Three lectures – will plan to discuss

Lect I : Fusion of ions: motivation and introductory remarks, concepts, terminology, models and indicators of fusion, reaction dynamics, barriers, coupled channels - assisted tunnelling, barrier distributions and optical potentials. Experience.

Lect II: Weakly-bound systems, methods for break-up calculations, fusion in few-body models of break-up reactions. Many open questions.

Lect III: Partial/incomplete fusion at higher incident energies, applications to knockout of one- and two nucleons and applications for spectroscopy of exotic nuclei

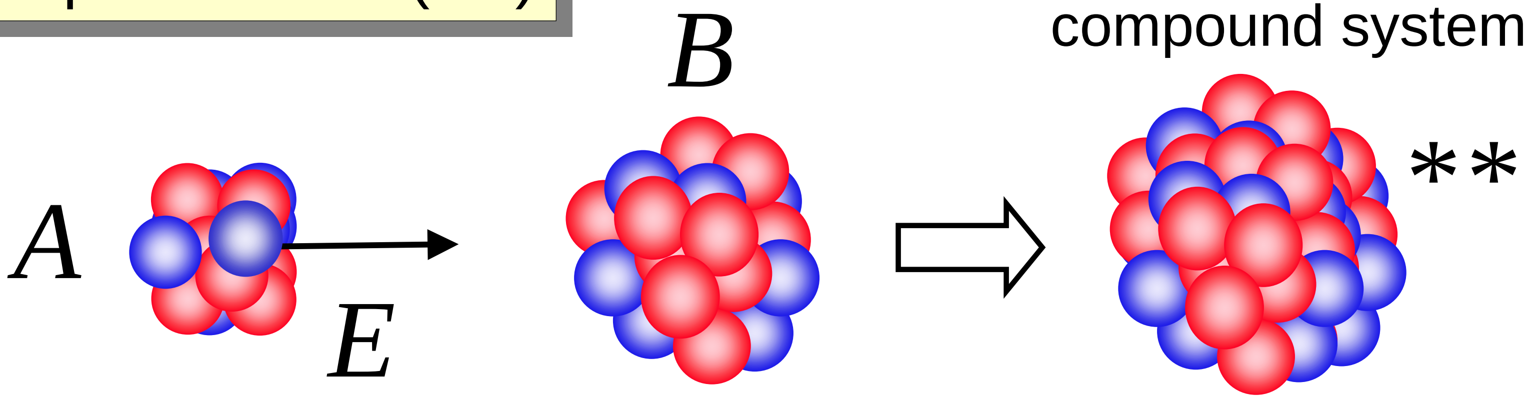
Handful of useful papers and topical conferences

- Fusion03: *From a Tunnelling Nuclear Microscope to Nuclear Processes in Matter,* Progress of Theoretical Physics Supplement **154**, 2004.
- A.B. Balantekin and N. Takigawa, *Quantum Tunnelling in Nuclear Fusion,* Rev. Mod. Phys. 70 (1998) 77-100.
- M. Dasgupta et al., *Measuring Barriers to Fusion,* Ann. Rev. Nucl. Part. Phys. 48 (1998) 401-461
- Workshop: *Heavy-ion Collisions at Energies Near the Coulomb Barrier* 1990, IoP Conference Series, Vol 110 (1990).
- S.G. Steadman et al., ed. *Fusion Reactions Below the Coulomb Barrier*, Springer Verlag (1984)
- M.E. Brandan and G.R. Satchler, *The Interaction between Light Heavy-ions and what it tells us,* Phys. Rep. **285** (1997) 143-243.
- M. Beckerman, *Sub-barrier Fusion of Two Nuclei,* Rep. Prog. Phys. **51** (1988) 1047-1103.
- M.S. Hussein and K.W. McVoy, *Inclusive Projectile Fragmentation in the Spectator Model,* Nucl. Phys. **A445** (1985) 124-139.
- M. Ichimura, *Theory of Inclusive Break-up Reactions,* Int. Conf on Nucl. React. Mechanism. World Scientific (Singapore), 1989, 374-381.
- plus enormous volume of relevant literature – much of which is cited in the above

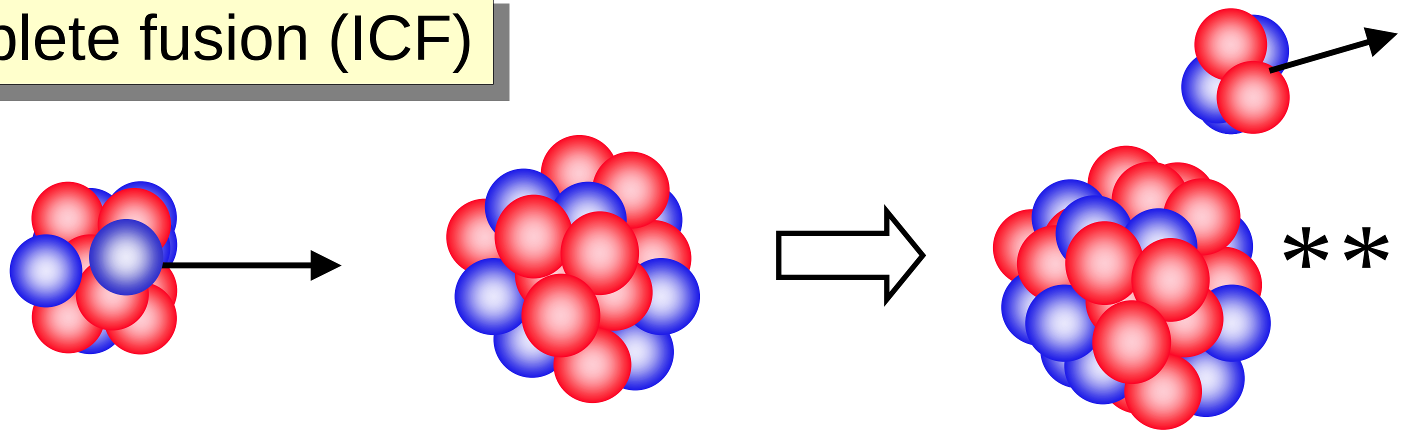
Fusion reaction processes – ion-ion systems

Complete fusion (CF)

Highly excited compound system



Incomplete fusion (ICF)

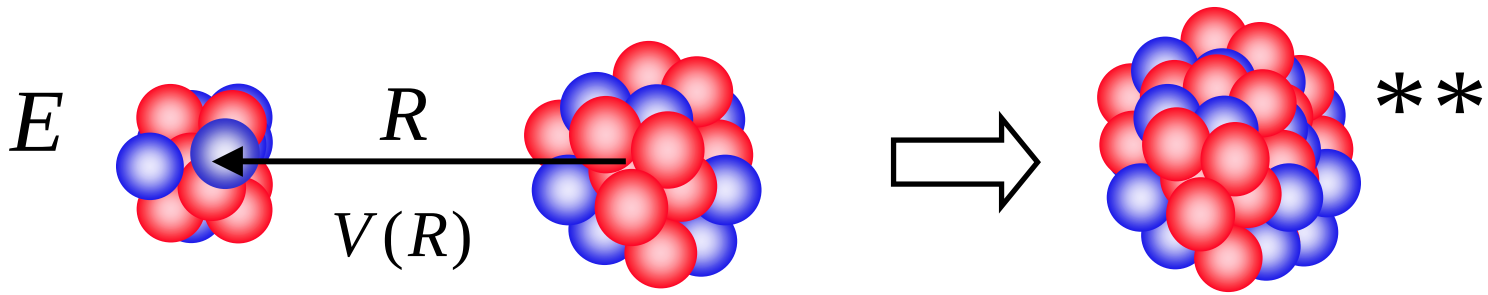


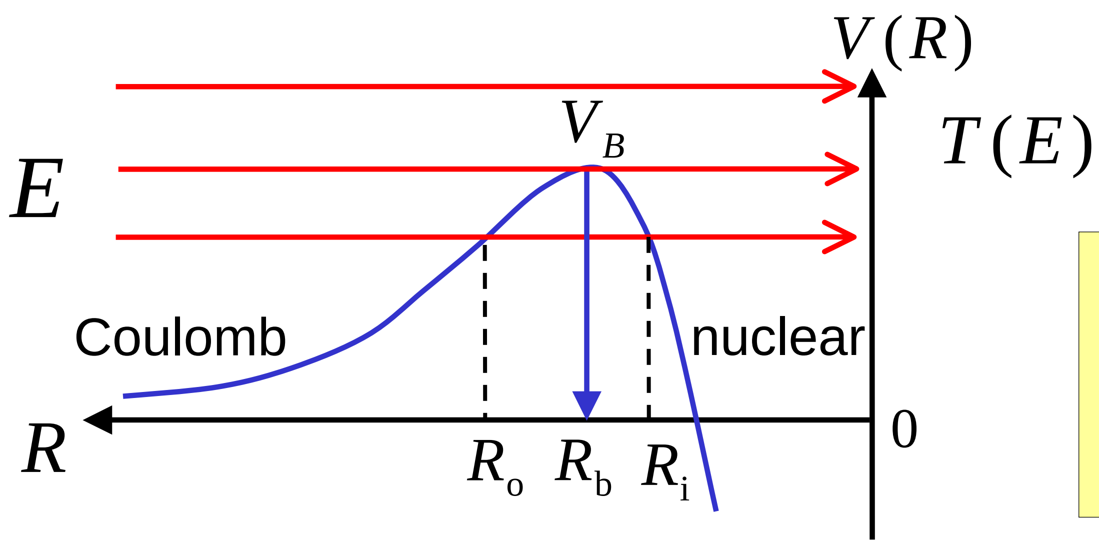
Total fusion (TF) $\sigma_{TF} = \sigma_{CF} + \sigma_{ICF}$

Complete fusion process – static picture

Nuclear astrophysics

(Super)Heavy element synthesis

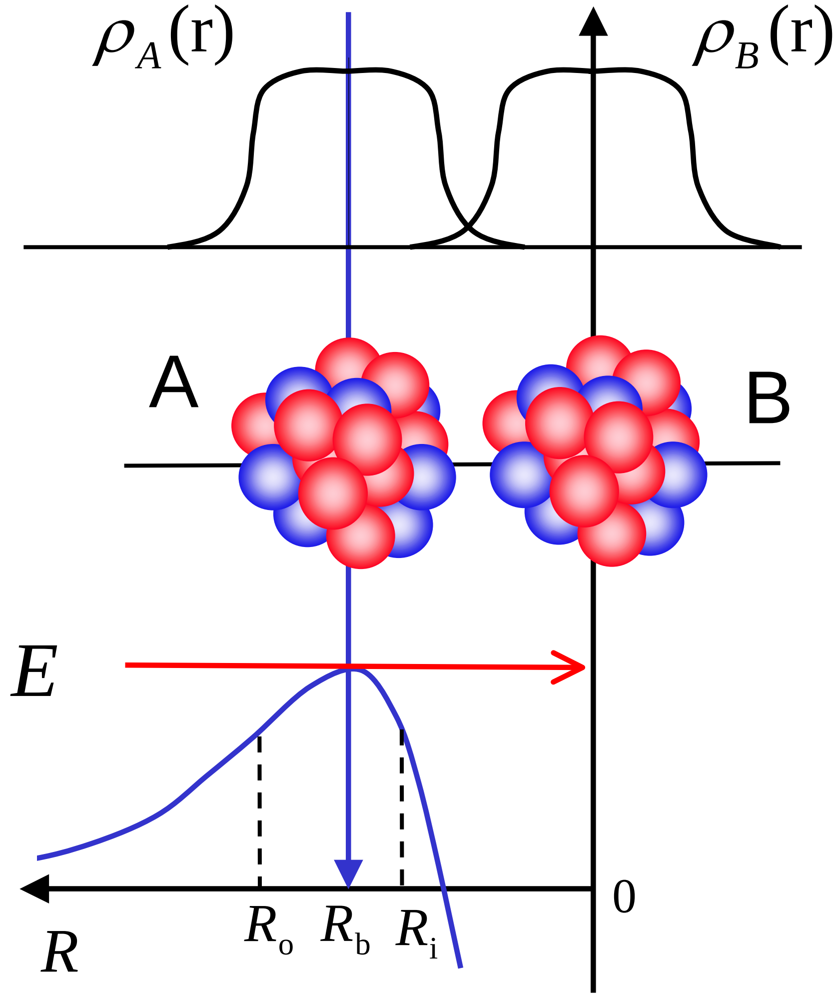




transmission probability:

Dominated by interaction *V(R)* near 'surface', R_b

Barrier radii and nuclear densities - surfaces



Fusion will be probe and be sensitive to:

nuclear binding (tails of the nuclear densities),

nuclear structure (tails of the single particle wave functions)

but also expect sensitivity and complications due to the reaction dynamics –

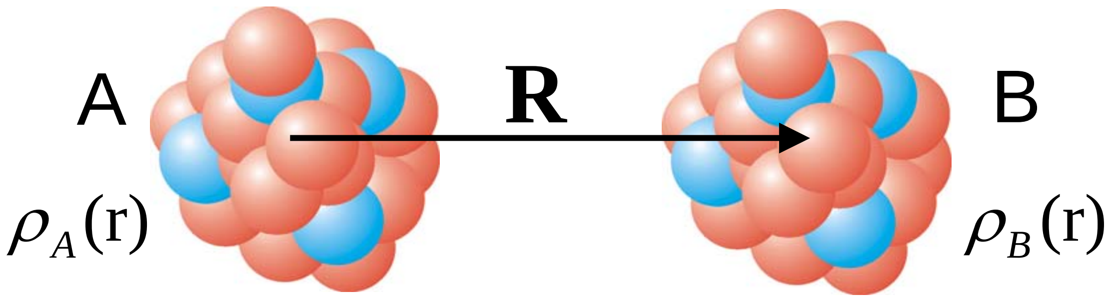
intrinsically surface dominated

Effective interactions – Folding models

Double folding

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(|\,\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1\,|)$$

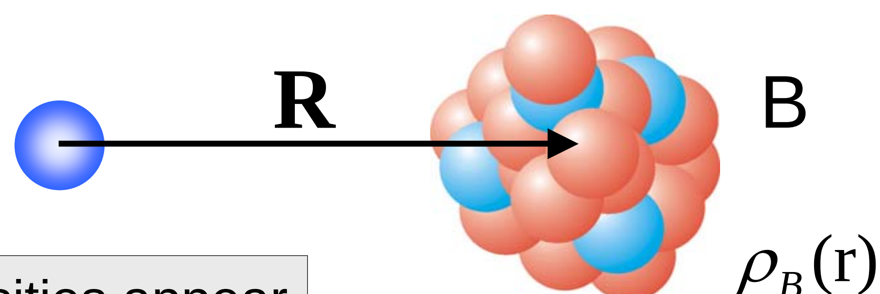
V_{AB}



Single folding

$$V_{NB}(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(|\,\mathbf{R} + \mathbf{r}_2\,|)$$

V_{NB}



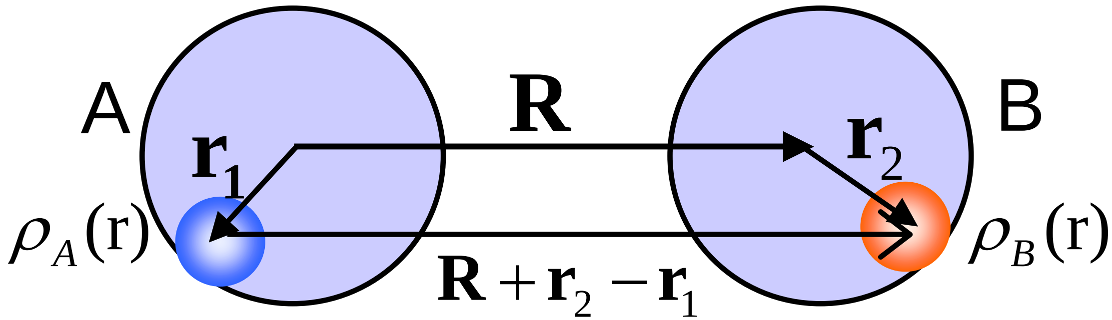
Only ground state densities appear

Effective interactions – Folding models

Double folding

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

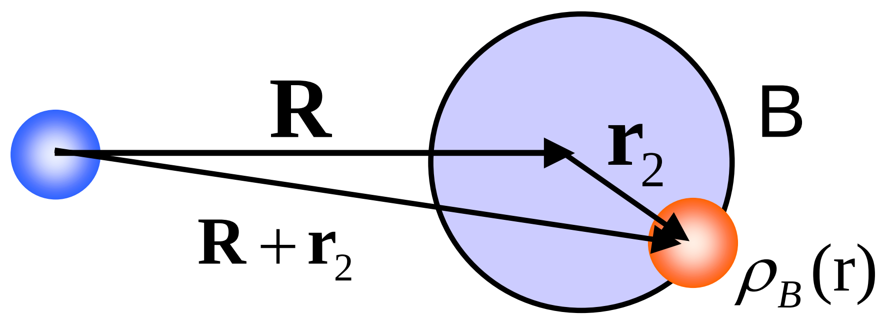
V_{AB}



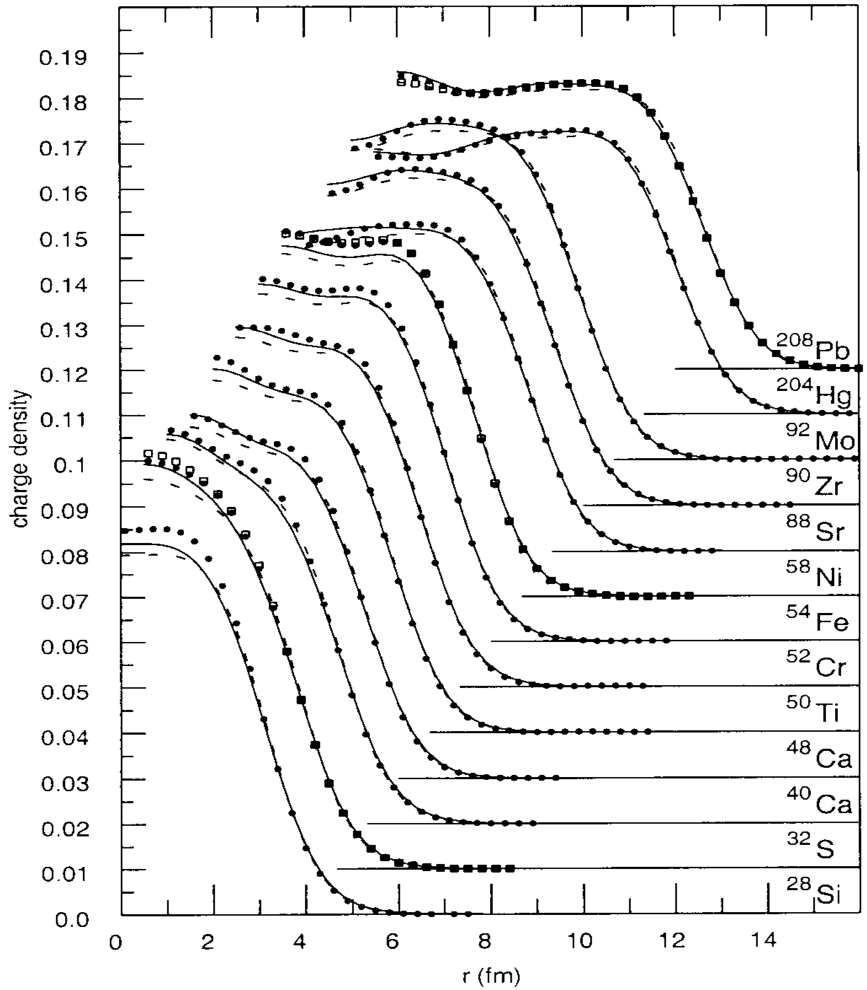
Single folding

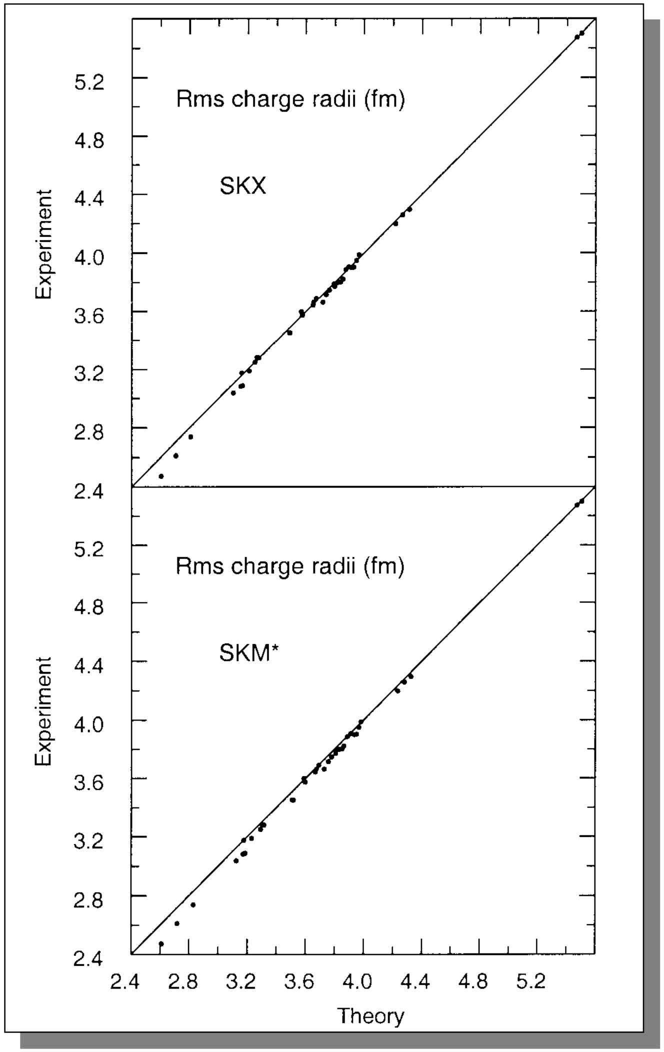
$$V_{NB}(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_2)$$

V_{NB}



Skyrme Hartree-Fock charge radii and densities

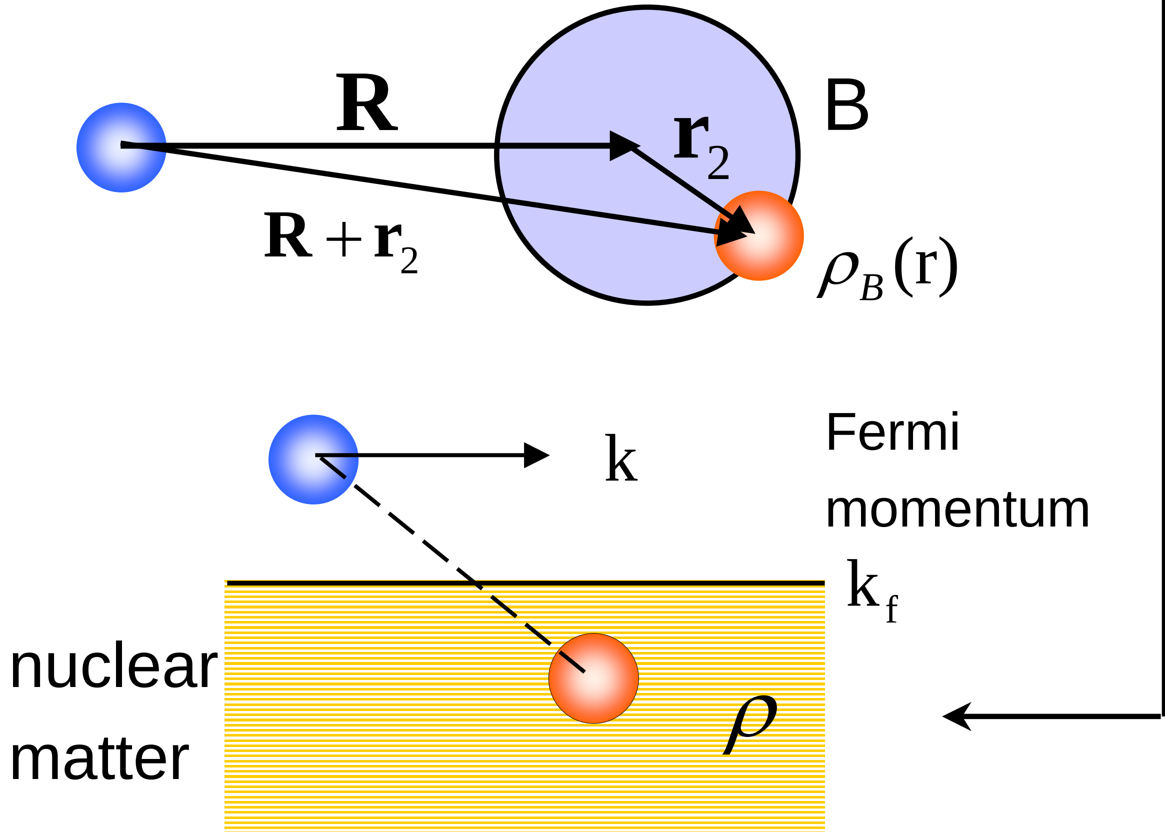




W.A. Richter and B.A. Brown, Phys. Rev. **C67** (2003) 034317

Effective NN interactions – not free interactions

$$V_{NB}(R) = \int d\mathbf{r}_2\ \rho_B(r_2)\ v_{NN}(\mathbf{R}+\mathbf{r}_2)$$



include the effect of NN interaction in the “nuclear medium” – Pauli blocking of pair scattering into occupied states

