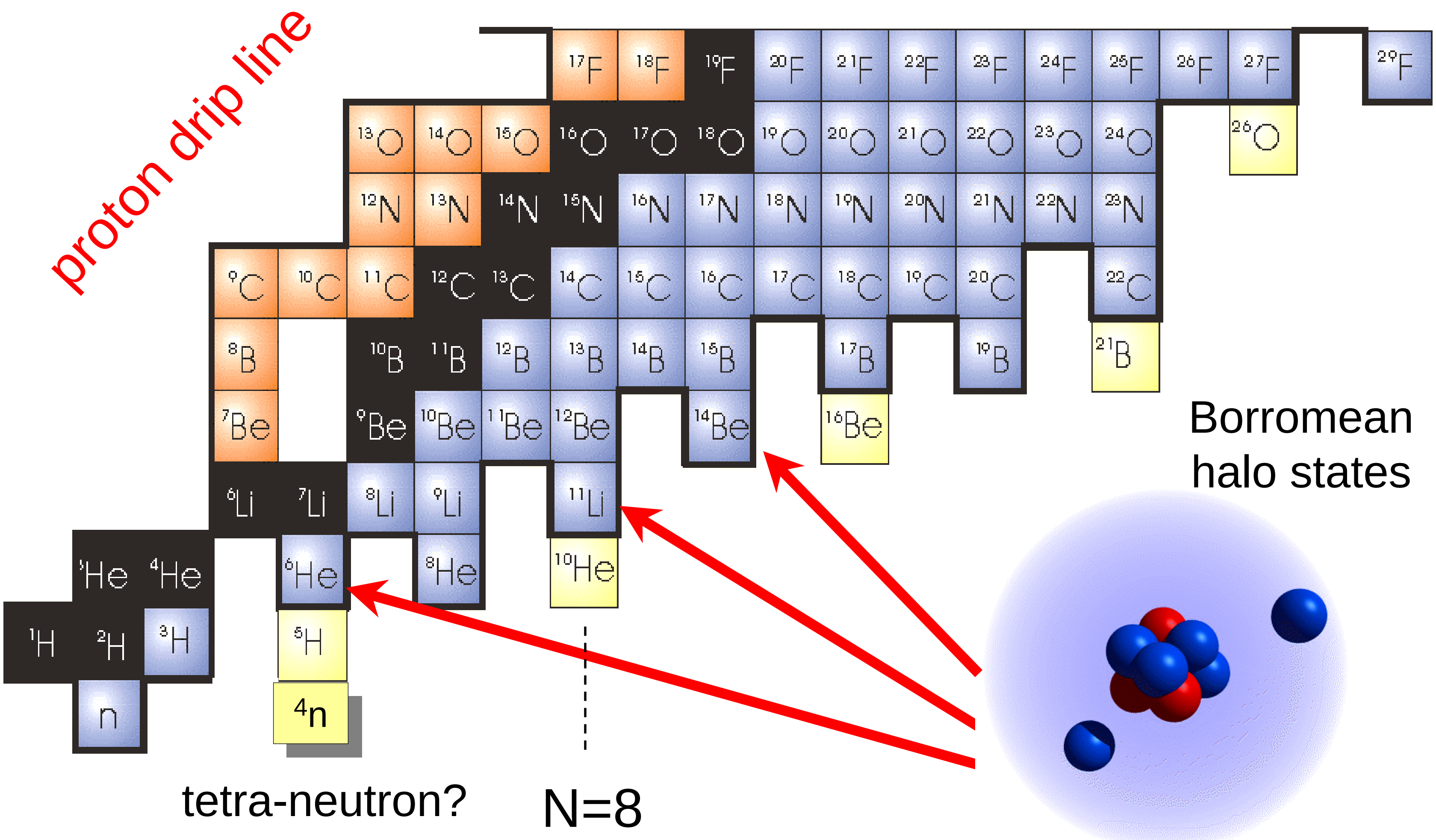
Three lectures – will plan to discuss

Lect I : Fusion of ions: motivation and introductory remarks, concepts, terminology, models and indicators of fusion, reaction dynamics, barriers, coupled channels - assisted tunnelling, barrier distributions and optical potentials. Experience.

Lect II: Weakly-bound systems, methods for break-up calculations, fusion in few-body models of break-up reactions. Many open questions.

Lect III: Partial/incomplete fusion at higher incident energies, applications to knockout of one- and two nucleons and applications for spectroscopy of exotic nuclei

The driplines in light nuclei – exotic nuclei



Weakly-bound and exotic nuclear systems

Stable systems

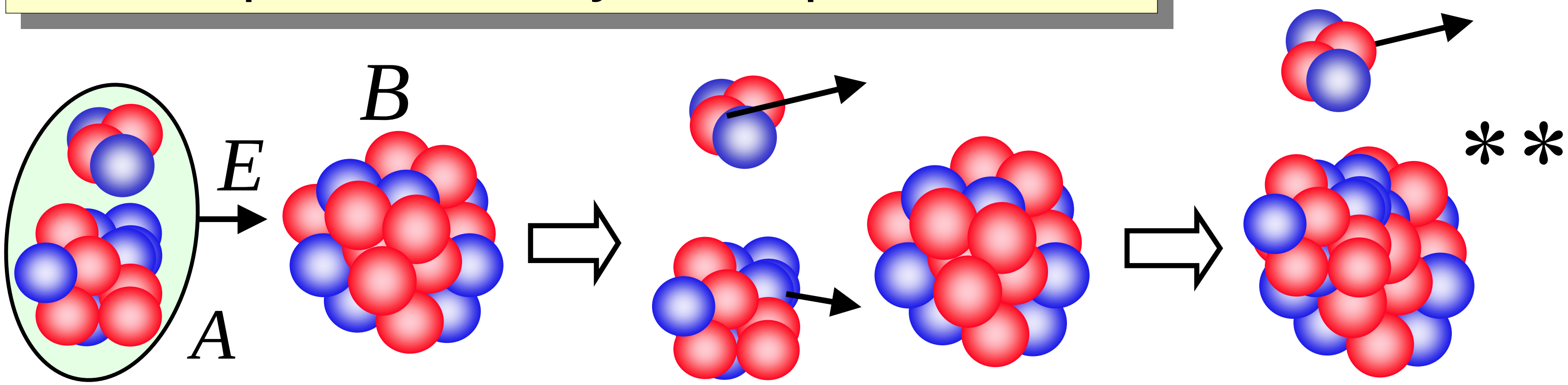
^{6}Li	$\rightarrow$	$^{4}He + d$	$S_\alpha = 1.48$ MeV
^{7}Li	$\rightarrow$	$^{4}He + t$	$S_\alpha = 2.45$ MeV
^{9}Be	$\rightarrow$	$^{8}Be + n \rightarrow {}^{4}He + {}^{4}He + n$	$S_n = 1.67$ MeV

Unstable (exotic) systems

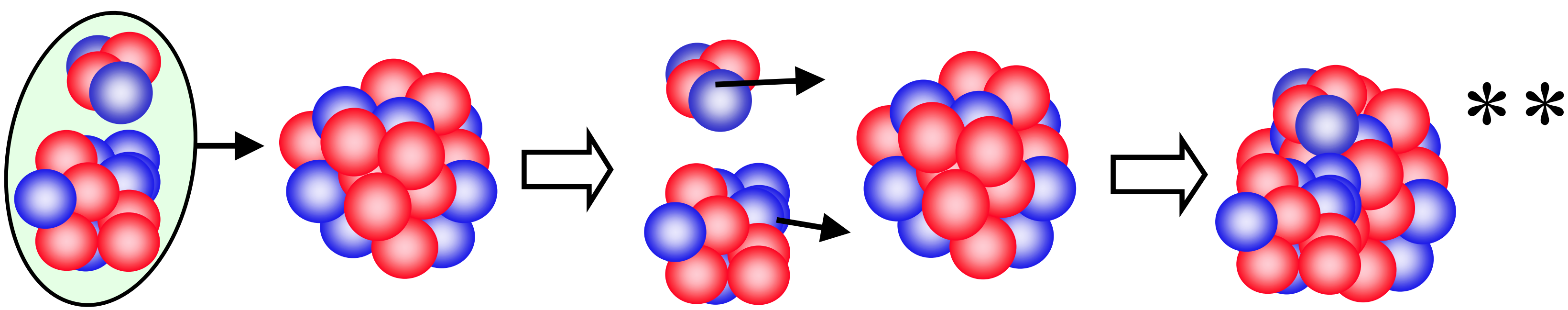
^{6}He	$\rightarrow$	$^{4}He + 2n$	$S_{2n} = 0.98$ MeV
^{11}Be	$\rightarrow$	$^{10}Be + n$	$S_n = 0.50$ MeV
^{11}Li	$\rightarrow$	$^{9}Li + 2n$	$S_{2n} = 0.33$ MeV

