

Direct reactions at low energies: Part III - Transfer Spectroscopy

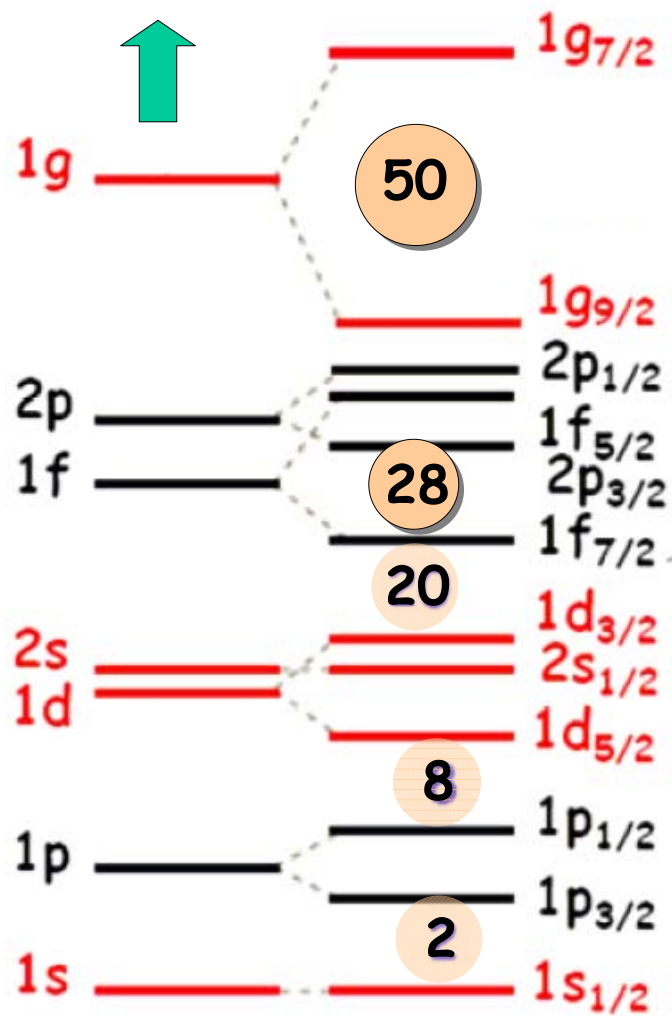
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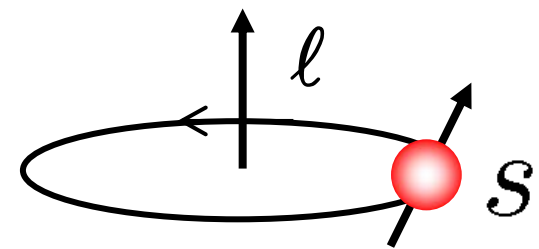
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What is involved in realistic reaction calculations?



$$\ell, s = 1/2 \begin{cases} j_{<} = \ell - 1/2 \\ j_{>} = \ell + 1/2 \end{cases}$$

$^{23}\text{O}(p,d)$
 $Z=8$
 $N=15$

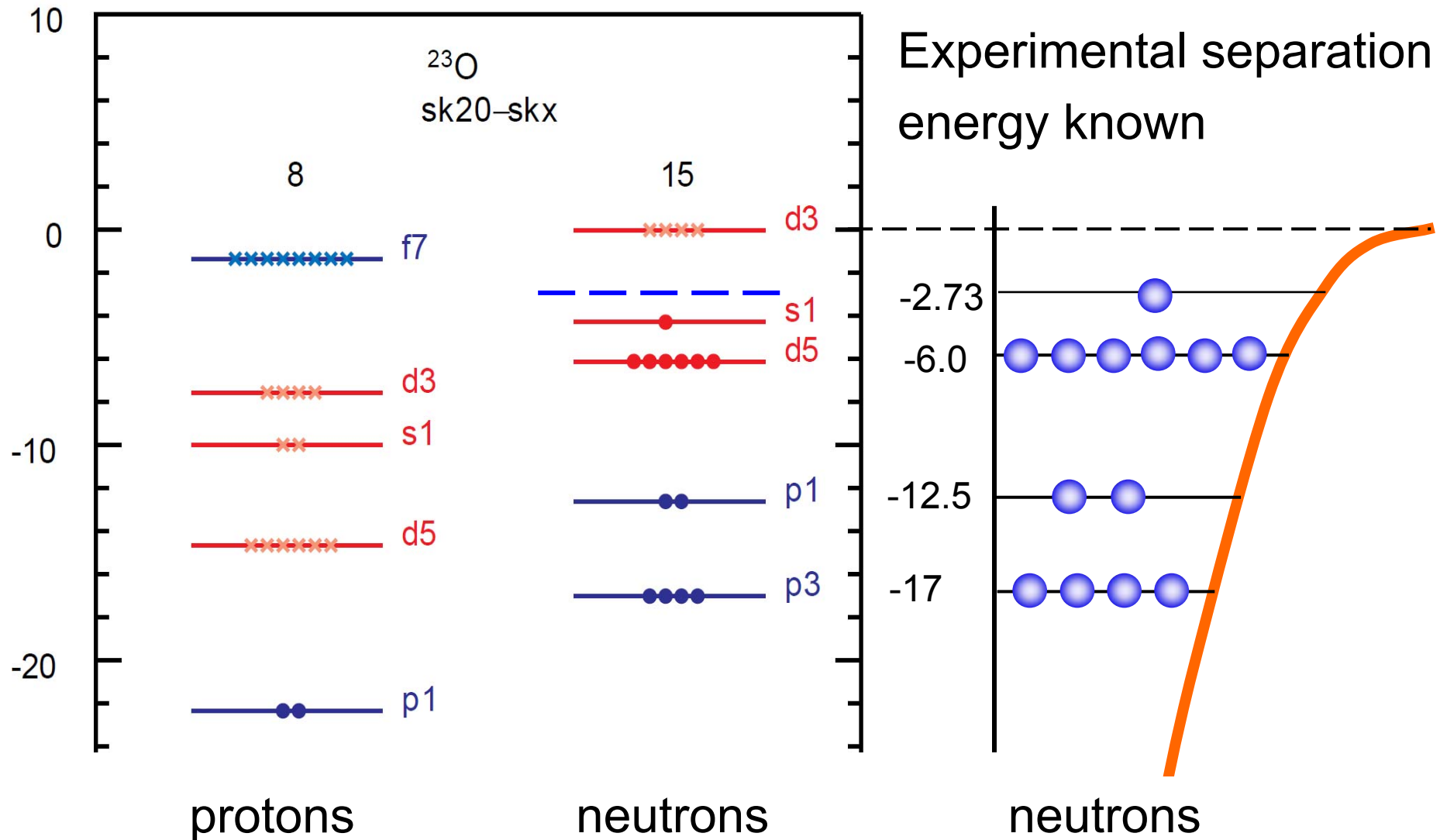


$$V_{\ell s}(r) \vec{\ell} \cdot \vec{s}$$

$$V(r) + V_{so}(r) \vec{\ell} \cdot \vec{s}$$

$$V_{so}(r) < 0$$

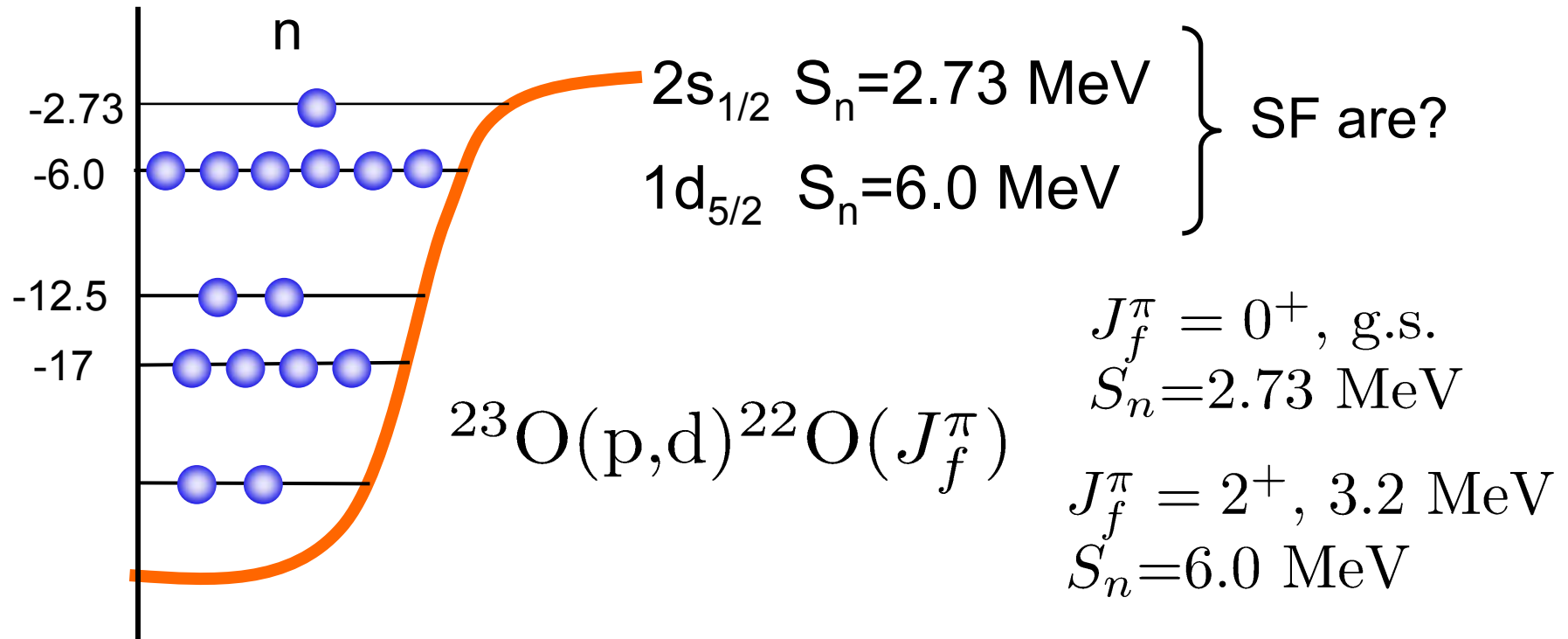
Example: What is involved – take neutron from ^{23}O



