}(t),\mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t),\mathbf{R}) = i\hbar\frac{\partial\Phi}{\partial t}$$

Adiabatic step assumes

$\mathbf{r}(t) \approx \mathbf{r}(0)=\mathbf{r}$=fixed or $\Lambda=1$ for the collision time t_{coll}

Adiabatic equation

$$[T_R + U(\mathbf{r},\mathbf{R})]\Phi(\mathbf{r},\mathbf{R}) = (E+\varepsilon_0)\Phi(\mathbf{r},\mathbf{R})$$

requires

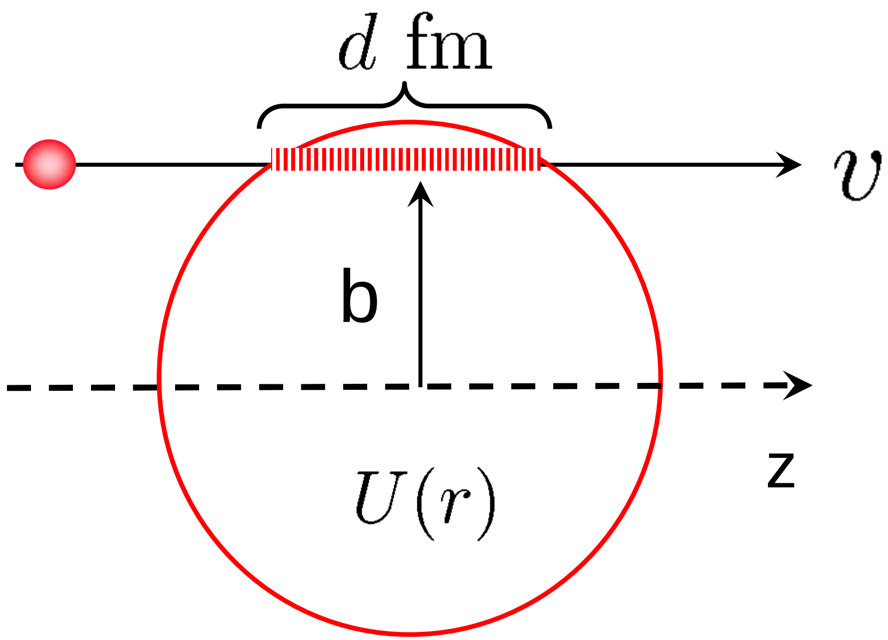
$$(H_p+\varepsilon_0)t_{coll}/\hbar \ll 1$$

Reaction timescales – surface grazing collisions

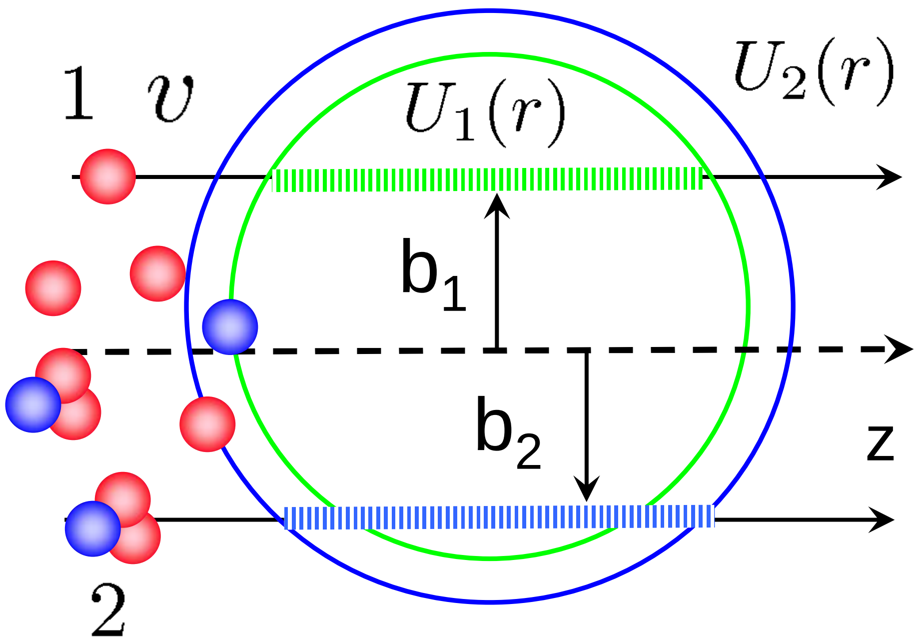
For 100 and 250 MeV/u incident energy:

$$\gamma = 1.1, \quad v/c = 0.42, \qquad \gamma = 1.25, \quad v/c = 0.6,$$

$$\Delta t = 7.9 \times d \times 10^{-24} s, \qquad \Delta t = 5.6 \times d \times 10^{-24} s$$



Adiabatic approximation: composite projectile



$$\chi_i(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_i(r)dz$$

Total interaction energy

$$U(r_1,\ldots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp\left[i\chi_i(b_i)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_i(r_i)dz'\right]$$

$$\chi(b_1,\ldots) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}\sum_i U_i(r_i)dz$$

with composite systems: get products of the S-matrices

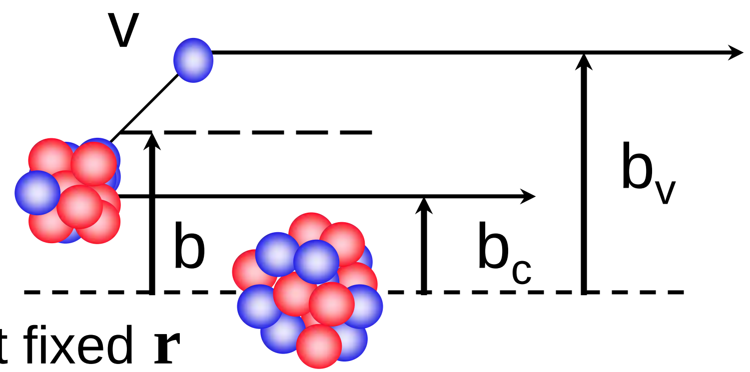
$$\exp[i\chi(b_1,\ldots)] = \prod_i S_i(b_i)$$

Few-body eikonal model amplitudes

So, after the collision, as $Z \to \infty$ $\quad \omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\, S_v(b_v)$

$$\Psi^{\text{Eik}}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) \to e^{i\mathbf{K}\cdot\mathbf{R}}\, S_c(b_c)\, S_v(b_v)\, \phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



at fixed $\mathbf{r}$
adiabatic

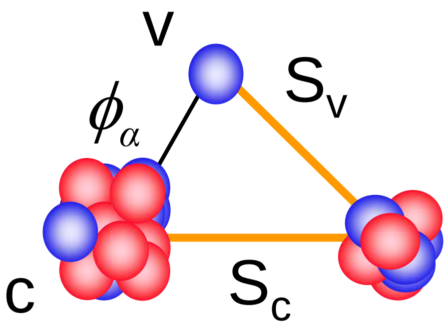
So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)}\, |\, \phi_0 \rangle_{\mathbf{r}}$$

averaged over position probabilities of c and v ← amplitude that c,v survive interaction with b_c and b_v

Eikonal theory - dynamics and structure

Independent scattering information of c and v from target



scattering

$$S_{\alpha\beta}(b) = \langle \phi_\beta \,|\, \overbrace{S_c(b_c)\, S_v(b_v)}^{\text{scattering}} \,|\, \phi_\alpha \rangle$$

structure

Use the <u>best available</u> few- or many-body wave functions

<u>More generally,</u>

$$S_{\alpha\beta}(b) = \langle \varphi_\beta \,|\, S_1(b_1)\, S_2(b_2) \ldots\ldots\, S_n(b_n) \,|\, \varphi_\alpha \rangle$$

for any choice of 1,2 ,3, ….. n clusters for which a most realistic wave function φ is available