New challenge is presented by weak binding

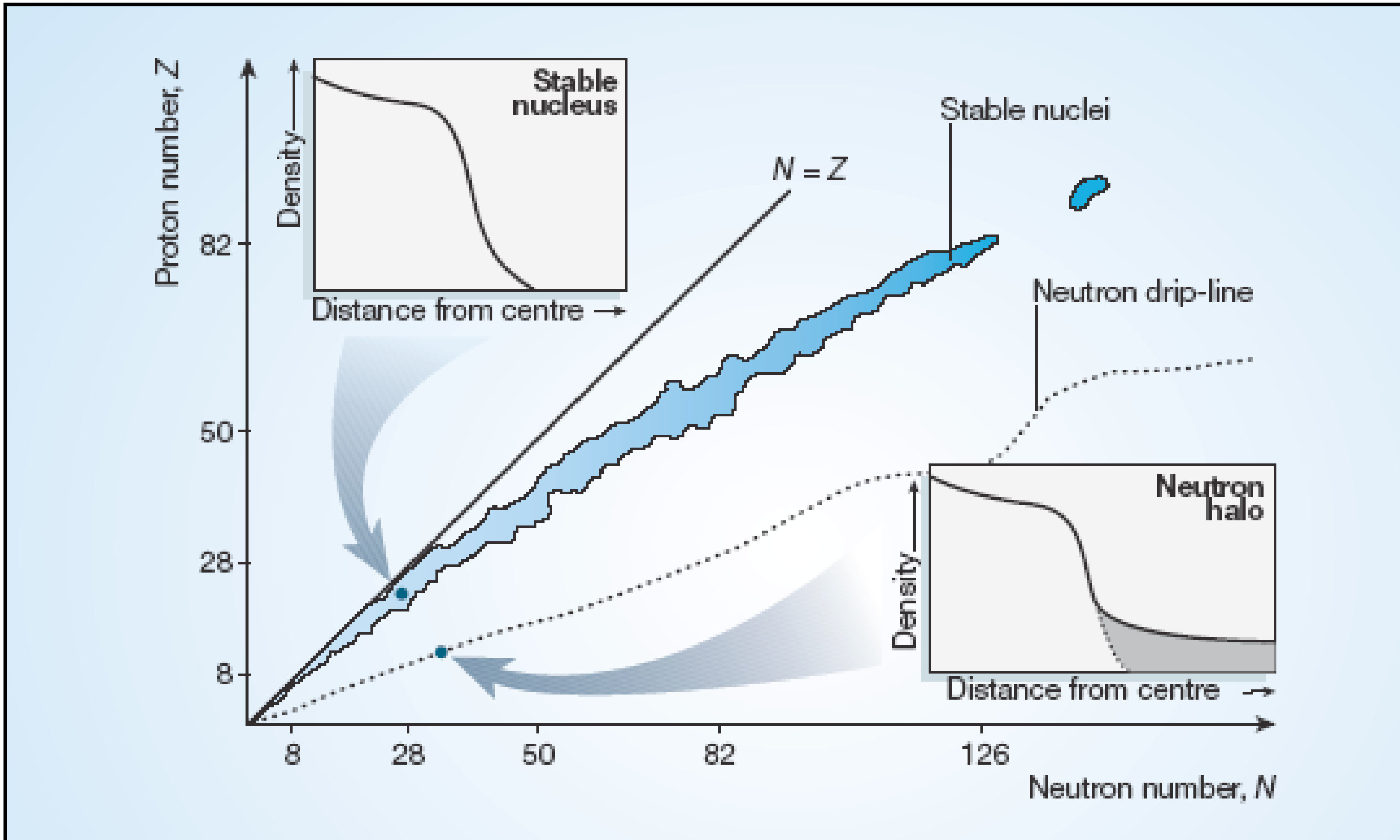
Break-up, followed by incomplete fusion



Break-up, followed by complete fusion



Neutron rich phenomena

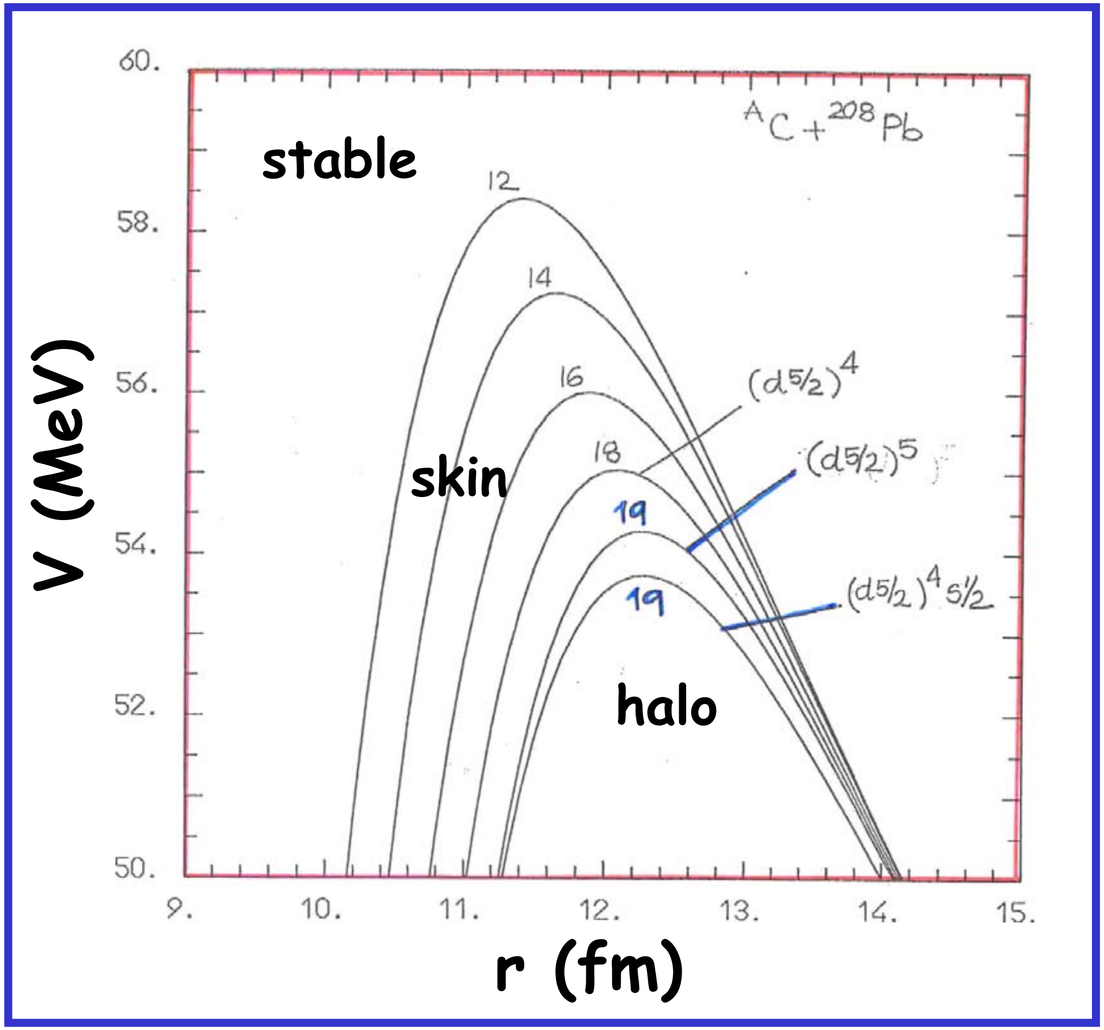


D. Hinde et al., Nature. **431** (2004) 748

Static effects – barriers for n-rich Carbon isotopes

$^{A}C + ^{208}Pb$

HF predictions

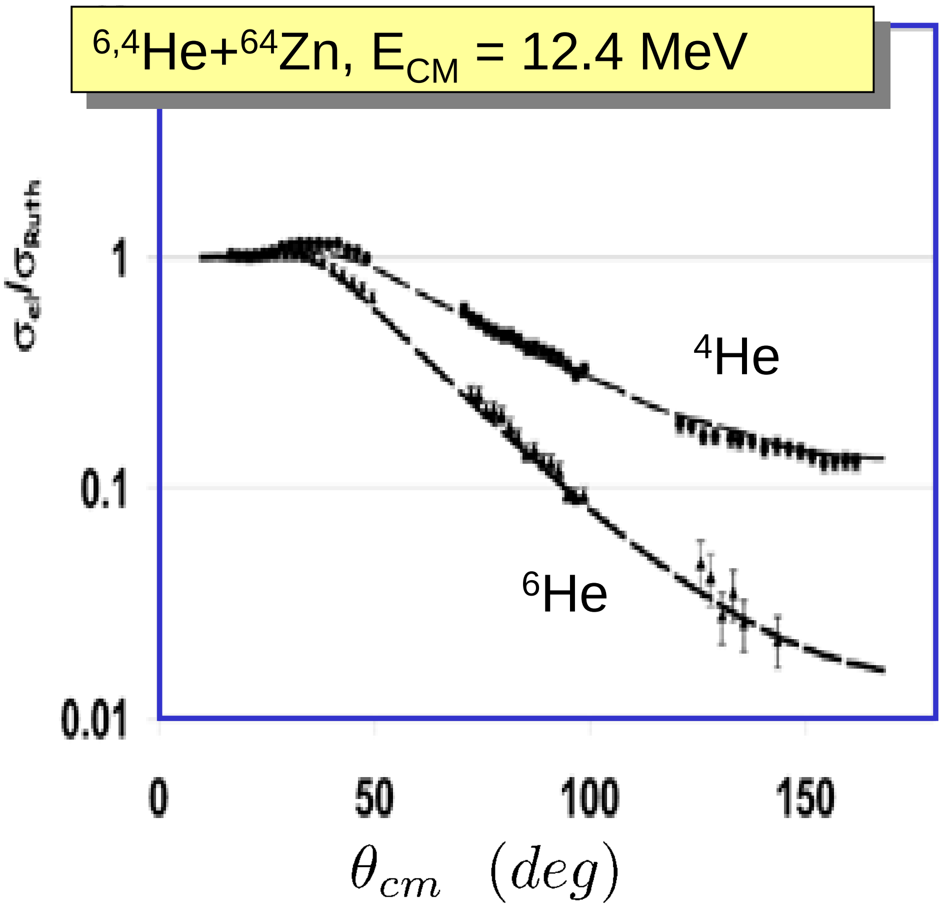


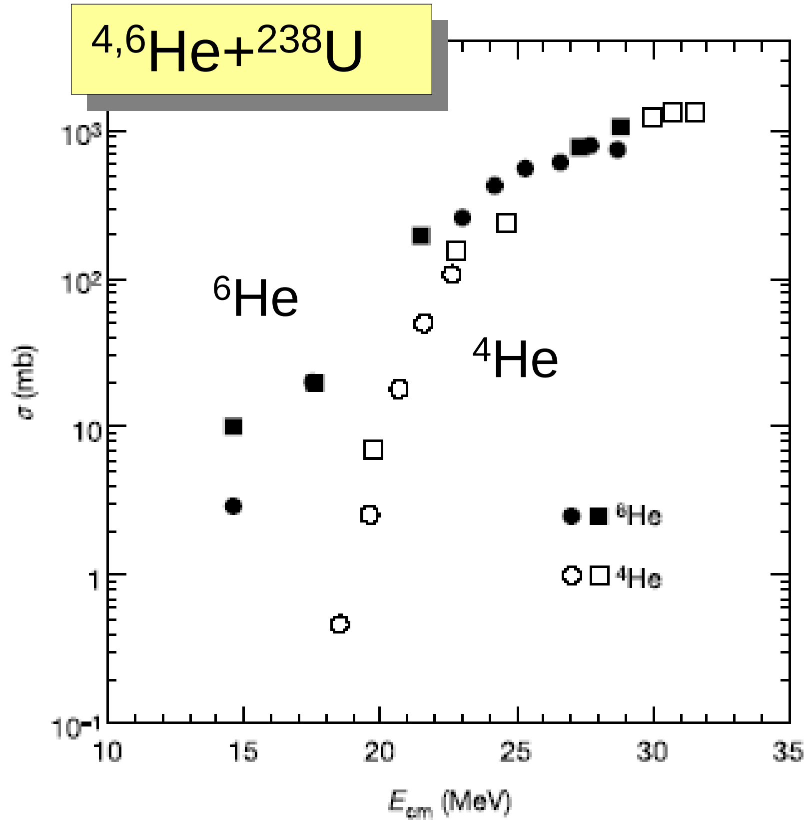
A. Vitturi, NUSTAR'05, Surrey January 2005

What are considerations for weakly-bound nuclei

- Static effects due to tails in density distribution - longer tails in ion-ion potential, lowering of Coulomb barrier - larger sub-barrier fusion probabilities
- Dynamical effects due to coupling to states in the continuum (break-up processes), polarization term in optical potential – larger sub-barrier fusion
- Breakup is due to the different forces acting on the fragments, that then separate – reduced expectation of total fusion
- Weak binding leads typically to large +ve Q-values for nucleon transfers

Elastic scattering reflects loose binding





A. Di Pietro et al., Europhys. Lett. **64** (2003) 309

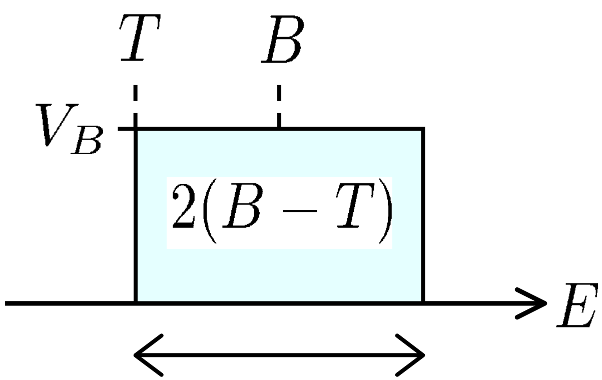
M. Trotta et al., PRL **84** (2000) 2342
R. Raabe et al., Nature **431** (2004) 823

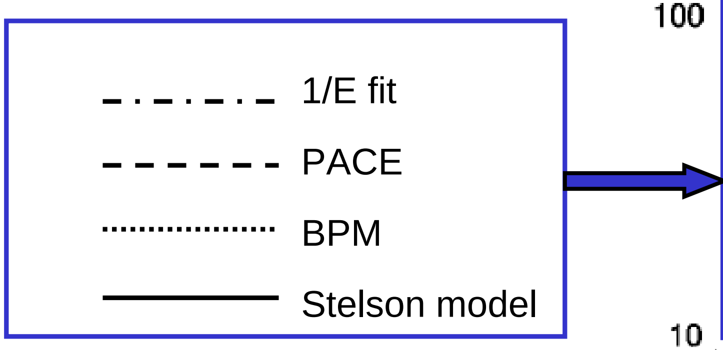
Qualitative features of loosely bound systems

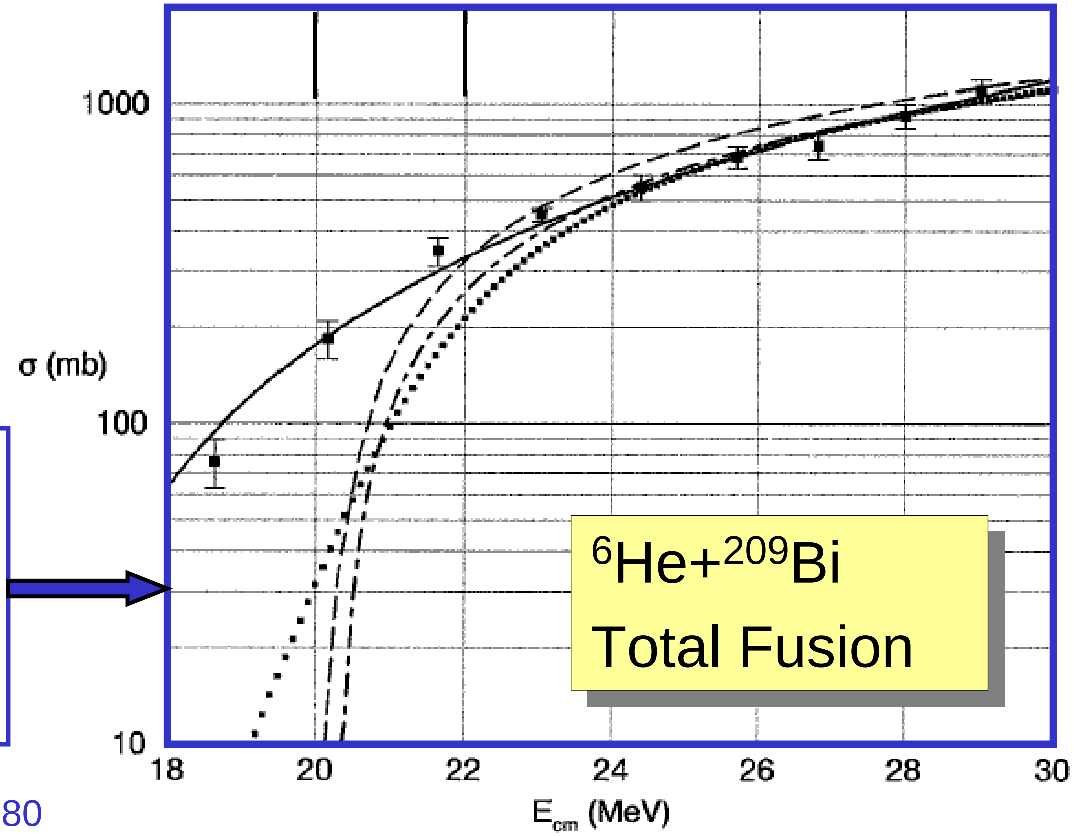
$$\sigma(E) = \pi R_b^2 (E-T)^2/[4(B-T)E]$$

