Topics in Nuclear Fusion

XII SSTNPJAS School, São Paulo, Brasil
13-20th February 2005

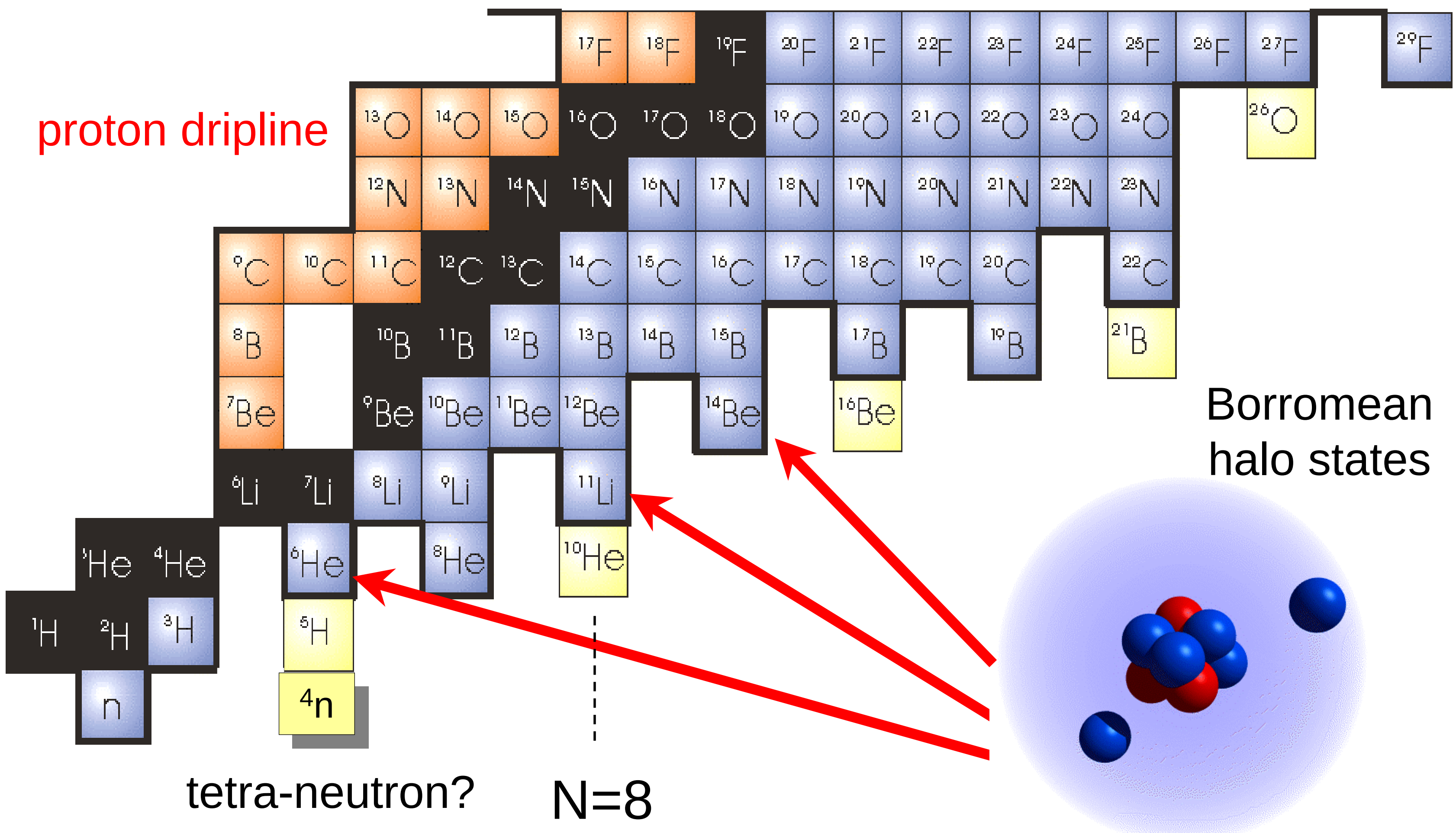
Three lectures – will plan to discuss

Lect I : Fusion of ions: motivation and introductory remarks, concepts, terminology, models and indicators of fusion, reaction dynamics, barriers, coupled channels - assisted tunnelling, barrier distributions and optical potentials. Experience.

Lect II: Weakly-bound systems, methods for break-up calculations, fusion in few-body models of break-up reactions. Many open questions.

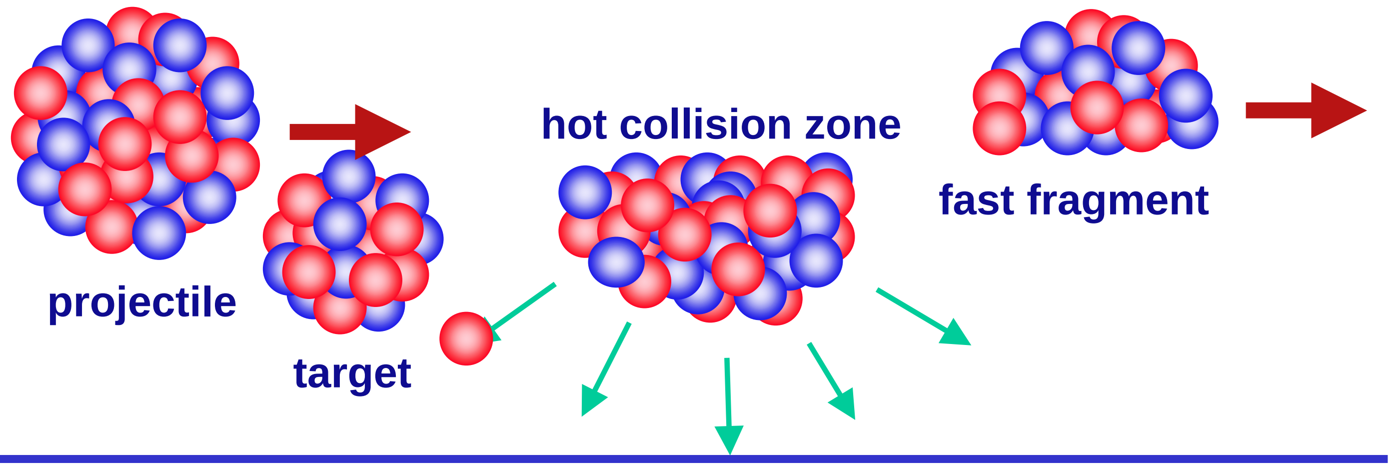
Lect III: Partial/incomplete fusion at higher incident energies, applications to knockout of one- and two nucleons and applications for spectroscopy of exotic nuclei

The neutron dripline in light nuclei

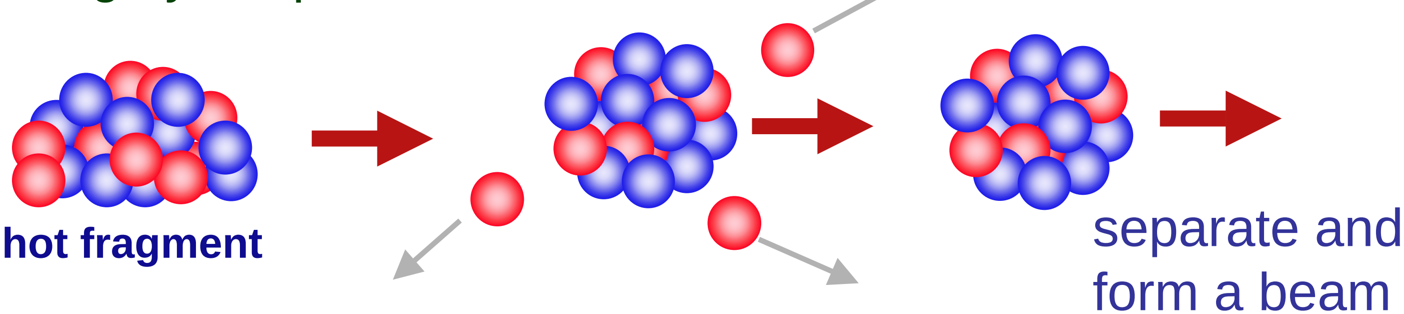


High energy - projectile fragmentation RIBs

Random removal of protons and/or neutrons from heavy projectile in peripheral collisions at high energy - 100 MeV per nucleon or more

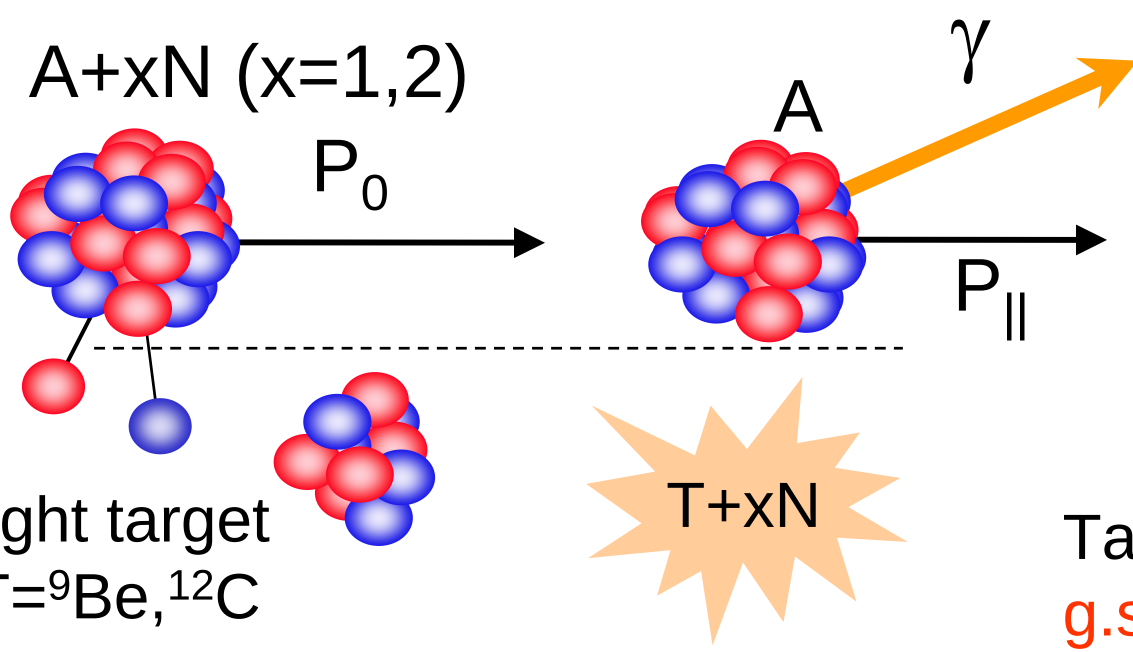


Cooling by evaporation.



One- and two-nucleon knockout reactions

Peripheral collisions (E ≥ 50A MeV; MSU, RIKEN, GSI)



heavy mass A residue is detected, often with coincident γ-ray detection

light target $T={}^{9}Be, {}^{12}C$

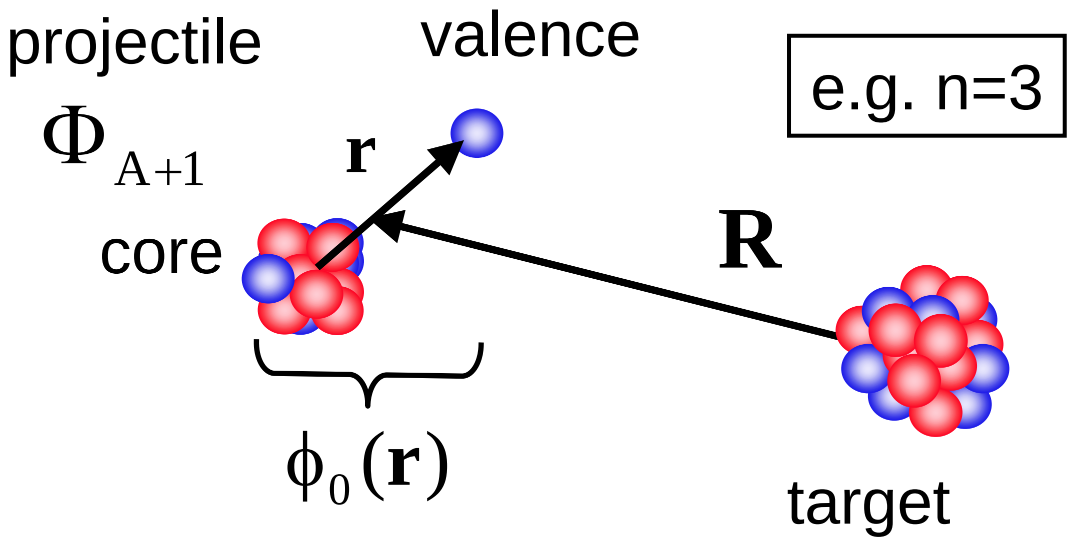
Stripping = Incomplete fusion

Target T left in its g.s. or excited state

Events contributing will be both break-up and stripping both of which leave a mass A residue in the final state