Hartree-Fock mean field calculation

Independent particle – (p,d) reaction

Single neutron removal from $^{23}\text{O} \equiv [1d_{5/2}]^6 [2s_{1/2}]$



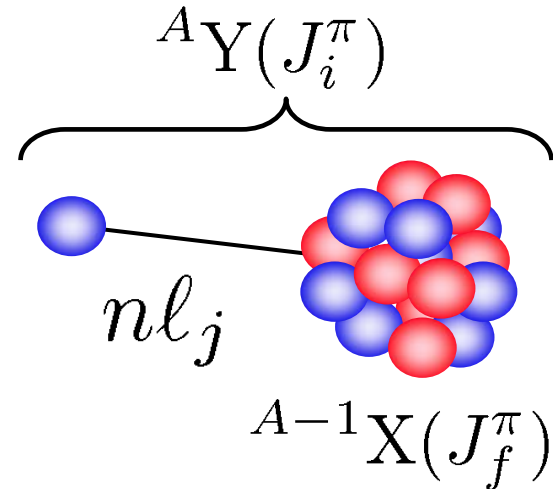
transfer reaction code(s) available at:
<http://www.nucleartheory.net/NPG/code.htm>

Bound states – spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.

$$\phi_{n\ell j}^m(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma | jm) \frac{u_{n\ell j}(r)}{r} Y_{\ell}^{\lambda}(\hat{r}) \chi_s^{\sigma},$$

$$\int_0^{\infty} [u_{n\ell j}(r)]^2 dr = 1$$



The potential model wave function approximates the overlap function of the A and A–1 body wave functions (A and A–n in the case of an n-body cluster) i.e. the overlap

$$\langle \ell j, \vec{r}, A-1 X(J_f^\pi) | A Y(J_i^\pi) \rangle \rightarrow I_{\ell j}(r), \quad \int_0^{\infty} [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

$S(\dots)$ is a spectroscopic factor, that scales the normalised single-particle wave function/overlap/form-factor

Connection to many-body structure calculations (1)

$$\langle \alpha, \vec{r}, {}^{A-1}X(J_f^\pi) | {}^A Y(J_i^\pi) \rangle$$

If we describe many body states by single Slater determinants, since these must be antisymmetric

$$\langle 1 \dots A | {}^A Y \rangle \equiv \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_1(1) & \phi_2(1) & \dots & \phi_A(1) \\ \phi_1(2) & \phi_2(2) & \dots & \phi_A(2) \\ \dots & \dots & \dots & \dots \\ \phi_1(A) & \phi_2(A) & \dots & \phi_A(A) \end{vmatrix}$$

then, for A identical particles (isospin) [or if (n,p), then N or Z]

$$\langle \alpha, \vec{r}, {}^{A-1}X(J_f^\pi) | {}^A Y(J_i^\pi) \rangle = \frac{1}{\sqrt{A}} \phi_\alpha(\vec{r})$$

The A factor is not usually carried: it cancels in cross sections that have an A multiplier to account for each identical particle.

Connection to many-body structure calculations (2)

$$\langle \alpha, \vec{r}, {}^{A-1}X(J_f^\pi) | {}^A Y(J_i^\pi) \rangle = \frac{1}{\sqrt{A}} \phi_\alpha(\vec{r})$$

Here the radial wave function (form factor) is normalised. In a reaction that removes a nucleon from a given orbital then, if a sub-shell is filled in the initial nucleus there are $(2j+1)$ nucleons available with a given (j, ℓ) to contribute.

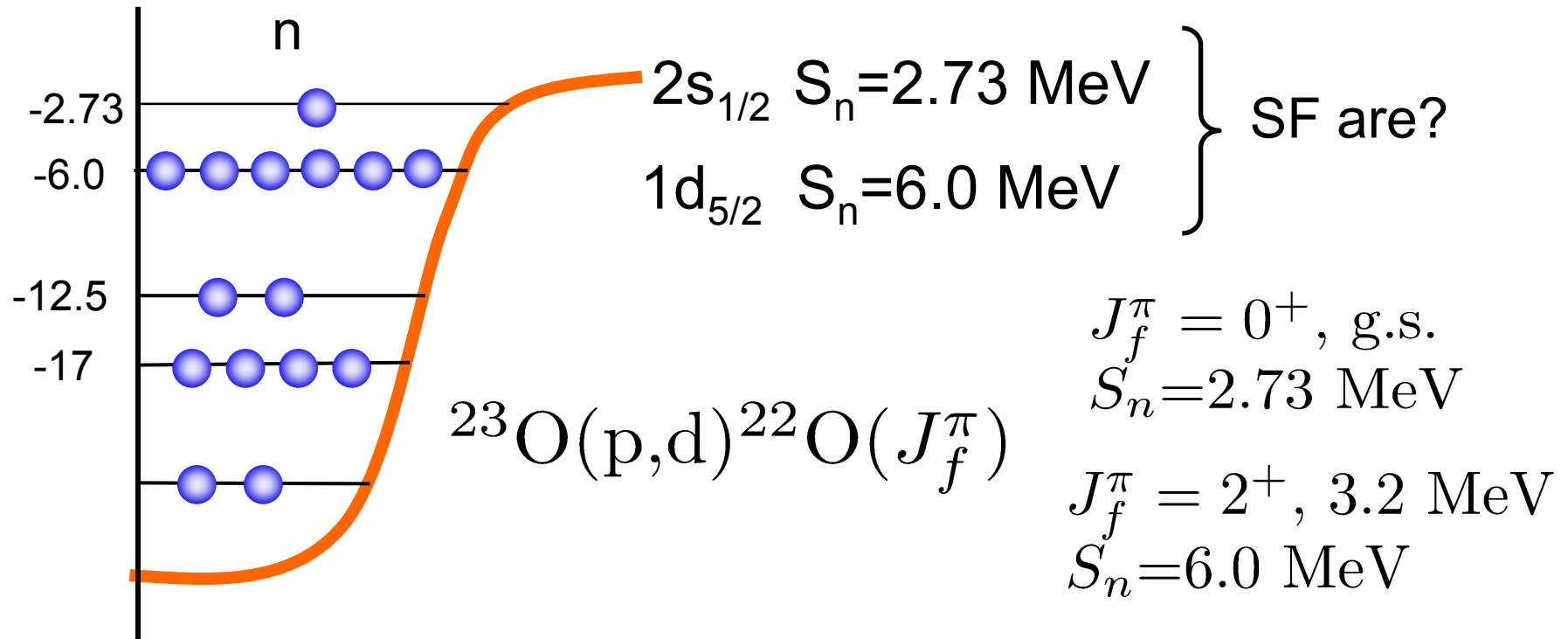
So, more generally (non-single Slater determinant) many-body structure models calculate and provided overlaps as:

$$\langle j\ell, \vec{r}, {}^{A-1}X(J_f^\pi) | {}^A Y(J_i^\pi) \rangle = \frac{\sqrt{S(J_i, J_f j\ell)}}{\sqrt{A}} \phi_{j\ell m}(\vec{r})$$

So, S multiplies the cross section calculated with a normalised form-factor. The S are defined so that (for given n, j, ℓ quantum numbers) their sum over final states is the mean number of nucleons occupying the given sub-shell (sum-rule).

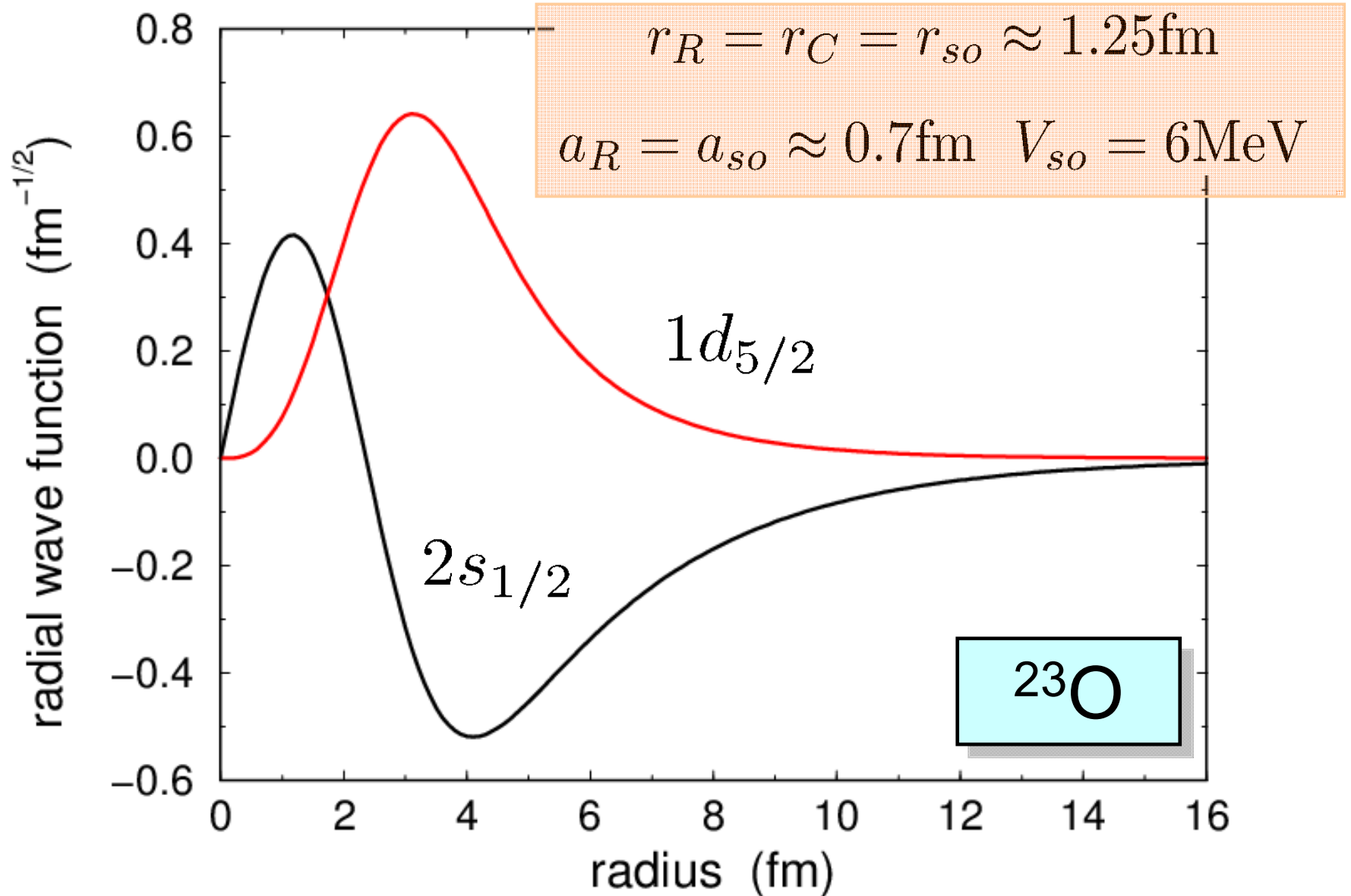
Independent particle – (p,d) reaction

Single neutron removal from $^{23}\text{O} \equiv [1d_{5/2}]^6 [2s_{1/2}]$



transfer reaction code(s) available at:
<http://www.nucleartheory.net/NPG/code.htm>

