Direct reactions at low energies: Part I – Background and concepts

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100 years of nuclei – scattering from the beginning

[669]

LXXIX. *The Scattering of α and β Particles by Matter and the Structure of the Atom.* *By* Professor E. RUTHERFORD, *F.R.S., University of Manchester* *.

§ 1. IT is well known that the α and β particles suffer deflexions from their rectilinear paths by encounters with atoms of matter. This scattering is far more marked for the β than for the α particle on account of the much smaller momentum and energy of the form
There seems to be no doubt that such swiftly
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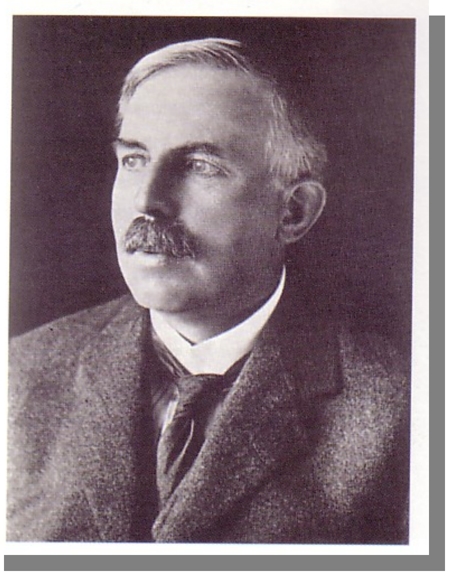
deflexions observed are due to the strong e
traversed within the atomic system. It has ge



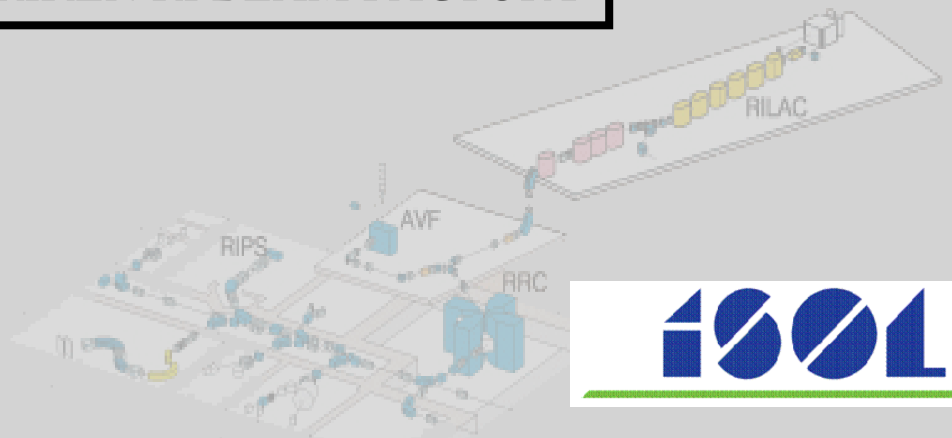
Philosophical Magazine, volume **21** (1911),
pages 669-688

Radioactive ion-beams: facilities – and future plans

FAIR Facility for Antiproton and Ion Research

GSI

RIKEN RI BEAM FACTORY



---A Dream Factory for Particle Beams---

TRIUMF ARIEL

ADVANCED RARE ISOTOPE LABORATORY

ISOLDE

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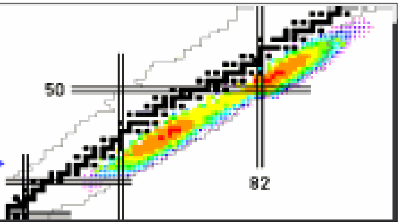
Spiral2



NSCL

FRIB

There are several good reaction theory texts: e.g.

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) Norman Austern

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages) G R Satchler

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) L S Rodberg, R M Thaler

Direct Nuclear Reactions (World Scientific Publishing, 396 pages) Norman K. Glendenning

Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages) C A Bertulani, P Danielewicz

Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages) Herman Feshbach

Introduction to Nuclear Reactions (Oxford University Press, 332 pages) G R Satchler

Nuclear Reactions for Astrophysics (Cambridge University Press, 2010) Ian Thompson and Filomena Nunes

Some other notes/resources available at:

http://www.nucleartheory.net/DTP_material

Please let me know if there are problems.

Exotic Beams Summer School 2011 (at NSCL):

Ian Thompson

Lawrence Livermore National Laboratory

Nuclear Reactions (Theory)

http://www.nscl.msu.edu/~zegers/ebss2011/thompson.pdf

Read e.g. Thompson's EBSS 2011 lecture 1 …

… for a short discussion of the characteristics of direct (fast) and compound (massive energy sharing) nuclear reactions.

Direct reactions: Reactions in which nuclei make glancing contact and then separate rapidly. Projectile may exchange some energy and / or angular momentum, or have one or more nucleons transferred to it or removed from it.

Direct reactions: mostly take place at or near the nuclear surface and at larger impact parameters

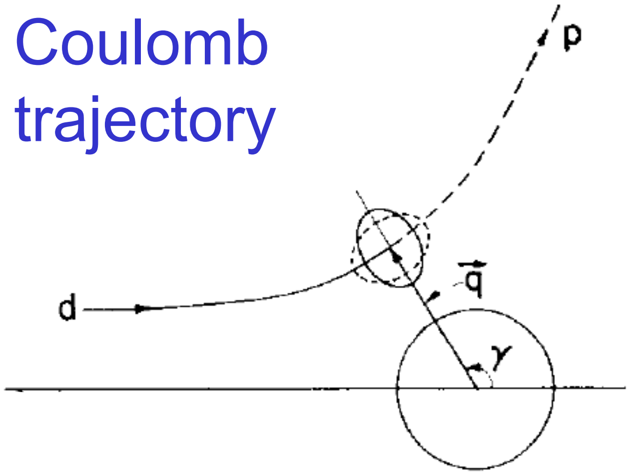
Direct reaction products tend to be strongly forward peaked as projectile continues to move in general forward direction

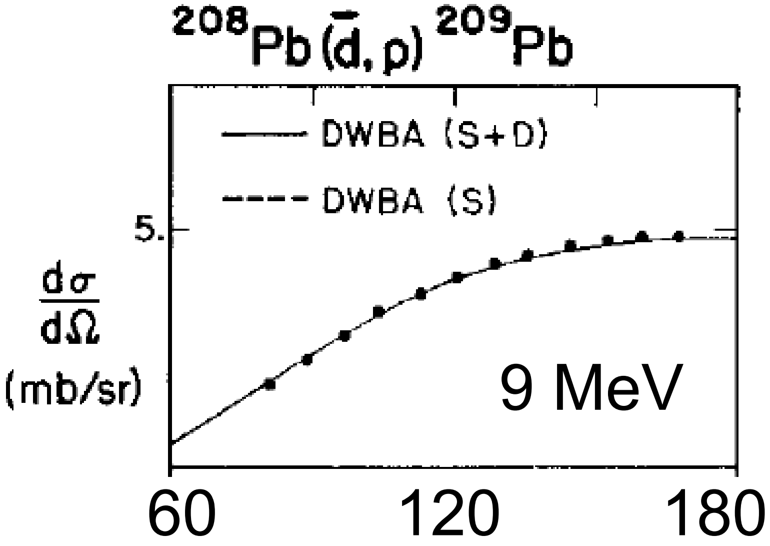
Direct reactions take place on a short timescale (we will need to quantify) – a timescale that reduces with increasing energy of the projectile beam (that allows extra approximations)

Direct reaction clock ticks in units of $\sim 10^{-22}$ s – timescale for a nucleon's transit across in a typical nucleus

Direct reactions at low energies need care

1. Energies near the Coulomb barriers of the reacting systems [$E_{cm} \sim Z_1 * Z_2 * e^2/(R_1+R_2)$]
2. So, incident energies of 1 to a few MeV per nucleon
3. Are such energies high enough and timescales short enough for reactions to be <u>fast</u> and <u>direct</u>?
4. Reaction products may not be forward peaked, e.g. (d,p) transfers near/below the Coulomb barrier





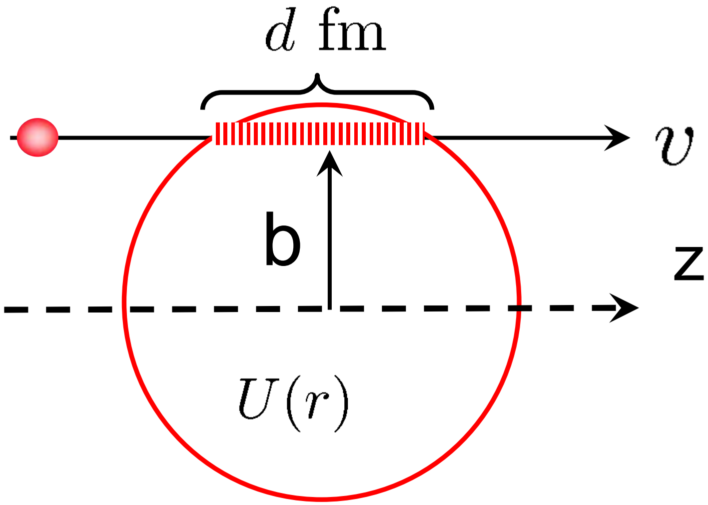
L.D. Knutson, Ann. Phys. **106**, 1 (1977)

Reaction timescales – surface grazing collisions

For say 10 and 100 MeV/u incident energy:

$$\gamma = 1.01, \quad v/c = 0.14, \qquad \gamma = 1.1, \quad v/c = 0.42,$$

$$\Delta t = 2.4 \times d \times 10^{-23} s, \qquad \Delta t = 7.9 \times d \times 10^{-24} s$$



d<nuclear diameter (a few fm) for strong interactions: but if Coulomb effects are important or weakly bound systems with extended wave functions – extended interaction times and probable - likely <u>higher-order effects</u>

First part aims:

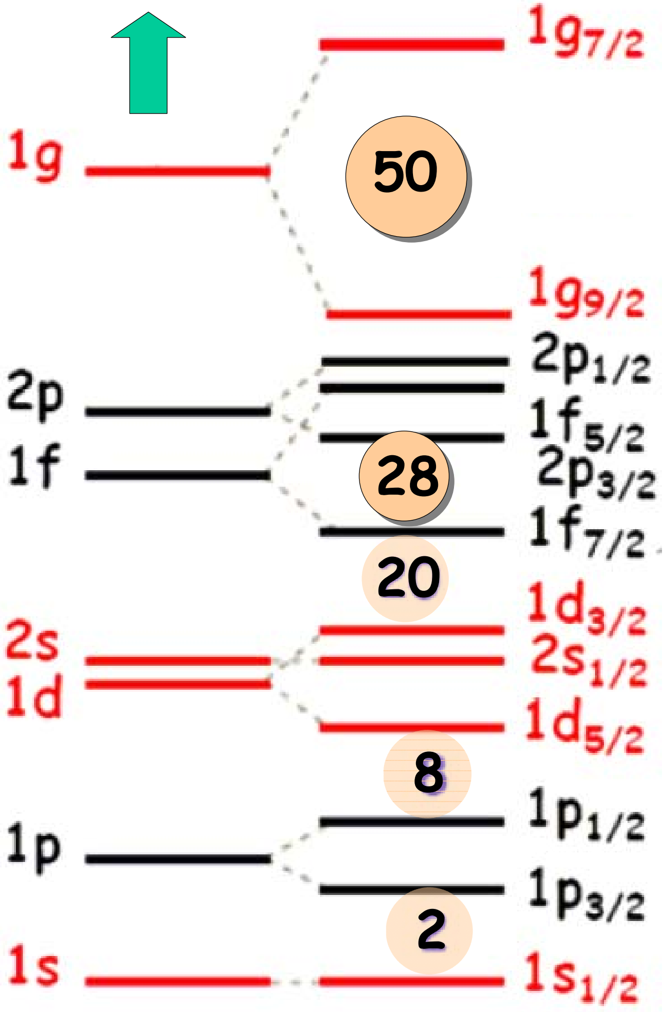
To discuss: 1. solutions of the Schrodinger equation for states of **two** bodies with specific quantum numbers over a wide range of energies – the need for bound, resonant, continuum (and continuum bin) states.