Few-body models of nuclear reactions

There are no practical <u>many-body</u> reaction theories - we construct model 'effective' few-body models (n=2,3,4 …)

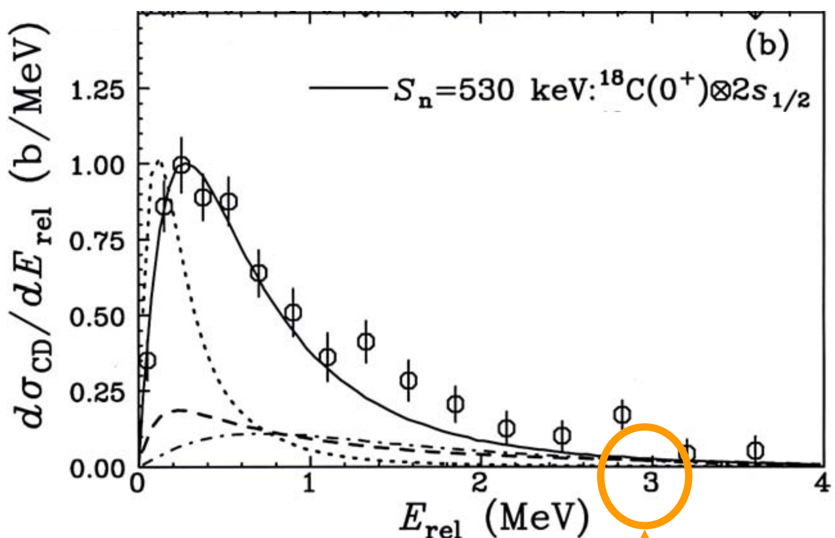


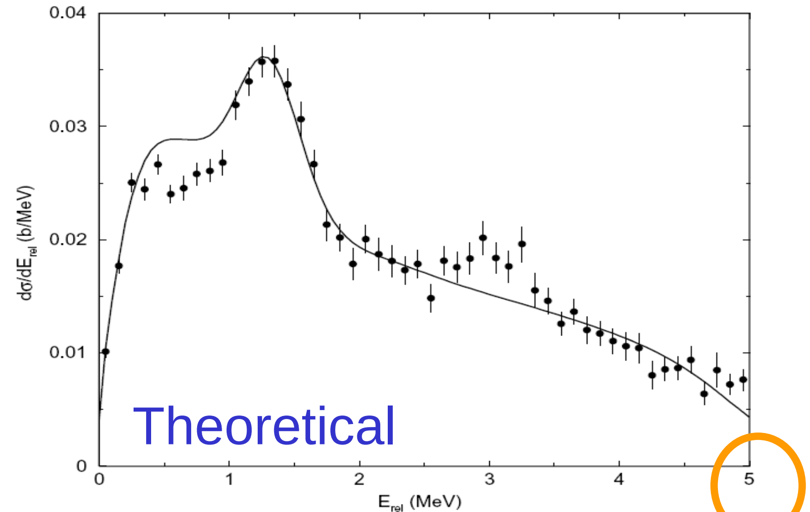
Construct an <u>effective</u> Hamiltonian H and solve as best we can the Schrödinger equation: $\mathrm{H}\Psi = \mathrm{E}\Psi$

Break-up continua from nuclear and Coulomb

T. Nakamura et al, PRL **83** (1998) 1112

J.A. Tostevin et al, PRC **66** (2002) 02460
N. Fukuda et al., PRC **70** (2004) 054606





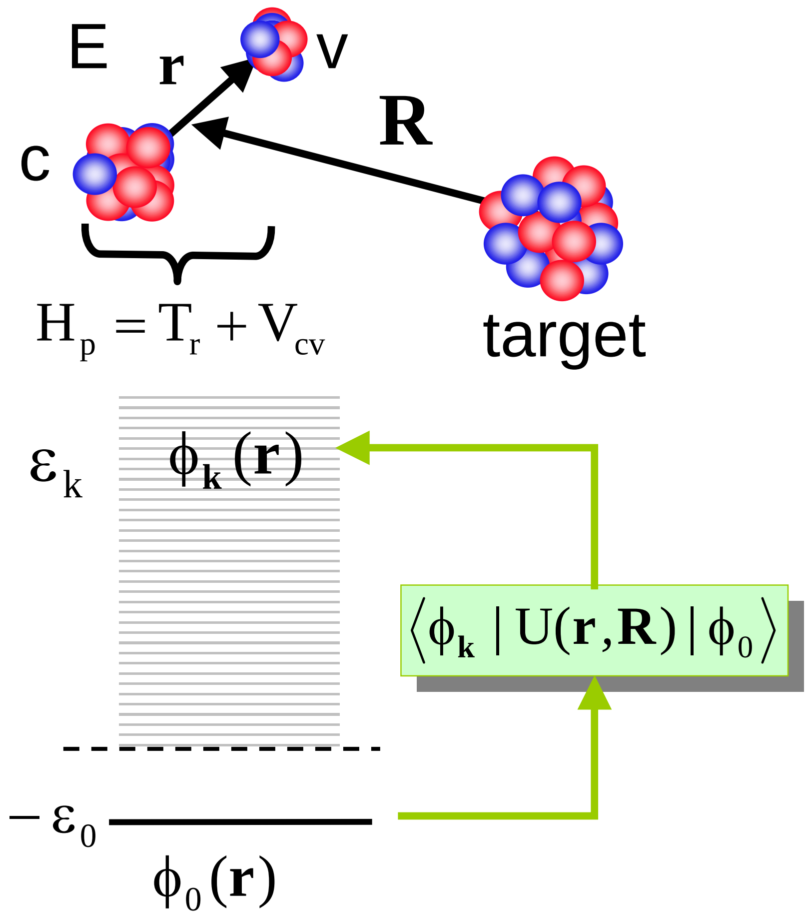
Experimental

$^{19}C+Pb \rightarrow ^{18}C+n+X$
E= 67A MeV = 1.33 GeV
Coulomb dominated

Experimental

$^{11}Be + {}^{12}C \rightarrow ^{10}Be+n+X$
E= 67A MeV = 737 MeV
Nuclear dominated

Energetics of few-body composite systems



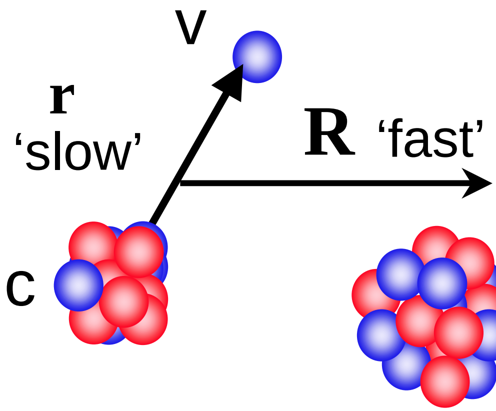
$$H = H_p + T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R})$$

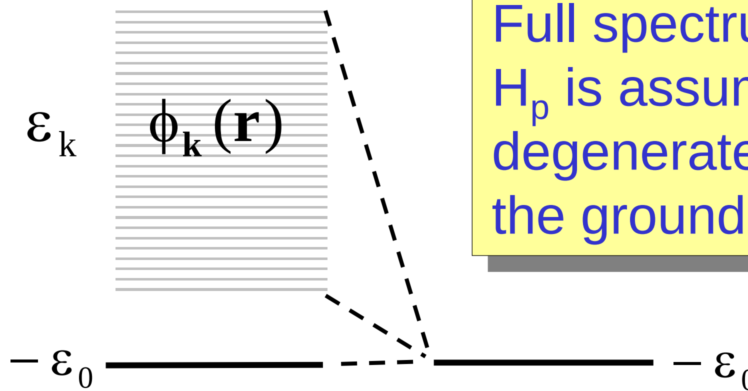
The tidal forces $U(\mathbf{r},\mathbf{R}) = V_{cT} + V_{vT}$ between c and v and the target cause excitation of the projectile to excited states of c+v and to the continuum states

$$H_p \phi_{\mathbf{k}}(\mathbf{r}) = \varepsilon_k \phi_{\mathbf{k}}(\mathbf{r})$$

Which $\phi_{\mathbf{k}}(\mathbf{r})$ are excited?

Adiabatic reaction model for few-body projectiles





Full spectrum of H_p is assumed degenerate with the ground state

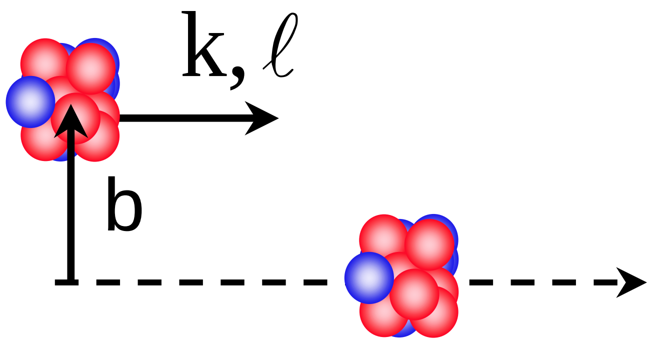
Freeze internal co-ordinate $\mathbf{r}$ then scatter c+v from target and compute f(θ,$\mathbf{r}$) for all required fixed values of $\mathbf{r}$

Physical amplitude for break-up to state $\phi_k(\mathbf{r})$ is then,

$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_{\mathbf{r}}$$

$$H = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) + H_p \quad\longrightarrow\quad H^{AD} = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - \varepsilon_0$$

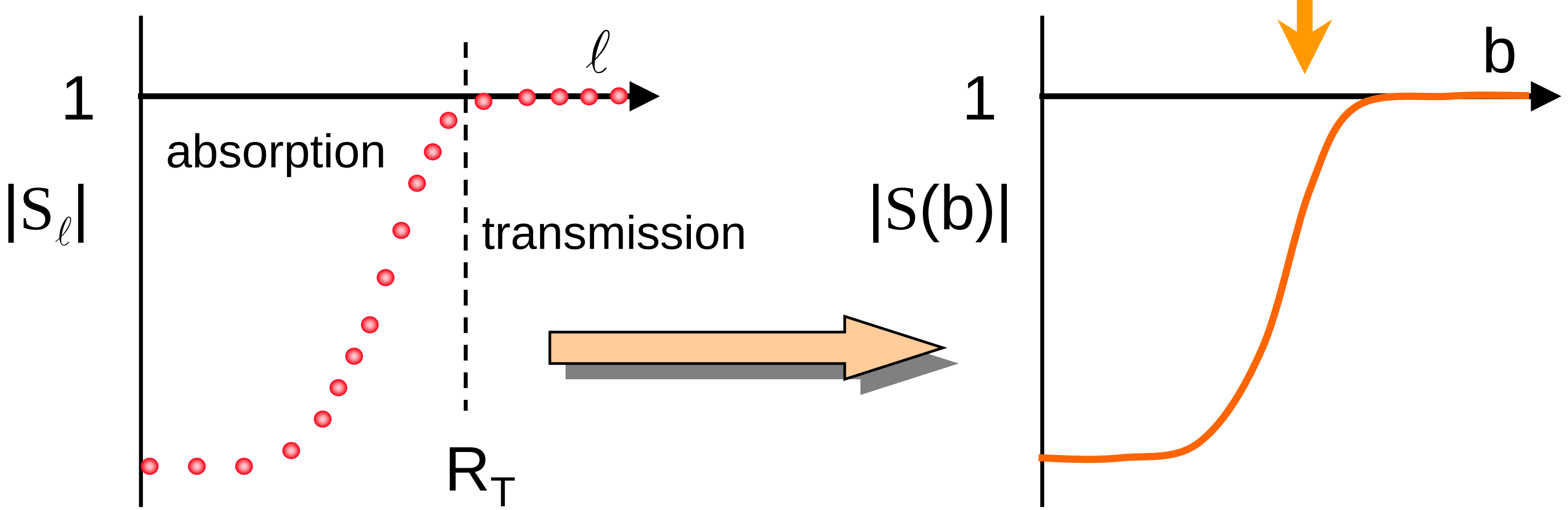
The semi-classical S-matrix - S(b)



b=impact parameter

for high energy/or large mass, semi-classical ideas are good

$kb \cong \ell$, actually $\Rightarrow \ell+1/2$



Point particle scattering - observables

All experimental observables can be computed from the S-matrix, in either representation, for example,

$$\sigma_{el} = \frac{\pi}{k^2} \sum_{\ell} (2\ell+1) \,|1 - S_\ell\,|^2 \rightarrow \int d\mathbf{b}\, |1 - S(b)\,|^2$$

