# TWO-NUCLEON KNOCKOUT SPECTROSCOPY

A probe of pair correlations in rare nuclei?

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**Abstract.** The cross sections for the knockout of two correlated nucleons from light nuclei are considered. These reactions may offer a means for populating and identifying low-lying states of very exotic nuclear species and also a probe of two-nucleon spectroscopy and correlations in rare nuclei. A calculation scheme that combines the full shell model two-nucleon spectroscopic amplitudes with eikonal reaction theory is discussed. The predictions of the method are compared with existing data on inclusive two-neutron and two-proton removal from <sup>12</sup>C and new, more exclusive data on two-proton removal from <sup>28</sup>Mg. The combined shell model structure amplitudes and reaction dynamics predictions in these exploratory studies are in good agreement with the available measurements.

### 1. Introduction

Single-nucleon knockout reactions, both with [1, 2, 3, 4] and without [5, 6, 7] coincident gamma-ray detection, have been the subject of numerous studies. Since their first application to the phosphorus isotopes [1] single-nucleon knockout experiments (with gamma-ray detection) have been tested extensively and used to study the single-nucleon spectroscopy of light [8, 9, 10, 11, 12, 4, 13] and medium-mass nuclei [14, 15, 16], in particular the relative and absolute single-nucleon spectroscopy of neutron- and proton-rich nuclei [17, 14, 13, 15, 16]. Recent reviews can be found in [3, 2, 11].

Single-nucleon knockout reactions using fast intermediate energy exotic beams in inverse kinematics are highly peripheral reactions. The residual nuclei, having had one nucleon removed via the diffractive dissociation (elastic breakup) or stripping (target absorption) mechanisms on a thick, absorptive light nuclear target, are detected in the extreme forward direction with velocities close to those of the incident beam particles. The method has been demonstrated to be highly sensitive [3]. Analyses of such data using eikonal few-body reaction theory have also been shown to yield results of high accuracy [18, 19, 20, 7] and with practical advantages over alternative direct reaction approaches [11]. This is allowing a systematic study of effective interaction theory predictions, such as from the shell-model, and of correlation effects on weakly and strongly-bound neutron-and proton-orbitals in nuclei.

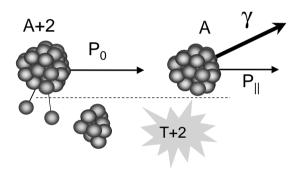


Figure 1. Schematic of the inelastic breakup mechanism in the peripheral, sudden two-nucleon knockout reaction. The A-body residue is left in one of a number of final states and the target nucleus is excited by the two removed nucleons, denoted by T+2.

Generalization of these methods to two-nucleon knockout, Fig. 1, and the magnitudes of the associated cross sections are of interest for two reasons. First, to populate and identify the ground- and low-lying excited-states of exotic asymmetric nuclei. Second, as a potential spectroscopic probe of two-nucleon correlations in exotic systems and hence a means to assess modern nuclear structure calculations of these effects. It was recently proposed that two-proton removal reactions from nuclei on the slightly-neutron-rich side of the valley of beta stability, at high energy, do proceed as direct processes. The separation energies and nucleon evaporation thresholds in such systems suggest very strongly that direct two-proton removal will be the only significant path to bound residue final states. Compelling experimental evidence was also offered by both the inclusive cross section and the parallel momentum distribution of the reaction residues measured in the two-proton knockout from <sup>28</sup>Mg [21].

Unlike single-nucleon knockout spectroscopy, two-nucleon removal reaction theories do not factorize simply into a structural (spectroscopic) factor

and a dynamical (single-particle) cross section. The reaction dynamics and structure are now strongly linked, the reaction amplitudes being a coherent linear superposition of the contributing two-nucleon configuration terms, e.g. [22]. In this paper we present an eikonal model scheme for the calculation of the stripping (inelastic breakup) contribution to two-nucleon removal. Our approach combines the two-nucleon spectroscopic amplitudes from the shell-model with a generalization of the few-body eikonal-based reaction theory. The latter has been discussed extensively in the case of one-nucleon knockout reactions [3, 11].

These formal developments are presented in the next section and the quantitative theoretical predictions of the model are then compared with existing inclusive measurements for <sup>12</sup>C [23] and the new more exclusive data for two-proton knockout from <sup>28</sup>Mg [21].