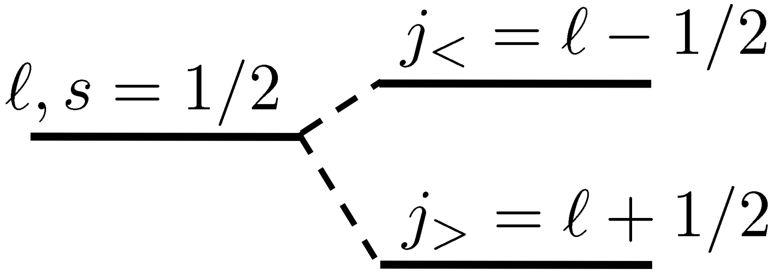
2. The form of these two-body problem solutions at large separations and their relationships to nuclear structure, absorption, reaction and scattering observables.

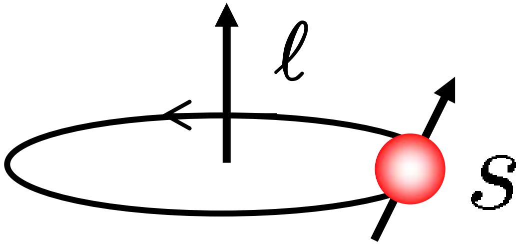
3. The constraints on two-body potentials and their parameters. Parameter conventions. The need to cross reference to known nuclear structures, resonances, nuclear sizes and experiment whenever possible in constraining parameter choices for calculations.

4. Connection of structure and reactions – overlap functions

Single-particle aspects of structure from reactions







$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

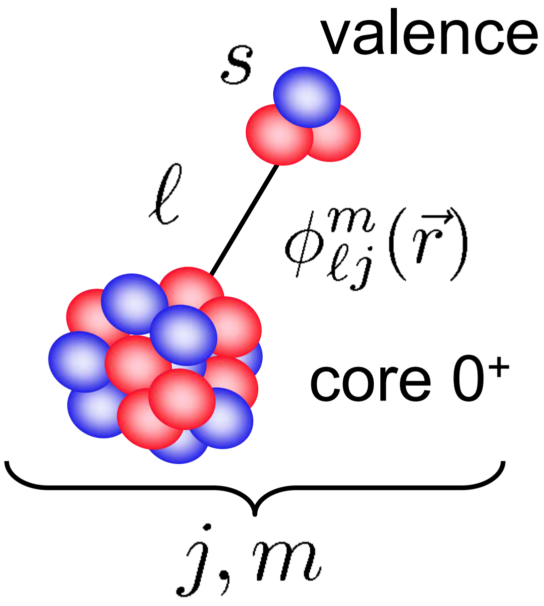
$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V_{so}(r) < 0$$

Underpinnings of direct reaction methods

Solutions of Schrodinger's equation for (pairs of) nuclei interacting via a potential energy function of the form*

$$U(r) = \underbrace{V_C(r)}_{\text{Coulomb}} + \underbrace{V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}}_{\text{Nuclear}}$$



Need descriptions of wave functions of:

(1) Bound states of nucleons or clusters (valence particles) to a core (that is assumed for now to have spin zero).

(2) Unbound scattering or resonant states at low energy

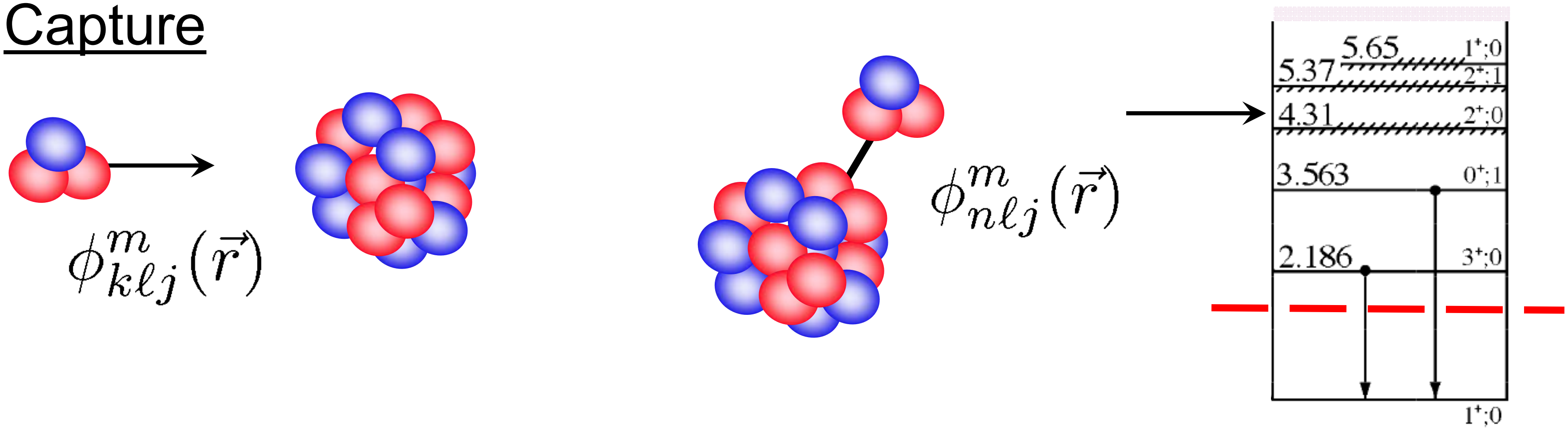
(3) Distorted waves for such bodies in complex potentials

$$U(r) = V_C(r) + V(r) + \boxed{iW(r)} + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

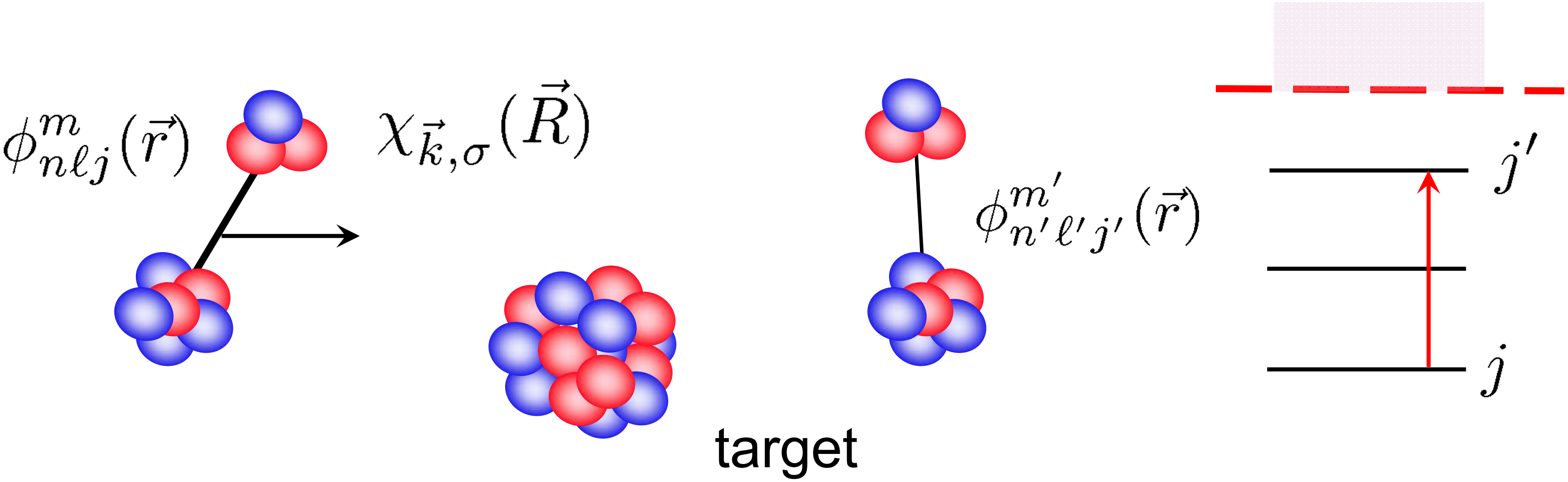
*Additional, e.g. tensor terms, when s=1 or greater neglected

Direct reactions – types and characteristics

Capture

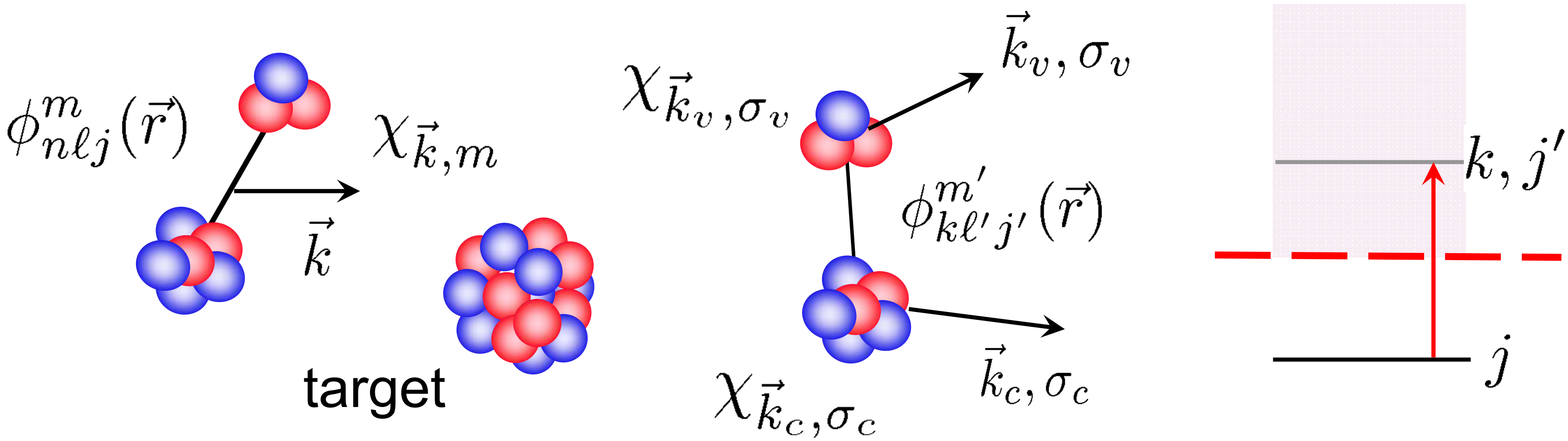


Inelastic excitations (bound to bound states) DWBA, Coulex

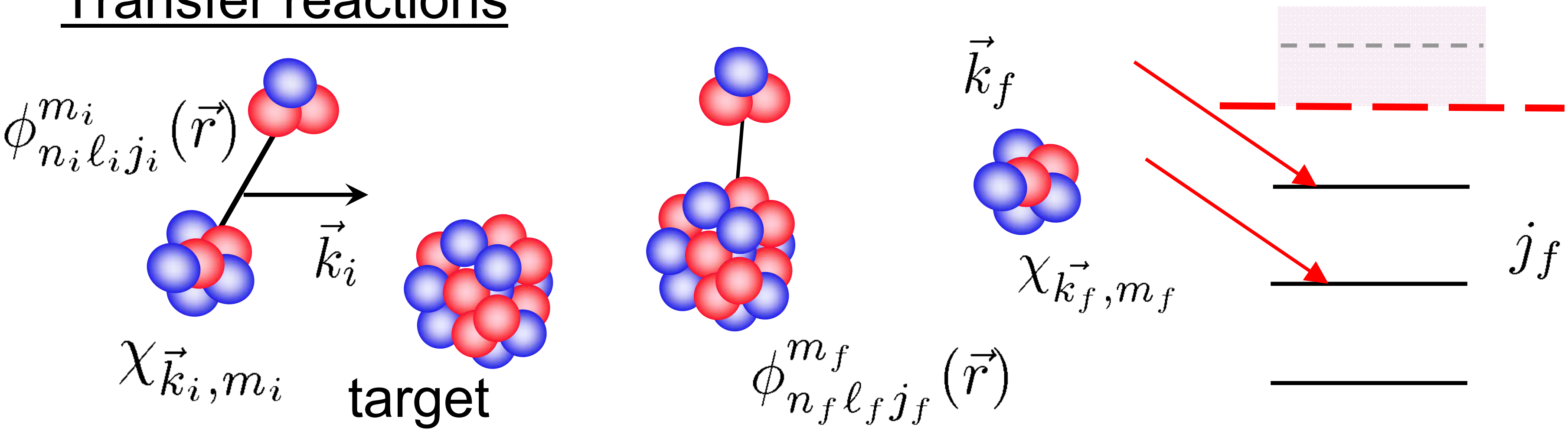


Direct reactions – types and characteristics

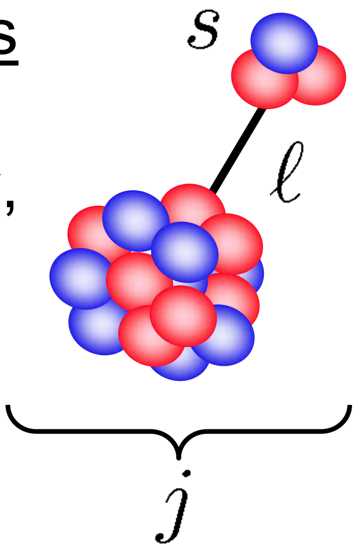
Inelastic excitations (breakup), Coulomb dissociation



Transfer reactions



Direct reactions – requirements (1)



Description of wave functions of **bound** systems (both nucleons or clusters) – (a) can take from structure theory, if available or, (b) more usually, use a real potential model to bind system with the required experimental separation energy.