$$T = 15.4\,MeV,\ (B-T) = 5.14\,MeV$$

P.H. Stelson et al.,
PRC **41** (1990) 1584

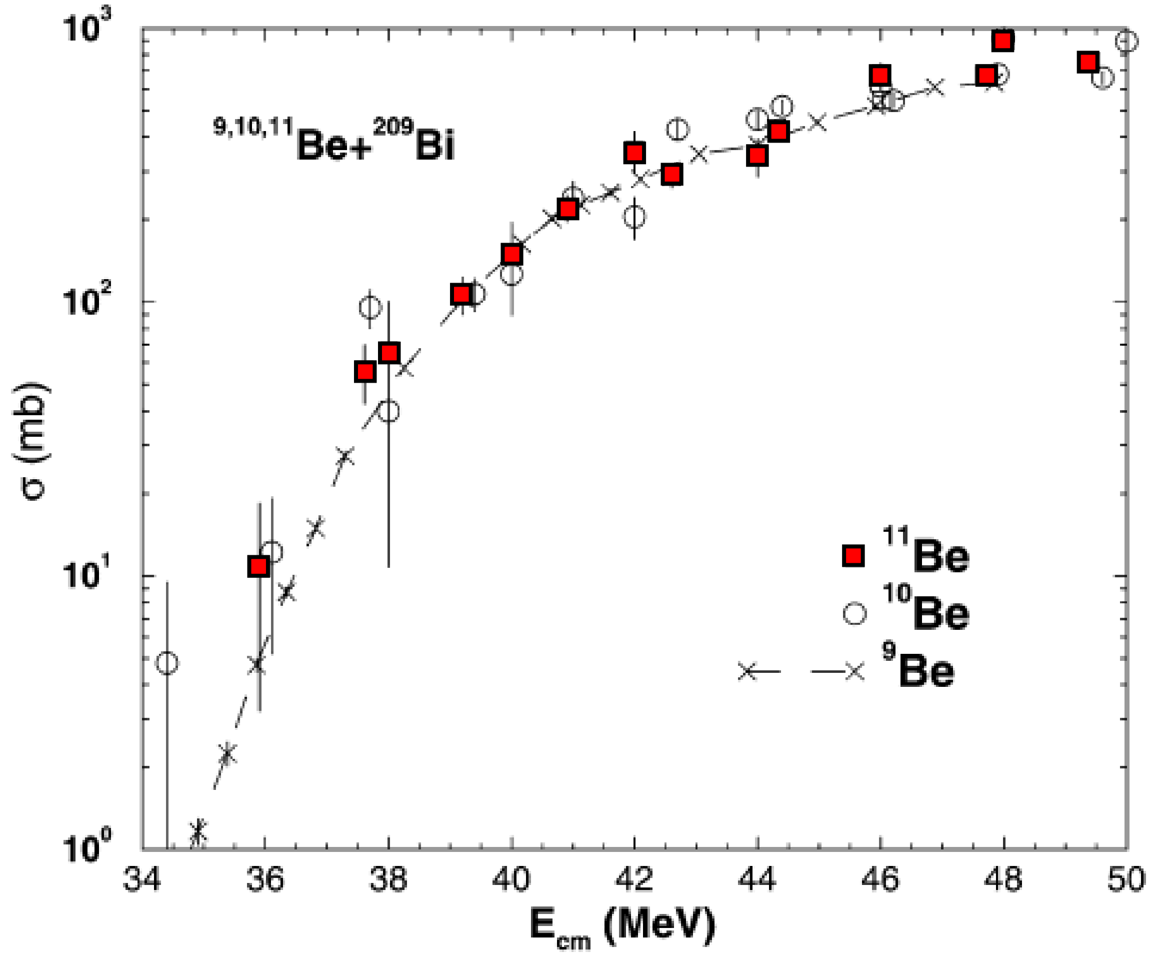






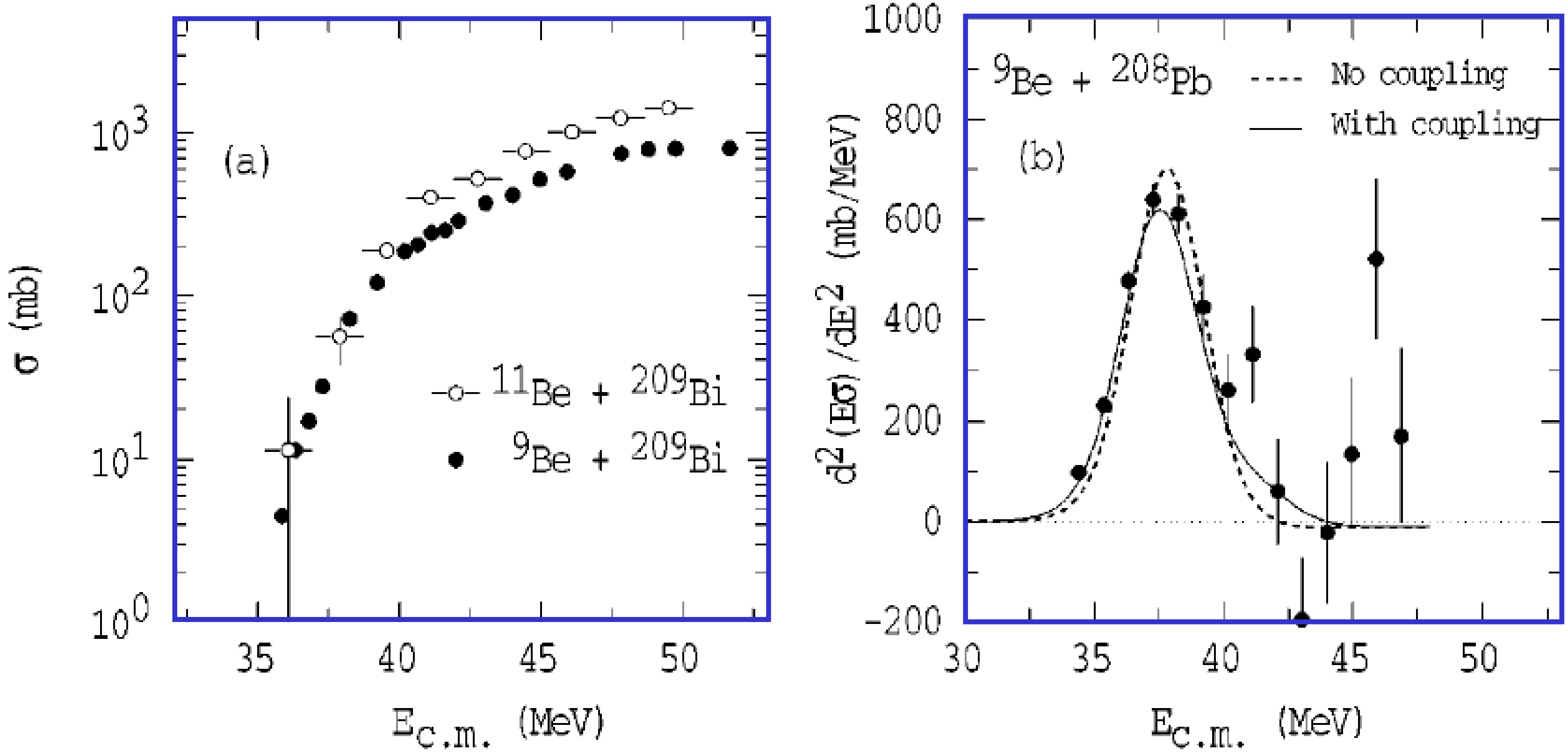
J.J. Kolata et al., PRL **81** (1998) 4580

Beryllium isotopes – 11Be a halo nucleus case



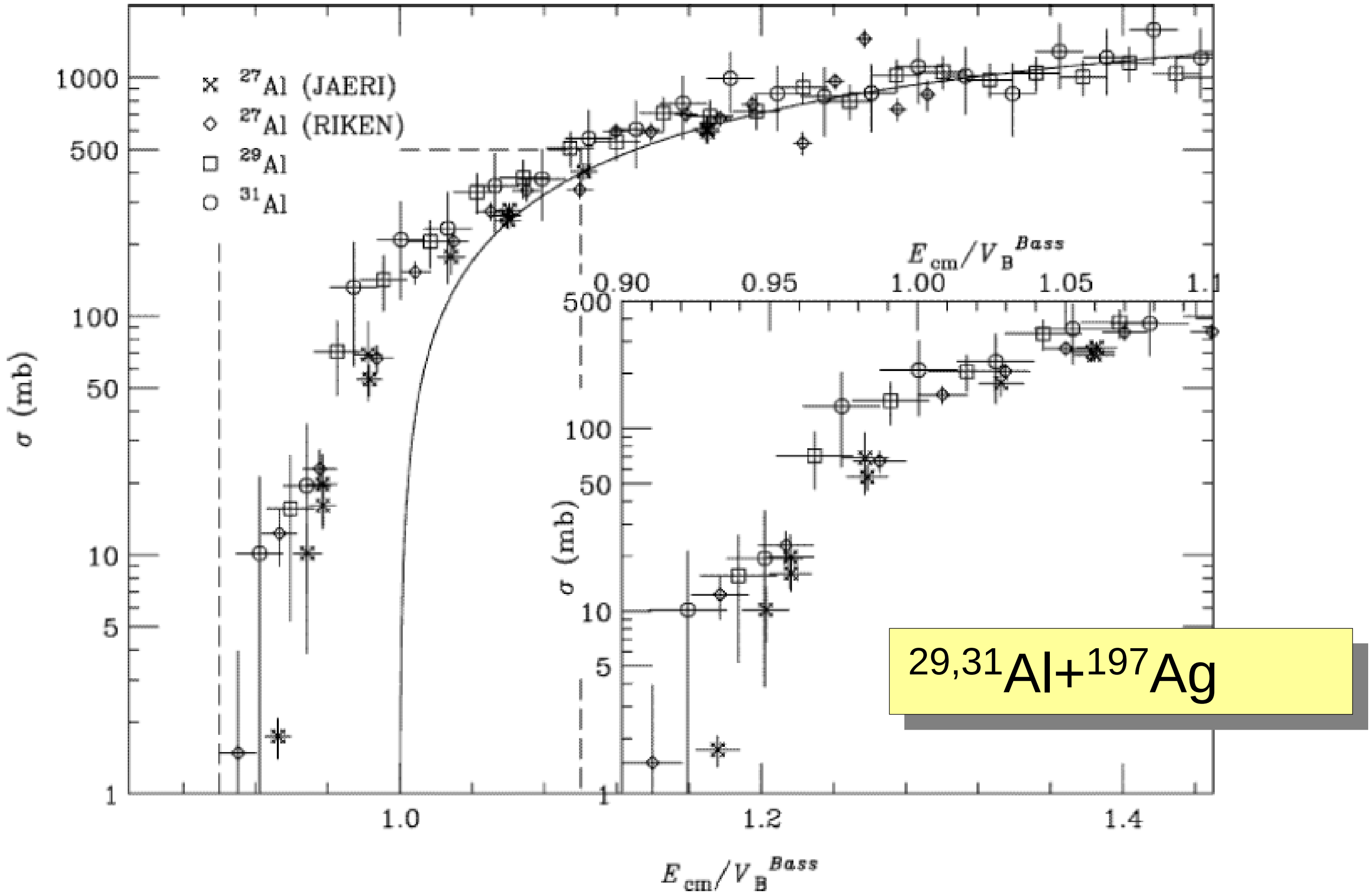
C. Signorini, Nucl.Phys. **A735** (2004) 329

Fusion of the Beryllium isotopes



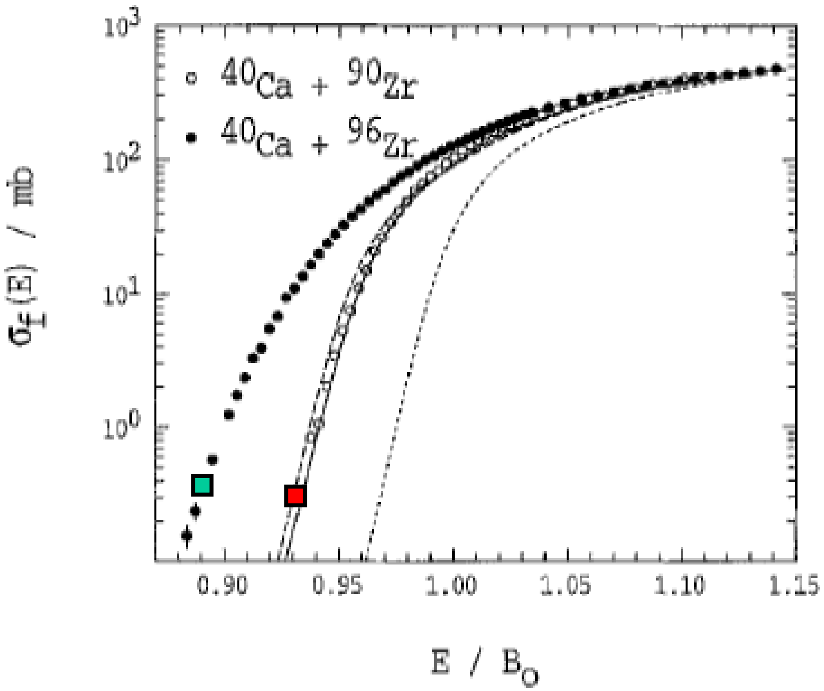
Data: C. Signorini, Eur. Phys. J. A **13** (2002) 129

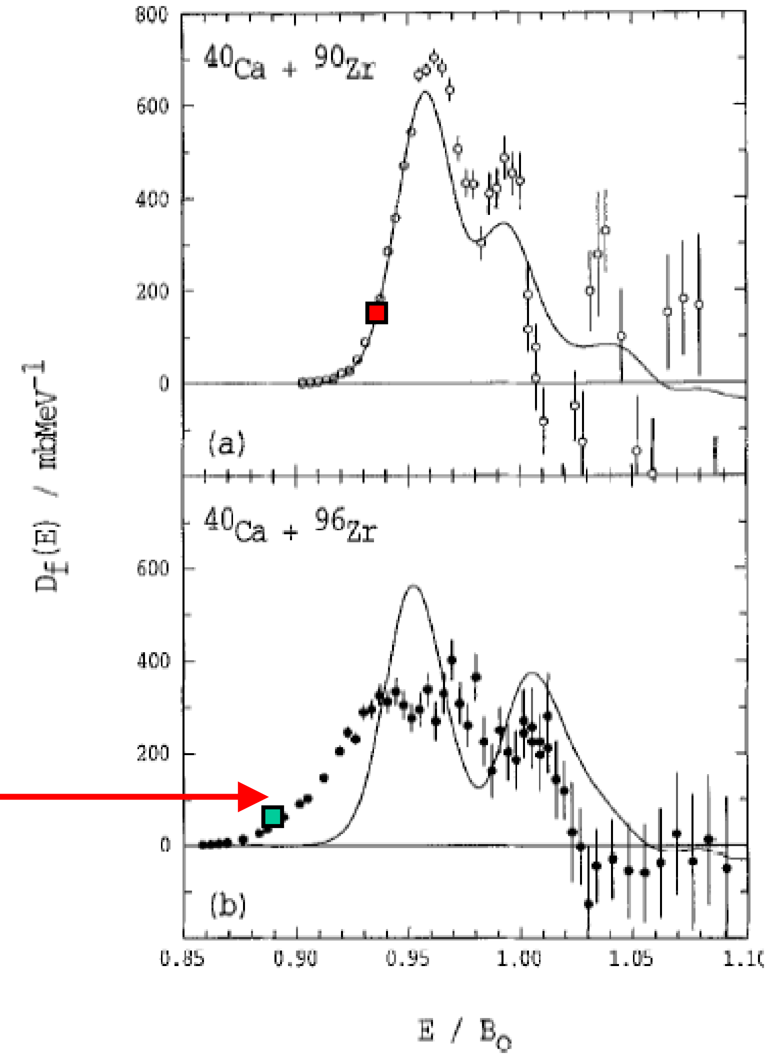
Heavier n-rich systems – no enhancement?