### 2. Formalism

We discuss two-nucleon knockout from a primary or secondary projectile beam at intermediate energy. The projectile is an antisymmetrized A+2 nucleon system with many-body wave function  $\Psi(A,1,2)$ . Specifically,  $\Psi(A,1,2)$  represents the shell model ground state of the nuclei of the beam with total angular momentum and isospin  $J_i$  and  $T_i$  and projections  $M_i$  and  $\tau_i$ . Following the sudden removal of two-nucleons in a peripheral high speed collision with the target, the A-body residual (or core) nucleus will be found in one of a set of final states  $\Phi(A)$  with spin and isospin  $J_f$  and  $T_f$  with projections  $M_f$ ,  $\tau_f$ . The isospin and angular momentum couplings involved are summarized in Fig. 2 where the two active removed nucleons are assumed to couple to an total angular momentum eigenstate  $I, \mu$  with total isospin  $T, \tau$ .

# 2.1. TWO-NUCLEON AMPLITUDES

The two removed nucleons (1 and 2) are assumed to be stripped from a set of active and partially occupied single-particle orbitals  $\phi_j$ . These have spherical (shell model) single-particle quantum numbers  $n(\ell s)j, m$ .

The shell model two-nucleon overlap functions of these two nucleons in the projectile ground state, relative to a specified residue or core nucleus final state f, is a coherent sum over the contributing two-particle configurations, as

$$\Psi_{J_{i}M_{i}}^{(f)}(1,2) \equiv \langle \Phi_{J_{f}M_{f}}(A)|\Psi_{J_{i}M_{i}}(A,1,2)\rangle 
= \sum_{I\mu\alpha} C_{\alpha}^{J_{i}J_{f}I} (I\mu J_{f}M_{f}|J_{i}M_{i}) [\overline{\phi_{j_{1}}(1)\otimes\phi_{j_{2}}(2)}]_{I\mu} , \quad (1)$$

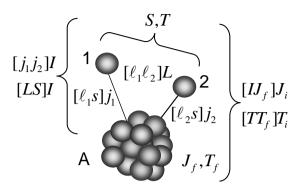


Figure 2. Schematic of the angular momentum couplings used in the description of the two-nucleon knockout reaction.

where  $\alpha \equiv \{n_1\ell_1j_1, n_2\ell_2j_2\}$  labels the available nucleon-orbital pairs which contribute. In this equation

$$[\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I\mu} = -N_{12}\langle 1, 2| \left[ a_{j_1}^{\dagger} \otimes a_{j_2}^{\dagger} \right]_{I\mu} |0\rangle 
= D_{\alpha} \sum_{m_1 m_2} (j_1 m_1 j_2 m_2 | I\mu) \left[ \phi_{j_1}^{m_1}(1) \phi_{j_2}^{m_2}(2) - \phi_{j_1}^{m_1}(2) \phi_{j_2}^{m_2}(1) \right]$$
(2)

is a normalized, antisymmetrized nucleon-pair wave function and  $D_{\alpha} = N_{12}/\sqrt{2} = 1/\sqrt{2(1+\delta_{12})}$ . So as not to complicate the notation we do not show the isospin labels and coupling explicitly but comment later on how to include these into the formalism, for completeness. The  $C_{\alpha}^{J_i J_f I}$  in Eq. (1) are the signed two-nucleon amplitudes which carry the structure calculation details; in particular, the information on the parentage and phase of each of the participating two-nucleon configurations in the projectile ground state with respect to the final states f of the residue.

### 2.2. EIKONAL MODEL OF TWO-NUCLEON STRIPPING

We will calculate the dominant stripping contribution to the two-nucleon removal cross section. This is the following projectile ground state average of the nucleon absorption and residue transmission factors,

$$\sigma_{str} = \frac{1}{2J_i + 1} \sum_{M_i} \int d\vec{b} \langle \Psi_{J_i M_i} | |\mathcal{S}_f|^2 (1 - |\mathcal{S}_1|^2) (1 - |\mathcal{S}_2|^2) |\Psi_{J_i M_i} \rangle , \quad (3)$$

integrated over all projectile center-of-mass (cm) impact parameters b. Here the  $S_i$  are the eikonal S-matrices [18] for the scattering of the two nucleons (1,2) and of the A-body residue f from the target. Each is a function of the impact parameter of that constituent. These  $S_i$  will be assumed to be spin-independent. This cross section expression reflects the stripping (inelastic breakup) mechanism in which the residue interacts at most elastically with the target, survives the collision, and escapes to infinity: reflected by  $|S_f|^2$ . The two removed nucleons interact inelastically with the target and are absorbed from the elastic channel: as described by their absorption probabilities  $(1 - |S_1|^2)$  and  $(1 - |S_2|^2)$ .