Refer to core and valence particles

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\boxed{\frac{u_{n\ell j}(r)}{r}}Y^\lambda_\ell(\hat{r})\chi^\sigma_s, \quad \int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$

Usually just one or a few such states are needed.

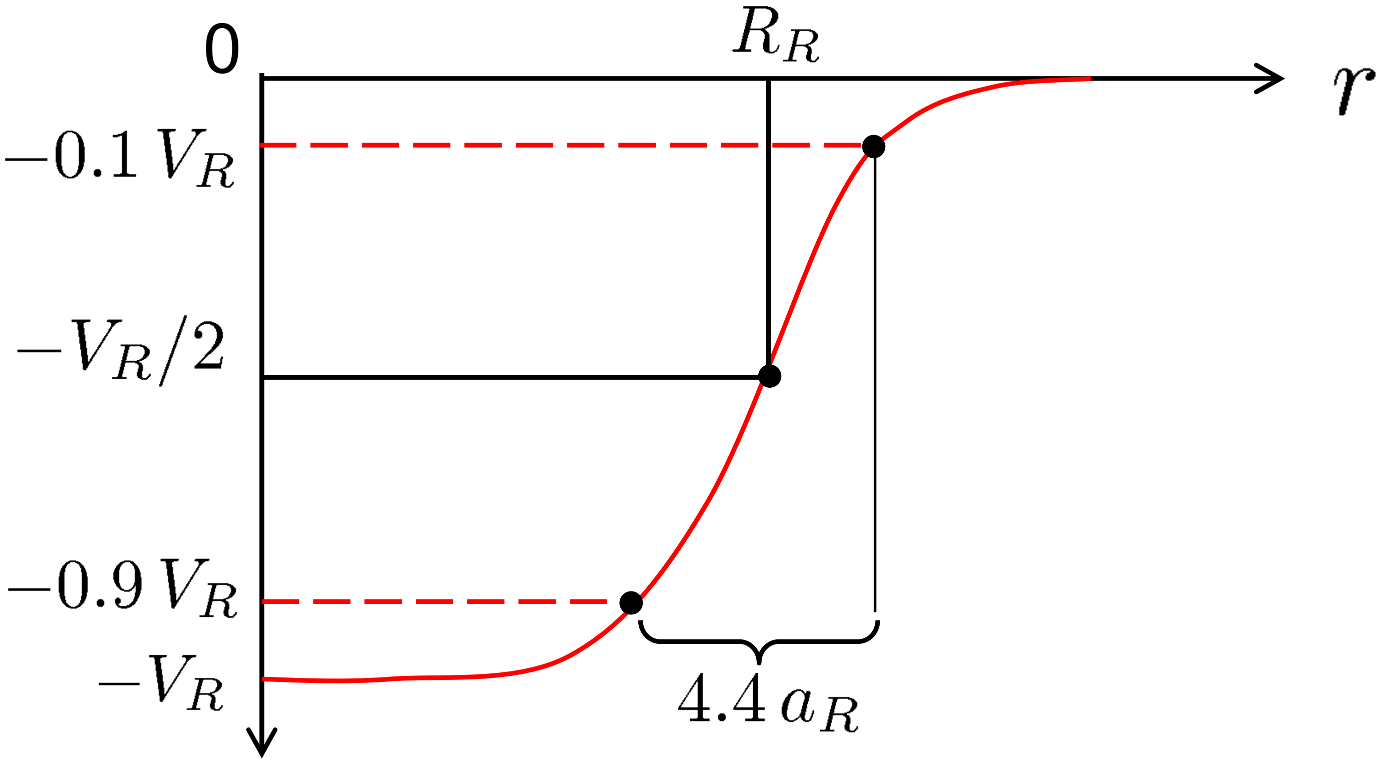
Separation energies/Q-values: many sites, e.g.

http://ie.lbl.gov/toi2003/MassSearch.asp

Bound states – real potentials

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\,, \qquad X_R = \frac{r - R_R}{a_R}$$



Bound states potential parameters - nucleons

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r - R_i}{a_i}$$

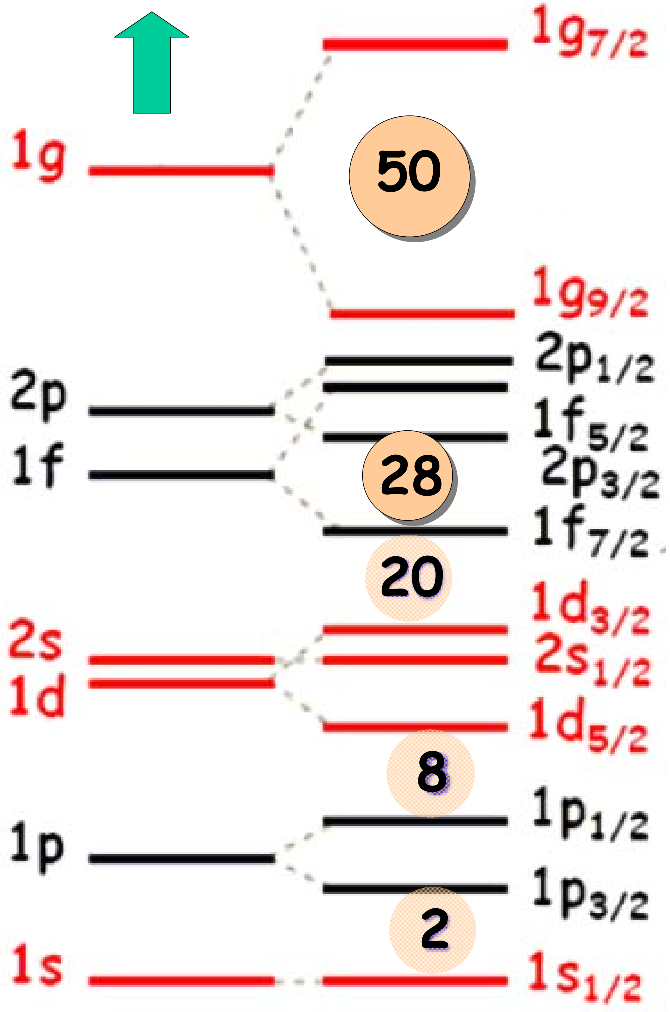
$$V_{so}(r) = -\frac{4\ V_{so}}{r\, a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

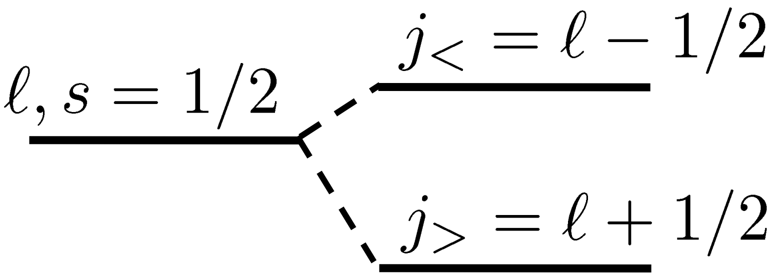
$$R_i = r_i\, A_c^{1/3}$$

$$r_R = r_C = r_{so} \approx 1.25\text{fm}$$

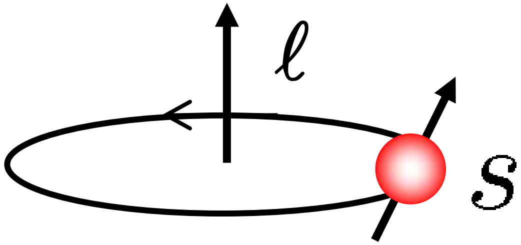
$$a_R = a_{so} \approx 0.7\text{fm} \quad V_{so} = 6\text{MeV}$$

Bound states – single particle quantum numbers





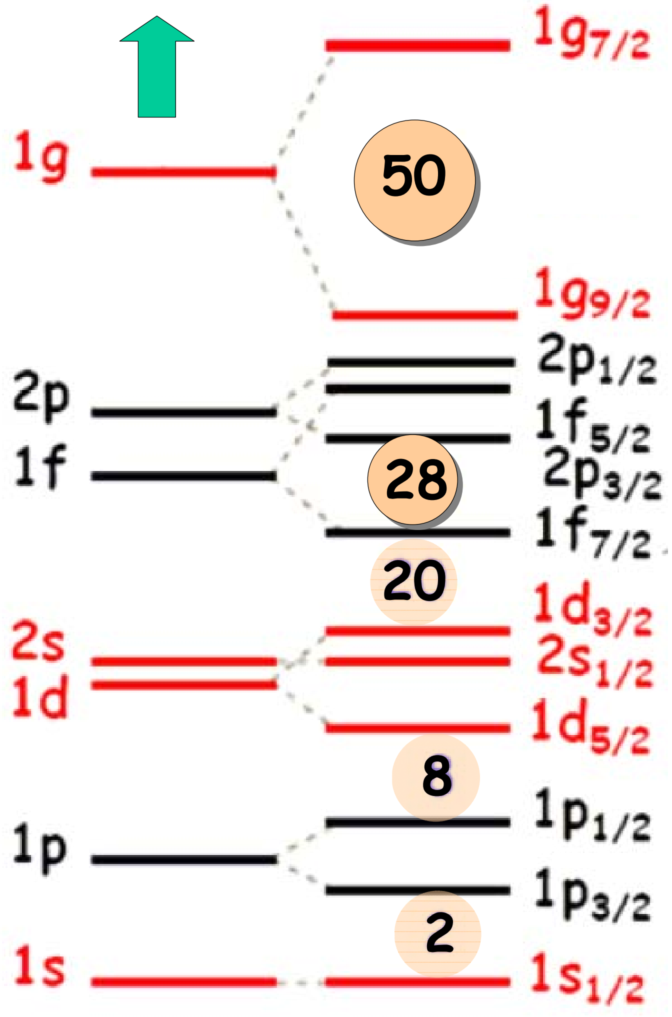
^{24}O?



$$V_{\ell s}(r)\vec{\ell} \cdot \vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell} \cdot \vec{s}$$
$$V_{so}(r) < 0$$

Bound states – for nucleons - conventions



Conventions $\phi^{m}_{n\ell j}(\vec{r})$

With this potential, and using sensible parameters, we will obtain the independent-particle shell model level orderings, shell closures with spin-orbit splitting.

NB: In diagram $2d_{5/2}$ means the second $d_{5/2}$ state. Defined this way, $n>0$ and $n-1$ is the number of nodes in the radial wave function. Reaction codes can ask for n, or n-1 (the actual number of nodes). Care is needed.