Absorptive cross sections - target excitation

Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels → $|S(b)|^2 \leq 1$

$$\sigma_{abs} = \sigma_R - \sigma_{diff} = \int d\mathbf{b}\ \langle \phi_0 \,|\, \underbrace{1 - |S_c S_v|^2}\, |\, \phi_0 \rangle$$

$$\left\{ \begin{array}{l} |S_v|^2 (1 - |S_c|^2) + \\ |S_c|^2 (1 - |S_v|^2) + \\ (1 - |S_c|^2)(1 - |S_v|^2) \end{array} \right.$$

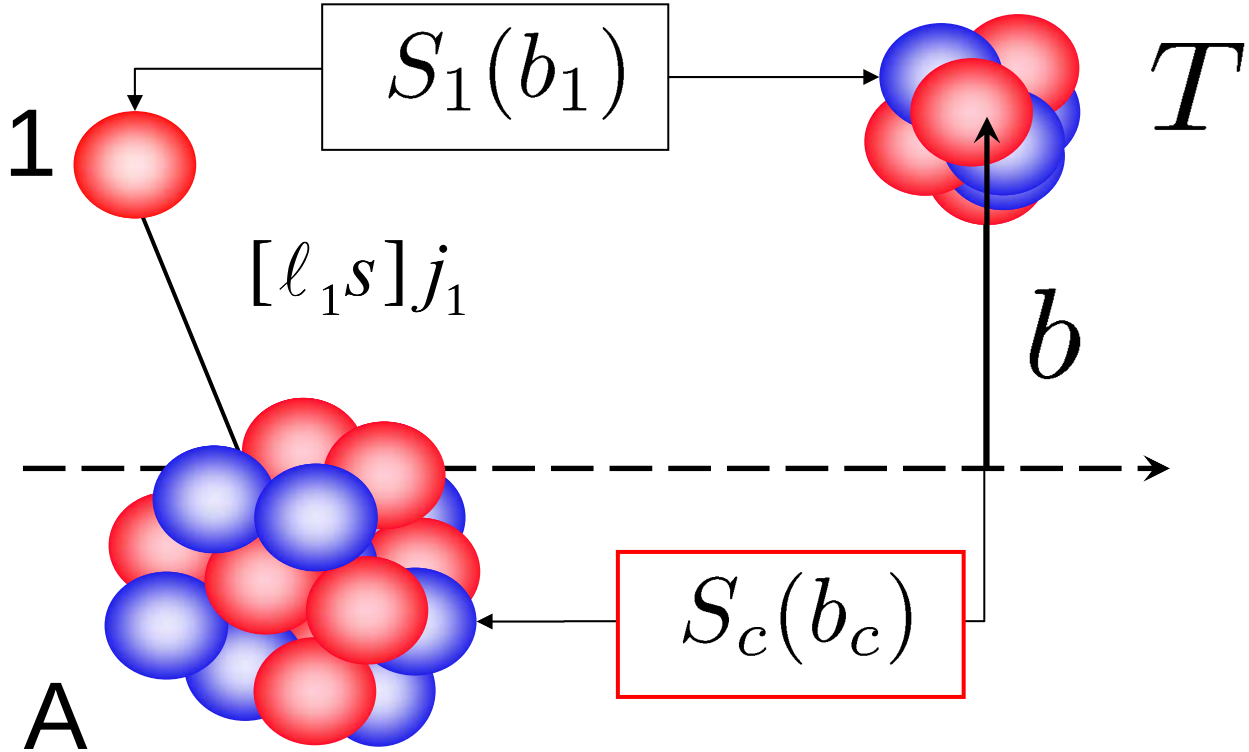
- $|S_v|^2 (1 - |S_c|^2)$: v survives, c absorbed
- $|S_c|^2 (1 - |S_v|^2)$: v absorbed, c survives
- $(1 - |S_c|^2)(1 - |S_v|^2)$: v absorbed, c absorbed

stripping of v from projectile exciting the target. c scatters at most elastically with the target

$$\sigma_{strip} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1 - |S_v|^2) \,|\, \phi_0 \rangle$$

Related equations exist for the differential cross sections, etc.