Y. Watanabe et al., EJPA. **10** (2001) 373

Static effects – barriers for n-rich systems

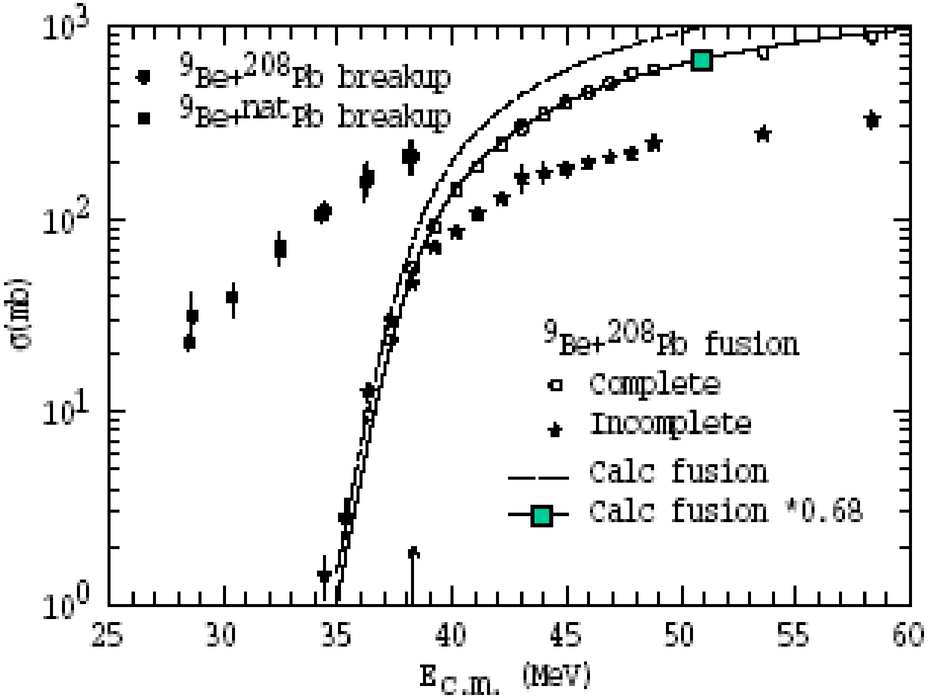


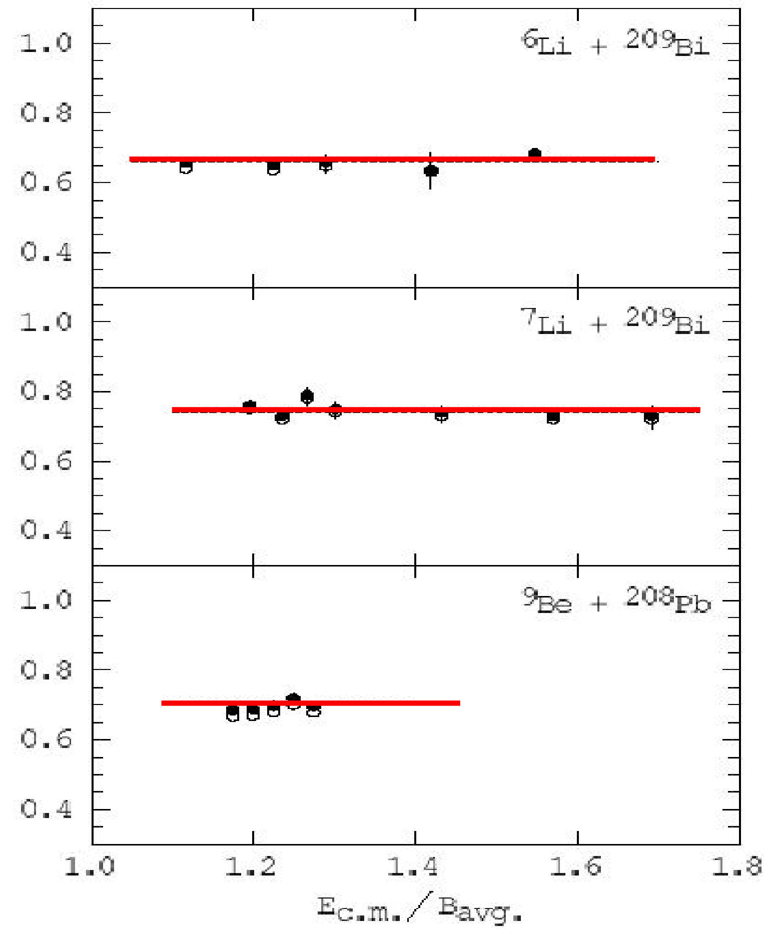


Extended sub-barrier tail – transfer reactions?

H Timmers et al., Phys Lett. B **399** (1997) 35

Break-up suppressing fusion above the barrier?





several examples suggesting break-up channels suppress the expected complete fusion cross section above the barrier

D.J. Hinde et al., PRL **89** (2002), 272701

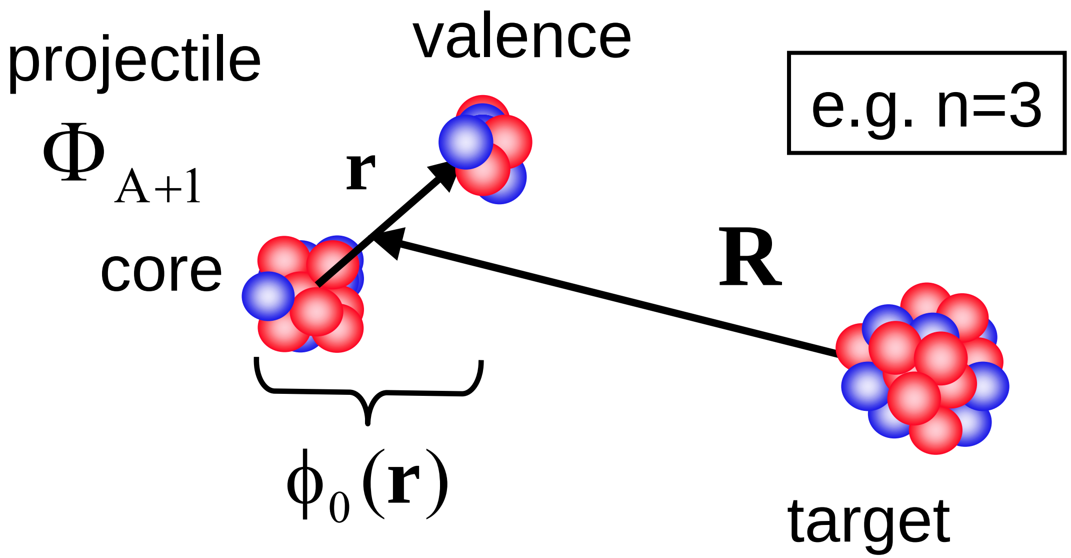
M. Dasgupta et al., PRC **70** (2004), 024606

Practical difficulties - for making detailed tests

- Theoretical definition:
  - Complete fusion: capture all projectile fragments, CF
  - Incomplete: capture only some of fragments, ICF
- Experimentally:
  - Complete fusion: capture all projectile charge, ChF
  - Incomplete: capture only some of the charge, IChF
  - What is measured in each case, TF, ICF, CF?
  - Differentiate between ICF and transfers?
- For ^{6}Li, ^{7}Li reactions these definitions agree (mostly)
- Weakly bound neutron rich nuclei and halos: What happens to the dissociated neutrons?

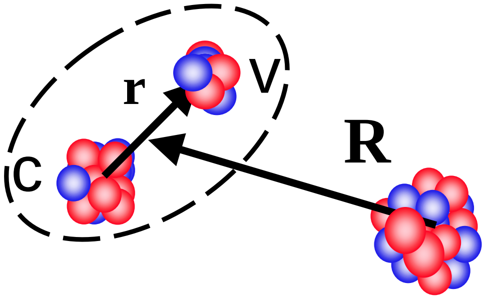
Few-body models of nuclear reactions

There are no practical many-body reaction theories - we construct model 'effective' few-body models (n=2,3,4 …)



Construct an effective Hamiltonian H and solve as best we can the Schrödinger equation: $\mathrm{H}\Psi = \mathrm{E}\Psi$

Few-body models - effective interactions



$$H = T_{\mathbf{r}} + V_{vc} + T_{\mathbf{R}} + \underbrace{V_{cT} + V_{vT}}$$

binds projectile

effective (complex) interactions of c and v individually with target (nuclear + Coulomb potentials)

(a) From experiment: potentials fitted to available data for c+T or v+T scattering at the appropriate energy per nucleon

(b) From theory: folding models or multiple scattering, for example

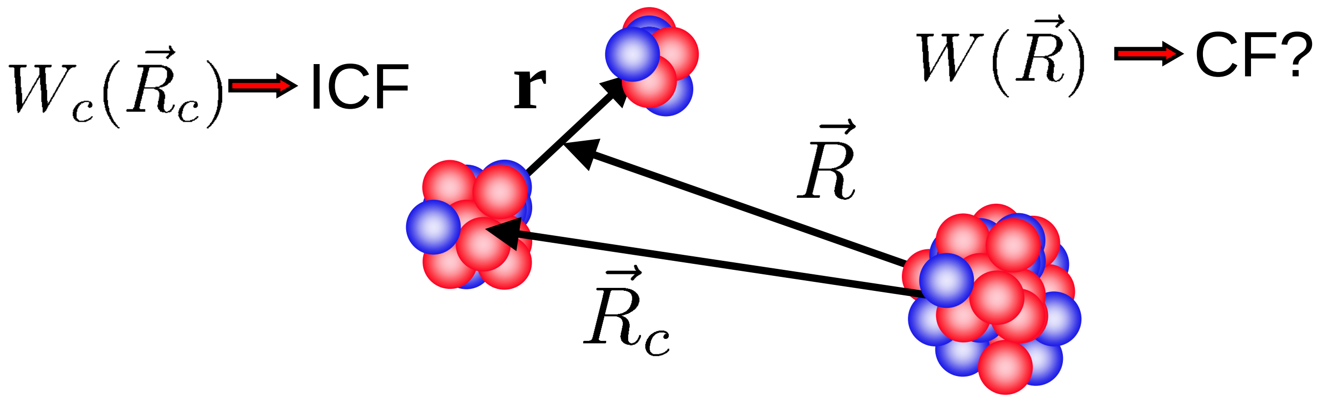
$$V_{cT}(\mathrm{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \underbrace{\rho_c(\mathrm{r}_1)\, \rho_T(\mathrm{r}_2)} \; \underbrace{\mathrm{v}_{\mathrm{NN}}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)}$$

core and target densities

nucleon-nucleon t-matrix or effective NN interaction

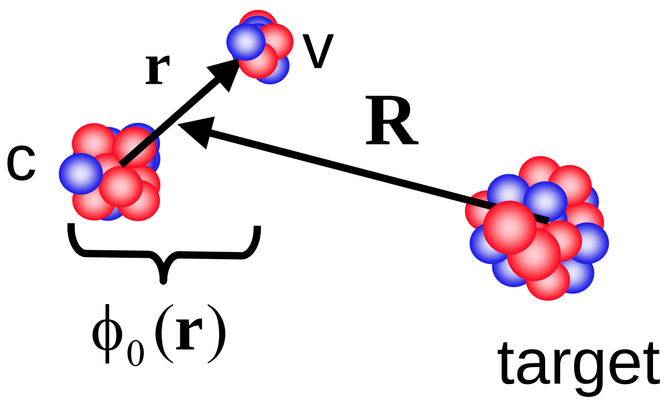
Three-body models - questions

Theoretically: hard to separate complete and incomplete fusion – as absorption collapses wave function if 'any' fragment is absorbed – formulation is of wave function when target remains in its ground state, so, Total fusion



Where to put imaginary potentials? – in fragment-target potentials (usual) – or in projectile-target coordinate? Problem: Have 'two-body' and not 'three-body' potentials

Few-body reaction theory - definitions - notation



$\phi_0(\mathbf{r})$

$$H = \underbrace{T_{\mathbf{r}} + V_{vc}}_{\text{projectile } H_p} + T_{\mathbf{R}} + \underbrace{V_{cT} + V_{vT}}_{U(\mathbf{r},\mathbf{R})}$$

projectile H_p

$U(\mathbf{r},\mathbf{R})$

total interactions with the target: excites projectile

excited continuum of c+v

$-\varepsilon_0$

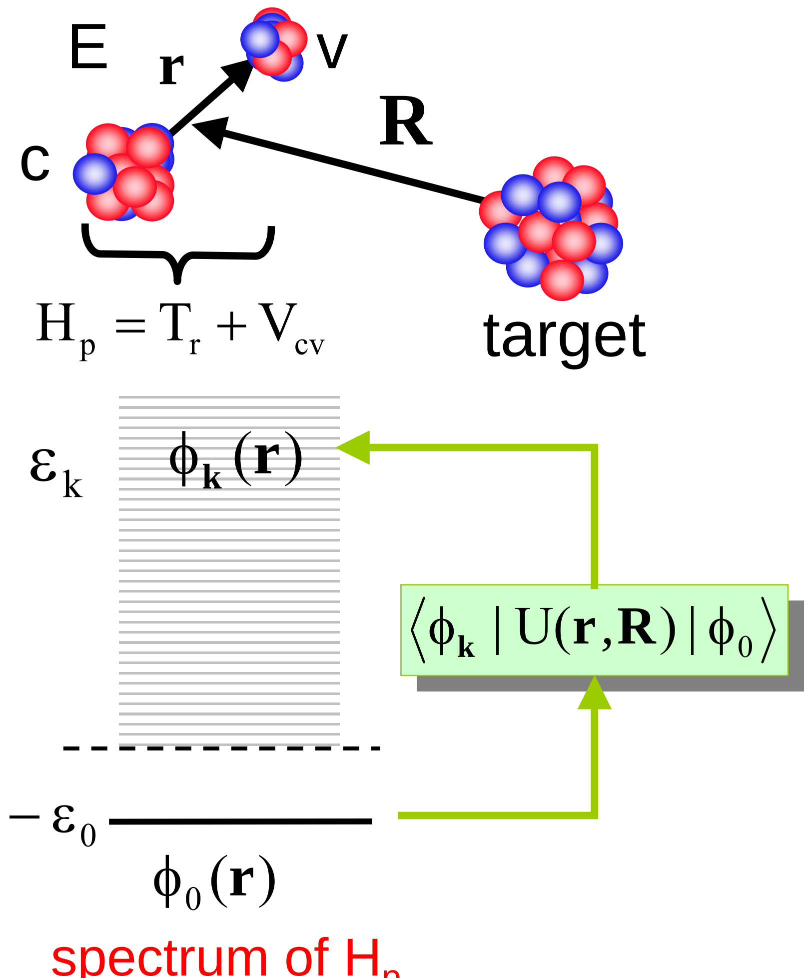
$$H_p \phi_0(\mathbf{r}) = -\varepsilon_0 \phi_0(\mathbf{r})$$

$$H\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R}) = E\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R})$$

with boundary conditions:

$$\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R}) \to e^{i\mathbf{K}\cdot\mathbf{R}}\phi_0(\mathbf{r}) + \text{outgoing waves}$$

Energetics of few-body composite systems



$$H = H_p + T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R})$$

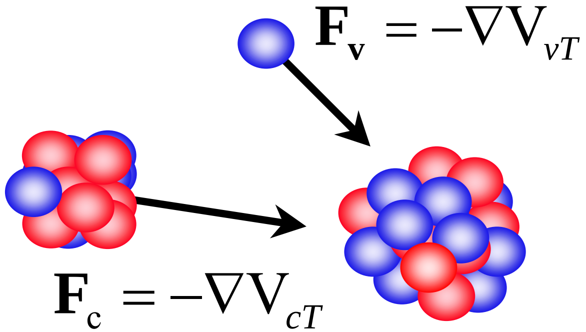
The tidal forces
$U(\mathbf{r},\mathbf{R}) = V_{cT} + V_{vT}$
between c and v and the target cause excitation of the projectile to excited states of c+v and to the continuum states

$$H_p \phi_{\mathbf{k}}(\mathbf{r}) = \varepsilon_k \phi_{\mathbf{k}}(\mathbf{r})$$

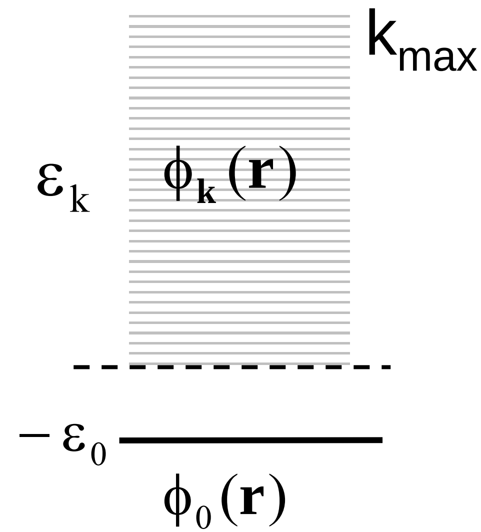
Which $\phi_{\mathbf{k}}(\mathbf{r})$ are excited?