We make two reasonable but simplifying approximations. We first assume that the residue-target S-matrix is diagonal with respect to different final states f of the residue and that this diagonal interaction is the same as that for the residue ground state (denoted  $S_c$ ) for all final states f. This has been termed the spectator-core approximation when used in single-nucleon knockout [24]. It assumes that the amplitudes for dynamical excitation of the core during the collision are small.

We also neglect explicit recoil effects associated with the heavy mass A residue. It follows that

$$\langle \Phi_{J_f'M_f'}(A)||\mathcal{S}_f|^2|\Phi_{J_fM_f}(A)\rangle = |\mathcal{S}_c(b)|^2\delta_{ff'}\delta_{J_fJ_f'}\delta_{M_fM_f'}, \qquad (4)$$

with b the projectile cm impact parameter. As we consider here only nucleon knockout from deeply-bound single-particle states, we do not calculate other possible contributions to the two-nucleon removal cross section, which we assume are small. These involve diffraction dissociation processes in which one or both nucleons are dissociated from the projectile by their elastic collisions with the target.

Having made the spectator-core and the no-recoil approximation, the inclusive stripping cross section is the incoherent sum  $\sigma_{str} = \sum_f \sigma_{str}^{(f)}$  of the exclusive residue final state cross sections. Defining  $\hat{J}^2 = (2J+1)$ , these are

$$\sigma_{str}^{(f)} = \int d\vec{b} \, |\mathcal{S}_c|^2 \frac{1}{\hat{J}_i^2} \sum_{M_i} \langle \Psi_{J_i M_i}^{(f)} | (1 - |\mathcal{S}_1|^2) (1 - |\mathcal{S}_2|^2) | \Psi_{J_i M_i}^{(f)} \rangle , \qquad (5)$$

where the bra-ket denotes integration over the spatial coordinates of the two removed nucleons,  $\vec{r}_1$  and  $\vec{r}_2$ , and the integration over all spin variables, denoted by

$$\langle \Psi_{J_i M_i}^{(f)} | \dots | \Psi_{J_i M_i}^{(f)} \rangle = \int d\vec{r}_1 \int d\vec{r}_2 \langle \Psi_{J_i M_i}^{(f)} | \dots | \Psi_{J_i M_i}^{(f)} \rangle_s \quad . \tag{6}$$

Since all the particle-target S-matrices are assumed spin-independent we require only the spin-average of the two-nucleon wave functions, that is

$$\frac{1}{\hat{J}_{i}^{2}} \sum_{M_{i}} \langle \Psi_{J_{i}M_{i}}^{(f)} | \Psi_{J_{i}M_{i}}^{(f)} \rangle_{s} = \frac{1}{\hat{J}_{i}^{2}} \sum_{M_{f}M_{i}I\mu\alpha I'\mu'\alpha'} C_{\alpha'}^{J_{i}J_{f}I'} C_{\alpha}^{J_{i}J_{f}I} \\
\times (I'\mu'J_{f}M_{f}|J_{i}M_{i})(I\mu J_{f}M_{f}|J_{i}M_{i}) \\
\times \langle [\overline{\phi_{j_{1}'}(1) \otimes \phi_{j_{2}'}(2)}]_{I'\mu'} | [\overline{\phi_{j_{1}}(1) \otimes \phi_{j_{2}}(2)}]_{I\mu} \rangle_{s} .$$
(7)