Bound states – can also use mean field information

```
                 ***********************
                 *                     *
INPUT VALUES     * IA,IZ =    24     8 *
                 *                     *
                 ***********************
```

----- Neutron bound state results -----

k	n	l	j	e	IE	OCC		
1	1	s	1/2	-26.757	1	2.00	36.70	35.28
2	1	p	3/2	-16.883	1	4.00	36.70	35.80
3	1	p	1/2	-12.396	1	2.00	36.70	36.04
4	1	d	5/2	-6.166	1	6.00	36.70	36.37
5	1	d	3/2	-0.109	1	0.00	36.70	36.69
6	2	s	1/2	-3.360	1	2.00	36.70	36.52
7	1	f	7/2	-0.200	3	0.00	46.02	46.01
8	1	f	5/2	-0.200	3	0.00	60.56	60.55
9	2	p	3/2	-0.200	3	0.00	48.10	48.09

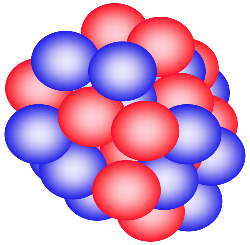
----- Neutron single-particle radii -----

				R(2)	R(4)	OCC	rho(8.9)	rho(9.9)	rho(10.9)
1	1	s	1/2	2.274	2.575	2.000	0.848E-09	0.706E-10	0.600E-11
2	1	p	3/2	2.863	3.133	4.000	0.188E-07	0.244E-08	0.325E-09
3	1	p	1/2	2.954	3.268	2.000	0.727E-07	0.122E-07	0.210E-08
4	1	d	5/2	3.434	3.757	6.000	0.524E-06	0.129E-06	0.327E-07
5	1	d	3/2	4.662	6.063	0.000	0.131E-04	0.675E-05	0.371E-05
6	2	s	1/2	4.172	4.895	2.000	0.769E-05	0.278E-05	0.102E-05
7	1	f	7/2	3.865	4.440	0.000	0.324E-05	0.134E-05	0.600E-06
8	1	f	5/2	3.890	4.477	0.000	0.341E-05	0.141E-05	0.631E-06
9	2	p	3/2	6.815	8.635	0.000	0.451E-04	0.270E-04	0.167E-04

But must make small corrections as HF is a fixed centre calculation

^{24}O

$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$

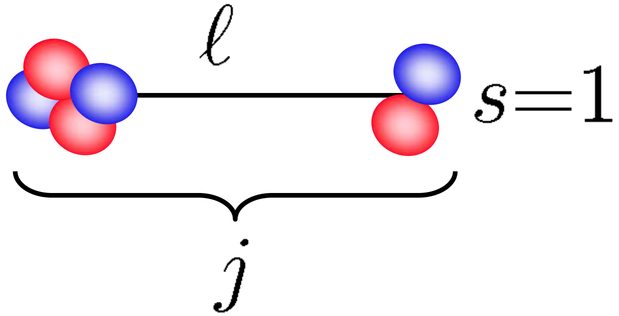


$^{24}O(g.s.)$

Direct reactions – requirements (2)

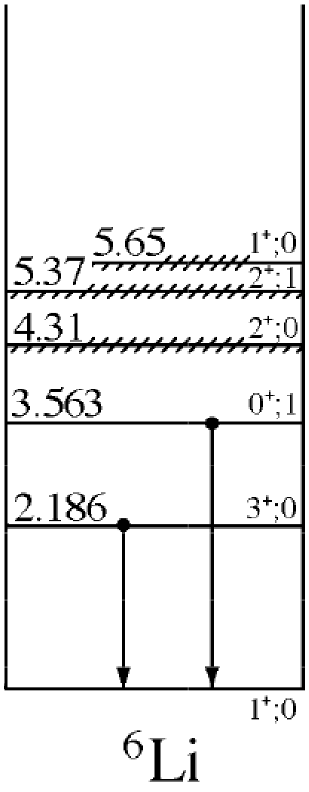
Description of wave functions for unbound (often light) systems (nucleons or clusters) with low relative energy: Usually have low nuclear level density of isolated resonances. Use the same real potential model as binds the system → scattering wave functions in this potential. (Also 'bin' wave functions)

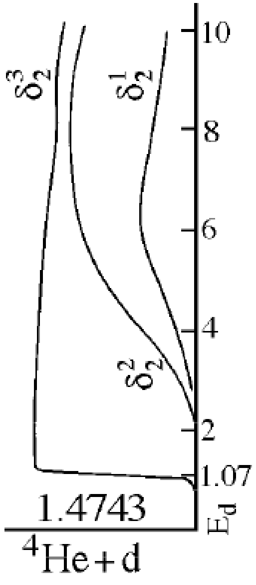
^{6}Li (α+d)



$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

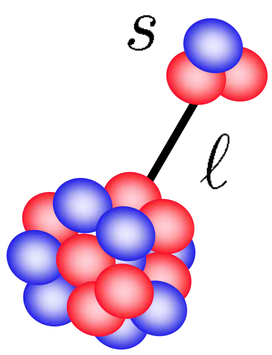
$$\phi^m_{k\ell j}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm)\frac{u_{k\ell j}(r)}{kr} Y^\lambda_\ell(\hat{r})\chi^\sigma_s$$





Completeness and orthogonality - technical piont

Given a fixed two-body Hamiltonian



$$H = T + U(r) = T + V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

the set of all of the bound and unbound wave functions

$$\{\phi^m_{n\ell j}(\vec{r}),\ \phi^m_{k\ell j}(\vec{r})\}$$

form a complete and orthogonal set, and specifically

$$\langle\phi^m_{n\ell j}(\vec{r})|\,\phi^m_{k\ell j}(\vec{r})\rangle = 0$$

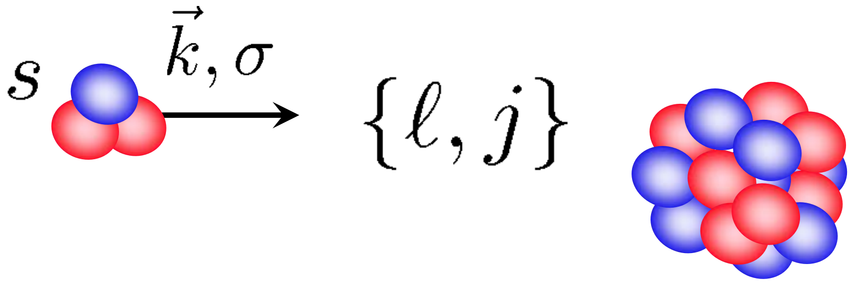
When including both bound to unbound states it is essential to use a fixed Hamiltonian for both the bound and unbound states (in each $\ell\, j$ channel) else we lose the orthogonality and the states will couple even without any perturbation or interactions with a reaction target.

Direct reactions – requirements (3)

Description of wave functions for scattering of nucleons or clusters from a heavier target and/or at higher energies: (a) high nuclear level density and broad overlapping resonances, (b) many open reaction channels, inelasticity and absorption. Use a complex (absorptive) optical model potential – from theory or ‘simply’ fitted to a body of elastic scattering data for a system and energy near that of interest.

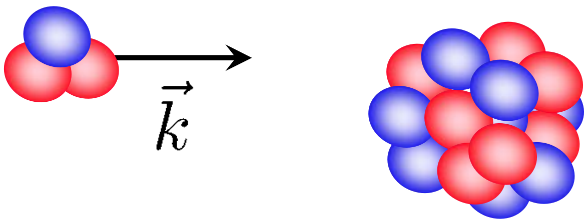
Distorted waves:

$$\chi_{\vec{k},\sigma}(\vec{r})$$



$$U(r) = V_C(r) + V(R) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

Optical potential – formal– Feshbach P's and Q's



Elastic channel $|\Psi_0\rangle$ describes motion when both projectile and target in their ground states

$$|\Psi\rangle = |\Psi_0\rangle + |\Psi_1\rangle + |\Psi_2\rangle \ldots = |\Psi_0\rangle + |\Psi_{in}\rangle$$

$$H|\Psi\rangle = E|\Psi\rangle \qquad P|\Psi\rangle = |\Psi_0\rangle \qquad Q|\Psi\rangle = |\Psi_{in}\rangle$$

Orthogonality of states of H: $PQ = QP = 0,\ P + Q = 1$

P and Q are <u>projection</u> operators: $PP = P,\ QQ = Q$

$$H(P+Q)|\Psi\rangle = E(P+Q)|\Psi\rangle$$

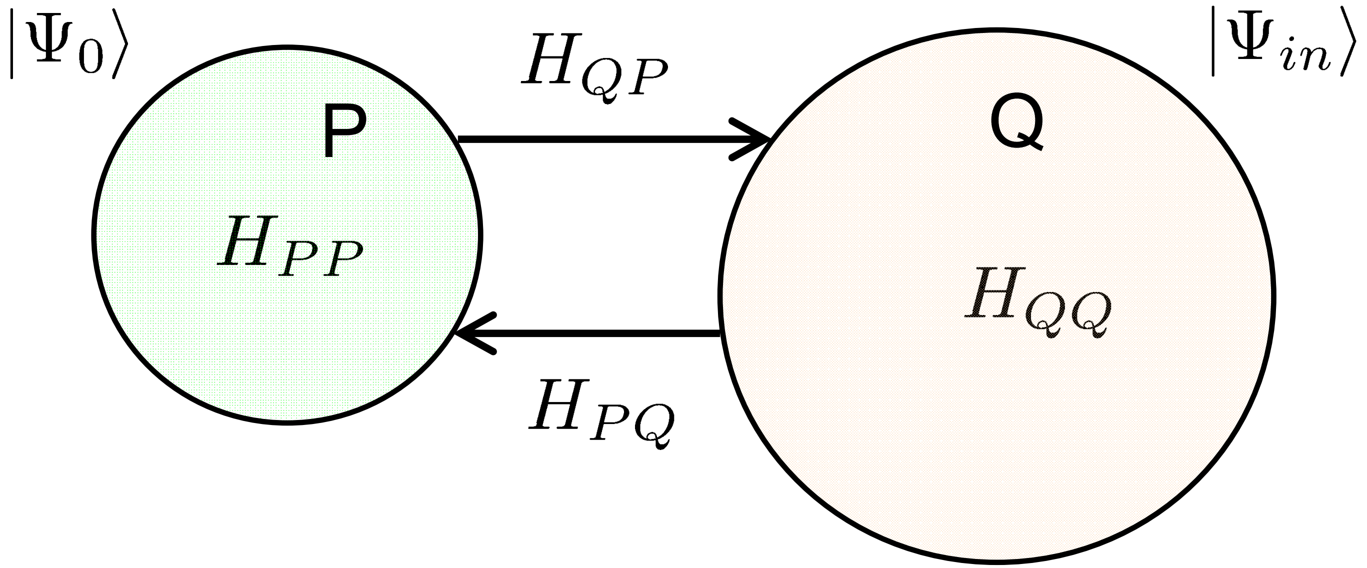
$$PH(P+Q)|\Psi\rangle = PE(P+Q)|\Psi\rangle = EP|\Psi\rangle = E|\Psi_0\rangle$$

$$QH(P+Q)|\Psi\rangle = QE(P+Q)|\Psi\rangle = EQ|\Psi\rangle = E|\Psi_{in}\rangle$$