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell} (2\ell+1) [1 - |S_\ell\,|^2\,] \rightarrow \int d\mathbf{b}\, [1 - |S(b)\,|^2\,]$$

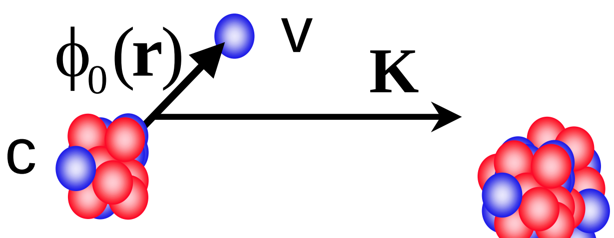
$$\sigma_{tot} = \sigma_R + \sigma_{el} = 2\int d\mathbf{b}\, [\,1 - \mathrm{Re.}S(b)\,], \quad \text{etc.}$$

and where $\int d\mathbf{b} \equiv 2\pi \int b\, db$

Eikonal solution of the few-body model

Practical application of adiabatic approximation: $H_p \to -\varepsilon_0$

$$H = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) + H_p \;\longrightarrow\; H^{AD} = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - \varepsilon_0$$



$$K = \sqrt{2\mu(E+\varepsilon_0)/\hbar^2}$$

substituting the eikonal form solution

$$\Psi_{\mathbf{K}}^{AD}(\mathbf{r},\mathbf{R}) = \underbrace{e^{i\mathbf{K}\cdot\mathbf{R}}\phi_0(\mathbf{r})}_{\text{incident wave}}\;\underbrace{\omega(\mathbf{r},\mathbf{R})}_{\text{modulating function}}$$

$$[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - (E+\varepsilon_0)]\,\Psi_{\mathbf{K}}^{AD}(\mathbf{r},\mathbf{R}) = 0$$

and neglecting the curvature term $\nabla_R^2\omega(\mathbf{r},\mathbf{R}) << 2\nabla_R\omega\cdot\mathbf{K}$

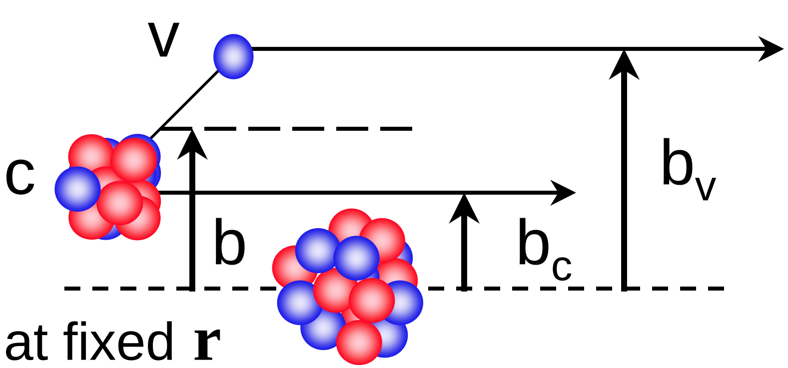
$$\omega(\mathbf{r},\mathbf{R}) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{Z} dZ'\, U(\mathbf{r},\mathbf{R}')\right\} \;\longrightarrow\; V_{cT} + V_{vT}$$

Few-body eikonal model amplitudes

So, after the collision, as $Z \to \infty$ $\quad \omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\, S_v(b_v)$

$$\Psi_{\mathbf{K}}^{\text{Eik}}(\mathbf{r},\mathbf{R}) \to e^{i\mathbf{K}\cdot\mathbf{R}}\, S_c(b_c)\, S_v(b_v)\, \phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



at fixed $\mathbf{r}$

adiabatic

So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)} \,|\, \phi_0 \rangle_{\mathbf{r}}$$

averaged over position probabilities of c and v ← amplitude that c,v survive interaction with b_c and b_v

Dynamics and structure - formal transparency

Independent scattering information of c and v from target

$$S_{\alpha\beta}(b) = \langle \phi_\beta \,|\, \overbrace{S_c(b_c)\, S_v(b_v)}^{\text{dynamics}} \,|\, \phi_\alpha \rangle$$

structure

Use the best available few- or many-body wave functions

More generally,

$$S_{\alpha\beta}(b) = \langle \varphi_\beta \,|\, S_1(b_1)\, S_2(b_2) \ldots\ldots\, S_n(b_n) \,|\, \varphi_\alpha \rangle$$

for any choice of 1,2 ,3, ….. n clusters for which a realistic wave function φ is available

Break-up of composite systems

The total cross section for removal of the valence particle from the projectile due to the break-up (or diffractive dissociation) mechanism is the break-up amplitude, summed over all final continuum states

$$\sigma_{\text{diff}} = \int d\mathbf{k} \int d\mathbf{b}\, |\langle \phi_{\mathbf{k}} \,|\, S_c(b_c)\, S_v(b_v) \,|\, \phi_0 \rangle|^2$$

but, using completeness of the break-up states

$$\int d\mathbf{k}\, |\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}| = 1 - |\phi_0\rangle\langle\phi_0| - |\phi_1\rangle\langle\phi_1| \;\ldots\ldots$$

If > 1 bound state

can (for a weakly bound system with a single bound state) be expressed in terms of only the projectile ground state wave function as:

$$\sigma_{\text{diff}} = \int d\mathbf{b} \left\{ \langle \phi_0 \,||\, S_c\, S_v \,|^2 \,|\, \phi_0 \rangle - |\langle \phi_0 \,|\, S_c\, S_v \,|\, \phi_0 \rangle|^2 \right\}$$

Absorptive cross sections - target excitation

Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels → $|S(b)|^2 \leq 1$

$$\sigma_{abs} = \sigma_R - \sigma_{diff} = \int d\mathbf{b}\ \langle \phi_0 \,|\, \underbrace{1 - |S_c S_v|^2}\, |\, \phi_0 \rangle$$

$$\left\{ \begin{array}{l} |S_v|^2 (1 - |S_c|^2) + \\ |S_c|^2 (1 - |S_v|^2) + \\ (1 - |S_v|^2)(1 - |S_v|^2) \end{array} \right.$$

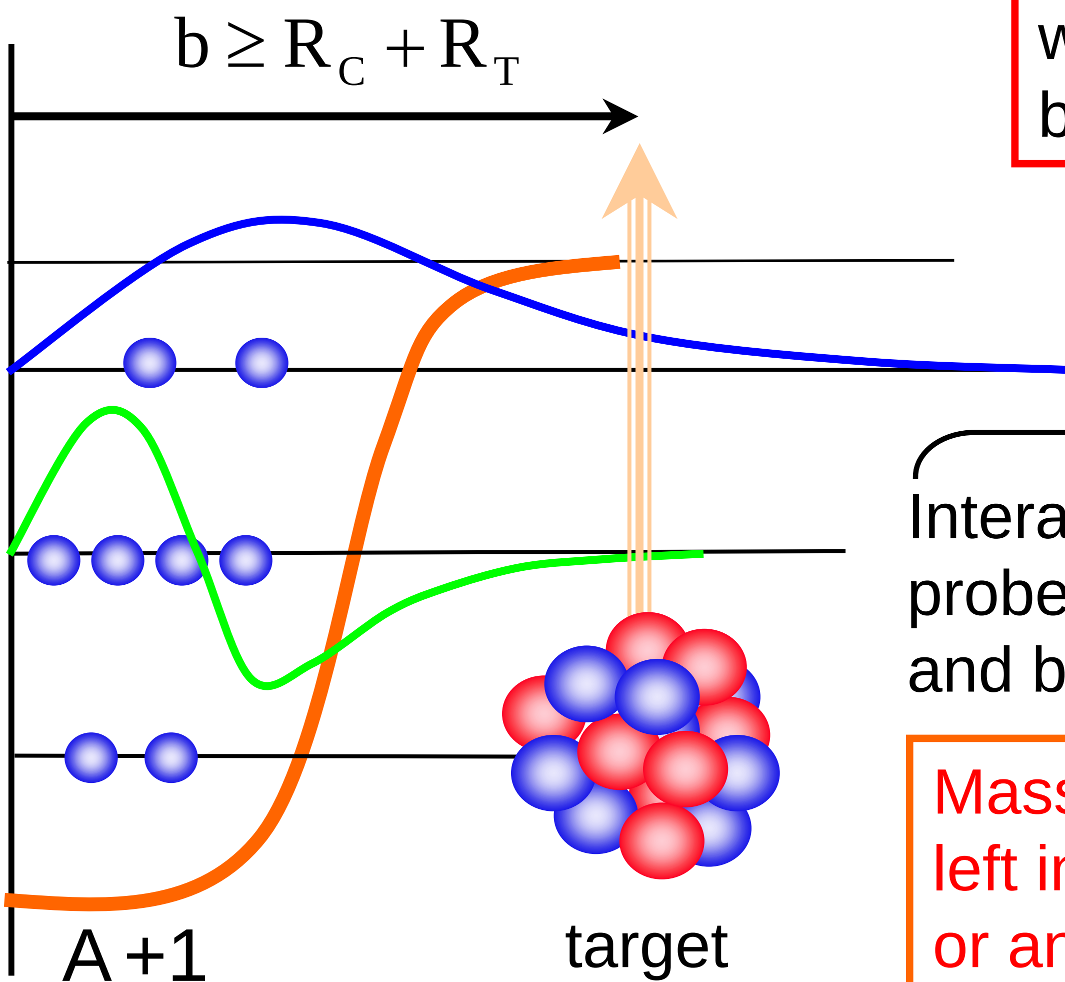
- $|S_v|^2 (1 - |S_c|^2)$: v survives, c absorbed
- $|S_c|^2 (1 - |S_v|^2)$: v absorbed, c survives
- $(1 - |S_v|^2)(1 - |S_v|^2)$: v absorbed, c absorbed

stripping of v from projectile exciting the target. c scatters at most elastically with the target

$$\sigma_{strip} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1 - |S_v|^2) \,|\, \phi_0 \rangle$$

Related equations exist for the differential cross sections, etc.

Probing the surface and tails of wave functions



with sensitivity to both ℓ and κ

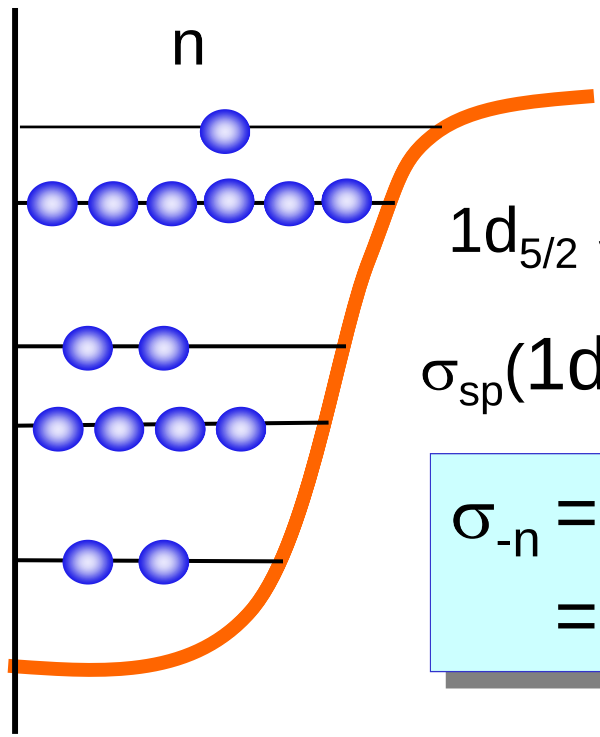
$h_\ell(i\kappa r)$

Interaction with the target probes $\phi_0(\mathbf{r})$ at surface and beyond $b \geq R_C + R_T$

Mass A residue will be left in the ground state or an excited state

Example for orientation - extreme sp model

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$2s_{1/2}$ S_n=2.7 MeV

$1d_{5/2}$ S_n=5.5 MeV

$\sigma_{sp}(1d_{5/2})$=23 mb $\sigma_{sp}(2s_{1/2})$=64 mb

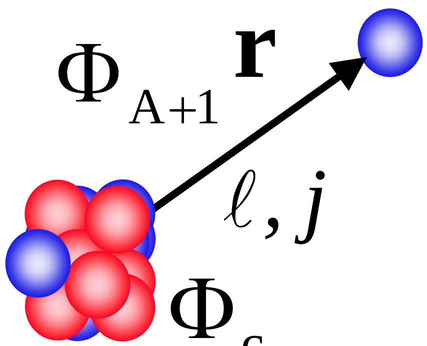
$$\sigma_{-n} = 6\,\sigma_{sp}(1d_{5/2}) + \sigma_{sp}(2s_{1/2})$$
$$= 202 \text{ mb}$$