$\rightarrow v_{NN}(\rho, \mathbf{r})$

(e.g. M3Y, JLM)

But as E → high

$v_{NN} \rightarrow v_{NN}^{free}$

Information from the elastic scattering channel

Folding model (including account of non-localities**) often used to provide the radial shape and approximate strength of the real part of the potential, call it $F_E(R)$, Then, at each E

$$U_E(R) = [N_R(E) + iN_I(E)]\, F_E(R)$$

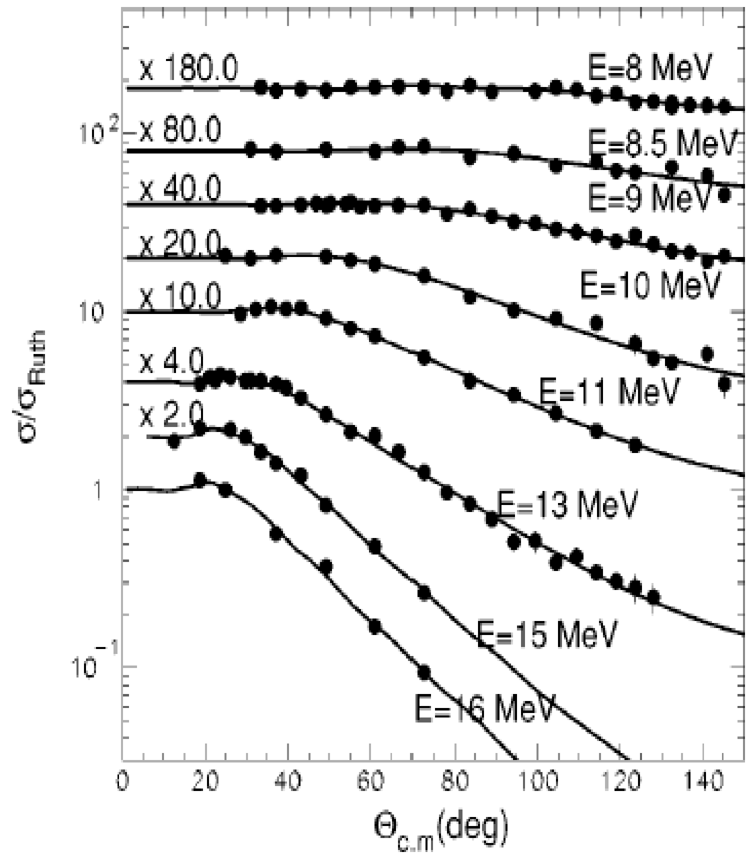
the N_R and N_I are fitted to data with N_R of order unity. (e.g. S. Paulo potential)

Quite generally, for most systems***

$$N_R(E) = 1.0 \pm 0.15$$

$$N_I(E) = 0.8 \pm 0.15$$

$^{7}Li + ^{28}Si$



** L.C. Chamon et al., PRC 66 (2002) 014610

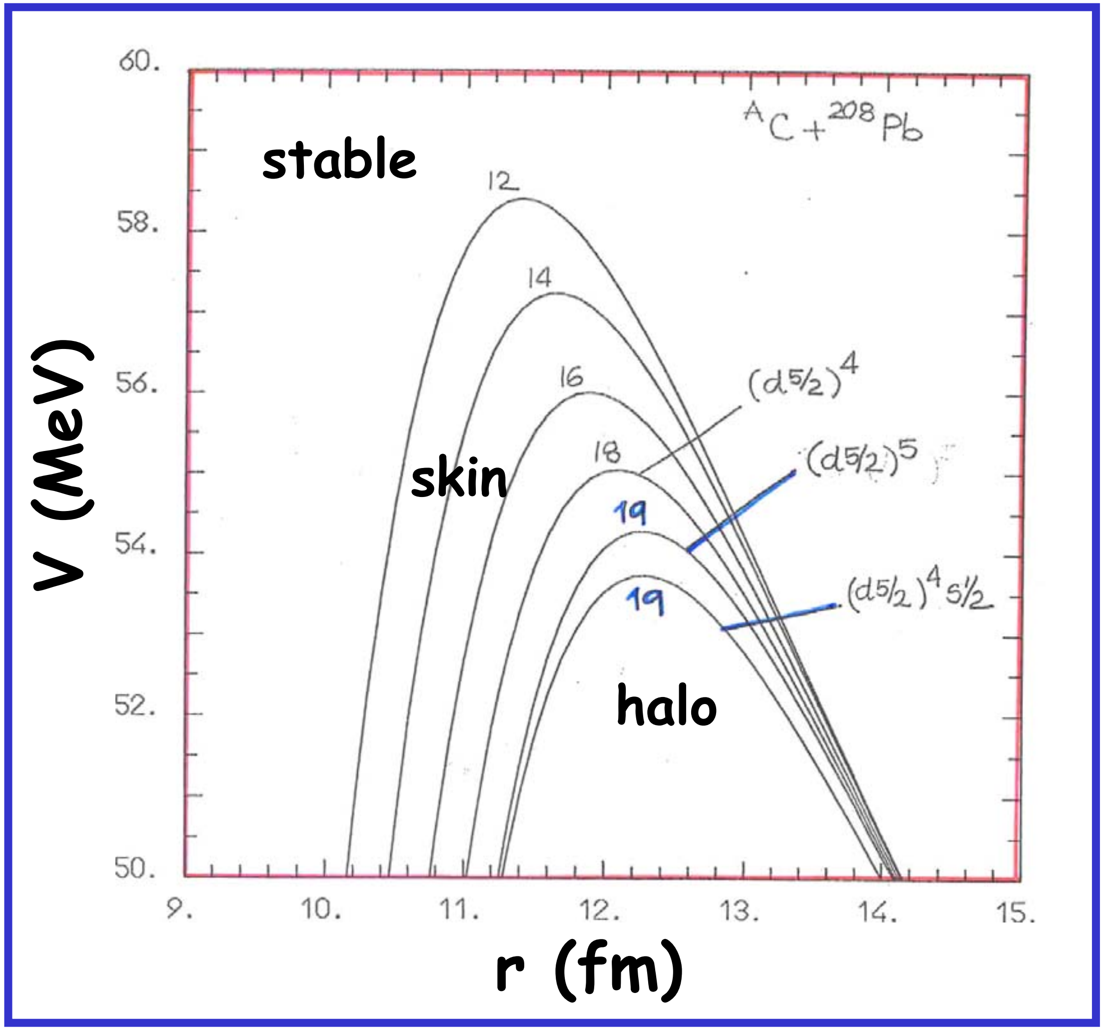
*** G.R. Satchler and W.G. Love, Phys. Rep. **55** (1979) 183

A. Pakou et al., PRC 69 (2004) 054602

Static effects – barriers for n-rich Carbon isotopes

$^{A}C + ^{208}Pb$

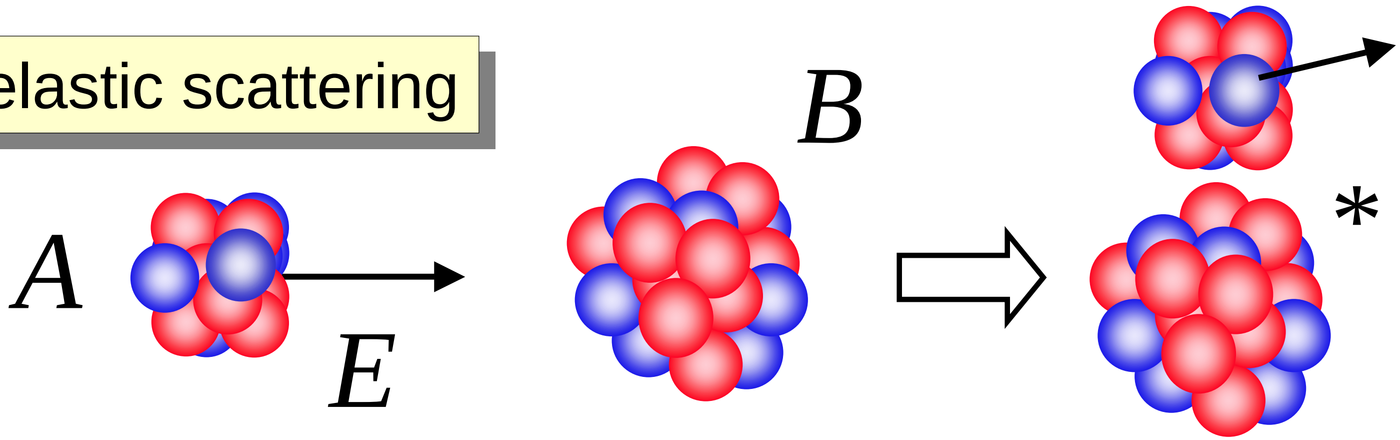
HF predictions



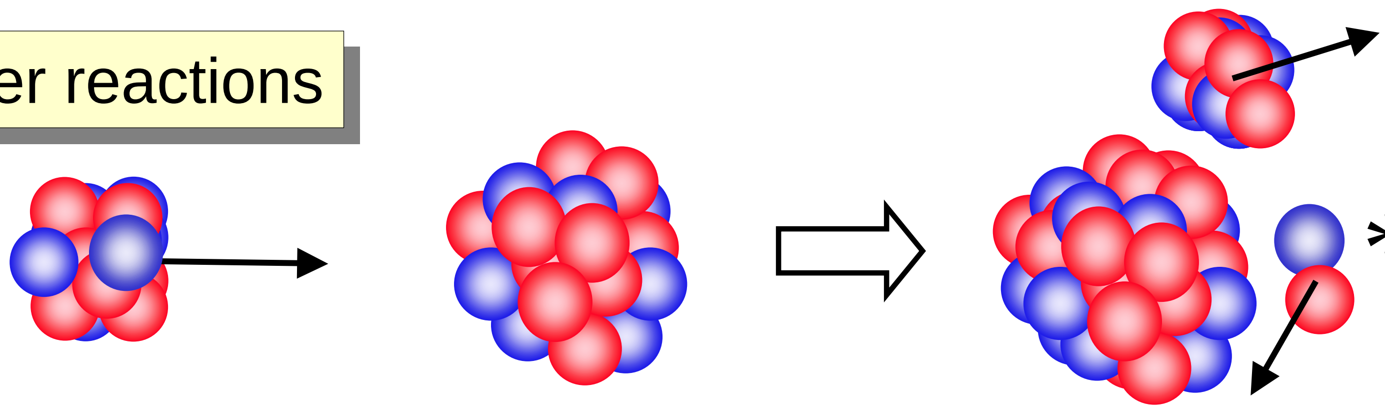
A. Vitturi, NUSTAR'05, Surrey January 2005

