

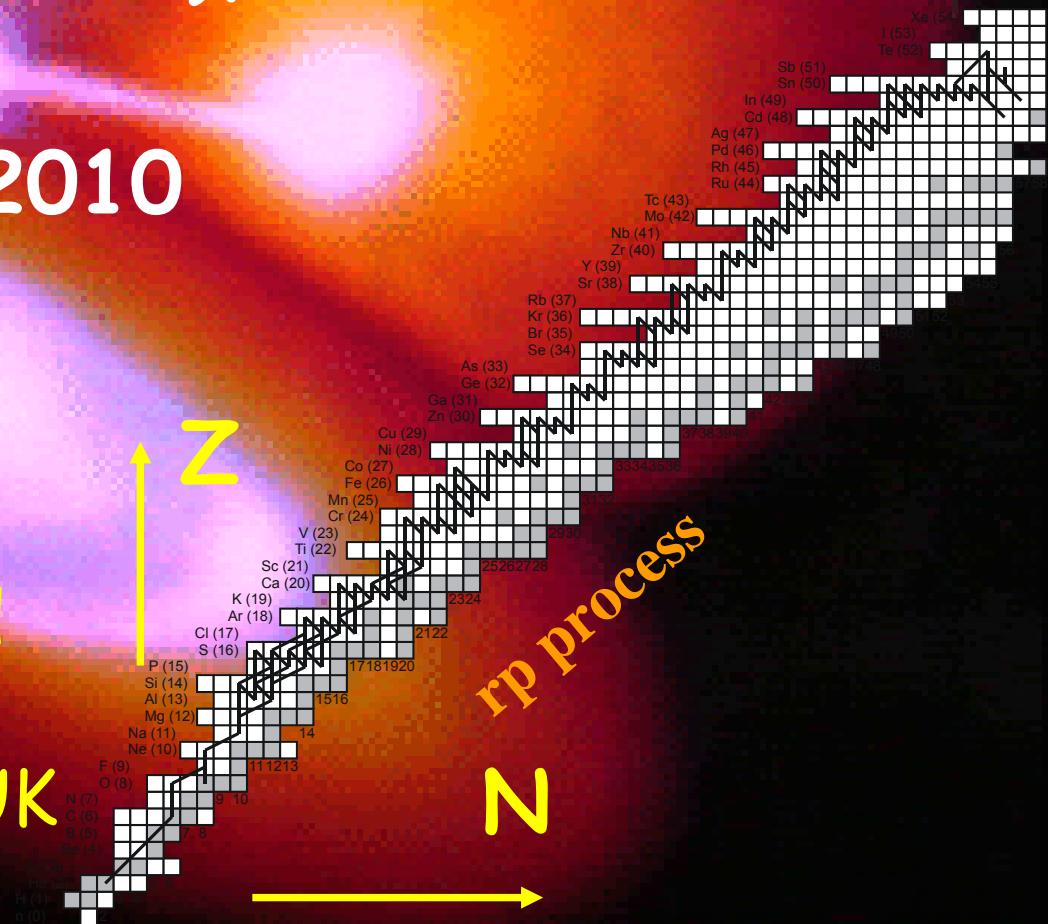
Reaction tools for the study of exotic nuclei: theory and applications - II

Hadrons and Nuclei under Extreme Conditions (HANE2010),

TIT, O-Okayama

16-17 September 2010

Jeff Tostevin, TIT and
Department of Physics
University of Surrey, UK



Lectures plan

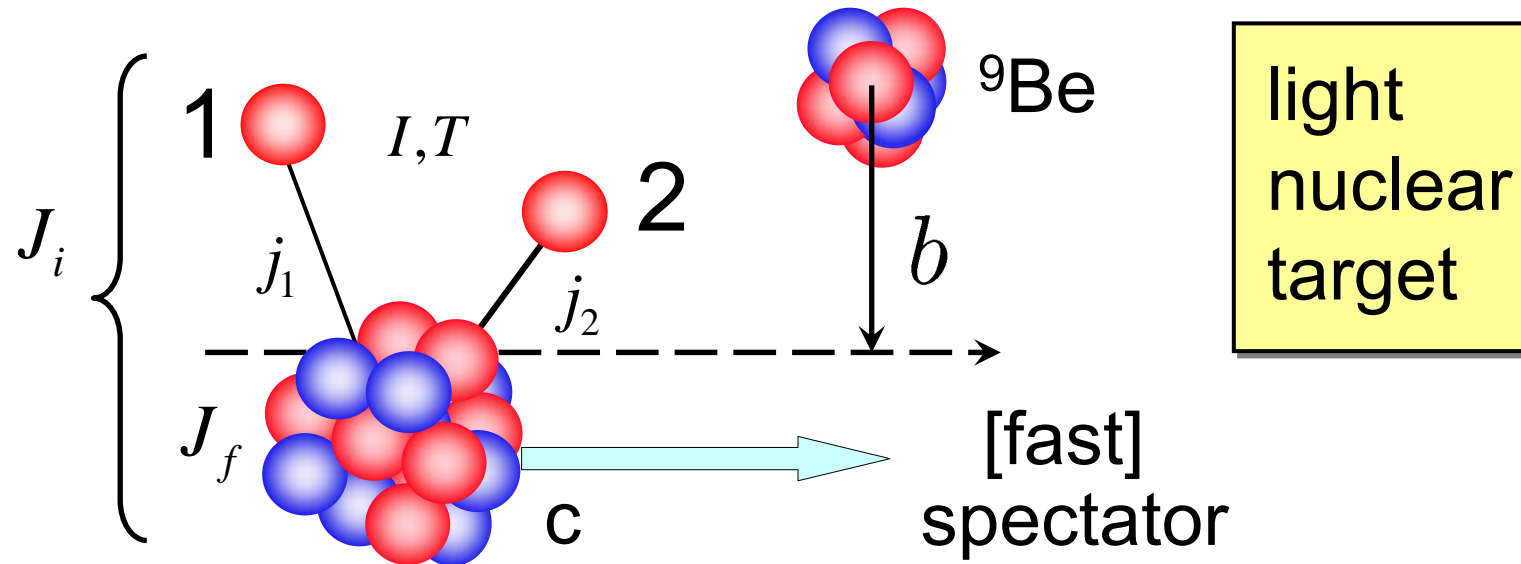
Lecture 1: Introduction: history and key ideas

- nuclear shell structure, old and new
- Reactions at high-energy (removal)
- approximations, simplifications
- connection/interface with nuclear structures
- observables and what they tell us

Lecture 2: Illustrative and topical applications

- determining exotic structures
- testing of structure models and of shell model effective interactions
- nucleon and pair 'correlations'

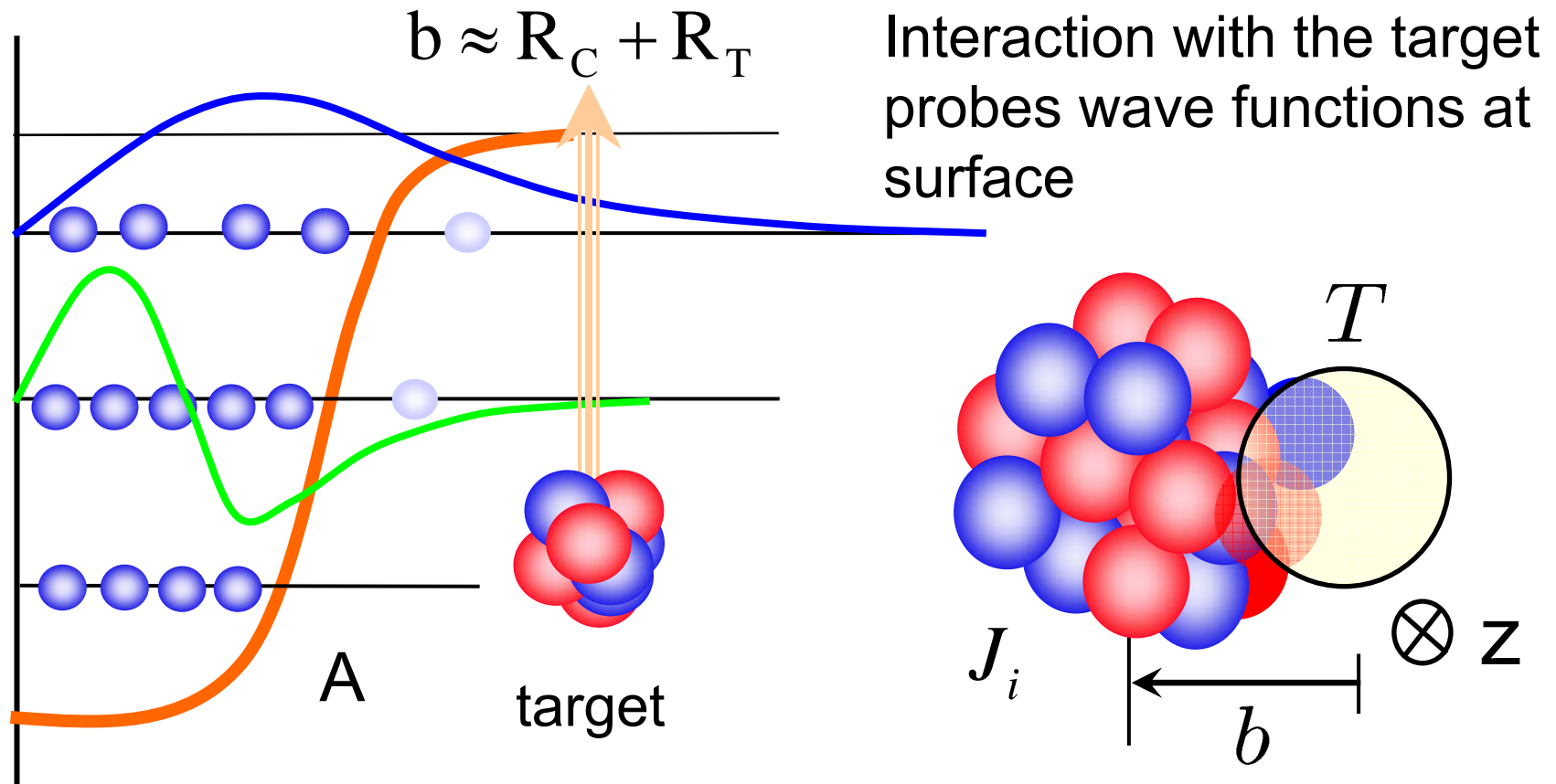
One and two nucleon knockout, >100 MeV/u



Experiments are inclusive (with respect to the target final states). Core final state measured – using gamma rays – whenever possible – and the momenta of the residues

Cross sections are large and they include both: Break-up (elastic) and stripping (inelastic/absorptive) interactions of the removed nucleon(s) with the target

Target drills a cylindrical volume at projectile surface



Residue momentum distributions after knockout

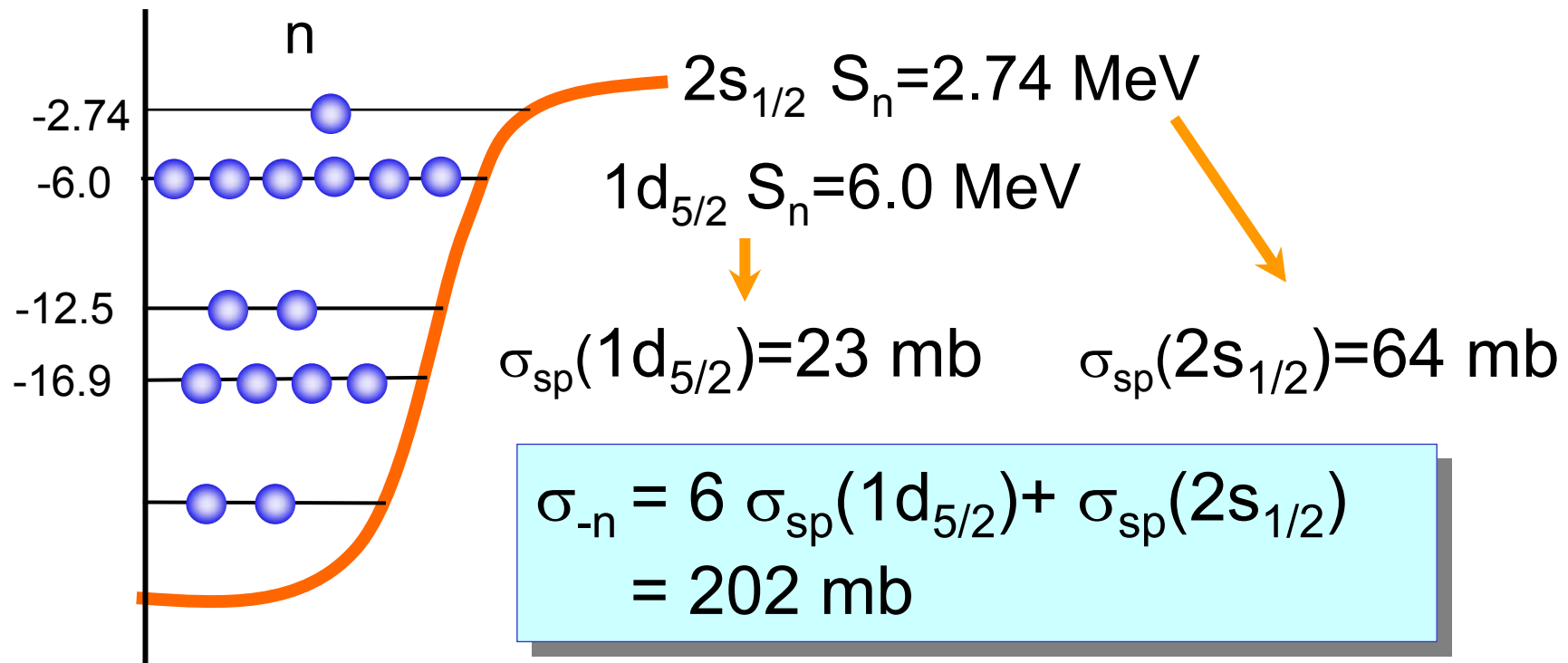
$$\begin{aligned}
 \sigma_{str} &= \frac{1}{2l+1} \sum_m \int d^2b \langle \psi_{lm} | |S_c(b_c)|^2 (1 - |S_n(b_n)|^2) | \psi_{lm} \rangle \\
 &= \frac{1}{2l+1} \sum_m \int d^2b_n (1 - |S_n(b_n)|^2) \langle \psi_{lm} | S_c^* S_c | \psi_{lm} \rangle \\
 &\quad \underbrace{\frac{1}{(2\pi)^3} \int d\vec{k}_c |\vec{k}_c\rangle \langle \vec{k}_c|}_{=1}
 \end{aligned}$$

In projectile rest frame:

$$\begin{aligned}
 \frac{d\sigma_{str}}{d^3k_c} &= \frac{1}{(2\pi)^3} \frac{1}{2l+1} \sum_m \int d^2b_n [1 - |S_n(b_n)|^2] \\
 &\quad \times \left| \int d^3r e^{-i\mathbf{k}_c \cdot \mathbf{r}} S_c(b_c) \psi_{lm}(\mathbf{r}) \right|^2
 \end{aligned}$$

Orientation - extreme sp model – inclusive sigma

Single neutron removal from $^{23}\text{O} \equiv [1d_{5/2}]^6 [2s_{1/2}]$



Measurement at RIKEN [Kanungo *et al.* PRL **88** ('02) 142502]
at 72 MeV/nucleon on a ^{12}C target; $\sigma_{-n} = 233(37)\text{mb}$

Bound states – the mean field helps our intuition

```

*****
*
* IA,IZ =    24    8 *
*
*****

INPUT VALUES

----- Neutron bound state results -----

k n l j      e  IE  OCC
1 1 s 1/2 -26.757 1  2.00  36.70  35.28
2 1 p 3/2 -16.883 1  4.00  36.70  35.80
3 1 p 1/2 -12.396 1  2.00  36.70  36.04
4 1 d 5/2 -6.166 1  6.00  36.70  36.37
5 1 d 3/2 -0.109 1  0.00  36.70  36.69
6 2 s 1/2 -3.360 1  2.00  36.70  36.52
7 1 f 7/2 -0.200 3  0.00  46.02  46.01
8 1 f 5/2 -0.200 3  0.00  60.56  60.55
9 2 p 3/2 -0.200 3  0.00  48.10  48.09

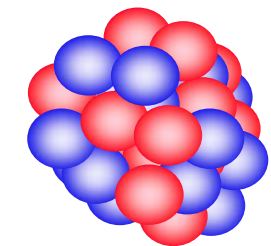
----- Neutron single-particle radii -----

```

				R(2)	R(4)	OCC	rho(8.9)	rho(9.9)	rho(10.9)
1	1	s	1/2	2.274	2.575	2.000	0.848E-09	0.706E-10	0.600E-11
2	1	p	3/2	2.863	3.133	4.000	0.188E-07	0.244E-08	0.325E-09
3	1	p	1/2	2.954	3.268	2.000	0.727E-07	0.122E-07	0.210E-08
4	1	d	5/2	3.434	3.757	6.000	0.524E-06	0.129E-06	0.327E-07
5	1	d	3/2	4.662	6.063	0.000	0.131E-04	0.675E-05	0.371E-05
6	2	s	1/2	4.172	4.895	2.000	0.769E-05	0.278E-05	0.102E-05
7	1	f	7/2	3.865	4.440	0.000	0.324E-05	0.134E-05	0.600E-06
8	1	f	5/2	3.890	4.477	0.000	0.341E-05	0.141E-05	0.631E-06
9	2	p	3/2	6.815	8.635	0.000	0.451E-04	0.270E-04	0.167E-04

The mean field – e.g. spherical HF - gives an excellent estimate and guide

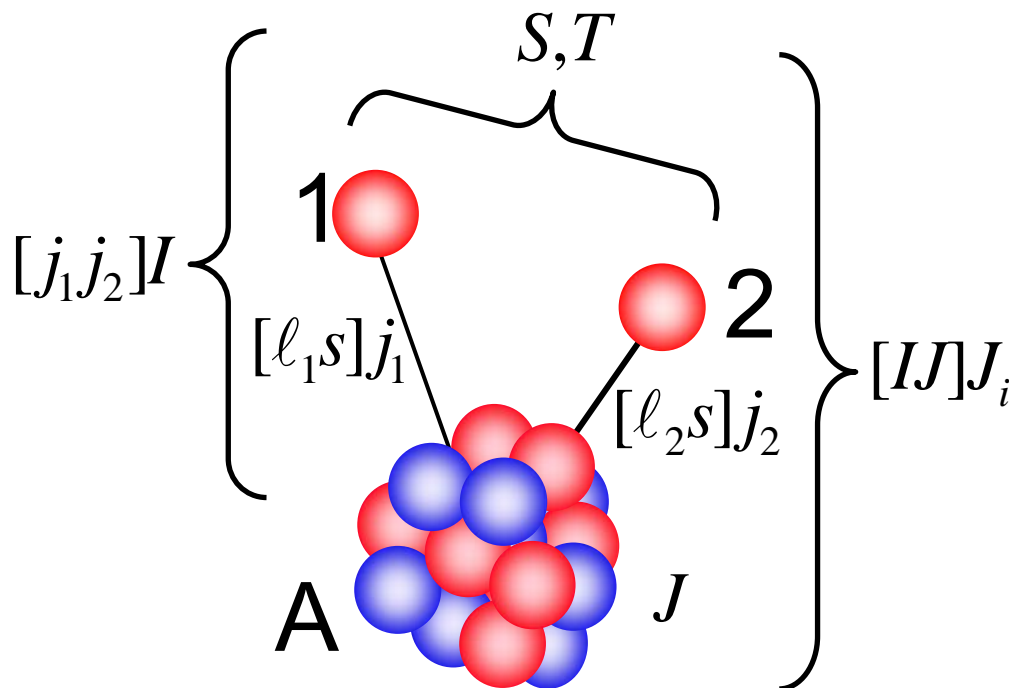
$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



$^{24}\text{O}(g.s.)$

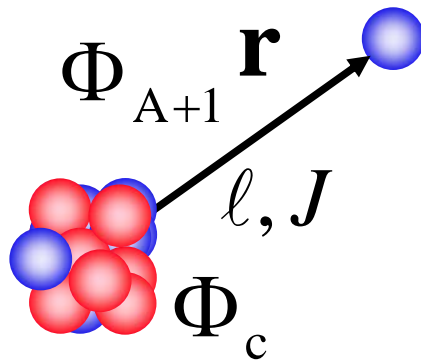
Sudden removal from the residue as a spectator

In both the one- and two-nucleon removal cases there is an assumption that the A-nucleon core is a spectator and does not change its state during the collision – is not dynamically excited. So, if the the residue is found in a given state, it is because this component existed in the projectile ground state.



So, reaction probes the one/two nucleon overlap and (in general) there are several active configurations – the 2N overlap is determined by the (**TNA**) two nucleon amplitudes from the shell model.

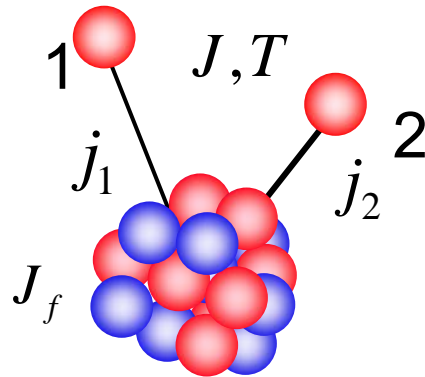
Target probes the one- and two-nucleon overlaps



$$F_{JM}(\vec{r}) = \langle \vec{r}, \Phi_c | \Phi_{A+1} \rangle$$

$$S_N = E_{A+1} - E_c$$

$$F_{JM}(\vec{r}) = C(J) \phi_{JM}(\vec{r})$$



$$C^2 S(J) = |C_J|^2$$

Spectroscopic
factor/strength

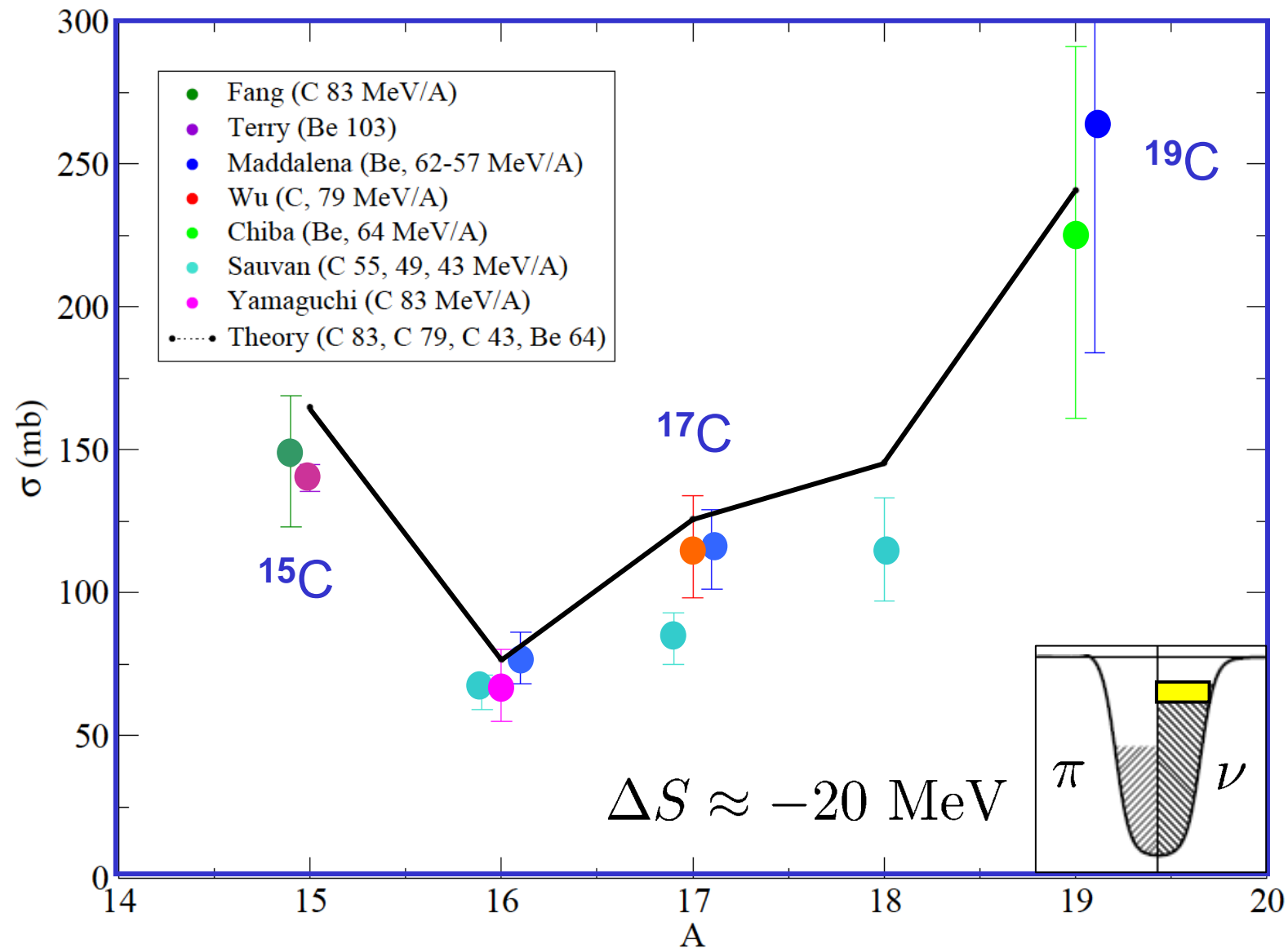
In two-nucleon case there are (in general) several coherent 2N configurations – the two-nucleon motions are correlated

$$F_{JM}(1, 2) = \sum_{j_1 j_2} (-)^{J+M} C(j_1 j_2 J) / \hat{J} [\overline{\phi_{j_1 m_1} \otimes \phi_{j_2 m_2}}]_{J-M}$$