Optical potential – formal – Feshbach P's and Q's

$$[E - PHP]|\Psi_0\rangle = PHQ|\Psi_{in}\rangle$$

$$[E - QHQ]|\Psi_{in}\rangle = QHP|\Psi_0\rangle$$

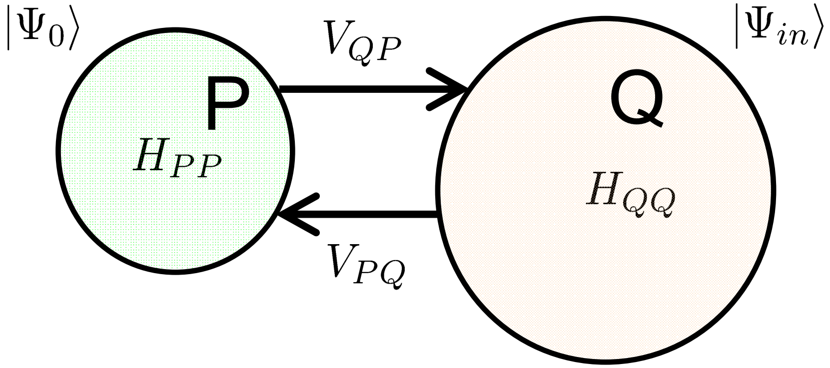


$$H_{PP} = PHP = T + PVP = T + V_{PP}, \text{ etc.}$$

Optical potential – formal – Feshbach P's and Q's

$$[E - T - V_{PP}]|\Psi_0\rangle = V_{PQ}|\Psi_{in}\rangle$$

$$[E^{(+)} - T - V_{QQ}]|\Psi_{in}\rangle = V_{QP}|\Psi_0\rangle$$

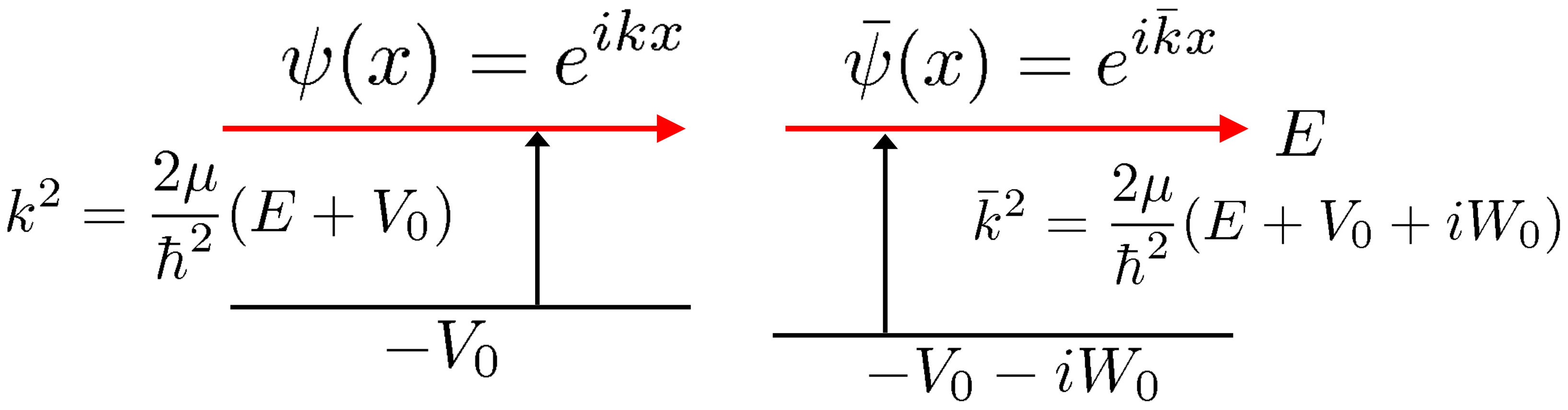


$$|\Psi_{in}\rangle = [E^{(+)} - T - V_{QQ}]^{-1} V_{QP}|\Psi_0\rangle$$

$$[E - T - V_{PP}^{opt}]|\Psi_0\rangle = 0$$

$$V_{PP}^{opt} = V_{PP} + V_{PQ}[E^{(+)} - T - V_{QQ}]^{-1} V_{QP}$$

Optical potentials –imaginary part – mean free path



$$\bar{k}^2 = \frac{2\mu}{\hbar^2}(E + V_0 + iW_0) = \frac{2\mu}{\hbar^2}(E + V_0)\left[1 + \frac{iW_0}{E + V_0}\right]$$

$$\bar{k} = k\left[1 + \frac{iW_0}{E + V_0}\right]^{1/2} \approx k\left[1 + \frac{iW_0}{2(E + V_0)}\right], \quad W_0 \ll E, V_0$$

So, for $W_0 > 0$, $\bar{k} = k + ik_i/2$, $k_i = kW_0/(E + V_0) > 0$,

$$\bar{\psi}(x) = e^{i\bar{k}x} = e^{ikx}e^{-\frac{1}{2}k_i x}, \qquad |\bar{\psi}(x)|^2 = e^{-k_i x}$$

Optical potentials - parameter conventions

$$U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

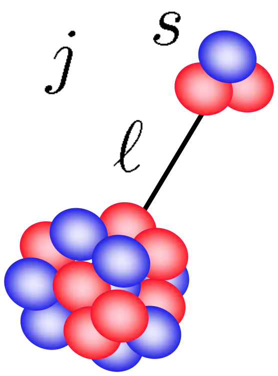
$$V_{so}(r) = -\frac{4V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

usual conventions

$$W(r) = -\frac{W_V}{[1+\exp(X_V)]} - \frac{4W_S\exp(X_S)}{[1+\exp(X_S)]^2}\ ,$$

$$R_i = r_i\,A_2^{1/3} \quad \text{or} \quad R_i = r_i\left[A_1^{1/3} + A_2^{1/3}\right]$$

The Schrodinger equation (1)



So, using usual notation

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\phi_{\ell j}^m(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

and defining $\phi_{\ell j}^m(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{\ell j}(r)}{r}Y_\ell^\lambda(\hat{r})\chi_s^\sigma$

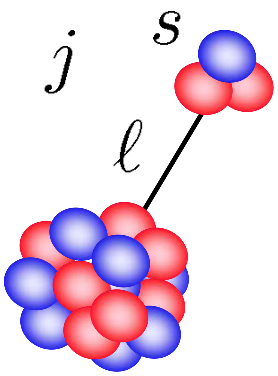
$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right)u_{\ell j}(r) = 0$$

bound states $E_{cm} < 0$ scattering states $E_{cm} > 0$

With $U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$

$$\begin{aligned} U_{\ell j}(r) &= V_C(r) + V(r) + iW(r) \\ &+ V_{so}(r)[j(j+1) - \ell(\ell+1) - s(s+1)]/2 \end{aligned}$$

The Schrodinger equation (2)



Must solve

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right) u_{\ell j}(r) = 0$$

<u>bound states</u> $E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

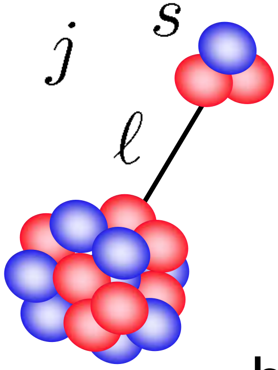
Discrete spectrum

<u>scattering states</u> $E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

Continuous spectrum

Large r: The Asymptotic Normalisation Coefficient



Bound states

$$E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

but beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta_b\kappa_b}{r} - \kappa_b^2\right) u_{n\ell j}(r) = 0, \quad \eta_b = \frac{\mu Z_c Z_v e^2}{\hbar\kappa_b}$$

$$u_{n\ell j}(r) \rightarrow C_{\ell j} W_{-\eta_b,\ell+1/2}(2\kappa_b r) \underset{r\rightarrow\infty}{\longrightarrow} C_{\ell j}\exp(-\kappa_b r)$$

Whittaker function

$C_{\ell j}$ ANC completely determines the wave function outside of the range of the nuclear potential – only requirement if a reaction probes only these radii

Large r: The phase shift and partial wave S-matrix

Scattering states

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr),\ G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\begin{aligned} u_{k\ell j}(r) &\rightarrow e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right] \\ &\rightarrow (i/2)\left[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)\right] \end{aligned}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm i F_\ell(\eta, kr)$$

Phase shift and partial wave S-matrix

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}} \underbrace{[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)]}$$

If *U(r)* is real, the phase shifts $\delta_{\ell j}$ are real, and [...] also

$$u_{k\ell j}(r) \quad \rightarrow \quad (i/2)[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)]$$

$S_{\ell j} = e^{2i\delta_{\ell j}}$

Ingoing waves — outgoing waves

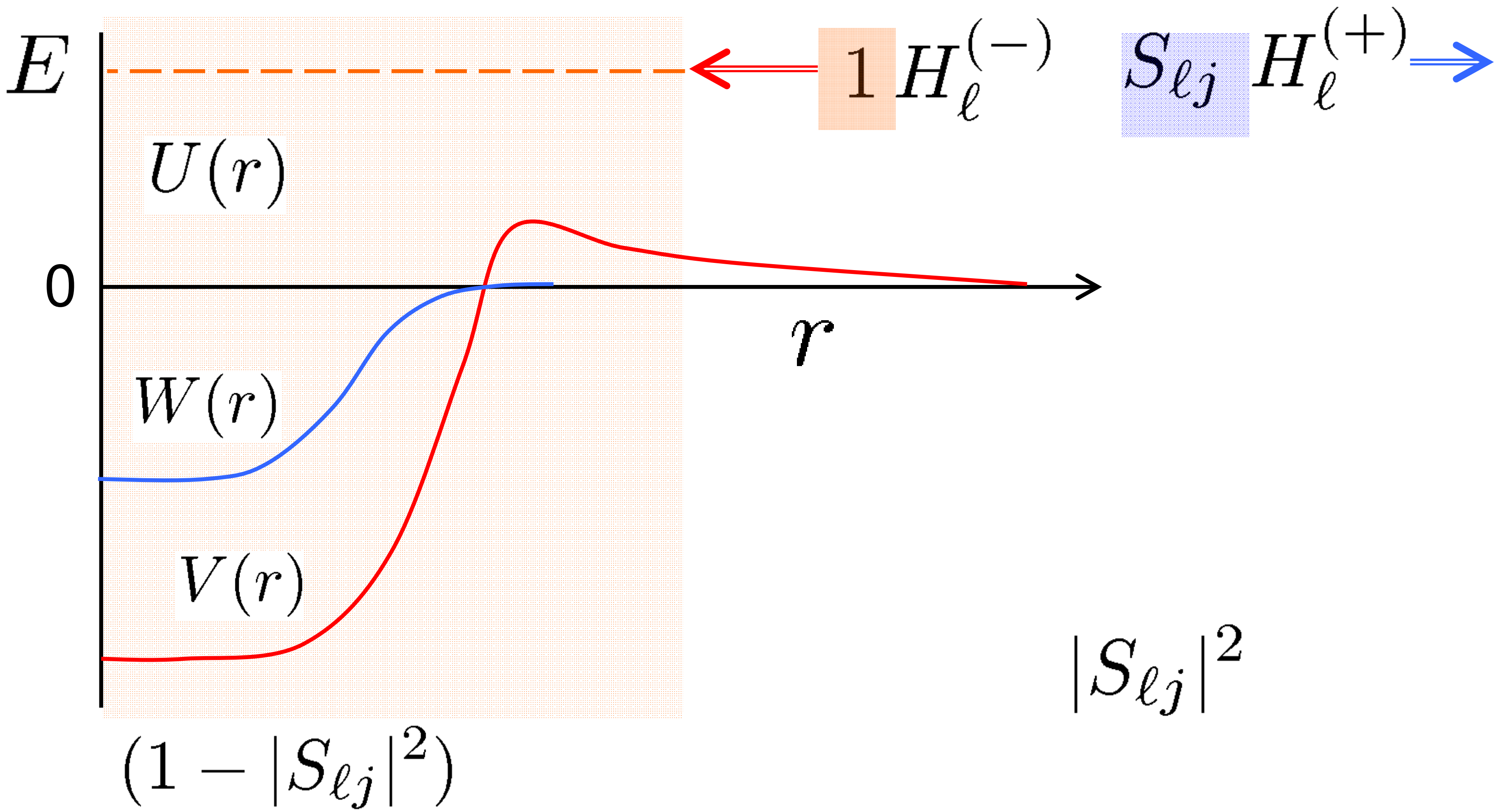
$|S_{\ell j}|^2$ survival probability in the scattering

$(1 - |S_{\ell j}|^2)$ absorption probability in the scattering

Having calculate the phase shifts and the partial wave S-matrix elements we can then compute all scattering observables for this energy and potential (but later).

