Direct reactions at low energies: Part II – Interactions and couplings

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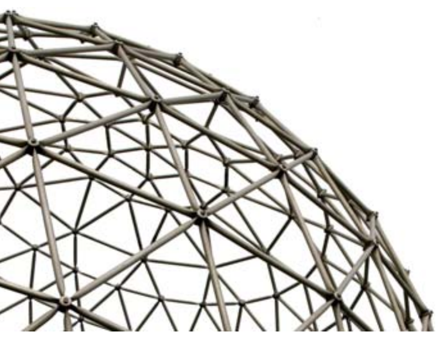
Jeff Tostevin, NSCL, MSU, East Lansing, MI and
Department of Physics, Faculty of Engineering and
Physical Sciences University of Surrey, UK



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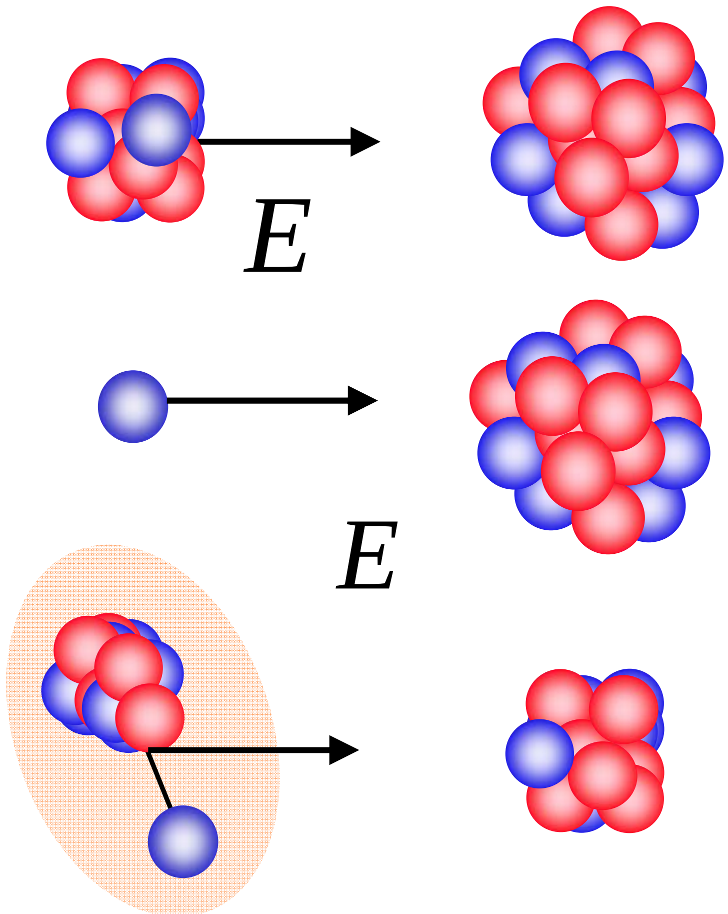
Interactions of composite systems

Reactions require us to make *educated* assumptions about the interactions between a wide variety of systems at different energies – that describe the elastic/inelastic mix

nucleus-nucleus

nucleon-nucleus

Cluster-nucleus



Can involve nucleus -nucleus and nucleon -nucleus depending on the clusters, e.g. for ^{11}Be halo nucleus

Elastic scattering determines only the asymptotics

$$u_{k\ell}(r) \quad \rightarrow \quad (i/2)[H_\ell^{(-)}(\eta, kr) - S_\ell H_\ell^{(+)}(\eta, kr)]$$

Fitting elastic scattering data can determine a set of S_ℓ (not without ambiguity) that reproduce the cross section angular distribution – but <u>not</u> the wave function at the nuclear surface

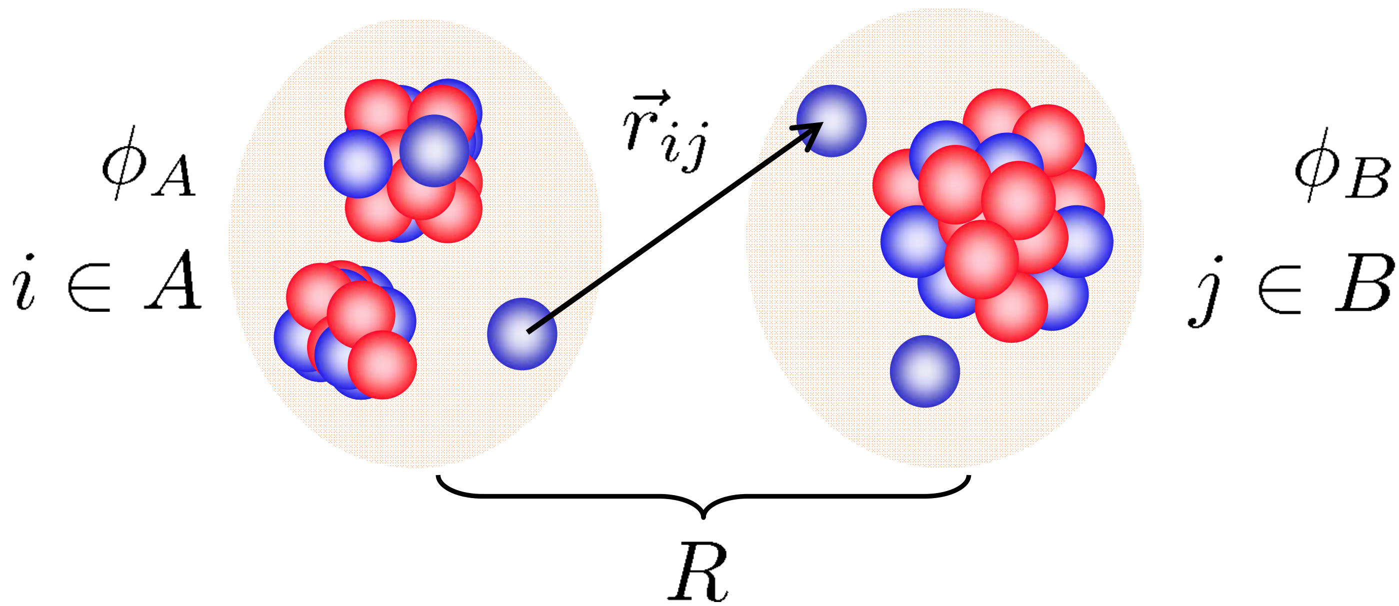
$$\frac{d\sigma_{el}}{d\Omega} = |f_{el}(\theta)|^2 \ , \ f_{el}(\theta) = f_C(\theta) + f_n(\theta)$$

$$f_n(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell+1) e^{2i\sigma_\ell(\eta)} [S_\ell^n - 1] P_\ell(\cos\theta)$$

Wave functions are obtained by using theoretically-motivated potential shapes and forms, calculating the S_ℓ, and adjusting parameters iteratively – there is potential ambiguity - <u>always</u>

Folding models are a general procedure

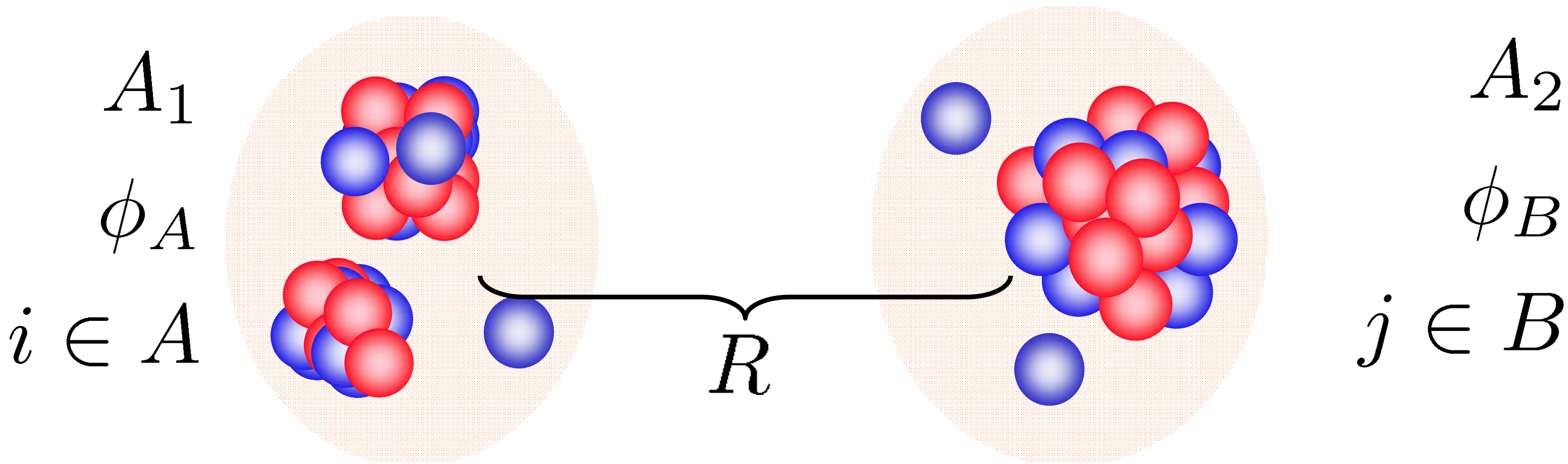
Diagonal interactions



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} V_{ij}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

Pair-wise interactions integrated (averaged) over the internal motions of the two composites – like interaction between two extended charge denstities

Double folding models – useful identities



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} v_{NN}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

$$J_{V_F} = A_1\, A_2\, J_{v_{NN}}$$

$$\langle r^2 \rangle_{V_F} = \langle r^2 \rangle_A + \langle r^2 \rangle_B + \langle r^2 \rangle_{v_{NN}}$$

$$J_f = \int d\vec{r}\, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r}\, r^2 f(r) / J_f$$

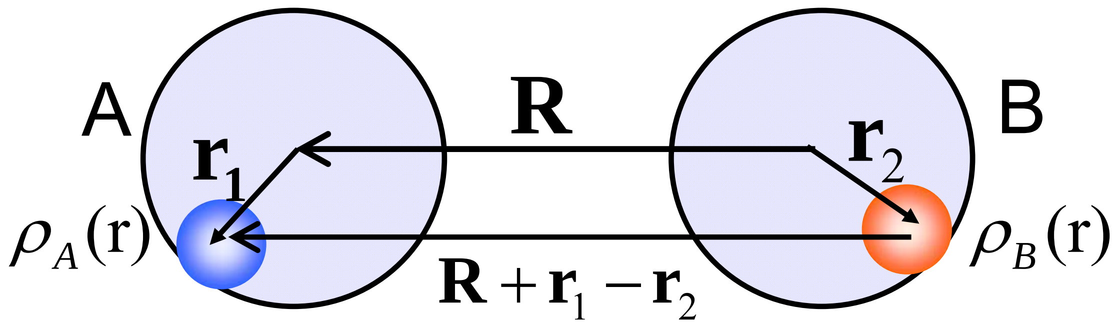
proofs by taking Fourier transforms of each element