Measurement at RIKEN [Kanungo et al PRL **88** ('02) 142502] at 72 MeV/nucleon on a ^{12}C target; σ_{-n} = 233(37)mb

Structure information - nucleon wave function

Nucleon removal from Φ_{A+1} will leave mass A residue in the ground or an excited state - even in extreme sp model

More generally:



amplitude for finding nucleon with sp quantum numbers ℓ,j, about core state Φ_c in Φ_{A+1} is

$$F^c_{\ell j}(\mathbf{r}_c) = \langle \mathbf{r}, \Phi_c \,|\, \Phi_{A+1} \rangle, \; S_N = E_{A+1} - E_c$$

$$\int d\mathbf{r}\,|F^c_{\ell j}(\mathbf{r})|^2 = C^2S(\ell j)$$

Spectroscopic factor ~ occupancy of the state

Usual to write

$$F^c_{\ell j}(\mathbf{r}) = \sqrt{C^2S(\ell j)}\;\phi^c_{\ell j}(\mathbf{r})\,; \qquad \int d\mathbf{r}\,|\phi^c_{\ell j}(\mathbf{r})|^2 = 1$$

with $\phi(\mathbf{r})$ calculated in a potential model (Woods-Saxon)

Of course we need to do this carefully

Energy (MeV)	I^π	ℓ	C^2S	σ_{sp} (mb)	σ_{1n} (mb)
0	0^+	0	0.797	64.2	51.2
♦ 3.38	2^+	2	2.130	22.8	48.6
4.62	0^+	0	0.115	32.0	3.7
♦ 4.83	3^+	2	3.079	20.4	62.9
5.32	1^-	1	0.851	17.8	15.2
5.93	0^-	1	0.332	16.9	5.6
6.50	2^+	2	0.242	18.0	4.4
				Sum:	191

p

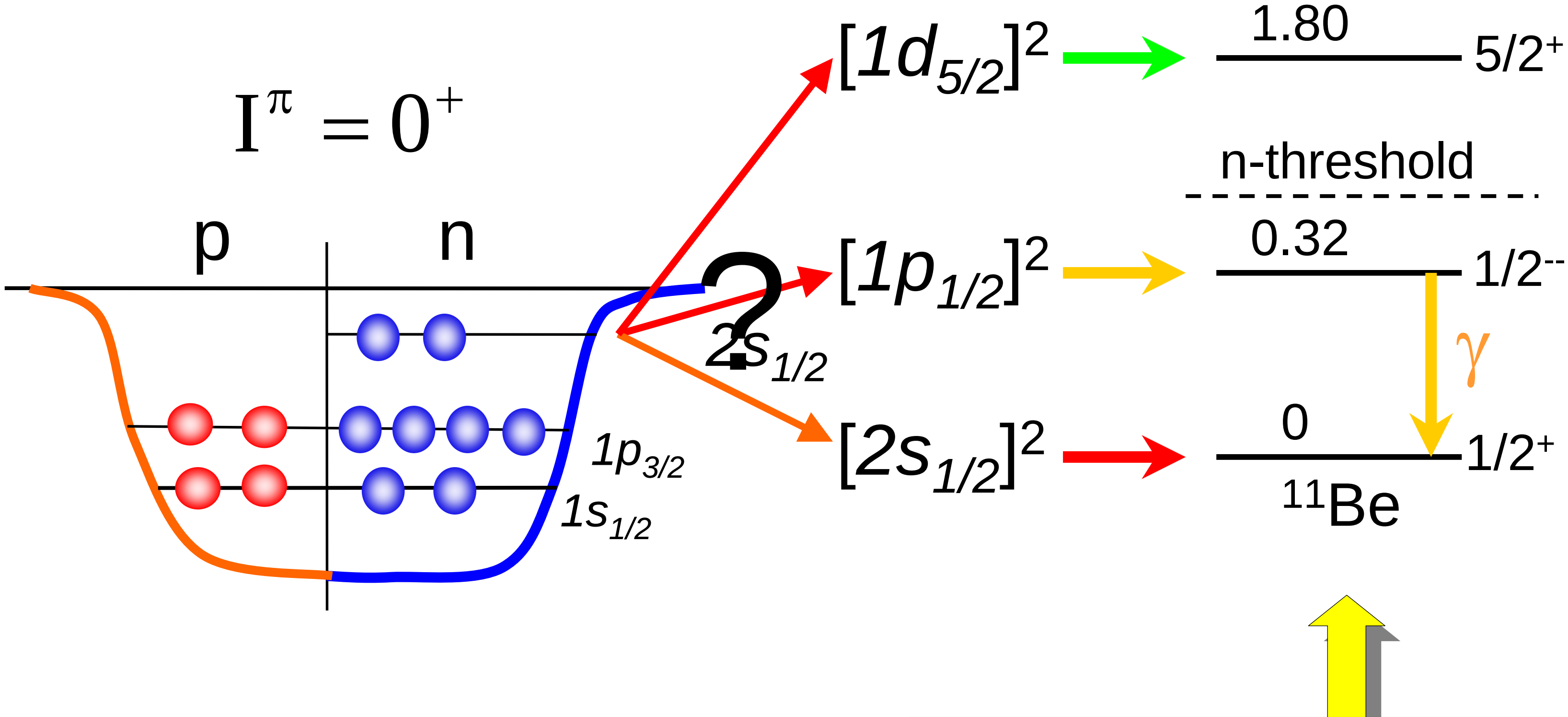
datum 233(37)mb

Shell model (Brown)

^{22}O final states below n-threshold

♦ $[d_{5/2} \times s_{1/2}]_J$ { $C^2S(2^+)=2.5$, $C^2S(3^+)=3.5$

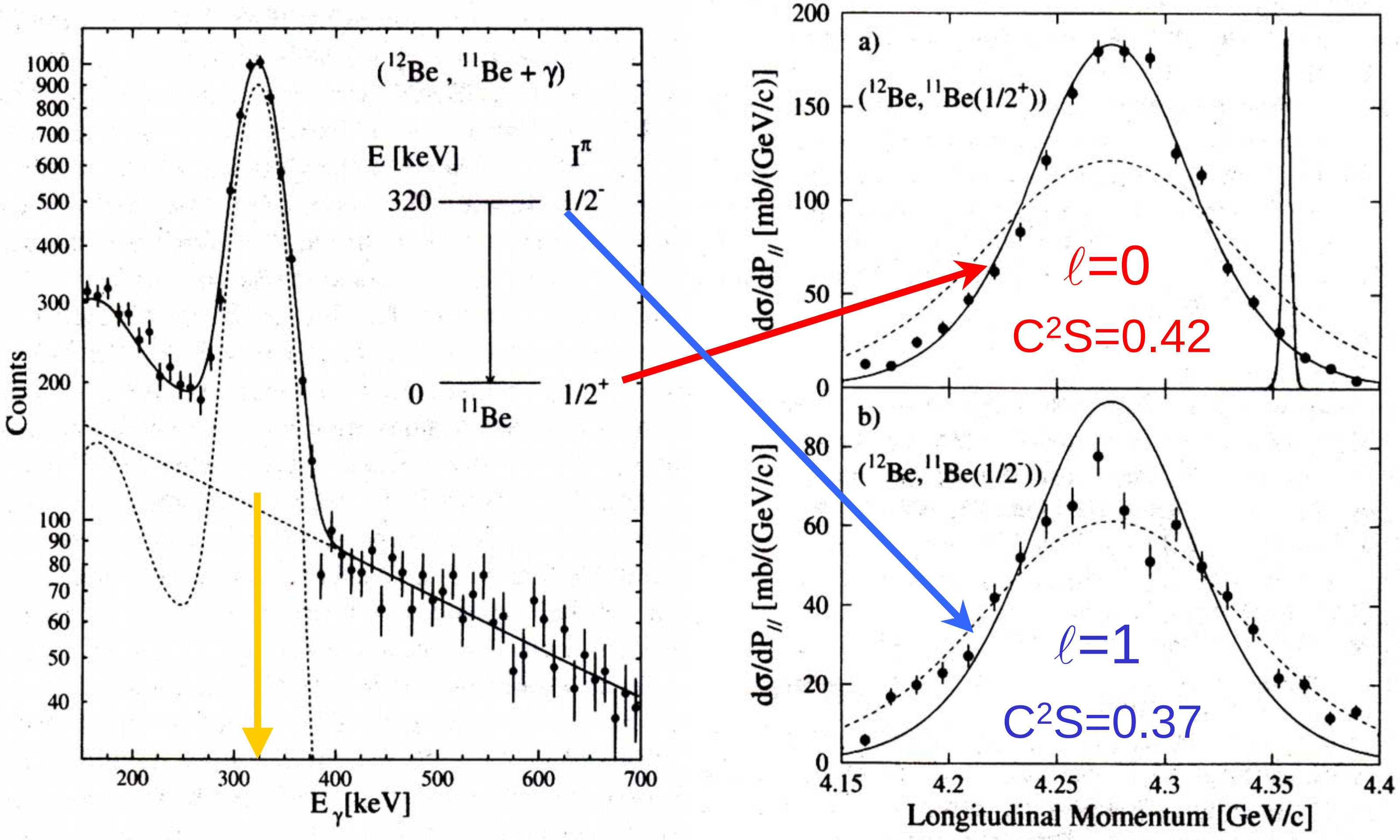
N=8 neutron shell closure (magic no.) in ^{12}Be?



is psd-shell ordering restored in ^{12}Be?

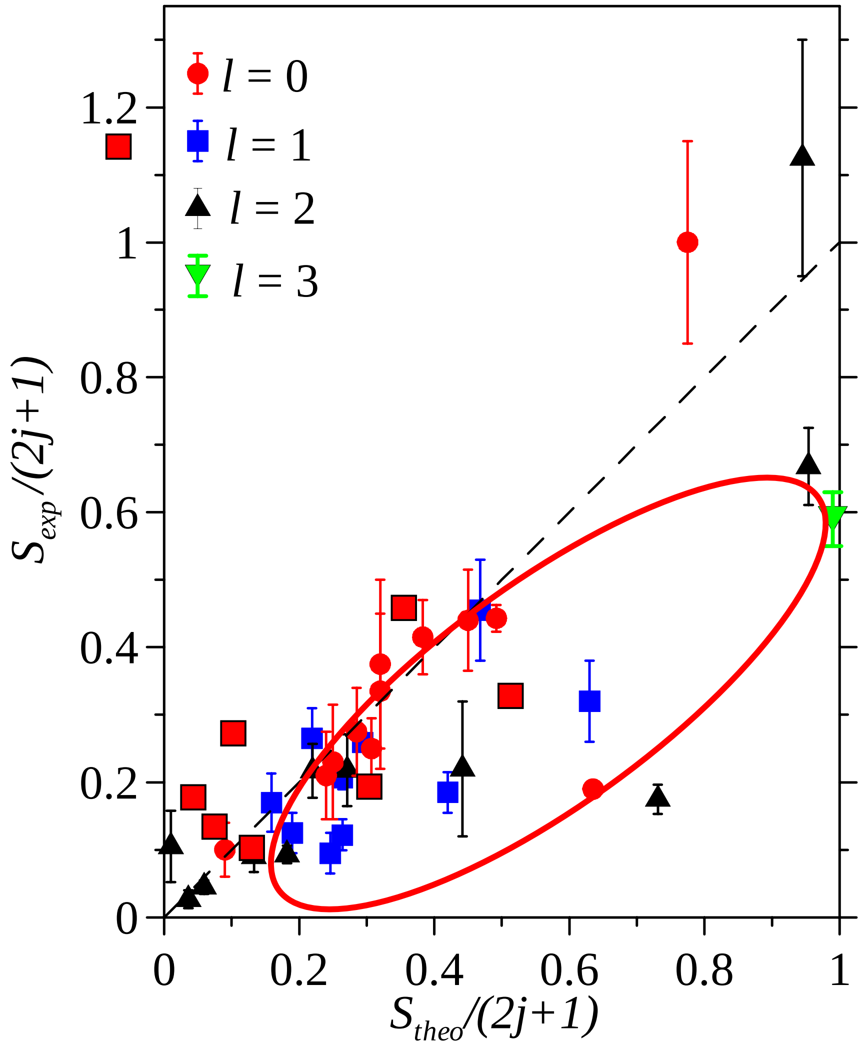
level inversion in ^{11}Be established - halo state

N=8 neutron shell closure in ^{12}Be?