Upon using Eqs. (1) and (2) this reduces to

$$\frac{1}{\hat{J}_{i}^{2}} \sum_{M_{i}} \langle \Psi_{J_{i}M_{i}}^{(f)} | \Psi_{J_{i}M_{i}}^{(f)} \rangle_{sp} = \sum_{I\alpha\alpha'} 2D_{\alpha}D_{\alpha'} \frac{C_{\alpha'}^{J_{i}J_{f}I}C_{\alpha}^{J_{i}J_{f}I}}{\hat{I}^{2}} \times \sum_{m_{1}m_{2}m'_{1}m'_{2}\mu} (j_{1}m_{1}j_{2}m_{2}|I\mu)(j'_{1}m'_{1}j'_{2}m'_{2}|I\mu) \times \left[ \langle \phi_{j'_{1}}^{m'_{1}} | \phi_{j_{1}}^{m_{1}} \rangle_{s} \langle \phi_{j'_{2}}^{m'_{2}} | \phi_{j_{2}}^{m_{2}} \rangle_{s} - \langle \phi_{j'_{1}}^{m'_{1}} | \phi_{j_{2}}^{m_{2}} \rangle_{s} \langle \phi_{j'_{2}}^{m'_{2}} | \phi_{j_{1}}^{m_{1}} \rangle_{s} \right]. \quad (8)$$

We will refer to terms from the first product in the last bracket as being direct and terms from the second product as exchange. The general form of this spin-average for each single-particle state (with the nucleon spin s = 1/2 understood) has the following multipole expansion [11]

$$\langle \phi_{j'}^{m'} | \phi_{j}^{m} \rangle_{s} = \sum_{kq} (j'm'kq|jm) \left[ \frac{\hat{\ell}\hat{\ell}'\hat{j}'}{\sqrt{4\pi}} (-1)^{2s+j+j'-\ell} (\ell 0\ell'0|k0) \right] \times W(jsk\ell';\ell j') u_{j'\ell'}(r) u_{j\ell}(r) Y_{kq}(\hat{r}) \right],$$

$$\equiv \sum_{kq} (j'm'kq|jm) \langle \langle j'\ell' | \mathcal{O}_{kq}(\vec{r}) | j\ell \rangle \rangle, \qquad (9)$$

where the  $u_{j\ell}(r)$  are the single-particle radial wave functions and  $\langle \langle ... \rangle \rangle$  is used as shorthand for the square-bracketed expression. These single-particle spin averages actually enter the stripping calculation as a product with their corresponding nucleonic absorption factors  $A_i = (1 - |S_i|^2)$  and are integrated over the appropriate single particle position coordinate. Explicitly,

$$\int d\vec{r} \,\mathcal{A}\langle\phi_{j'}^{m'}|\phi_{j}^{m}\rangle_{sp} = \sum_{kq} (j'm'kq|jm) \int d\vec{r} \,\mathcal{A}\,\langle\langle j'\ell'|\mathcal{O}_{kq}(\vec{r})|j\ell\rangle\rangle$$

$$\equiv \sum_{kq} (j'm'kq|jm) \left\{j'\ell'|\mathcal{F}_{kq}(b)|j\ell\right\}, \qquad (10)$$

which defines the brackets  $\{j'\ell'|\mathcal{F}_{kq}(b)|j\ell\}$  that are now functions only of the angular momenta indicated and the projectile cm impact parameter b. Upon

simplifying the remainder of the angular momentum coupling coefficients we can write

$$\frac{1}{\hat{J}_{i}^{2}} \sum_{M_{i}} \langle \Psi_{J_{i}M_{i}}^{(f)} | \mathcal{A}_{1} \mathcal{A}_{2} | \Psi_{J_{i}M_{i}}^{(f)} \rangle = \sum_{\alpha \alpha' I} 2D_{\alpha} D_{\alpha'} C_{\alpha'}^{J_{i}J_{f}I} C_{\alpha}^{J_{i}J_{f}I} \hat{j}_{1} \hat{j}_{2}$$

$$\times \sum_{KQ} \frac{(-)^{Q}}{\hat{K}^{2}} \left[ \text{direct - exchange} \right], \quad (11)$$

where

direct 
$$\equiv (-)^{I-j_1-j'_2}W(j_1j'_1j_2j'_2;KI)\{j'_1\ell'_1|\mathcal{F}_{K-Q}(b)|j_1\ell_1\}$$
  
 $\times \{j'_2\ell'_2|\mathcal{F}_{KQ}(b)|j_2\ell_2\},$  (12)  
exchange  $\equiv (-)^{j'_2-j_1}W(j_1j'_2j_2j'_1;KI)\{j'_2\ell'_2|\mathcal{F}_{K-Q}(b)|j_1\ell_1\}$   
 $\times \{j'_1\ell'_1|\mathcal{F}_{KQ}(b)|j_2\ell_2\}.$  (13)