Competing 'direct reaction' dynamical processes

Inelastic scattering



Transfer reactions



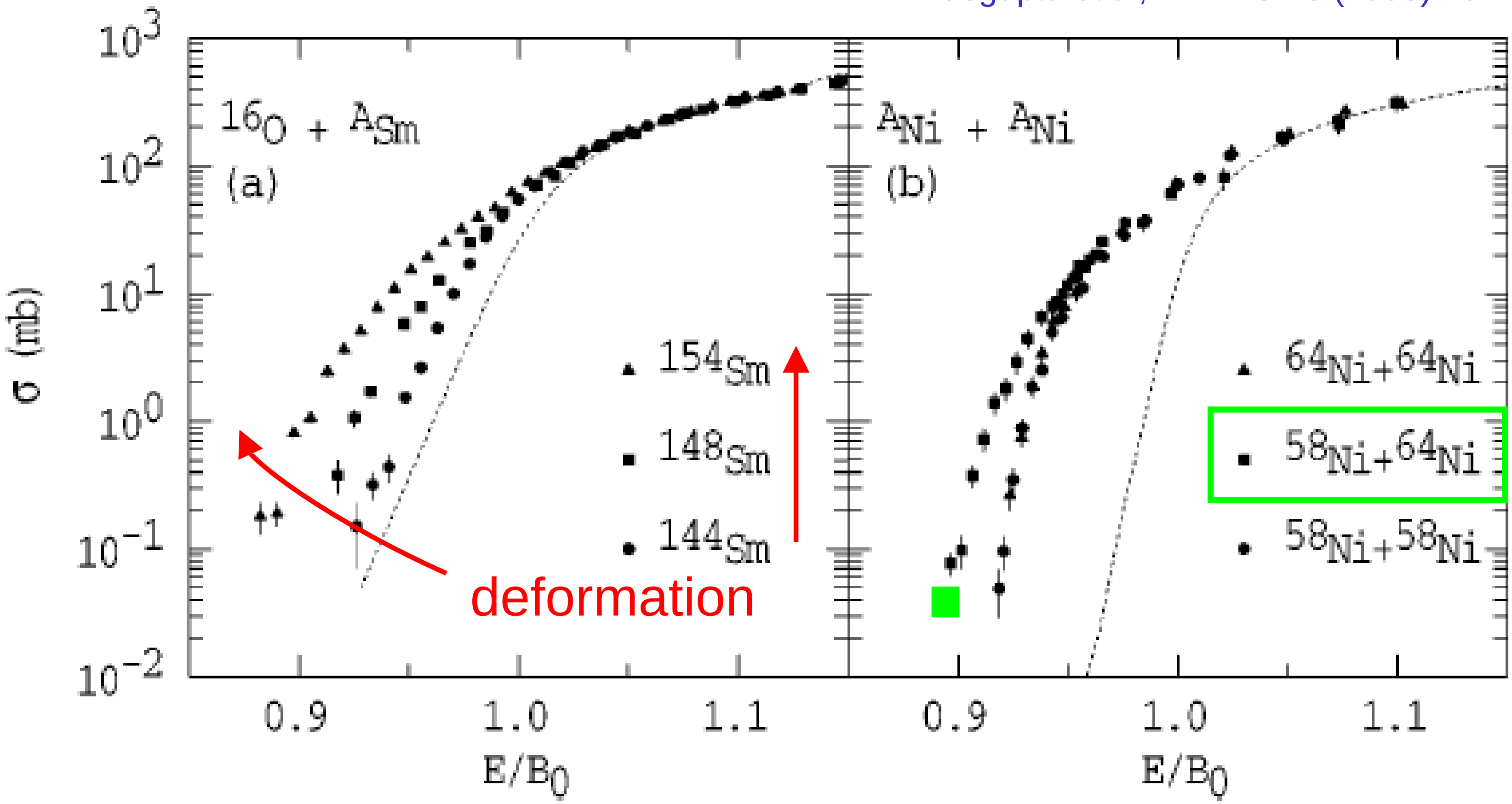
<u>Surface dominated</u> and will 'renormalize' bare ion-ion interaction
Channel-assisted/suppressed tunnelling – general phenomenon

Challenges – potentials, thresholds and dynamics

- Expect a complex interplay of *static*, density driven, and surface, *dynamical* effects
- Far below the barrier, for normally bound nuclei, direct reaction channels switch off – have opportunity to study *threshold effects* as reaction channels open and evolve as a function of energy
- Fusion expected to be a severe test of our models of nuclear structures and of treatments of direct reaction dynamics
- Facilities available for sophisticated and very precise experiments - ANU (Canberra), USP, INFN Legnaro, etc.
- Weakly bound systems are different – do break-up channels turn off below the barrier? What can we learn?

Channel coupling – classic examples

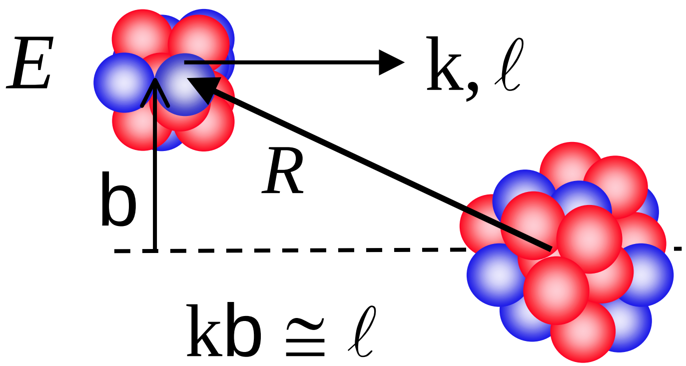
M.Dasgupta et al, ARNPS **48** (1998) 401

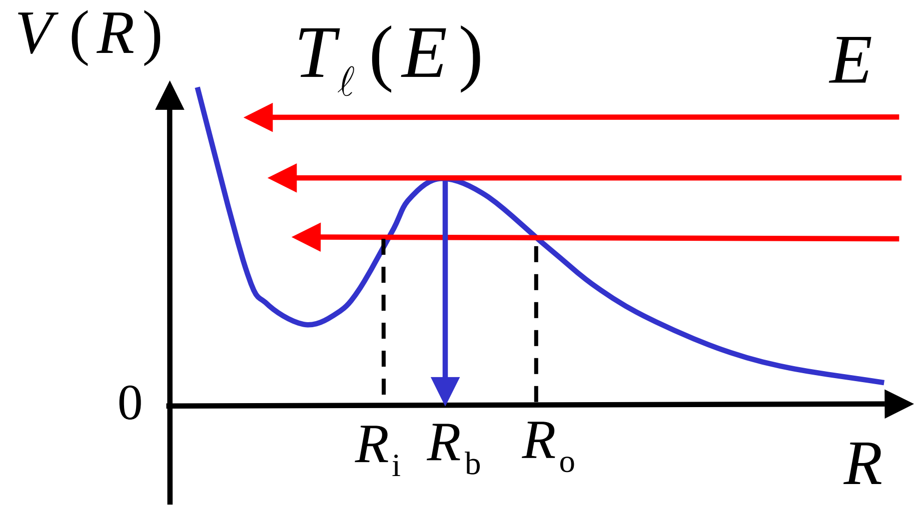


R.G. Stokstad et al, PRL **41** (1978) 465, PRC **21** (1980) 2427.

M. Beckerman et al, PRL **45** (1980) 1472, PRC **23** (1981) 1581, PRC **25** (1982) 837.

Complete fusion - expectations – static model



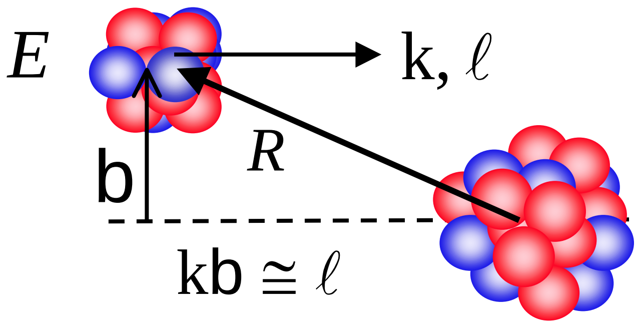


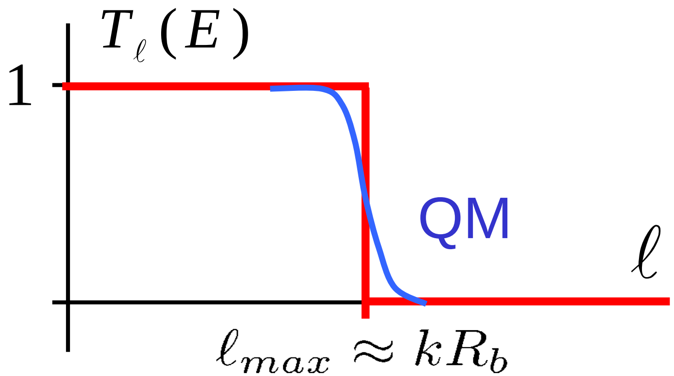
$$(A_1, Z_1) + (A_2, Z_2)$$

$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) T_\ell(E)$$

$$\frac{d^2 u_\ell(R)}{dR^2} + \frac{2\mu}{\hbar^2} \left[E - V(R) - \frac{\ell(\ell+1)}{R^2} \right] u_\ell(R) = 0$$

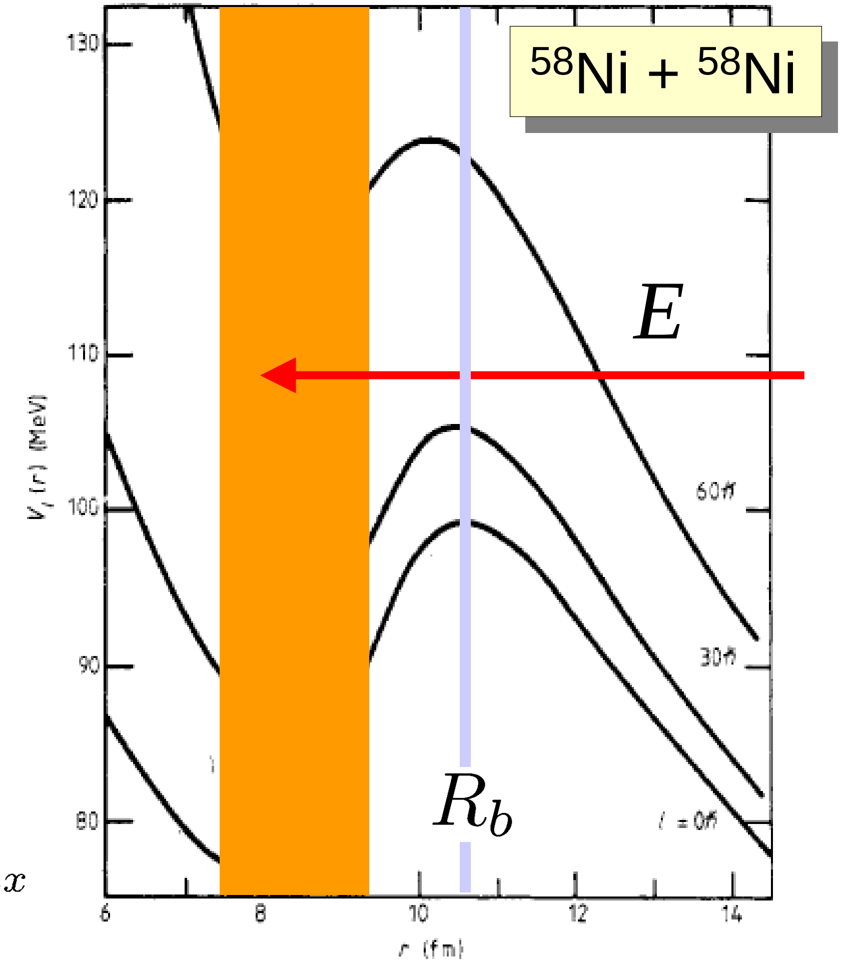
Angular momentum dependence of the barrier





$$\sigma_R(E) \approx \frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{max}}(2\ell+1) \approx \frac{\pi}{k^2}\ell_{max}^2$$

$$\sigma^{cf}(E) \approx \sigma_R \approx \pi R_b^2$$

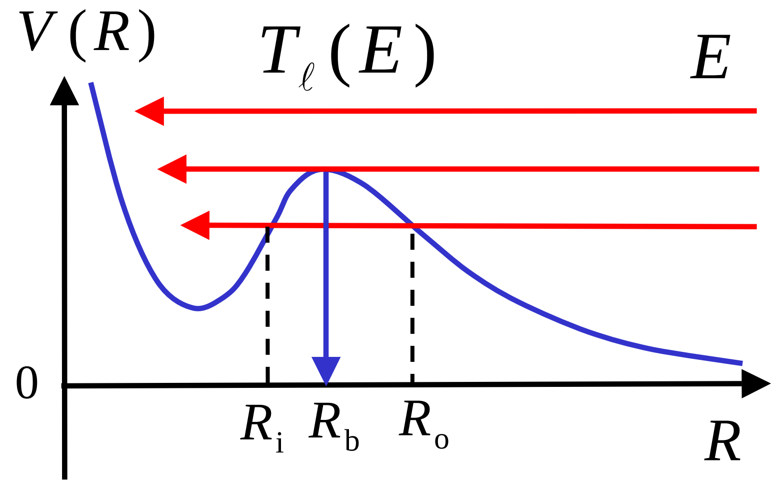


M. Beckerman, Rep. Prog. Phys. **51** (1988) 1047

Quantum mechanical barrier penetration

$$\frac{d^2u_\ell(R)}{dR^2} + \frac{2\mu}{\hbar^2}\left[E - V(R) - \frac{\ell(\ell+1)}{R^2}\right]u_\ell(R) = 0$$