Inclusive neutron removal – $^{15-19}\text{C}$ isotopes

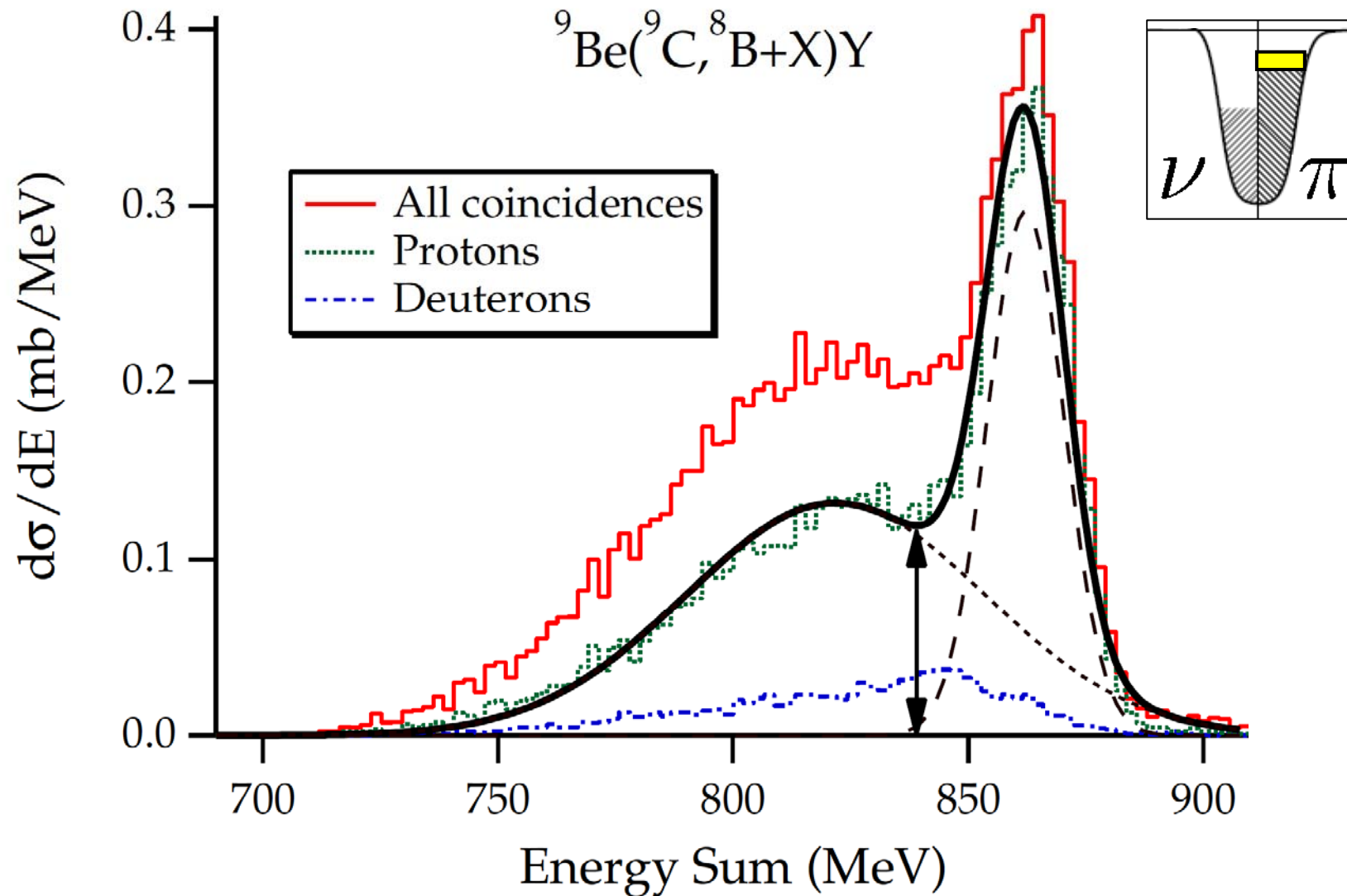
Reaction	E(MeV)	J^π	ℓ	σ_{str} (mb)	σ_{diff} (mb)	C ² S	σ_{-1n} (mb)	σ_{exp} (mb)	R_s
$(^{15}\text{C}, ^{14}\text{C})$ [5] $\Delta S=19.86\text{MeV}$	0.000	0^+	0	66.3	35.5	0.983	114.8	100.8(44)	0.88
	6.094	1^-	1	21.1	5.4	1.197	36.4	27.4(41)	
	6.903	0^-	1	20.1	5.0	0.457	13.2	6.5(9)	
	7.012	2^+	2	21.2	4.9	0.016	0.5	5.5(17)	
	sum						164.9	140.2 \pm 4.6	
$(^{16}\text{C}, ^{15}\text{C})$ [5] $\Delta S=18.32\text{MeV}$	0.000	$1/2^+$	0	42.4	19.7	0.508	35.9		0.85
	0.740	$5/2^+$	2	25.3	8.7	1.054	40.8		
	sum						76.7	65 $^{+15}_{-10}$	
$(^{17}\text{C}, ^{16}\text{C})$ [7] $\Delta S=22.60\text{MeV}$	0.000	0^+	2	38.9	18.8	0.035	2.3		0.92
	1.766	2^+	2	29.6	12.0	1.415	66.5		
			0	53.2	28.4	0.162	14.9		
	3.986	2^+	0	41.6	19.9	0.225	15.6		
	4.100	$2^+, 3^{(+)}, 4^+$	2	24.0	8.5	0.721	26.5		
	sum						126	116 \pm 18	
$(^{18}\text{C}, ^{17}\text{C})$ [4] $\Delta S=21.95\text{MeV}$	0.000	$3/2^+$	2	22.5	9.7	0.103	3.7		0.81
	0.032	$5/2^+$	2	22.5	9.6	2.8	100.8		
	0.295	$1/2^+$	0	37.2	19.1	0.65	41.0		
	sum						145.5	118 \pm 18	
$(^{19}\text{C}, ^{18}\text{C})$ [9] $\Delta S=26.62\text{MeV}$	0.000	0^+	0	93.8	65.6	0.58	103.0		0.94
	2.144	2^+	2	25.6	10.4	0.47	18.9		
	3.639	2^+	2	22.0	8.2	0.087	2.9		
	3.988	0^+	0	34.0	15.8	0.32	17.8		
	4.437	1^-	1	20.4	7.9	0.12	3.8		
			1	20.4	7.9	0.67	21.1		
	4.915	3^+	2	19.9	7.0	1.53	45.9		
	4.975	2^+	2	19.9	6.9	0.92	27.5		
	sum						240.8	226 \pm 65	

Inclusive neutron removal – $^{15-19}\text{C}$ isotopes

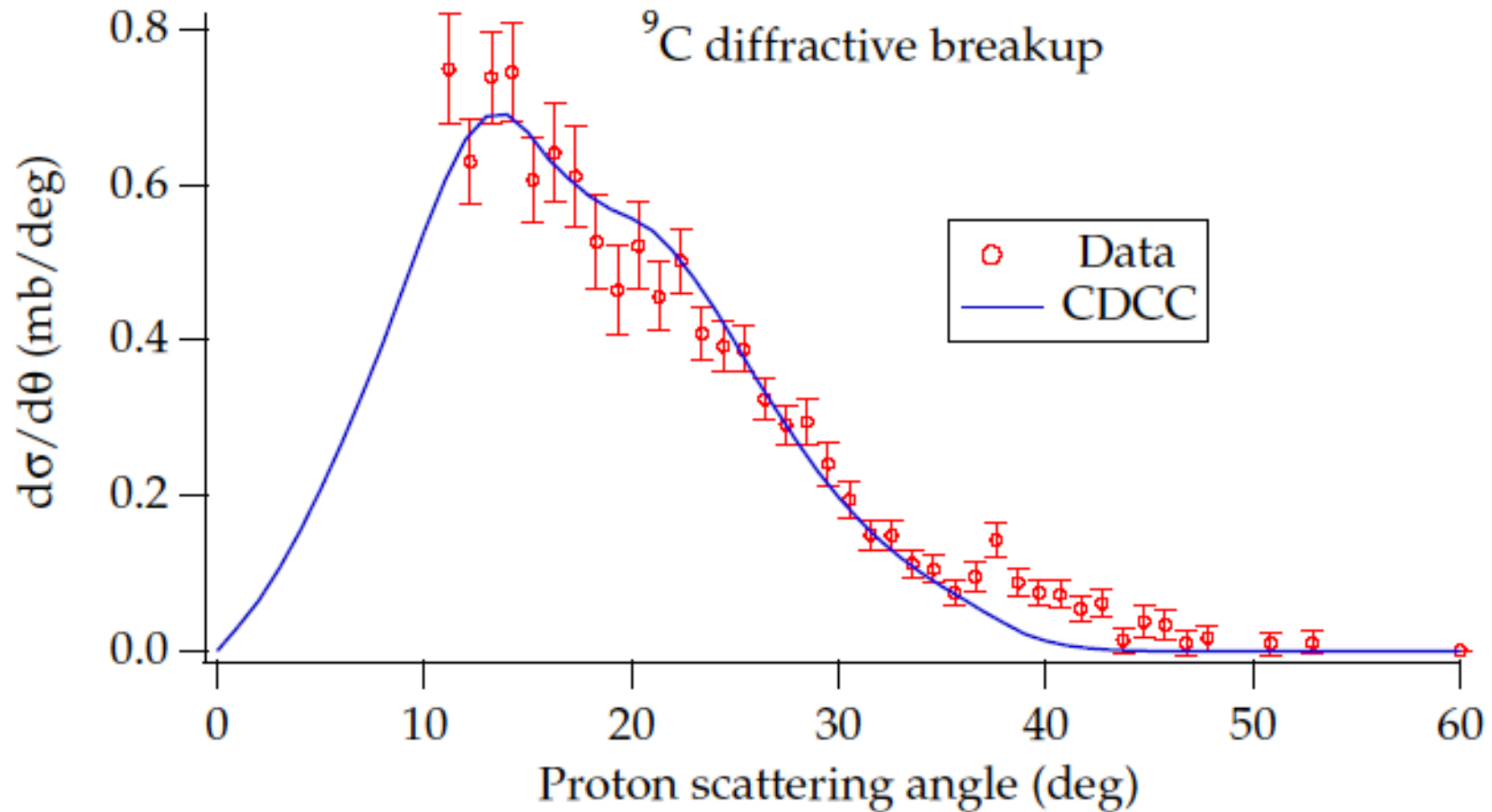


E.C. Simpson and J.A. Tostevin, PRC 79 024616 (2009)

Test of the mechanisms – stripping and diffraction

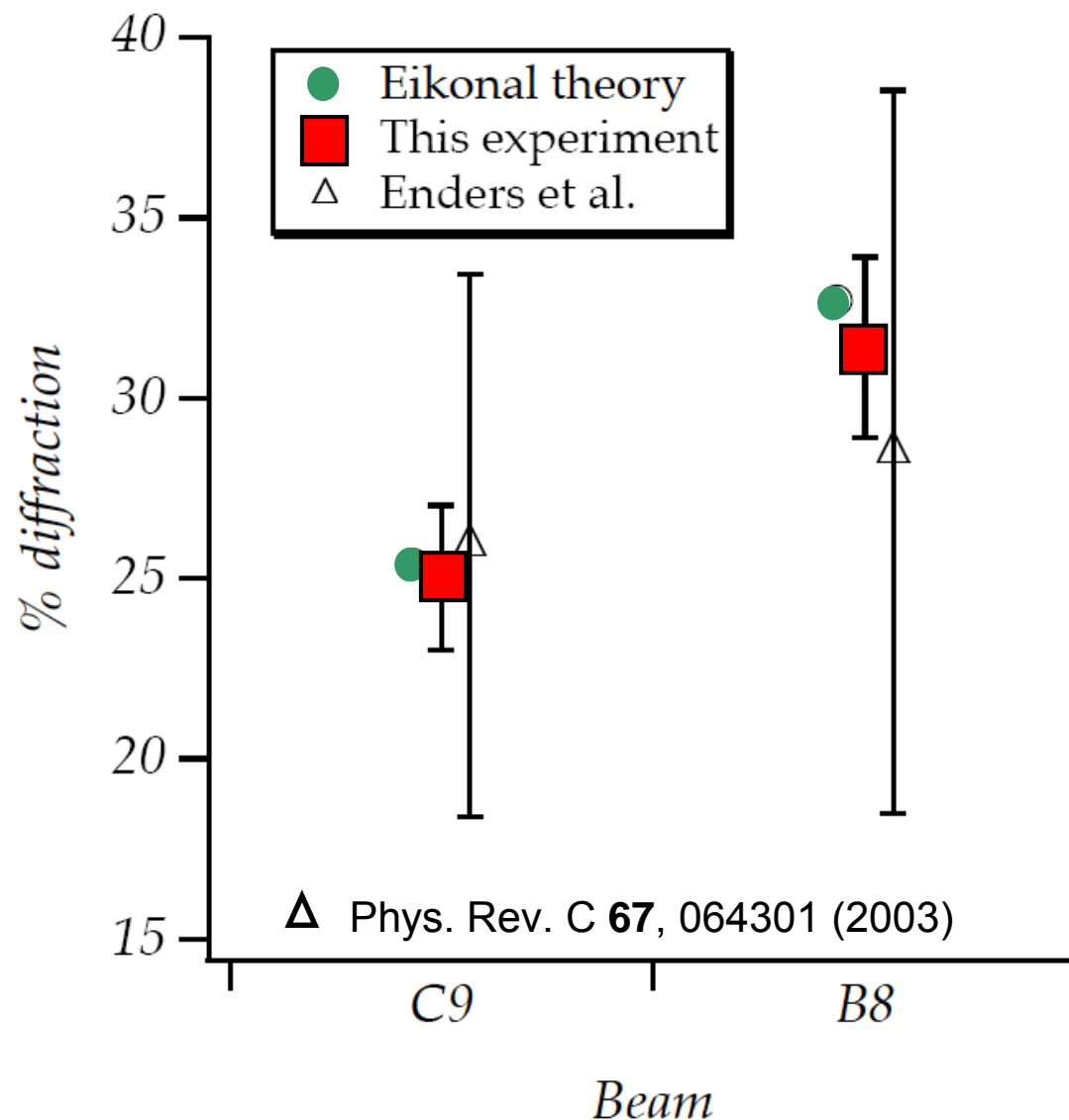


CDCC diffraction (elastic breakup component)



D. Bazin et al., Phys. Rev. Lett. **102**, 232501 (2009)

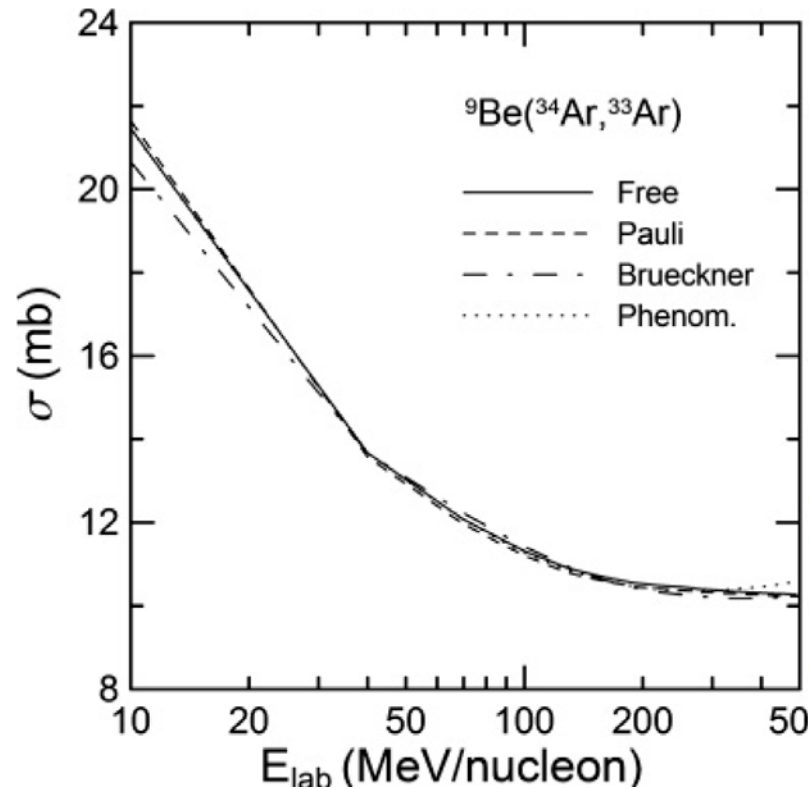
The two mechanisms – stripping and elastic breakup



In these cases, ${}^9\text{C}(-p)$ and ${}^8\text{B}(-p)$, where the two mechanisms have been quantified, the eikonal dynamical description does a (very) good job. Only the sum is measured usually – since only the heavy residue is detected.

D. Bazin et al., Phys. Rev. Lett. **102**, 232501 (2009)

Pauli blocking of the free NN interaction



$$\sigma_{NN}(k, \rho_1, \rho_2) = \int \frac{d^3 k_1 d^3 k_2}{(4\pi k_{F1}^3/3)(4\pi k_{F2}^3/3)} \times \frac{2q}{k_0} \sigma_{NN}^{\text{free}}(q) \frac{\Omega_{\text{Pauli}}}{4\pi},$$

TABLE I. Cross sections in mb at 40 MeV/nucleon for nucleon knockout of a few selected reactions.

Reaction	σ	Free	Pauli	Brueckner	Pheno.
${}^9\text{Be}({}^{11}\text{Be}, {}^{10}\text{Be})$	σ_{dif}	47.6	36.9	45.7	45.2
	σ_{str}	151	144	139	149
	σ_{tot}	198	181	185	194
${}^9\text{Be}({}^{15}\text{C}, {}^{14}\text{C})$	σ_{dif}	25.3	19.9	21.3	24.0
	σ_{str}	99.8	95.8	96.5	98.5
	σ_{tot}	125	116	118	123
${}^9\text{Be}({}^{34}\text{Ar}, {}^{33}\text{Ar}(1/2^+))$	σ_{dif}	2.69	2.63	2.66	2.68
	σ_{str}	11.0	10.9	11.0	11.0
	σ_{tot}	13.6	13.5	13.6	13.6

C.A. Bertulani and C. de Conti,
Phys. Rev. C **81**, 064603 (2010)

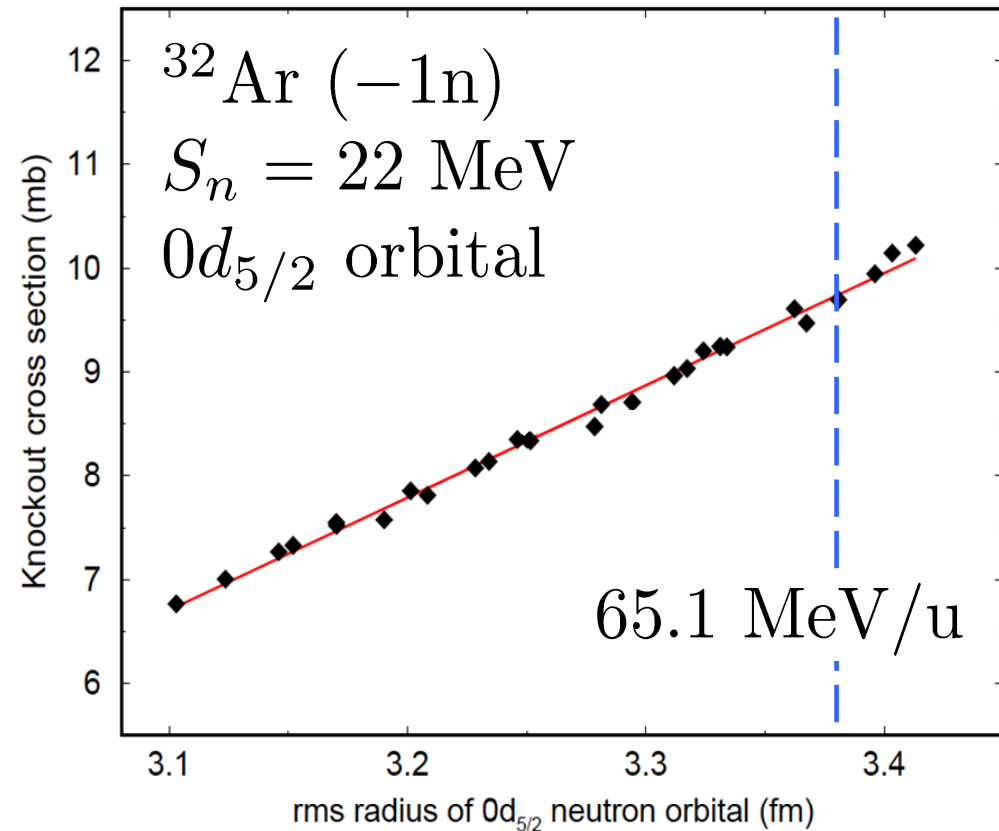
Geometry considerations: Hartree Fock for ‘sizes’

The rms radii of single particle formfactors are the sole requirement for determining the cross section calculations – to high precision. We constrain these to Hartree-Fock or other theoretical values

$$\sigma(r_0, a_0, V_{so}, \beta_{NL})$$



$$\sigma(\langle r^2 \rangle^{1/2})$$



Reaction description
between different exotic
systems is very ‘robust’

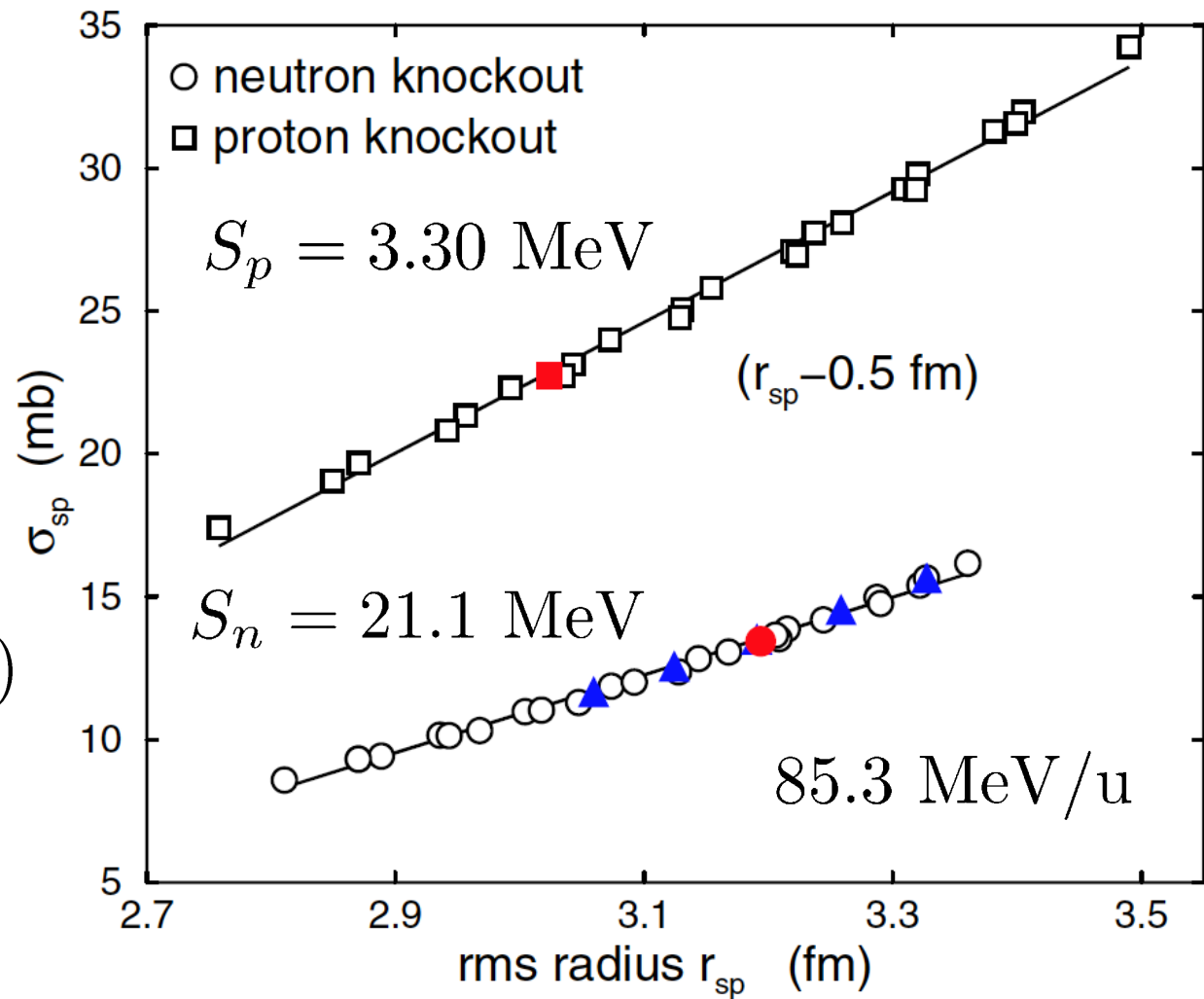
Overlap function sensitivity: Hartree Fock 'sizes'