Stripping of a nucleon – nucleon 'absorbed'



$$\sigma_{\text{strip}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_C|^2\,(1 - |S_1|^2)\,|\,\phi_0 \rangle$$

And for two-nucleon removal

$$\sigma_{abs} \quad \rightarrow \quad 1 - |S_c|^2 |S_1|^2 |S_2|^2$$

$$\begin{aligned} 1 \quad &= \quad [\, \boxed{|S_c|^2} + \cancel{(1 - |S_c|^2)}\,] \\ &\times \quad [\, |S_1|^2 + (1 - |S_1|^2)\,] \\ &\times \quad [\, |S_2|^2 + (1 - |S_2|^2)\,] \end{aligned}$$

core survival and nucleon “ removal ”

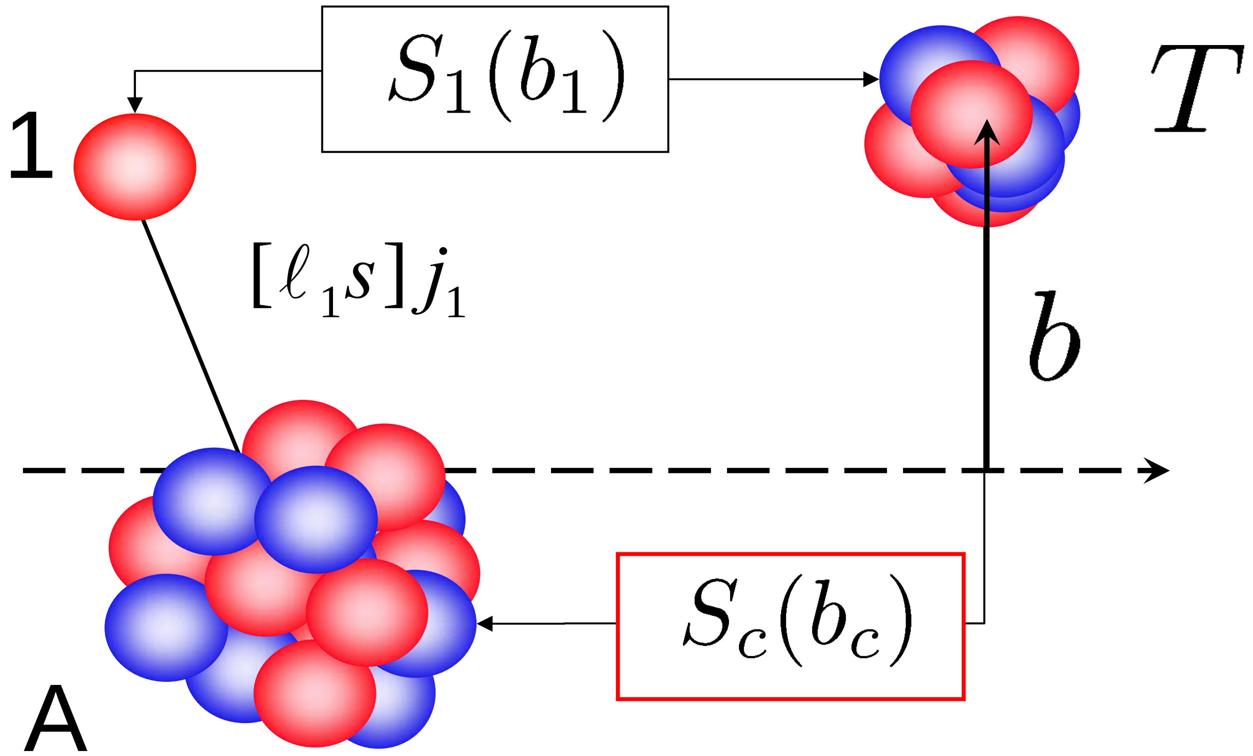
$$\begin{aligned} \sigma_{abs}^{\mathrm{KO}} \quad &\rightarrow \quad \boxed{|S_c|^2} (1 - |S_1|^2)(1 - |S_2|^2) \\ &+ \quad \boxed{|S_c|^2} |S_1|^2 (1 - |S_2|^2) \\ &+ \quad \boxed{|S_c|^2} (1 - |S_1|^2) |S_2|^2 \end{aligned}$$