Continuum excitations and interactions

A major simplification to the reaction dynamics is possible if $\varepsilon_k << E$ but not applicable for reactions near barrier



Those states excited (to k_{max}) are dictated by the geometry of the interactions



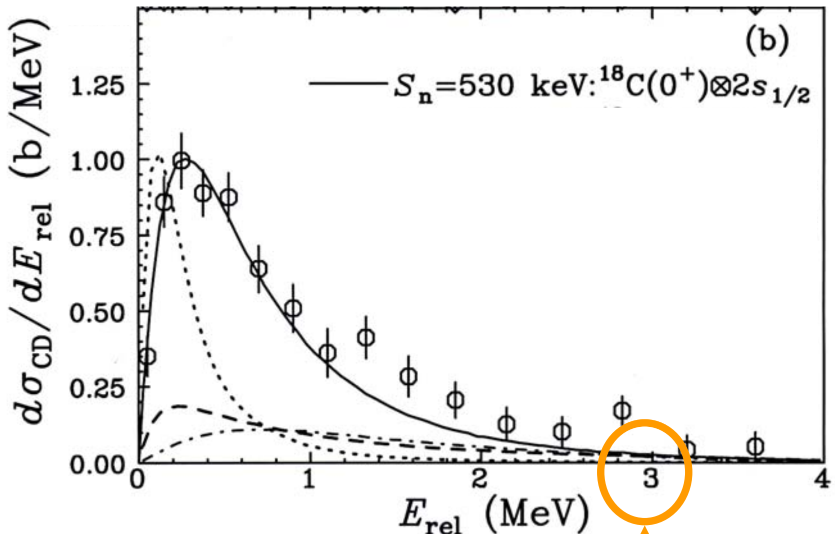
Nuclear forces, sharp surfaces, large $\mathbf{F}$, larger ε_k, universally, given surface diffuseness of nuclear potentials $\varepsilon_k \leq 20$ MeV

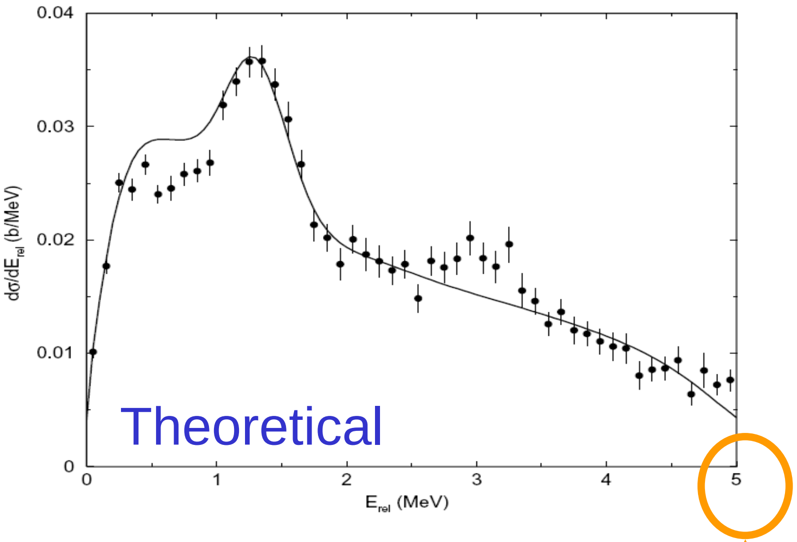
Coulomb forces,slow spatial changes, small $\mathbf{F}$, typically $\varepsilon_k \leq 4$ MeV (Nakamura et al, PRL **83** (1998) 1112)

Break-up continua from nuclear and Coulomb

T. Nakamura et al, PRL **83** (1998) 1112

J.A. Tostevin et al, PRC **66** (2002) 02460
N. Fukuda et al., PRC **70** (2004) 054606





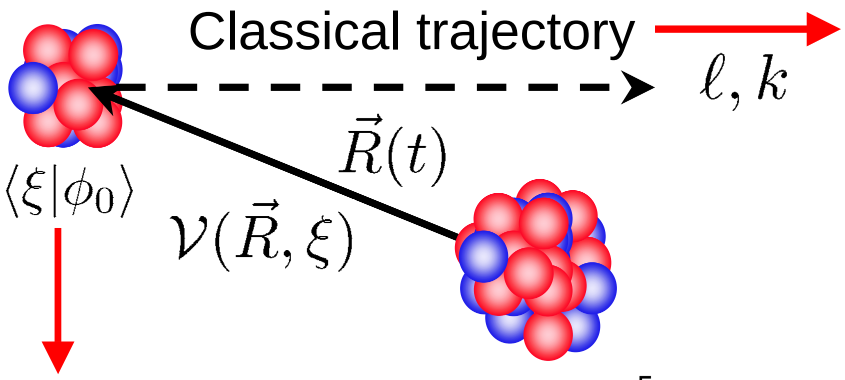
Experimental

$^{19}C+Pb \rightarrow {}^{18}C+n+X$
E= 67A MeV = 1.33 GeV
Coulomb dominated

Experimental

$^{11}Be + {}^{12}C \rightarrow {}^{10}Be+n+X$
E= 67A MeV = 737 MeV
Nuclear dominated

Semi-classical – Alder-Winther theory



$$V(R) = \langle \phi_0 | \mathcal{V}(\vec{R}, \xi) | \phi_0 \rangle$$

includes static effects

$$H = H_0(\xi) + \mathcal{V}(\vec{R}, \xi)$$

$$H_0(\xi) |\phi_\alpha\rangle = \mathcal{E}_\alpha |\phi_\alpha\rangle$$

$$a_\alpha(\ell, t \to -\infty) = \delta_{\alpha 0}$$

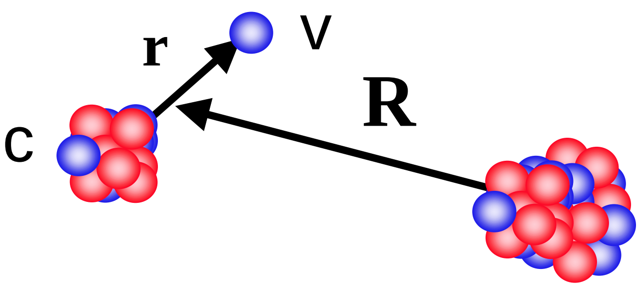
$$\left[H_0(\xi) + \mathcal{V}(\vec{R}(t), \xi) \right] \psi_\ell(\xi, t) = i\hbar \frac{\partial \psi_\ell(\xi, t)}{\partial t}$$

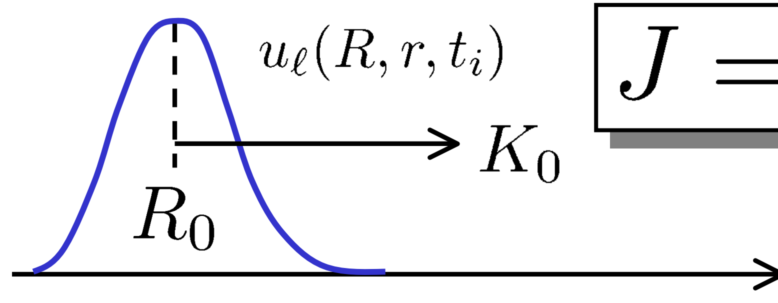
$$\psi_\ell(\xi, t) = \sum_\alpha a_\alpha(\ell, t) \phi_\alpha(\xi) \exp(-i\mathcal{E}_\alpha t/\hbar)$$

$$i\hbar \dot{a}_\alpha(\ell, t) = \sum_\beta \langle \phi_\alpha | \mathcal{V}(\vec{R}, \xi) | \phi_\beta \rangle \exp(i[\mathcal{E}_\alpha - \mathcal{E}_\beta]/\hbar) \; a_\beta(\ell, t)$$

$$\sigma_F = \sum_\alpha \sigma_F(\alpha) \qquad \sigma_F(\alpha) \approx \sum_\ell (2\ell + 1) \, T_\ell \, |a_\alpha(\ell, t = 0)|^2$$

Wave packet solution of Yabana





$$J=0$$

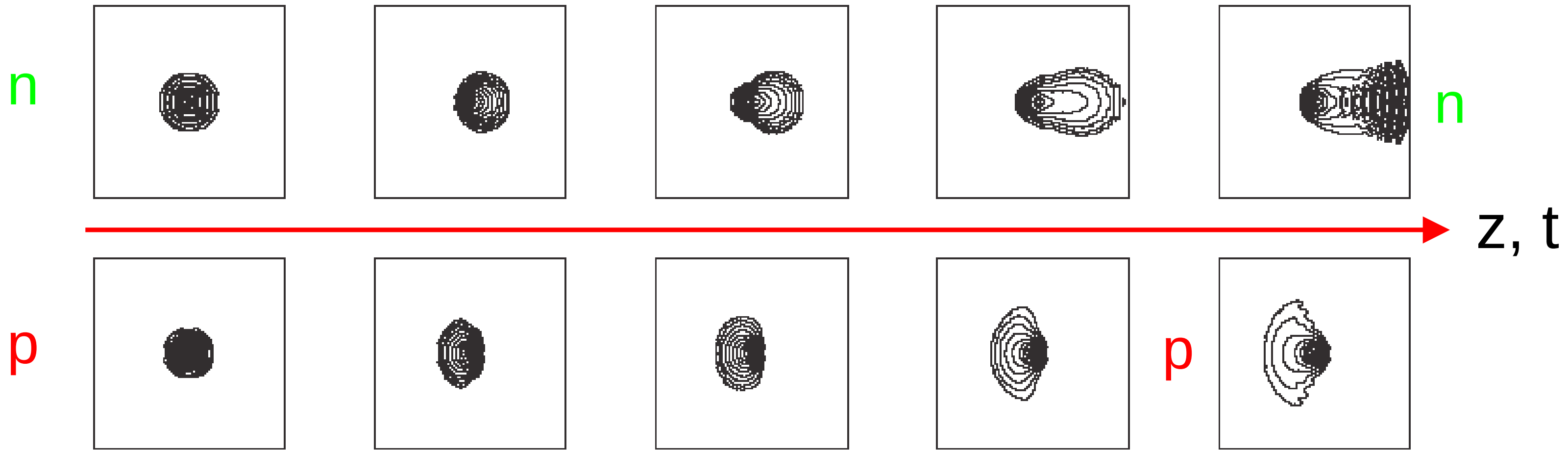
$$\left\{-\frac{\hbar^2}{2\mu}\nabla_R^2 + H_0(\vec{r}) + V_{vT}(R_{vT}) + V_{cT}(R_{cT})\right\}\Psi(\vec{R},\vec{r},t) = i\hbar\frac{\partial\Psi(\vec{R},\vec{r},t)}{\partial t}$$

$$u_\ell(R,r,t_i) = \delta_{\ell 0}\, u_{Cv}(r)\exp[-\gamma(R-R_0)^2]\exp(-iK_0R)$$

$$\Psi^{J=0}(\vec{R},\vec{r},t) = \sum_{\ell=0}^{\ell_{max}} \frac{\sqrt{2\ell+1}}{4\pi}\frac{u_\ell(R,r,t)}{Rr}P_\ell(\cos\theta)$$

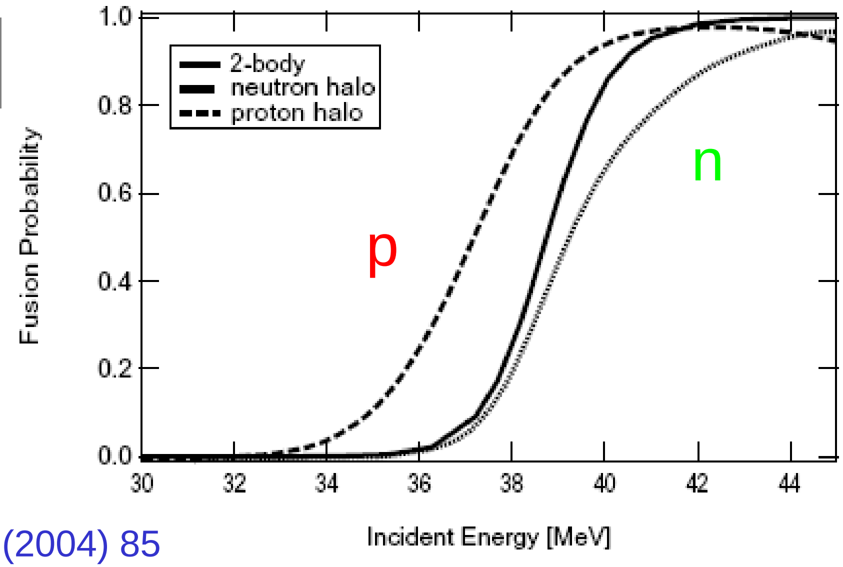
K. Yabana et al., Nucl. Phys. **A722** (2003) 261c,
T. Nakatsukasa, et al, PTP Supplement **154** (2004) 85