$^{24}\text{Si} (-1\text{N})$
 $1d_{5/2}$

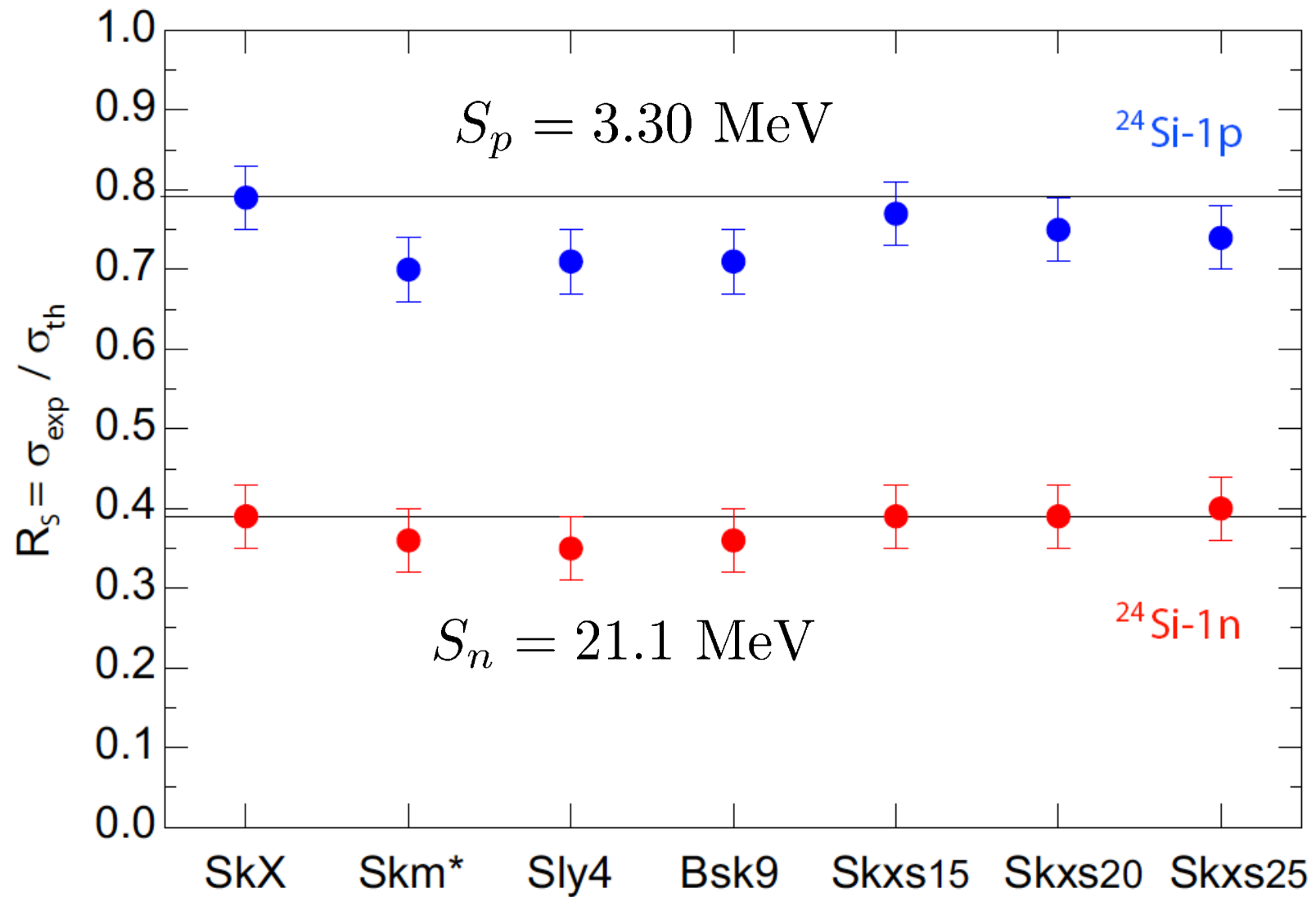
$$\sigma(\langle r^2 \rangle^{1/2})$$



$$\sigma(r_0, a_0, V_{so}, \beta_{NL})$$

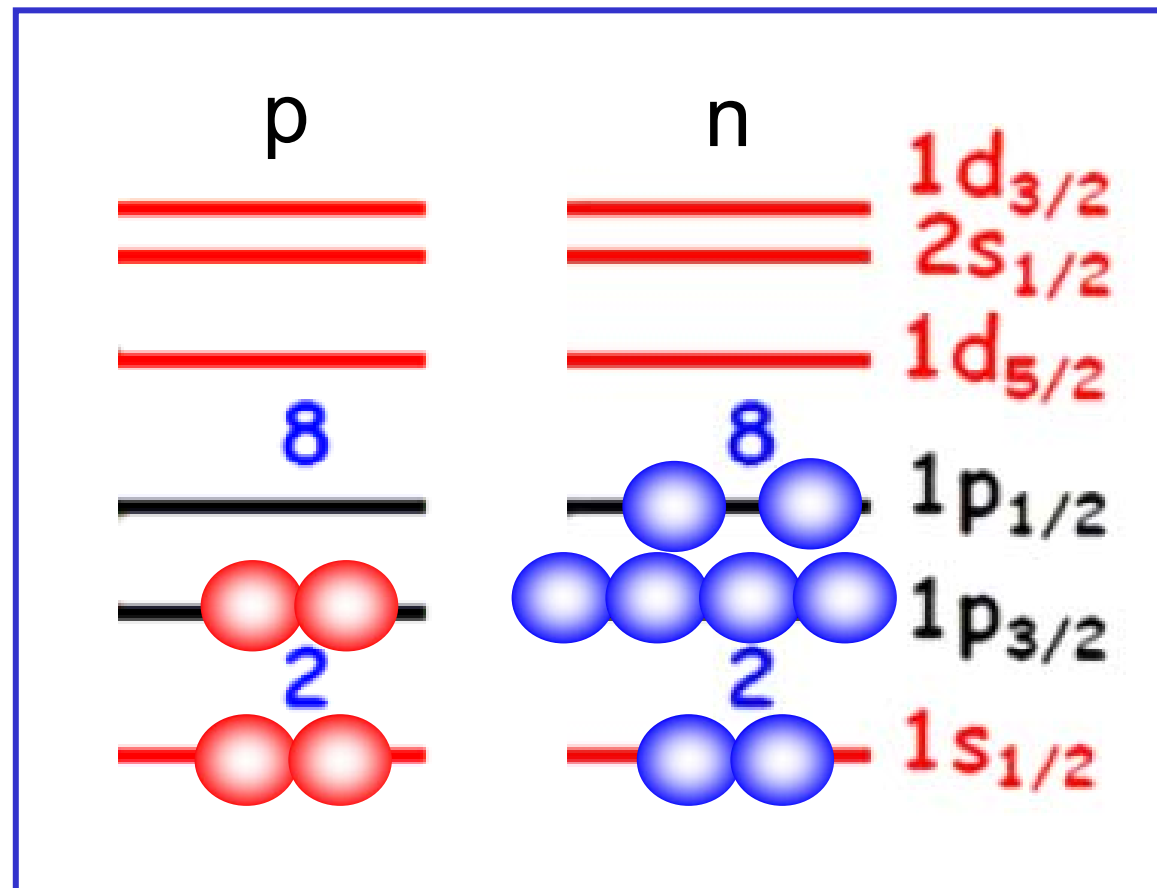


Removal strengths: Skyrme (in)sensitivity



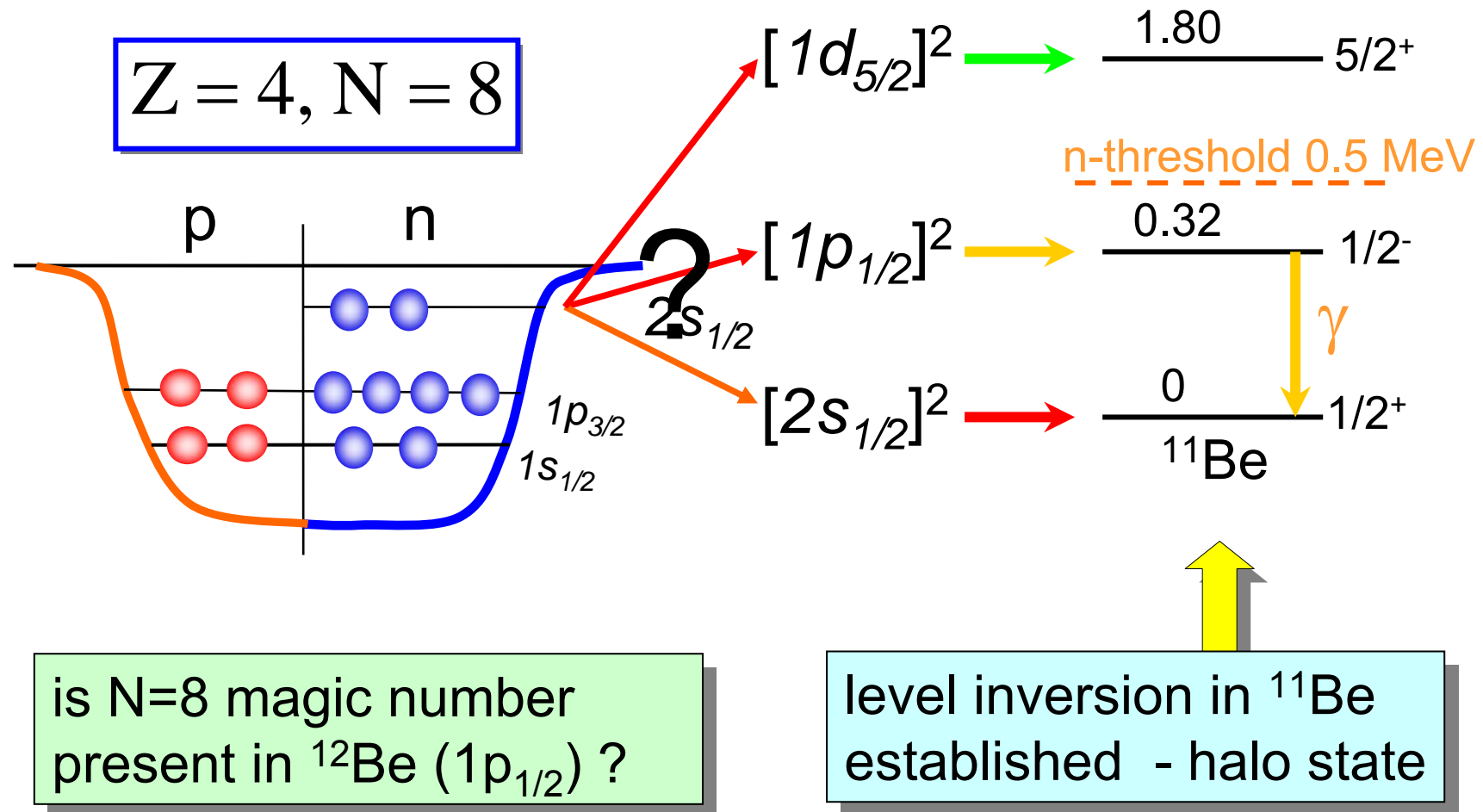
N=8 neutrons – still a magic number in ^{12}Be ?

^{12}Be



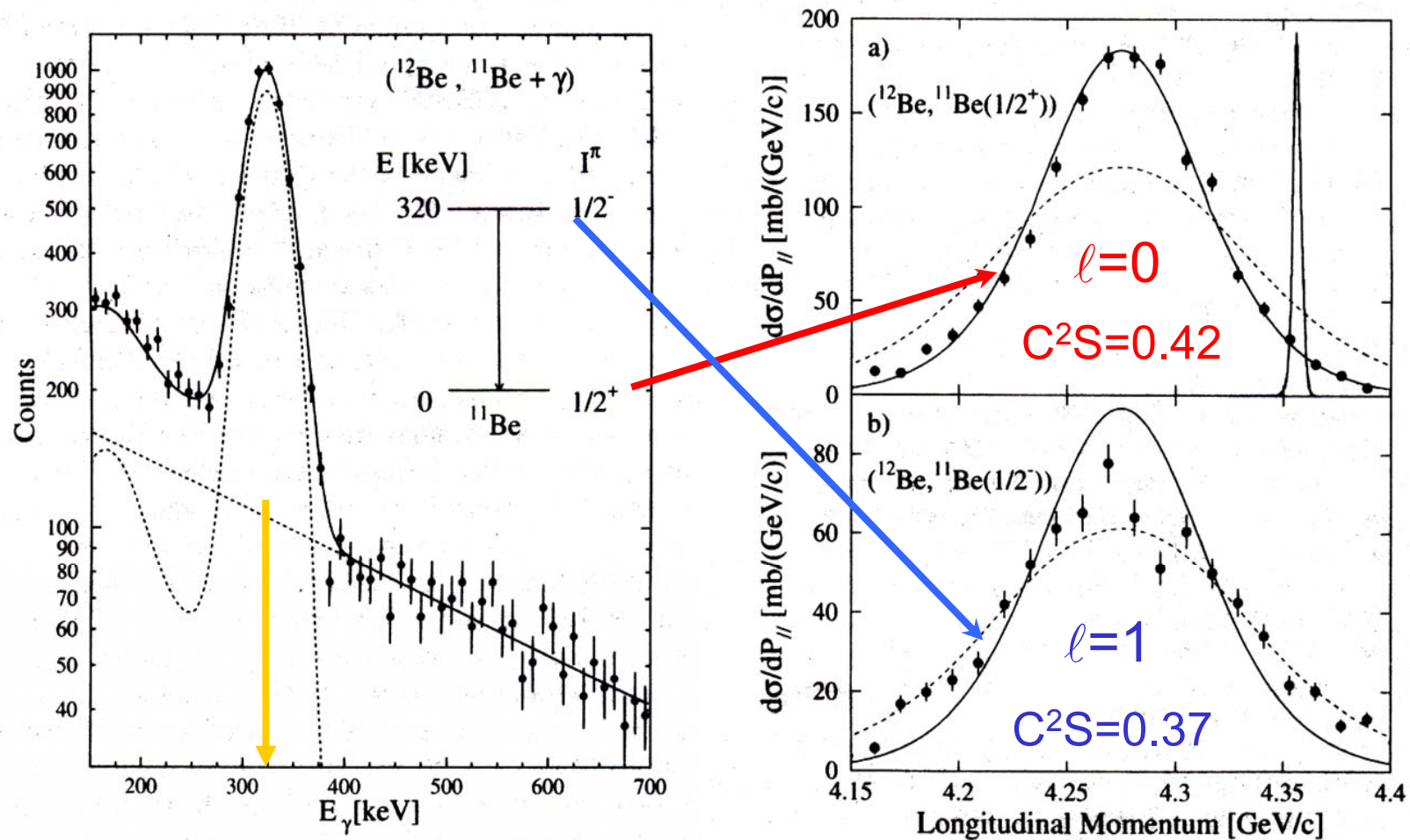
$Z = 4, N = 8$

N=8 neutrons – still a magic number in ^{12}Be ?



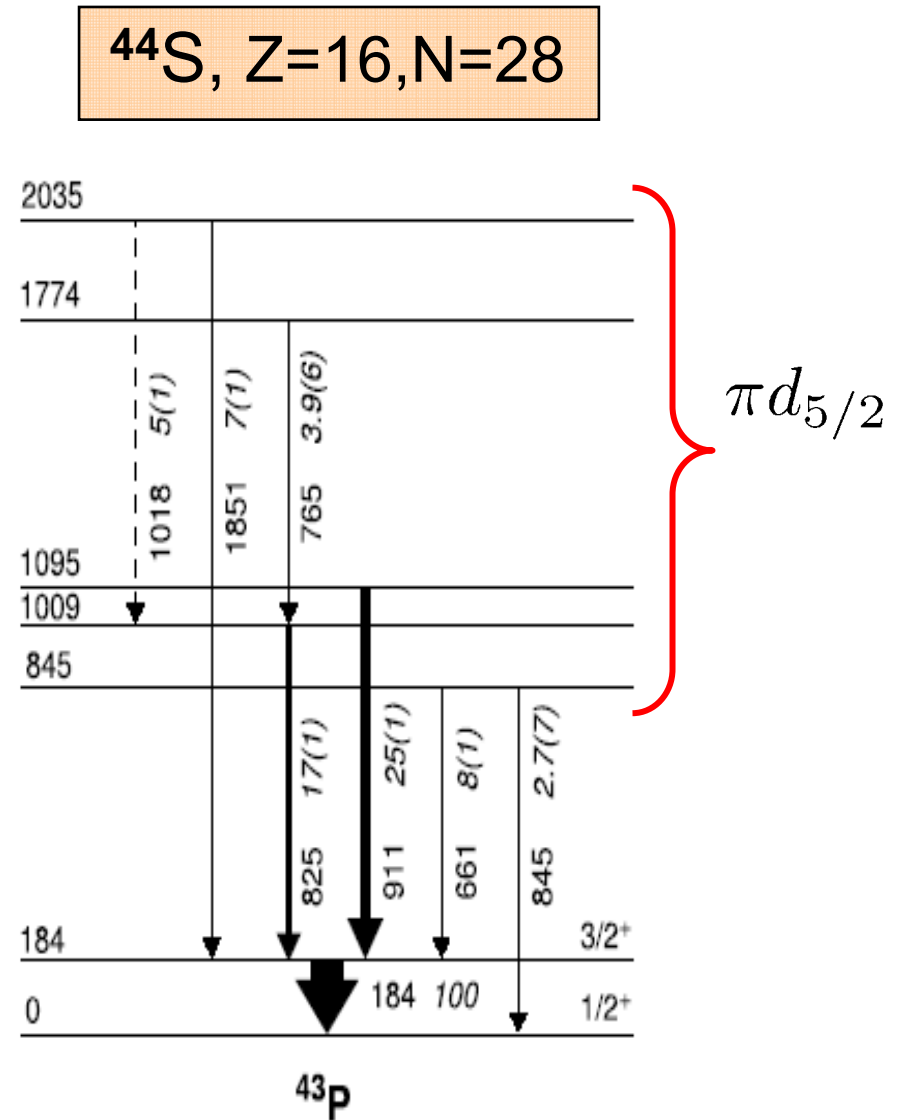
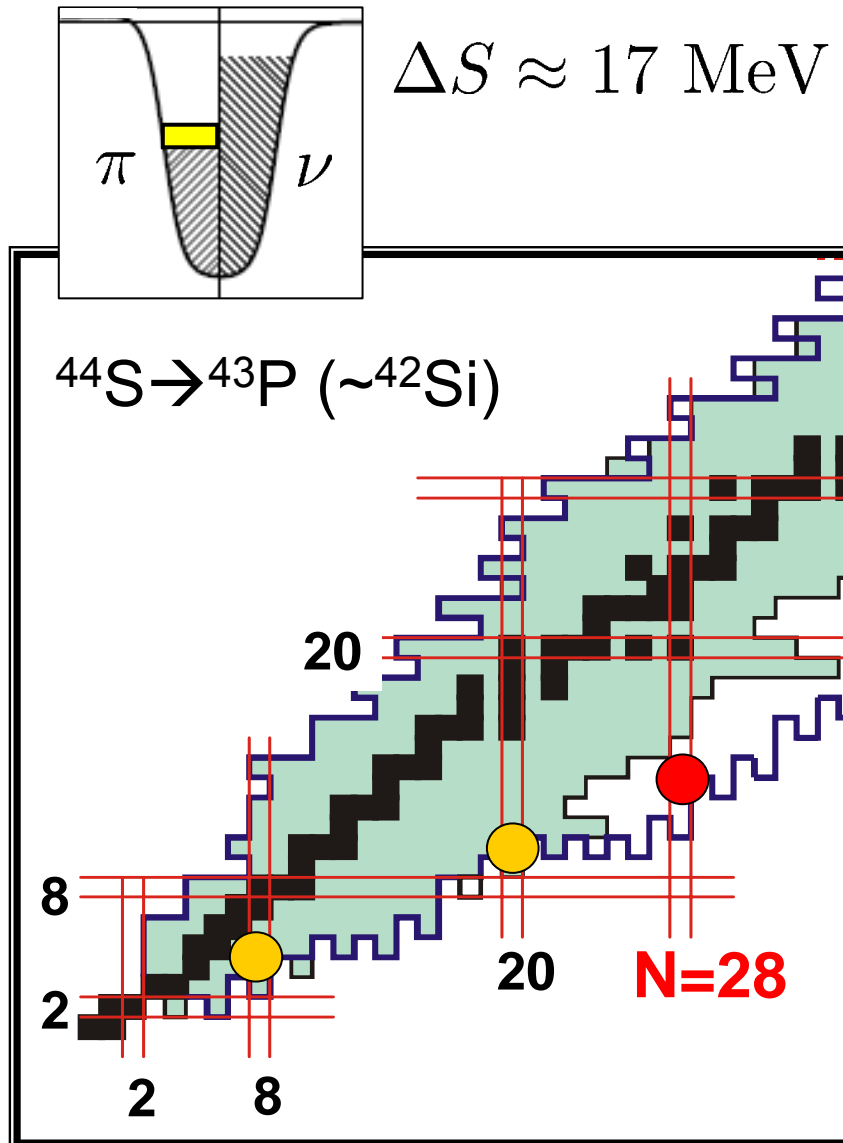
A. Navin et al., PRL **85** (2000) 266
 S. Pain et al., PRL **96** (2006) 032502

N=8 magic number has disappeared in ^{12}Be

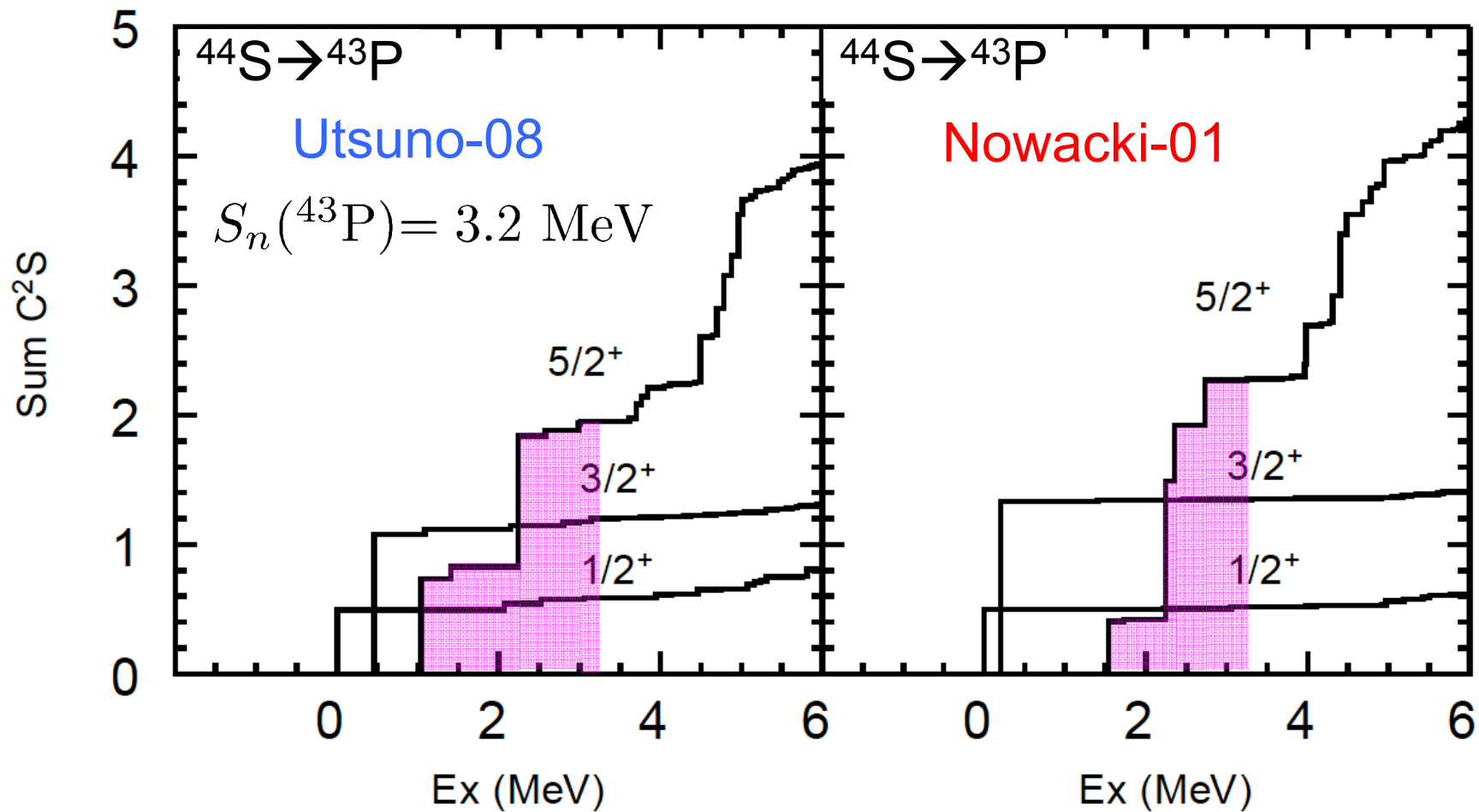


A. Navin et al., PRL **85** (2000) 266

The case of proton removal from ^{44}S ($^{44}\text{S} \rightarrow ^{43}\text{P}$)



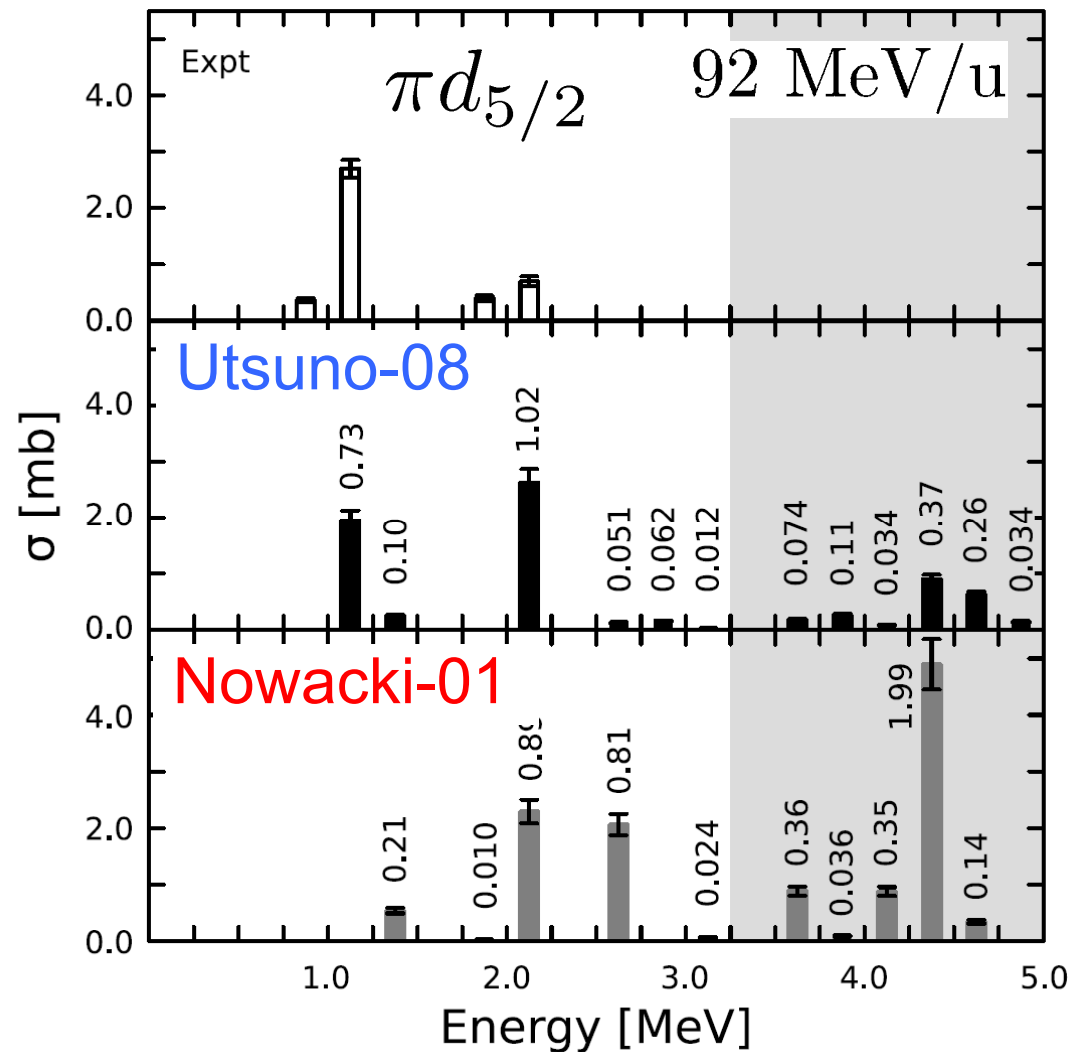
Testing predictions of shell model interactions



$^{42}\text{Si}(2^+)$ at 0.87 MeV

$^{42}\text{Si}(2^+)$ at 1.49 MeV

Tests of predictions of shell model interactions



This lowered proton $d_{5/2}$ strength is correlated with a lower 2^+ energy in ^{42}Si - from 1.49 MeV with [Nowacki-01](#) to 0.87 MeV with [Utsuno-08](#), to be compared to the recent experimental value** of **0.77 MeV**.

Adds supporting evidence for an (oblate) deformation in ^{42}Si .