Numerical solutions of this QM barrier penetration problem, the solution of the radial equation for *u(R)* and the transmission prob. - and later, more complex (coupled channels) examples, account for fusion by one of two methods:



(i) the $u_\ell(R)$ have ingoing wave boundary conditions for $R<R_i$
No flux transmitted through the barrier is reflected

(ii) the absorptive (imaginary) part in *V(R)* at short distances absorbs all flux transmitted through the barrier

Theoretical expression for the cross section

$$\sigma_R(E) = \frac{2}{\hbar v}\langle \mathcal{X}^+ | W_E(r) | \mathcal{X}^+ \rangle$$

the projectile-target distorted wave function is $\mathcal{X}^+$

where $W_E(R)$ is total absorptive part of the optical potential

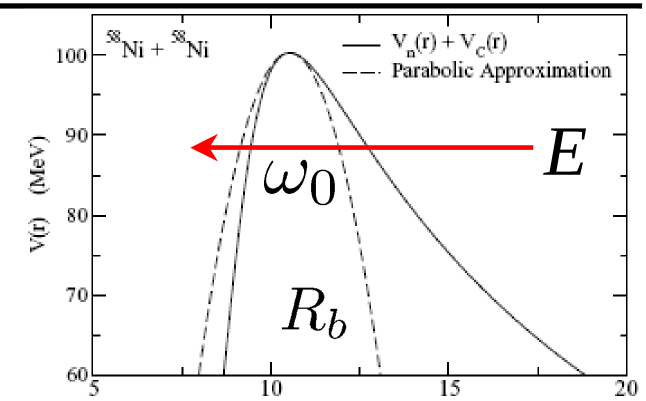
$$\sigma_F(E) = \frac{2}{\hbar v}\langle \mathcal{X}^+ | W_F(r) | \mathcal{X}^+ \rangle$$

where $\mathrm{W_F(R)}$ is that part of the absorption responsible for fusion

Formula of Wong – quadratic form barrier

$$V_\ell(R) = V_b - \frac{1}{2}\mu\omega_0^2(R-R_b)^2 + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2}$$

$$T_\ell(E) = \{1+\exp[(2\pi/\hbar\omega_\ell)(V_\ell - E)]\}^{-1}$$



Assuming $\hbar\omega_\ell = \hbar\omega_0$

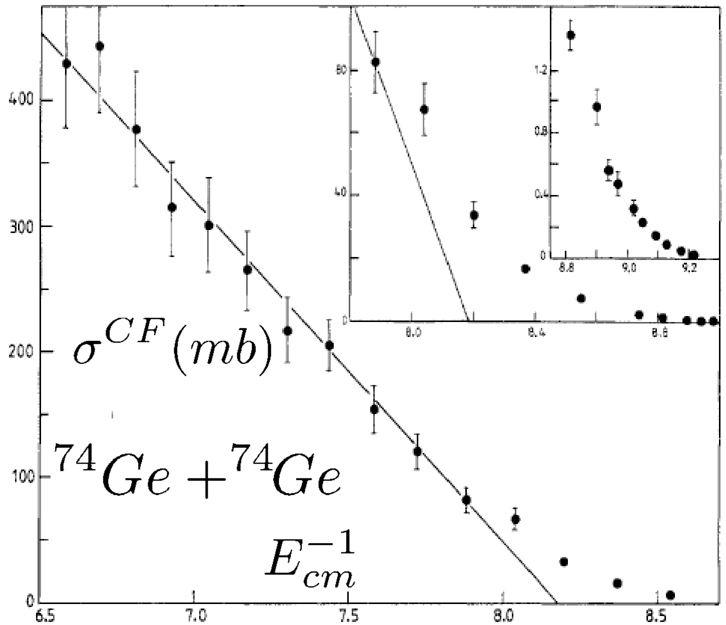
$$V_\ell = V_b + \ell(\ell+1)\hbar^2/2\mu R_b^2$$

$$\sigma^{cf}(E) = \frac{R_b^2\hbar\omega_0}{2E}\ln(1+e^x)$$

$$x = (2\pi/\hbar\omega_0)(E-V_b)$$

and for $E \gg V_b$

$$\sigma^{cf}(E) = \pi R_b^2(1 - V_b/E)$$



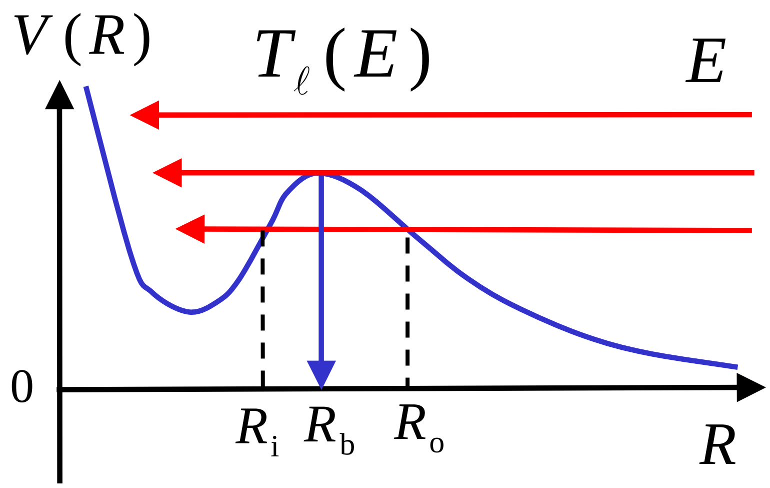
C.Y. Wong, PRL **31** (1973) 766

Making connection with empirical cross sections

$$T_\ell(E) \approx \left[1 + \exp\sqrt{\frac{8\mu}{\hbar^2}} \int_{R_i(\ell)}^{R_o(\ell)} dR \left\{ V(R) + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2} - E \right\}^{1/2} \right]^{-1}$$

Localised barrier of height (for ℓ=0) of $V_B = V(R_b)$

$$\frac{\ell(\ell+1)}{R^2} \approx \frac{\ell(\ell+1)}{R(E)^2} \quad \rightarrow \quad T_\ell(E) \approx T_0\left(E - \frac{\ell(\ell+1)\hbar^2}{2\mu R(E)^2}\right), \quad R(E) \approx R_b$$



$$\sigma(E) = \sum_\ell \sigma_\ell(E) \rightarrow \int d\ell\ \sigma(\ell, E)$$

$$E\sigma(E) = \pi R(E)^2 \int_0^E dE'\ T_0(E')$$

A.B. Balantekin, Rev. Mod. Phys. **70** (1998) 77

Distribution of barriers – directly from the data

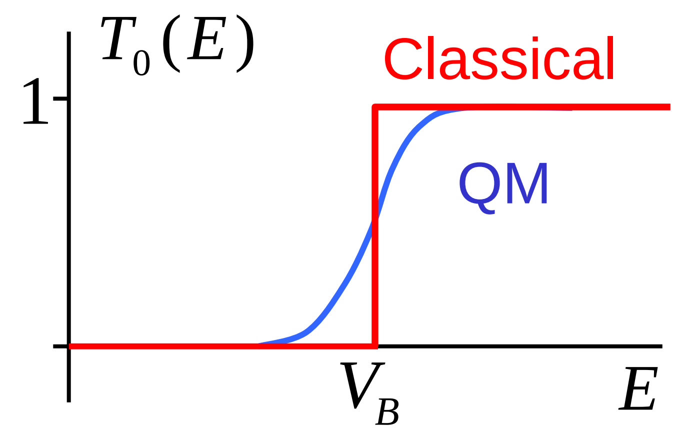
$$E\sigma(E) = \pi R(E)^2 \int_0^E dE' \; T_0(E')$$

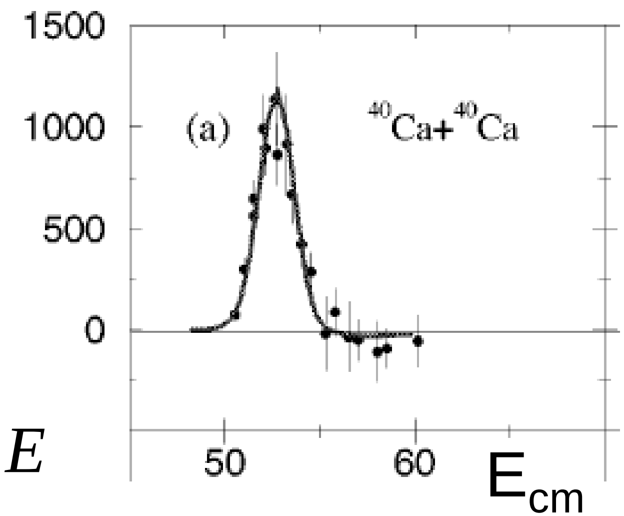
<u>Classically</u>

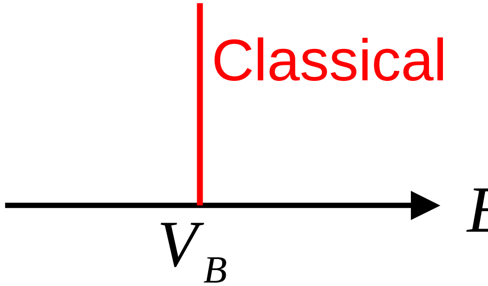
$$R(E) \equiv R_b$$

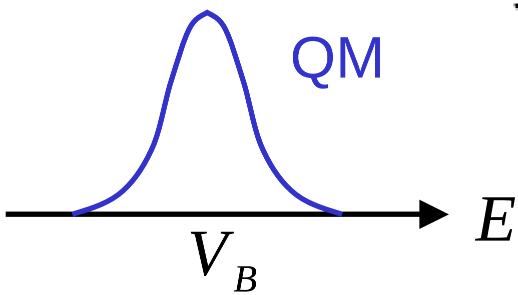
$$\begin{aligned} E\sigma(E) &= \pi R_b^2 (E - V_B), \;\; E > V_B \\ &= 0, \quad E < V_B \end{aligned}$$

$$\frac{d^2}{dE^2}[E\sigma(E)] = \pi R_b^2 \delta(E - V_B)$$









A.B. , Rev. Mod. Phys. **70** (1998) 77

M.Dasgupta et al, ARNPS **48** (1998) 401

Coupled channels effects on barrier distribution

Model problem

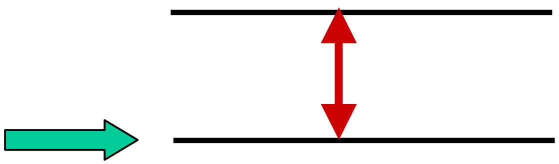
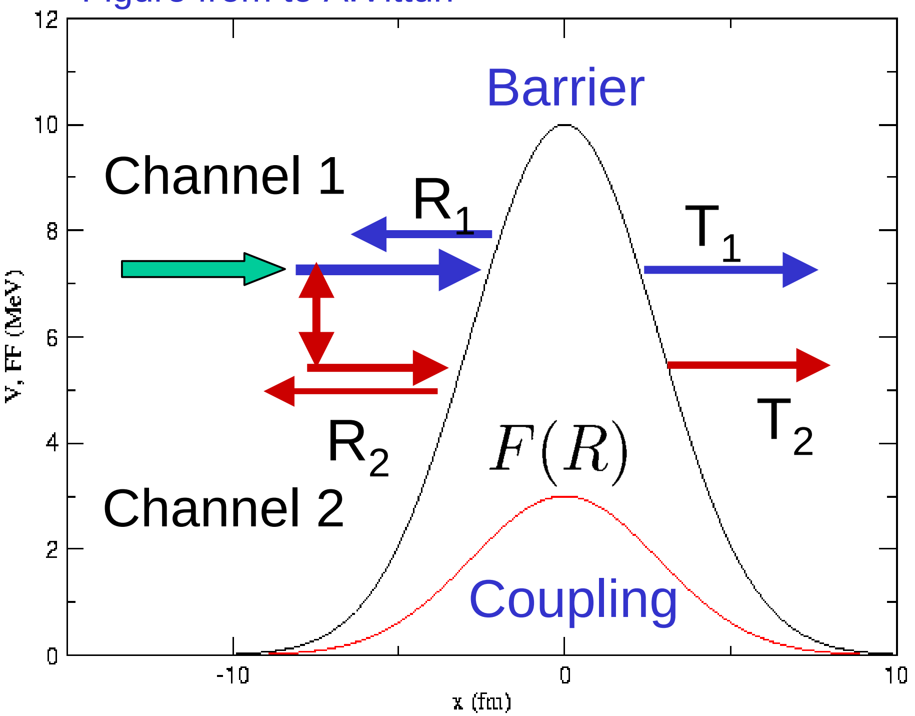


Figure from to A.Vitturi



Coupling of two channels 1,2 assumed degenerate for simplicity - coupling *F(R)* – incident waves in channel 1.