Wave packet, but total J=0 only, calculations



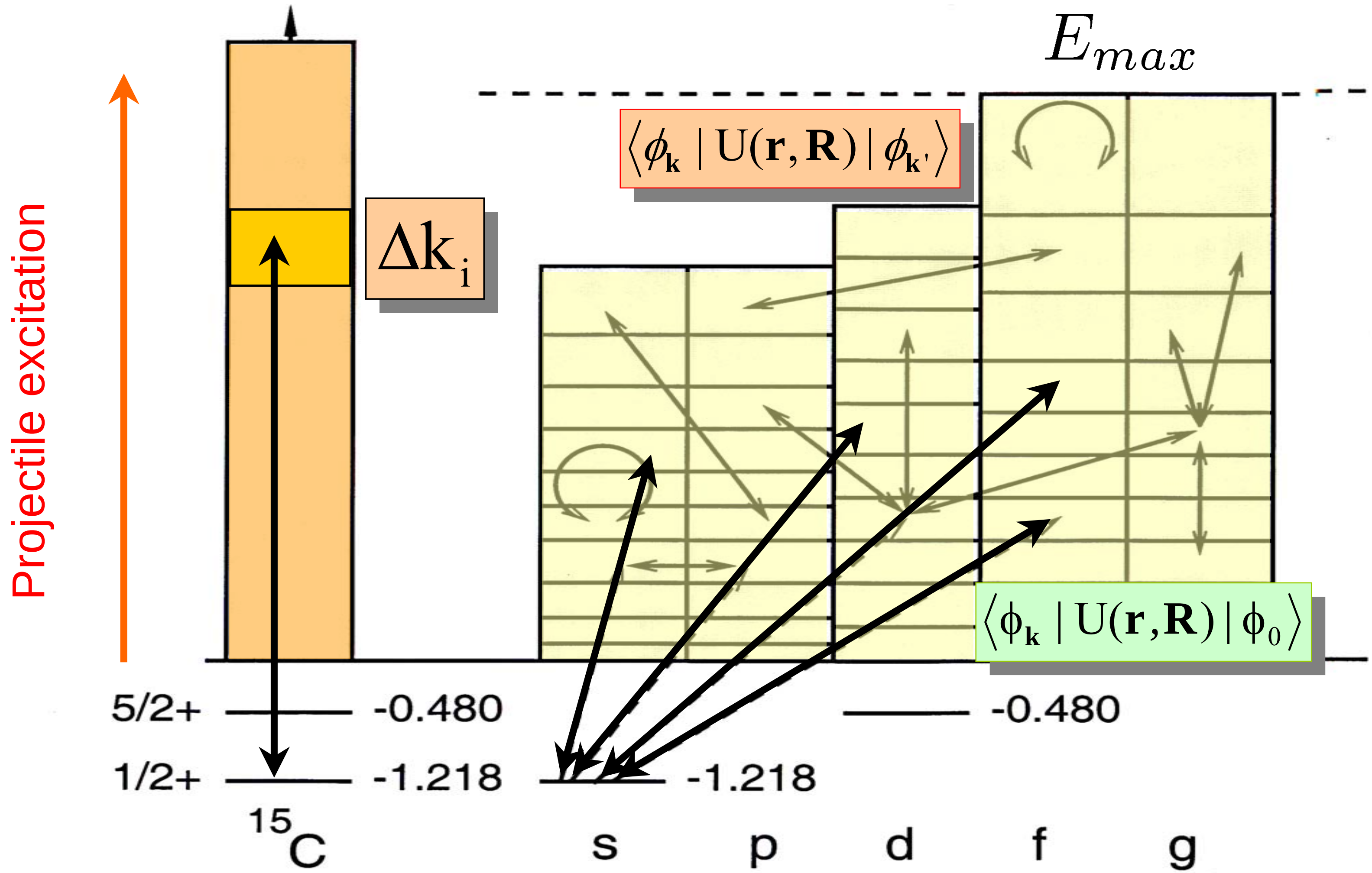
[^{10}Be+N]+^{208}Pb, 20 MeV

These calculations are of the fusion of the ^{10}Be, with no strong N+target interaction, i.e. Coulomb plus core-target interaction



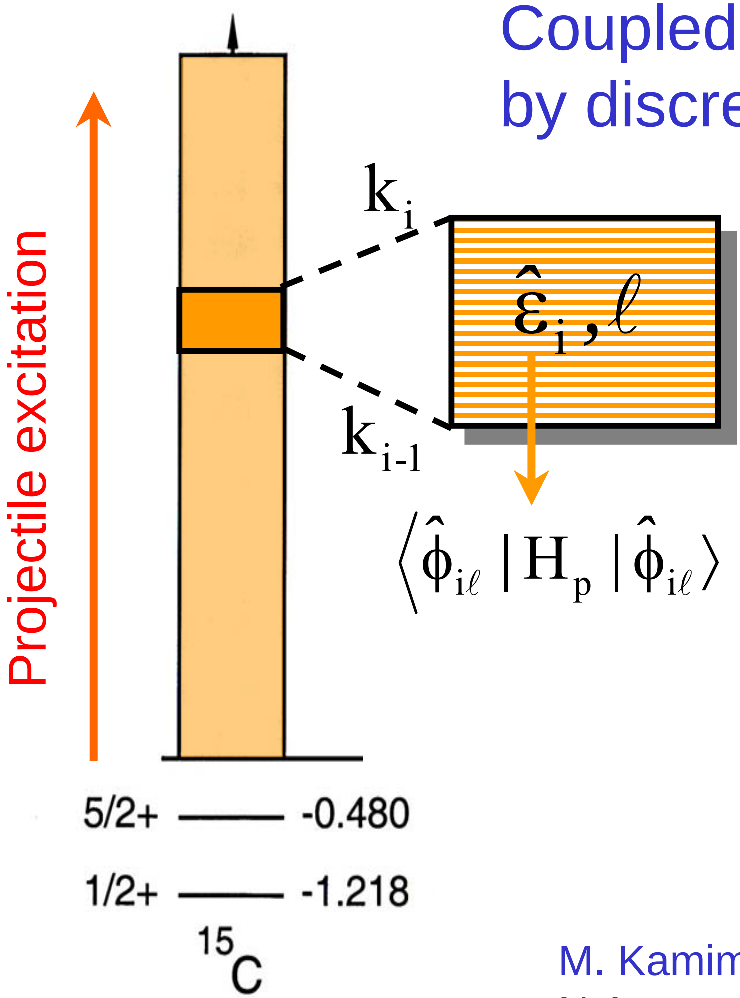
T. Nakatsukasa, et al, PTP Supplement **154** (2004) 85

Problem of continuum-continuum couplings



Coupled Discretised Continuum Channels

Coupled channels solution of break-up by discretisation of the continuum



$$\hat{\phi}_{i\ell m}(\mathbf{r}) = u_{i\ell}(r) Y_{\ell m}(\hat{\mathbf{r}})$$

complete orthonormal set of states in relative motion co-ordinate $\mathbf{r}$

$$u_{i\ell}(r) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} dk \; f_i(k)\, \phi_\ell(k,r)$$

$$\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R}) = \sum_i \hat{\chi}_i(\mathbf{R})\, \hat{\phi}_i(\mathbf{r})$$

M. Kamimura et al, Prog Theor Phys (Suppl) **89** (1986), 1
N.Austern et al., Phys. Rep. **154** (1987), 125

Properties of CDCC bin (basis) states

bin states

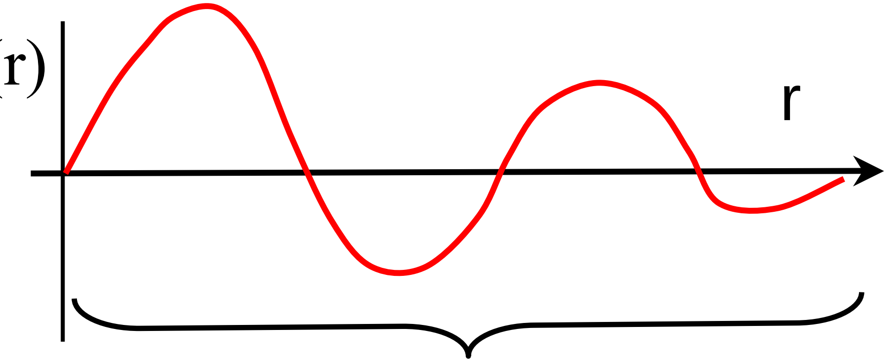
$\hat{\phi}_{i\ell m}(\mathbf{r})$

$$u_{i\ell}(r) = \sqrt{\frac{2}{\pi N_i}} \int_{\Delta k_i} dk \; f_i(k)\, \phi_\ell(k,r)$$

normalised and orthogonal

$\hat{\varepsilon}_i, \ell$ — Δk_i

$u_{i\ell}(r)$



r_{bin}

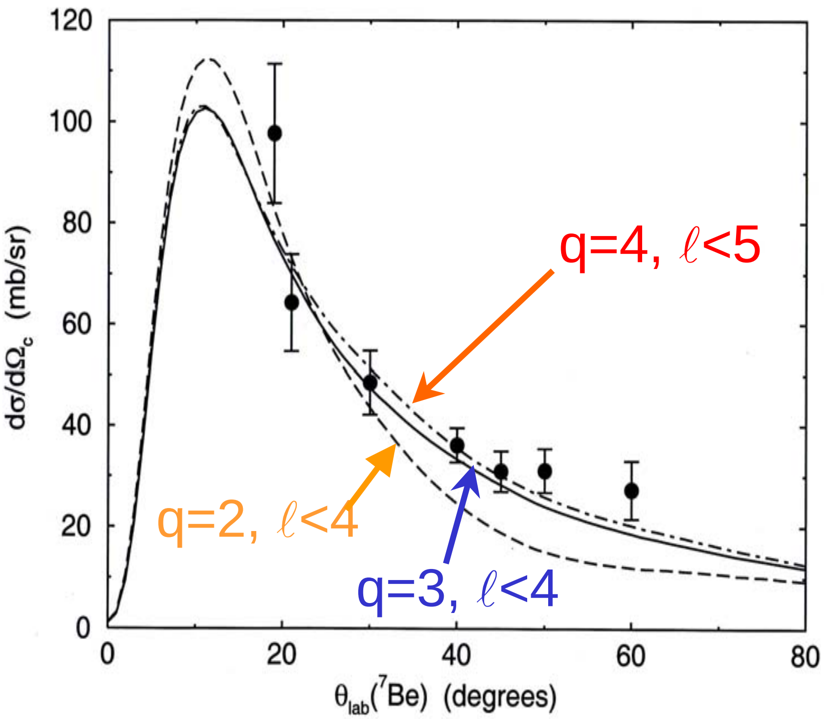
Uncertainty principle: so these must be chosen carefully

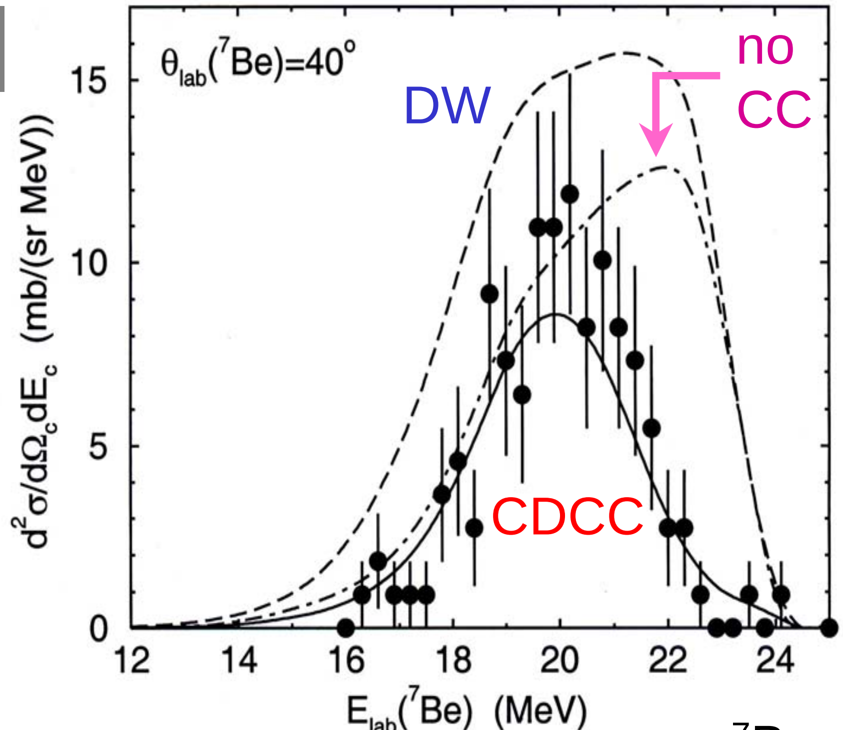
Couplings between bin states (channels) are

$$V_{ij}(\mathbf{R}) = \langle \hat{\phi}_i \,|\, U(\mathbf{r},\mathbf{R}) \,|\, \hat{\phi}_j \rangle_{\mathbf{r}}$$

CDCC can reproduce data at low energy

$^{8}B + {}^{57}Ni \rightarrow {}^{7}Be + X$, 25.8 MeV





J.A. Tostevin et al., Phys Rev C **63** (2001) 024617
J. Kolata et al., Phys Rev C **63** (2001) 024616

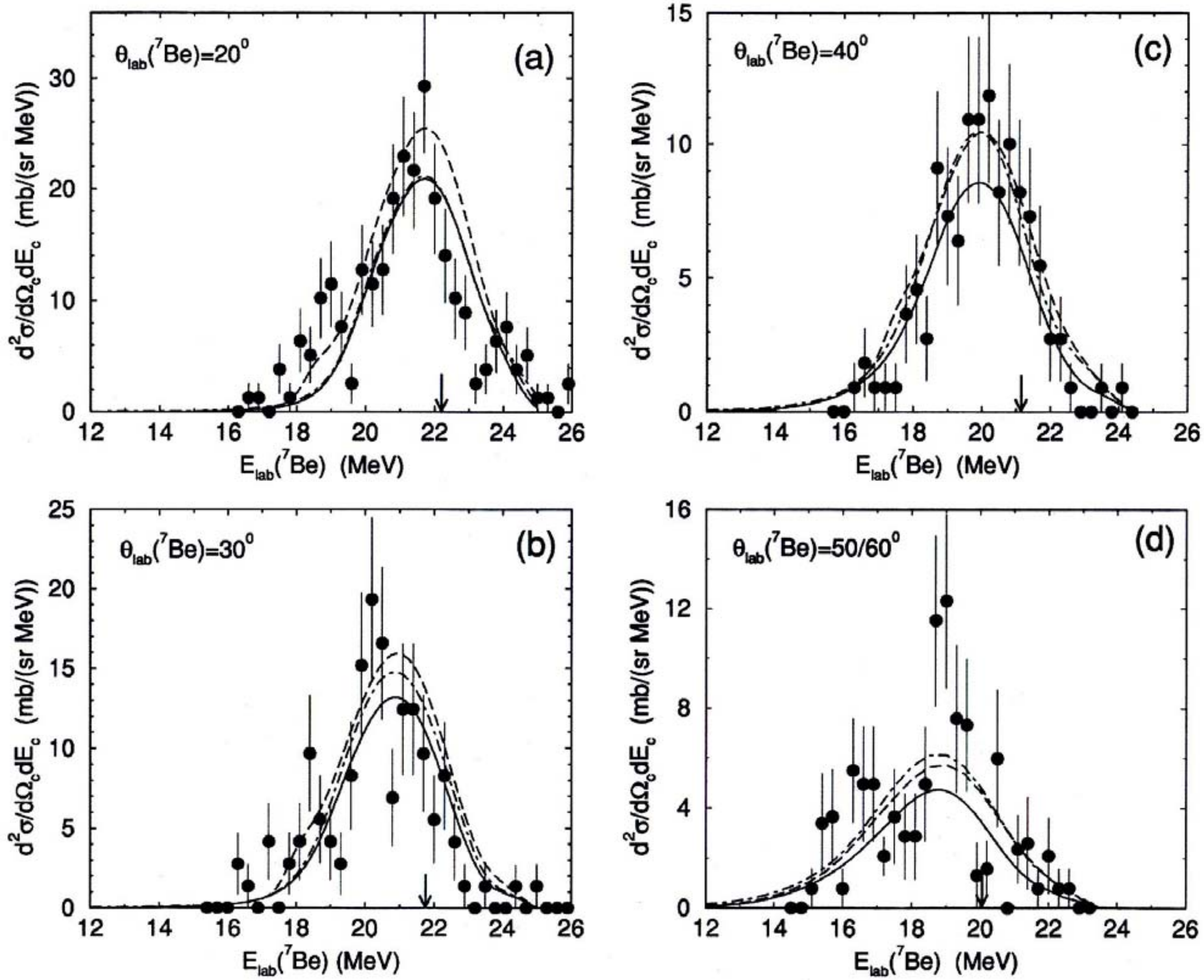
Double differential cross sections for breakup