L.A. Riley et al., PhysRev C 78, 011303(R) 2008

** B. Bastin et al., PRL **99**, 022503 (2007).

Breakdown of N=28 gap in $^{45}\text{Cl}(\text{gs})$ [$\rightarrow ^{44}\text{Cl}(\text{gs})$]

SDPF-NR (Numella et al.
PRC 63 044316 (2001)):

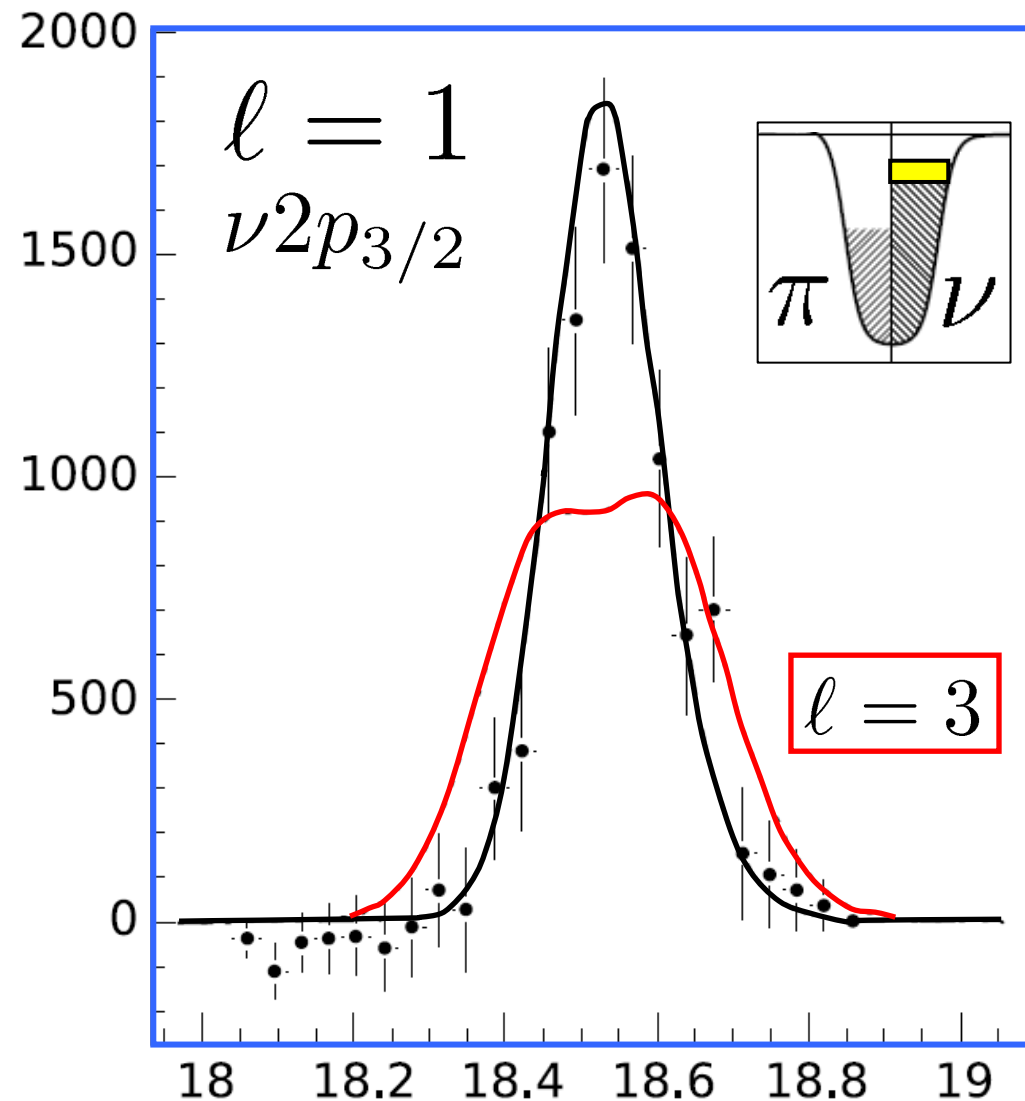
$$^{44}\text{Cl}(\text{gs}) = 2^-$$

$$^{45}\text{Cl}(\text{gs}) = 1/2^+$$

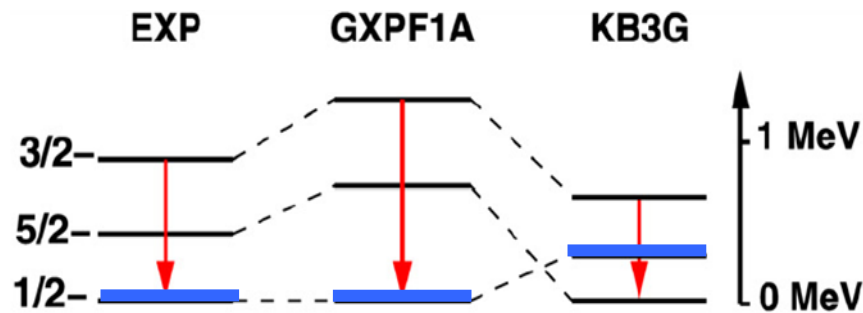
(unpaired $\pi 2s_{1/2}$)

n SF of: 6.63 (f 7/2),
0.23 (f 5/2), 1.03 (p
3/2), 0.10(p1/2)

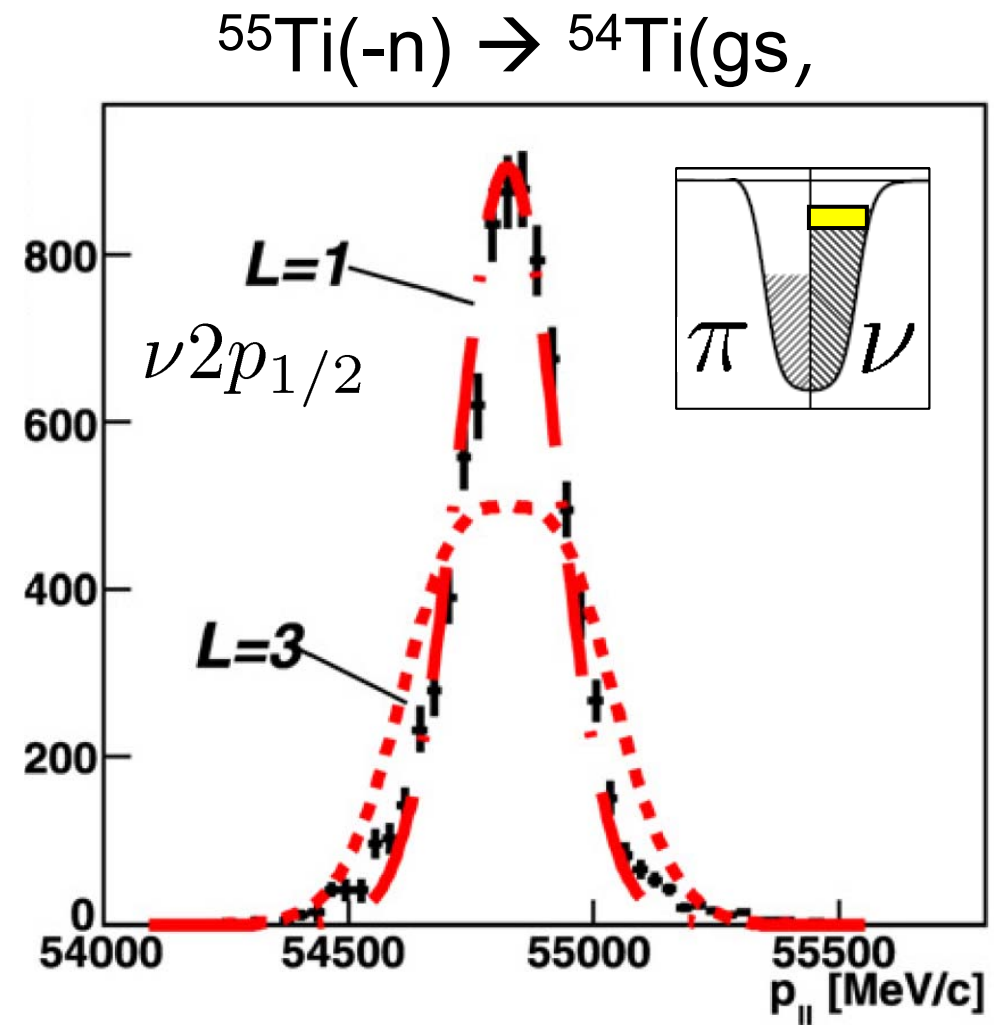
the gs transition
strength indicates
greater low lying p3/2
strength



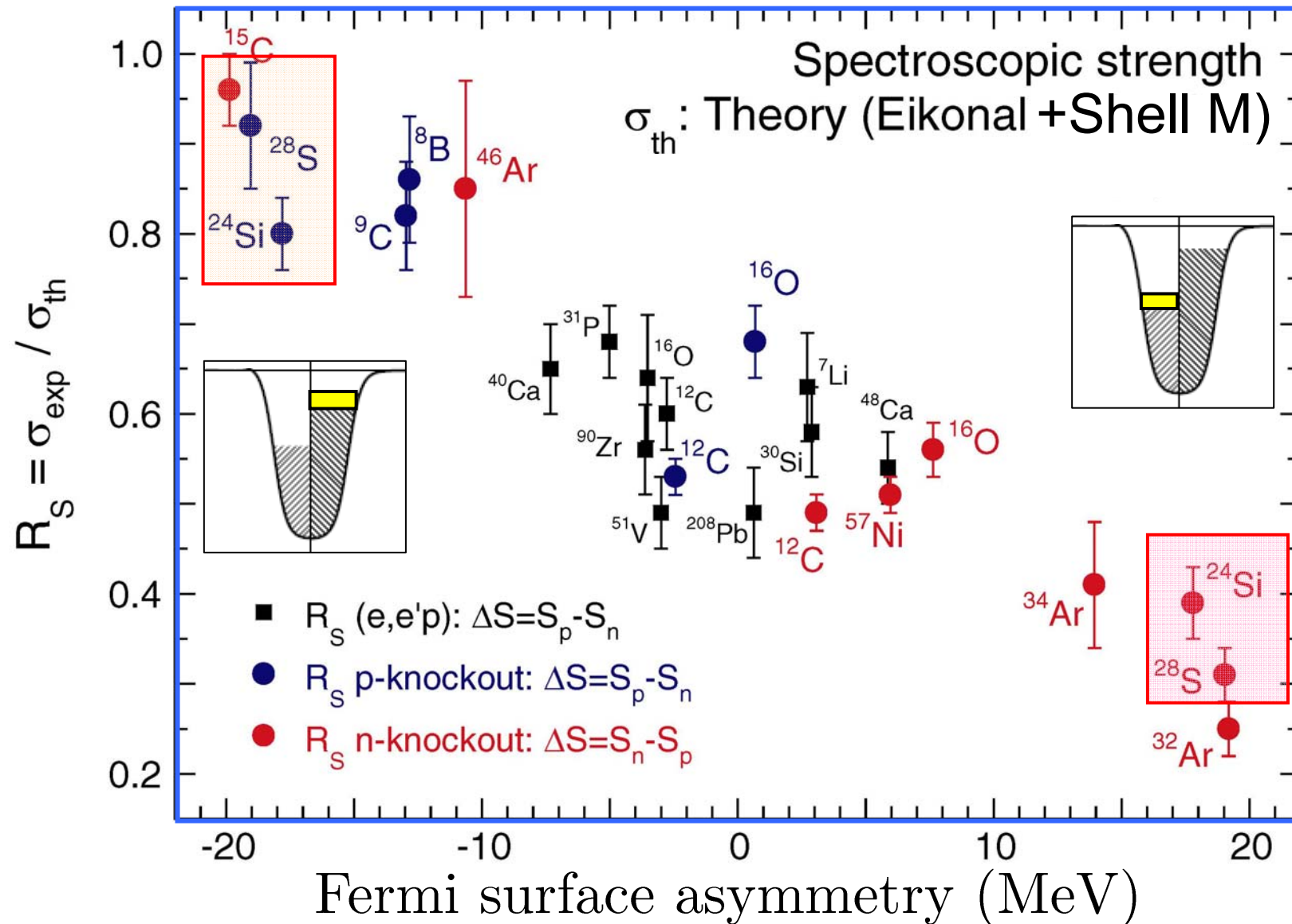
Shell model at the N=34 gap in ^{55}Ti : relativistic energy



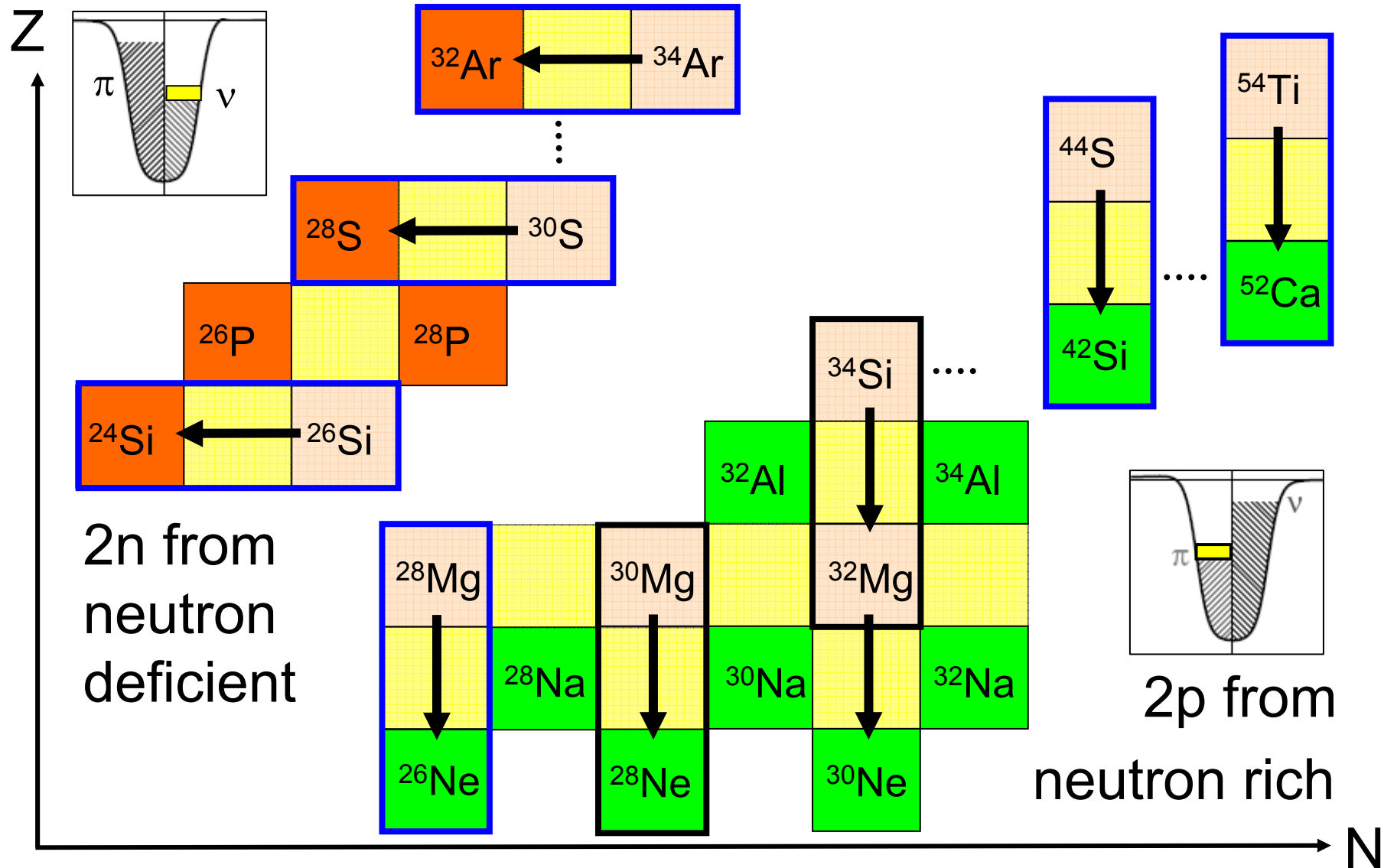
$^{55}\text{Ti}(-n)$, GSI, 450 MeV/u
 Observed the 955(6) keV
 $3/2^-$ neutron $p_{3/2}$ hole state
 in ^{55}Ti . Established the
 ground state of ^{55}Ti as $1/2^-$
 (as per GXPF1A) – that also
 predicts sizeable N=34 gap



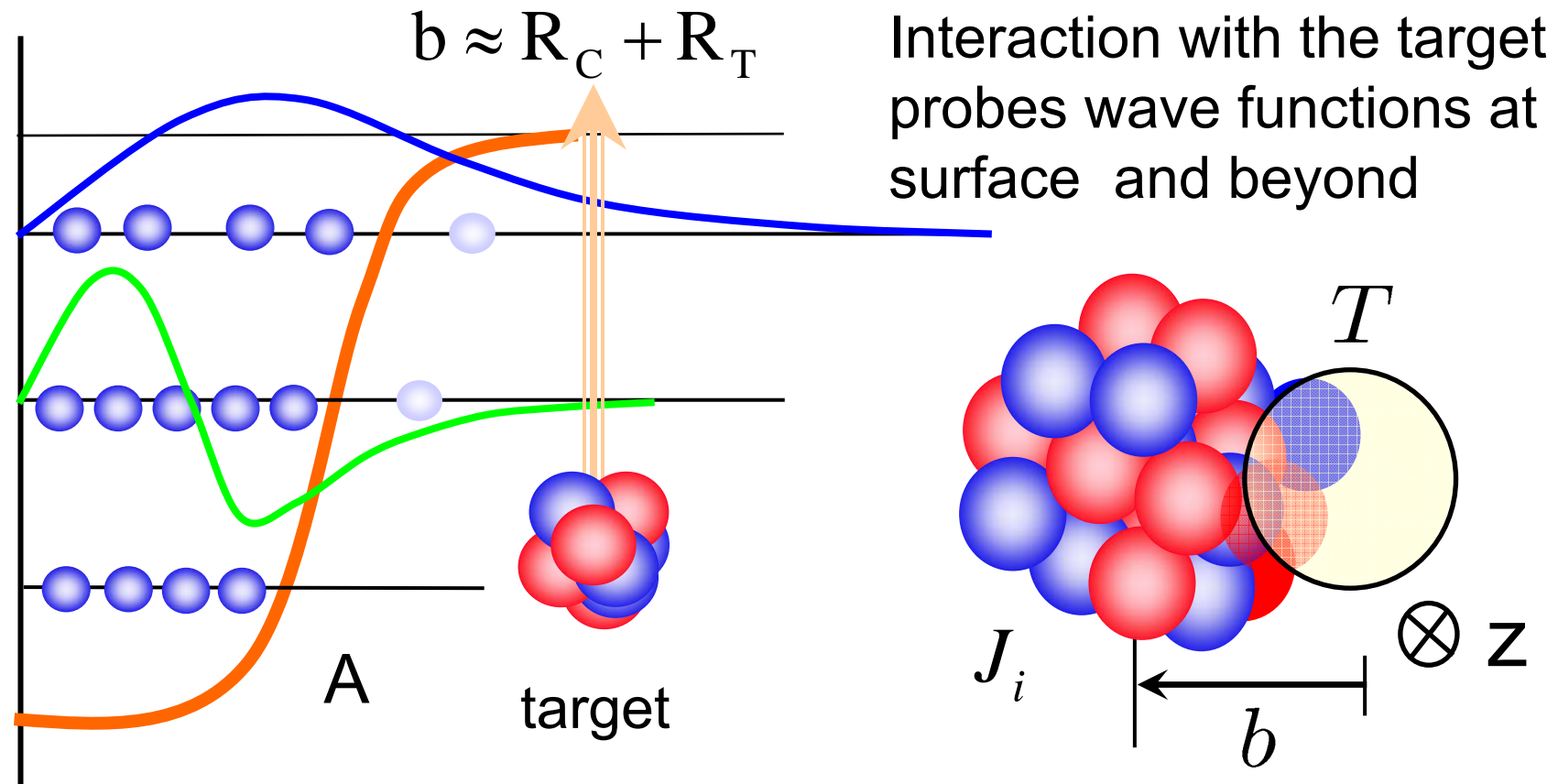
Removal strengths at the Fermi surface (2009)



Two nucleon knockout – direct reaction set



Sampling the two-nucleon wave function

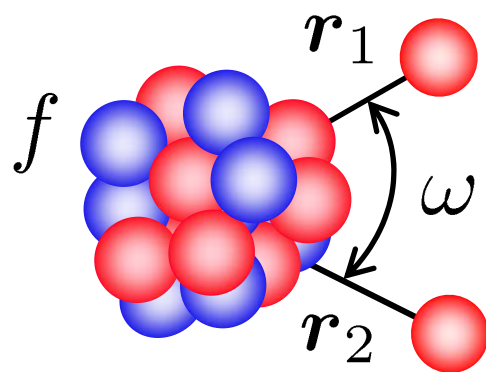
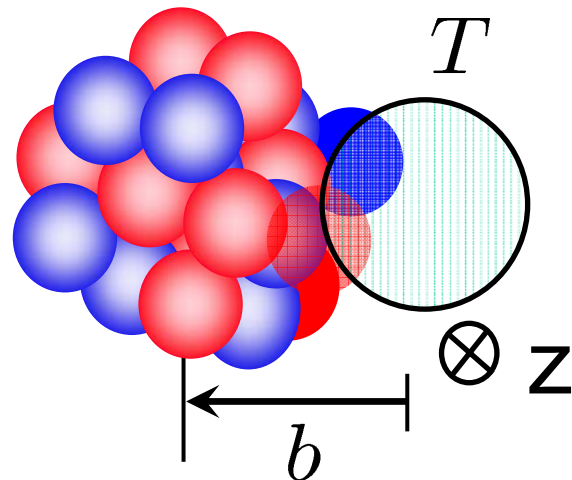


Shell model overlaps – for $0^+ \rightarrow$ heavy residue in state JM

$$F_{JM}(1, 2) = \sum_{j_1 j_2} (-)^{J+M} C(j_1 j_2 J) / \hat{J} [\overline{\phi_{j_1 m_1} \otimes \phi_{j_2 m_2}}]_{J-M}$$

Target drills a cylindrical volume at projectile surface

$$2N \text{ stripping} : \hat{O}(c, 1, 2) = |S_c|^2 (1 - |S_1|^2) (1 - |S_2|^2)$$



(i) 2N removal cross sections will be sensitive to the spatial correlations of pairs of nucleons near the surface

(ii) No spin selection rule (for $S=0$ versus $S=1$ pairs) in this 2N removal reaction mechanism

(iii) Expectation of the sensitivity to correlations can be predicted from 2N overlaps in the sampled volume

(iv) No linear or angular momentum mismatch – mechanism ‘sees’ ALL hole-like-state configurations

Two-nucleon direct reactions overlaps

$$\begin{aligned}\Psi_{J_i M_i}^{(f)}(1,2) &\equiv \langle \Phi_{J_f M_f}(A) | \Psi_{J_i M_i}(A, 1, 2) \rangle \\ &= \sum_{I \mu \alpha} C_{\alpha}^{J_i J_f I}(I \mu J_f M_f | J_i M_i) [\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I \mu}\end{aligned}$$

$$[\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I \mu} = -N_{12} \langle 1, 2 | [a_{j_1}^{\dagger} \otimes a_{j_2}^{\dagger}]_{I \mu} | 0 \rangle$$

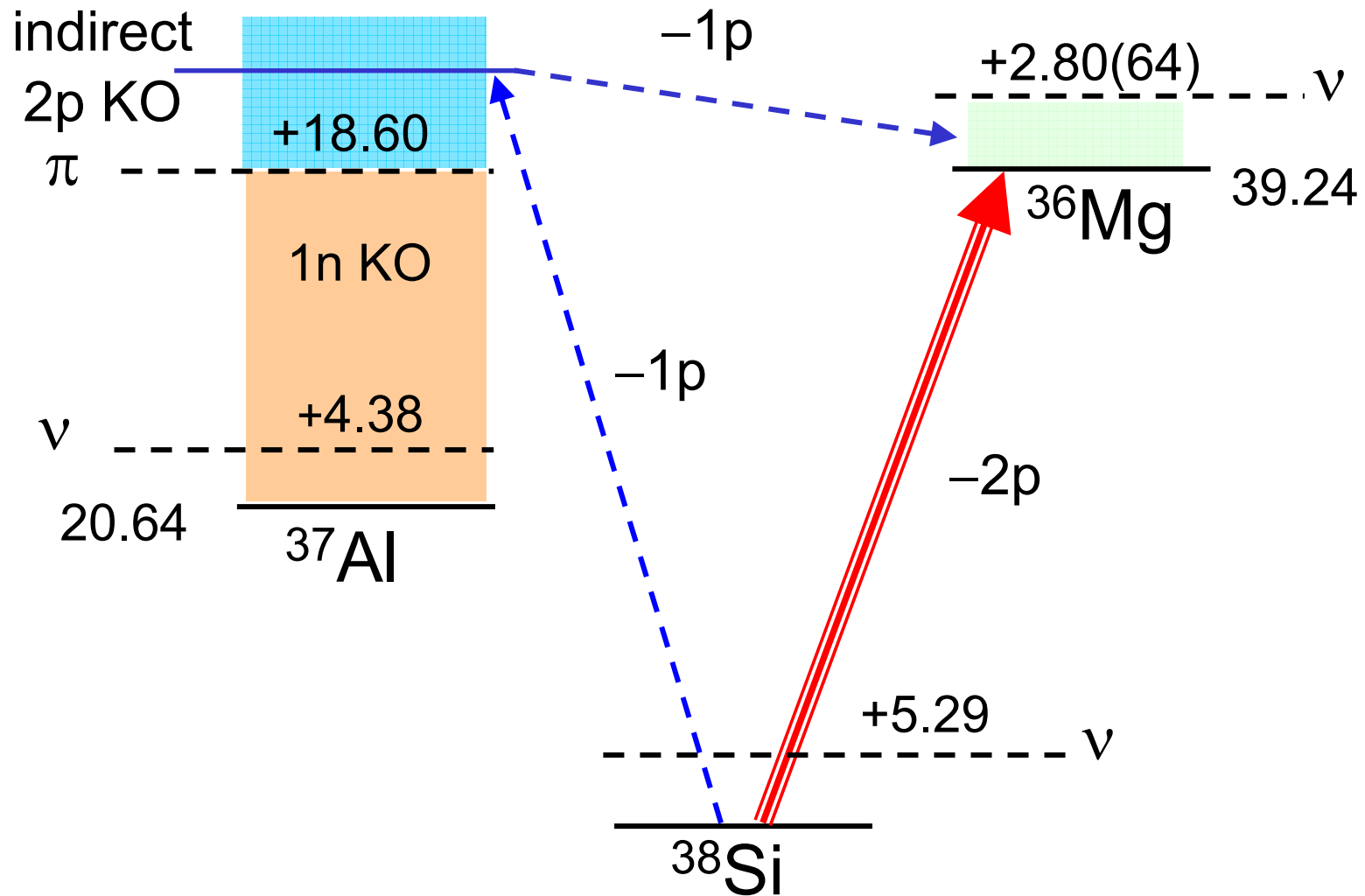
$$D_{\alpha} = N_{12} / \sqrt{2} = 1 / \sqrt{2(1 + \delta_{12})}$$

$$F_{IM}(1, 2) = \langle 1, 2, \Phi_{c, IM}(A) | \Phi_{A+2} \rangle \text{ and with } J_i = 0^+$$