Effective interactions – Folding models

Double folding

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$

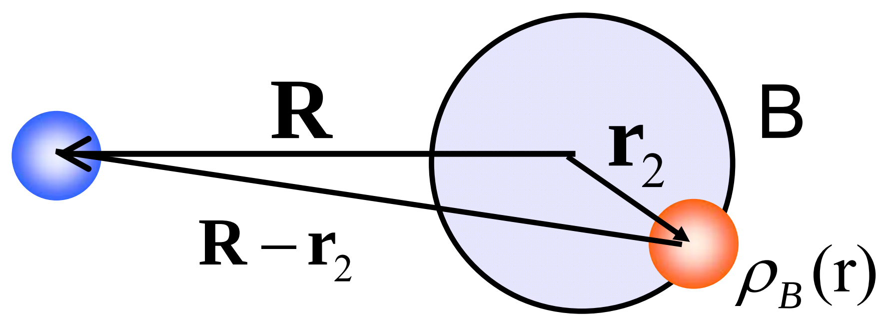
V_{AB}



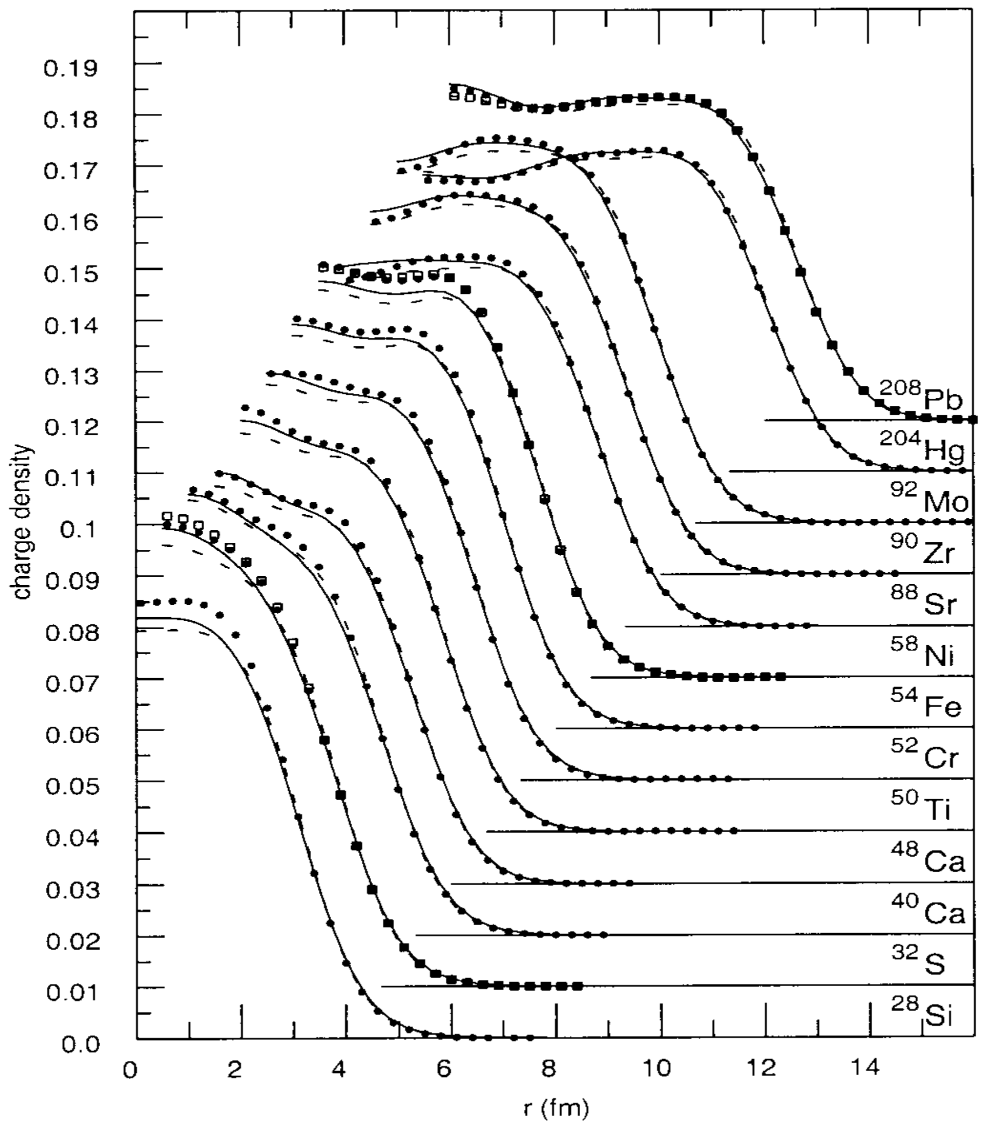
Single folding

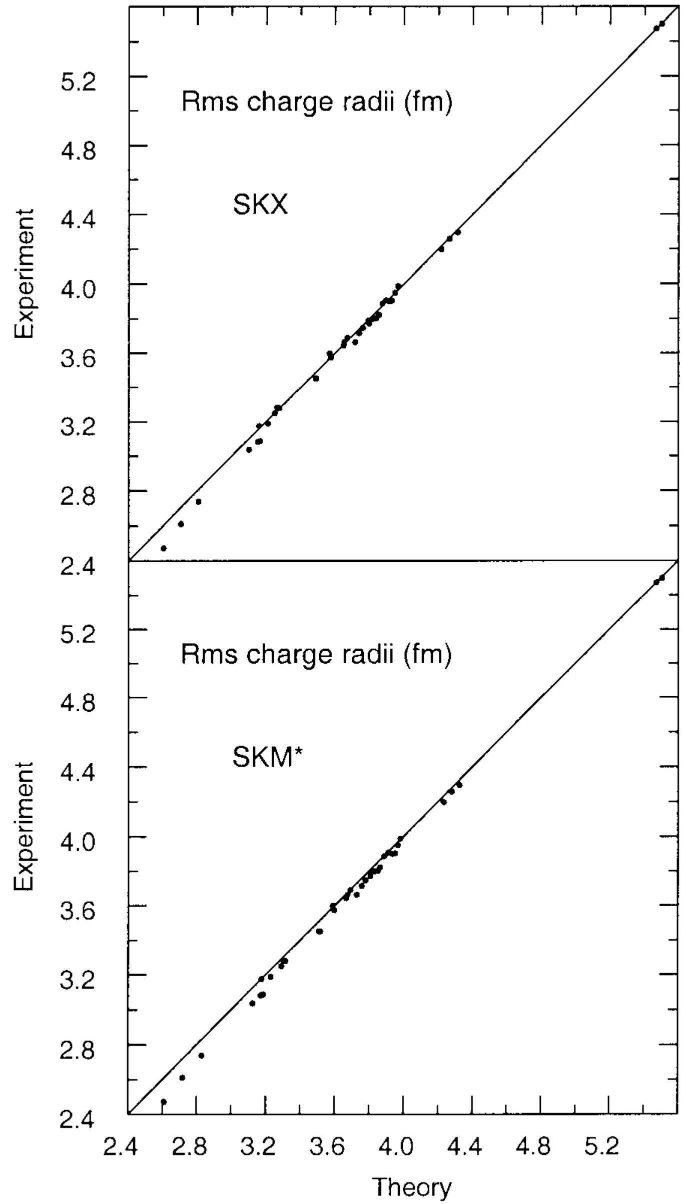
$$V_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} - \mathbf{r}_2)$$

V_B



Skyrme Hartree-Fock radii and densities





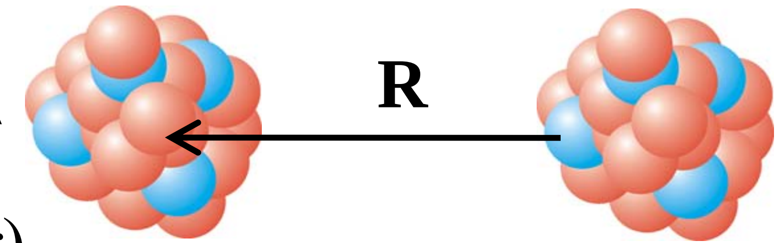
W.A. Richter and B.A. Brown, Phys. Rev. **C67** (2003) 034317

The M3Y interaction – nucleus-nucleus systems

Double folding

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

V_{AB}



M3Y interaction – Y=Yukawa

$$v_{NN}(r) = \left[7999\frac{e^{-4r}}{4r} - 2134\frac{e^{-2.5r}}{2.5r}\right] + \hat{J}(E)\delta(\vec{r})$$

$$\hat{J}(E) = -276[1 - 0.005(E/A)] \;\mathrm{MeVfm}^3$$

resulting in a REAL nucleus-nucleus potential

M.E. Brandan and G.R. Satchler, The Interaction between Light Heavy-ions and what it tells us, Phys. Rep. **285** (1997) 143-243.

Information from the elastic scattering channel

Folding model (including account of non-localities**) often used to provide the radial shape and approximate strength of the real part of the potential, call it $F_E(R)$, Then, at each E

$$U_E(R) = [N_R(E) + iN_I(E)]\, F_E(R)$$

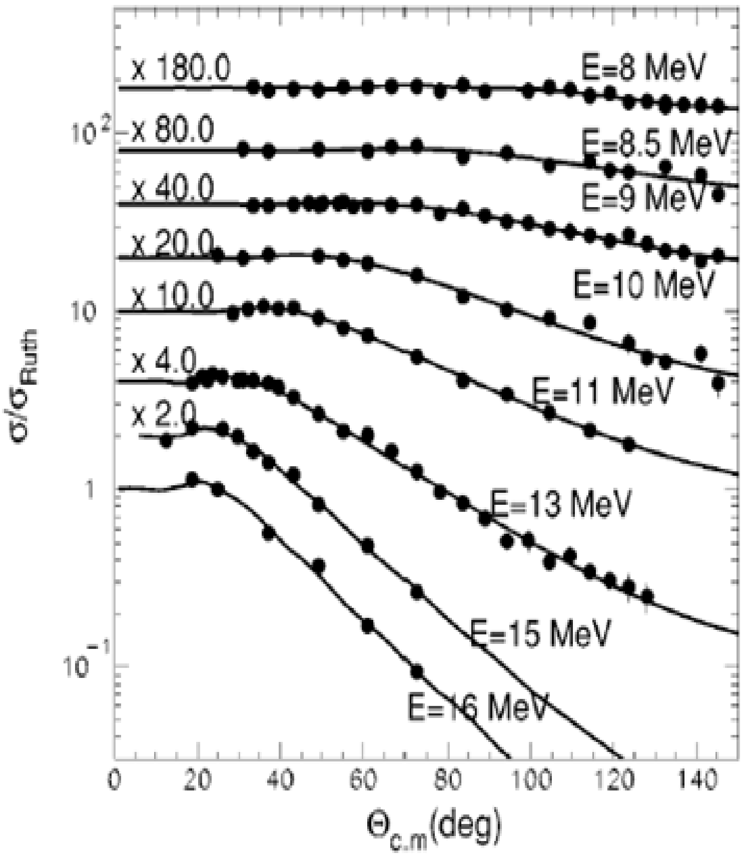
the N_R and N_I are fitted to data with N_R of order unity. (e.g. S. Paulo potential)

Quite generally, for many systems***

$$N_R(E) = 1.0 \pm 0.15$$

$$N_I(E) = 0.8 \pm 0.15$$

$^{7}Li + ^{28}Si$

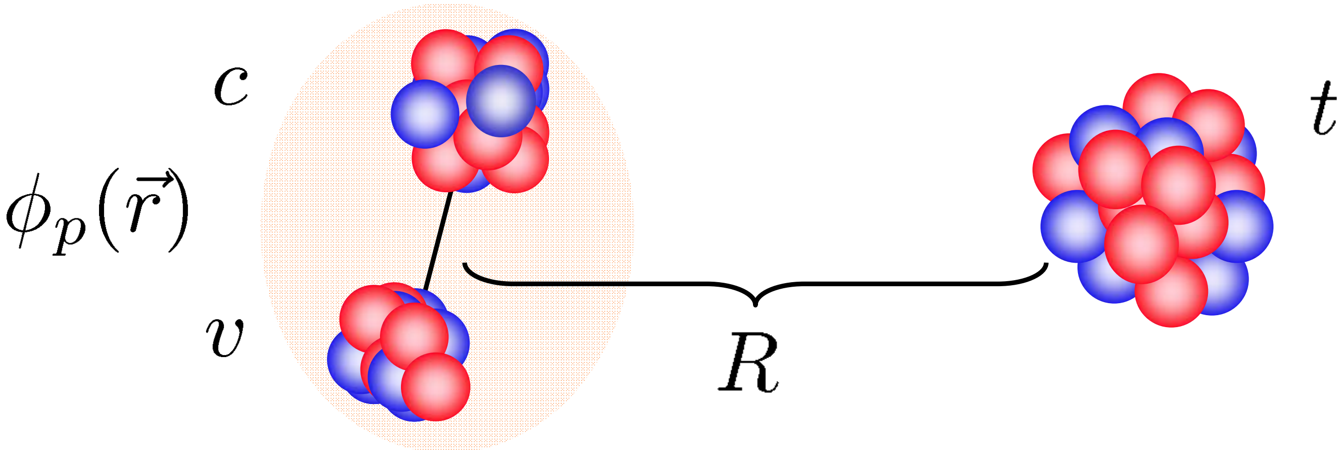


A. Pakou et al., PRC 69 (2004) 054602

** L.C. Chamon et al., PRC 66 (2002) 014610

*** G.R. Satchler and W.G. Love, Phys. Rep. **55** (1979) 183

Cluster folding models – useful identities



$$V_F(R) = \langle \phi_p | V_{ct}(\vec{r}_{ct}) + V_{vt}(\vec{r}_{vt}) | \phi_p \rangle$$

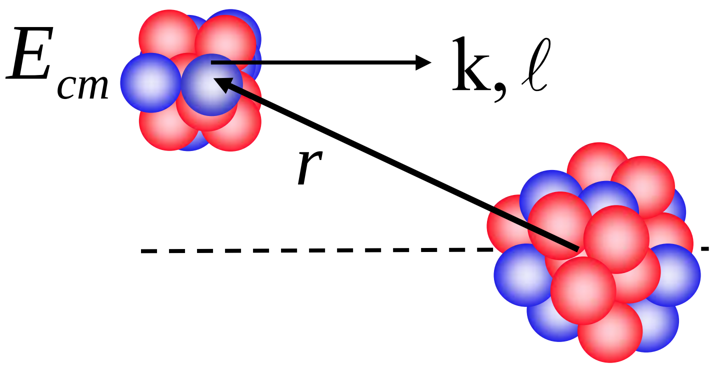
$$J_{V_F} = J_{V_{ct}} + J_{V_{vt}}$$

$$\langle r^2 \rangle_{V_F} = \frac{A_c}{A_p} \langle r^2 \rangle_{V_{ct}} + \frac{A_v}{A_p} \langle r^2 \rangle_{V_{vt}} + \frac{A_c A_v}{A_p^2} \langle r^2 \rangle_{\phi_p}$$

$$J_f = \int d\vec{r}\, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r}\, r^2 f(r) / J_f$$