A. Navin et al., PRL **85** (2000) 266

Experimental v shell model spectroscopic factors



Can define reduction factor

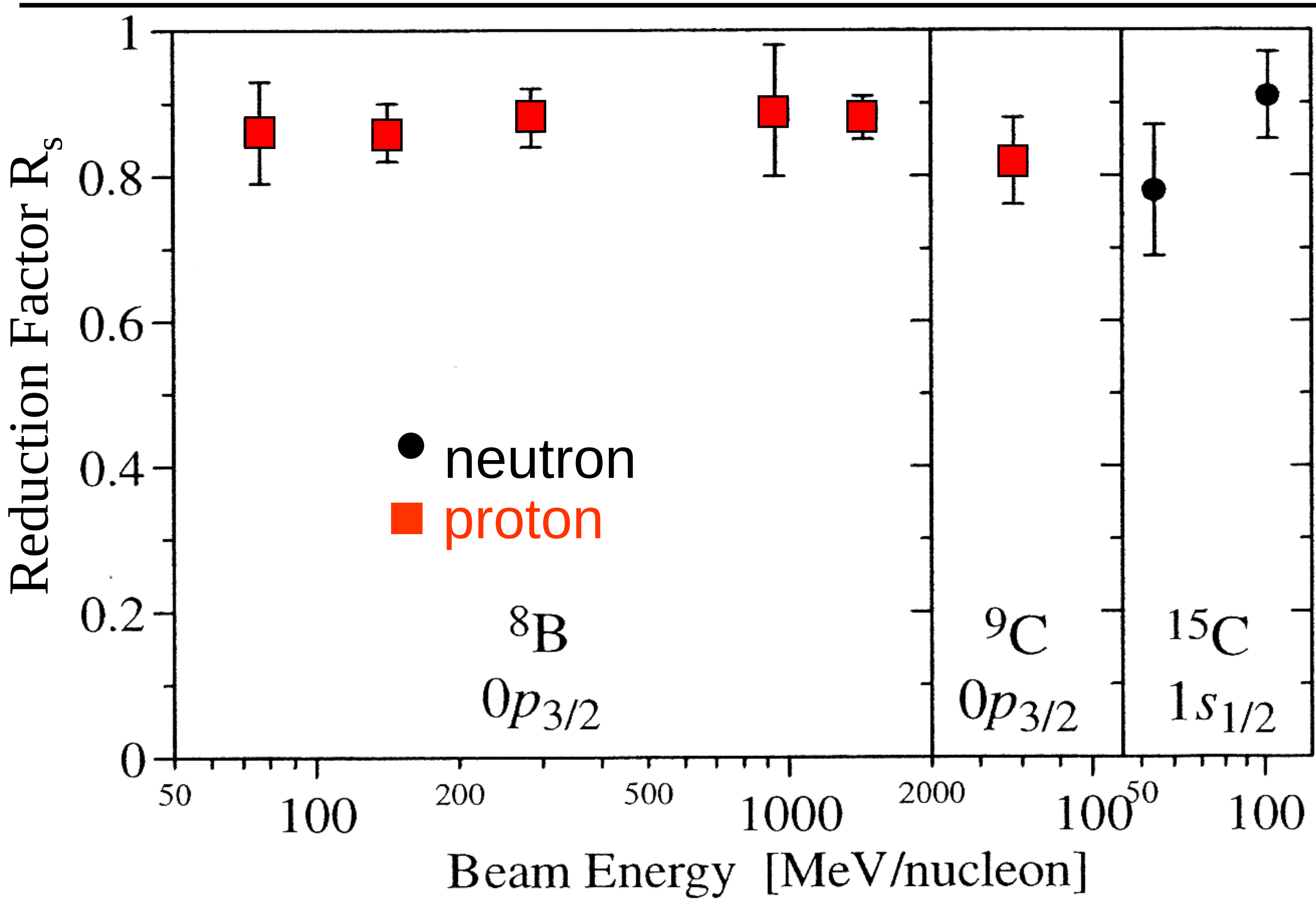
$$R_s = \frac{\sigma_{\exp}}{\sigma_{\text{th}}} \leq 1$$

th ≡ Shell model structure plus eikonal reaction

More bound systems

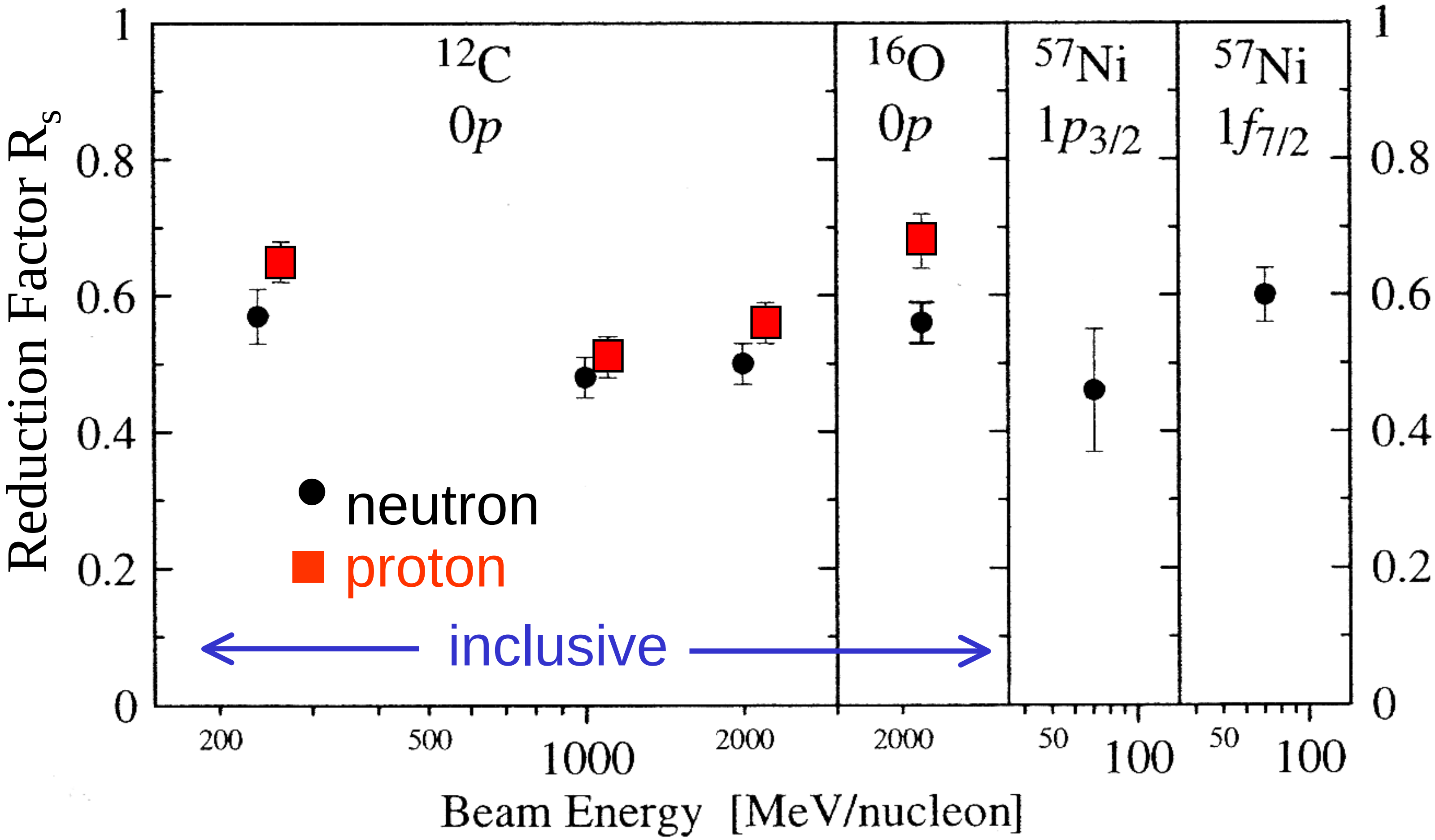
P.G. Hansen and J.A.Tostevin, ARNPS **53** (2003), 219

Weakly bound states – with good statistics



P.G. Hansen and J.A.Tostevin, ARNPS **53** (2003), 219

More strongly bound states – deep hole states



P.G. Hansen and J.A.Tostevin, ARNPS **53** (2003), 219

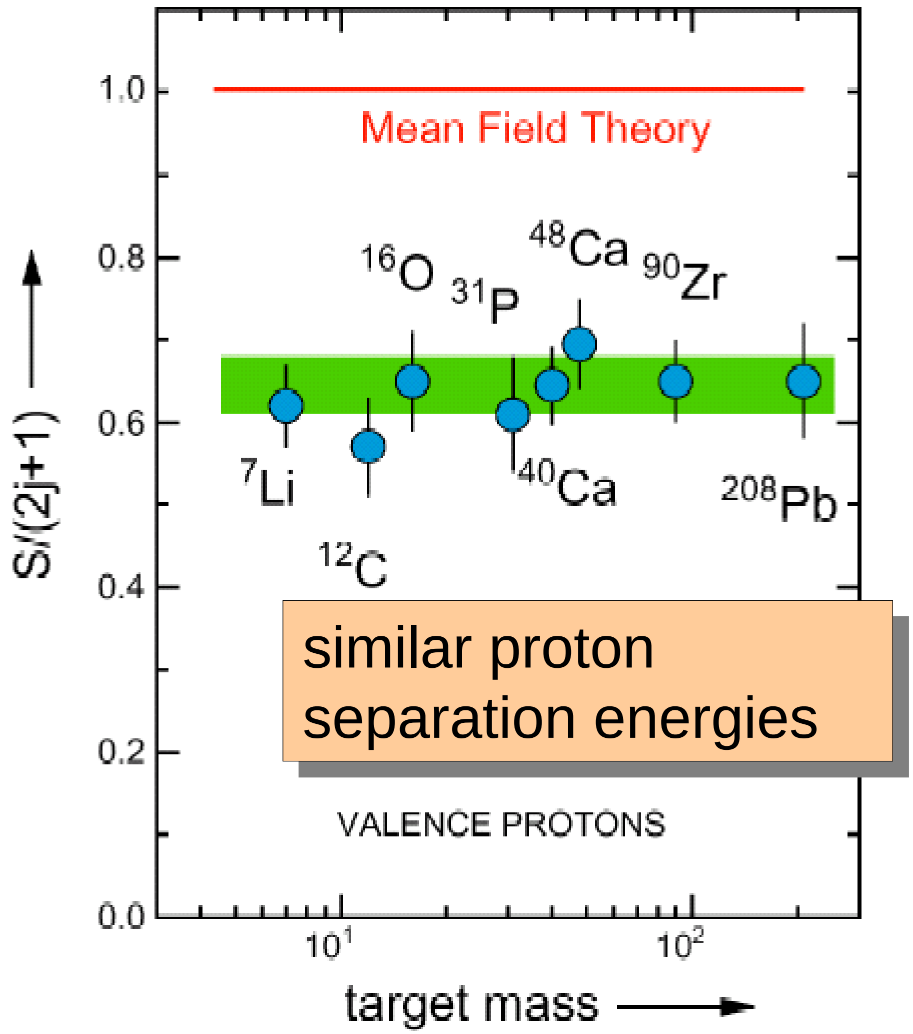
Results from electron scattering – stable nuclei

Departures of measured spectroscopic factors from the independent single-particle model predictions

Electron induced proton knockout reactions:

[A,Z] (e,e′p) [A-1,Z-1]

See only 60-70% of nucleons expected!