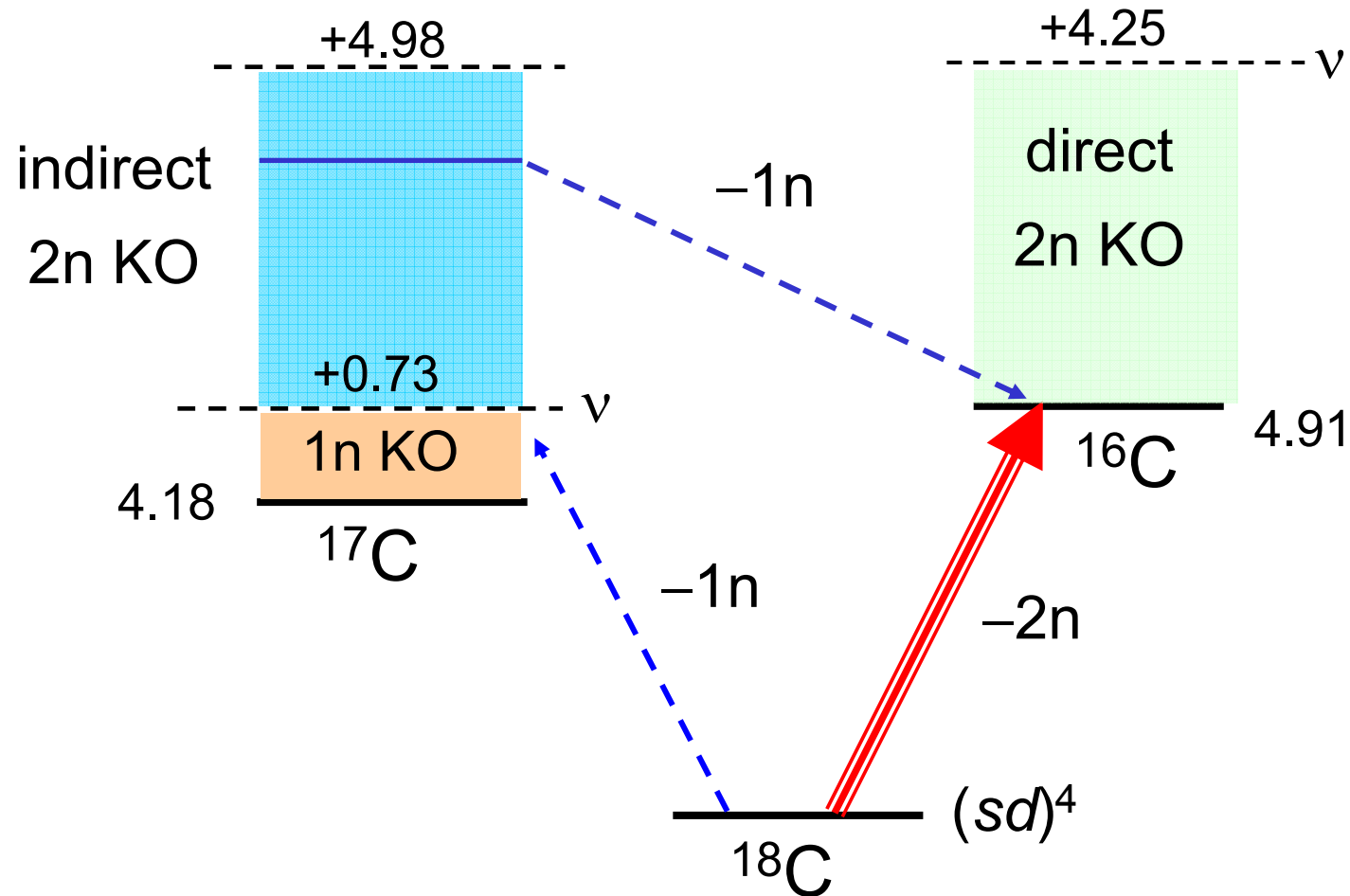
Two-nucleon amplitudes – TNA

$$F_{IM}(1, 2) = \sum_{j_1 j_2} (-)^{I+M} C(j_1 j_2 I) / \hat{I} [\overline{\phi_{j_1 m_1} \otimes \phi_{j_2 m_2}}]_{I-M}$$

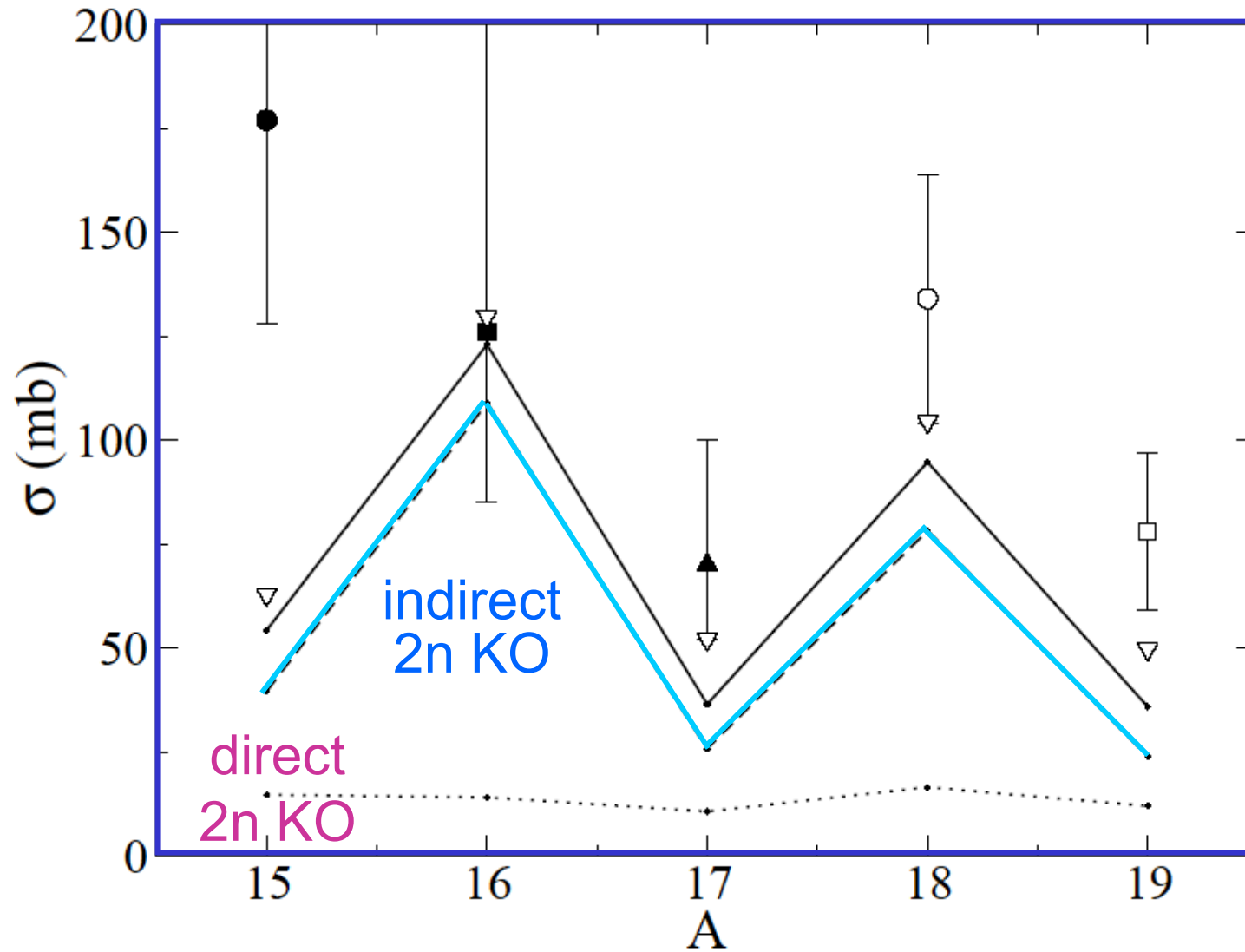
Two-proton knockout: $^{38}\text{Si} \rightarrow ^{36}\text{Mg}$



Two-neutron knockout: example $^{18}\text{C} \rightarrow ^{16}\text{C}$



Two neutron removal from neutron rich carbons



DT format, made from OXBASH binary output file: B460AW040.T2N

! Model-space name = SD

! Interaction name = W

!

!

Ai <-> Af	=	26	<->	28
Ji <-> Jf	=	2	<->	0
Ti <-> Tf	=	3	<->	2
Pi <-> Pf	=	+	<->	+
#i <-> #f	=	690	<->	874

!

!

! Two-nucleon spectroscopic amplitudes A(DJ,DT) =

! = -<f||| [a+(k1)a+(k2)]^(DJ,DT) |||i>/SQRT{(2Jf+1)(2Tf+1)[1+delta(k1,k2)]}

! with Edmonds (de-Shalit Talmi) reduced matrix element convention

!

! For n,l,j = 1.0 2.0 2.5 label k = 4

! For n,l,j = 1.0 2.0 1.5 label k = 5

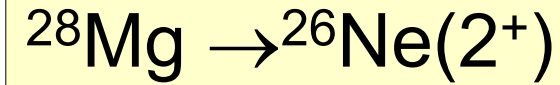
! For n,l,j = 2.0 0.0 0.5 label k = 6

!

Ji,	Jf,	Ti,	Tf,				
DJ,	Ni,	Nf,		Ef,	Ei,	Exi,	Exf,

!

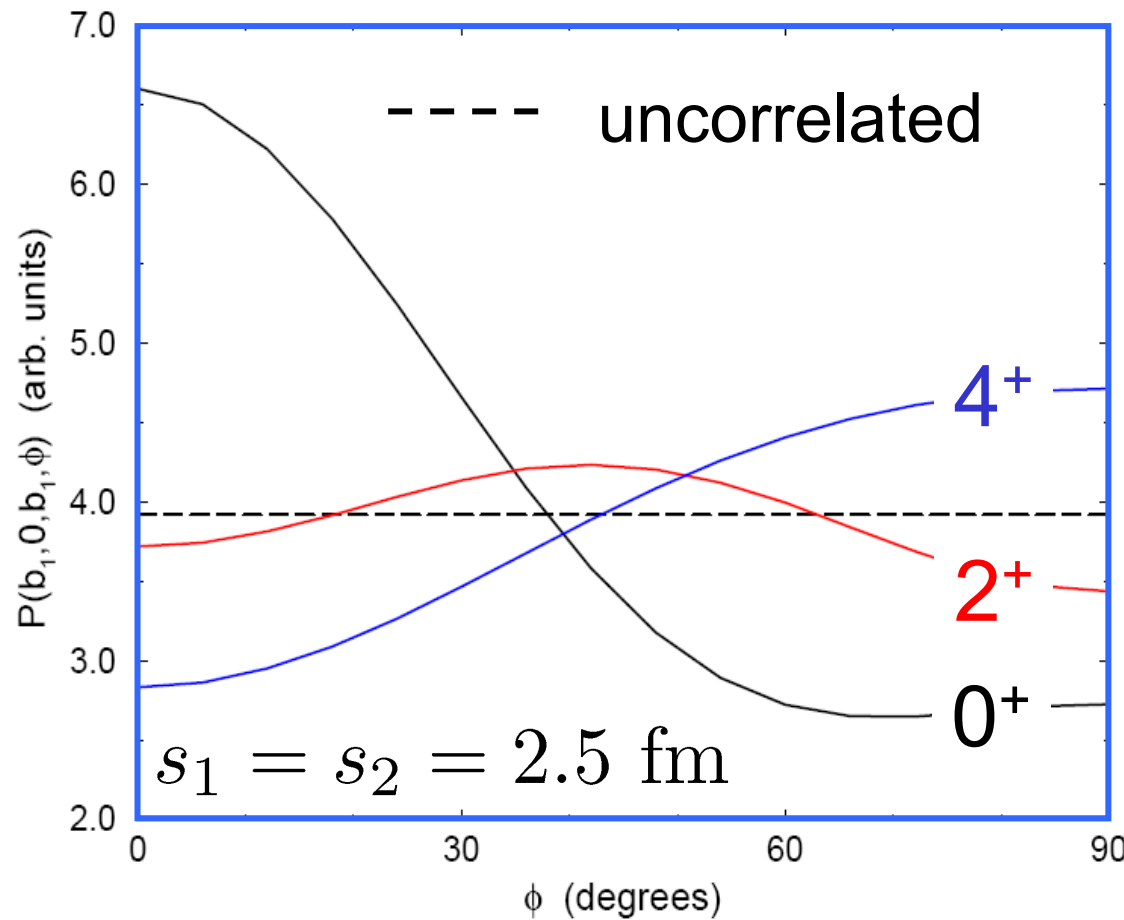
2.0,	0.0,	3.0,	2.0,	0.0,	0.0,		
2.0,	1.,	1.,	-120.533,	-79.613,	2.011,	0.000,	
5,	5,	0.00000,	-0.05026,		! k1,k2,A(DT=0),A(DT=1)		
4,	5,	0.00000,	0.37363,				
4,	4,	0.00000,	-0.63683,				
6,	5,	0.00000,	-0.06083,				
6,	4,	0.00000,	-0.13895,				



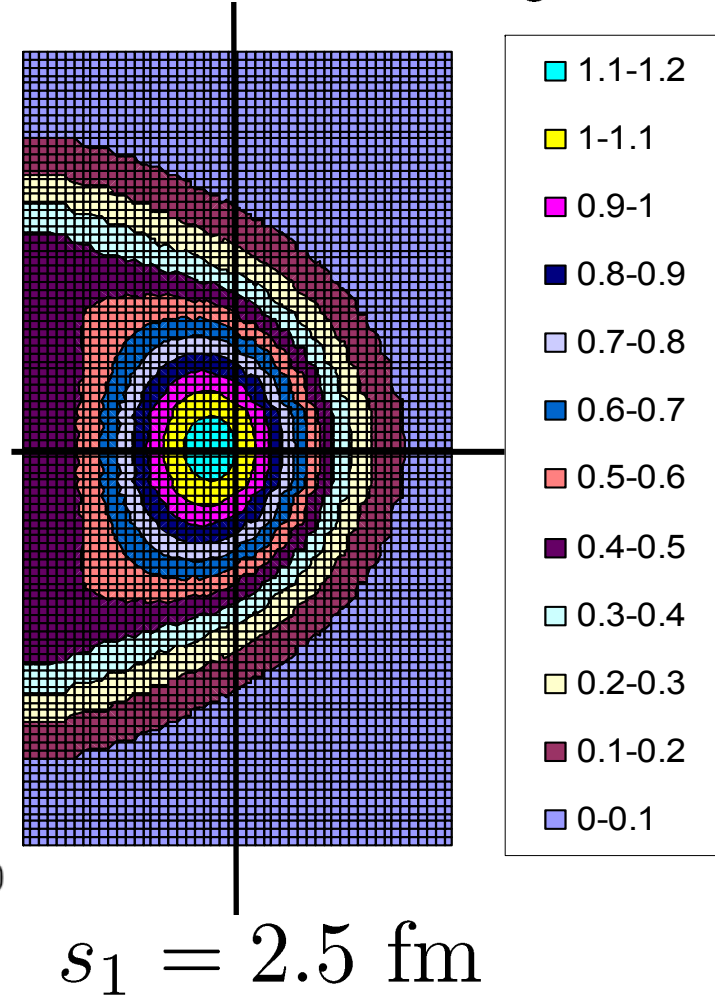
$$F_{JM}(1,2) = \sum_{j_1 j_2} (-)^{J+M} C(j_1 j_2 J) / \hat{J} [\overline{\phi_{j_1 m_1} \otimes \phi_{j_2 m_2}}]_{J-M}$$

Antisymmetrized $^{28}\text{Mg} \rightarrow ^{26}\text{Ne}$ removal of $\pi[1d_{5/2}]^2$ ³⁶

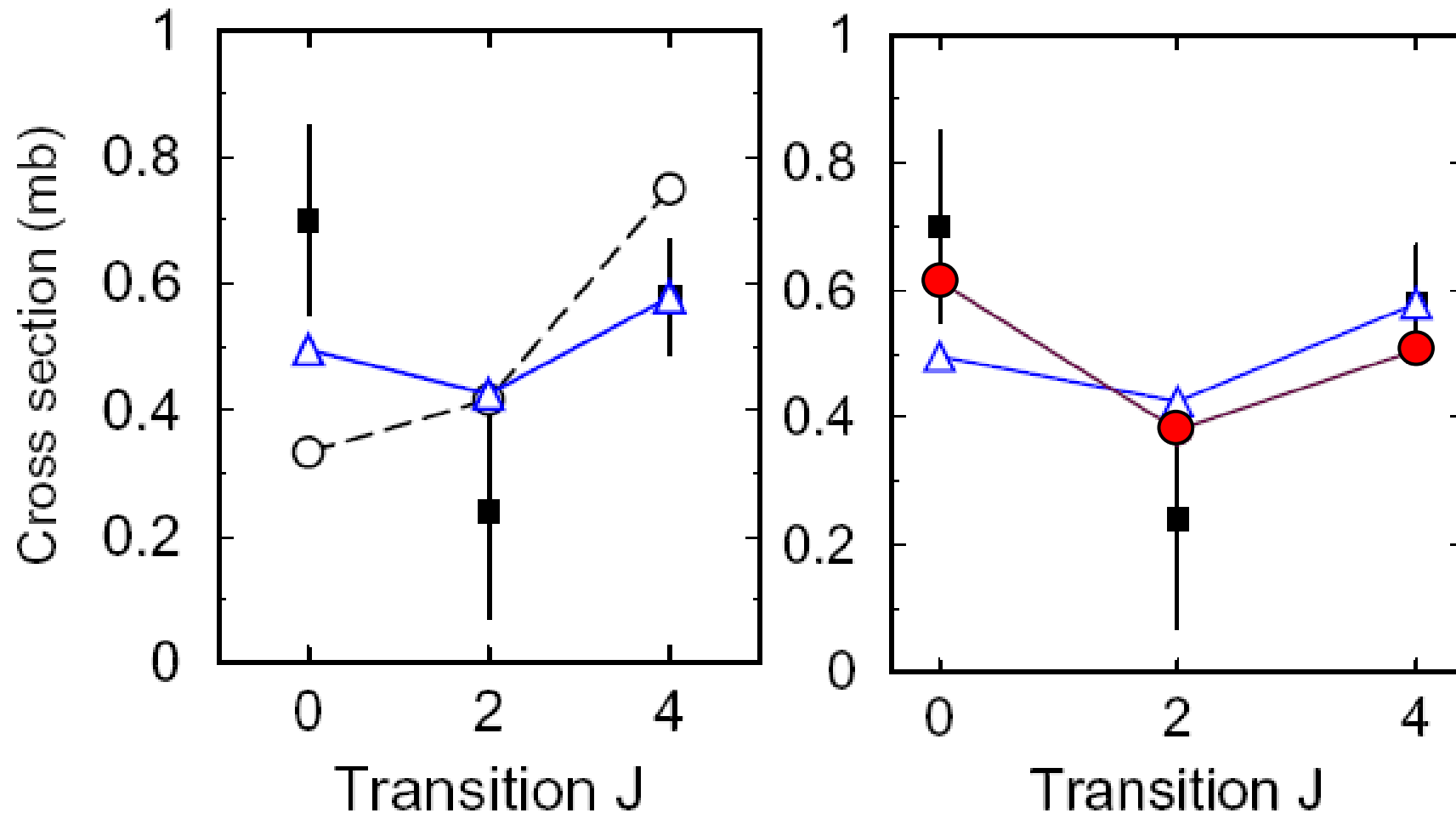
$$F_{JM} \propto [\overline{\phi_j} \otimes \phi_j]_{J-M}$$



$$P(\vec{s}_1, \vec{s}_2) \quad 0^+$$

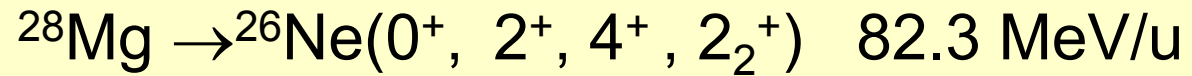


Correlated: $^{28}\text{Mg} \rightarrow ^{26}\text{Ne}(0^+, 2^+, 4^+)$, 82.3 MeV/u

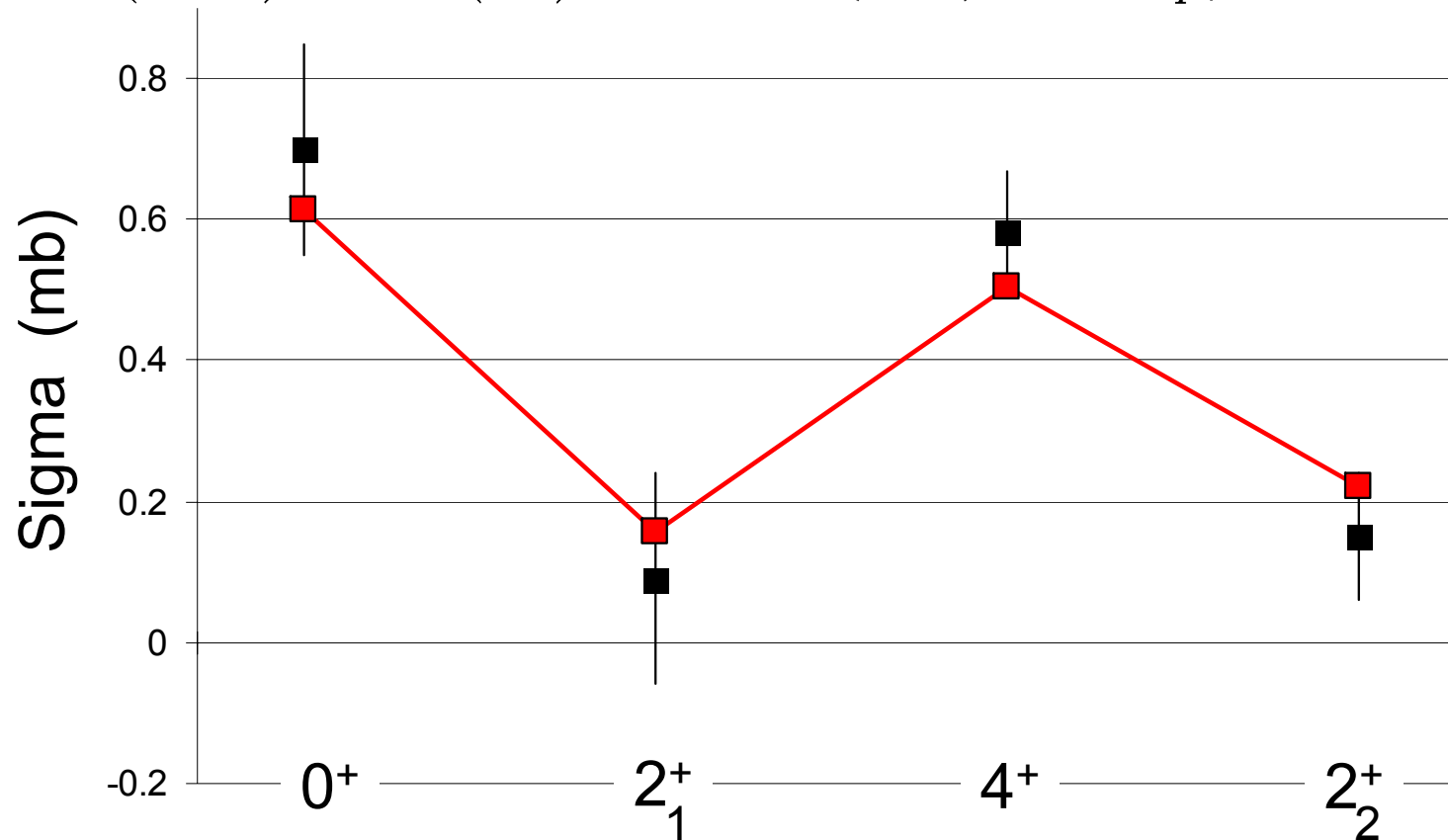


Data: D. Bazin et al., PRL **91** (2003) 012501

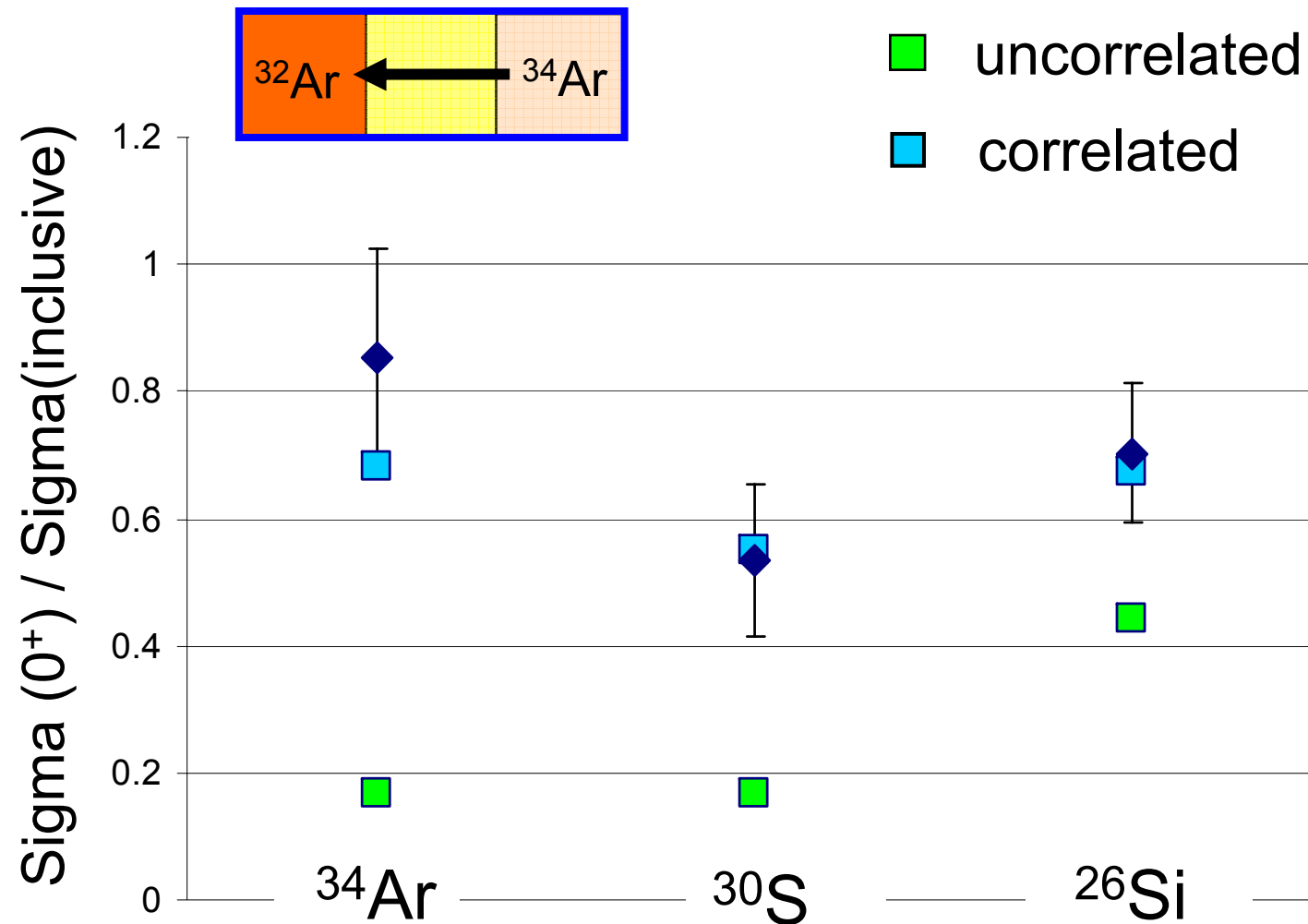
Knockout cross sections – correlated case



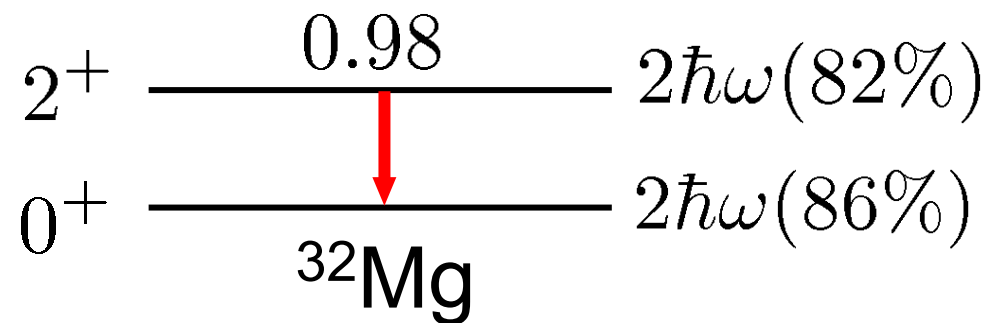
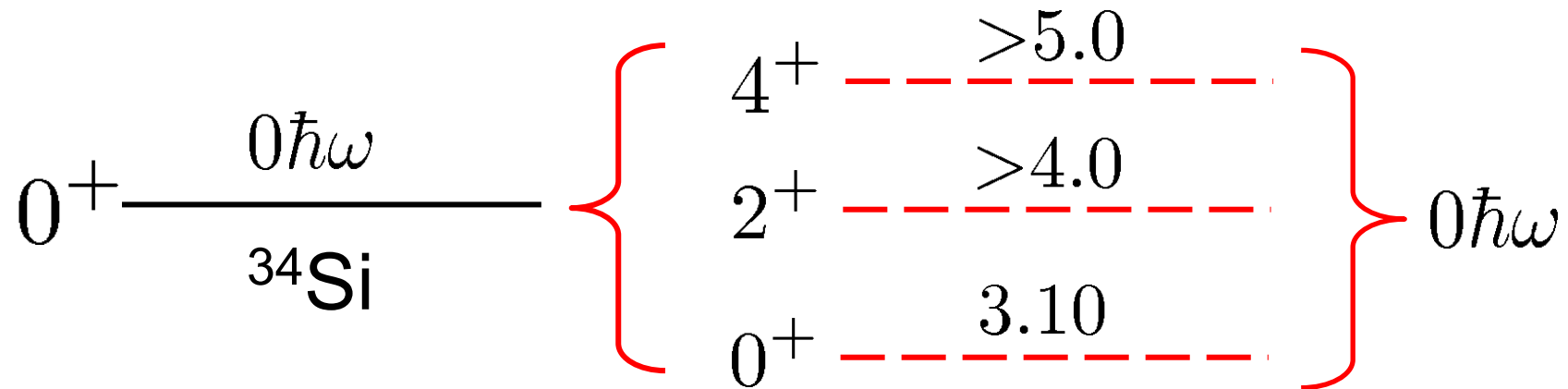
$$\sigma_{inc}(-2p) = 1.50(10) \text{ mb}, \quad R_s(2N) = \sigma_{\text{exp}}/\sigma_{\text{th}} = 0.52(4)$$