Neutron bound state wave functions



Transfer reactions e.g. (p,d) – coordinate systems

$$\vec{R} = [\vec{r}_n + \vec{r}_p]2$$

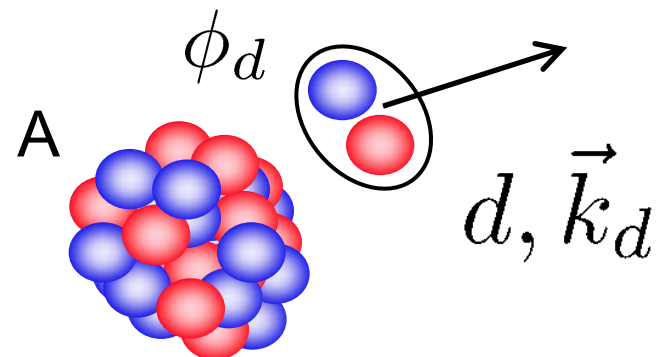
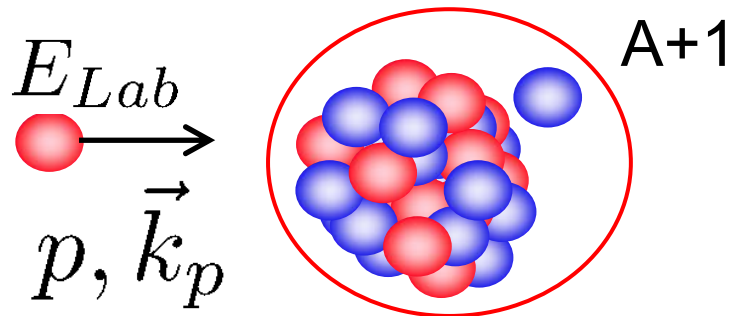
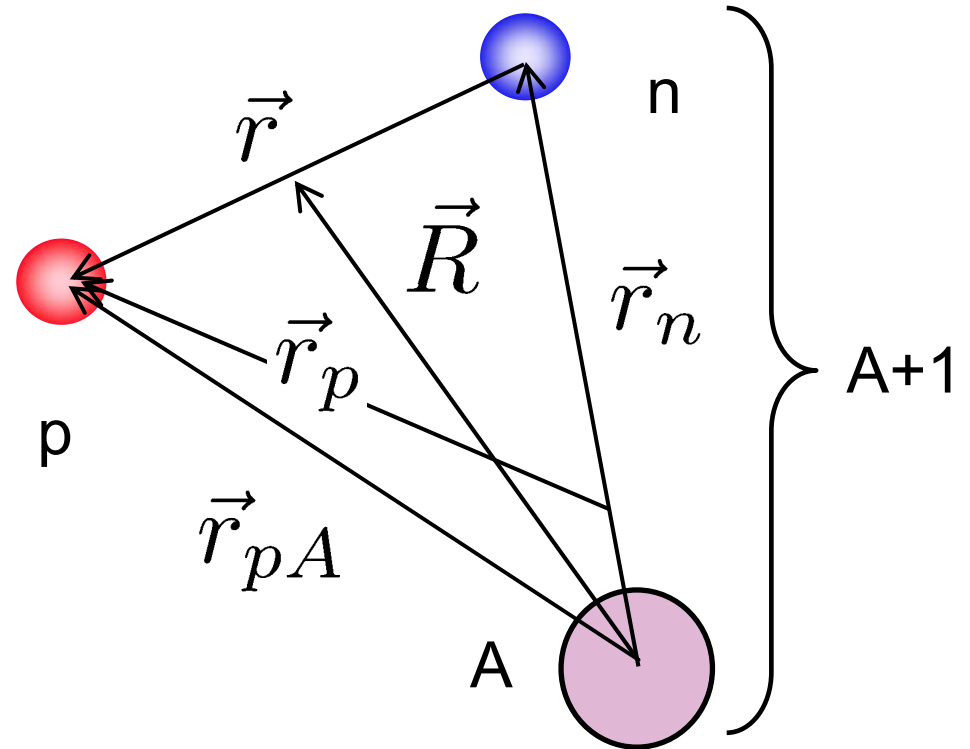
$$\vec{r}_{pA} = \vec{R} + \vec{r}/2$$

$$\vec{r}_n = \vec{R} - \vec{r}/2$$

$$V_p(\vec{r}_{pA}) = V_p(\vec{R} + \vec{r}/2)$$

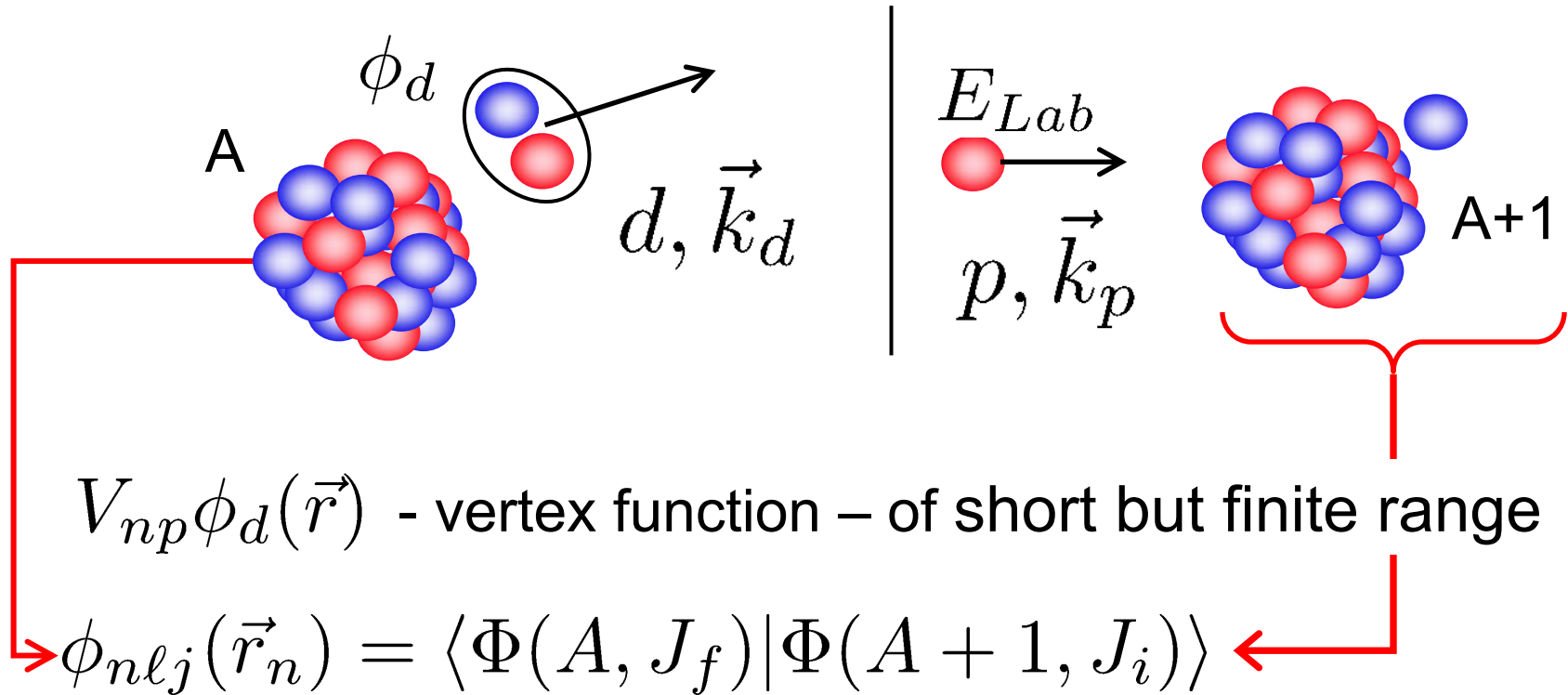
$$V_n(\vec{r}_n) = V_n(\vec{R} - \vec{r}/2)$$

$$V_{np}(\vec{r})$$



Transfer reaction transition amplitudes - DWBA

$$T(p, d) = \langle \underbrace{\chi_{d, \vec{k}_d}^{(-)} \phi_d \Phi(A, J_f)}_{\text{exit channel}} | V_{np} | \underbrace{\chi_{p, \vec{k}_p}^{(+)} \Phi(A + 1, J_i)}_{\text{entrance channel}} \rangle$$