Referring back to Eq. (5), we note that the stripping cross section  $\sigma_{str}^{(f)}$  to a given residue final state f, with angular momentum  $J_f$ , is now calculated using Eq. (11), since

$$\sigma_{str}^{(f)} = \frac{2\pi}{2J_i + 1} \sum_{M_i} \int db \, b \, |\mathcal{S}_c|^2 \langle \Psi_{J_i M_i}^{(f)} | \mathcal{A}_1 \mathcal{A}_2 | \Psi_{J_i M_i}^{(f)} \rangle. \tag{14}$$

# 2.3. ISOSPIN DEPENDENCE

The inclusion of isospin labels in Eq. (1) and the subsequent equations leads to simple modifications. An additional phase factor of  $(-)^{1+T}$  must be inserted in front of the exchange term in Eq. (11) and the two-nucleon amplitudes C also depend on T. The final expression for the stripping cross section, Eq. (14), must also be multiplied by the square of the overall isospin coupling Clebshe-Gordan coefficient  $(T\tau T_f \tau_f | T_i \tau_i)$ .

# 3. Two-Nucleon Knockout from <sup>12</sup>C

As a first orientation of our approach we consider two-neutron and twoproton knockout from  $^{12}$ C. In the next Section we argue, on the grounds of threshold energies, that the direct two-nucleon knockout reaction mechanism is expected to be applicable to studies of large regions of unstable nuclei. In the case of two-nucleon knockout from  $^{12}$ C the argument for the reaction being direct is somewhat different. We note that the shell model predicts that the total spectroscopic strength for p-state single-nucleon removal from  $^{12}$ C is exhausted in knockout to the bound states in the mass A=11 systems [17]. Non-direct routes of populating the A=10 residues, by knockout into the A=11 continuum followed by nucleon evaporation, are thus expected to be highly suppressed.

The reasons for considering  $^{12}$ C are twofold. First,  $^{12}$ C,  $^{10}$ C and  $^{10}$ Be have wave functions that are rather well known - the p-shell model having high predictive power. Second, the existing experimental cross sections for two-nucleon knockout from  $^{12}$ C [23] are accurate to  $\approx 10\%$  and were taken at high energies where the eikonal model will be at its most reliable. The data were obtained in high-energy  $^{12}$ C+ $^{12}$ C fragmentation at 250, 1050 and 2100 MeV per nucleon incident energy (see Table 1). Knockout of an np-pair is not considered here because (i) of the complexity of the  $^{10}$ B level scheme below the nucleon thresholds, and (ii) as many of the known  $^{10}$ B excited states are absent from the truncated-space shell model calculation used. The np-knockout cross sections are enhanced significantly over those for 2n and 2p and deserve further study.

The simplest of the reactions is 2n removal to  $^{10}$ C, with only two bound states below the nucleon threshold: the  $0^+(gs)$  and  $2^+(3.354 \text{ MeV})$  excited state [25]. In the 2p knockout reaction to  $^{10}$ Be we must include population of the  $0^+(gs)$ , the  $2^+$  states at 3.368 MeV and 5.958 MeV, and a second  $0^+$  state at 6.179 MeV, all of which lie below the nucleon threshold of 6.812 MeV [25]. The two nucleons are stripped from the  $p_{3/2}$  and  $p_{1/2}$  single-particle orbitals. Their radial wave functions are calculated in a Woods-Saxon potential with radius and diffuseness parameters  $r_0 = 1.310$  fm and a = 0.55 fm, as were used in the one-nucleon knockout calculations of [17]. Since the 2n and 2p separation energies are 31.841 MeV and 27.184 MeV an average two-nucleon separation energy of 29 MeV was assumed and the binding potential depth was adjusted to give a single-nucleon separation energy of 14.5 MeV. The shell-model two-nucleon spectroscopic amplitudes,  $C_{\alpha}^{J_i J_f I}$  were calculated for each  $^{10}$ Be and  $^{10}$ C final state.