$$\frac{d^2\sigma}{dE_c d\Omega_c}$$

$^8B + ^{57}Ni \rightarrow$
$^7Be + X$
25.8 MeV

J. Tostevin et al, Phys Rev C **63** (2001) 024617

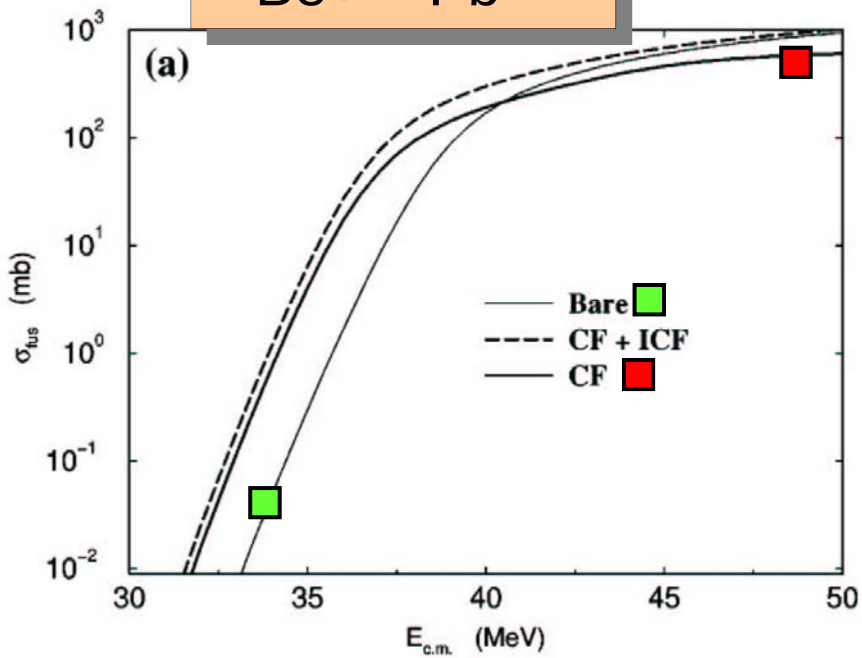
J. Kolata et al., Phys Rev C **63** (2001) 024616

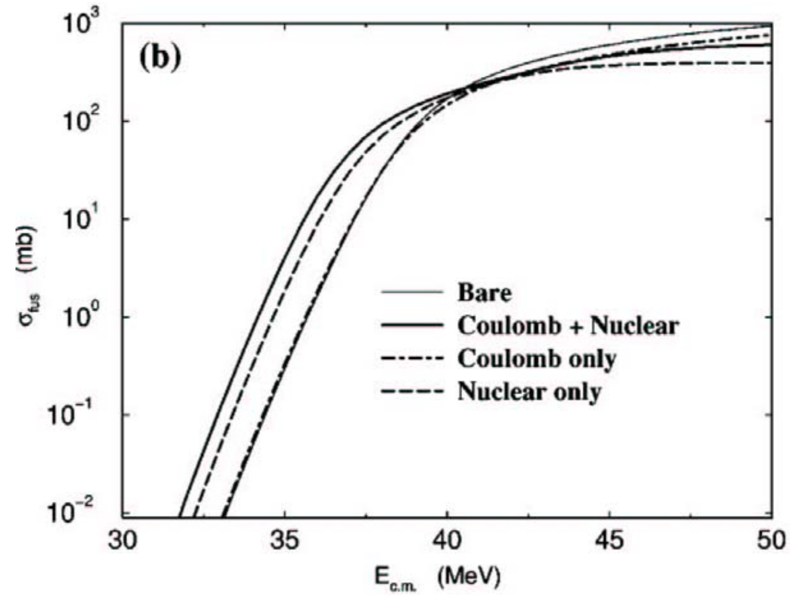


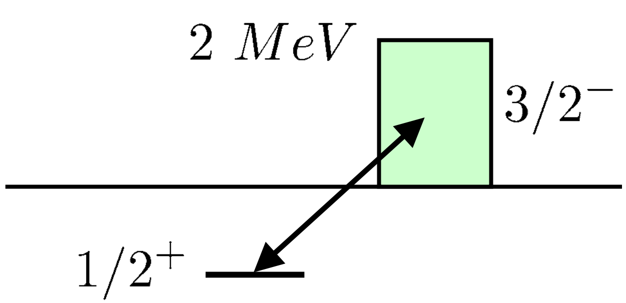
Coupled channels calculations – small space

$^{11}Be+^{208}Pb$

K. Hagino, et al, PRC **61** (2000) 036702







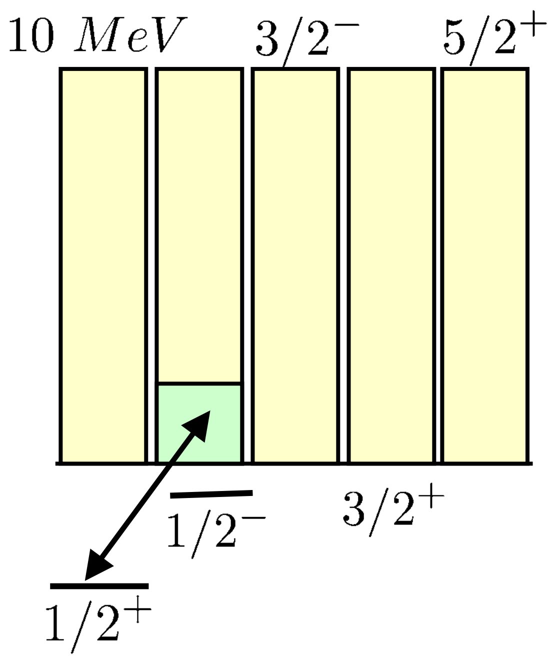
No continuum-continuum couplings
No coupling to bound p1/2 state
No ‘static’ changes – breakup only

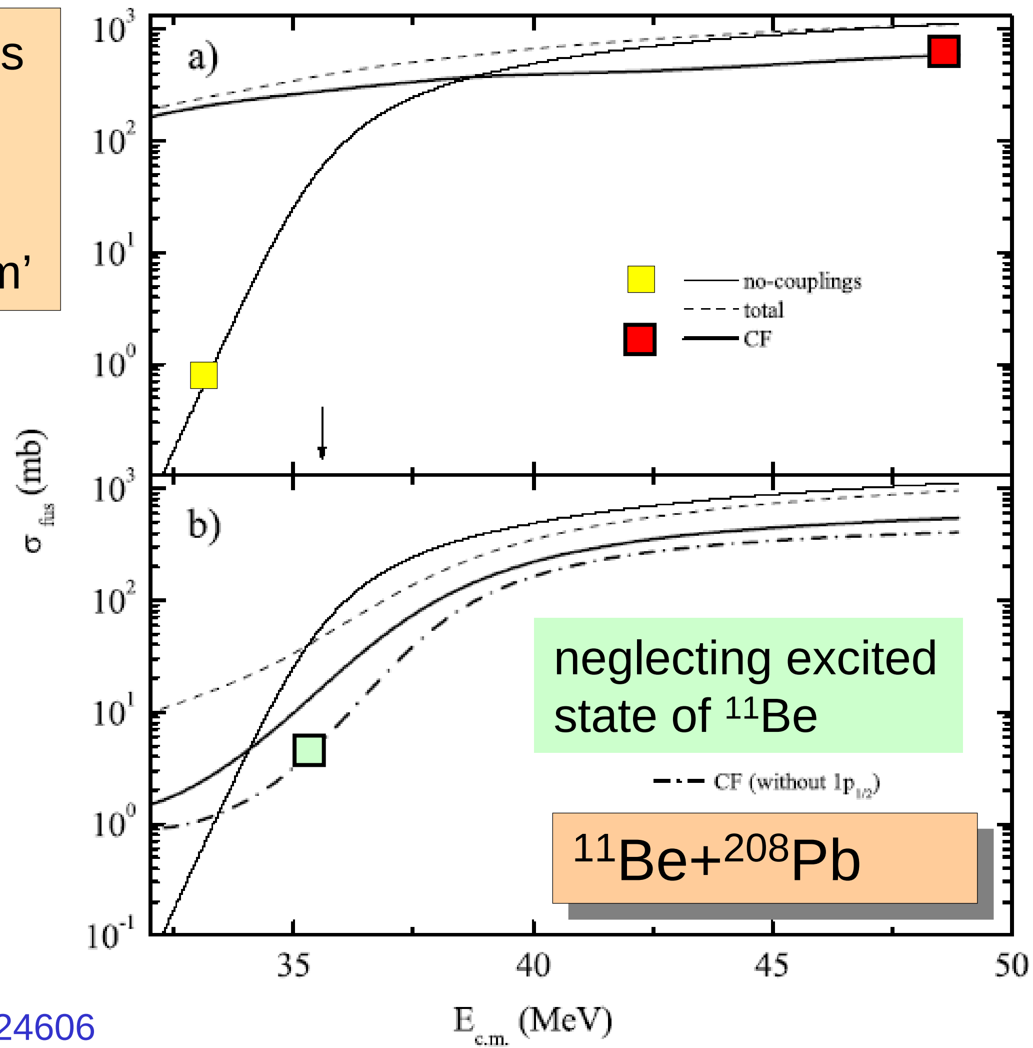
Enhanced sub-barrier
ICF is ‘fusion from p3/2 continuum’

Coupled channels calculations – large space

Continuum-continuum couplings
Coupling to bound p1/2 state
'Static' changes

ICF is 'fusion from all continuum'

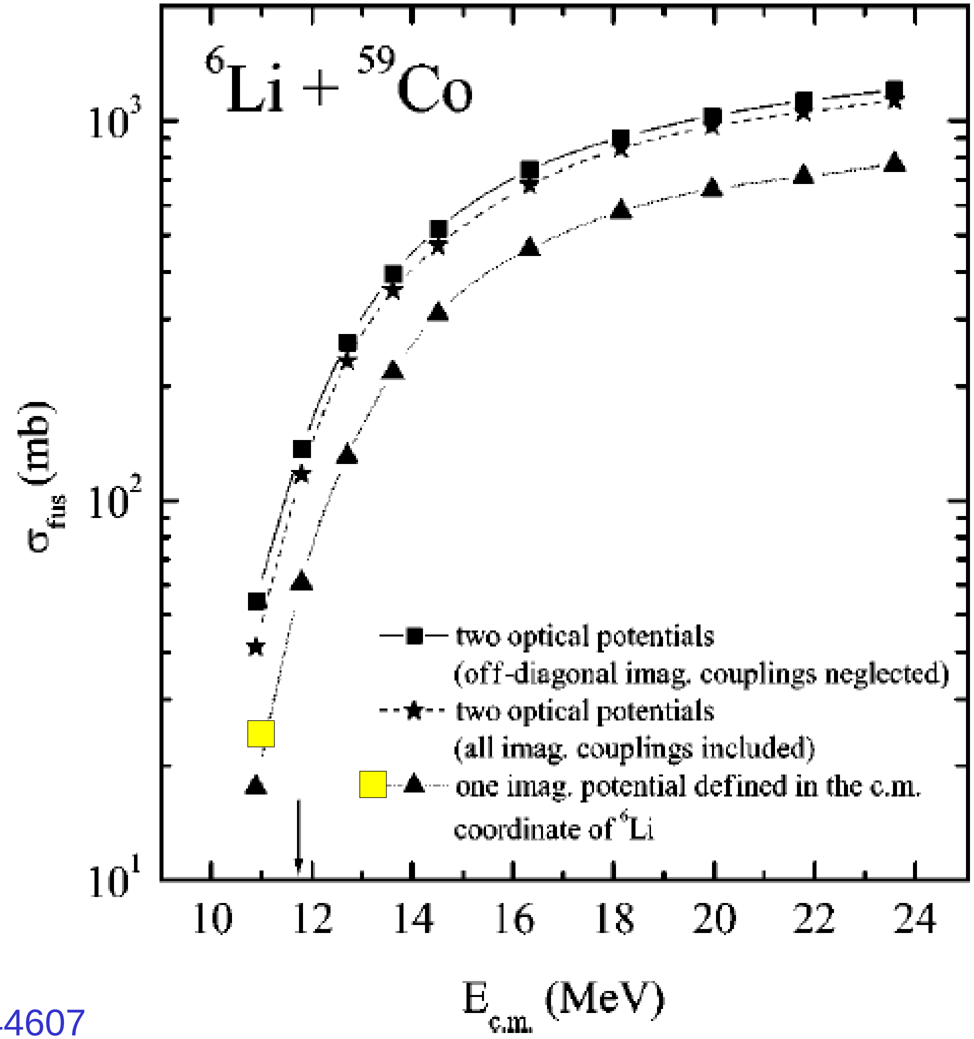




A. Diaz-Torres, et al, PRC **65** (2002) 024606

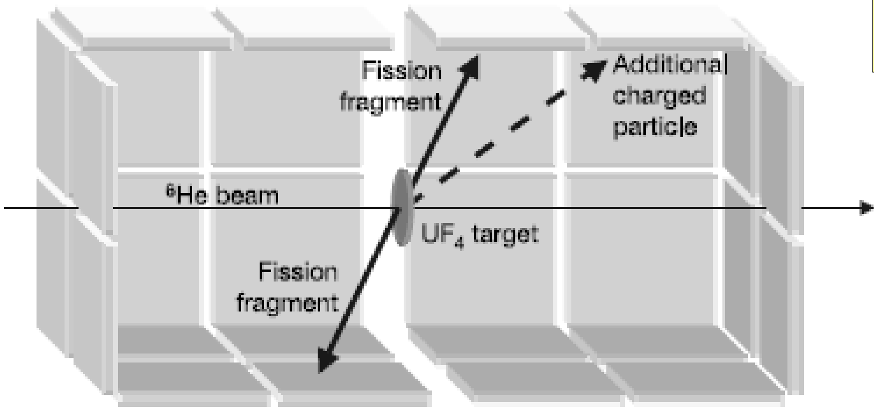
Role of fragment versus projectile absorption

Many events where one of the fragments of ^{6}Li is captured, but the centre of mass of the projectile does not enter into the absorption (fusion) region