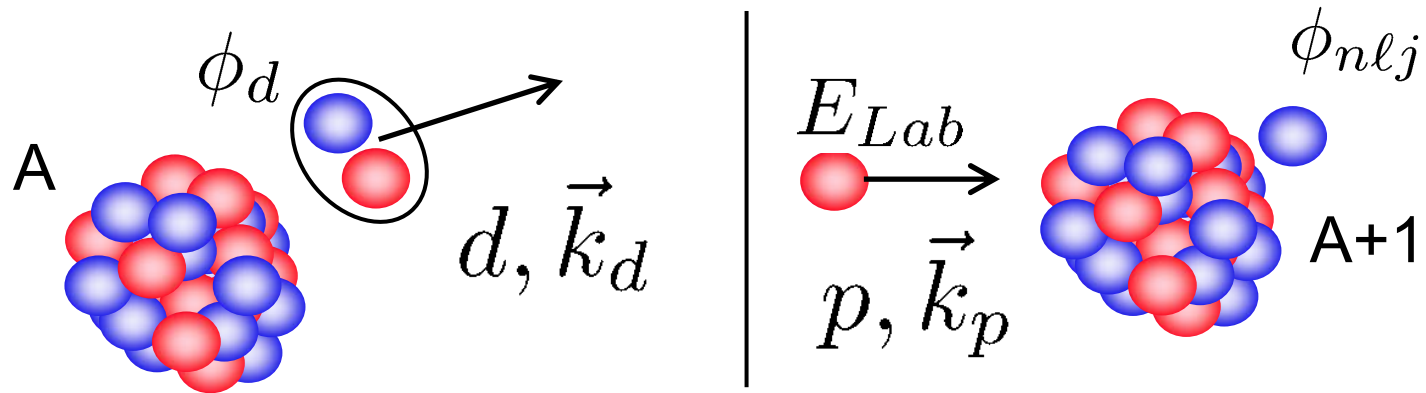


Transfer reaction – involve structure via overlaps

$$T(p, d) = \langle \underbrace{\chi_{d, \vec{k}_d}^{(-)} \Phi(A, J_f)}_{\phi_d} | V_{np} | \underbrace{\chi_{p, \vec{k}_p}^{(+)} \Phi(A+1, J_i)}_{\phi_{n\ell j}} \rangle$$

$$\phi_{n\ell j}(\vec{r}_n) = \langle \Phi(A, J_f) | \Phi(A+1, J_i) \rangle$$

$$T(p, d) = \langle \chi_{d, \vec{k}_d}^{(-)}(\vec{R}) \phi_d(r) | V_{np} | \chi_{p, \vec{k}_p}^{(+)}(\vec{r}_p) \phi_{n\ell j}(\vec{r}_n) \rangle$$

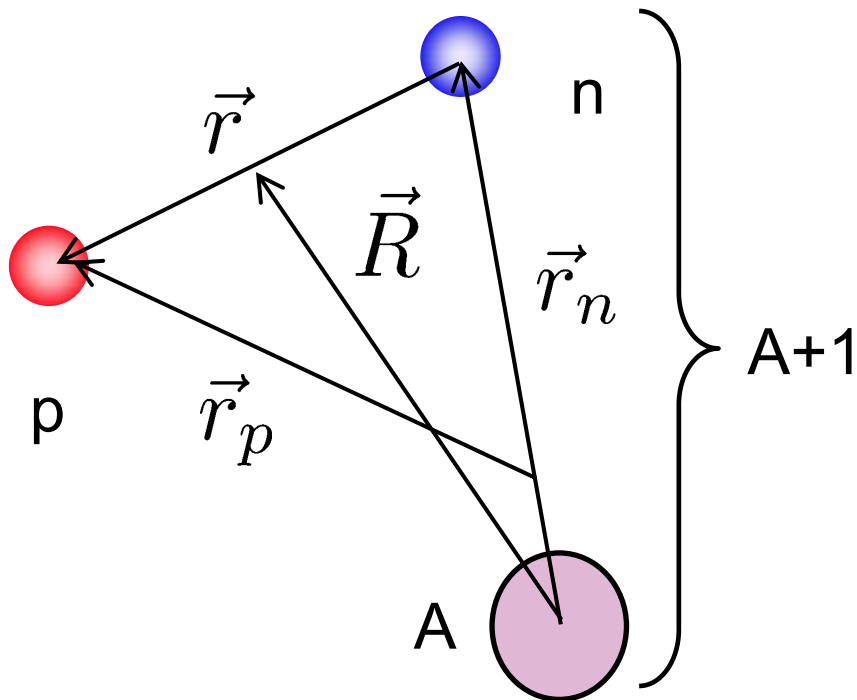


Transfer reaction – plane waves for insight (1)

$$T(p, d) = \langle \chi_{d, \vec{k}_d}^{(-)}(\vec{R}) \phi_d(r) | V_{np} | \chi_{p, \vec{k}_p}^{(+)}(\vec{r}_p) \phi_{n\ell j}(\vec{r}_n) \rangle$$

when using plane waves for p and d – i.e. ‘weak’ distortions

$$T_{pw} \approx \int d\vec{r}_n \int d\vec{r} e^{-i\vec{k}_d \cdot \vec{R}} V_{np} \phi_d(r) e^{i\vec{k}_p \cdot \vec{r}_p} \phi_{n\ell j}(\vec{r}_n)$$



$$\vec{r}_p = \gamma \vec{r}_n + \vec{r}$$

$$\gamma = A/(A+1) \approx 1$$

$$\vec{R} = \vec{r}_n + \vec{r}/2$$

$$\vec{k}_p \cdot \vec{r}_p - \vec{k}_d \cdot \vec{R} =$$

$$[\gamma \vec{k}_p - \vec{k}_d] \cdot \vec{r}_n +$$

$$[\vec{k}_p - \vec{k}_d/2] \cdot \vec{r}$$

Transfer reaction – plane waves for insight (2)

$$T_{pw} \approx \int d\vec{r}_n \int d\vec{r} e^{-i\vec{k}_d \cdot \vec{R}} V_{np} \phi_d(r) e^{i\vec{k}_p \cdot \vec{r}_p} \phi_{n\ell j}(\vec{r}_n)$$

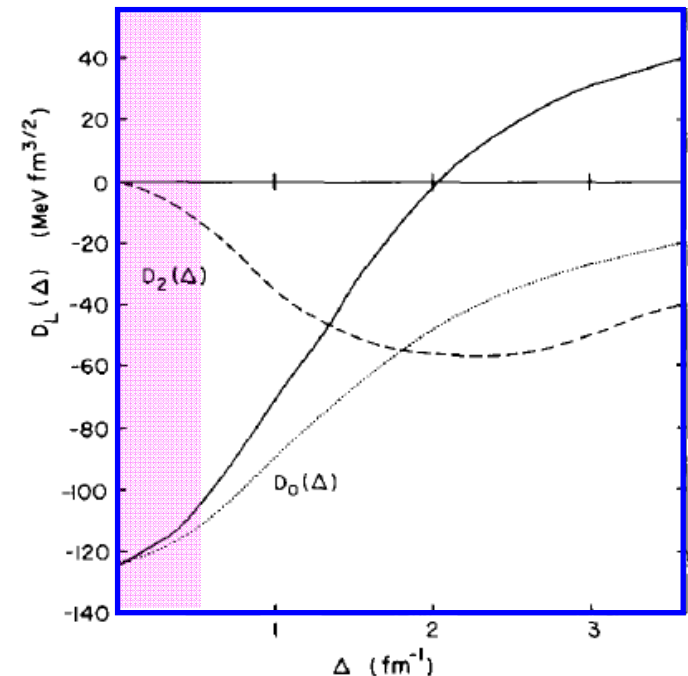
$$\begin{aligned} \vec{k}_p \cdot \vec{r}_p - \vec{k}_d \cdot \vec{R} &= [\gamma \vec{k}_p - \vec{k}_d] \cdot \vec{r}_n + [\vec{k}_p - \vec{k}_d/2] \cdot \vec{r} \\ &= \vec{q}_n \cdot \vec{r}_n + \vec{q} \cdot \vec{r} \end{aligned}$$

$$T_{pw} \approx \int d\vec{r}_n e^{i\vec{q}_n \cdot \vec{r}_n} \phi_{n\ell j}(\vec{r}_n) \times \int d\vec{r} e^{i\vec{q} \cdot \vec{r}} V_{np} \phi_d(r)$$

$$\sigma_{pw} \propto |F(\vec{q}_n)|^2 D^2(q)$$

and expanding $e^{i\vec{q} \cdot \vec{r}}$ for small q
 $D(q) = D_0[1 - \beta^2 q^2 + \dots]$

$$\begin{aligned} D_0 &= -122.5 \text{ MeV fm}^{3/2} \\ \beta &\approx 0.75 \text{ fm} \end{aligned}$$



Transfer reactions – “matching” considerations

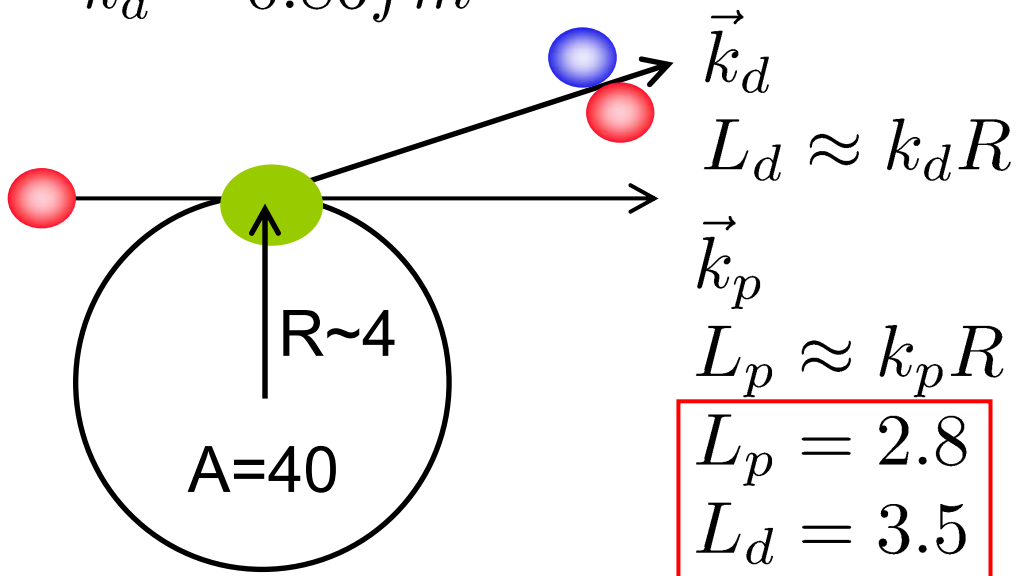
$$\vec{q}_n = [\gamma \vec{k}_p - \vec{k}_d] \quad \vec{q} = [\vec{k}_p - \vec{k}_d/2] \cdot \vec{r}$$

$$T_{pw} \approx \int d\vec{r}_n e^{i\vec{q}_n \cdot \vec{r}_n} \phi_{n\ell j}(\vec{r}_n) \times \int d\vec{r} e^{i\vec{q} \cdot \vec{r}} V_{np} \phi_d(r)$$

e.g. (p,d) on A=40 at $E_p=10$ MeV, say $S_n=4$ MeV, $R_{40} \sim 4$ fm

$$k_p = 0.68 \text{ fm}^{-1} \quad \Delta L = |L_d - L_p| \approx 0.7$$

$$k_d = 0.86 \text{ fm}^{-1}$$



with $E_p=20$ MeV

$$k_p = 0.96 \text{ fm}^{-1}$$

$$k_d = 1.27 \text{ fm}^{-1}$$

$$L_p = 3.9$$

$$L_d = 5.2$$

$$\Delta L = |L_d - L_p| \approx 1.3$$

Phenomenological optical potentials – where?