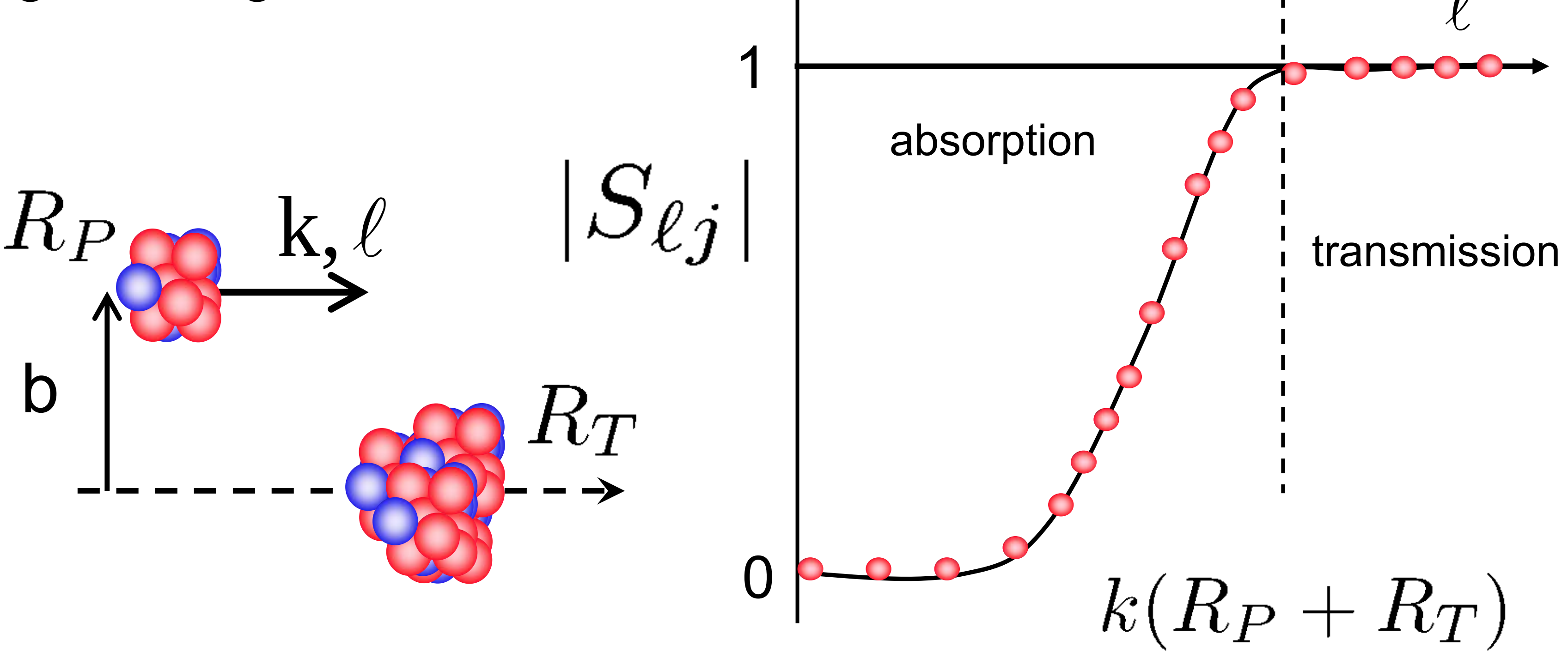
Ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[1\, H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$



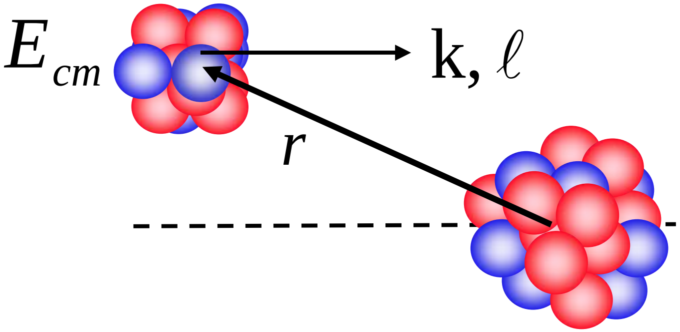
Semi-classical approximations – many ℓ-values

large k – e.g. nucleus-nucleus



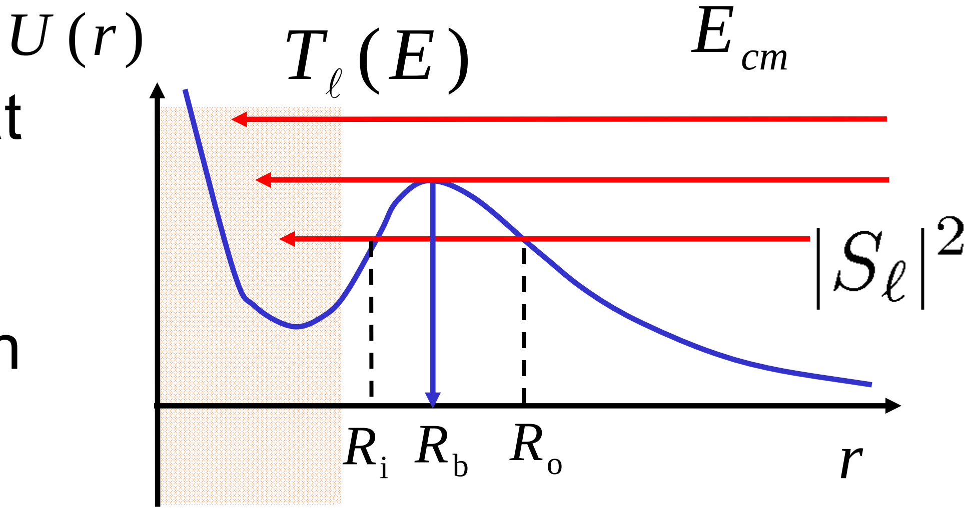
$$\text{semi}-\text{classical}: \ S(b), \quad \ell = kb$$

Barrier passing models of fusion (in Part II)



Gives basis also for simple (barrier passing) models of nucleus-nucleus fusion reactions

an imaginary part in *U(r)*, at short distances, can be included to absorb all flux that passes over or through the barrier – assumed to result in fusion



$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-|S_\ell|^2)$$

Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or semi-classical (impact parameter) representation, for example (spinless case):

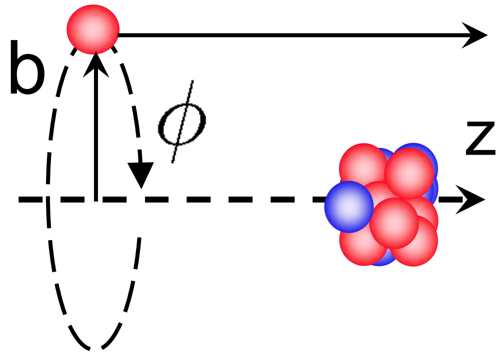
$$\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\;|1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\;(1-|S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el}+\sigma_R = 2\int d^2\vec{b}\;[1-\mathrm{Re}.S(b)]$$ etc.

and where (cylindrical coordinates)

$$\int d^2\vec{b} \equiv \int_0^\infty bdb\int_0^{2\pi}d\phi = 2\pi\int_0^\infty bdb$$



Elastic scattering determines only the asymptotics

Fitting elastic scattering data can determine a set of S_ℓ (not without ambiguity) that reproduce the cross section angular distribution – but not the wave function at the nuclear surface

$$\frac{d\sigma_{el}}{d\Omega} = |f_{el}(\theta)|^2 \ , \ f_{el}(\theta) = f_C(\theta) + f_n(\theta)$$

$$f_n(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)e^{2i\sigma_\ell(\eta)}[S_\ell^n - 1]P_\ell(\cos\theta)$$

Wave functions are obtained by using theoretically-motivated potential shapes and forms, calculating the S_ℓ, and adjusting parameters iteratively – there is potential ambiguity - always

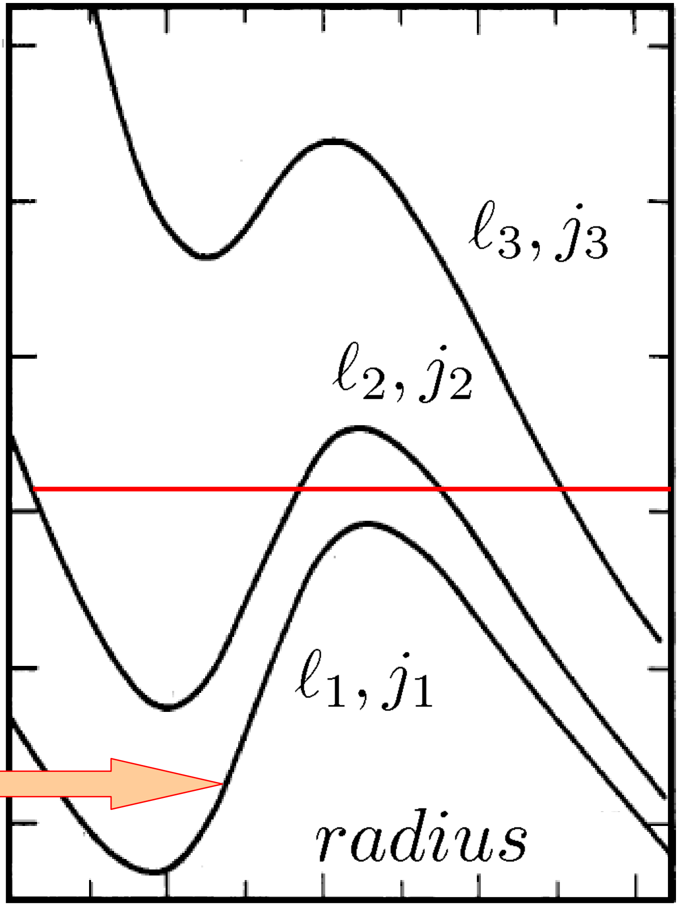
$$u_{k\ell}(r) \quad \rightarrow \quad (i/2)[H_\ell^{(-)}(\eta, kr) - S_\ell H_\ell^{(+)}(\eta, kr)]$$

… but interactions should be cross referenced against available differential and reaction cross section data.

Phase shifts and S-matrix: Resonant behaviour

In real potentials, at low energies, the combination of an attractive nuclear, repulsive Coulomb and centrifugal terms can lead to potential pockets and resonant behaviour – the system being able to trapped in the pocket for some (life)time τ.