Two-neutron removal – g.s. branching ratios

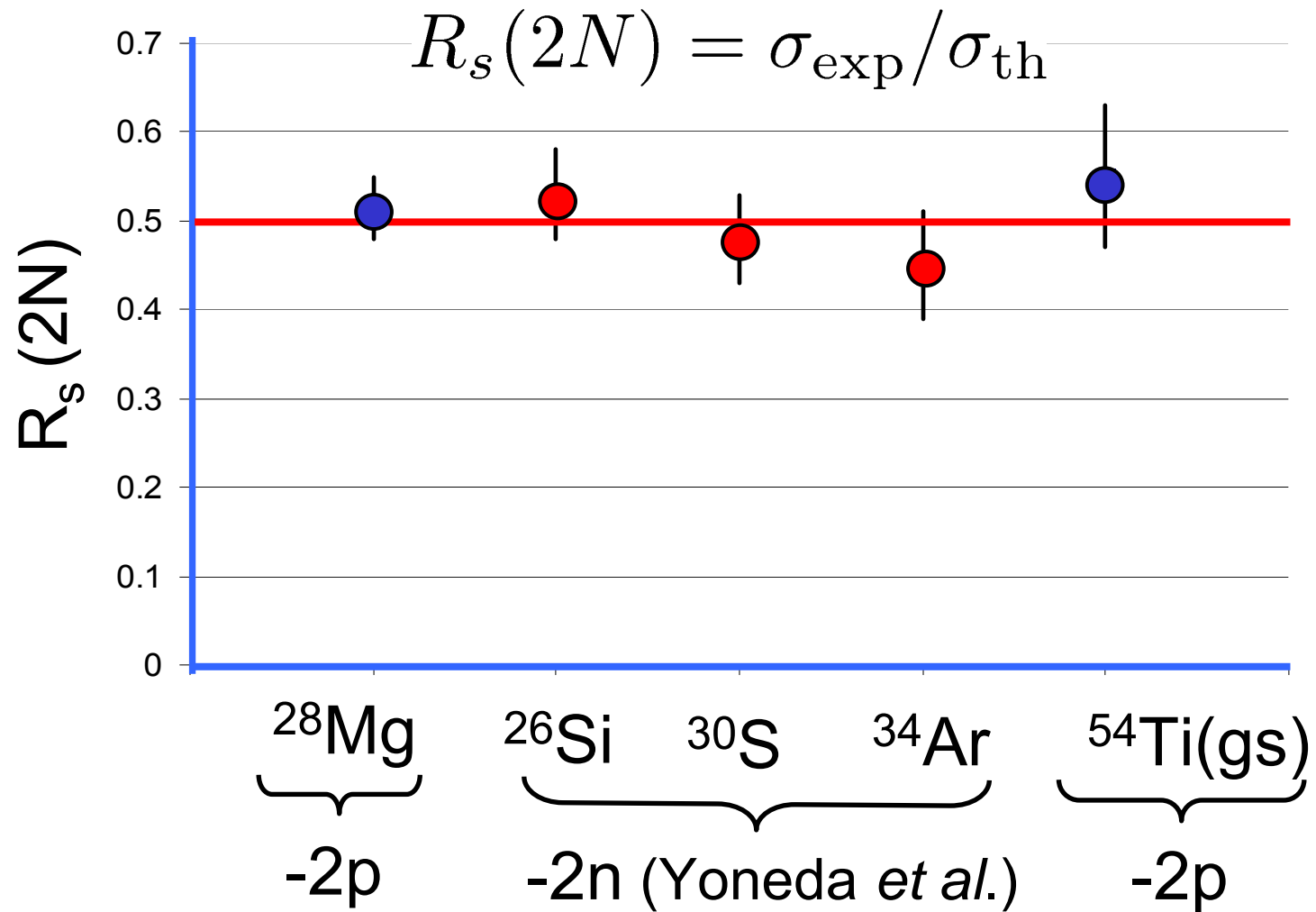


Rapid structural change: $^{34}\text{Si} \rightarrow ^{32}\text{Mg}$, $S_{2p}=33.6 \text{ MeV}$ ⁴⁰



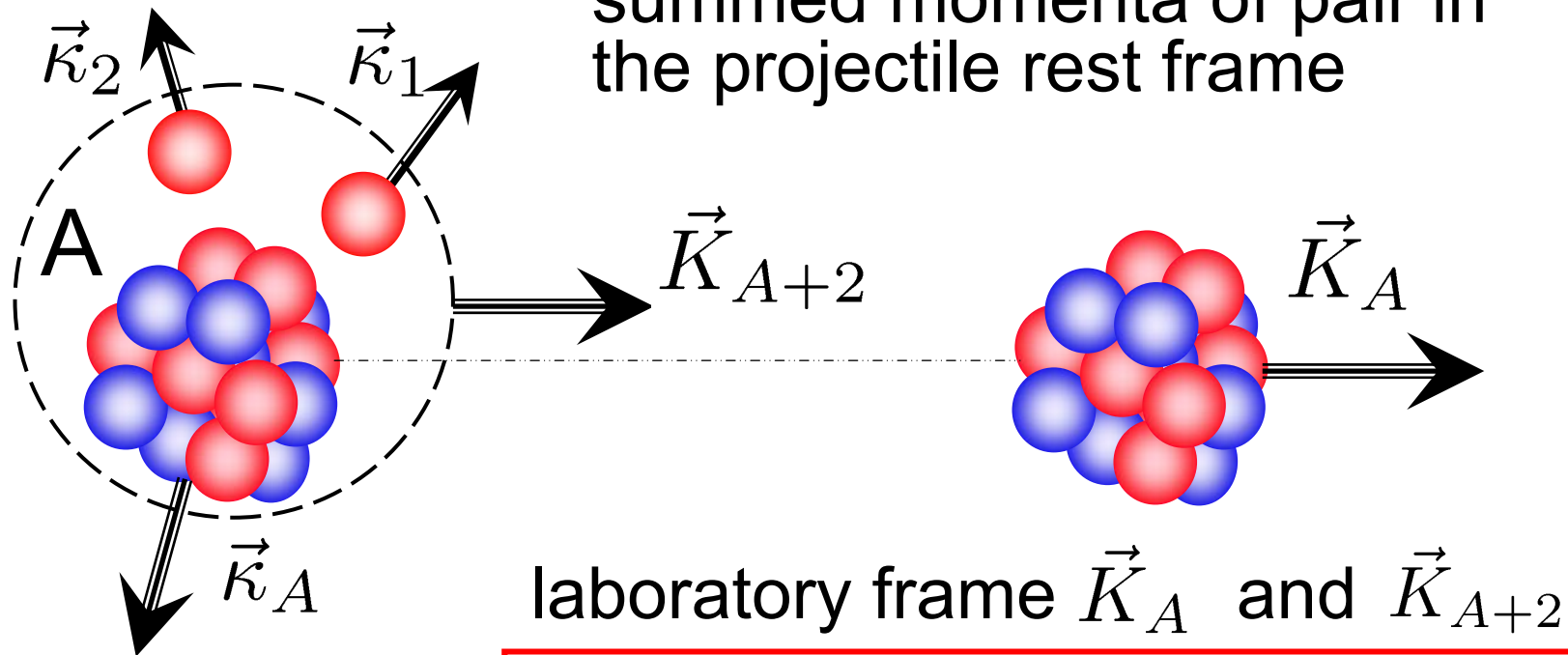
MCSM

Two-nucleon removal – suppression - $R_s(2N)$



Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame

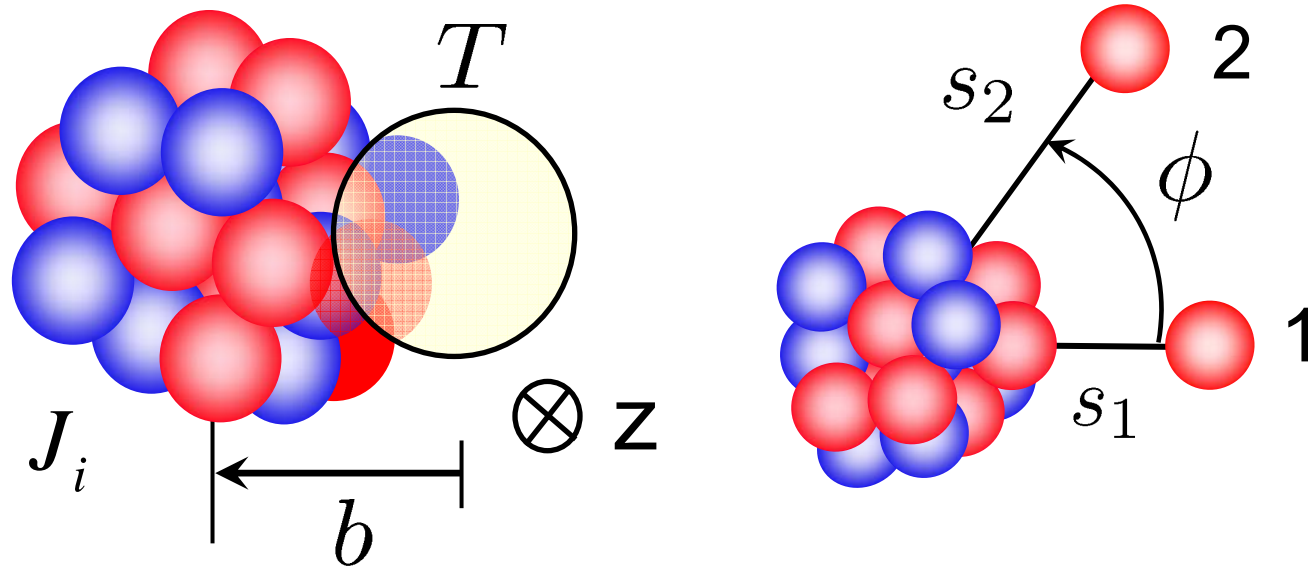


Projectile rest frame

$$\vec{K}_A = \frac{A}{A+2} \vec{K}_{A+2} - [\vec{k}_1 + \vec{k}_2]$$

and component equations

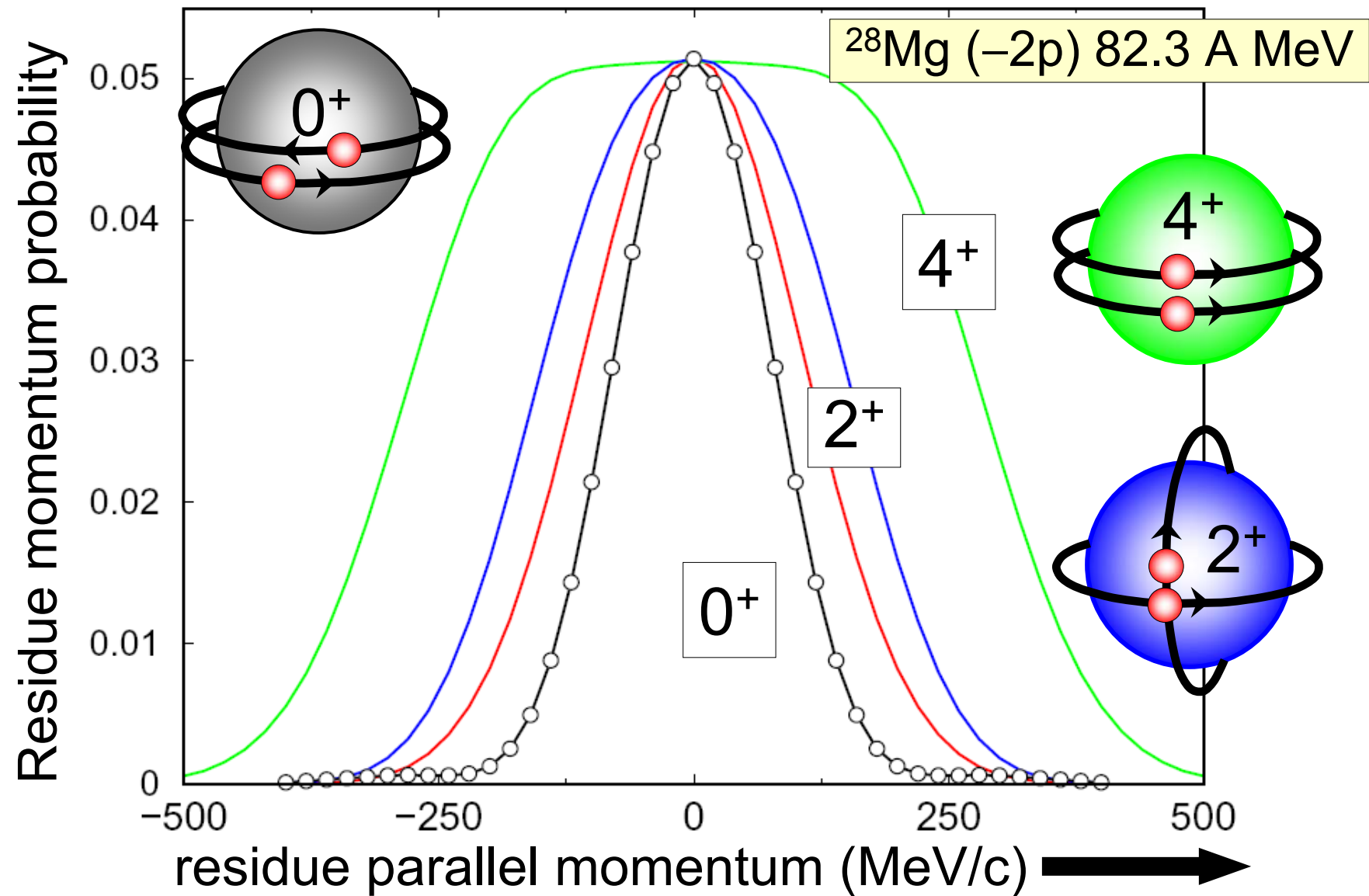
Look at momentum content of sampled volume



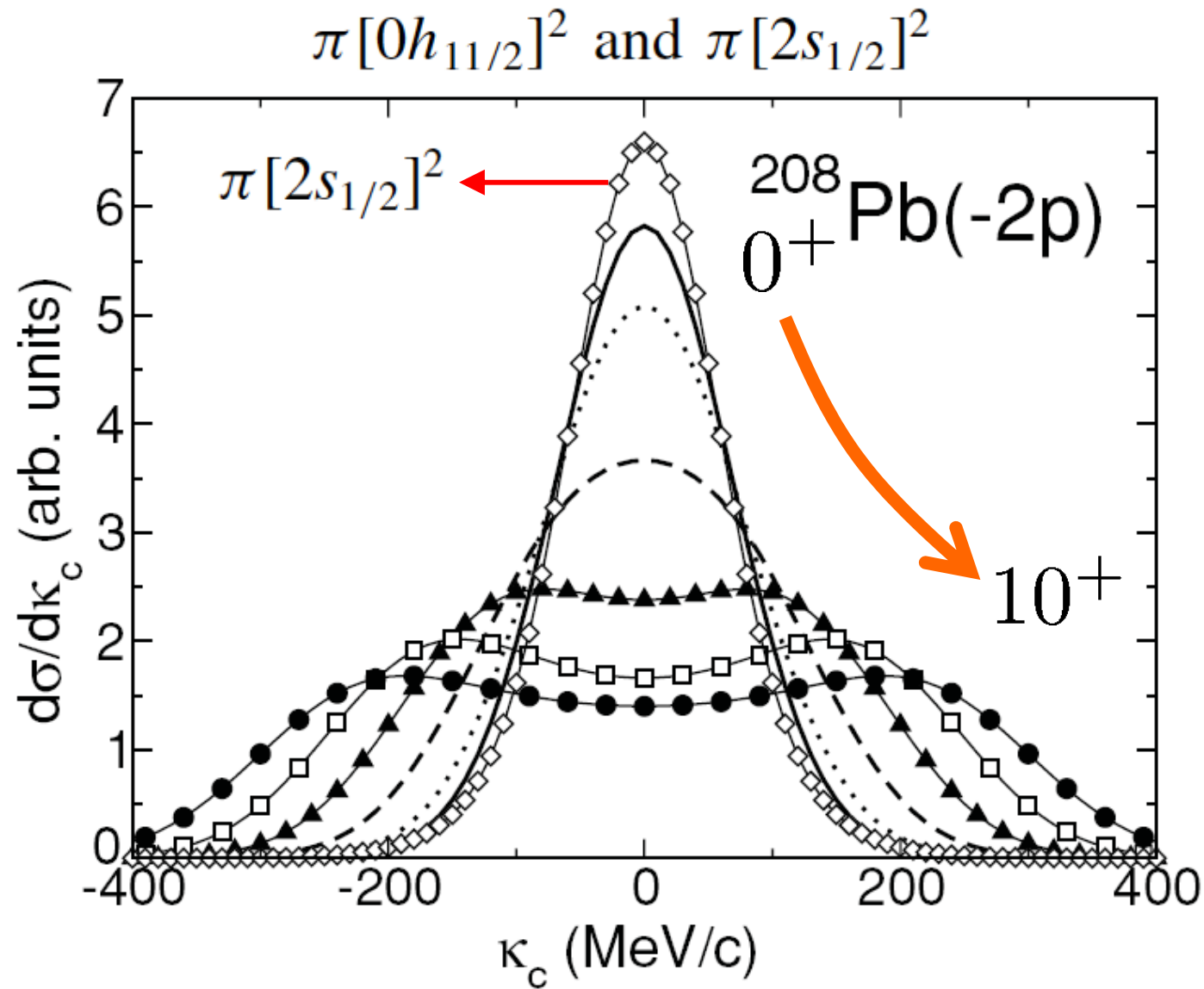
Probability of a residue with parallel momentum K

$$P(K, \vec{s}_1, \vec{s}_2) = \sum_M \left\langle \int dk_1 \int dk_2 \delta(K + k_1 + k_2) \right. \\ \left. \times \left| \int dz_1 \int dz_2 e^{ik_1 z_1} e^{ik_2 z_2} F_{JM}(1, 2) \right|^2 \right\rangle_{sp}$$

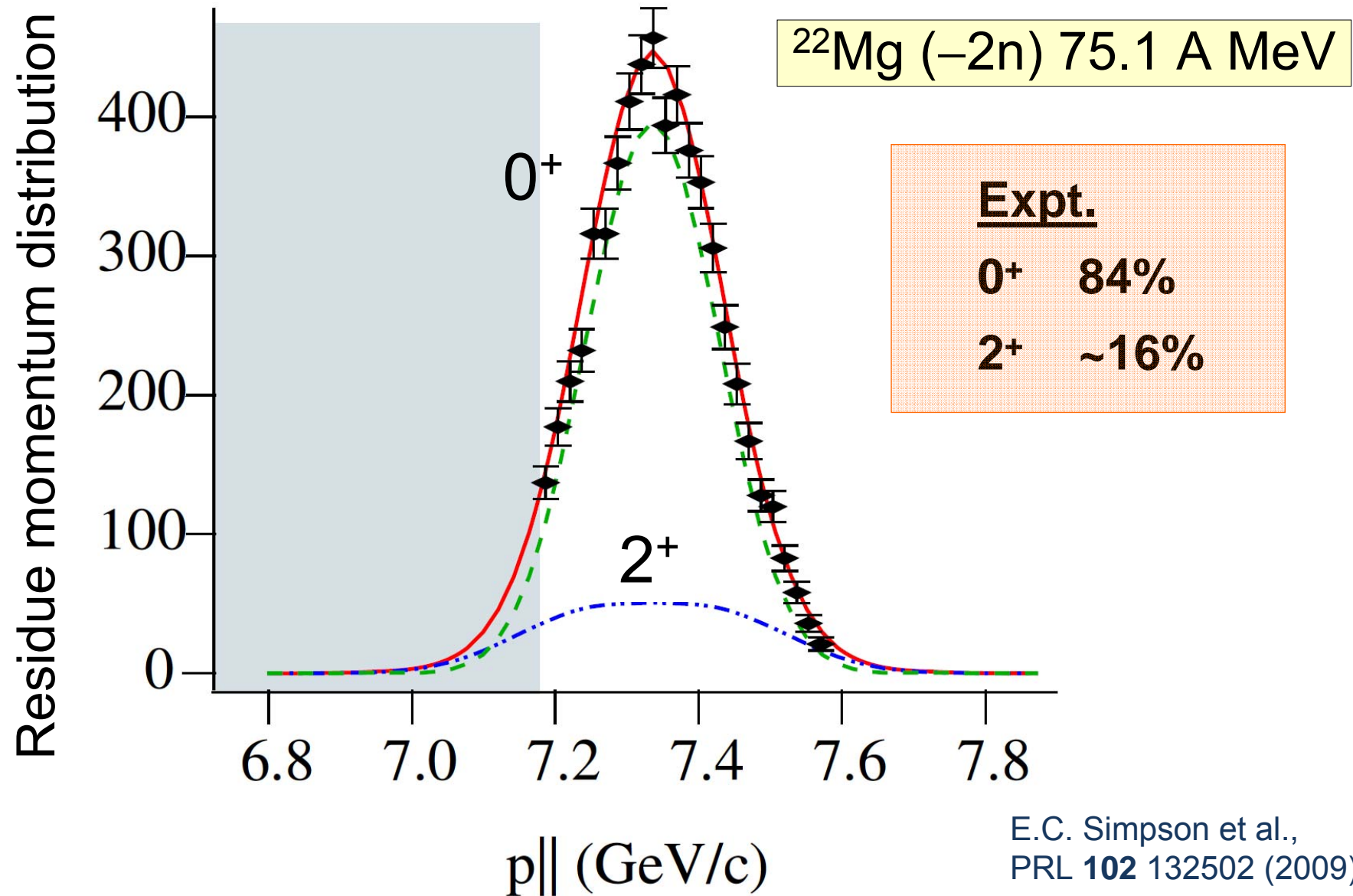
Two nucleon KO – predicted p_{\parallel} J-dependence



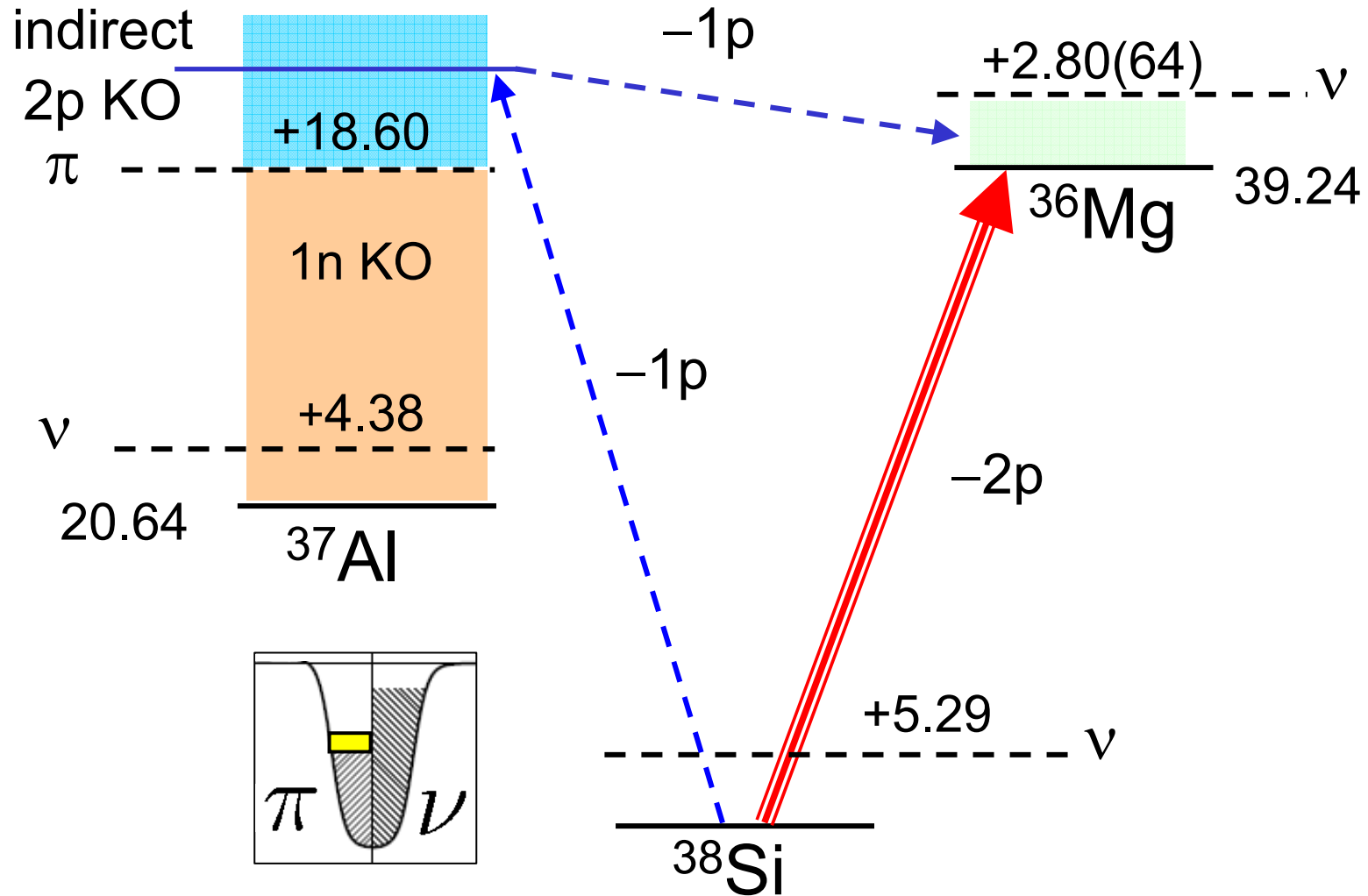
I-dependence of 2N removal p// distributions