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+V(R)-E\right]\phi_1(R)=F(R)\phi_2(R)$$

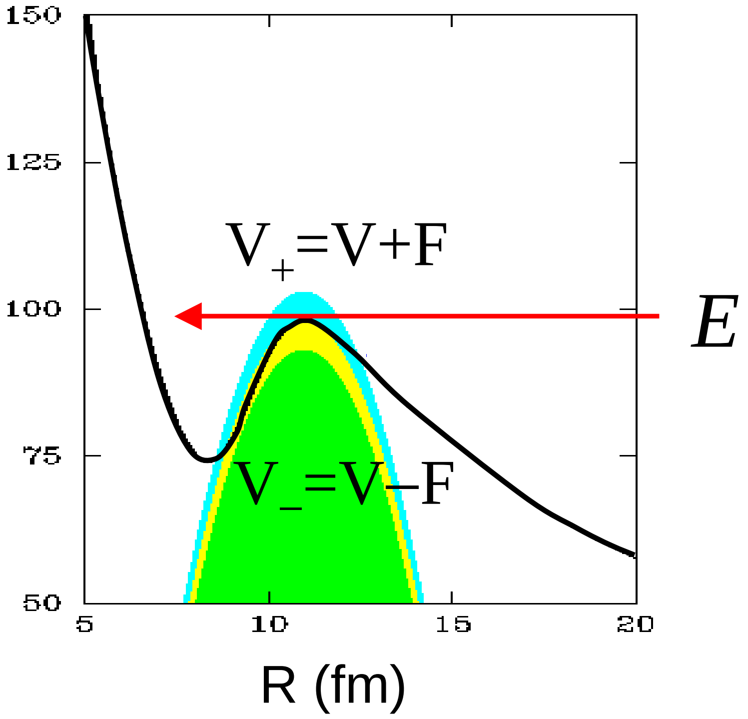
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+V(R)-E\right]\phi_2(R)=F(R)\phi_1(R)$$

Decoupled by addition and subtraction

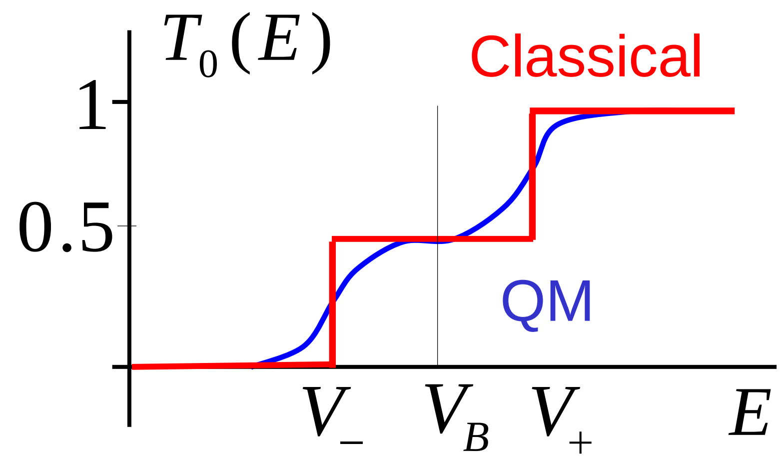
Decoupled, two barriers problem

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \{V(R) \pm F(R)\} - E\right] \mathcal{X}_{\pm}(R) = 0$$

$$\mathcal{X}_{\pm}(R) = \left[\phi_1(R) \pm \phi_2(R)\right]/\sqrt{2} \qquad \left|\langle \mathcal{X}_{\pm}|\phi_1\rangle\right|^2 = 1/2$$

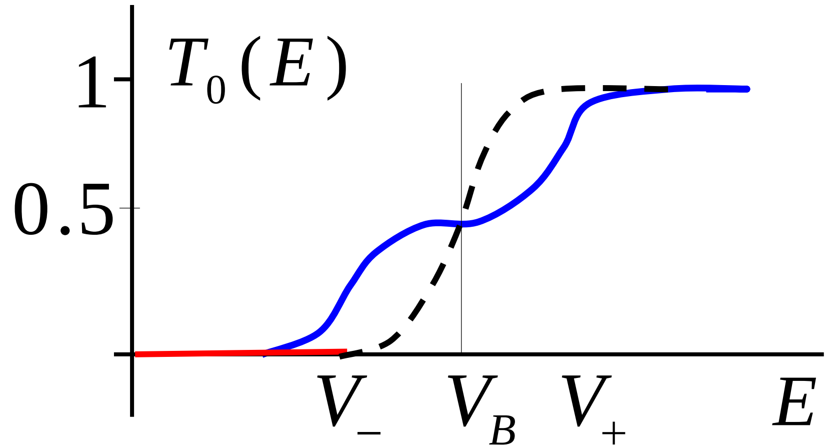


$$T_0(E) = \frac{1}{2}\left[T_+(E) + T_-(E)\right]$$



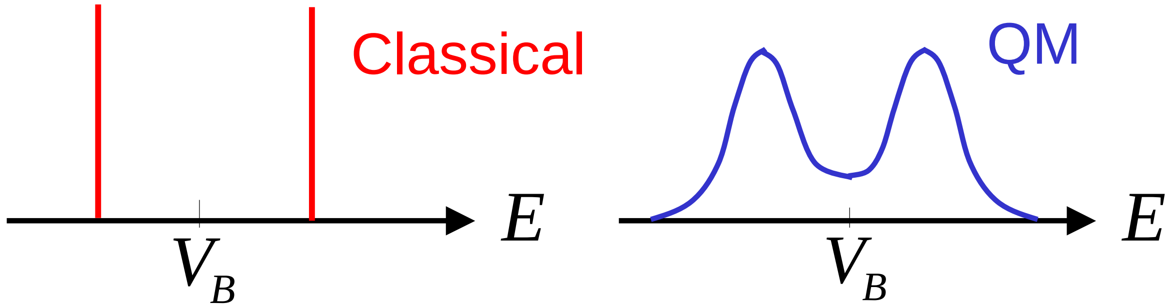
Barrier distributions will reflect channel couplings

In this simple model, channel coupling (no matter what the sign of the coupling potential) enhances fusion below and hinders fusion above the barrier – quite general result



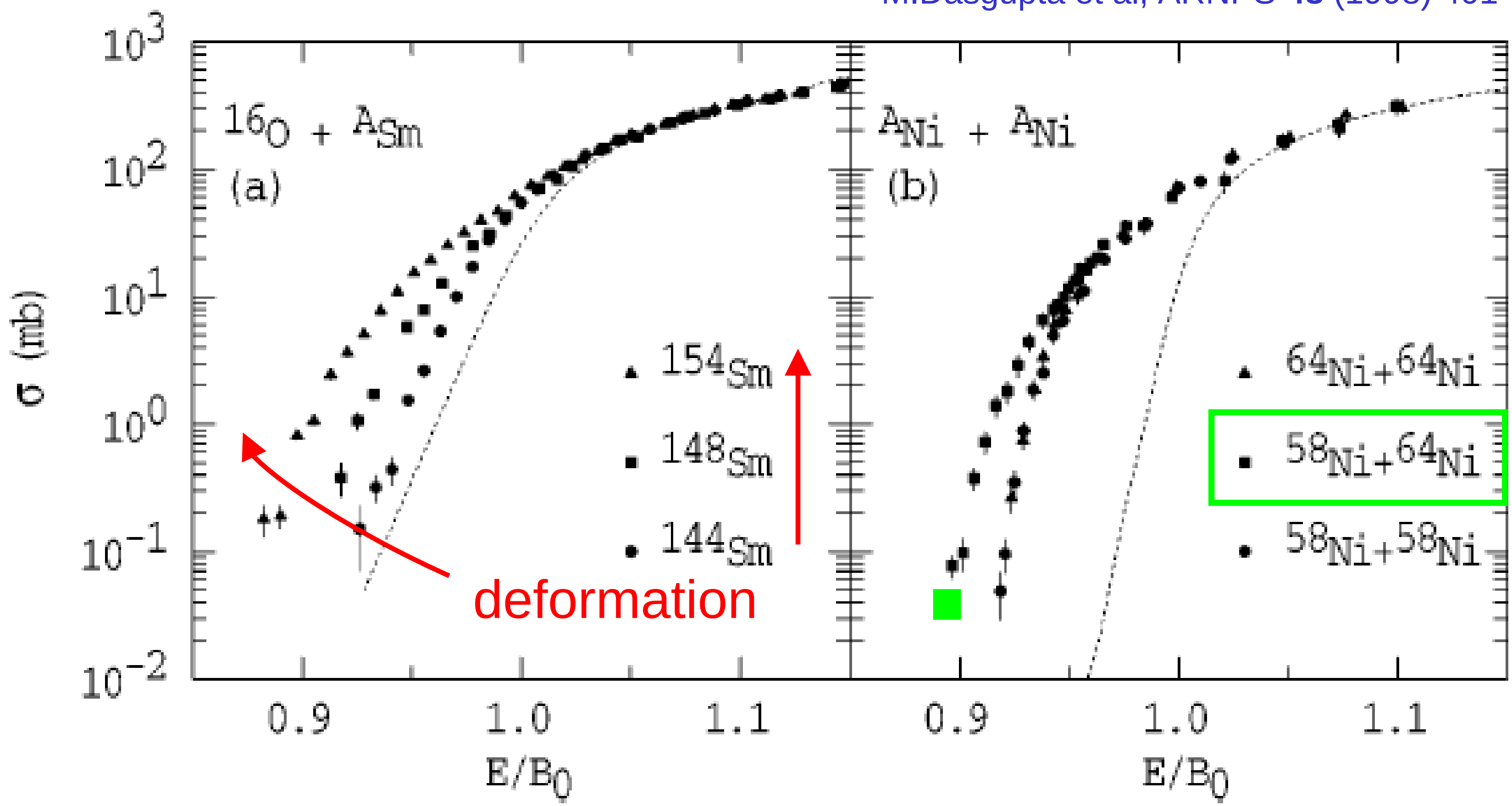
Non-degeneracy of the channels divides the flux incident on the barriers in a more complex way in the different channels (e.g. Beckerman, Rep. Prog. Phys. **51** (1988) 1047)

$$\frac{d^2}{dE^2}[E\sigma(E)] = \frac{\pi R_b^2}{2}\left[\delta(E - V_-) + \delta(E - V_+)\right]$$



Channel coupling – classic examples

M.Dasgupta et al, ARNPS **48** (1998) 401



R.G. Stokstad et al, PRL **41** (1978) 465, PRC **21** (1980) 2427.