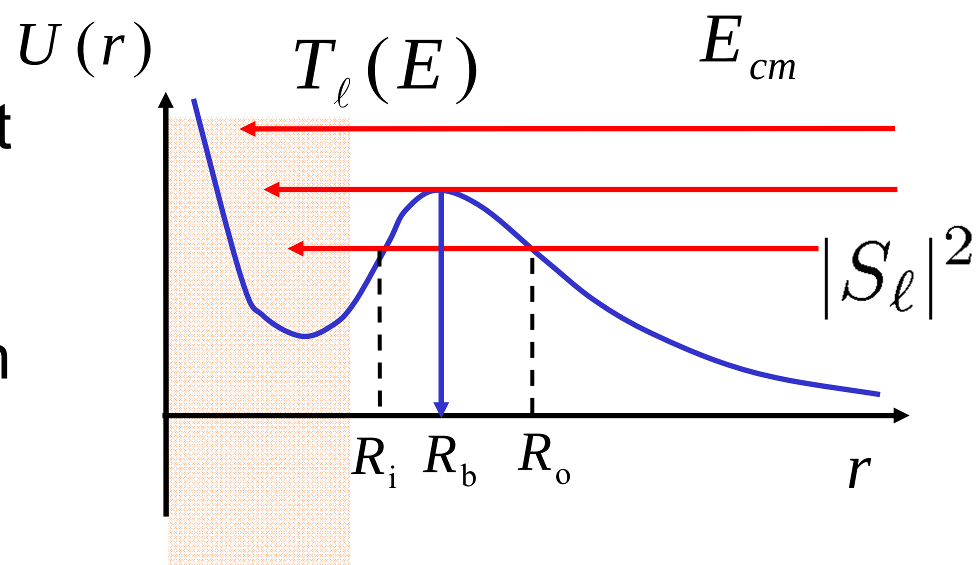
proofs by taking Fourier transforms of each element

Barrier passing models of fusion



Theoretical ideas for simple (barrier passing) models of nucleus-nucleus fusion reactions

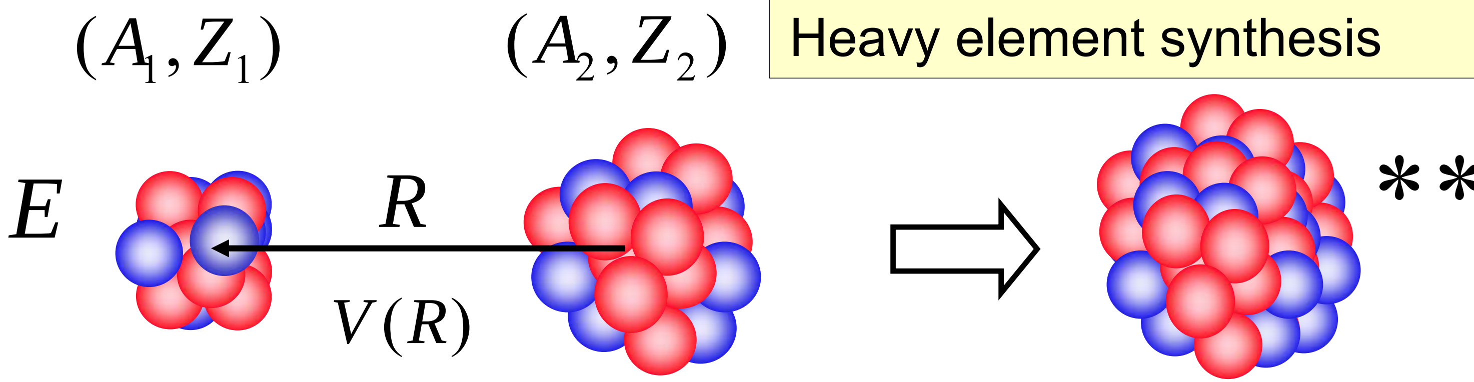
an imaginary part in *U(r)*, at short distances, can be included to absorb all flux that passes over or through the barrier – assumed to result in fusion

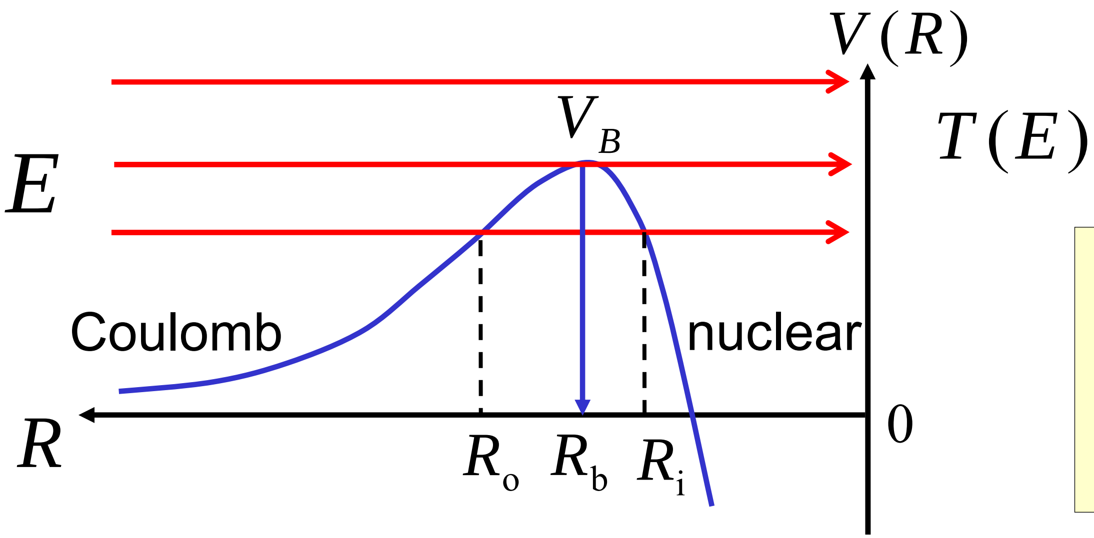


$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-|S_\ell|^2)$$

Complete fusion process – static picture

Nuclear astrophysics
Heavy element synthesis

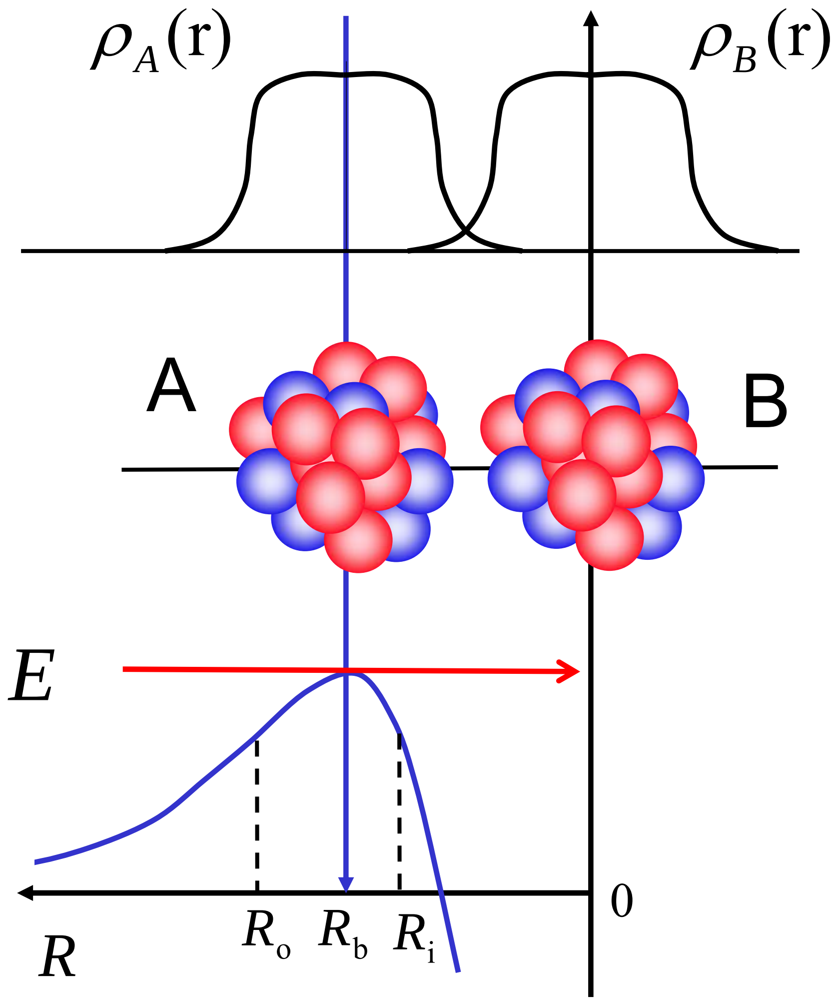




transmission probability:

Dominated by interaction *V(R)* near 'surface', R_b

Barrier radii and nuclear densities - surfaces



Fusion will be probe and be sensitive to:

nuclear binding (tails of the nuclear densities), nuclear structure (tails of the single particle wave functions)

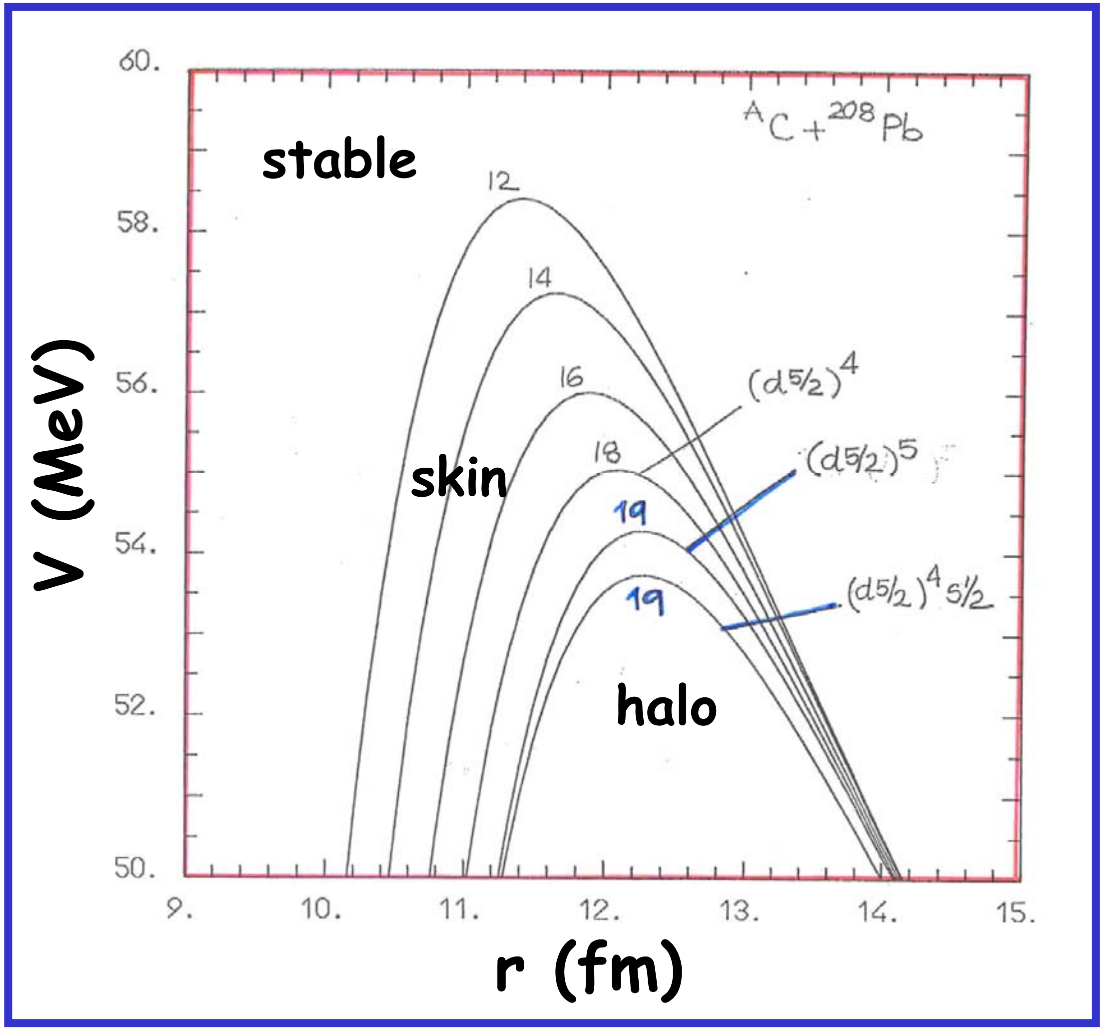
but also expect sensitivity and complications due to the reaction dynamics –