[C.M. Perey and F.G. Perey](#), At. Data Nucl. Data Tables 17, 1 (1976)
Compilation for many systems

[J.J.H. Menet, E.E. Gross, J.J. Malanify, and A. Zucker](#), Phys. Rev. C 4, 1114 (1971) – for nucleons

[R.L. Varner, W.J. Thompson, T.L. McAbee, E.J. Ludwig, and T.B. Clegg](#), Phys. Rep. 201, 57 (1991) – Chapel Hill 89 potential – for nucleons

[F.D. Becchetti, Jr. and G.W. Greenlees](#), Phys. Rev. 182, 1190 (1969)
– ‘old faithful’ parameterisation – for nucleons

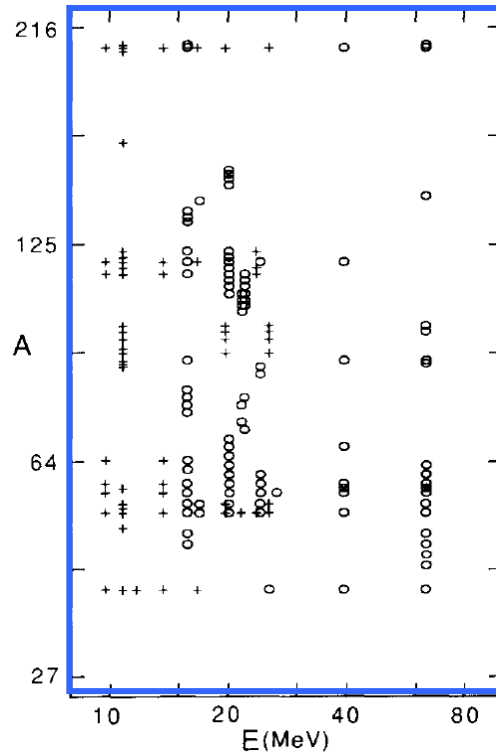
[W.W. Daehnick, J.D. Childs, and Z. Vrcelj](#), Phys. Rev. C 21, 2253 (1980) – good parameter set – for deuterons

[J.M. Lohr and W. Haeberli](#), Nucl. Phys. A232, 381 (1974) – for low energy deuterons

..... and many many more ... but many many gaps

<http://www-nds.iaea.org/RIPL-2/optical.html>

Global optical potentials – e.g. CH91 for nucleons



A GLOBAL NUCLEON OPTICAL MODEL POTENTIAL*

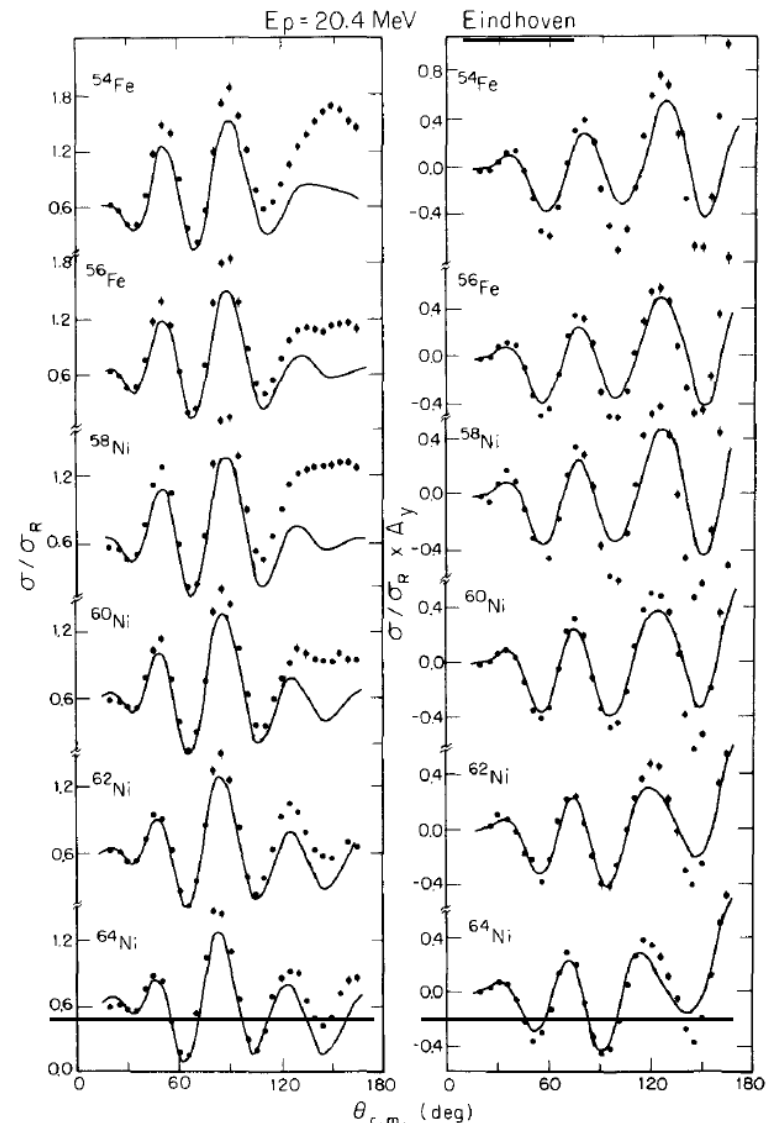
R.L. VARNER

*Oak Ridge National Laboratory, Oak Ridge, TN 37831-6368, USA
and Triangle Universities Nuclear Laboratory, Duke University, Durham, NC 27706, USA*

and

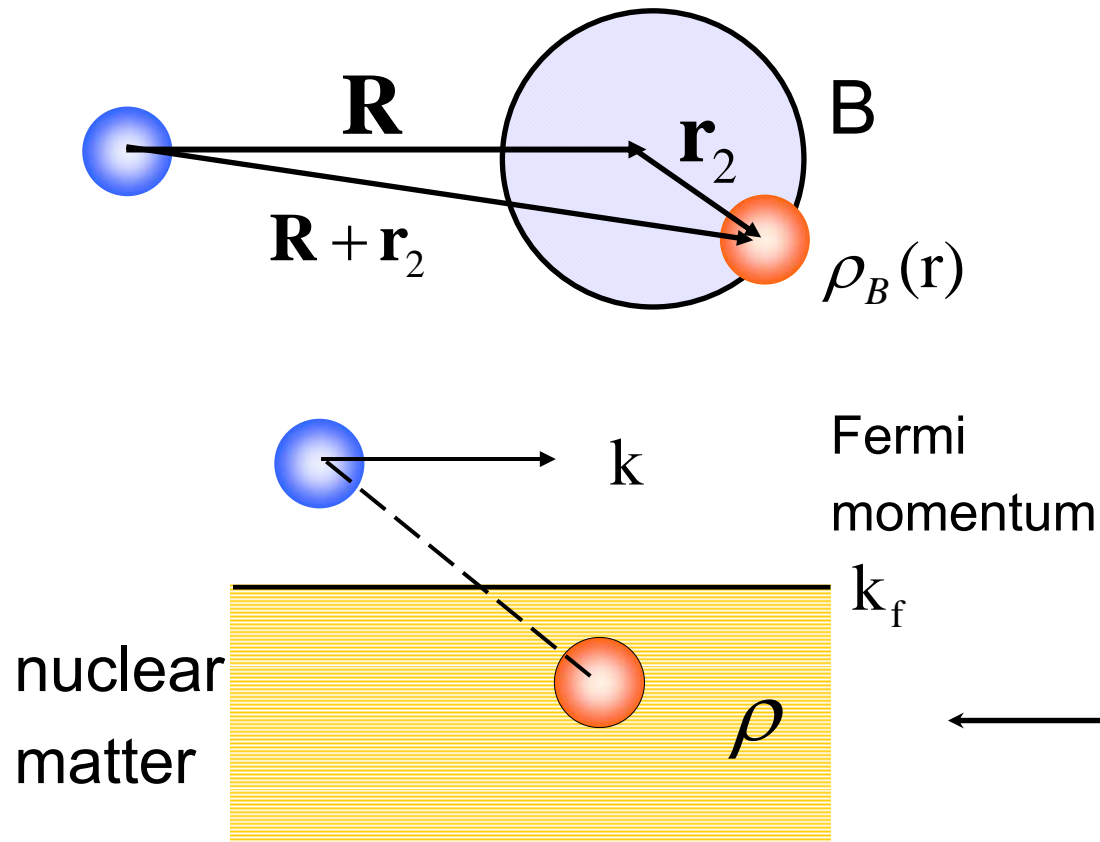
W.J. THOMPSON, T.L. McABEE**, E.J. LUDWIG and T.B. CLEGG

PHYSICS REPORTS (Review Section of Physics Letters) 201, No. 2 (1991) 57–119. North-Holland