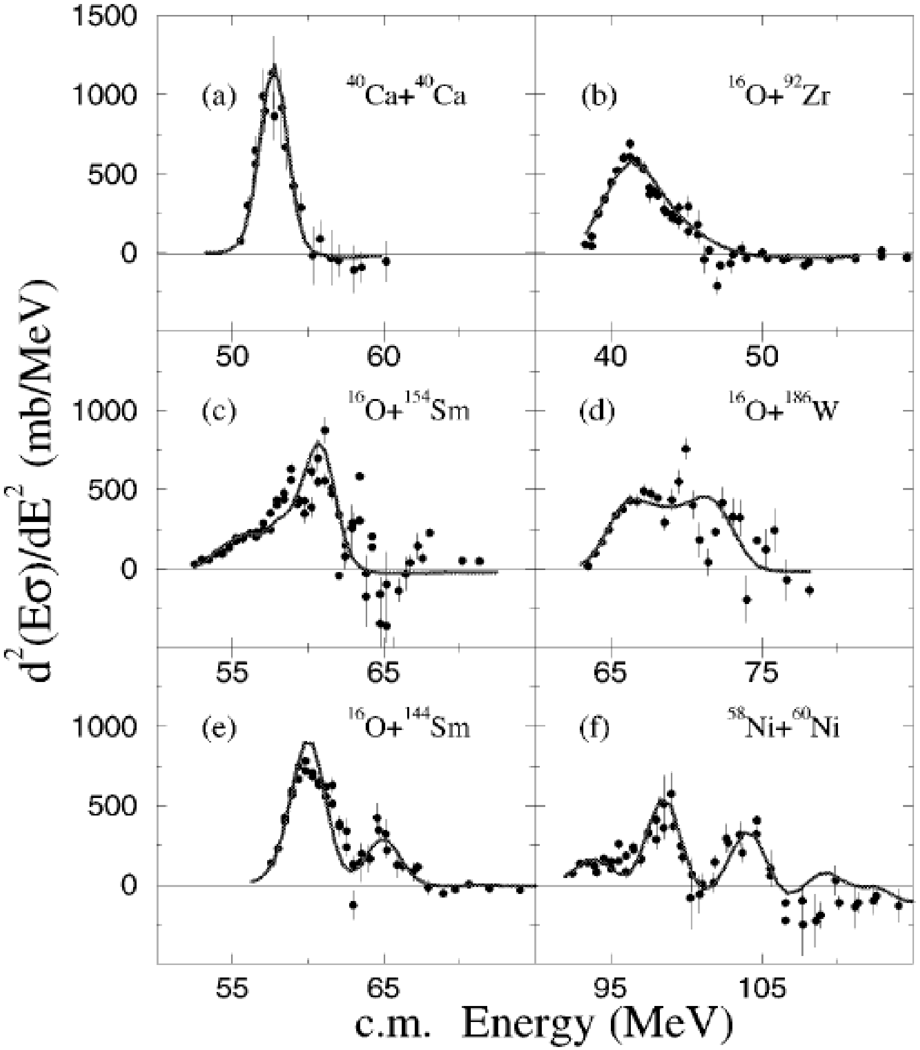
M. Beckerman et al, PRL **45** (1980) 1472, PRC **23** (1981) 1581, PRC **25** (1982) 837.

Empirical and calculated barrier distributions

For data of sufficiently high accuracy and precision, one can compare the values of

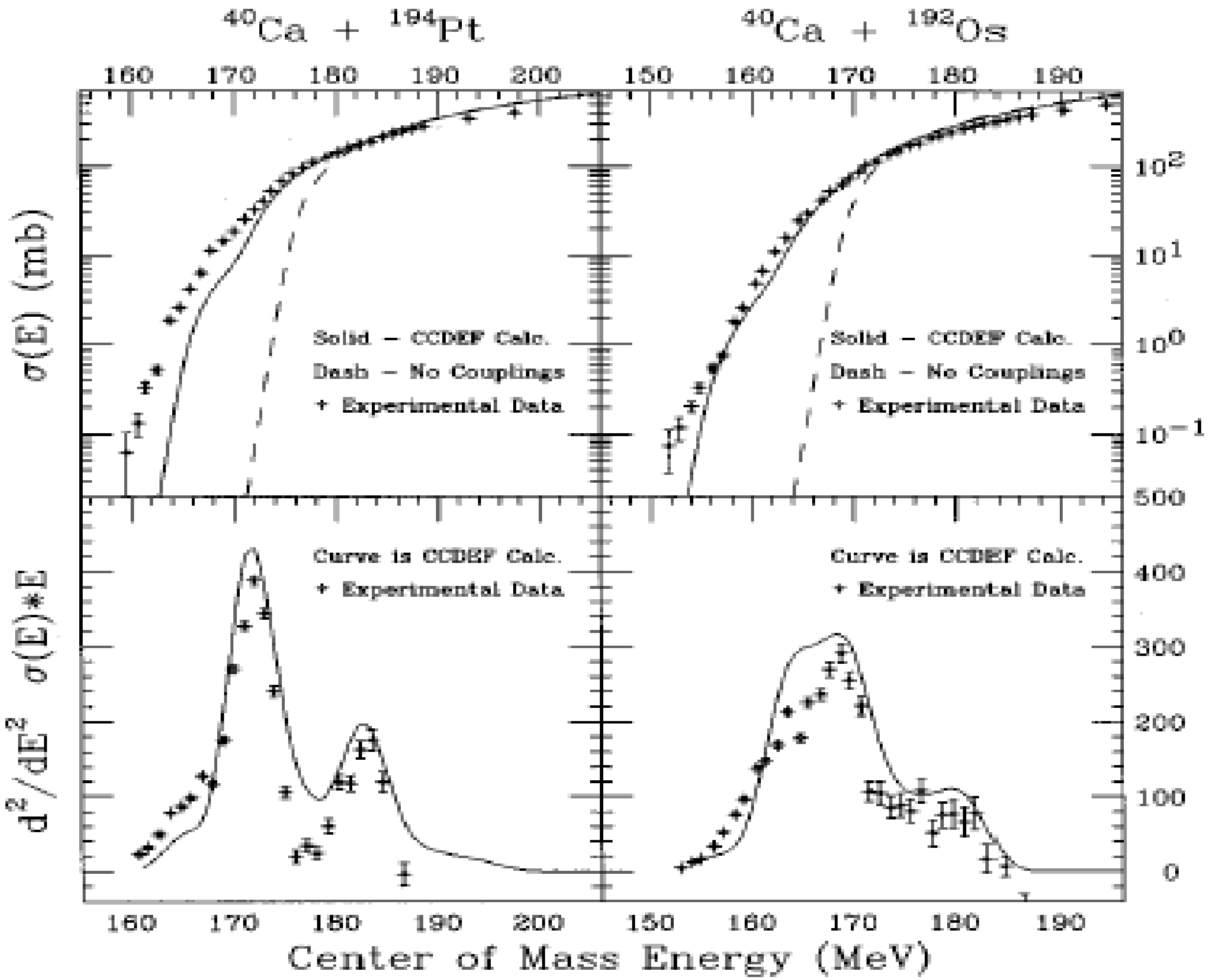
$$\frac{d^2}{dE^2}[E\sigma(E)]$$

deduced from the data and from detailed coupled channels calculations, including rotational, vibrational single particle or transfer couplings



M.Dasgupta et al, ARNPS **48** (1998) 401

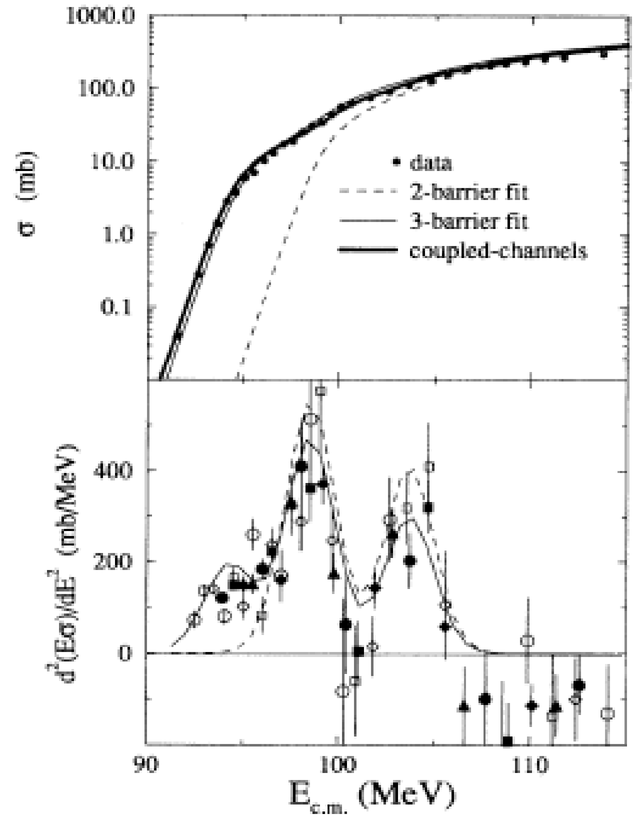
Fusion reaction processes

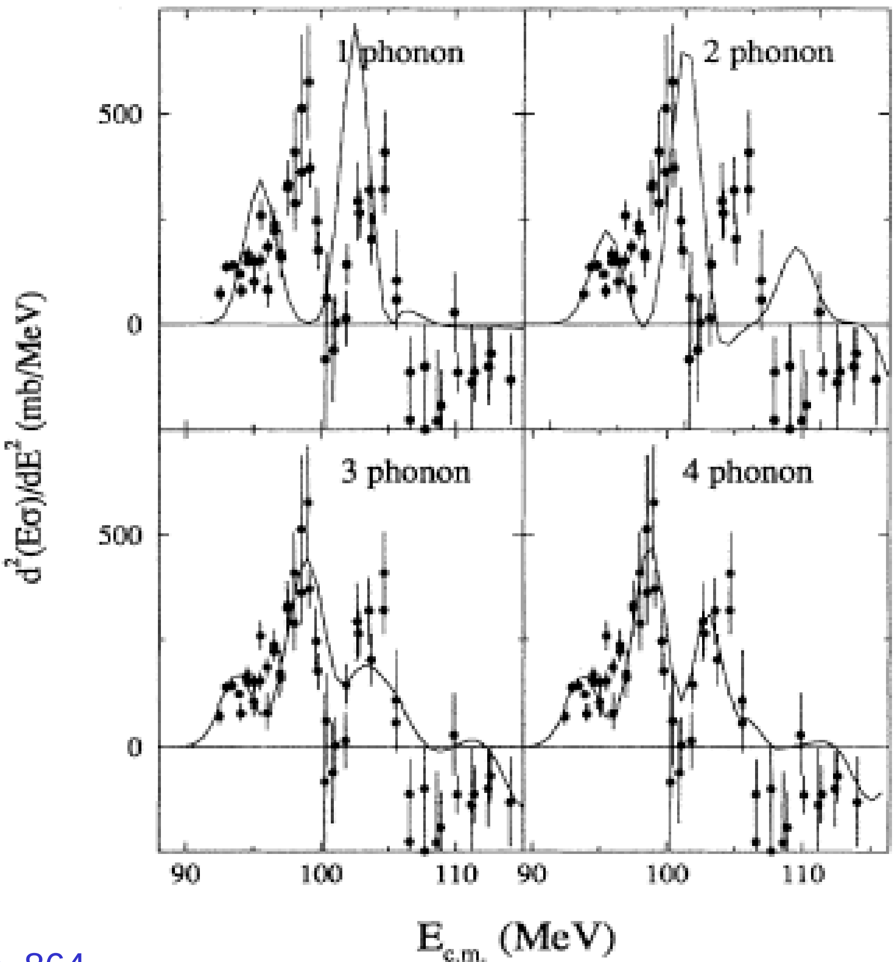


A.B. Balantekin and N.Takigawa, Rev. Mod. Phys. **70** (1998), 77-100

Coupling-assisted tunnelling - vibrational excitations

$^{58}Ni + ^{60}Ni$





A.M. Stefanini et al., Phys. Rev. Lett. **74** (1995), 864

Dispersion relations – threshold phenomena

Onset of inelastic processes with increasing energy develops absorption and perturbs the diffractive (real) part of the optical potential (assumed local for simplicity) - causality and unitarity

$$\begin{aligned} U_E(R) &= V_0(E,R) + \Delta U_E(R) \\ \Delta U_E(R) &= \Delta V_E(R) + iW_E(R) \end{aligned}$$

These terms are intimately connected through a dispersion-type relation (e.g. Feshbach, Ann Phys **5** (1958) 357**)**

$$\begin{aligned} \Delta V_E(R) &= +\frac{\mathcal{P}}{\pi}\int \frac{W_{E'}(R)}{E'-E}dE' \\ W_E(R) &= -\frac{\mathcal{P}}{\pi}\int \frac{\Delta V_{E'}(R)}{E'-E}dE' \end{aligned}$$

Other energy dependence, e.g. from non-locality, is not dispersive and is removed from relationship into $V_0(E,R)$