2N stripping

1N stripped

1N diffracted

Diffractive (breakup) removal of a nucleon



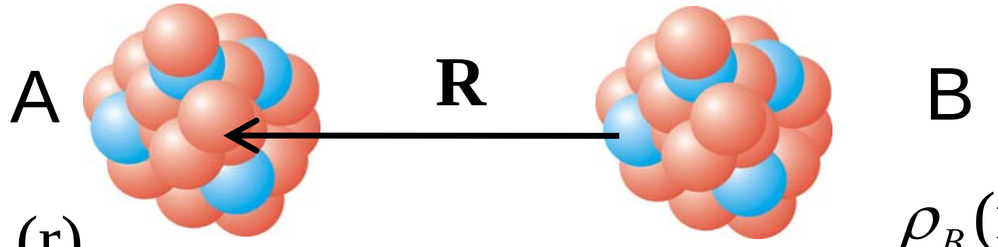
$$\sigma_{\text{diff}} = \int \mathbf{db} \left\{ \langle \phi_0 \,||\, \mathrm{S_c}\, \mathrm{S_v}\,|^2 \,|\, \phi_0 \rangle - |\langle \phi_0 \,|\, \mathrm{S_c}\, \mathrm{S_v} \,|\, \phi_0 \rangle|^2 \right\}$$

Core-target effective interactions – for $S_c(b_c)$

Double folding

$$U_{AB}(\mathrm{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(\mathrm{r}_1)\, \rho_B(\mathrm{r}_2)\, \mathrm{t}_{\mathrm{NN}}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

$$U_{AB}$$



$\rho_A(\mathrm{r})$ $\rho_B(\mathrm{r})$

At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2} \sigma_{NN}(i + \alpha_{NN}) \right] f(r), \quad \int d\vec{r} f(r) = 1$$

e.g. $f(r) = \delta(r)$

nucleon-nucleon cross section

$$f(r) = (\sqrt{\pi} t)^{-3} \exp(-r^2/t^2)$$

resulting in a COMPLEX nucleus-nucleus potential

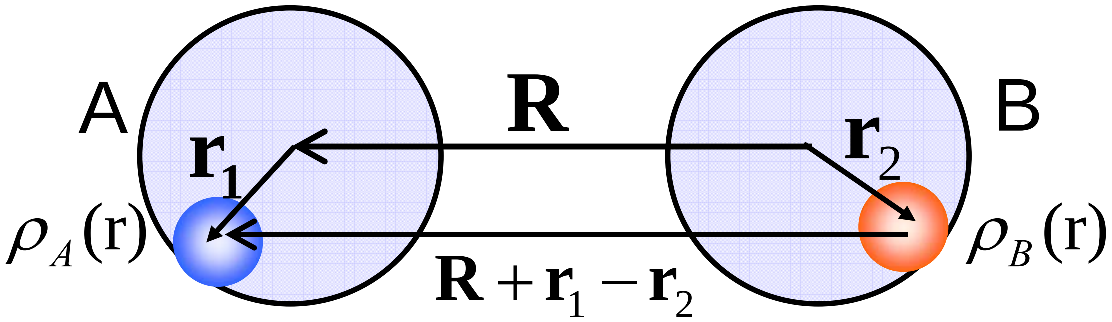
M.E. Brandan and G.R. Satchler, Phys. Rep. **285** (1997) 143-243.

Effective interactions – Folding models

Double folding

$$U_{AB}(\mathrm{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(\mathrm{r}_1)\, \rho_B(\mathrm{r}_2)\, \mathrm{v}_{\mathrm{NN}}(\mathbf{R}+\mathbf{r}_1-\mathbf{r}_2)$$