intrinsically surface dominated

Static effects – barriers for n-rich Carbon isotopes

$^{A}C + {}^{208}Pb$

HF predictions



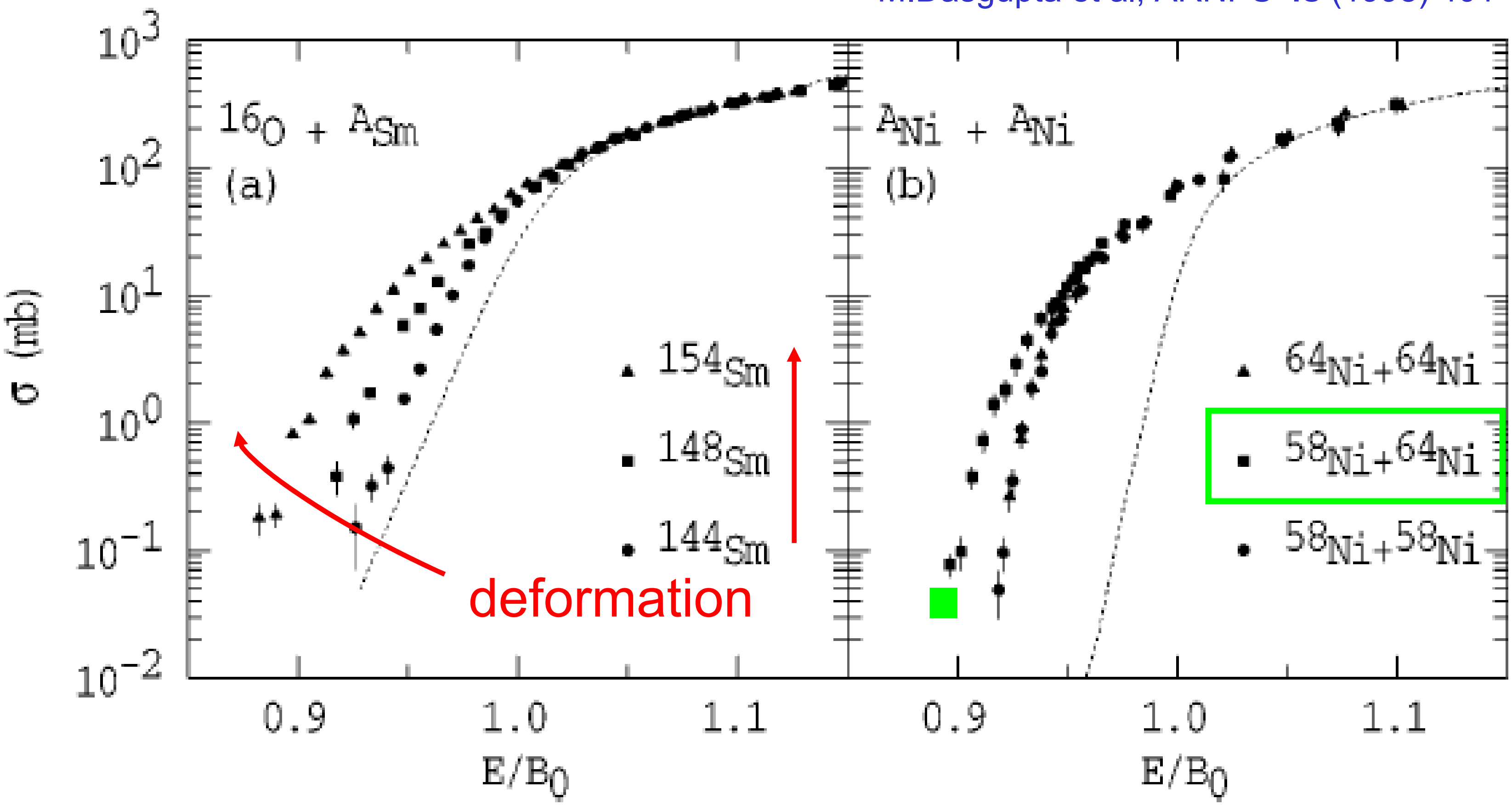
A. Vitturi, NUSTAR'05,

Challenges – potentials, thresholds and dynamics

- Expect a complex interplay of *static*, density driven, and surface, *dynamical* effects
- Far below the barrier, for normally bound nuclei, direct reaction channels switch off – have opportunity to study *threshold effects* as reaction channels open and evolve as a function of energy
- Fusion expected to be a severe test of our models of nuclear structures and of treatments of direct reaction dynamics
- Facilities available for sophisticated and very precise experiments - ANU (Canberra), USP, INFN Legnaro, etc.
- Weakly bound systems are different – do break-up channels turn off below the barrier? What can we learn?

Channel coupling – classic examples

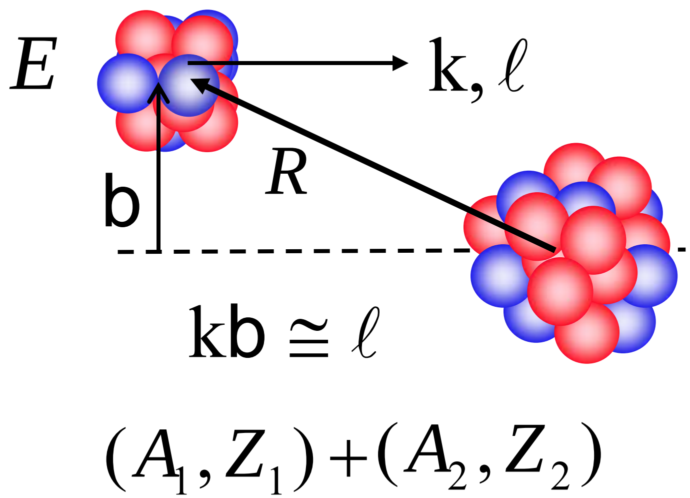
M.Dasgupta et al, ARNPS **48** (1998) 401

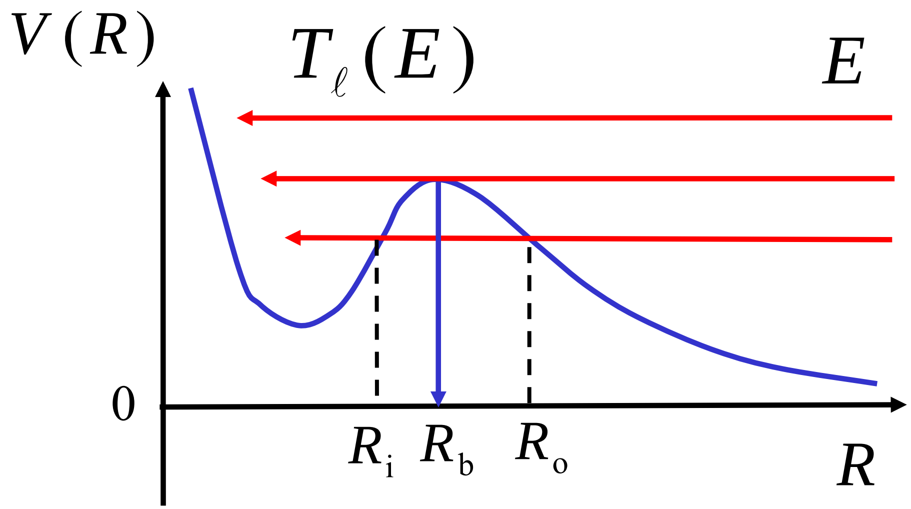


R.G. Stokstad et al, PRL **41** (1978) 465, PRC **21** (1980) 2427.

M. Beckerman et al, PRL **45** (1980) 1472, PRC **23** (1981) 1581, PRC **25** (1982) 837.

Complete fusion - expectations – static model





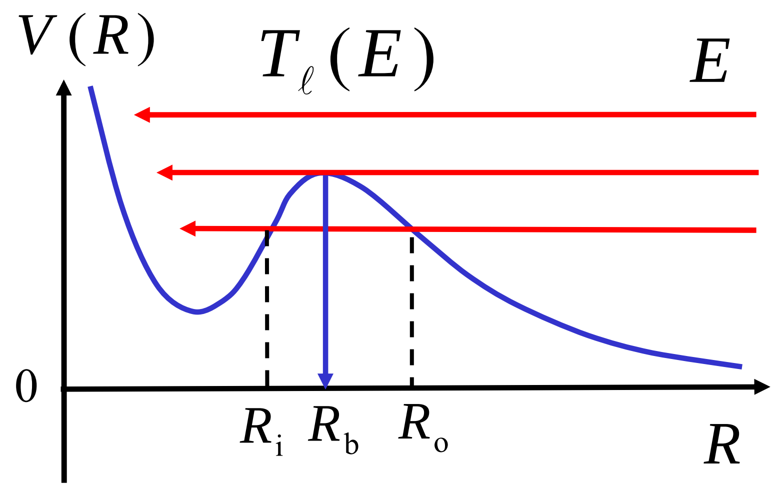
$$\sigma(E) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell(E), \quad T_\ell(E) = 1 - |S_\ell|^2$$

$$\frac{d^2u_\ell(R)}{dR^2} + \frac{2\mu}{\hbar^2}\left[E - V(R) - \frac{\ell(\ell+1)}{R^2}\right]u_\ell(R) = 0$$

Quantum mechanical barrier penetration

$$\frac{d^2 u_\ell(R)}{dR^2} + \frac{2\mu}{\hbar^2}\left[E - V(R) - \frac{\ell(\ell+1)}{R^2}\right] u_\ell(R) = 0$$

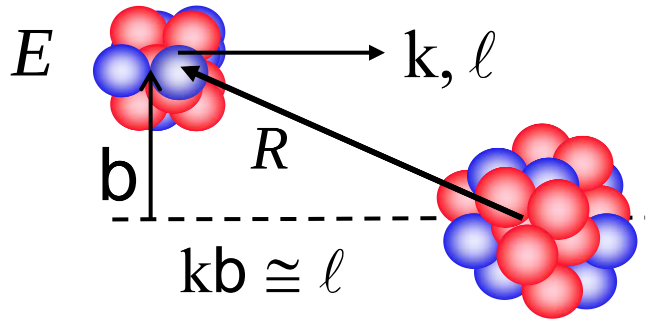
Numerical solutions of this QM barrier penetration problem, the solution of the radial equation for *u(R)* and the transmission prob. - and later, more complex (coupled channels) examples, account for fusion by one of two methods:

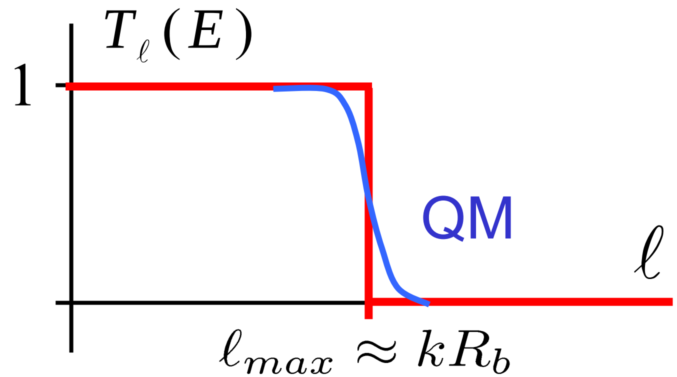


(i) the $u_\ell(R)$ have ingoing wave boundary conditions for an $R=R_0$
No flux transmitted through the barrier is reflected [exp(-ikR)]

(ii) an absorptive (imaginary) part in *V(R)* at short distances absorbs all flux transmitted through the barrier

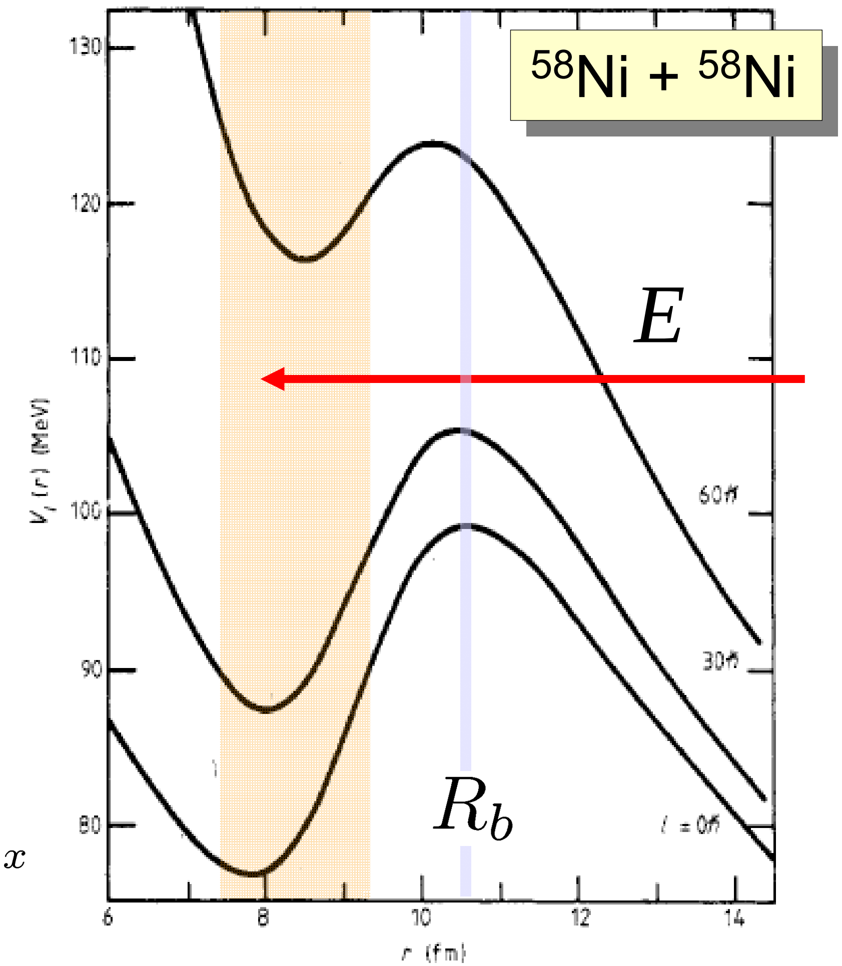
Angular momentum dependence of the barrier





$$\sigma_R(E) \approx \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell+1) \approx \frac{\pi}{k^2} \ell_{max}^2$$

$$\sigma^{cf}(E) \approx \sigma_R \approx \pi R_b^2$$



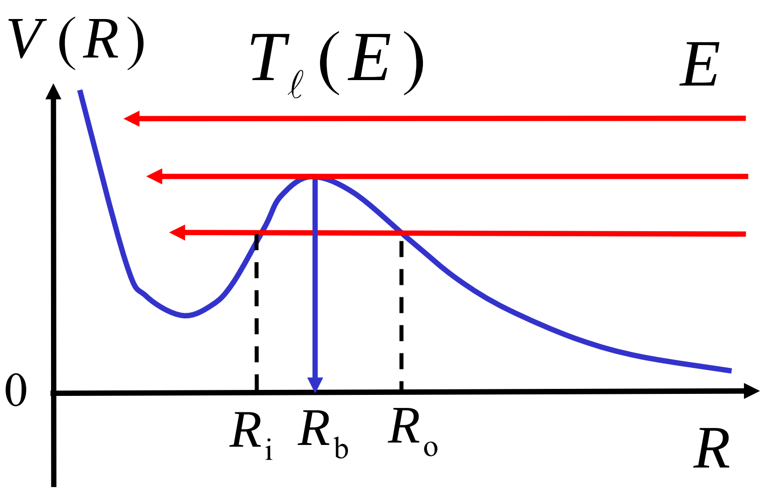
M. Beckerman, Rep. Prog. Phys. **51** (1988) 1047