The S-matrices in Eq. (14) have been calculated using the optical limit of Glauber theory [26]. The essential input parameters are the free nucleon-nucleon (NN) cross sections for the energies of interest, e.g. [27]. A zero-range interaction was assumed. The <sup>12</sup>C, <sup>10</sup>Be and <sup>10</sup>C density distributions were assumed to have a Gaussian shape with root mean squared (rms) matter radii of 2.32 fm, 2.30 and 2.30 fm.

The results are given in Table 1. The experimental cross sections are taken from [23]. There is good agreement with these inclusive experimental data in these stable p-shell test cases. In the next Section we show that more exclusive experiments, with data to specific residue final states, provide an opportunity to investigate the role of two-removed-nucleon correlations.

Energy/nucleon 250 MeV $1.05~{\rm GeV}$ 2.1 GeV<sup>10</sup>Be Theoretical 5.82 5.33 5.15 <sup>10</sup>Be Experimental 5.88 5.30(30)5.81(29)<sup>10</sup>C Theoretical 4.263.91 3.84 <sup>10</sup>C Experimental 5.33(81)4.44(24)4.11(22)

TABLE 1. Calculated and measured inclusive two-proton and two-neutron knockout cross sections from  $^{12}$ C (in mb) (on a  $^{12}$ C target) at 250, 1050 and 2100 MeV/nucleon. Calculations use the shell model two-nucleon amplitudes of reference [28]. The experimental values are taken from [23, 29].

# 4. Two-Nucleon Knockout from <sup>28</sup>Mg

It is proposed, following [21], that two-proton removal from exotic nuclei having even a modest neutron excess will proceed as a direct reaction. We discuss two-proton removal from  $^{28}$ Mg, with only two neutrons more than stable  $^{26}$ Mg. The energetics of these N=16 isotones, shown schematically in Fig. 3, suggest strongly that direct two-proton (-2p) removal is the only probable route to bound states of  $^{26}$ Ne. Supporting experimental evidence was offered by both the measured  $^{9}$ Be( $^{28}$ Mg, $^{26}$ Ne)X inclusive cross section and the observed parallel momentum distribution of the reaction residues [21]. Here we discuss and elaborate upon the calculations presented there with emphasis on the inclusive and partial knockout cross sections.

We consider the knockout of two protons from <sup>28</sup>Mg at 82 MeV per nucleon. The <sup>26</sup>Ne final states populated are the 0<sup>+</sup> ground state and the  $2^{+}$  (2.02 MeV),  $4^{+}$  (3.50 MeV) and second  $2^{+}$  (3.70 MeV) excited states [30, 31, 32, 21]. The measured cross sections are collected in Table 2. The calculations of the S-matrices at this lower energy assume a Gaussian NN effective interaction [8], the strength being determined by the free NN cross sections [27]. The densities of the target and the core were also assumed to have Gaussian shapes with rms matter radii of 2.36 for <sup>9</sup>Be and 2.90 fm for <sup>26</sup>Ne [33]. In our full calculations the removed protons are stripped from the  $0d_{5/2},\ 0d_{3/2}$  and  $1s_{1/2}$  orbitals. The spectroscopic coefficients  $C_{\alpha}^{J_iJ_fI}$ were calculated with the code OXBASH [34] in the sd-shell model space with the USD Hamiltonian [35]. The single-particle wave functions  $u_{i\ell}(r)$ are calculated in a Woods-Saxon potential well with radius and diffuseness parameters 1.25 fm and 0.70 fm. The strength of the binding potential is adjusted to reproduce the physical two-proton separation energy,  $S_{2p}$  = 30.03 MeV.

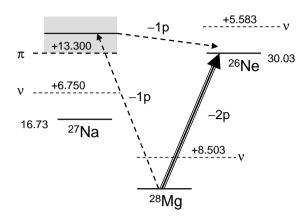


Figure 3. Energy diagram of the neutron-rich N=16 isotones <sup>28</sup>Mg, <sup>27</sup>Na and <sup>26</sup>Ne, showing the single-neutron ( $\nu$ ) and proton ( $\pi$ ) separation energies for each nucleus. The diagram shows that non-direct population of the bound states of <sup>26</sup>Ne, by one-proton removal to excited <sup>27</sup>Na followed by proton evaporation, would involve states high above the (much lower) neutron evaporation threshold and so is expected to be negligible.