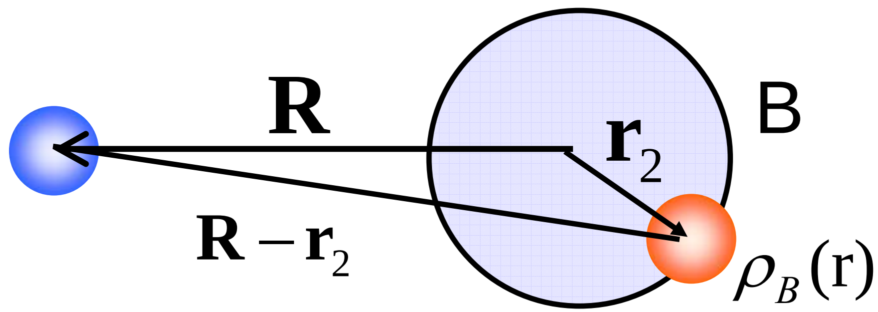
$$U_{AB}$$



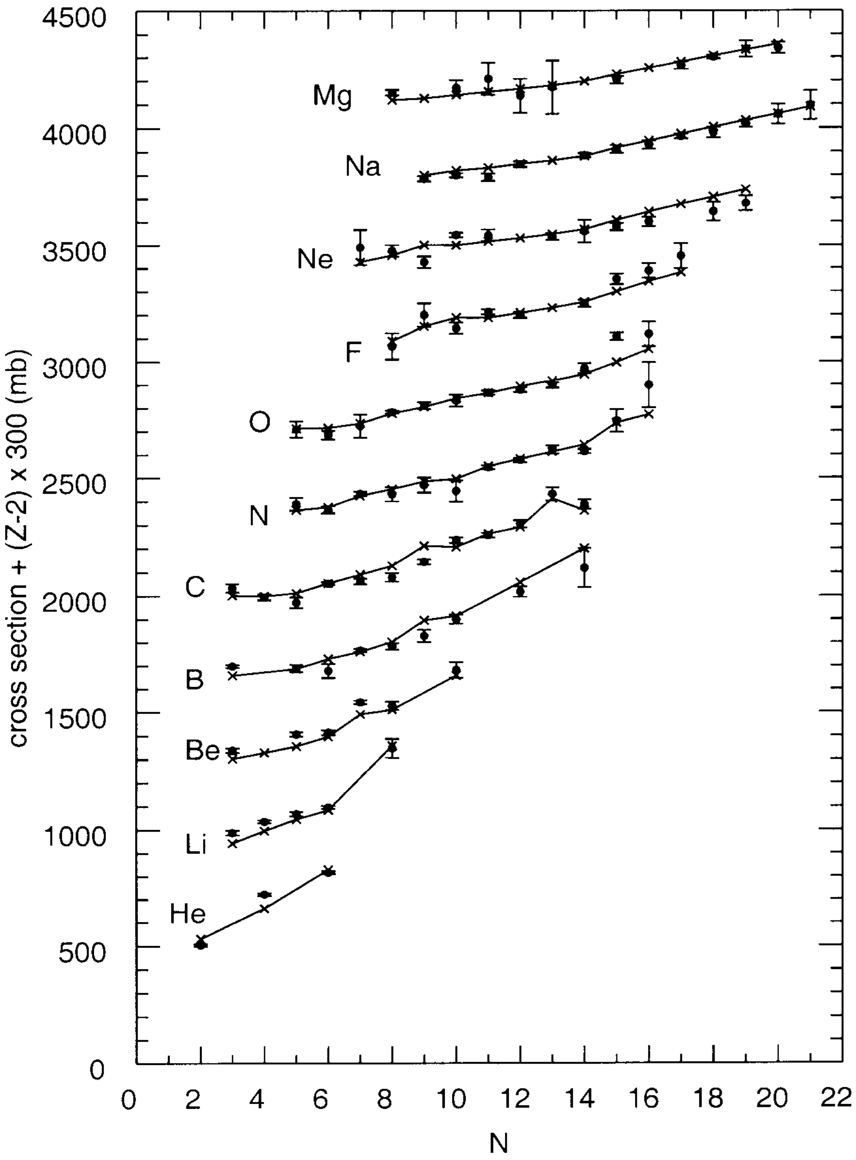
Single folding

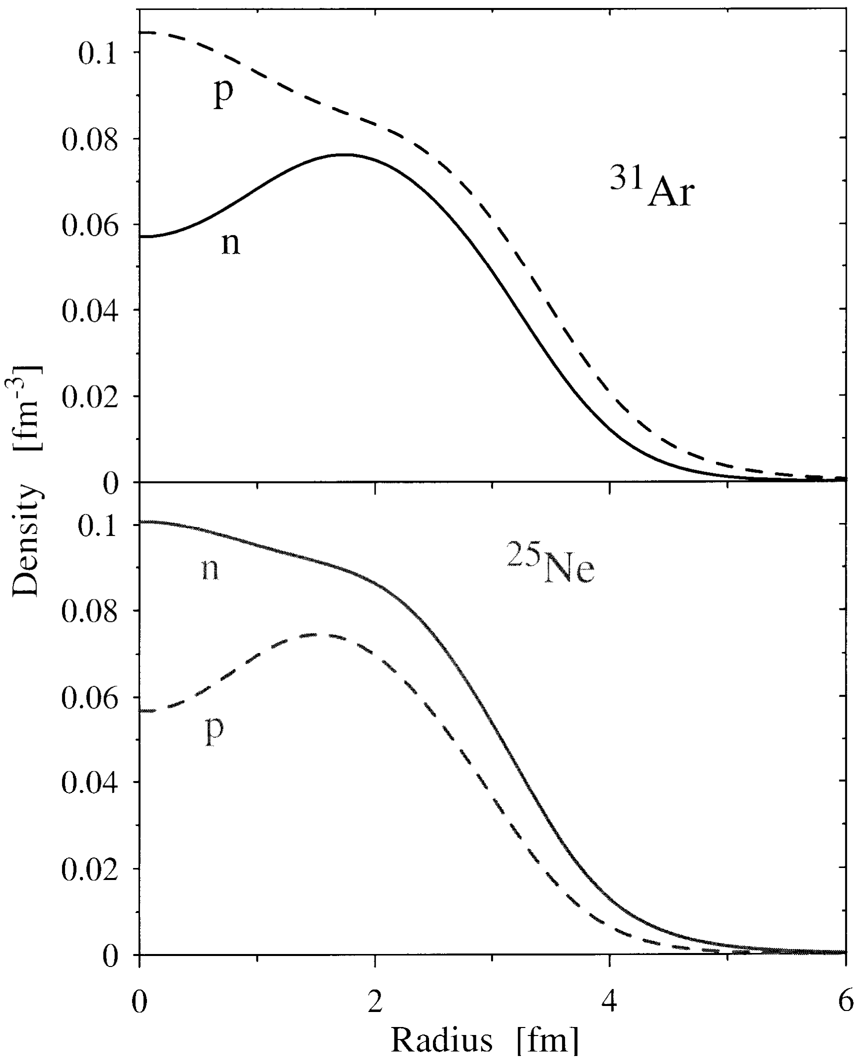
$$U_B(\mathrm{R}) = \int d\mathbf{r}_2 \; \rho_B(\mathrm{r}_2)\, \mathrm{v}_{\mathrm{NN}}(\mathbf{R}-\mathbf{r}_2)$$

$$U_B$$



Sizes - Skyrme Hartree-Fock radii and densities





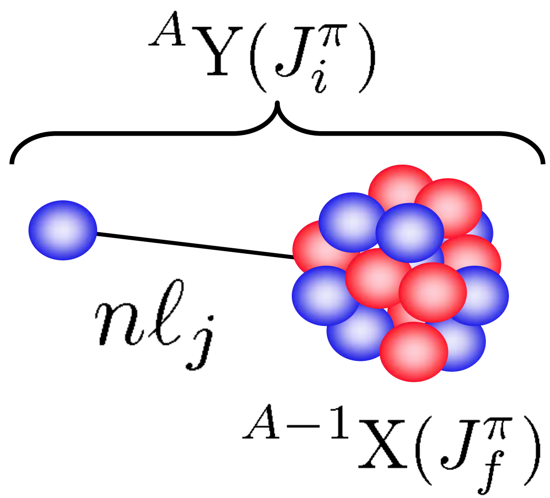
B.A. Brown, S. Typel, and W.A. Richter, Phys. Rev. **C65** (2002) 014612