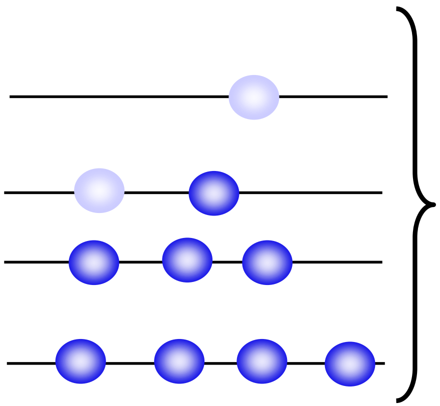
similar proton separation energies

W. Dickhoff and C. Barbieri, Prog. Nucl. Part. Sci., in press

Correlations and truncated shell model spaces

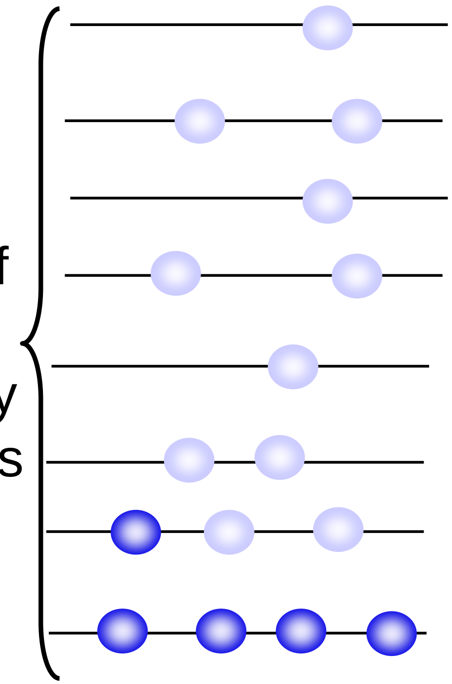
Truncated shell model space – with effective force

but reality is …



Few active orbitals, high occupancy e.g. sd shell

Greater distribution of nucleons to higher energy configurations

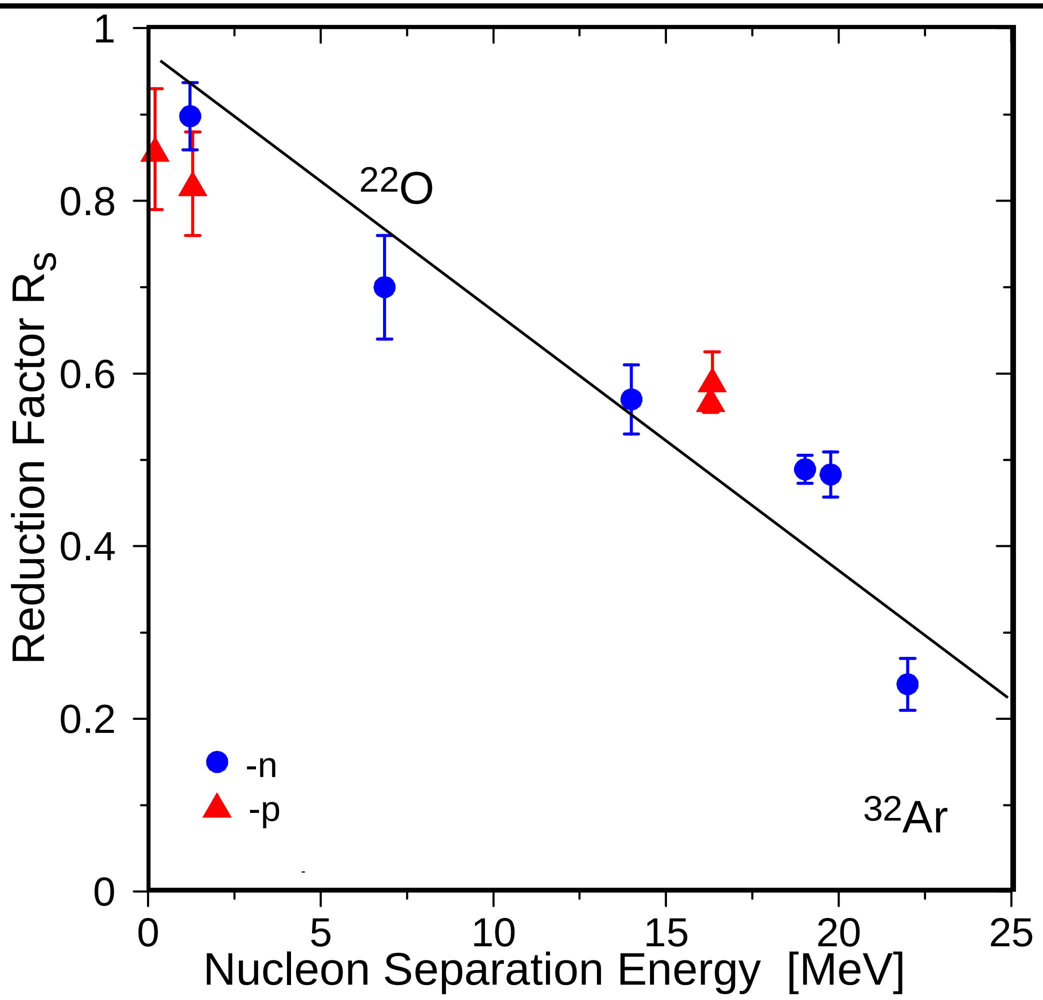


Filled core states e.g. 0s,1p

Short range, tensor force and collective correlations

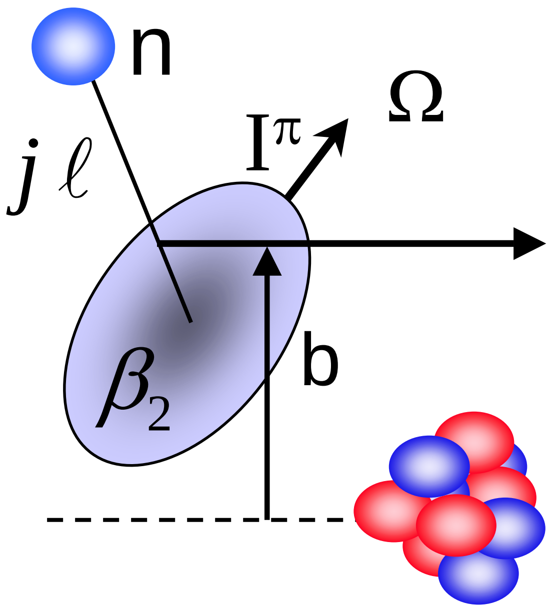
Filled core states e.g. 0s,1p

R_s factors – deviations from the shell model



A. Gade et al., PRL 93 (2004) 042501

Deformation - core degrees of freedom



$$\Psi_{JM}(\mathbf{r},\hat{\Omega}) = \sum_{\ell\, j\, I} [\phi_{j\ell}(\mathbf{r}) \otimes \phi_I(\hat{\Omega})]_{JM}$$

$$I = 0, 2, 4, \ldots\ldots.$$

weak-coupling n-deformed core model: this includes

- core excitation / de-excitation
- core reorientation effects

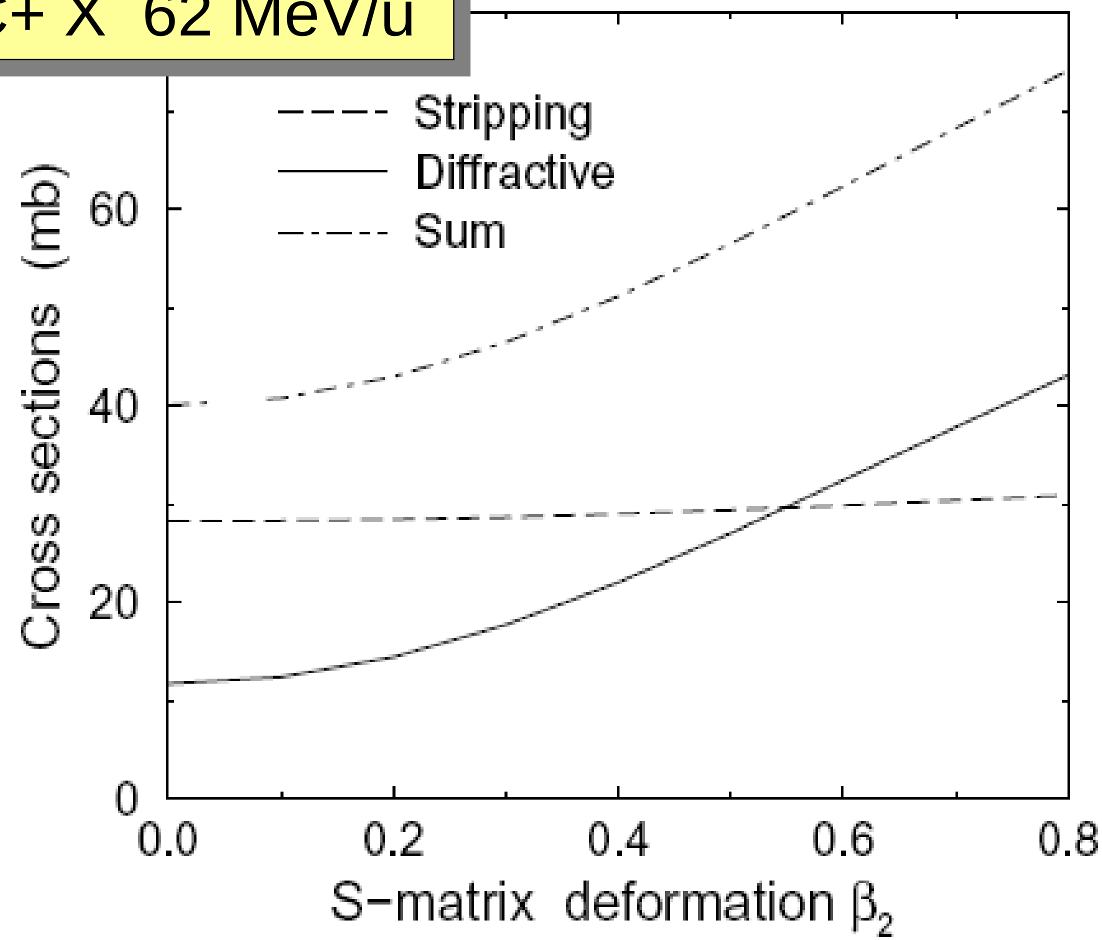
The inclusive stripping contribution now reads, e.g.

$$\sigma_{\text{strip}} = \sum_M \int d\mathbf{b} \int d\hat{\Omega} \langle \Psi_{JM} \,||\, S_c(\hat{\Omega})|^2 (1 - |S_n|^2) | \Psi_{JM} \rangle$$