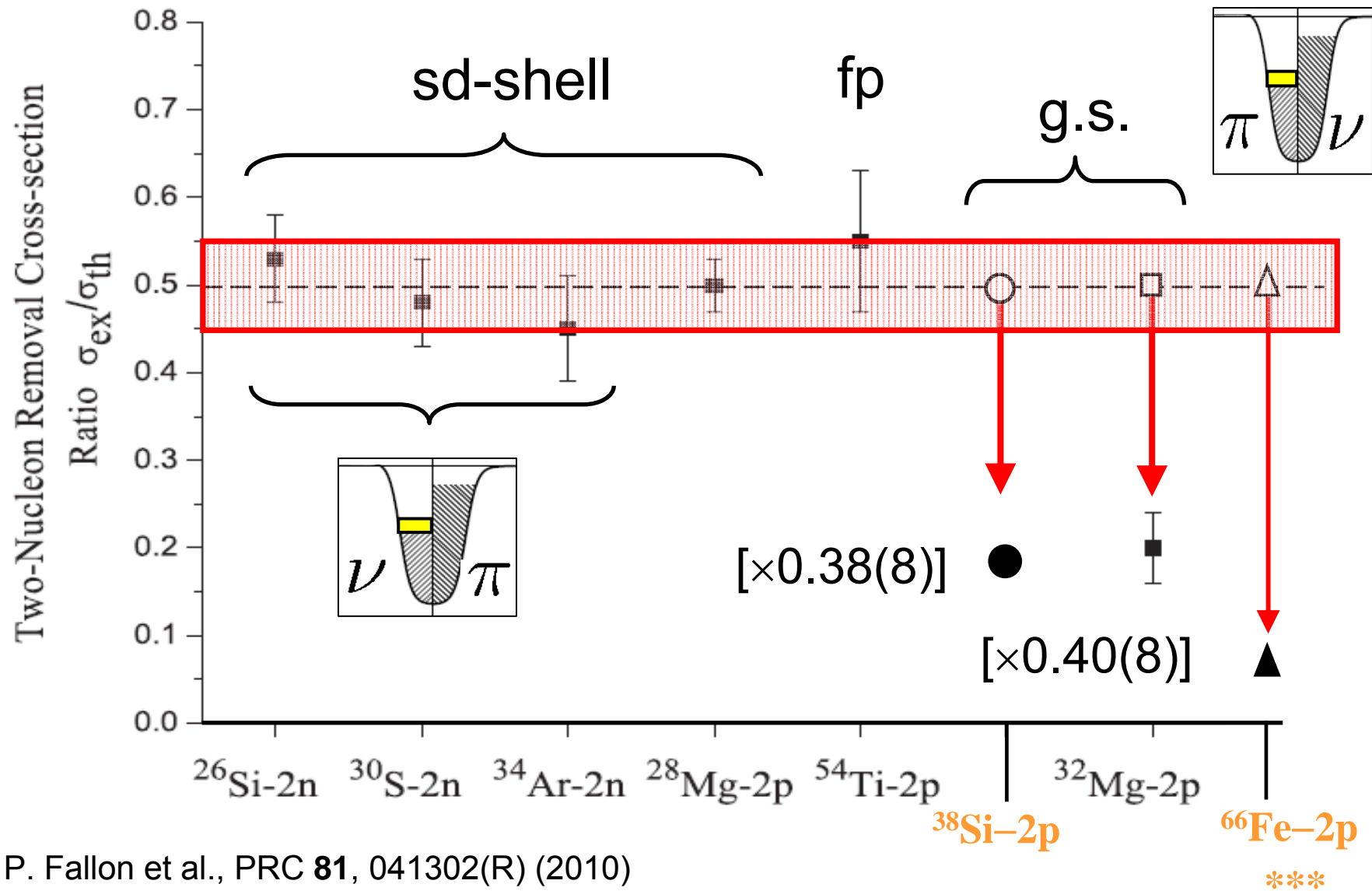
Two neutron knockout from $^{22}\text{Mg} \rightarrow ^{20}\text{Mg}(0^+, 2^+)$



Direct two-proton removal reaction mechanism



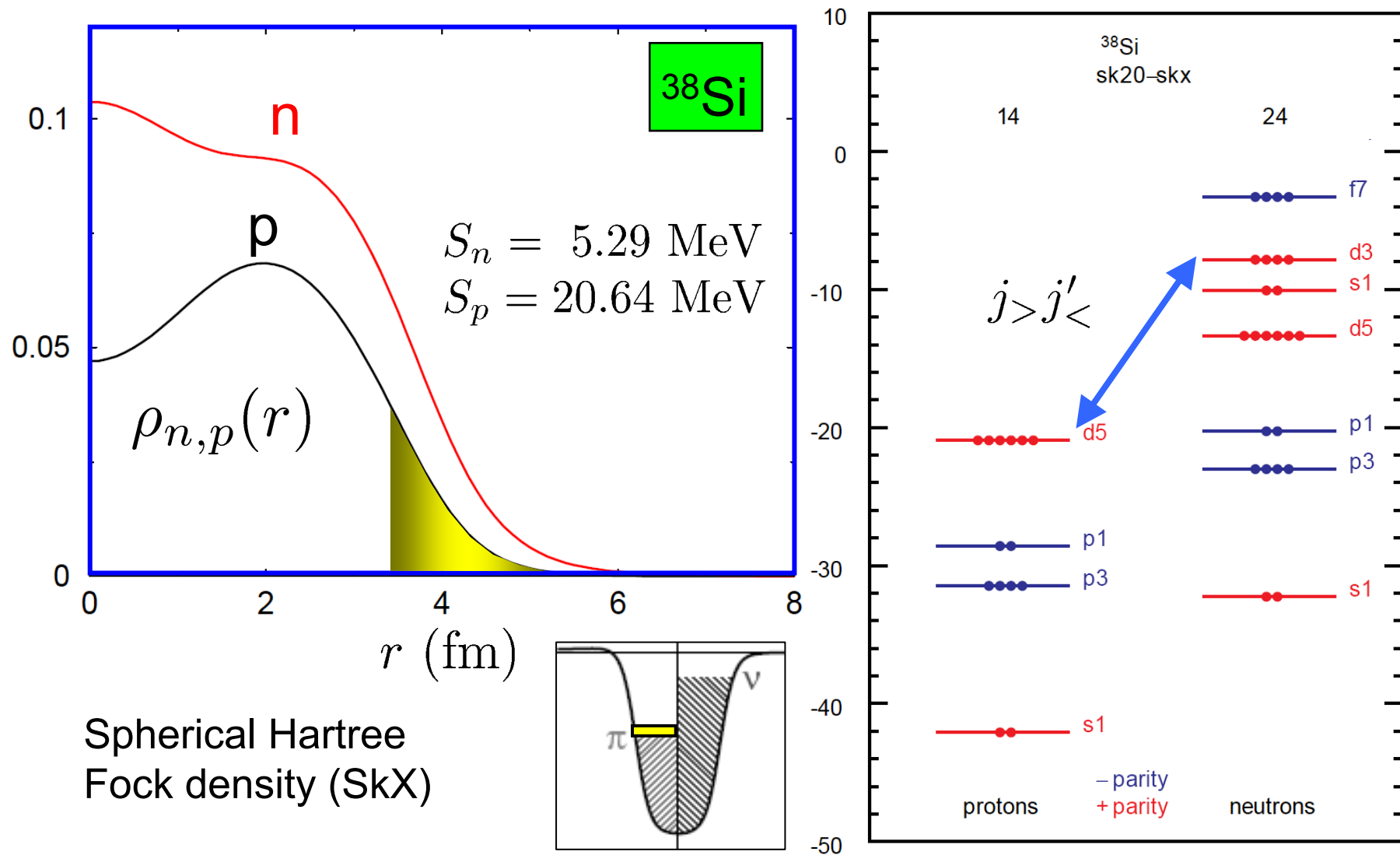
Mapping rapid changes of structure: a challenge



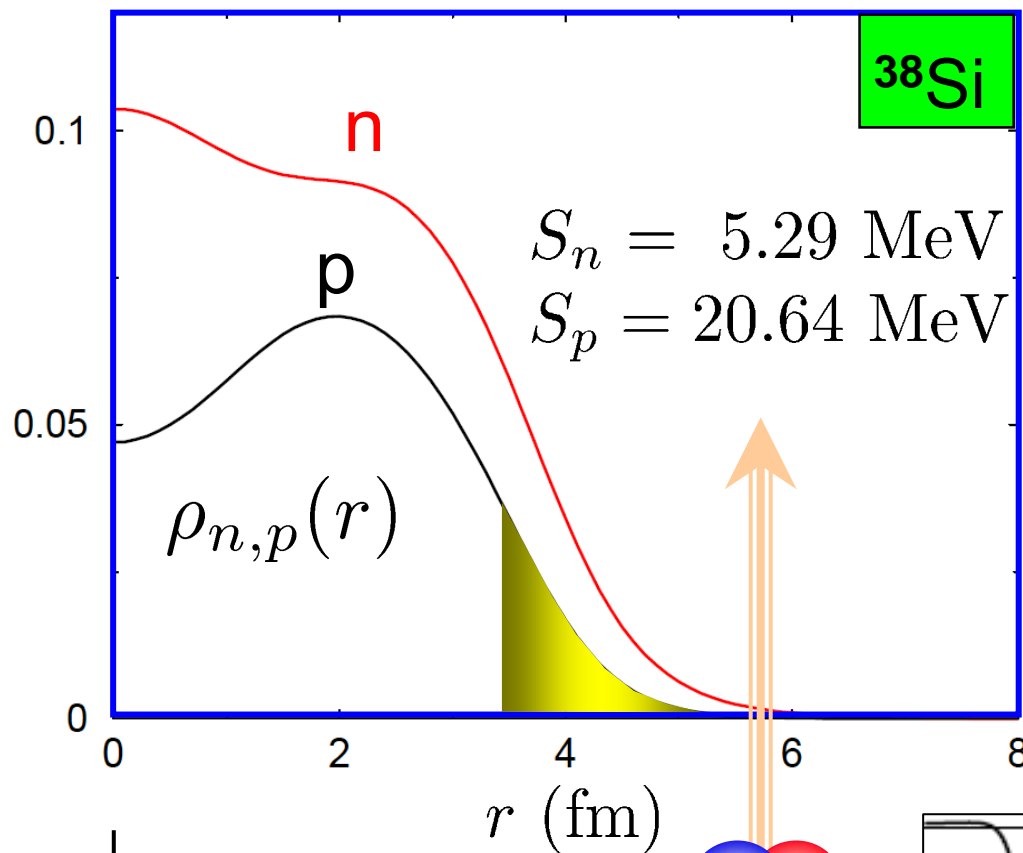
P. Fallon et al., PRC **81**, 041302(R) (2010)

P. Adrich et al., PRC **77**, 054306 (2008) ***

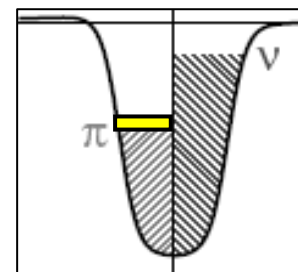
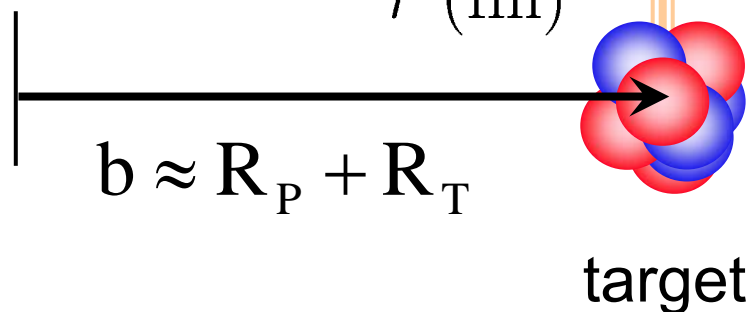
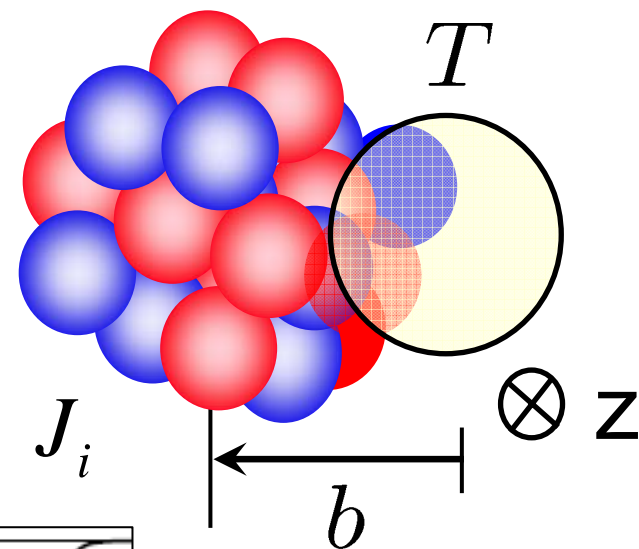
Otsuka: the np interaction tensor correlation



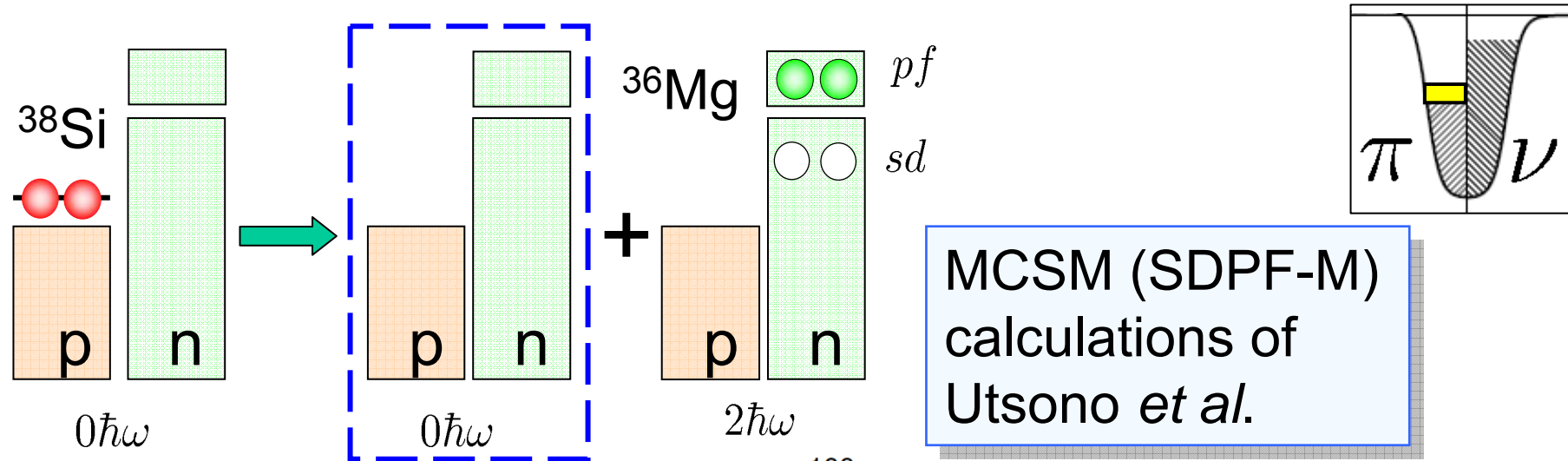
Removal probes single-nucleon wave functions



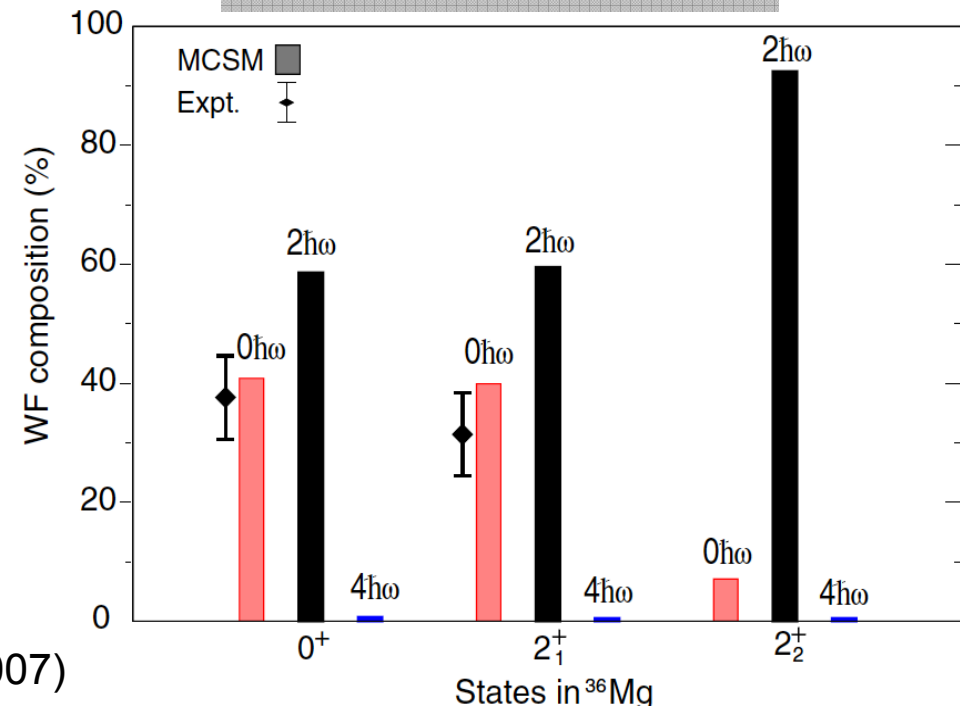
Interaction with the target probes wave functions at surface



The onset of deformation: as seen in $^{38}\text{Si}(-2p)$

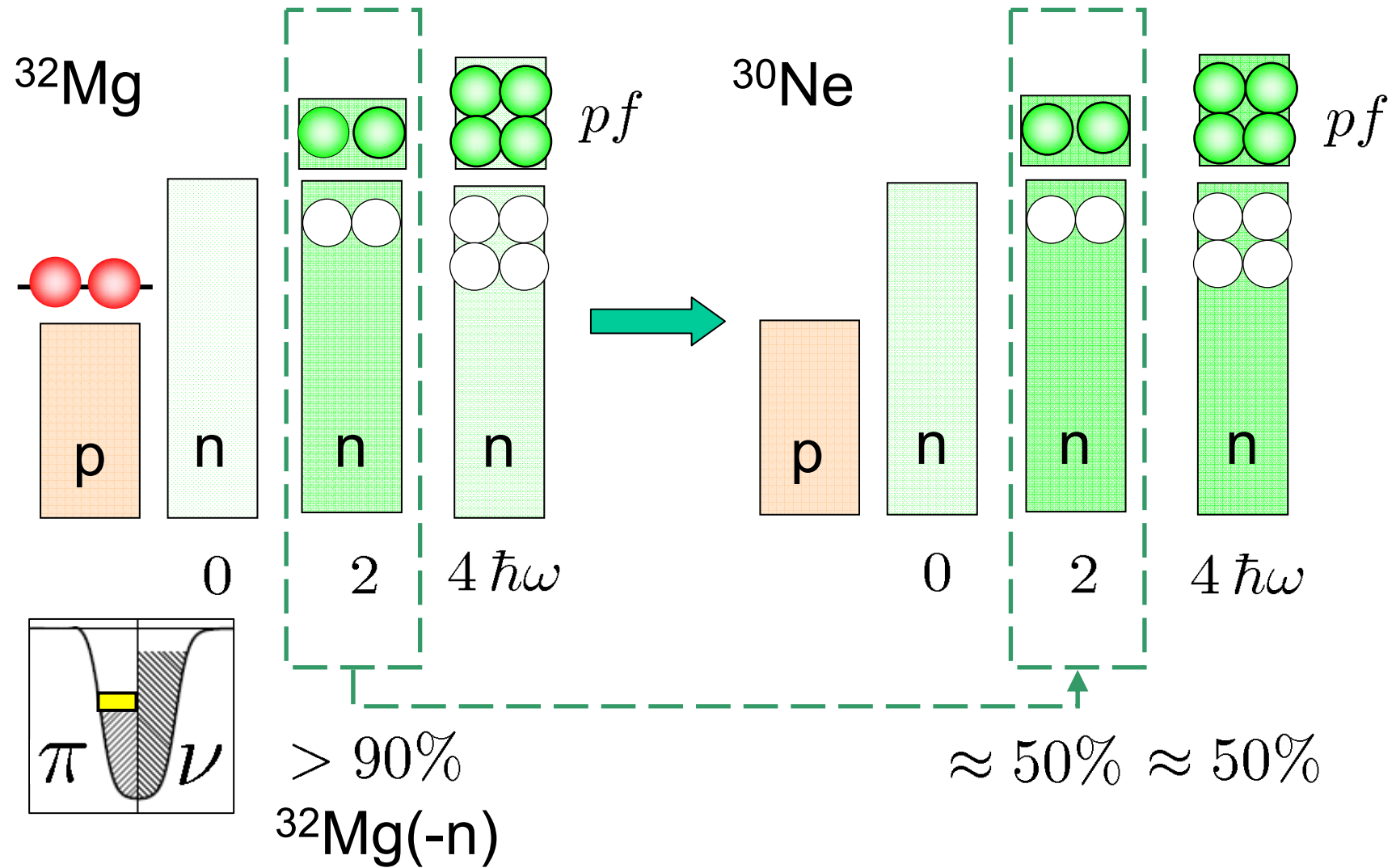


Measured cross sections are a fraction of those computed when assuming only $0\hbar\omega$ initial and final states. ^{38}Si is (reasonably) $0\hbar\omega$ so transition is to $0\hbar\omega$ components of final states (0^+ , 2^+ populated)

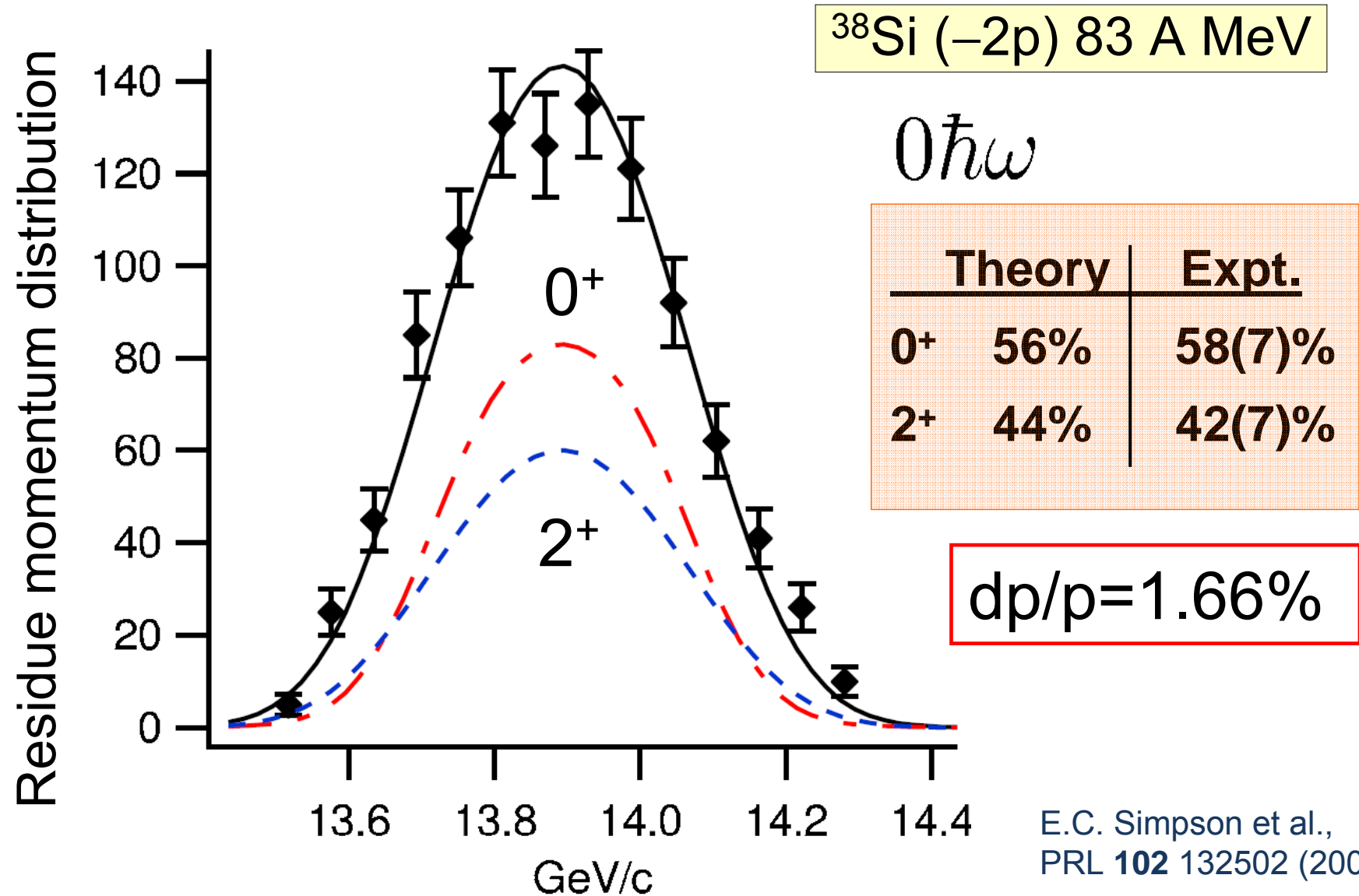


A. Gade et al., Phys. Rev. Lett. **99**, 072502 (2007)

Challenge for (spherical) shell model at $^{32}\text{Mg}(-2p)$



Two proton knockout from $^{38}\text{Si} \rightarrow ^{36}\text{Mg}(0^+, 2^+)$



Testing the (spherical) shell model at $^{32}\text{Mg}(-2p)$

TABLE II. Ground state neutron $0p0h$, $2p2h$, $4p4h$ probabilities (%) for ^{32}Mg , ^{30}Ne , $^{29,31}\text{F}$ calculated using the SDPF-M interaction in the Monte Carlo shell model code.