Connection to many-body – spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r} Y^\lambda_\ell(\hat{r})\chi^\sigma_s,$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$



The potential model wave function approximates the overlap function of the A and A–1 body wave functions (A and A–n in the case of an n-body cluster) i.e. the overlap

$$\langle \ell j, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^{A}\mathrm{Y}(J_i^\pi)\rangle \rightarrow I_{\ell j}(r), \quad \int_0^\infty [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

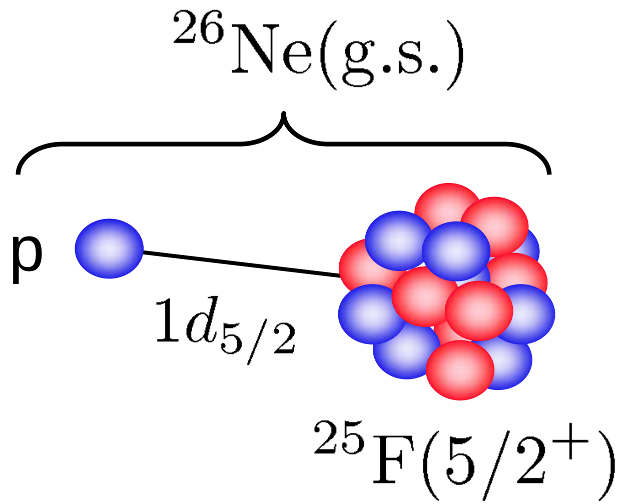
S(…) is the spectroscopic factor ← a structure calculation

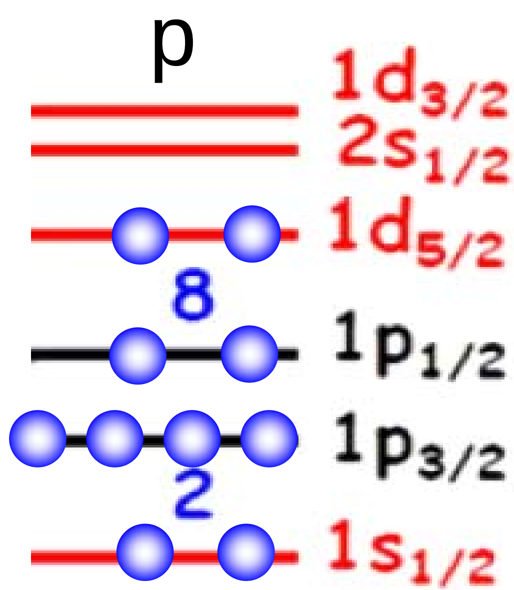
Many-body input – the shell model overlaps

$$\langle \vec{r}, {}^{25}\mathrm{Ne}(5/2^+, E^*) | {}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.}) \rangle$$

USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides spectroscopic factors but **not** the bound state radial wave function.

```
-- core state --- - overlap state -  (1 2  5) (
2j2t p    n      e  2j2t p    n      e     s
              -59.414               -81.625
 5 7 +    1    0.000 0 6 +    1     0.000 1.79039
 5 7 +    2    3.756 0 6 +    1     0.000 0.02316
 5 7 +    3    4.799 0 6 +    1     0.000 0.01084
 5 7 +    4    5.631 0 6 +    1     0.000 0.00012
 5 7 +    5    6.022 0 6 +    1     0.000 0.00589
 5 7 +    6    6.504 0 6 +    1     0.000 0.00044
 5 7 +    7    6.796 0 6 +    1     0.000 0.00002
 5 7 +    8    8.034 0 6 +    1     0.000 0.00006
 5 7 +    9    8.186 0 6 +    1     0.000 0.00097
 5 7 +   10    8.398 0 6 +    1     0.000 0.00006
total =                                   1.83196
centroid =    0.102     centroids =       0.000
centroid*= -22.313      centroids =     -22.211
```





We have the background we need to apply this in a serious context and confront experiment: tomorrow we look at a number of applications.