Making connection with empirical cross sections

$$T_\ell(E) \approx \left[1 + \exp\sqrt{\frac{8\mu}{\hbar^2}} \int_{R_i(\ell)}^{R_o(\ell)} dR \left\{V(R) + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2} - E\right\}^{1/2}\right]^{-1}$$

Localised barrier of height (for ℓ=0) of $V_B = V(R_b)$

$$\frac{\ell(\ell+1)}{R^2} \approx \frac{\ell(\ell+1)}{R(E)^2} \quad \rightarrow \quad T_\ell(E) \approx T_0\left(E - \frac{\ell(\ell+1)\hbar^2}{2\mu R(E)^2}\right), \quad R(E) \approx R_b$$



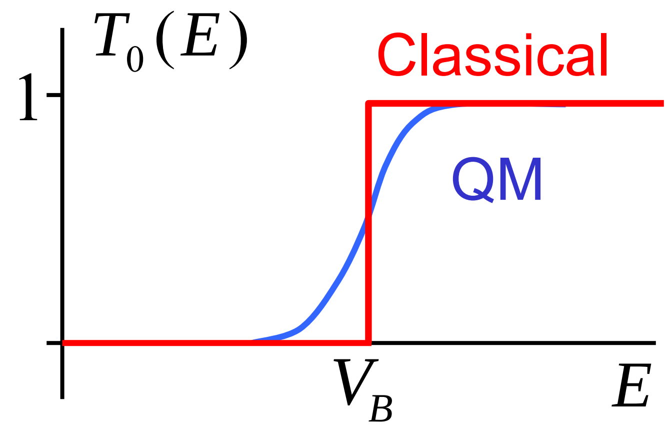
$$\sigma(E) = \sum_\ell \sigma_\ell(E) \rightarrow \int d\ell \; \sigma(\ell, E)$$

$$E\sigma(E) = \pi R(E)^2 \int_0^E dE' \; T_0(E')$$

A.B. Balantekin, Rev. Mod. Phys. **70** (1998) 77

Distribution of barriers – directly from the data

$$E\sigma(E) = \pi R(E)^2 \int_0^E dE' \, T_0(E')$$

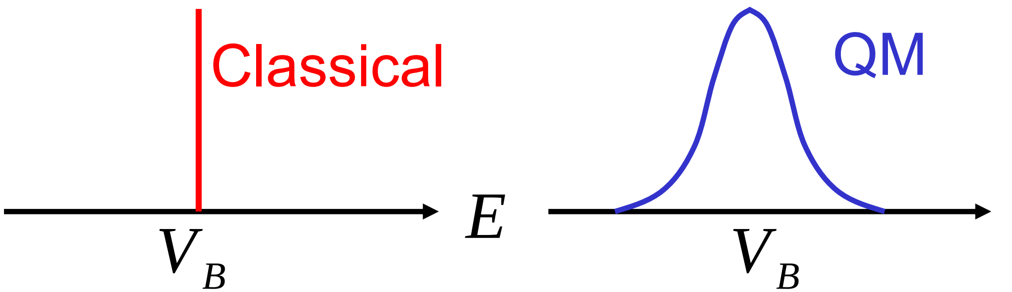


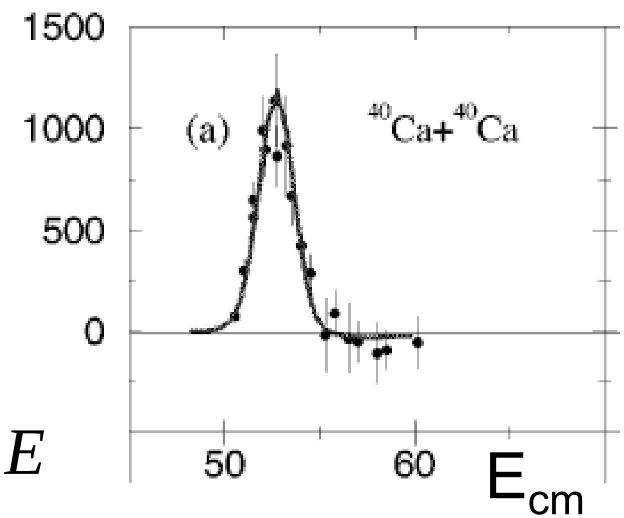
Classically

$$R(E) \equiv R_b$$

$$\begin{aligned} E\sigma(E) &= \pi R_b^2 (E - V_B), \; E > V_B \\ &= 0, \quad E < V_B \end{aligned}$$

$$\frac{d^2}{dE^2}[E\sigma(E)] = \pi R_b^2 \delta(E - V_B)$$





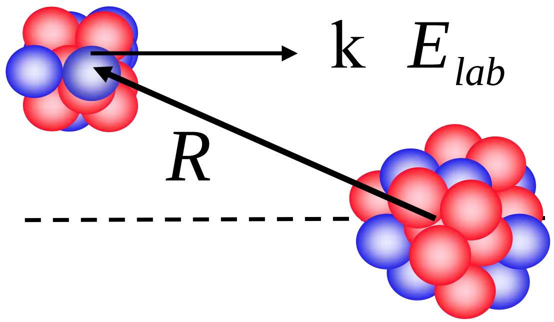
A.B. , Rev. Mod. Phys. **70** (1998) 77

M.Dasgupta et al, ARNPS **48** (1998) 401

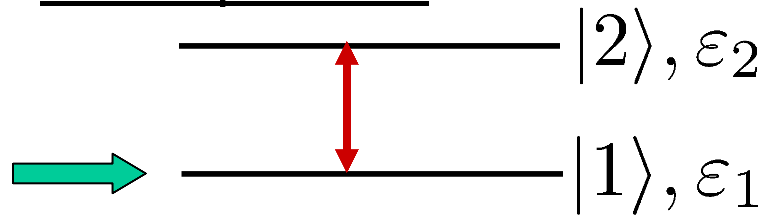
Coupled channels: for two-states problem

$$H = T_R + V(R) + h(\xi) + F(R,\xi)$$

$h(\xi)$



<u>Model problem</u>



Two channels 1,2 – incident waves in channel 1.

$$H|\Psi\rangle = E|\Psi\rangle \qquad \langle \vec{R},\xi|\Psi\rangle = \phi_1(\vec{R})\langle\xi|1\rangle + \phi_2(\vec{R})\langle\xi|2\rangle$$

$$\begin{aligned}\langle 1|H|\Psi\rangle &= [T_R + V(R) + \varepsilon_1 + F_{11}(R)]\phi_1(\vec{R}) + F_{12}(R)\phi_2(\vec{R}) \\ &= E\langle 1|\Psi\rangle = E\phi_1(\vec{R}), \qquad F_{ij}(R) = \langle i|F(R,\xi)|j\rangle\end{aligned}$$

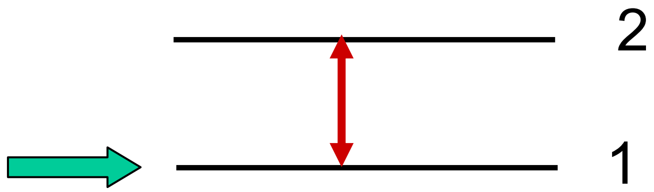
and similarly for the overlap with state 2, gives <u>coupled</u> equations

$$[E - \varepsilon_1 - T_R - V(R) - F_{11}(R)]\phi_1(\vec{R}) = F_{12}(R)\phi_2(\vec{R})$$

$$[E - \varepsilon_2 - T_R - V(R) - F_{22}(R)]\phi_2(\vec{R}) = F_{21}(R)\phi_1(\vec{R})$$

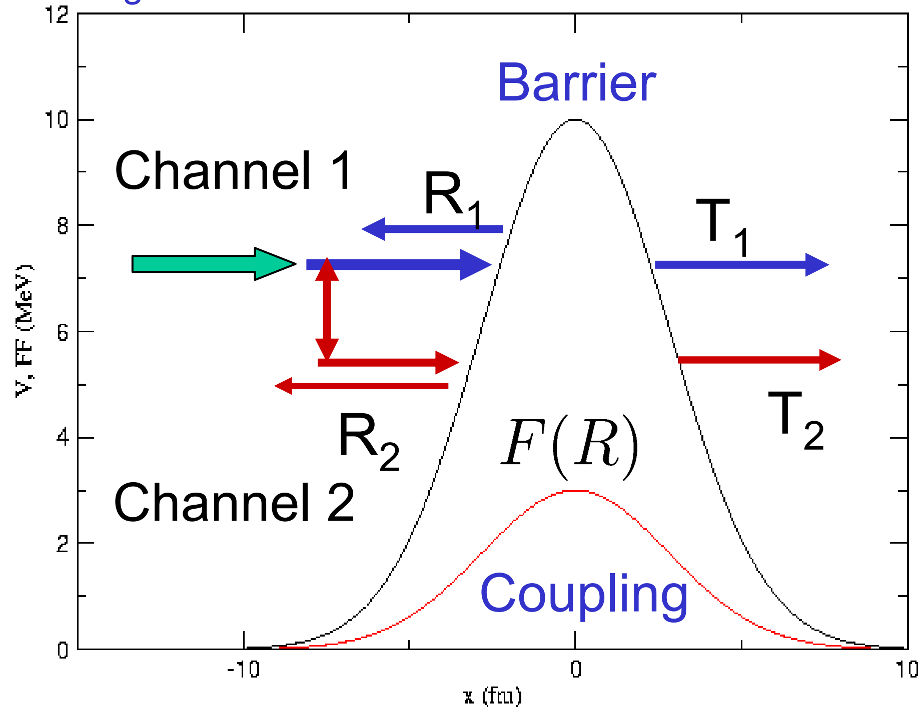
Coupled channels effects on barrier distribution

Model problem



Coupling of two channels 1,2 assumed degenerate for simplicity - coupling *F(R)* – incident waves in channel 1.