Theoretical nucleon potential – based on density

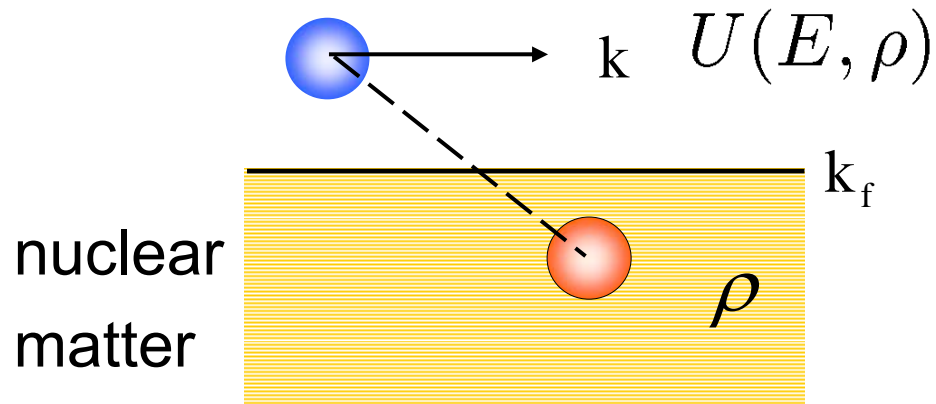
$$V_{NB}(\mathbf{R}) = \int d\mathbf{r}_2 \rho_B(\mathbf{r}_2) v_{NN}(\mathbf{R} + \mathbf{r}_2)$$



include the effect
of NN interaction
in the “nuclear
medium” – Pauli
blocking of pair
scattering into
occupied states
 $\rightarrow v_{NN}(\rho, \mathbf{r})$
(e.g. JLM)
But as $E \rightarrow$ high

$$v_{NN} \rightarrow v_{NN}^{\text{free}}$$

JLM interaction – local density approximation

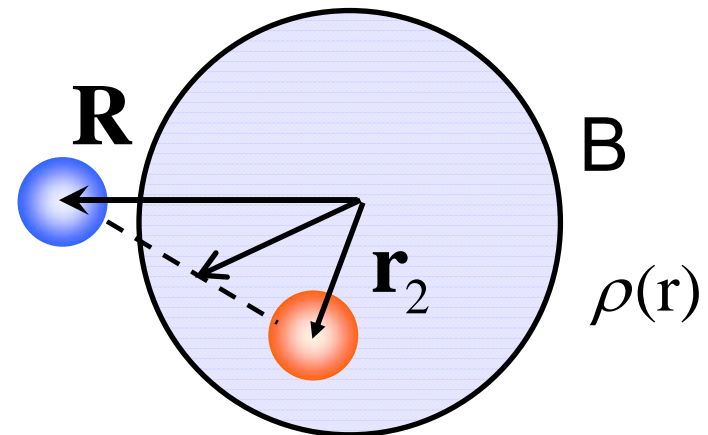


complex and density dependent interaction

$$v_{NN}(r) = \frac{U(E, \rho)}{\rho} f(r)$$

$$f(r) = (\sqrt{\pi}t)^{-3} \exp(-r^2/t^2)$$

For finite nuclei, what value of density should be used in calculation of nucleon-nucleus potential? Usually the local density at the mid-point of the two nucleon positions \mathbf{r}_x



$$U_B(R) = V_B(R) + iW_B(R) = \int d\vec{r}_2 \rho_B(r_2) \frac{U(E, \rho(r_x))}{\rho(r_x)} f(r)$$

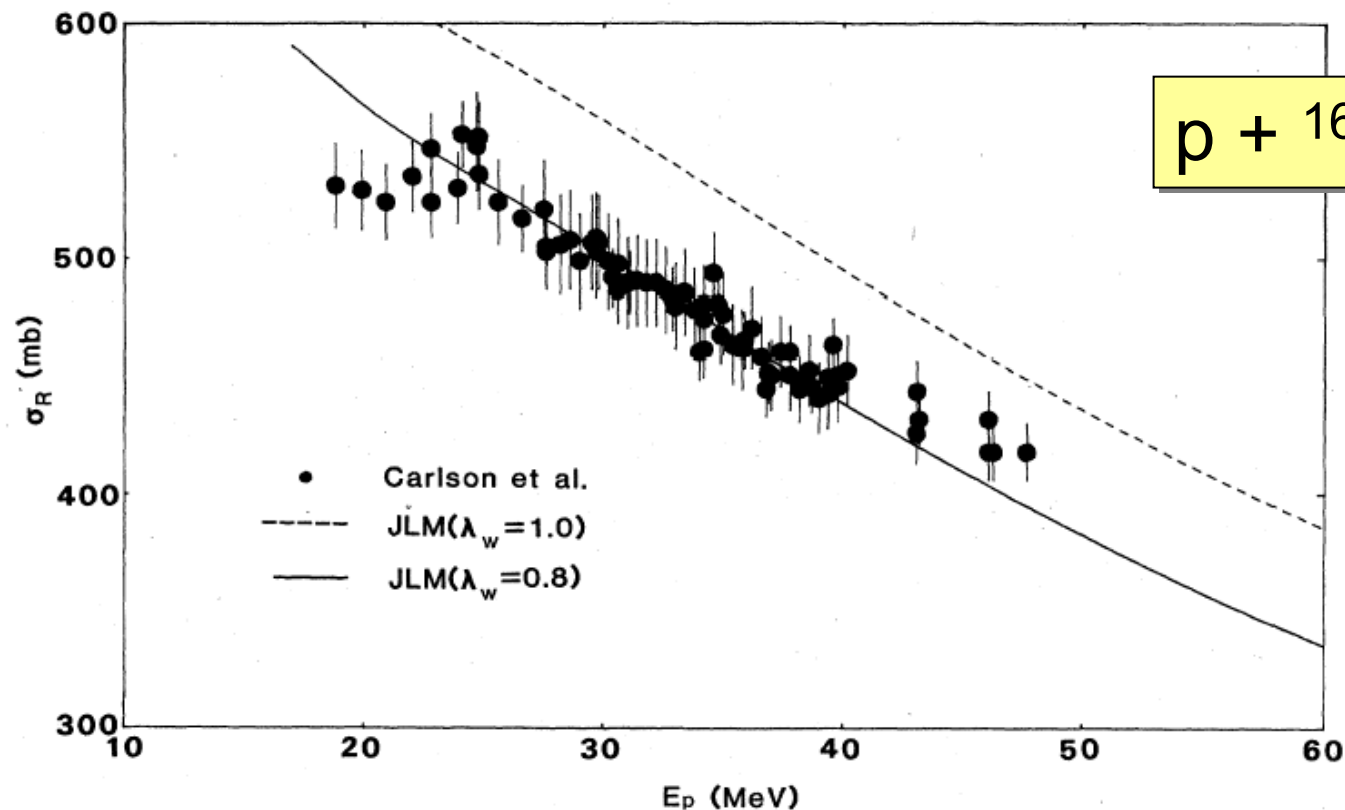
JLM interaction – fine tuning

Strengths of the real and imaginary parts of the potential can be adjusted based on experience of fitting data.

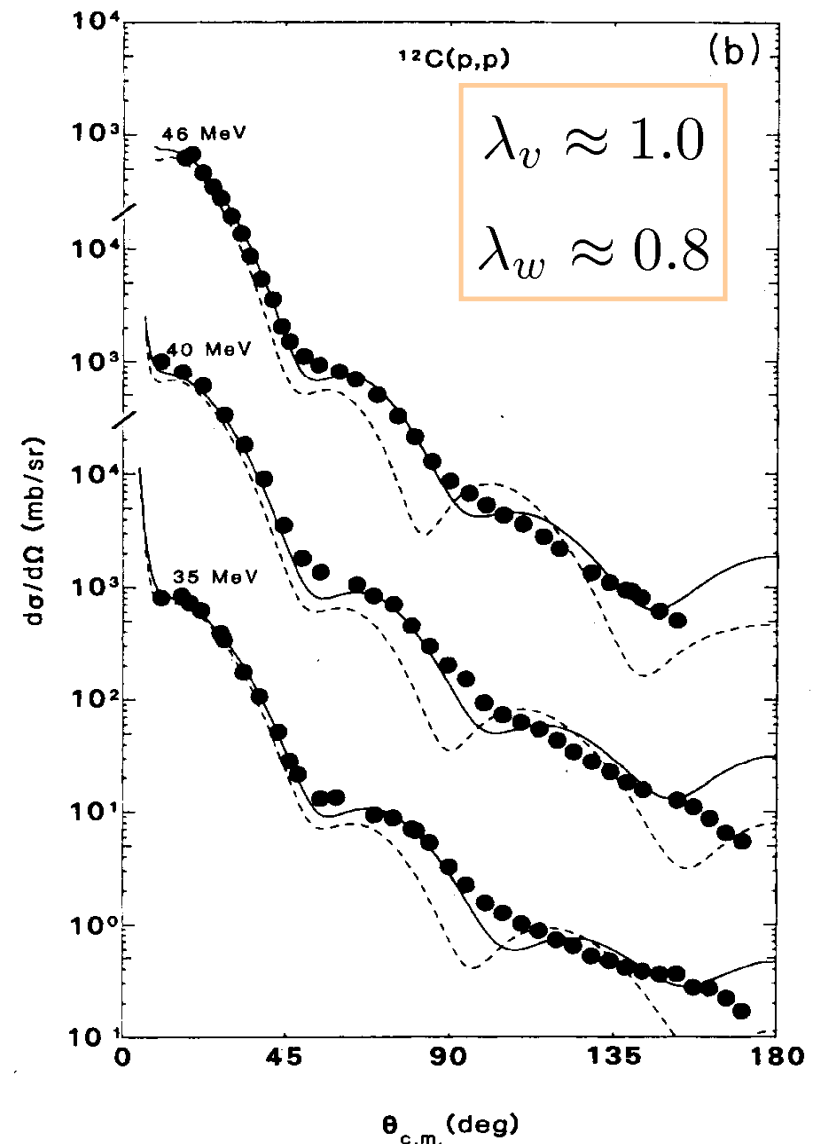
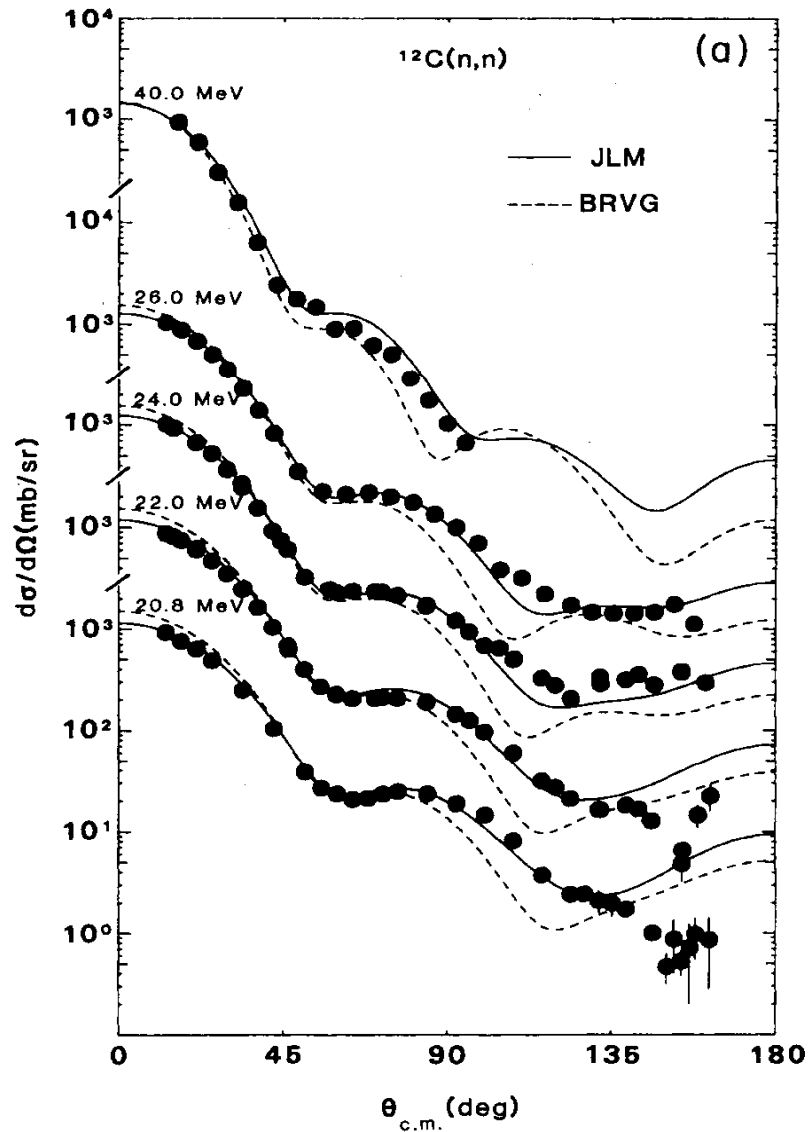
$$U_B(R) = \lambda_v V_B(R) + i\lambda_w W_B(R)$$