P.Batham, J.A. Tostevin and I.J. Thompson, submitted

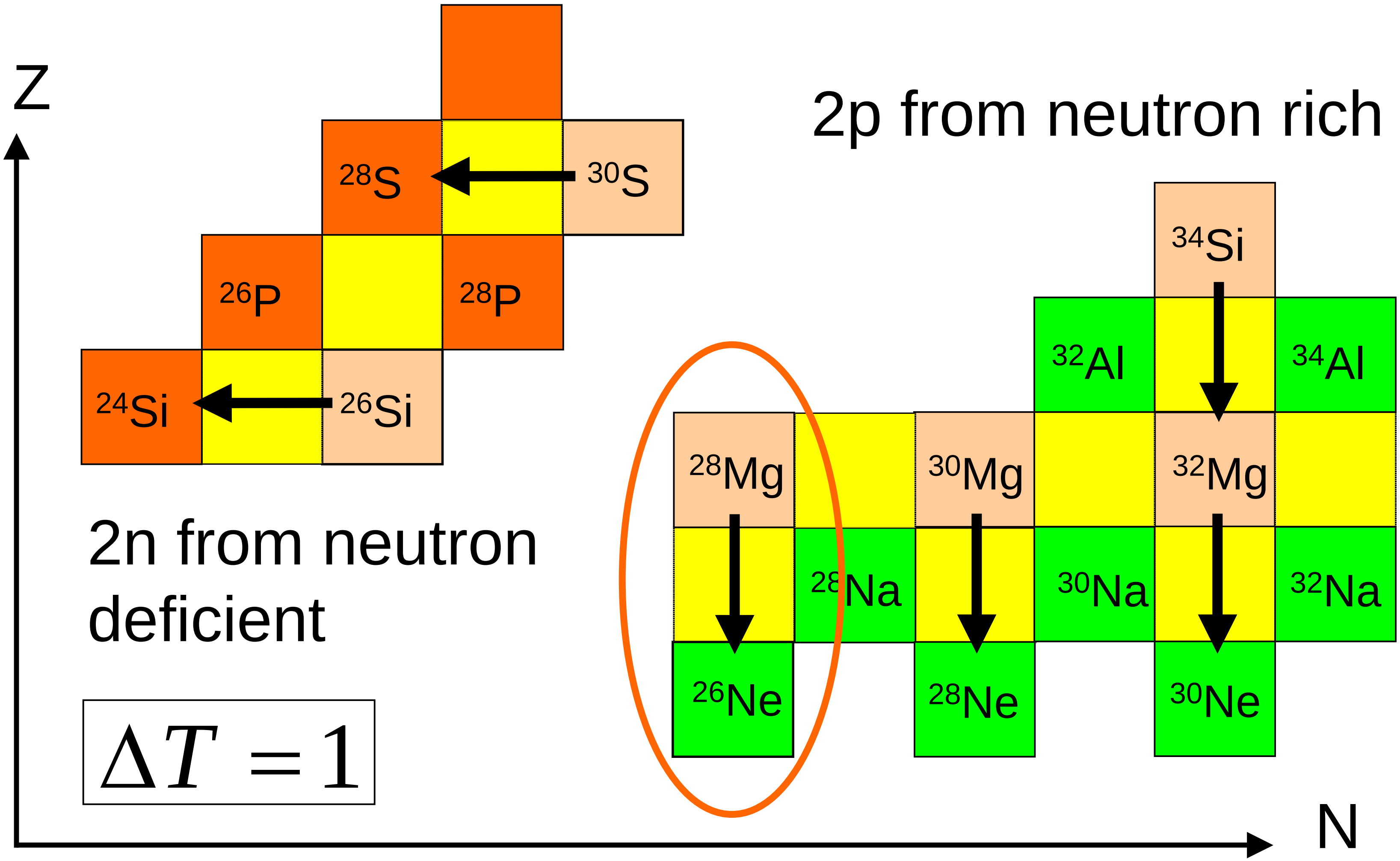
Deformation-assisted break-up

$^{17}C + {}^{9}Be \rightarrow {}^{16}C + X$ 62 MeV/u

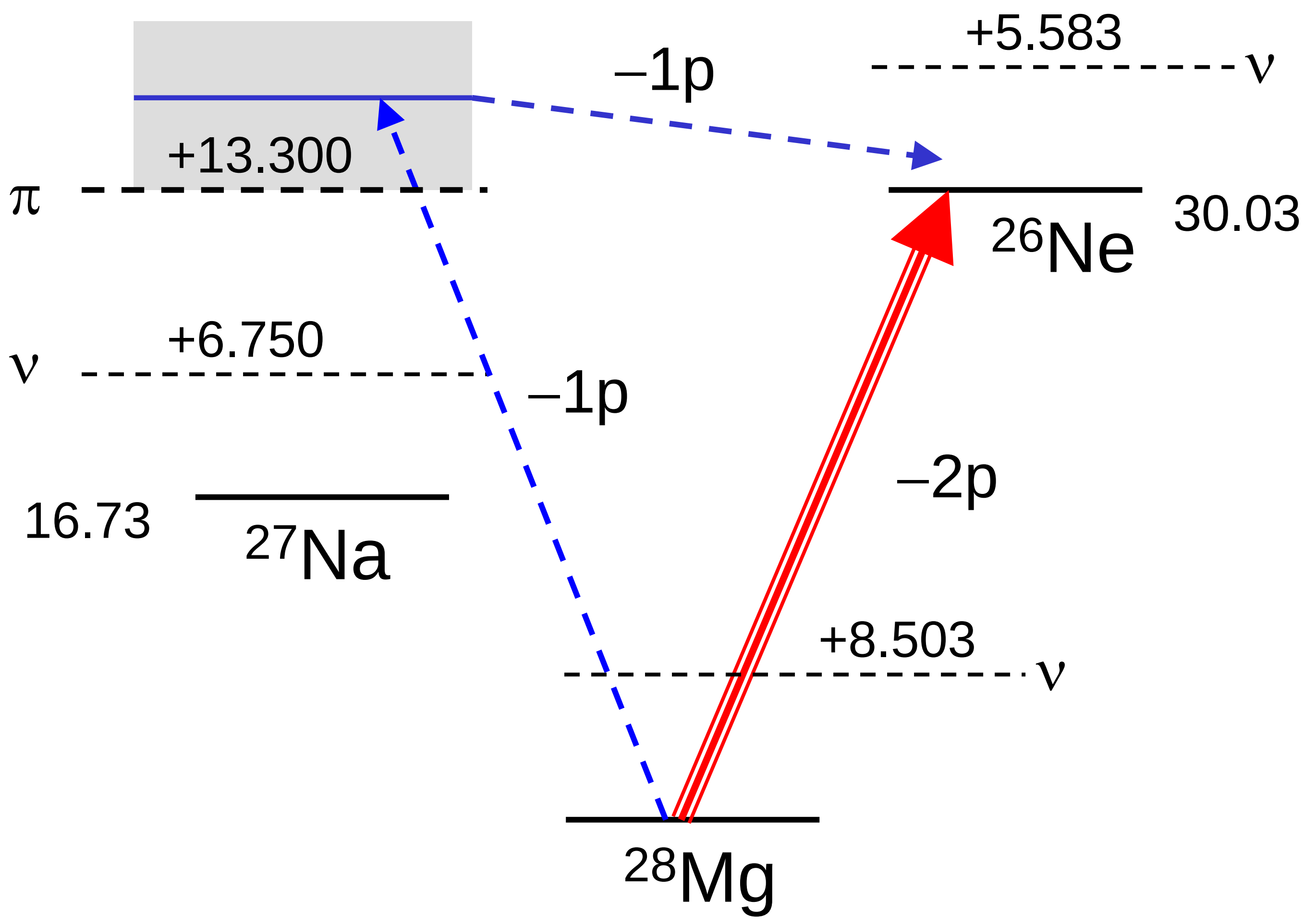


P.Batham, J.A. Tostevin and I.J. Thompson, submitted

Two nucleon knockout – go south, or west

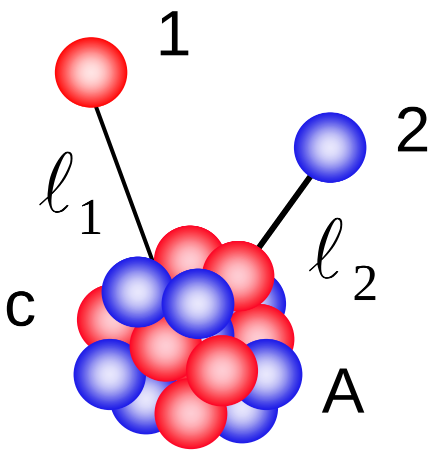


Two-proton knockout energies – magnesium 28



Direct two nucleon knockout – 2N correlations?

$$\sigma_{\mathrm{strip}} = \sigma_{-2\mathrm{N}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2\,(1-|S_1|^2)(1-|S_2|^2)\,|\,\phi_0\rangle$$



Estimate assuming removal of a pair of uncorrelated nucleons -

$$\phi_0(A,\mathbf{r}_1,\mathbf{r}_2) = \Phi_c(A)\phi_{\ell_1}(\mathbf{r}_1)\phi_{\ell_2}(\mathbf{r}_2)$$

$$\sigma_{\mathrm{strip}} \Rightarrow \sigma_{\mathrm{strip}}(\ell_1\ell_2)$$

contribution from direct 2N removal $\sigma_{-2\mathrm{N}}$

p particles ℓ_α

q particles ℓ_β

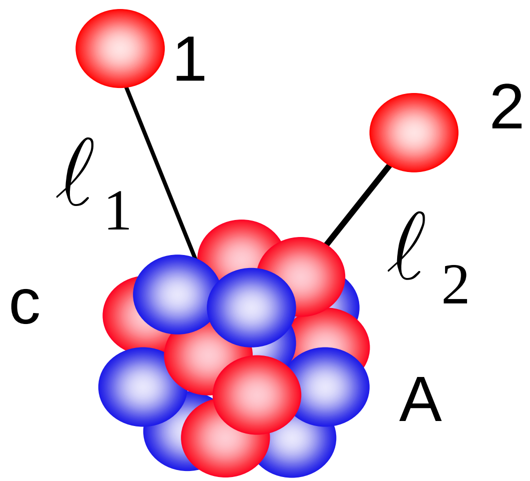
$$\sigma_{-2\mathrm{N}} = \frac{p(p-1)}{2}\sigma_{\mathrm{strip}}(\ell_\alpha\ell_\alpha) + \frac{q(q-1)}{2}\sigma_{\mathrm{strip}}(\ell_\beta\ell_\beta) + pq\,\sigma_{\mathrm{strip}}(\ell_\alpha\ell_\beta)$$

D. Bazin et al., PRL **91** (2003) 012501

Uncorrelated two proton removal

$^{28}Mg \rightarrow {}^{26}Ne$(inclusive)

D. Bazin et al., PRL **91** (2003) 012501



Assuming $(1d_{5/2})^4$ then

$$\sigma_{-2N} = \frac{4(4-1)}{2}\sigma_{strip}(22) \approx 1.8\,\text{mb}$$

Expt: 1.50(1) mb

$$\sigma_{strip}(22) = 0.29\text{ mb}$$

$$\sigma_{strip}(02) = 0.32\text{ mb}$$

$$\sigma_{strip}(00) = 0.35\text{ mb}$$

with weights to the ^{26}Ne final states

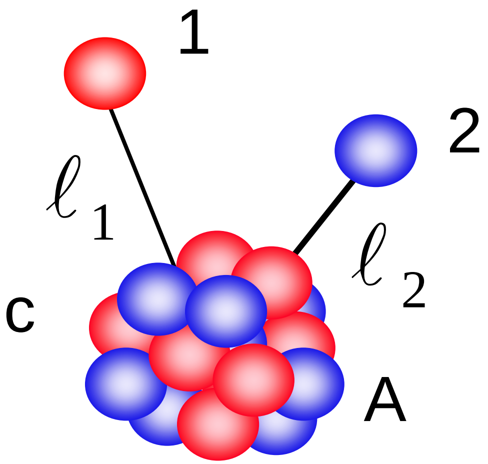
0^+: 1.33
2^+: 1.67
4^+: 3.00

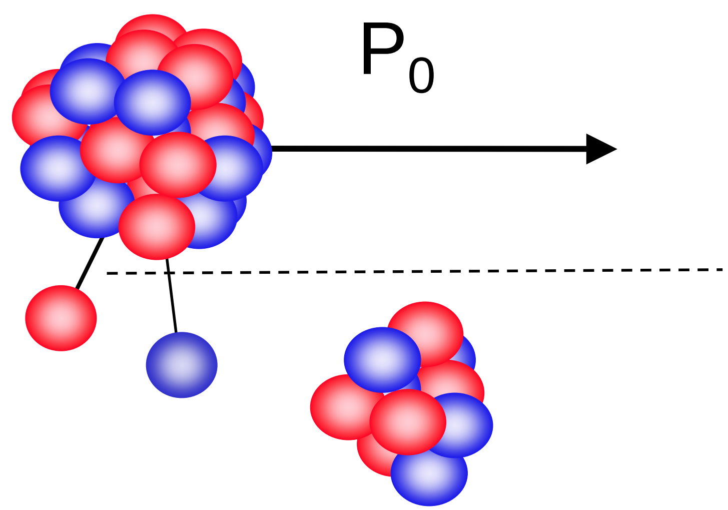
Two nucleon knockout – obvious generalisation

$$\sigma_{\text{strip}} = \sigma_{-2\text{N}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, \mathrm{S_c}\,|^2\,(1-|\,\mathrm{S_1}\,|^2)(1-|\,\mathrm{S_2}\,|^2)\,|\,\phi_0 \rangle$$

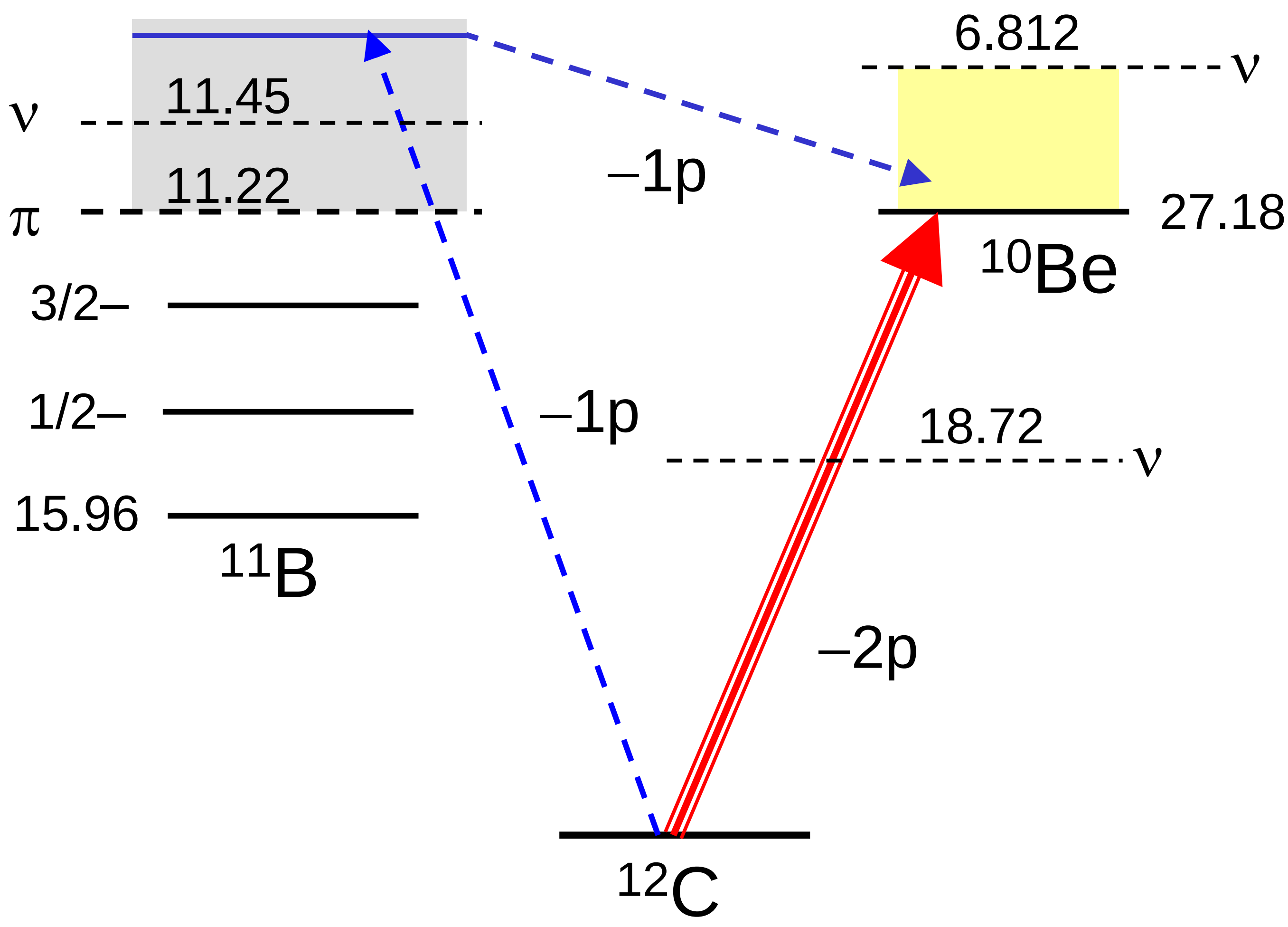
$$\phi_0 \equiv \Psi^{(c)}_{JM} = \sum_{\alpha\, I} C^{JIc}_{\alpha} [\overline{[\phi_{j_1\ell_1}(1) \otimes \phi_{j_2\ell_2}(2)]_I} \otimes \phi_c]_{JM}\,,$$

$$\alpha \equiv (j_1\ell_1,\, j_2\ell_2)$$





Two-proton knockout energies – carbon 12



Test case - earlier data from Berkeley (~10%)