Figure from to A.Vitturi



$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+V(R)-E\right]\phi_1(R)=F(R)\phi_2(R)$$

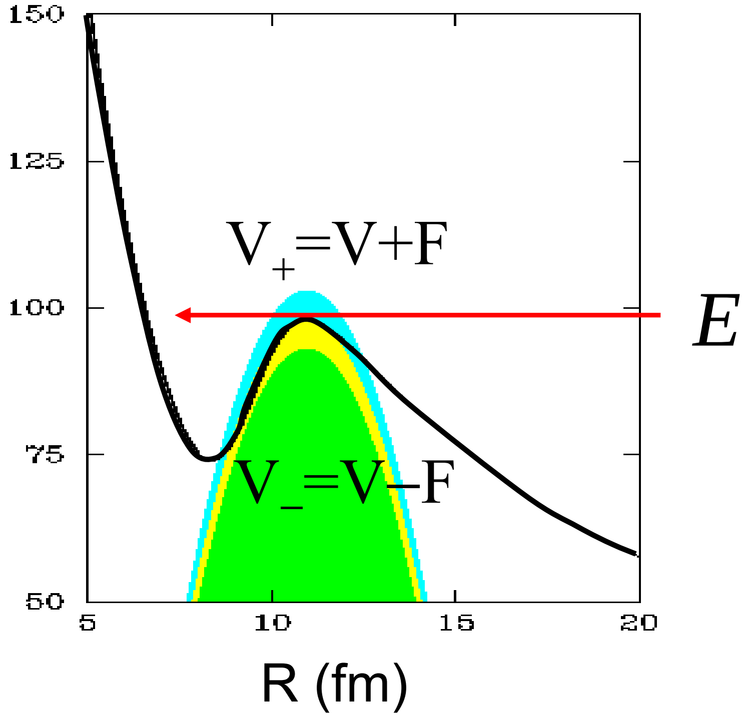
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+V(R)-E\right]\phi_2(R)=F(R)\phi_1(R)$$

Decoupled by addition and subtraction

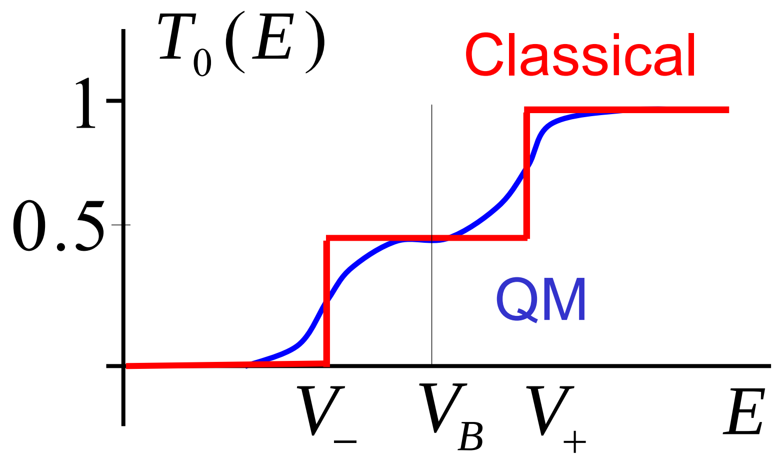
Decoupled, two barriers problem

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\{V(R)\pm F(R)\}-E\right]\mathcal{X}_\pm(R)=0$$

$$\mathcal{X}_\pm(R)=\left[\phi_1(R)\pm\phi_2(R)\right]/\sqrt{2} \qquad |\langle\mathcal{X}_\pm|\phi_1\rangle|^2=1/2$$

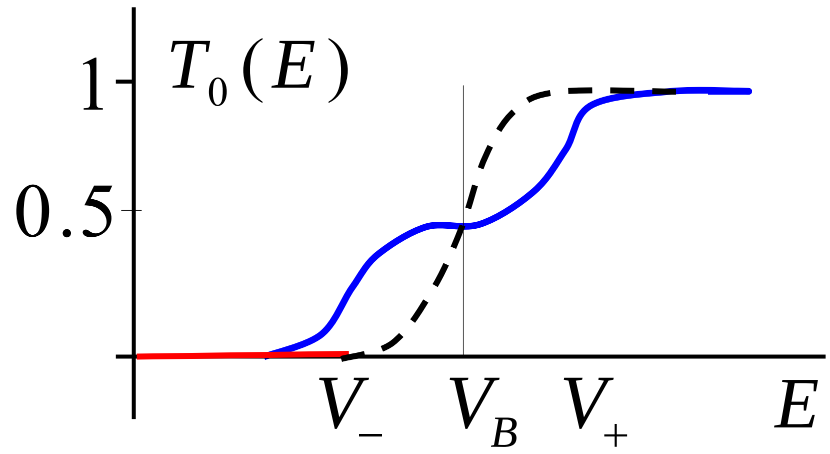


$$T_0(E)=\frac{1}{2}\left[T_+(E)+T_-(E)\right]$$



Barrier distributions will reflect channel couplings

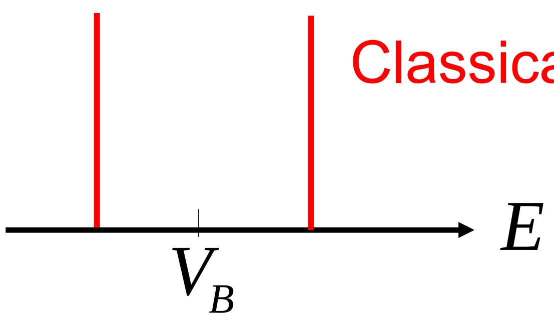
In this simple model, channel coupling (no matter what the sign of the coupling potential) enhances fusion below and hinders fusion above the barrier – quite general result



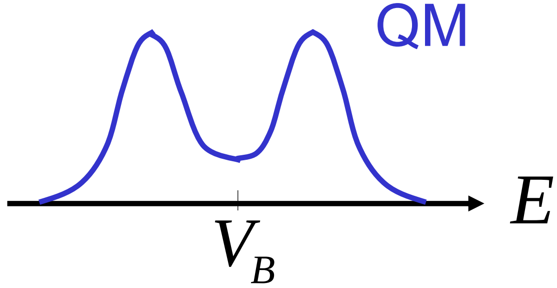
Non-degeneracy of the channels divides the flux incident on the barriers in a more complex way in the different channels (e.g. Beckerman, Rep. Prog. Phys. **51** (1988) 1047)

$$\frac{d^2}{dE^2}[E\sigma(E)] = \frac{\pi R_b^2}{2}\left[\delta(E - V_-) + \delta(E - V_+)\right]$$

Classical

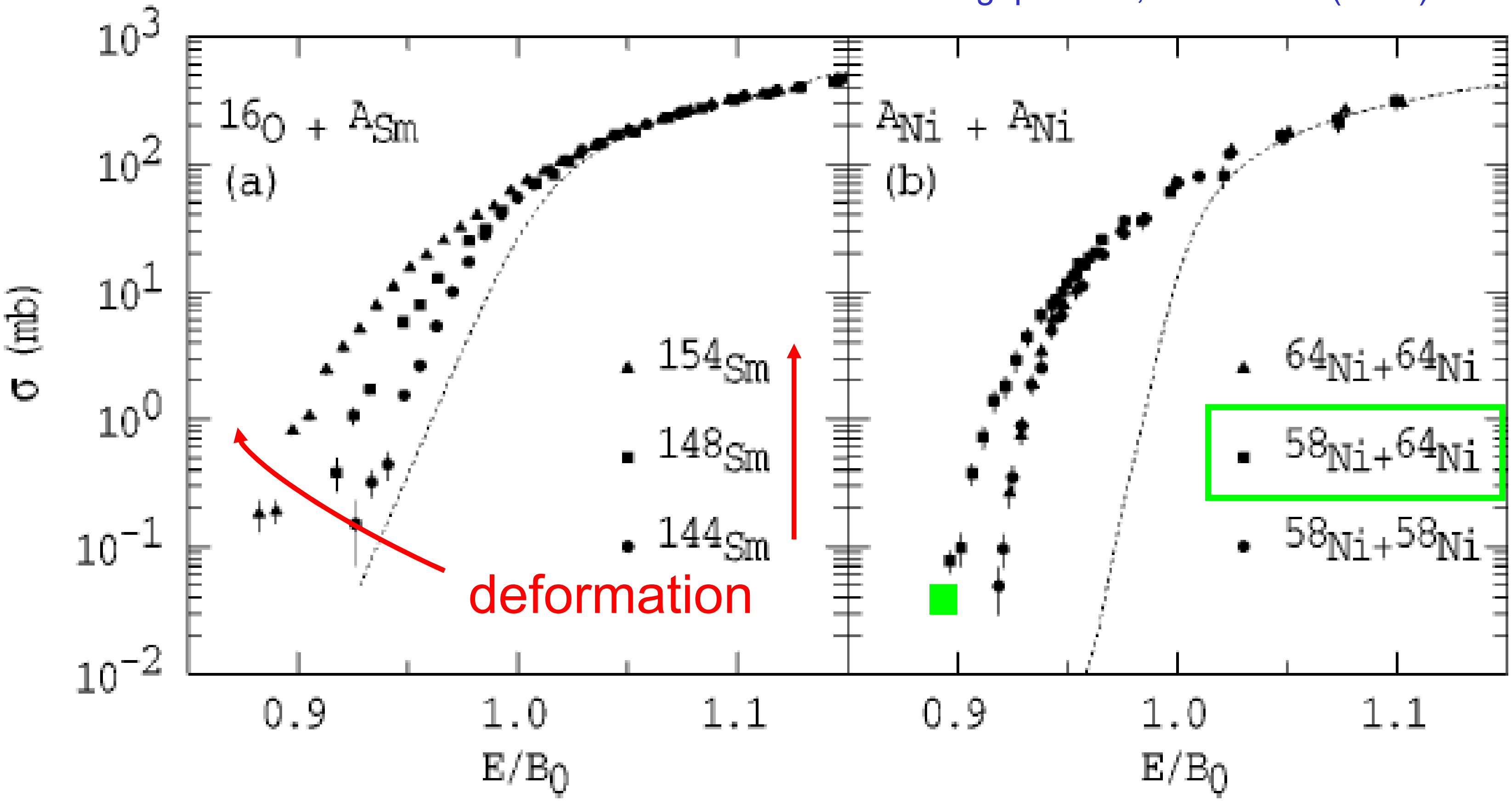


QM



Channel coupling – classic examples

M.Dasgupta et al, ARNPS **48** (1998) 401



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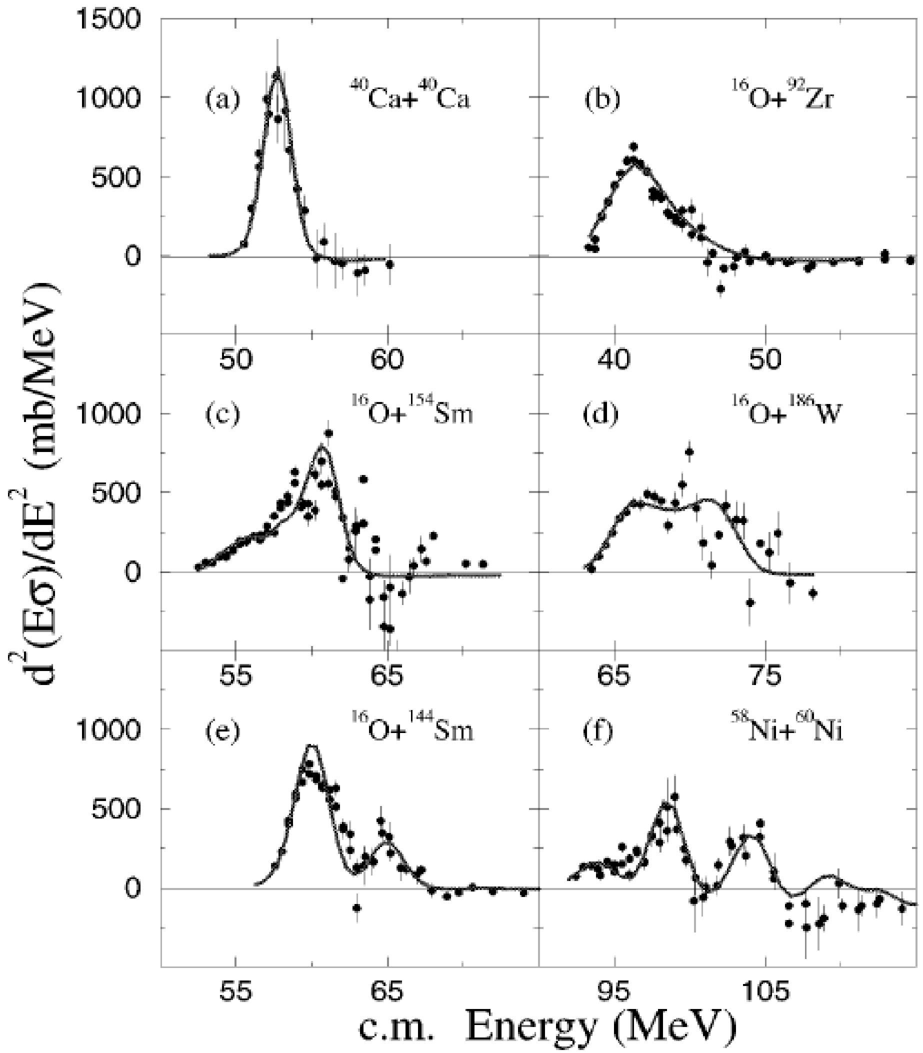
M. Beckerman et al, PRL **45** (1980) 1472, PRC **23** (1981) 1581, PRC **25** (1982) 837.