$$\lambda_v \approx 1.0$$

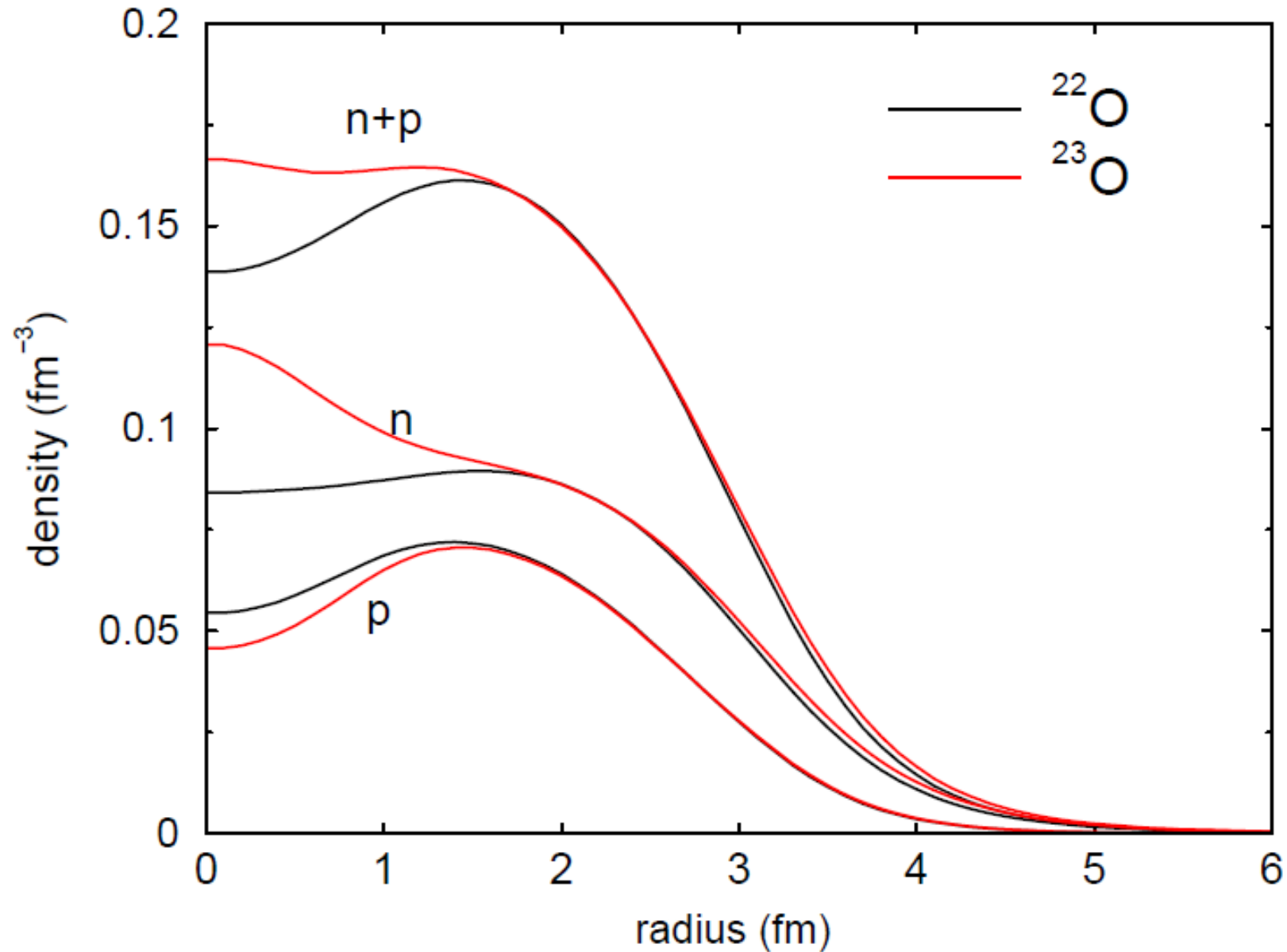
$$\lambda_w \approx 0.8$$



JLM folded nucleon-nucleus optical potentials



Neutron: proton: nucleon radial densities (HF)

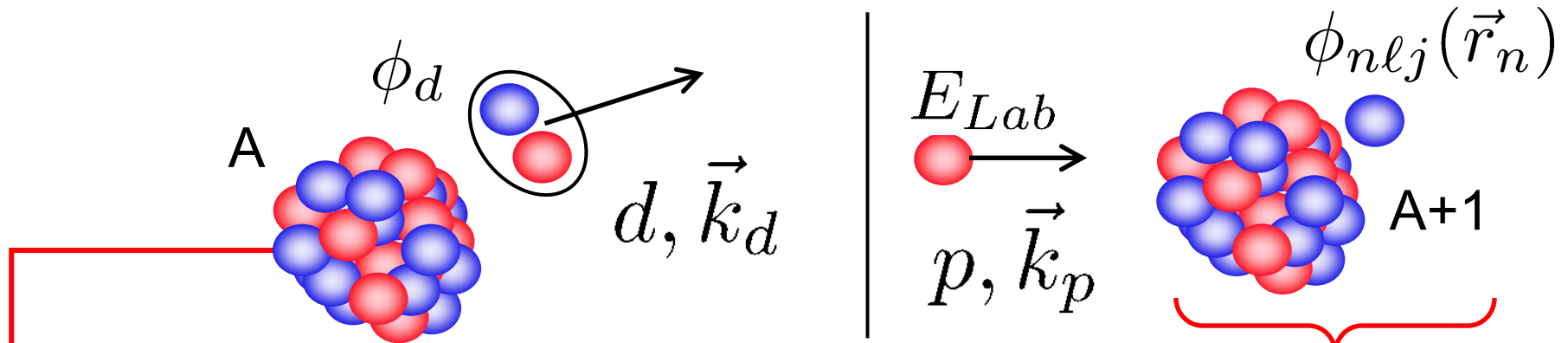


Transfer reaction transition amplitudes - DWBA

$$T(p, d) = \langle \underbrace{\chi_{d, \vec{k}_d}^{(-)}(\vec{R}) \phi_d(r)}_{\text{exit channel}} | V_{np} | \underbrace{\chi_{p, \vec{k}_p}^{(+)}(\vec{r}_p) \phi_{n\ell j}(\vec{r}_n)}_{\text{entrance channel}} \rangle$$

exit channel

entrance channel



$$V_{np} \phi_d(\vec{r}) \approx D_0 \delta(\vec{r}) \quad - \text{short range}$$

$$\rightarrow \phi_{n\ell j}(\vec{r}_n) = \langle \Phi(A, J_f) | \Phi(A+1, J_i) \rangle \leftarrow$$