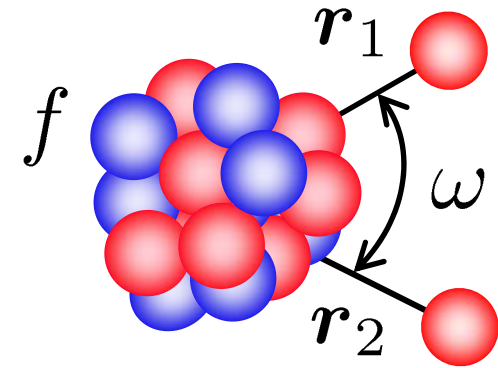
	$0p0h$	$2p2h$	$4p4h$
^{32}Mg	4.7	82.5	12.7
^{30}Ne	3.9	74.1	22.0
		0.61	0.03
		0.64	
^{32}Mg	5.0	95.0	0.0
^{30}Ne	5.0	47.5	47.5
		0.45	0.0
		0.45	Expt: 0.40(8)

Probing two-nucleon position correlations

After summing over the nucleon spins (to which we are insensitive) the two nucleon joint-position probability is:

$$\rho_f(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\hat{J}_i^2} \sum_{M_i M_f} \langle \Psi_i^{(F)} | \Psi_i^{(F)} \rangle_{sp}$$

$$\mathcal{P}_f(\mathbf{s}_1, \mathbf{s}_2) = \int dz_1 \int dz_2 \rho_f(\mathbf{r}_1, \mathbf{r}_2)$$



J_f^π	$[1p_{3/2}]^2$	$[1p_{1/2}, 1p_{3/2}]$	$[1p_{1/2}]^2$
1_1^+	0.69899	0.97868	-0.01067
1_2^+	-1.13385	0.22886	0.36314

J_f^π	σ_{01}	σ_{10}	σ_{11}	σ_{21}	σ_{str}
1_1^+	2.41	0.00	0.00	0.06	2.47
1_2^+	0.60	0.59	0.00	0.63	1.81

σ_{LS} (mb)

Two-nucleon correlations

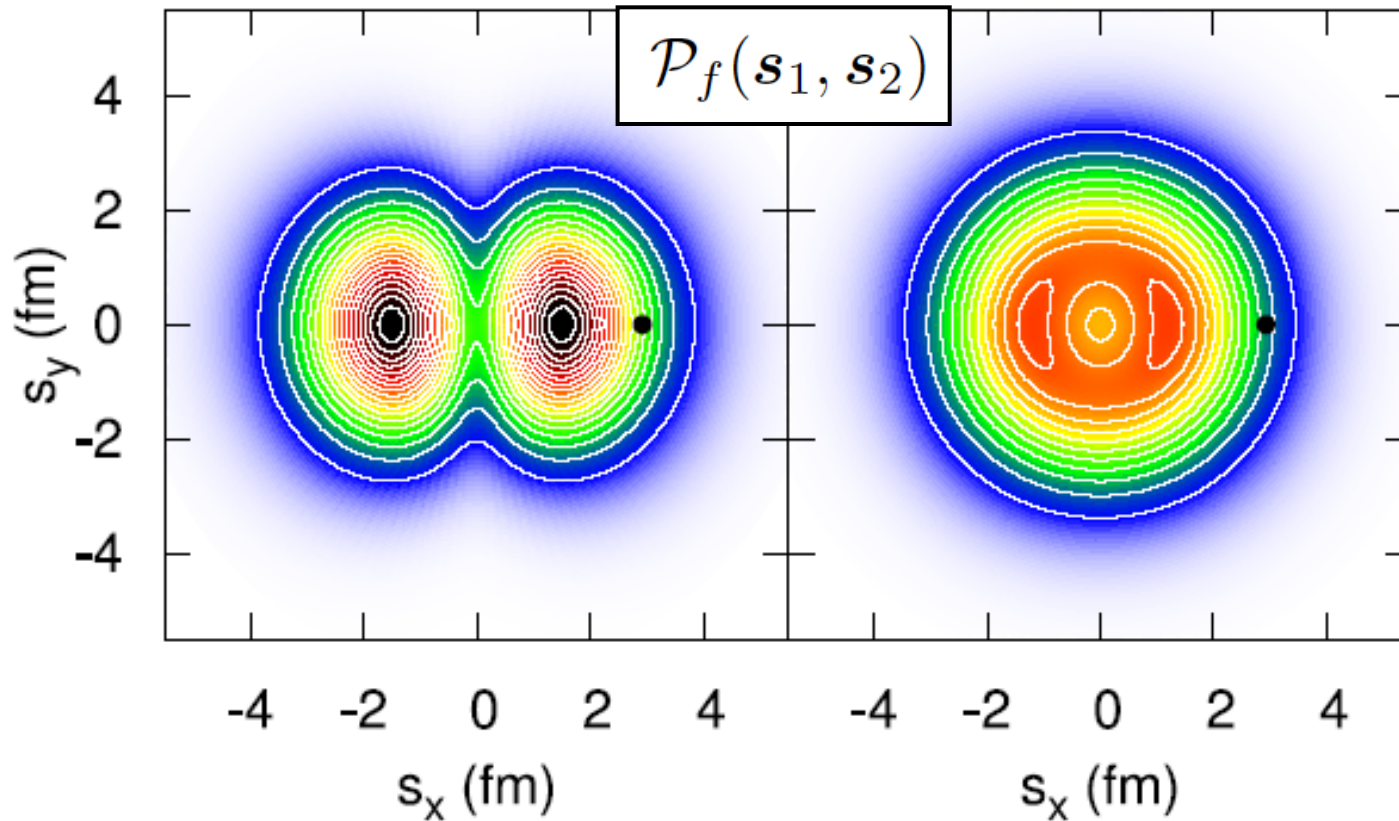


FIG. 4: Impact parameter plane-projected joint position probabilities for the first (left) and second (right) $T = 0$ $^{10}\text{B}(1^+)$ states populated via np knockout from ^{12}C .

Two-nucleon correlations

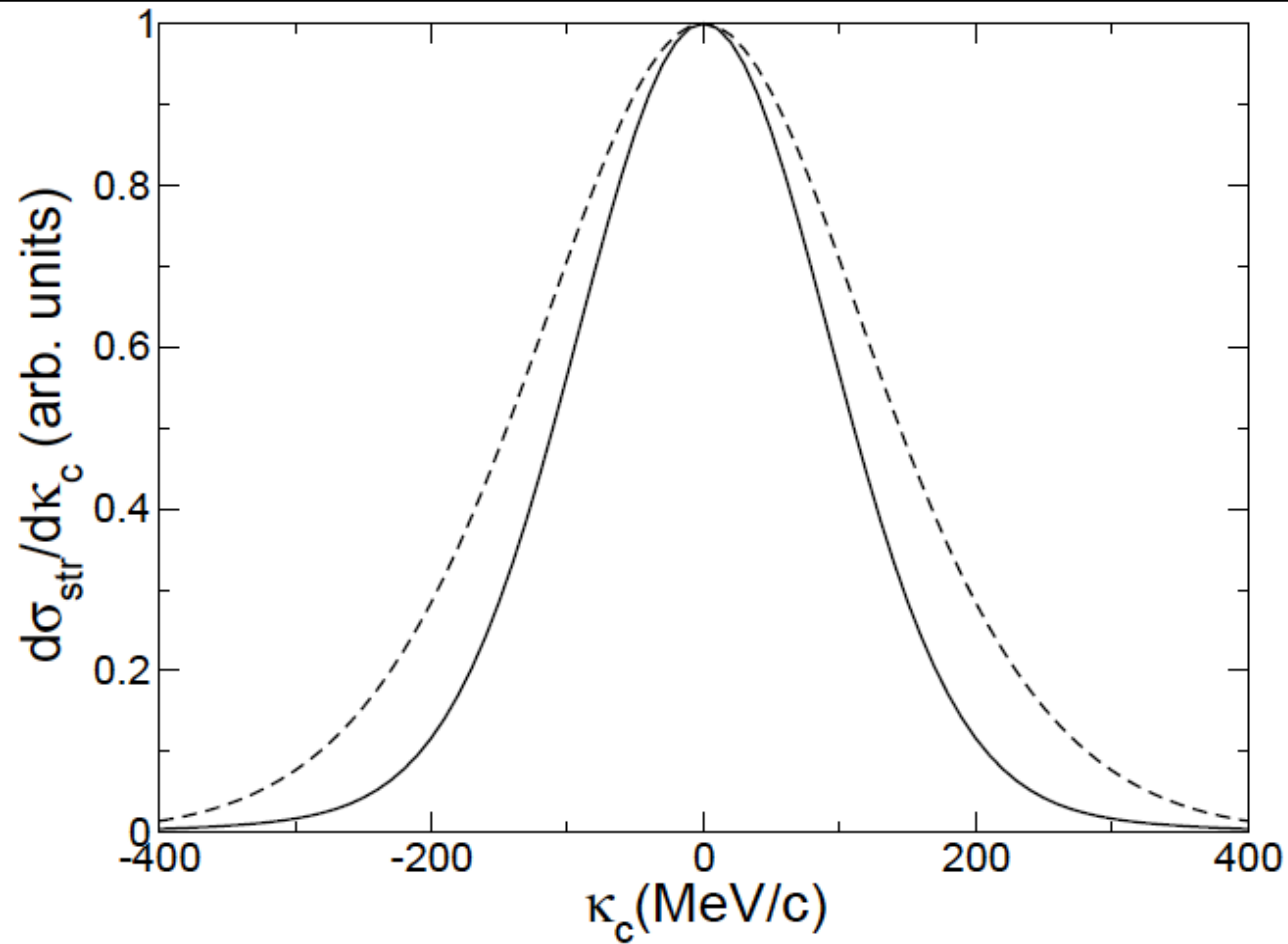


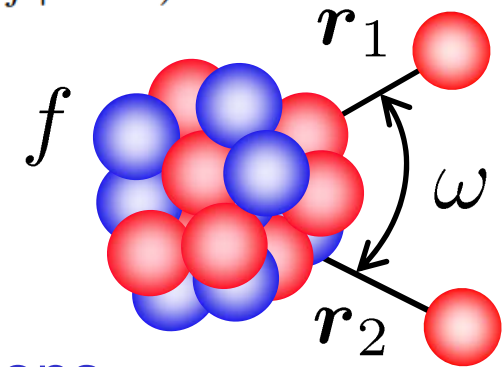
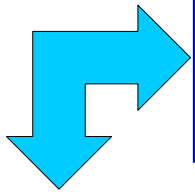
FIG. 3: Normalized residue momentum distributions for the first (solid) and second (dashed) $^{10}\text{B}(J_f=1^+)$ states populated in np knockout from ^{12}C at 2100 MeV per nucleon.

Angular correlations – and L-transfer sensitivity

After summing over the nucleon spins (to which we are insensitive) the two nucleon joint-position probability is:

$$\rho_f(\mathbf{r}_1, \mathbf{r}_2) = \sum_{LST} \sum_{I\alpha\alpha'} \frac{\mathfrak{C}_{\alpha LS}^{IT} \mathfrak{C}_{\alpha' LS}^{IT} D_{\alpha} D_{\alpha'}}{\hat{L}^2} (T\tau T_f \tau_f | T_i \tau_i)^2$$

$$\times \left[U_{\alpha\alpha'}^D(r_1, r_2) \Gamma^{L,D}(\omega) - (-)^{S+T} U_{\alpha\alpha'}^E(r_1, r_2) \Gamma^{L,E}(\omega) \right]$$



depends only on $L (= \ell_1 + \ell_2)$ of the two nucleons.

Structure calculation tells us strength of the L-content of the 2N overlap via the LS coupled two-nucleon amplitudes:

$$\mathfrak{C}_{\alpha LS}^{IT} = \hat{j}_1 \hat{j}_2 \hat{L} \hat{S} \left\{ \begin{matrix} \ell_1 & s & j_1 \\ \ell_2 & s & j_2 \\ L & S & I \end{matrix} \right\} C_{\alpha}^{IT} \longrightarrow \text{predict } p// \text{ distribution}$$

Final-state spin-value sensitivity: e.g. $^{54}\text{Ni}(-2n)$

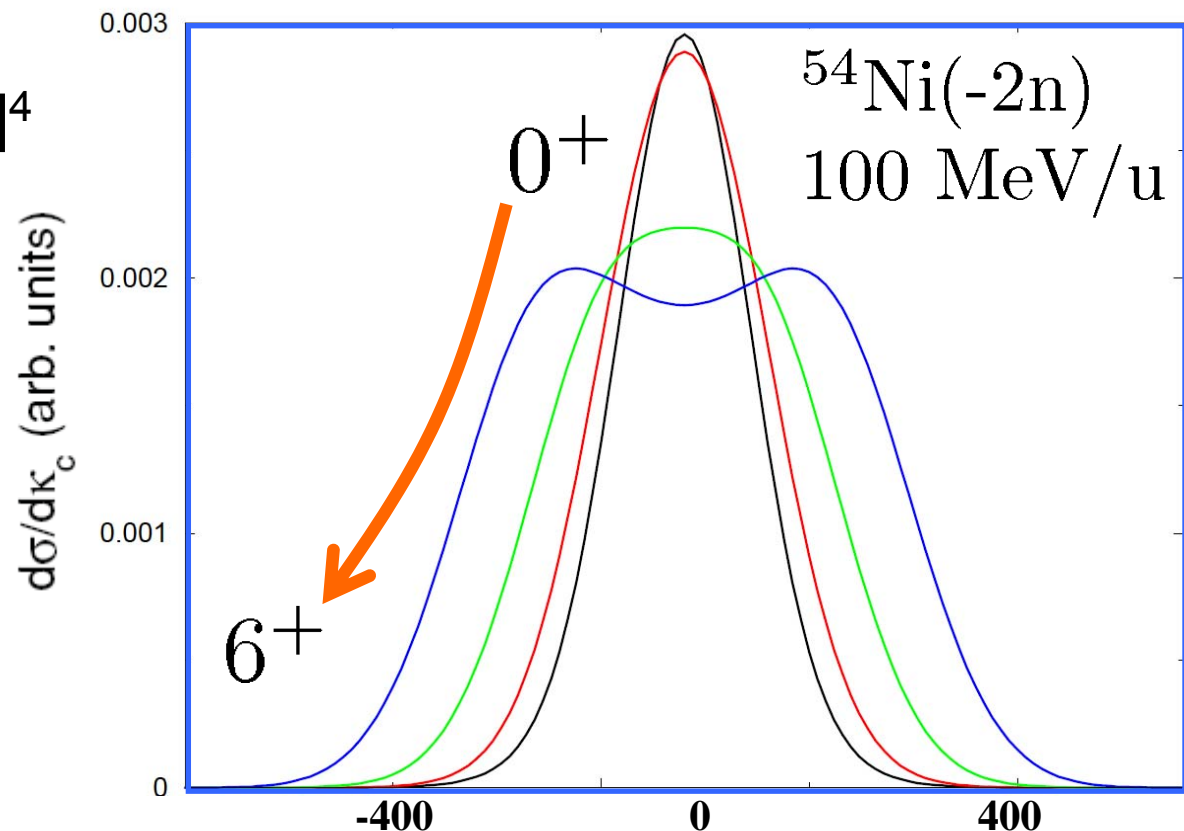
$$e_{\alpha LS}^{IT}$$



Relatively 'pure' 2N configurations give simple L (and I) – dependences – e.g. assuming $[f_{7/2}]^6 \rightarrow [f_{7/2}]^4$

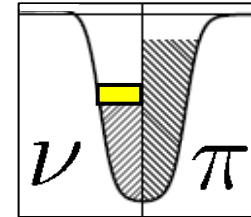
I	L-values
0	L= 0 , 1
2	L= 1 , 2 , 3
4	L= 3 , 4 , 5
6	L= 5 , 6

I	C (I=L)	C (L=I+1)	C (L=I-1)
0	0.571	0.428	0.0
2	0.510	0.122	0.367
4	0.367	0.034	0.598
6	0.142	0.0	0.857



Configuration-mixed, sd-shell example: $^{26}\text{Si}(-2n)$

J_f^π	$[1d_{5/2}]^2$	$[1d_{5/2}, 1d_{3/2}]$	
2_1^+	-0.70074	0.43499	
2_2^+	-0.38021	-0.12354	
	$[1d_{3/2}]^2$	$[2s_{1/2}, 1d_{3/2}]$	$[2s_{1/2}, 1d_{5/2}]$
	0.00594	-0.00188	-0.02781
	-0.12945	-0.15876	-0.58292



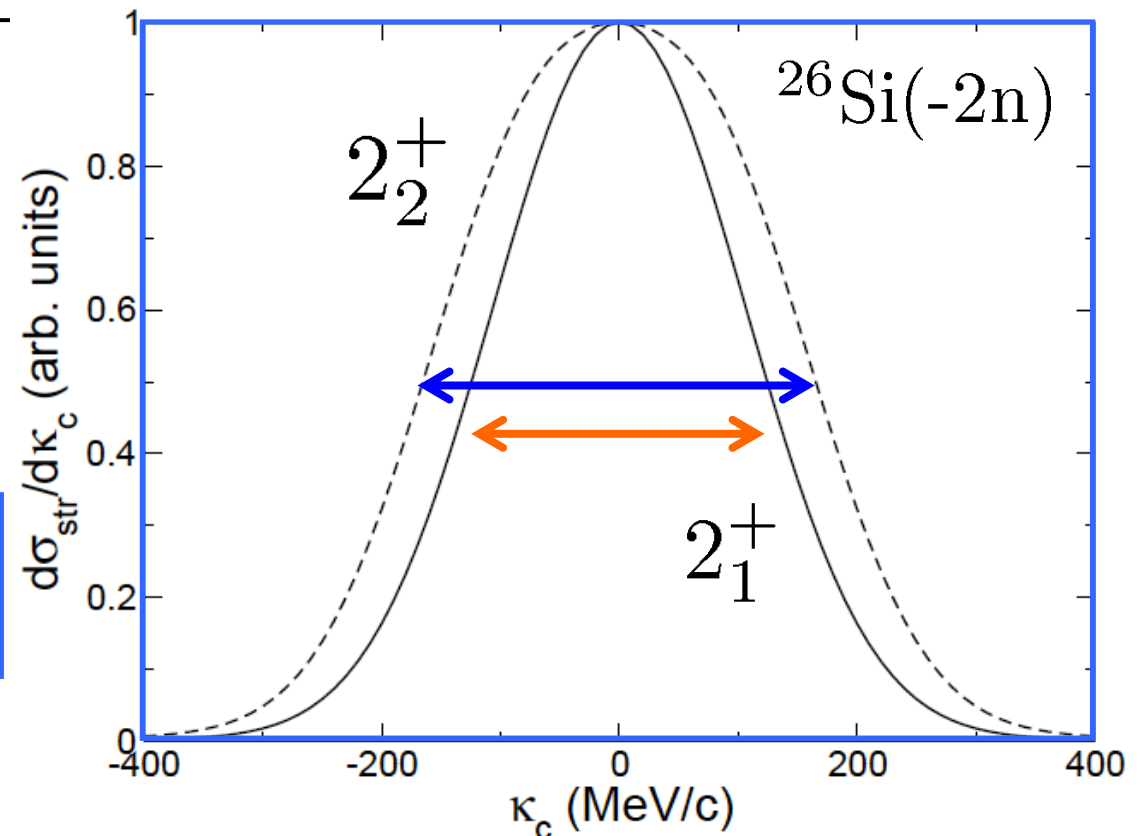
with cross sections:

σ_{LS} (mb)

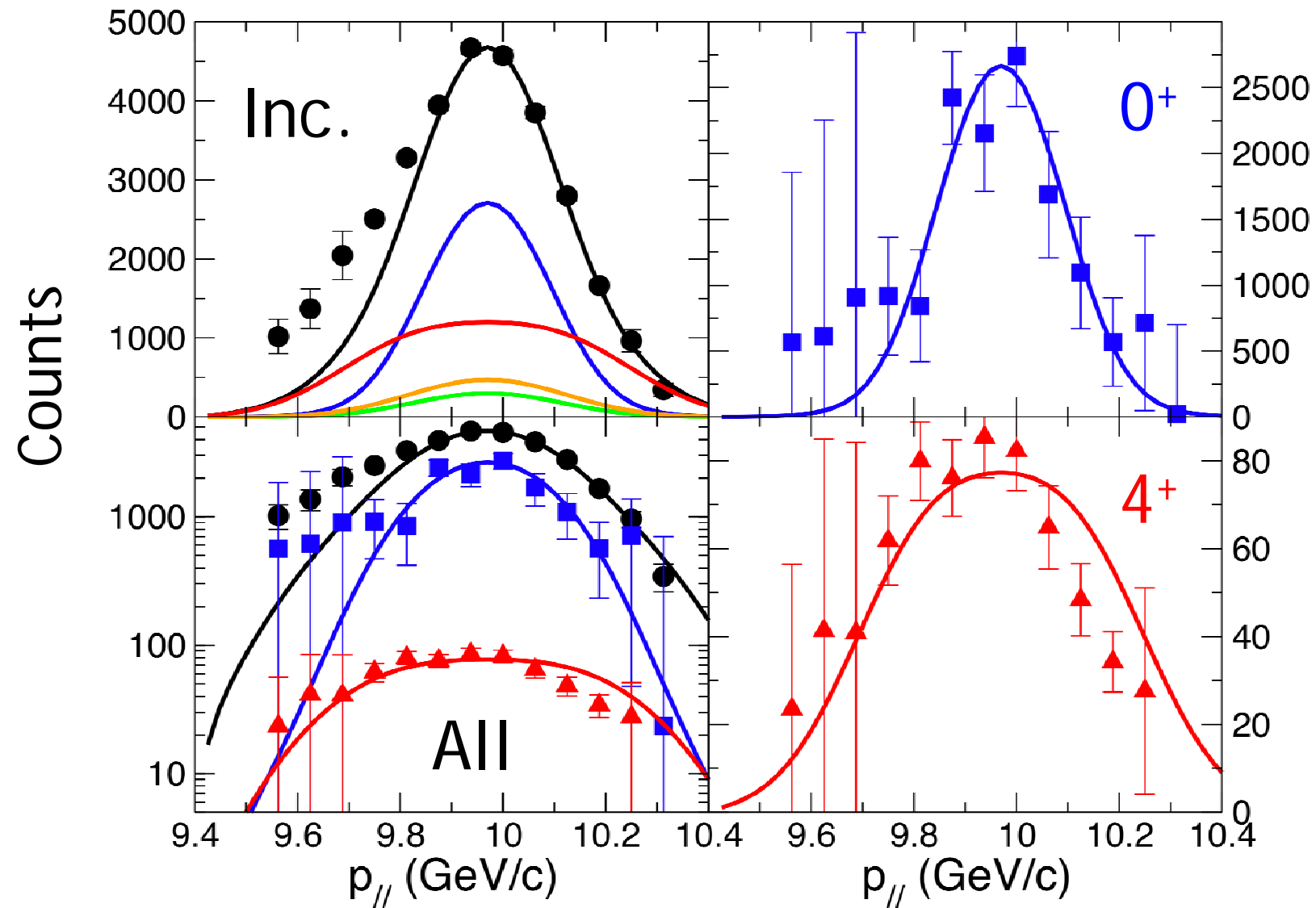
J_f^π	σ_{11}	σ_{20}	σ_{21}
2_1^+	0.17	0.02	0.00
2_2^+	0.01	0.17	0.01

$L = 1$
250 MeV/c

$L = 2$
330 MeV/c



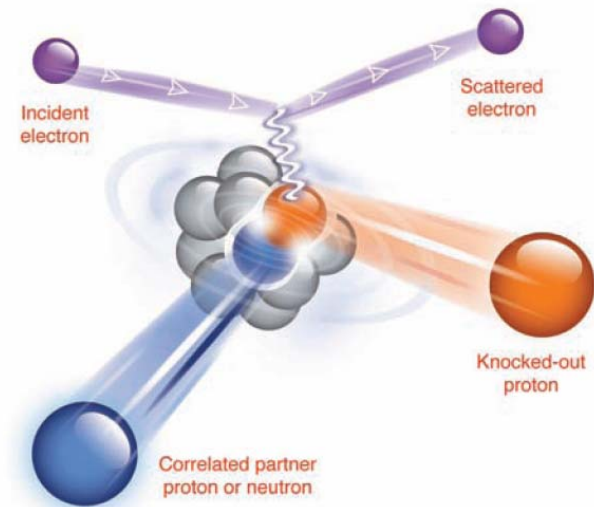
First final-state-exclusive p//: $^{28}\text{Mg}(-2p)$



np correlations are in the 'news' – stable nuclei

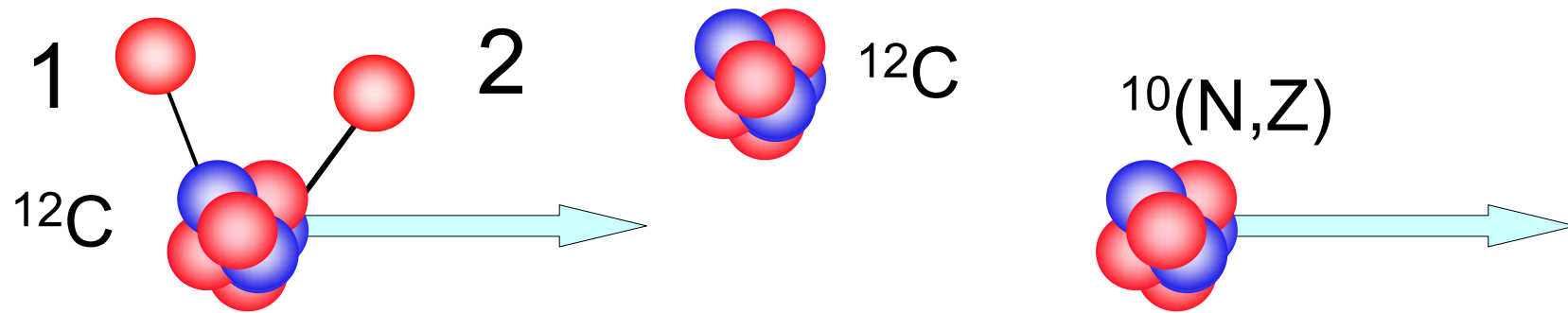
Probing Cold Dense Nuclear Matter

R. Subedi,¹ R. Shneor,² P. Monaghan,³ B. D. Anderson,¹ K. Aniol,⁴ J. H. Benaoum,^{7,8} F. Benmokhtar,⁹ W. Boeglin,¹⁰ J.-P. Chen,¹¹ Seonho B. Craver,¹⁴ S. Frullani,¹³ F. Garibaldi,¹³ S. Gilad,³ R. Gilman,^{11,15} C. J.-O. Hansen,¹¹ D. W. Higinbotham,^{11*} T. Holmstrom,¹⁷ H. Ibrahim,¹ C. W. de Jager,¹¹ E. Jans,²⁰ X. Jiang,¹⁵ L. J. Kaufman,^{9,21} A. Kelleher G. Kumbartzki,¹⁵ J. J. LeRose,¹¹ R. Lindgren,¹⁴ N. Liyanage,¹⁴ D. J. A. P. Markowitz,¹⁰ S. Marrone,²³ M. Mazouz,²⁴ D. Meekins,¹¹ R. Michael C. F. Perdrisat,¹⁷ E. Piasezky,² M. Potokar,²⁵ V. Punjabi,²⁶ Y. Qiang G. Ron,² G. Rosner,²⁷ A. Saha,¹¹ B. Sawatzky,^{14,28} A. Shahinyan,²⁹ S. P. Solvignon,²⁸ V. Sulkosky,¹⁷ G. M. Urciuoli,¹³ E. Voutier,²⁴ J. W. W. B. Wojtsekhowski,¹¹ S. Wood,¹¹ X.-C. Zheng,^{3,6,14} L. Zhu³¹



The protons and neutrons in a nucleus can form strongly correlated nucleon pairs. Scattering experiments, in which a proton is knocked out of the nucleus with high-momentum transfer and high missing momentum, show that in carbon-12 the neutron-proton pairs are nearly 20 times as prevalent as proton-proton pairs and, by inference, neutron-neutron pairs. This difference between the types of pairs is due to the nature of the strong force and has implications for understanding cold dense nuclear systems such as neutron stars.

In two nucleon removal/knockout data - one sees



Energy/nucleon	250 MeV	1.05 GeV	2.1 GeV

10Be Experimental	5.88	5.30 (30)	5.81 (29)
10C Experimental	5.33 (81)	4.44 (24)	4.11 (22)
.....			
10B Experimental	47.5 (24)	27.9 (22)	35.1 (34)

Based simply on combinatorics - and removal of the (4) $p_{3/2}$ nucleons we might expect scalings of:

$$\begin{array}{ll} \sigma \text{ (pp or nn)} & [4 \times 3]/2 = 6 \\ \sigma \text{ (np)} & 4 \times 4 = 16 \end{array}$$

A couple of take-away messages

1. It is possible with rather 'simple' measurements to
 - (i) track the positions of single particle states at both Fermisurfaces in asymmetric nuclei, and thus
 - (ii) study the evolution of the many-body system as a function of its isospin new shell structures,
 - (iii) assess the predictive power of modern shell model calculation (e.g. for nuclear astrophysics applications at extremes of N and Z) and the quality of CI model effective interactions.
2. We can understand/predict that exclusive residue momentum measurements, following two-nucleon removal, will be an excellent probe of predicted wave functions at an increasingly detailed level.