Empirical and calculated barrier distributions

For data of sufficiently high accuracy and precision, one can compare the values of

$$\frac{d^2}{dE^2}[E\sigma(E)]$$

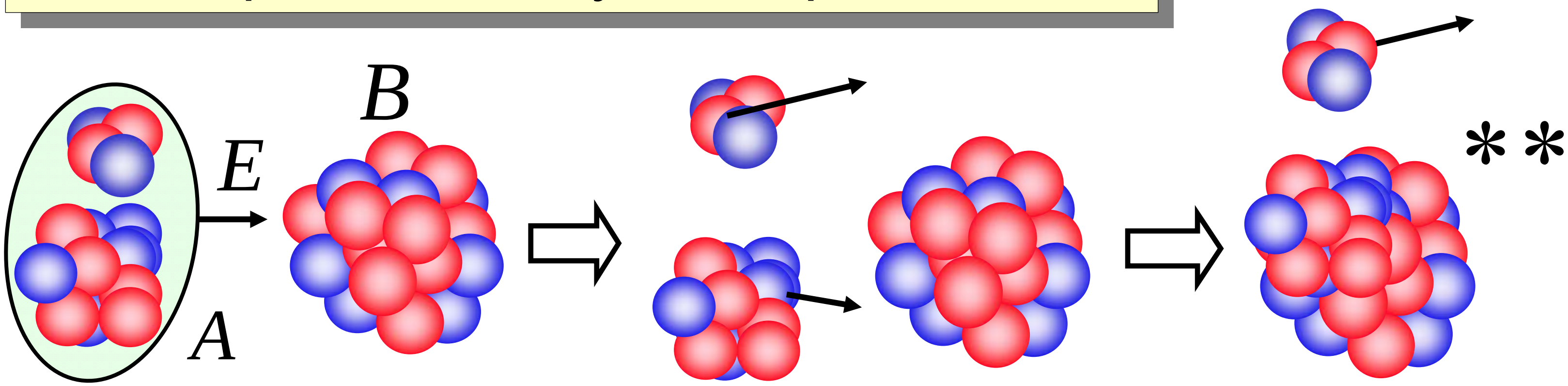
deduced from the data and from detailed coupled channels calculations, including rotational, vibrational single particle or transfer couplings



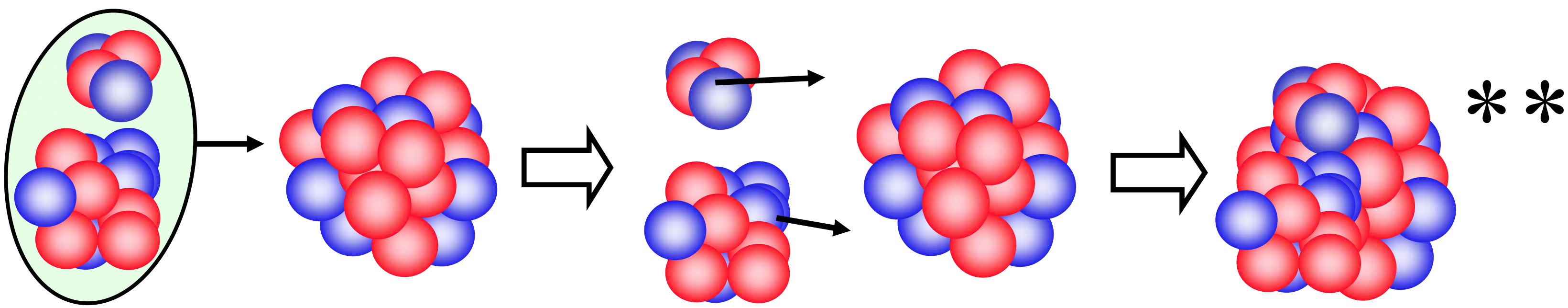
M.Dasgupta et al, ARNPS **48** (1998) 401

New challenge is presented by weak binding

Break-up, followed by incomplete fusion



Break-up, followed by complete fusion



Weakly-bound and exotic nuclear systems

Stable systems

^{6}Li	$\rightarrow$ $^{4}He + d$	$S_{\alpha} = 1.48$ MeV
^{7}Li	$\rightarrow$ $^{4}He + t$	$S_{\alpha} = 2.45$ MeV
^{9}Be	$\rightarrow$ $^{8}Be + n \rightarrow$ $^{4}He + {}^{4}He + n$	$S_{n} = 1.67$ MeV

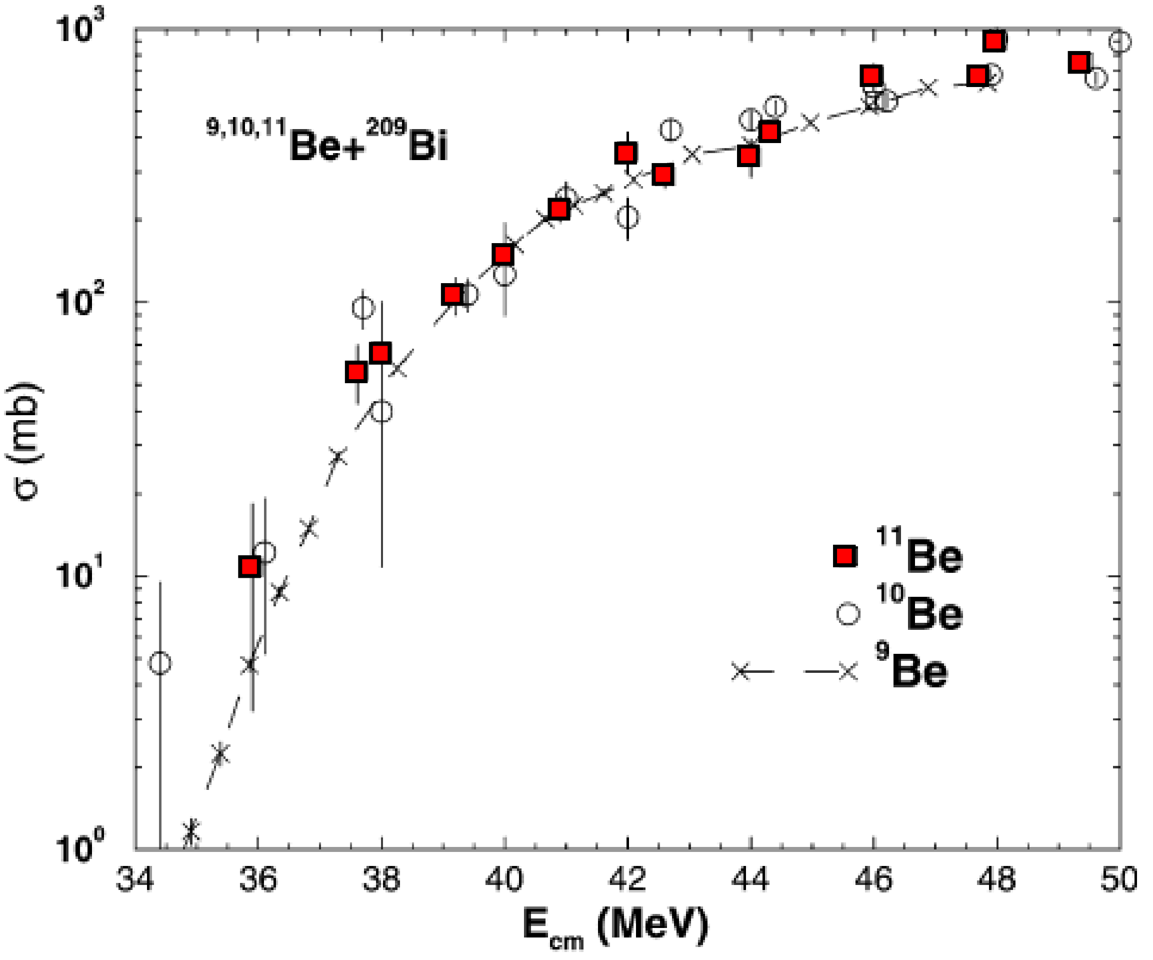
Unstable (exotic) systems

^{6}He	$\rightarrow$ $^{4}He + 2n$	$S_{2n} = 0.98$ MeV
^{11}Be	$\rightarrow$ $^{10}Be + n$	$S_{n} = 0.50$ MeV
^{11}Li	$\rightarrow$ $^{9}Li + 2n$	$S_{2n} = 0.33$ MeV

What are considerations for weakly-bound nuclei

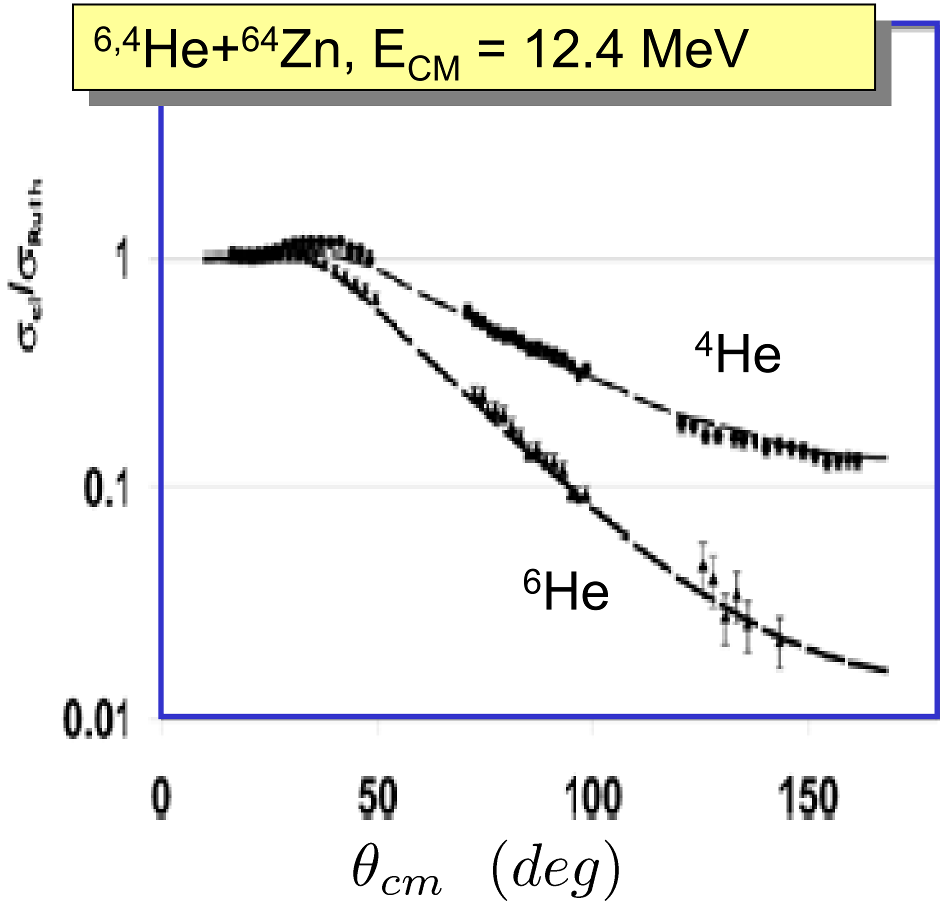
- Static effects due to tails in density distribution - longer tails in ion-ion potential, lowering of Coulomb barrier - larger sub-barrier fusion probabilities
- Dynamical effects due to coupling to states in the continuum (break-up processes), polarization term in optical potential – larger sub-barrier fusion
- Breakup is due to the different forces acting on the fragments, that then separate – reduced expectation of total fusion
- Weak binding leads typically to large +ve Q-values for nucleon transfers

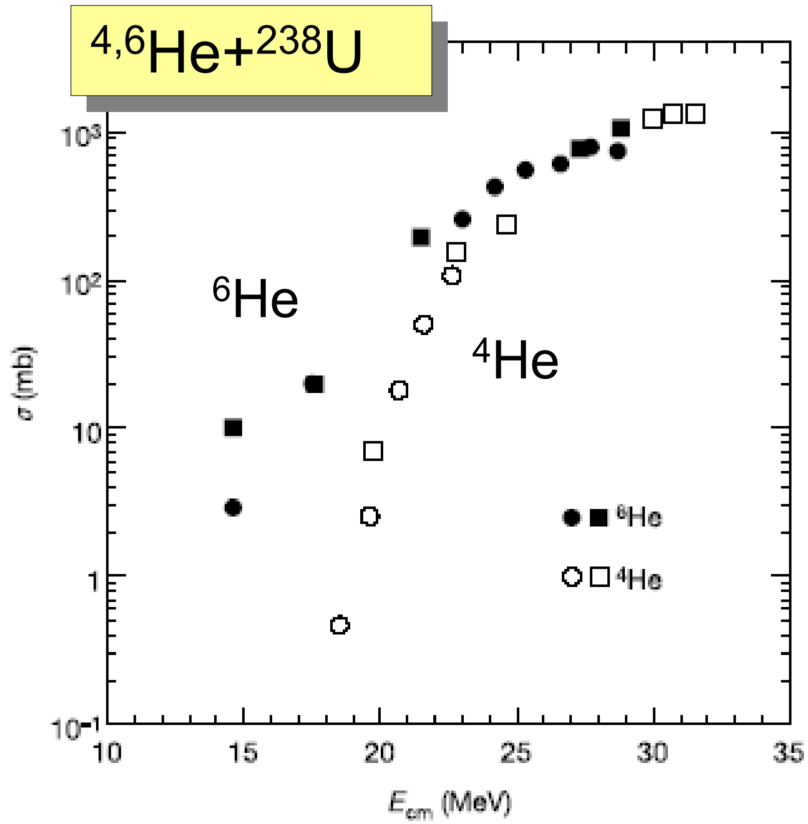
Beryllium isotopes – 11Be a halo nucleus case