Information from the elastic scattering channel

Folding model (including account of non-localities**) often used to provide the radial shape and approximate strength of the real part of the potential, call it $F_E(R)$, Then, at each E

$$U_E(R) = [N_R(E) + iN_I(E)]\, F_E(R)$$

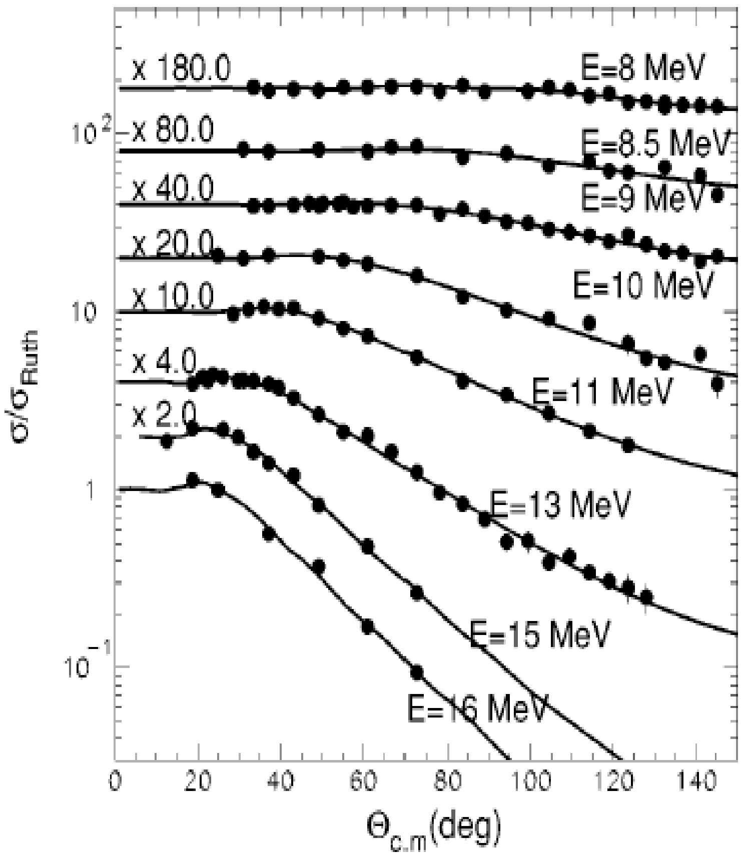
the N_R and N_I are fitted to data with N_R of order unity. (e.g. SP)

Else, entire potential

$$U_E(R) = V_E(R) + iW_E(R)$$

is fitted to elastic scattering data

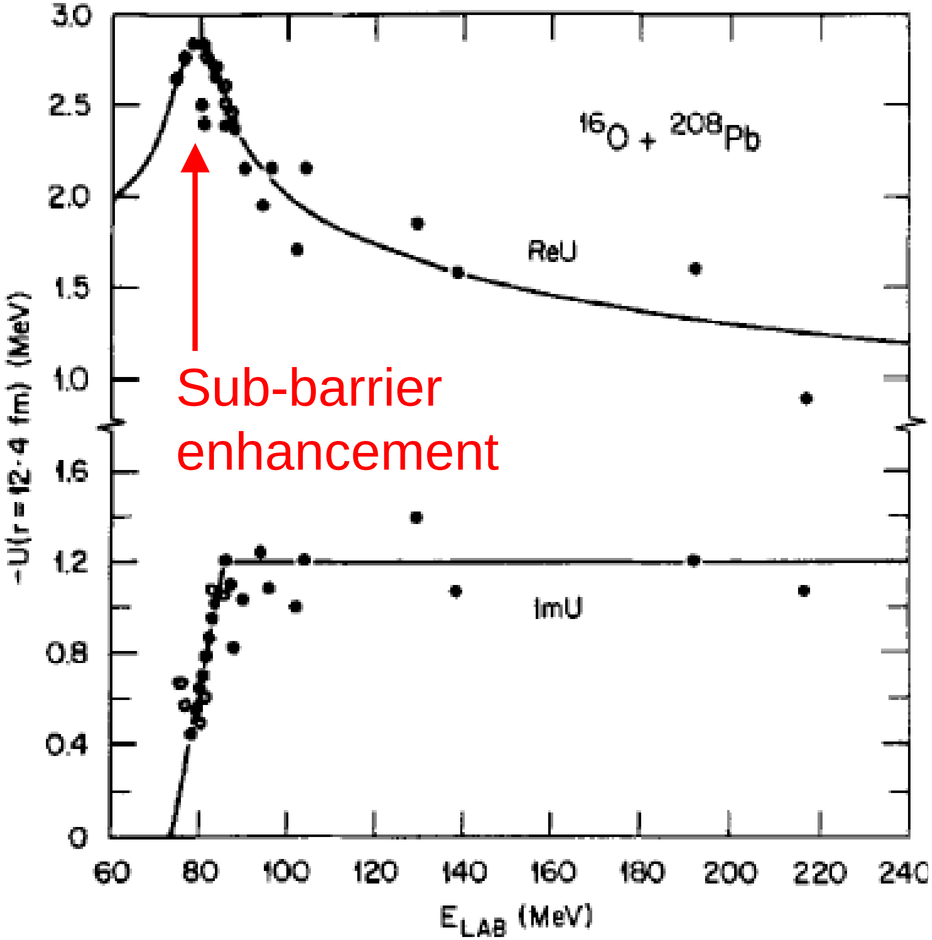
$^7Li + ^{28}Si$



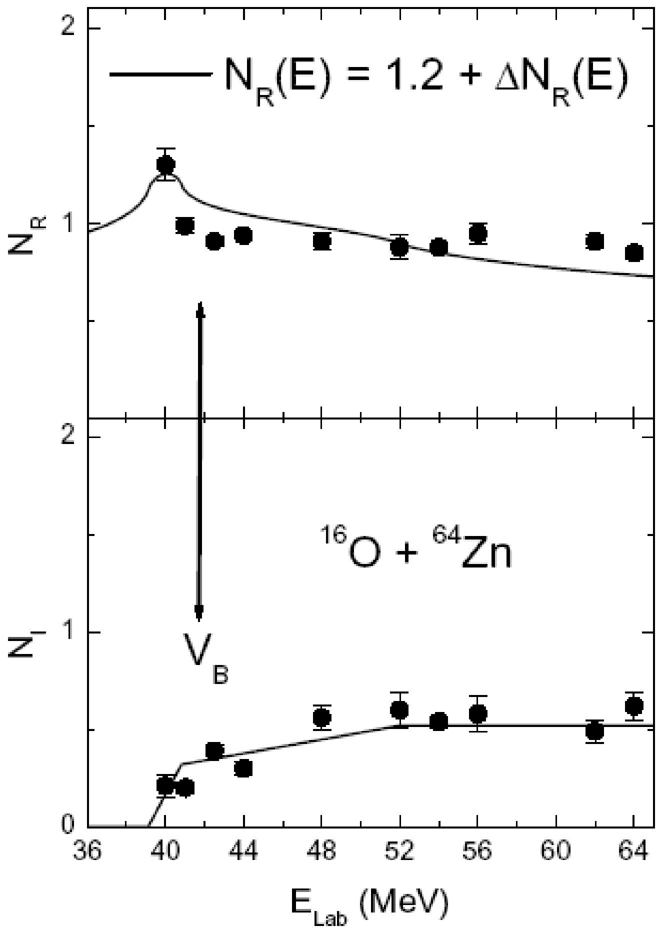
** L.C. Chamon et al., PRC 66 (2002) 014610

A. Pakou et al., PRC 69 (2004) 054602

Dispersion relations in comparison with data

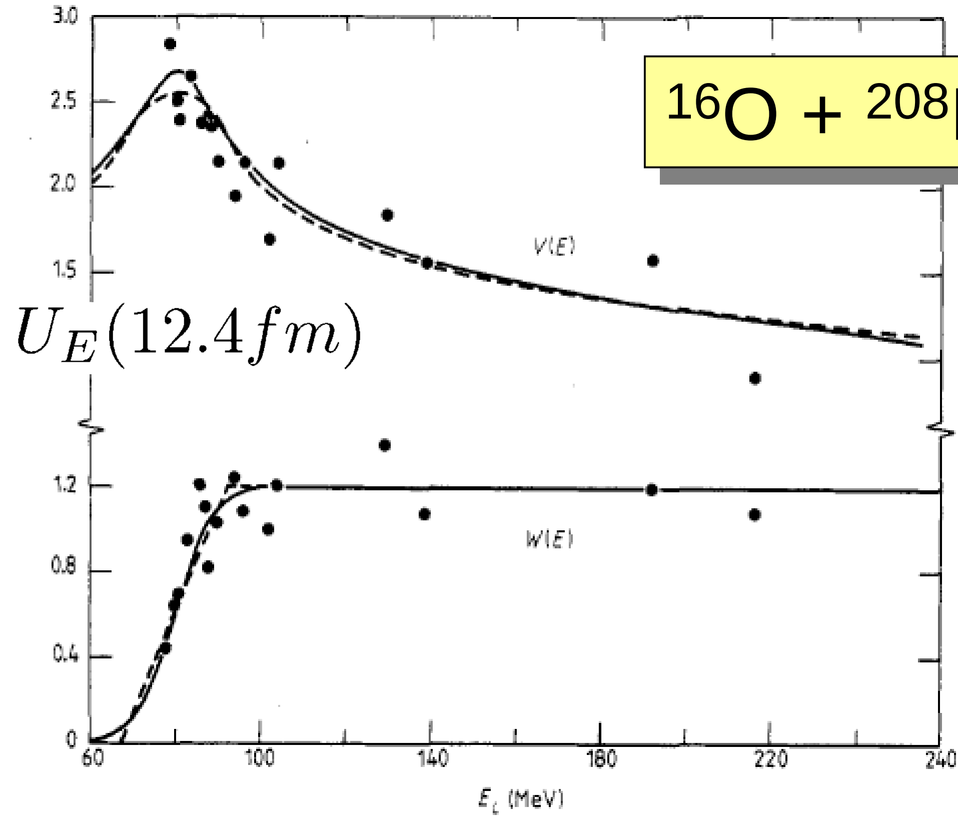


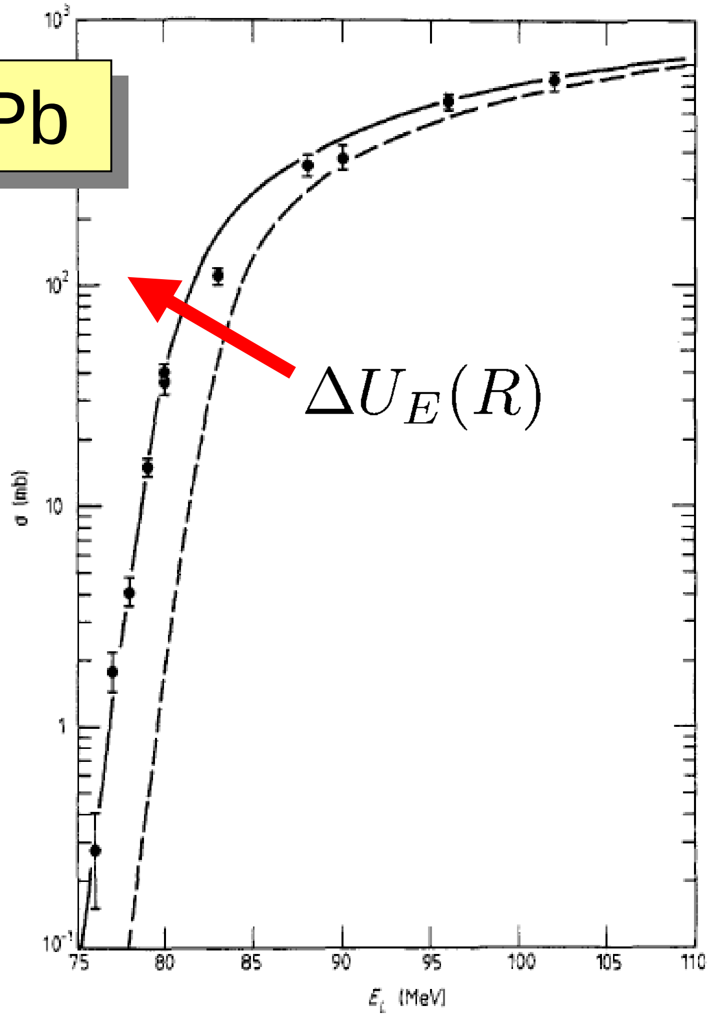
M.A. Nagarajan, C.C. Mahaux, and G.R. Satchler, PRL **54** (1985) 1136



L. Chamon, et al, NUSTAR05

Dispersion relation and sub-barrier enhancement





Barrier penetration calculations with and without $\Delta U_E(R)$