$$\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} + U_{\ell j}(r)$$

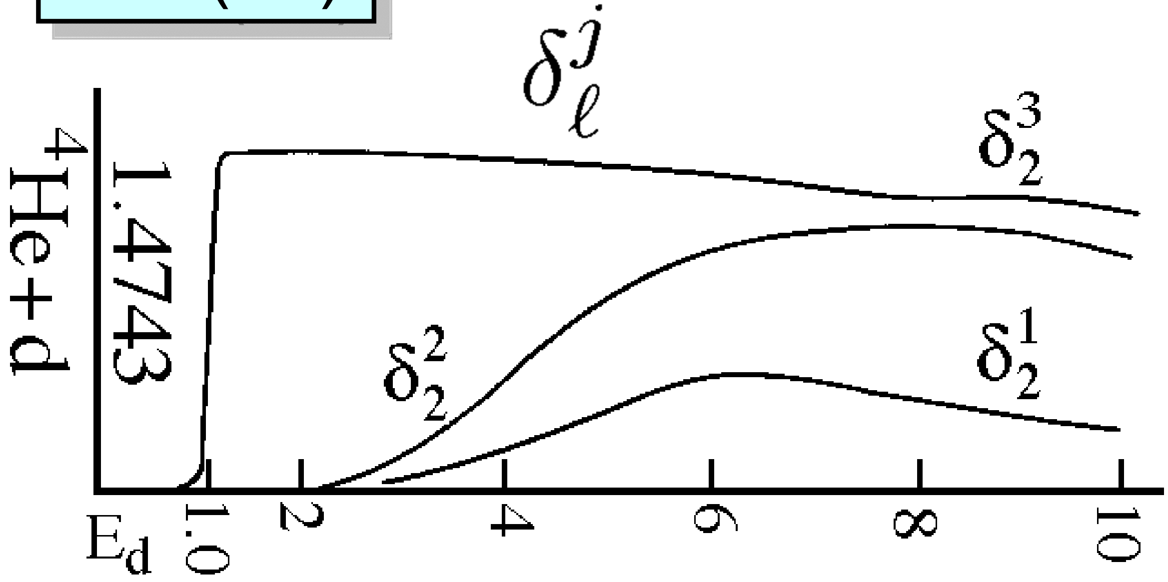


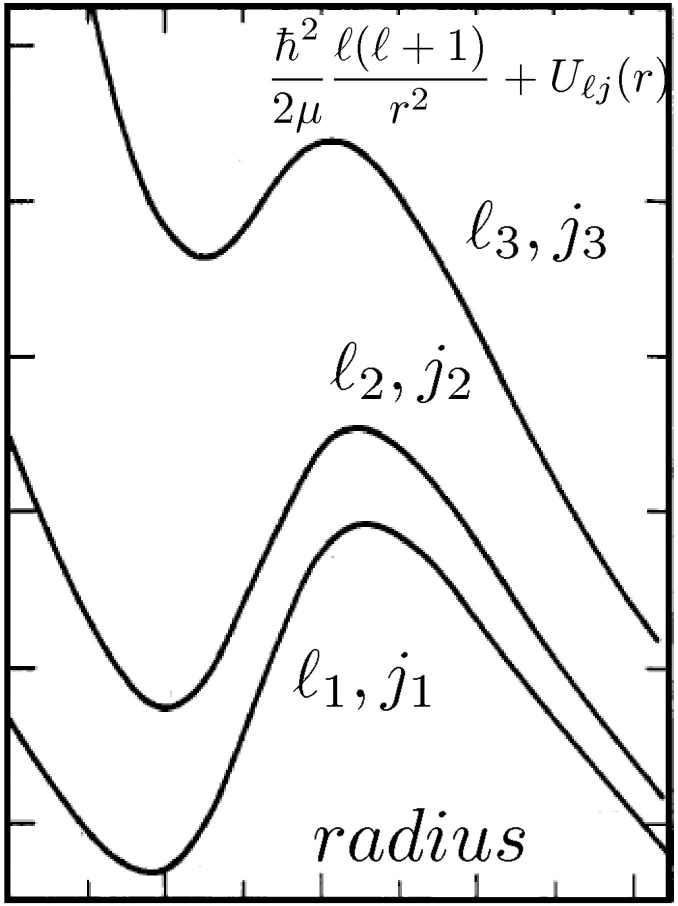
Phase shifts and S-matrix: Resonant behaviour

Potential pockets can lead to resonant behaviour – the system being able to trapped in the pocket for some (life)time τ.

A signal is the rise of the phase shift through 90 degrees.

α+d (^{6}Li)



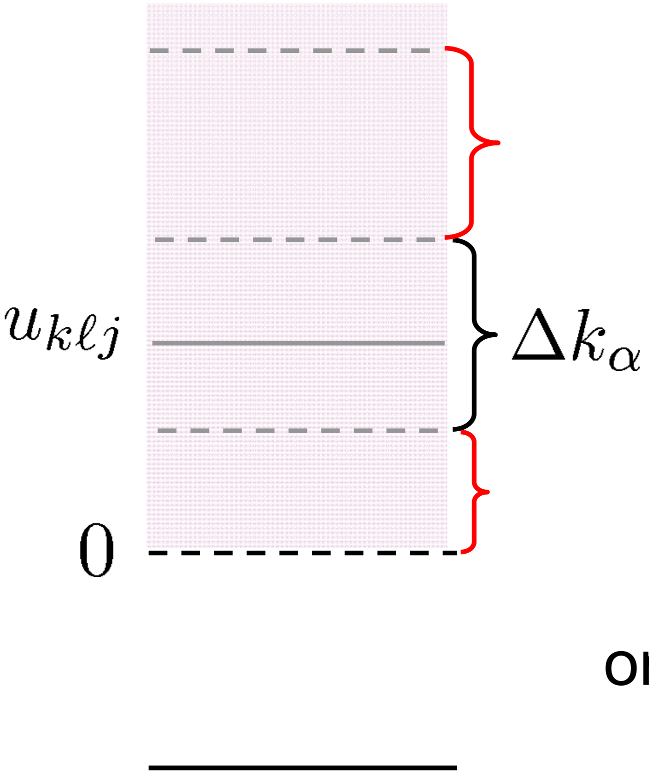


Potential parameters should describe any known resonances

Neither bound nor scattering – continuum bins

Scattering states

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right]$$



$$\int_0^\infty dr\, u_{k\ell j}(r)\, u^*_{k'\ell j}(r) = \frac{\pi}{2}\delta(k - k')$$

$$\hat{u}_{\alpha\ell j}(r) = \sqrt{\frac{2}{\pi N_\alpha}} \int_{\Delta k_\alpha} dk\, g(k)\, u_{k\ell j}(r)$$

$$N_\alpha = \int_{\Delta k_\alpha} dk\, [g(k)]^2$$

weight function

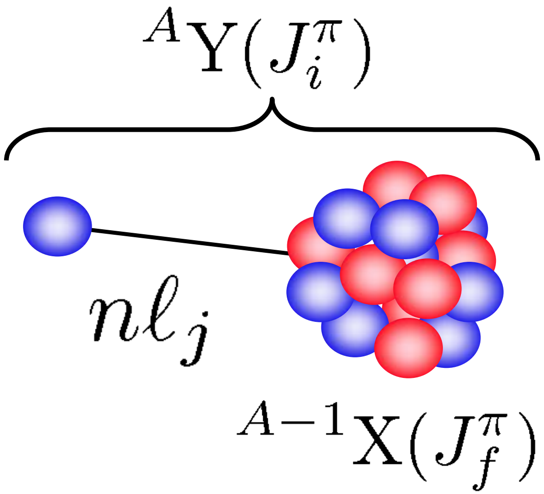
orthonormal set

$$\int_0^\infty dr\, \hat{u}^*_{\alpha\ell j}(r)\, \hat{u}_{\beta\ell j}(r) = \delta_{\alpha\beta}$$

$$g(k) = 1 \qquad g(k) = \sin\delta_{\ell j}$$

Bound states – spectroscopic factors

In a potential model it is natural to define <u>normalised</u> bound state wave functions.



$$\phi_{n\ell j}^{m}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r} Y_{\ell}^{\lambda}(\hat{r})\chi_s^{\sigma},$$

$$\int_0^{\infty} [u_{n\ell j}(r)]^2 dr = 1$$

The potential model wave function approximates the overlap function of the A and A–1 body wave functions (A and A–n in the case of an n-body cluster) i.e. the overlap

$$\langle \ell j, \vec{r}, {}^{A-1}\mathrm{X}(J_f^{\pi}) | {}^{A}\mathrm{Y}(J_i^{\pi})\rangle \rightarrow I_{\ell j}(r), \quad \int_0^{\infty} [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

S(…) is a spectroscopic factor, that scales the normalised single-particle wave function/overlap/form-factor

Connection to many-body structure calculations (1)

$$\langle \alpha, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^{A}\mathrm{Y}(J_i^\pi) \rangle$$

If we describe many body states by single Slater determinants, since these must be antisymmetric

$$\langle 1 \ldots A | {}^{A}\mathrm{Y} \rangle \equiv \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_1(1) & \phi_2(1) & \ldots & \phi_A(1) \\ \phi_1(2) & \phi_2(2) & \ldots & \phi_A(2) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_1(A) & \phi_2(A) & \ldots & \phi_A(A) \end{vmatrix}$$

then, for A identical particles (isospin) [or if (n,p), then N or Z]

$$\langle \alpha, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^{A}\mathrm{Y}(J_i^\pi) \rangle = \frac{1}{\sqrt{A}} \phi_\alpha(\vec{r})$$

The *A* factor is not usually carried: it cancels in cross sections that have an *A* multiplier to account for each identical particle.

Connection to many-body structure calculations (2)

$$\langle \alpha, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^A\mathrm{Y}(J_i^\pi) \rangle = \frac{1}{\sqrt{A}} \phi_\alpha(\vec{r})$$

Here the radial wave function (form factor) is normalised. In a reaction that removes a nucleon from a given orbital then, if a sub-shell is filled in the initial nucleus there are (2j+1) nucleons available with a given (j,ℓ) to contribute.

So, more generally (non-single Slater determinant) many-body structure models calculate and provided overlaps as:

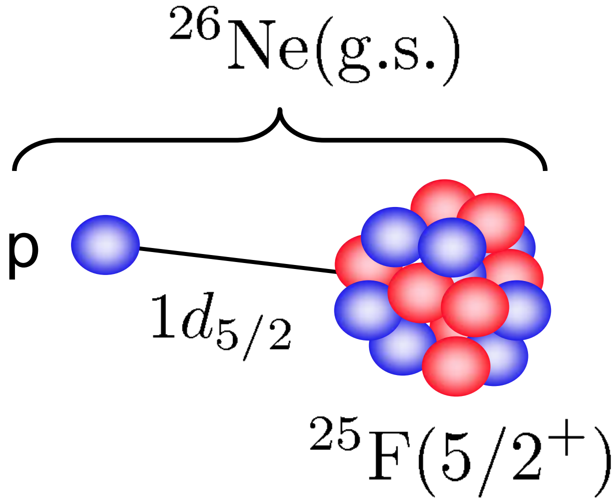
$$\langle j\ell, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^A\mathrm{Y}(J_i^\pi) \rangle = \frac{\sqrt{S(J_i, J_f j\ell)}}{\sqrt{A}} \phi_{j\ell m}(\vec{r})$$

So, *S* multiplies the cross section calculated with a normalised form-factor. The *S* are defined so that (for given n j,ℓ quantum numbers) their sum over final states is the number of nucleons occupying the given sub-shell (sum-rule).

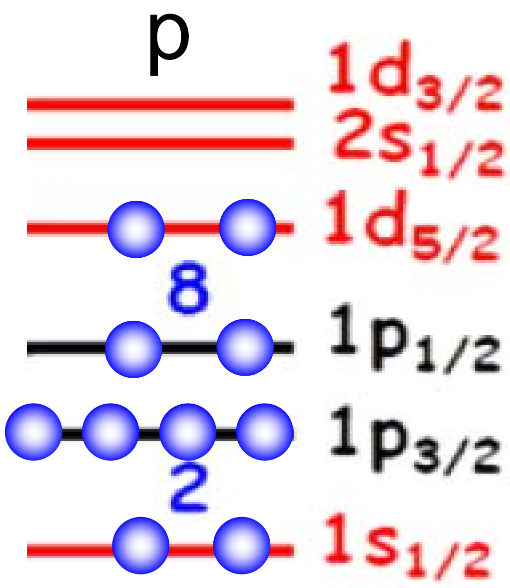
Bound states – shell model overlaps

$$\langle \vec{r}, {}^{25}\mathrm{Ne}(5/2^+, E^*) | {}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.}) \rangle$$

USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides spectroscopic factors but **not** the bound state radial wave function.