C. Signorini, Nucl.Phys. **A735** (2004) 329

Elastic scattering reflects loose binding

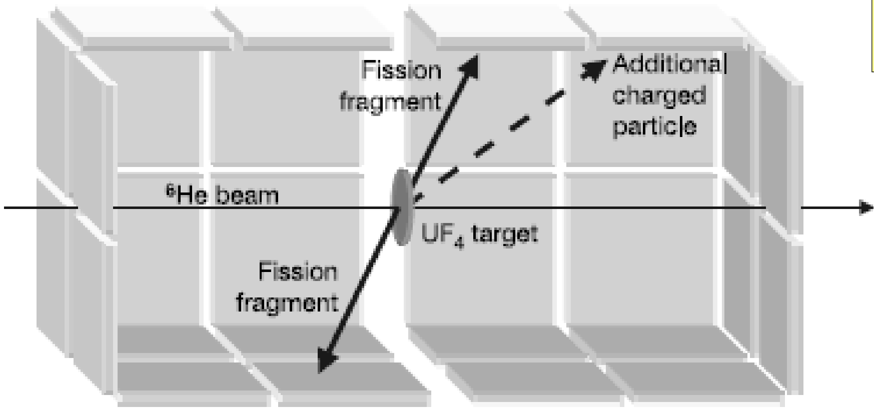




A. Di Pietro et al., Europhys. Lett. **64** (2003) 309

M. Trotta et al., PRL **84** (2000) 2342
R. Raabe et al., Nature **431** (2004) 823

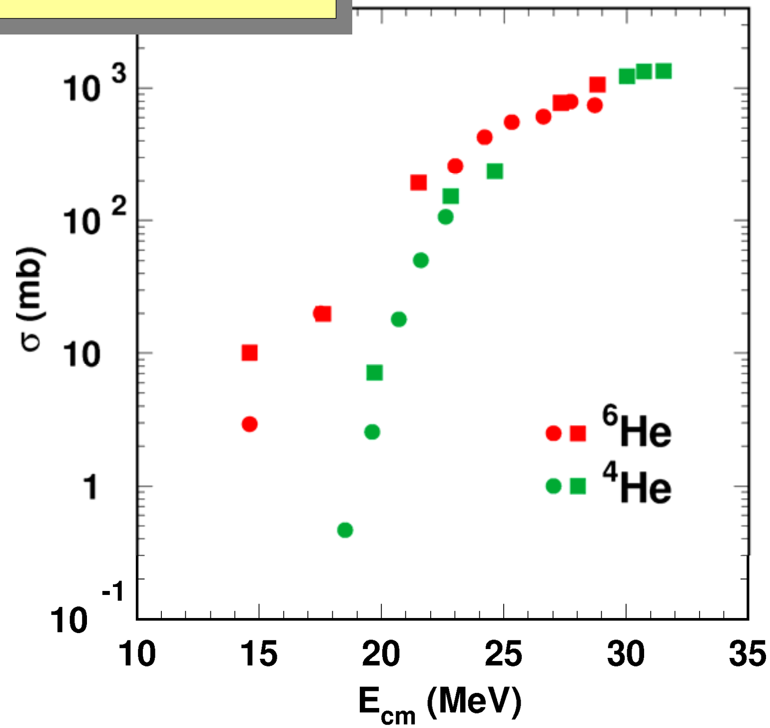
Exclusive measurements – transfer channels



No enhancement of fusion probability by the neutron halo of ^{6}He

R. Raabe[1,2], J. L. Sida[1*], J. L. Charvet[1], N. Alamanos[1], C. Angulo[3], J. M. Casandjian[4], S. Courtin[5], A. Drouart[1], D. J. C. Durand[1], P. Figuera[6], A. Gillibert[1], S. Heinrich[1], C. Jouanne[1], V. Lapoux[1], A. Lepine-Szily[7], A. Musumarra[6], L. Nalpas[1], D. Pierroutsakou[8], M. Romoli[8], K. Rusek[9] & M. Trotta[8]

$^{4,6}He+^{238}U$



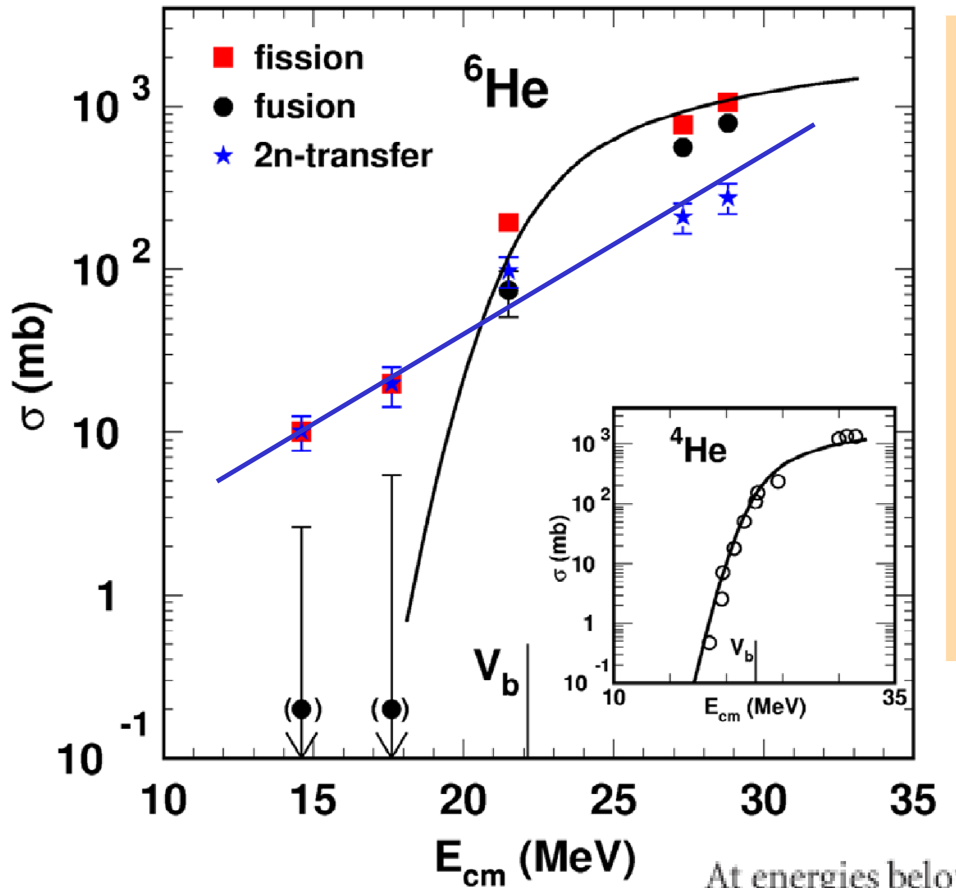
Transfer effects found to be larger than break-up for $^{6}He+^{65}Cu$ reactions

A. Navin et al., Phys Rev C **70**, 044601

M. Trotta et al., PRL **84** (2000) 2342

R. Raabe et al., Nature **431** (2004) 823

Two-neutron transfer and (no) enhancement



Measurement of coincidences with alpha-particles to clarify the role of 2n transfer (incomplete fusion)

No enhancement of the fusion cross section

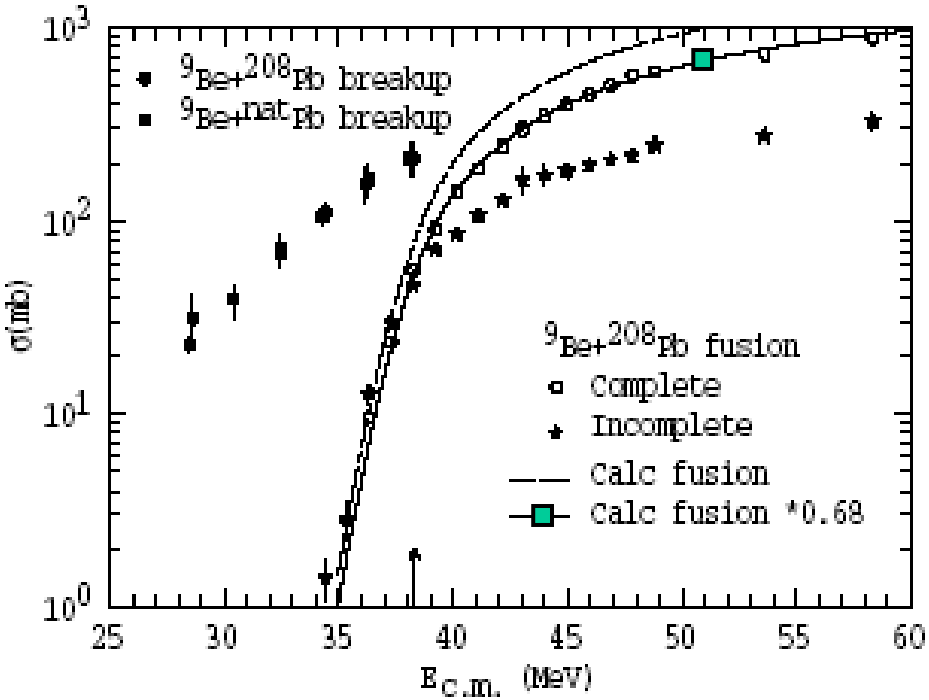
Below the barrier, the two-neutron transfer dominates

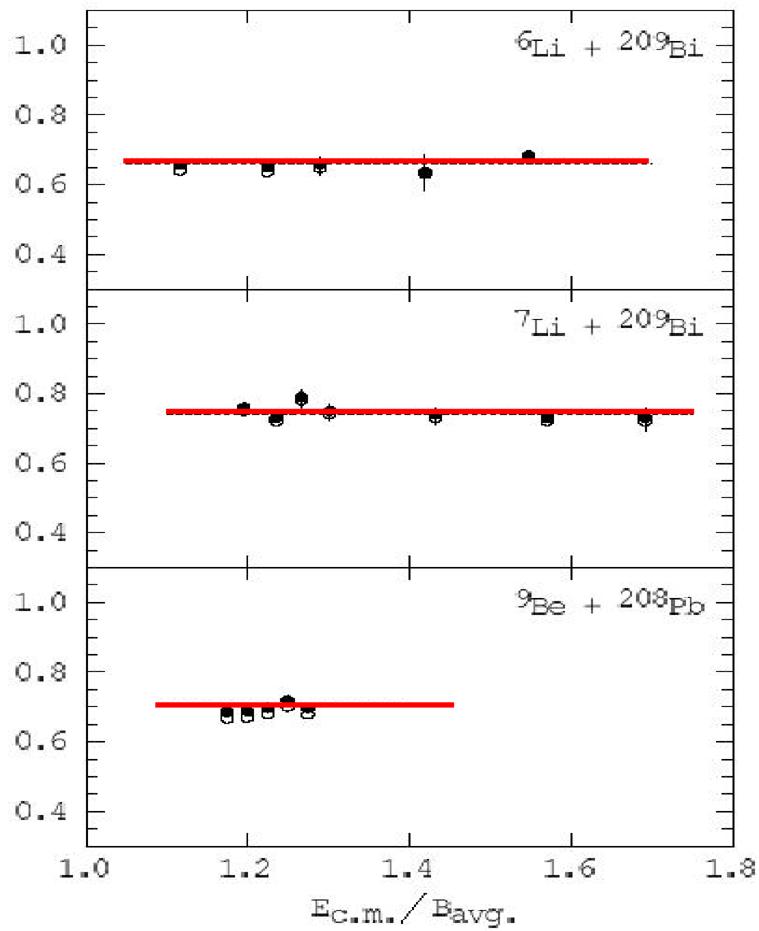
R. Raabe et al., et al, Nature **431** (2004) 823

$^{4,6}He+^{238}U$

At energies below the barrier, we find experimentally that there is no substantial enhancement of the fusion cross-section for the halo nucleus ^{6}He. The large observed yield for fission is entirely due to a direct process, the two-neutron transfer to the target nucleus.

Break-up suppressing fusion above the barrier?





several examples suggesting break-up channels suppress the expected complete fusion cross section above the barrier

D.J. Hinde et al., PRL **89** (2002), 272701

M. Dasgupta et al., PRC **70** (2004), 024606

Useful papers/reviews and conferences

- Fusion Conference series: for example
- Fusion03: *From a Tunnelling Nuclear Microscope to Nuclear Processes in Matter,* Progress of Theoretical Physics Supplement **154**, 2004.
- A.B. Balantekin and N. Takigawa, *Quantum Tunnelling in Nuclear Fusion,* Rev. Mod. Phys. 70 (1998) 77-100.
- M. Dasgupta et al., *Measuring Barriers to Fusion,* Ann. Rev. Nucl. Part. Phys. 48 (1998) 401-461
- Workshop: *Heavy-ion Collisions at Energies Near the Coulomb Barrier* 1990, IoP Conference Series, Vol 110 (1990).
- S.G. Steadman et al., ed. *Fusion Reactions Below the Coulomb Barrier*, Springer Verlag (1984)
- M.E. Brandan and G.R. Satchler, *The Interaction between Light Heavy-ions and what it tells us,* Phys. Rep. **285** (1997) 143-243.
- M. Beckerman, *Sub-barrier Fusion of Two Nuclei,* Rep. Prog. Phys. **51** (1988) 1047-1103.
- M.S. Hussein and K.W. McVoy, *Inclusive Projectile Fragmentation in the Spectator Model,* Nucl. Phys. **A445** (1985) 124-139.
- M. Ichimura, *Theory of Inclusive Break-up Reactions,* Int. Conf on Nucl. React. Mechanism. World Scientific (Singapore), 1989, 374-381.
- plus enormous volume of relevant literature – much of which is cited in the above