2N removal from ^{12}C
B.A. Brown, 2N amplitudes

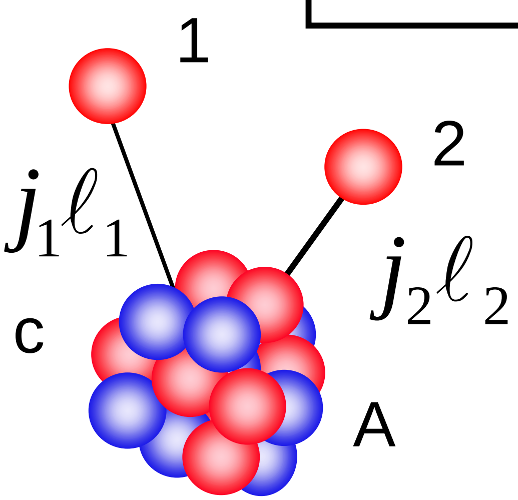
Kidd et al., Phys Rev C **37** (1988) 2613

Energy/nucleon	250 MeV	1.05 GeV	2.10 GeV
$^{12}C \rightarrow {}^{10}Be$ (2p) S(2p)=27.18 MeV	**5.82 mb** **5.88**	**5.33 mb** **5.30(30)**	**5.15 mb** **5.81(29)**
$^{12}C \rightarrow {}^{10}C$ (2n) S(2n)=31.84 MeV	**4.26 mb** **5.33(81)**	**3.91 mb** **4.44(24)**	**3.84 mb** **4.11(22)**

J.A. Tostevin et al., Nucl. Phys. A746 (2004) 166c.

Correlated two proton removal

$$\Psi_{JM}^{(c)} = \sum_{\alpha I} C_\alpha^{JIc} [\overline{[\phi_{j_1\ell_1}(1)\otimes\phi_{j_2\ell_2}(2)]}_I \otimes \phi_c]_{JM}, \quad \alpha \equiv (j_1\ell_1, j_2\ell_2)$$



amplitudes of nucleon-pair wave functions about core configurations c.

Spin states populated

$$|J - I| \le I_c \le J + I$$

$$|j_1 - j_2| \le I \le j_1 + j_2$$

There is now no SF factorisation

J.A. Tostevin, G. Podolyák, et al., PRC **70** (2004) 064602.

Shell model (sd-shell) 2N amplitudes

$^{28}Mg \rightarrow ^{26}Ne(0^+)$

$C(2s_{1/2})^2 = -0.305$

$C(1d_{3/2})^2 = -0.301$

$C(1d_{5/2})^2 = -1.05$

$^{28}Mg \rightarrow ^{26}Ne(2^+)$

$C(1d_{3/2})^2 = -0.050$

$C(d_{5/2},d_{3/2}) = +0.374$

$C(1d_{5/2})^2 = -0.637$

$C(s_{1/2},d_{5/2}) = -0.061$

$C(s_{1/2},d_{3/2}) = -0.139$

$^{28}Mg \rightarrow ^{26}Ne(4^+)$

$C(d_{5/2},d_{3/2}) = 0.331$

$C(1d_{5/2})^2 = 1.596$

J.A. Tostevin, G. Podolyák, et al., PRC **70** (2004) 064602.

Cross sections – correlated and uncorrelated

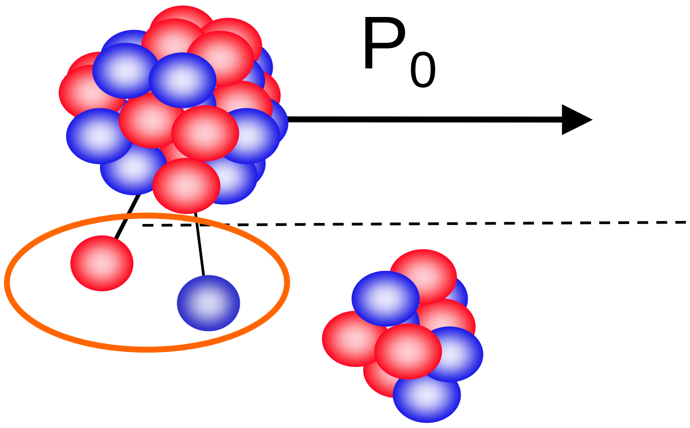
$^{28}Mg \rightarrow ^{26}Ne(0^+, 2^+, 4^+)$ $S = \sigma$(in mb) / 0.29

	S_{th} unc.	S_{exp}	S_{th} corr.	σ_{exp} (mb)	σ_{th} (mb)
0^+	1.33	**2.4(5)**	**1.83**	0.70(15)	0.532
2^+	1.67	**0.3(5)**	**0.54**	0.09(15)	0.157
4^+	3.00	**2.0(3)**	**1.79**	0.58(9)	0.518
2^+	-	**0.5(3)**	**0.78**	0.15(9)	0.225

Inclusive cross section (in mb) **1.50(10)** **1.43**

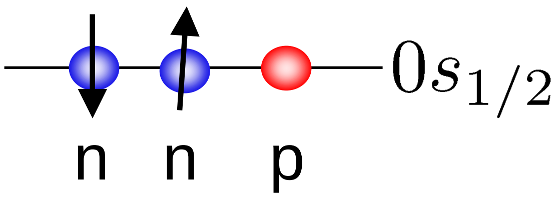
J.A. Tostevin, G. Podolyák, et al., PRC **70** (2004) 064602.

Nature of the two-nucleon correlations probed?



Removed nucleon pair are <u>spatially</u> correlated but no restriction on pair spin (S=0,1) or relative orbital angular momentum in formalism. All contributing pair wave functions are included.

Unlike, e.g. (p,t) reaction – <p|t> structure selects nn pair with S=0 in a relative s-state (ℓ =0)



Can assess by projecting S=0 component of knockout

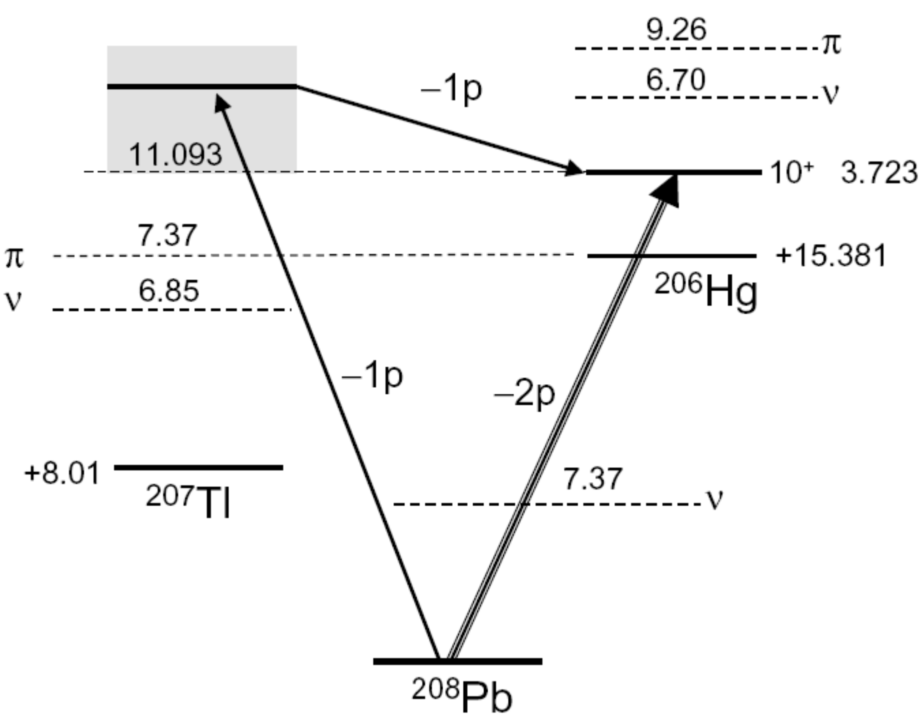
Spin-correlations – measure more than in transfer

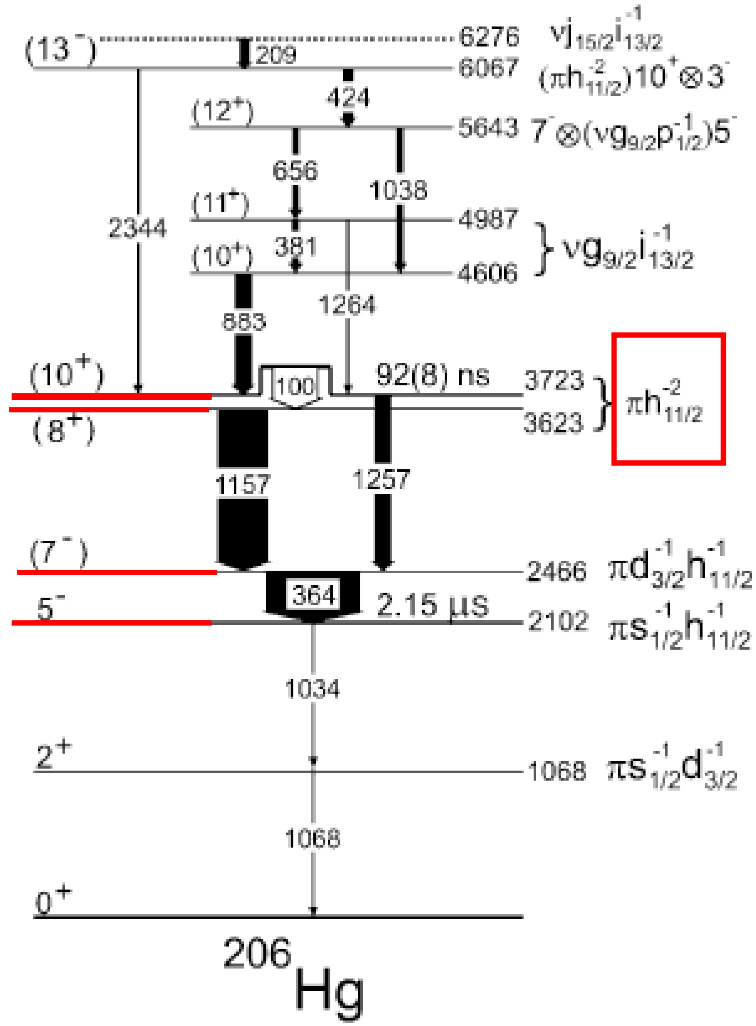
$^{28}Mg \rightarrow ^{26}Ne(0^+, 2^+, 4^+, 2^+)$

J_f^π	S_{unc}	S_{rel}	S'_{rel}	$S_{S=0}$	S_{exp}	S_{th}	σ_{th} (mb)	$\sigma_{S=0}$ (mb)
0^+	1.33	1.6	1.88	3.70	2.4(5)	1.83	0.532	0.484
2^+	1.67	0.14	0.15	0.26	0.3(5)	0.54	0.157	0.034
4^+	3.00	(2.0)	(2.0)	(2.0)	2.0(3)	1.79	0.518	0.259
2_2^+	-	0.46	0.43	0.95	0.5(3)	0.78	0.225	0.123

J.A. Tostevin, G. Podolyák, et al., PRC **70** (2004) 064602.

Seniority isomers in heavy nuclei





B. Fornal et al., PRL **87** (2001) 212501


thanks for your attention
and interest .....