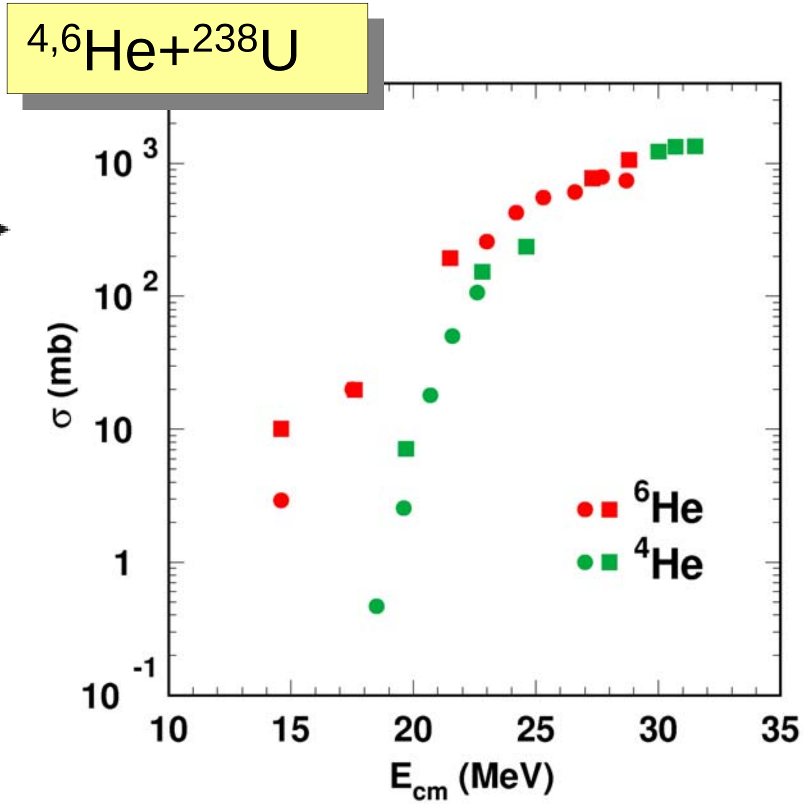
A. Diaz-Torres, et al, PRC **68** (2003) 044607

Exclusive measurements – transfer channels



No enhancement of fusion probability by the neutron halo of ^{6}He

R. Raabe[1,2], J. L. Sida[1*], J. L. Charvet[1], N. Alamanos[1], C. Angulo[3], J. M. Casandjian[4], S. Courtin[5], A. Drouart[1], D. J. C. Durand[1], P. Figuera[6], A. Gillibert[1], S. Heinrich[1], C. Jouanne[1], V. Lapoux[1], A. Lepine-Szily[7], A. Musumarra[6], L. Nalpas[1], D. Pierroutsakou[8], M. Romoli[8], K. Rusek[9] & M. Trotta[8]



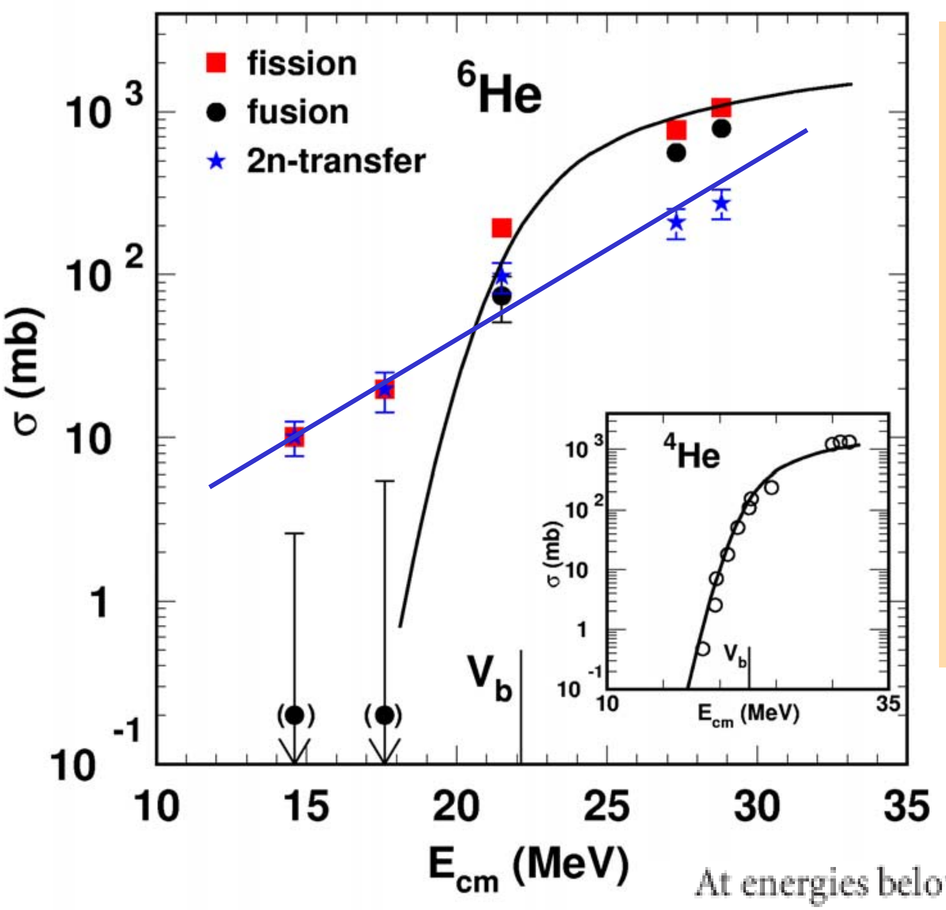
Transfer effects found to be larger than break-up for ^{6}He+^{65}Cu reactions

M. Trotta et al., PRL **84** (2000) 2342

A. Navin et al., Phys Rev C **70**, 044601

R. Raabe et al., Nature **431** (2004) 823

Two-neutron transfer and (no) enhancement



Measurement of coincidences with alpha-particles to clarify the role of 2n transfer (incomplete fusion)

No enhancement of the fusion cross section

Below the barrier, the two-neutron transfer dominates

R. Raabe et al., et al, Nature **431** (2004) 823

$^{4,6}He + ^{238}U$

At energies below the barrier, we find experimentally that there is no substantial enhancement of the fusion cross-section for the halo nucleus ^{6}He. The large observed yield for fission is entirely due to a direct process, the two-neutron transfer to the target nucleus.

Dispersion relations – threshold phenomena

Onset of inelastic processes with increasing energy develops absorption and perturbs the diffractive (real) part of the optical potential (assumed local for simplicity) - causality and unitarity

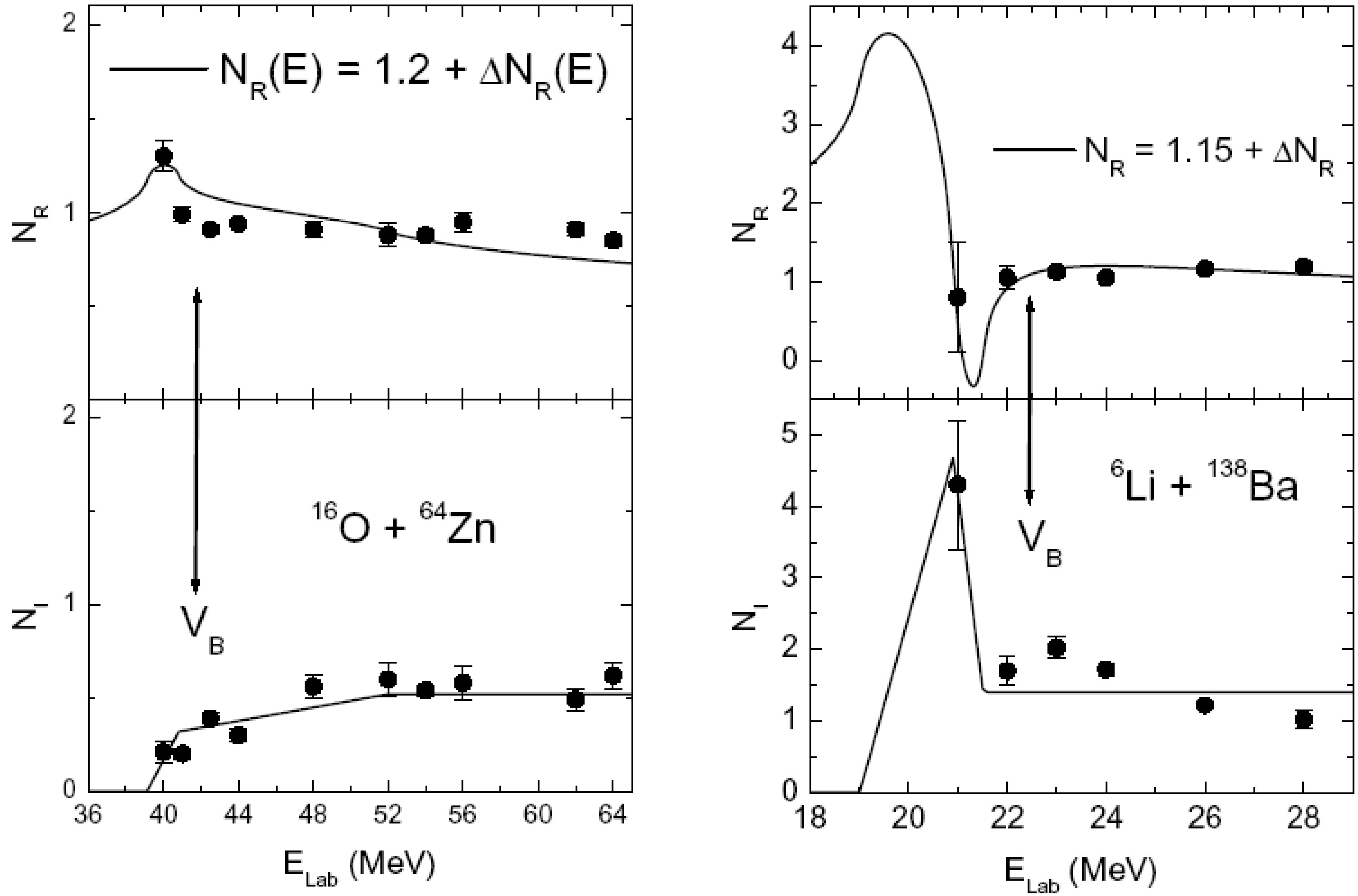
$$\begin{aligned} U_E(R) &= V_0(E,R) + \Delta U_E(R) \\ \Delta U_E(R) &= \Delta V_E(R) + iW_E(R) \end{aligned}$$

These terms are intimately connected through a dispersion-type relation (e.g. Feshbach, Ann Phys **5** (1958) 357**)**

$$\begin{aligned} \Delta V_E(R) &= +\frac{\mathcal{P}}{\pi}\int \frac{W_{E'}(R)}{E'-E}dE' \\ W_E(R) &= -\frac{\mathcal{P}}{\pi}\int \frac{\Delta V_{E'}(R)}{E'-E}dE' \end{aligned}$$

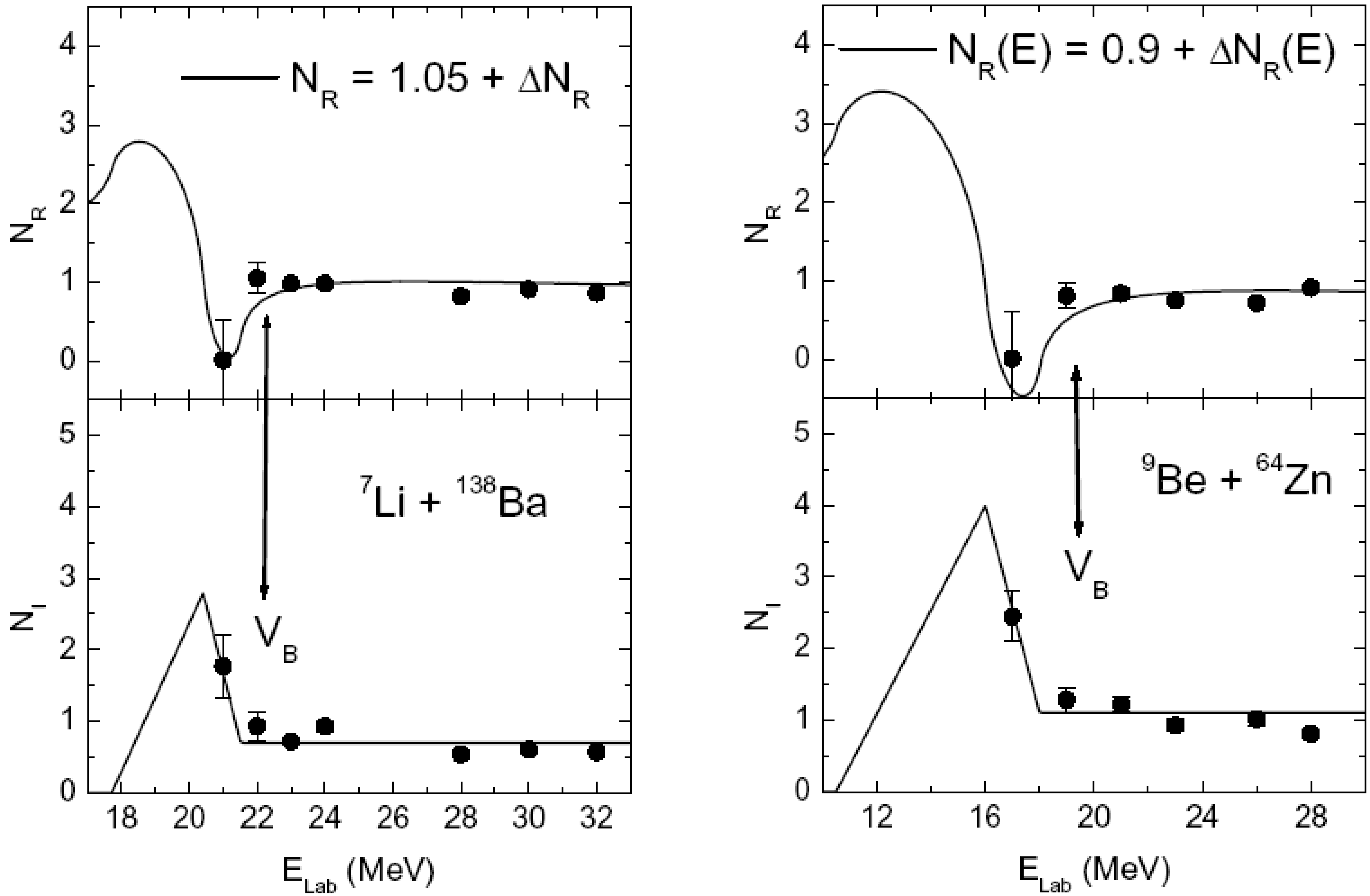
Other energy dependence, e.g. from non-locality, is not dispersive and is removed from relationship into $V_0(E,R)$

Dispersion relation analysis of elastic scattering



L. Chamon, et al, NUSTAR05, submitted

Dispersion relation analysis



L. Chamon, et al, NUSTAR05, submitted

Conclusion

Coupling to break-up channels predicts enhancement of fusion below the barrier when the reaction process is treated within the conventional coupled-channel approach, with proper modifications due to weak-binding nature of halo systems.

Data remains rather limited, and neutron transfer effects can be large and will have to be resolved to interpret experiments.

Dispersion relations approach from elastic scattering data are worthy of additional experimental effort.