### 4.1. UNCORRELATED STRIPPING

Assuming that the <sup>28</sup>Mg valence proton structure is the extreme singleparticle limit,  $[0d_{5/2}]^4$ , several results follow. Given our model parameters, the calculated (unit) cross section for removal of a  $[0d_{5/2}]^2$  pair is  $\sigma_{22}=0.29$ mb. This sets the scale for the anticipated cross section. Based on an assumed  $[0d_{5/2}]^n$  ground state (with n=4 for <sup>28</sup>Mg) this predicts an inclusive cross section of  $n(n-1)\sigma_{22}/2$  or 1.8 mb, in broad agreement with the measured value of 1.50(10) mb in Table 2. In this uncorrelated limit the cross sections (and associated spectroscopic strengths  $S_{unc}(J_f^{\pi})$ ), for removal of a pair from a  $0^+$ ,  $[j]^n$  occupied sub-shell, will also be spread between different  $J_f^{\pi}$  final states according to their coefficients of fractional parentage  $((j^{n-2})vJ_f,(j^2)J_f|(j^n)0)$ , where v is the seniority of the state [22]. This yields  $S_{unc}(0^+) = 4/3$ ,  $S_{unc}(2^+) = 5/3$  and  $S_{unc}(4^+) = 3$ , with  $\sum_{J_f} S_{unc}(J_f^{\pi}) = 6$ , as shown in Table 2. This distribution fails to reproduce the pattern of the measured <sup>26</sup>Ne partial cross sections. When multiplied by the unit cross section they overestimate both the expected  $\sigma(2^+)$  and  $\sigma(4^+)$  cross sections in comparison with the measured  $\sigma(0^+)$ . The low measured yield to the two 2<sup>+</sup> states presents a particular problem for this very simple model and suggests that the experimental data reflect the presence

of correlation effects.

TABLE 2. Two-proton knockout cross sections from a  $^{28}$ Mg projectile beam at 82 MeV/nucleon, to different final states of the  $^{26}$ Ne residue. The spectroscopic weights  $S_{exp}$  and  $S_{th}$  are the corresponding cross sections divided by the cross unit section  $\sigma_{22}=0.29$  mb, see text.

	$S_{unc}$	$S_{exp}$	$S_{th}$ corr	$\sigma_{exp}$ (mb)	$\sigma_{th}$ (mb)
0+	1.33	2.4(5)	1.83	0.70(15)	0.532
$2^{+}$	1.67	0.3(5)	0.54	0.09(15)	0.157
$4^{+}$	3.00	2.0(3)	1.79	0.58(9)	0.518
$2_{2}^{+}$	_	0.5(3)	0.78	0.15(9)	0.225
Inclusive	6	5.2(4)	4.94	1.50(10)	1.43

#### 4.2. CORRELATED STRIPPING

Calculations with the fully-correlated proton wave functions are shown (in mb) in Table 2. There is good agreement of the partial cross sections  $\sigma_{th}$ , and hence trivially of the theoretical spectroscopic factors  $S_{th}$ , with the corresponding experimental values. The calculated inclusive cross section to the four bound states is now 1.43 mb, also in good agreement with the measured value of 1.50(10) mb [21]. There is no scaling or renormalization of these cross sections, which are calculated in an absolute sense. A significant fraction of the integrated cross section expected, based on the  $[0d_{5/2}]^4$  uncorrelated estimate, of 1.8 mb, is accounted for in the measurements to the four  $^{26}$ Ne bound states, with  $\sum_{J_f} S_{th} = 4.94$ .

### 5. Conclusions

Single-nucleon knockout reactions are now an established technique for single-particle spectroscopy of both weakly-bound and deep-hole states, revealing the role of correlations in and beyond the shell model. We have presented a comprehensive calculation scheme to extend such studies to the direct two-nucleon stripping reaction. The inclusive cross sections for 2p and 2n removal from  $^{12}\mathrm{C}$  are consistent with measurements. For slightly neutron rich  $^{28}\mathrm{Mg}$  we obtain absolute predictions of the partial cross sections to different  $^{26}\mathrm{Ne}$  final states. These are in good agreement with available experimental data and show considerable improvement when compared to simple uncorrelated structure approximations. Our calculations provide

further evidence of the direct nature of two-proton knockout from neutronrich nuclei and of its potential for the study of two-nucleon correlations and spectroscopy.

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