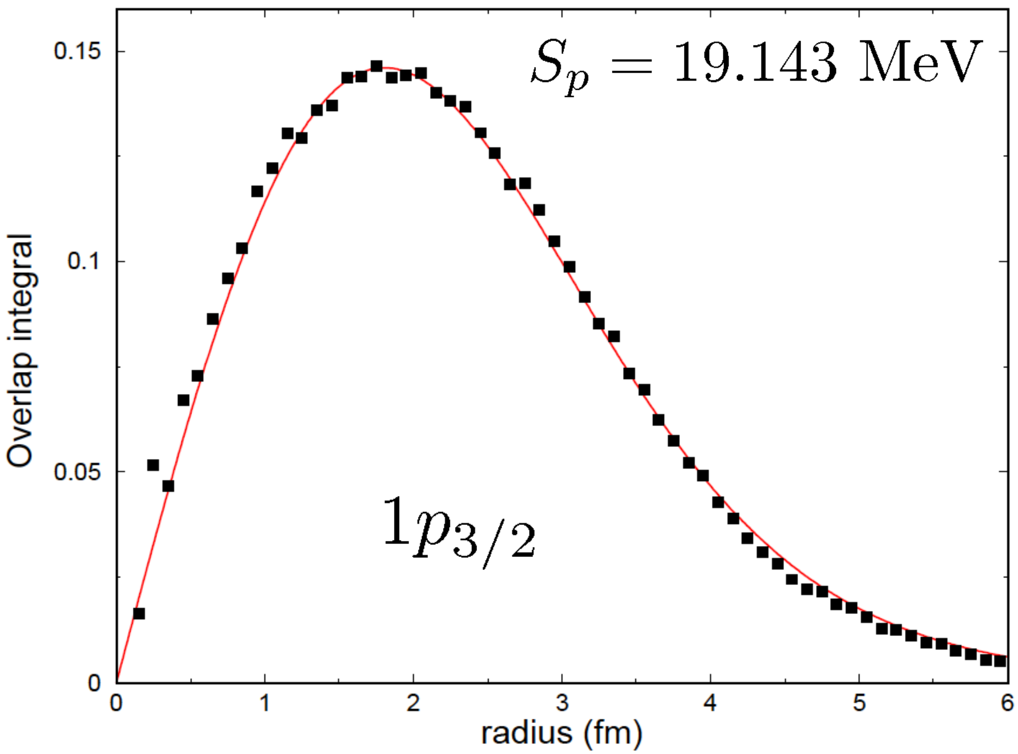


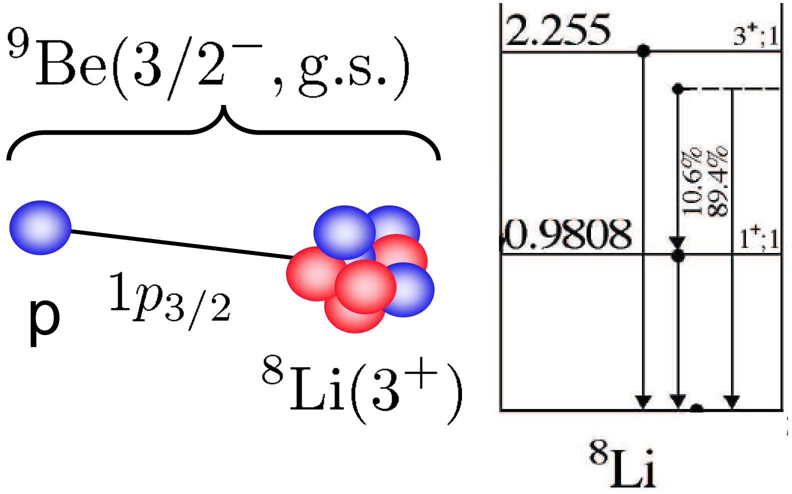
-- core state ---				- overlap state -				(1 2 5)(
2j2t	p	n	e	2j2t	p	n	e	s
			-59.414				-81.625	
5 7	+	1	0.000	0 6	+	1	0.000	1.79039
5 7	+	2	3.756	0 6	+	1	0.000	0.02316
5 7	+	3	4.799	0 6	+	1	0.000	0.01084
5 7	+	4	5.631	0 6	+	1	0.000	0.00012
5 7	+	5	6.022	0 6	+	1	0.000	0.00589
5 7	+	6	6.504	0 6	+	1	0.000	0.00044
5 7	+	7	6.796	0 6	+	1	0.000	0.00002
5 7	+	8	8.034	0 6	+	1	0.000	0.00006
5 7	+	9	8.186	0 6	+	1	0.000	0.00097
5 7	+	10	8.398	0 6	+	1	0.000	0.00006
total =								1.83196
centroid =			0.102	centroids =				0.000
centroid*=			-22.313	centroids =				-22.211



Bound states – microscopic overlaps for light nuclei

$\langle \vec{r}, {}^8\mathrm{Li}(3^+) | {}^9\mathrm{Be}(3/2^-, \mathrm{g.s.}) \rangle$





- Microscopic overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a several cases: at

http://www.phy.anl.gov/theory/research/overlap/

Normalised bound state in Woods-Saxon potential well x (0.23)$^{1/2}$ Spectroscopic factor

$$r_V = r_{so} = \text{fitted},\ a_V = a_{so} = \text{fitted},\ V_{so} = 6.0$$

Bound states – for clusters – conventions (1)

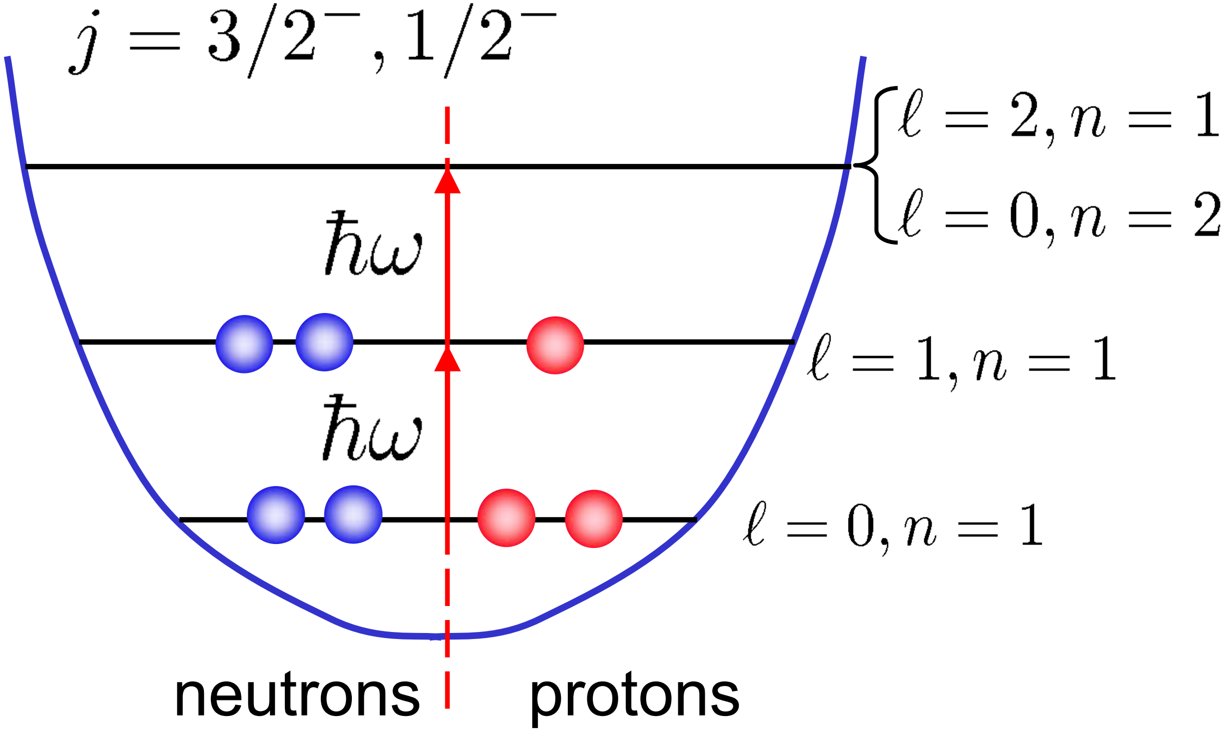
How many nodes for cluster states ?

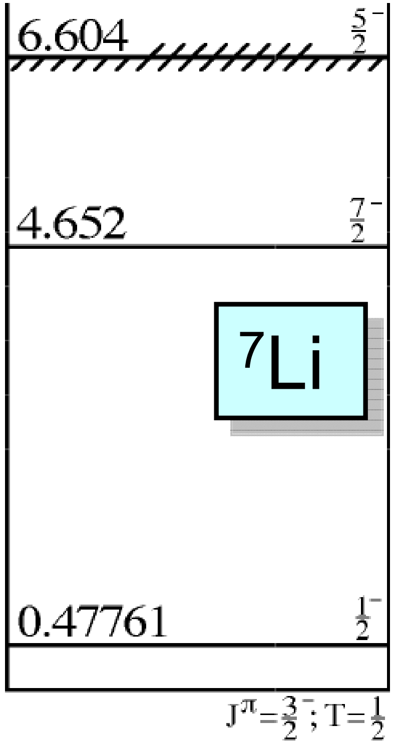
$$\phi^{m}_{n\ell j}(\vec{r})$$

Usually guided by what the 3D harmonic oscillator potential requires - so as not to violate the Pauli Principle.

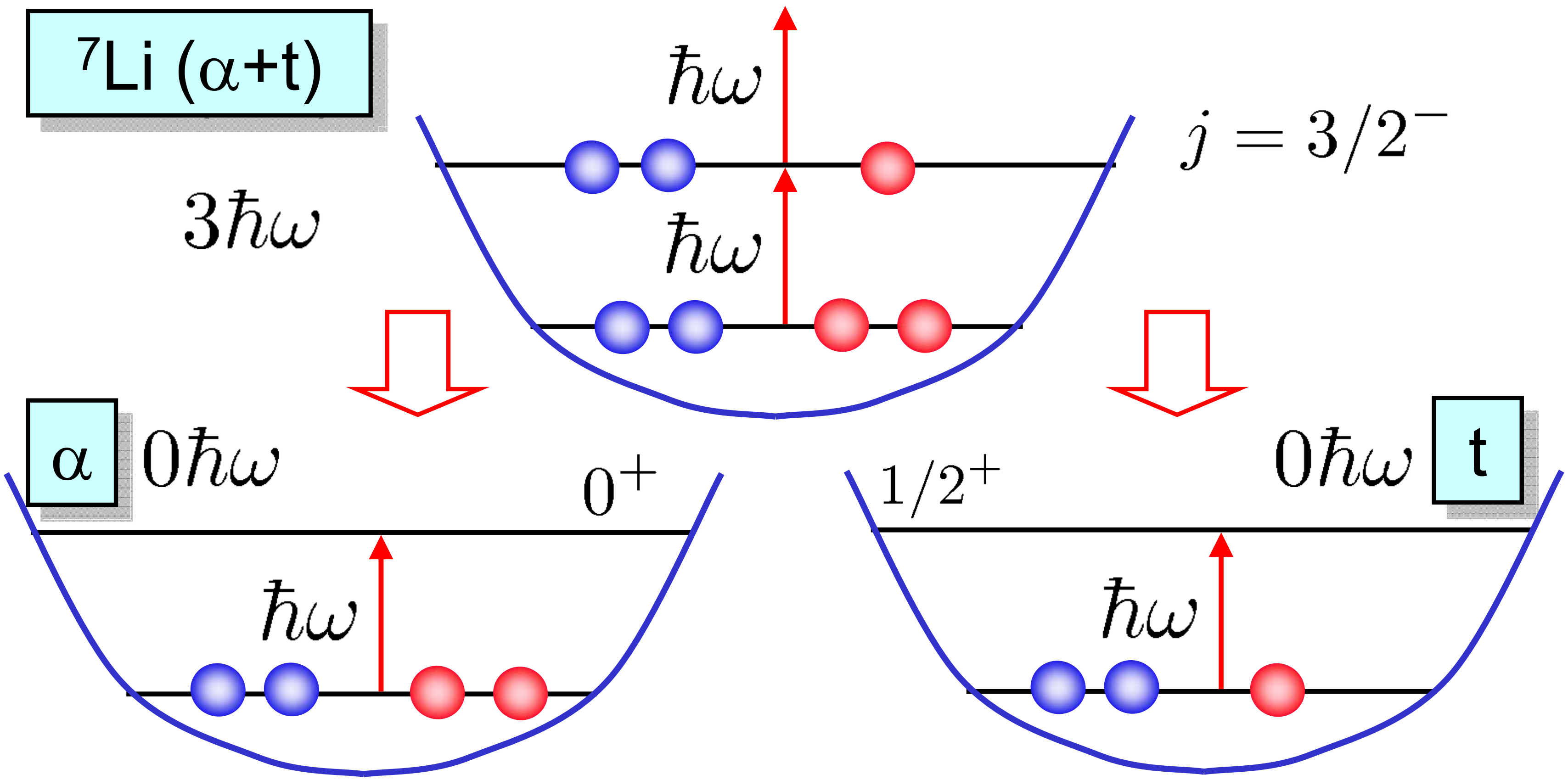
^{7}Li (α+t)

$$[2(n-1)+\ell\,]\hbar\omega \left\{ \text{excitation due to a nucleon each level} \right.$$





Bound states – for clusters - conventions (2)



$$[2(N-1)+L\,]\hbar\omega = 3\hbar\omega$$

⇒ must be associated with the α+t relative motion

$L = 1, N = 2$: $j = 3/2^-, 1/2^-,$

$L = 3, N = 1$: $j = 7/2^-, 5/2^-$