Global optical potentials – e.g. for deuterons

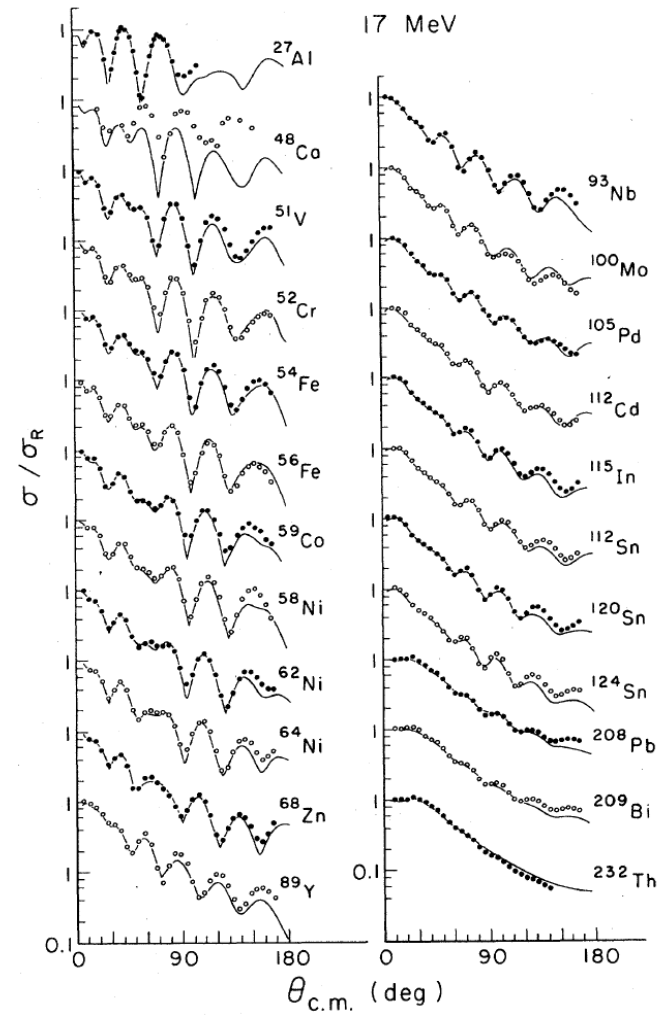
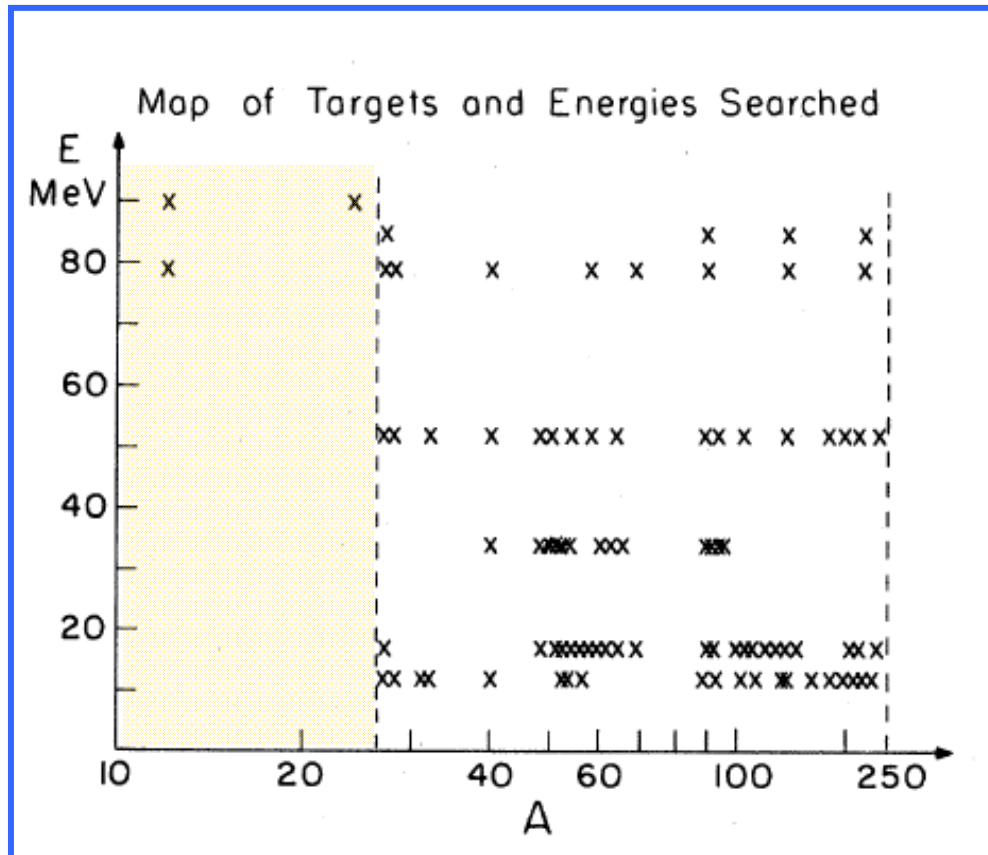
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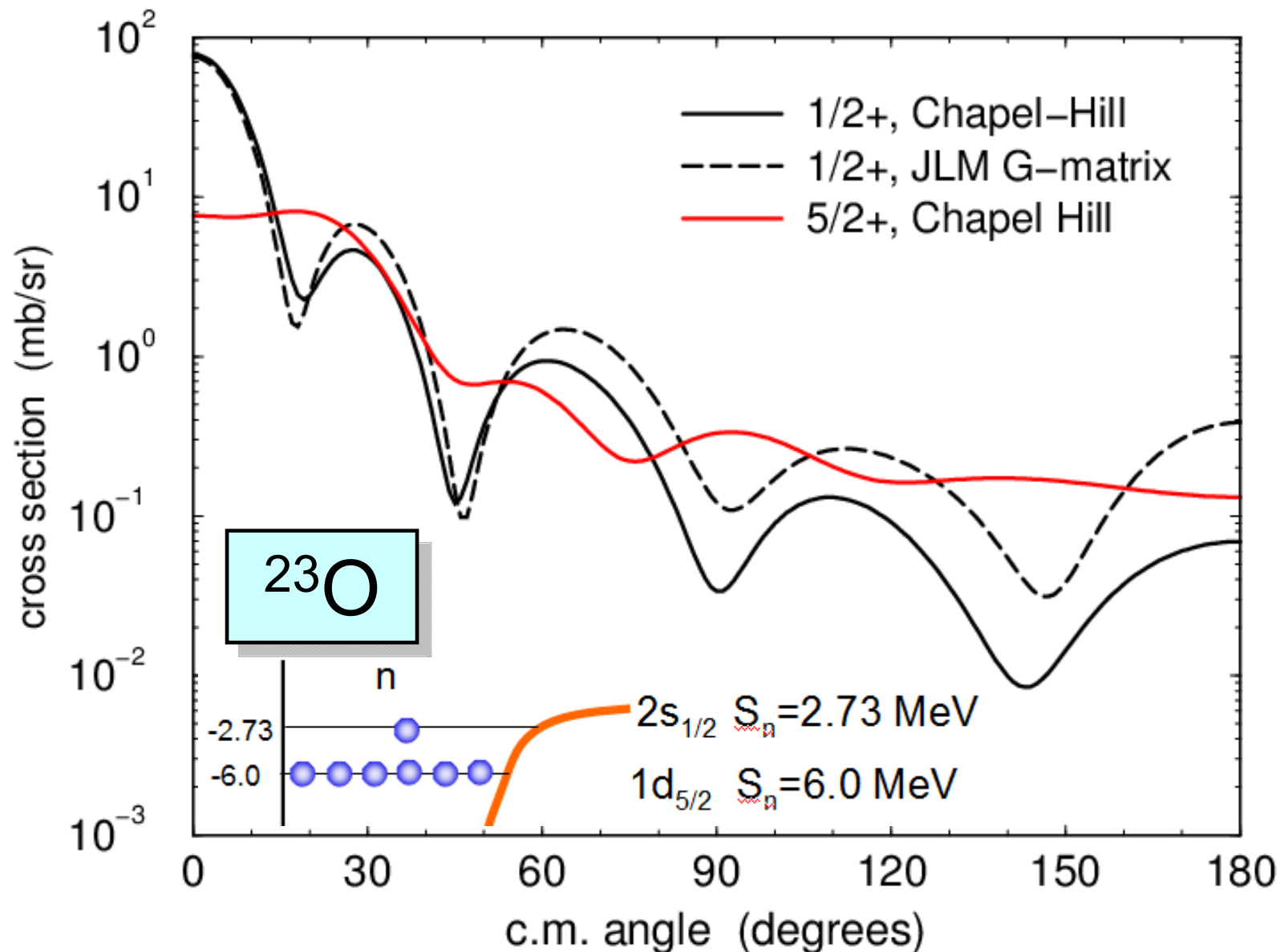
JUNE 1980

Global optical model potential for elastic deuteron scattering from 12 to 90 MeV

W. W. Daehnick, J. D. Childs,* and Z. Vrcelj



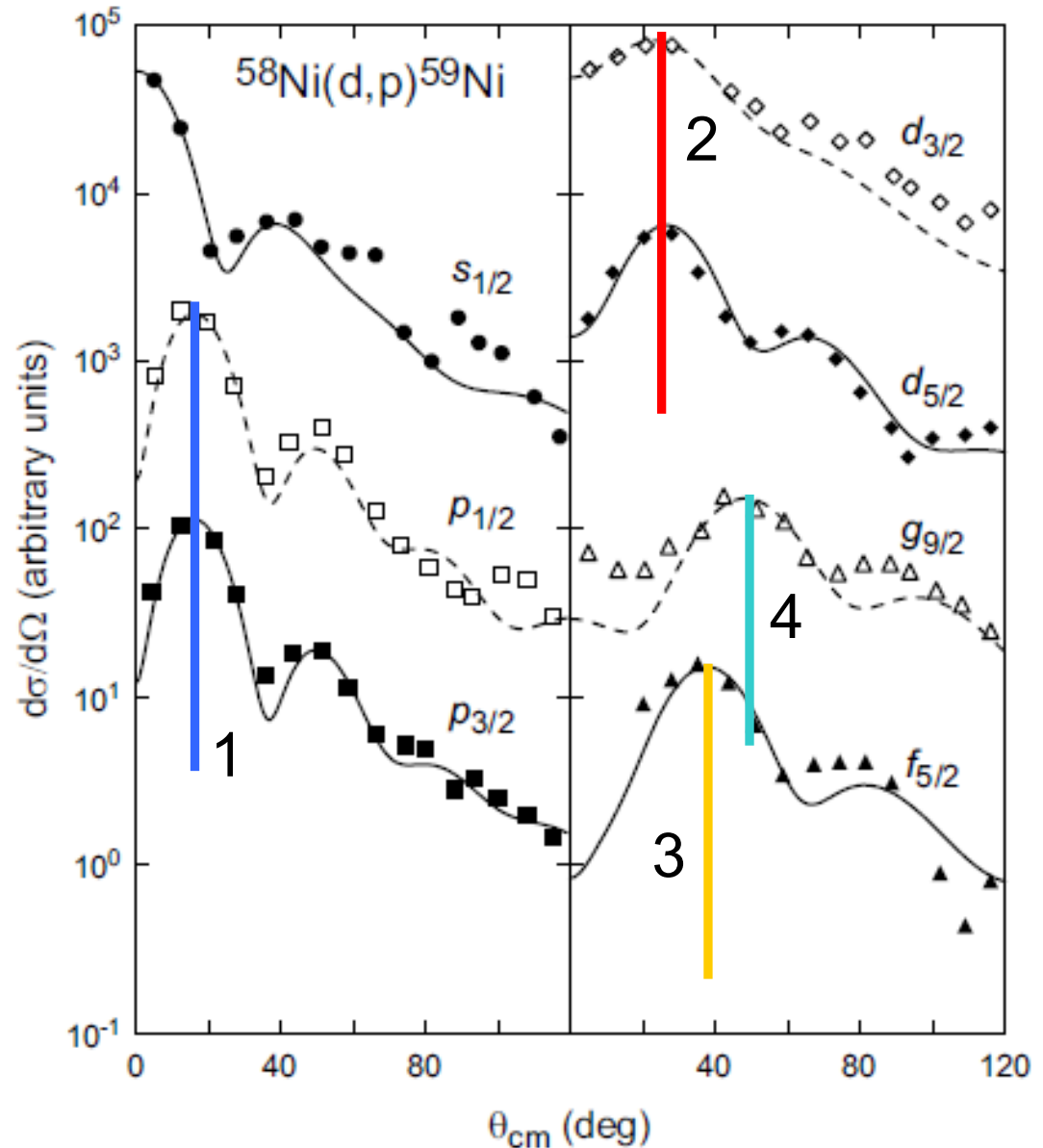
Calculated (p,d) transfer (pick-up) cross sections



Single-particle spectroscopy – angular distributions

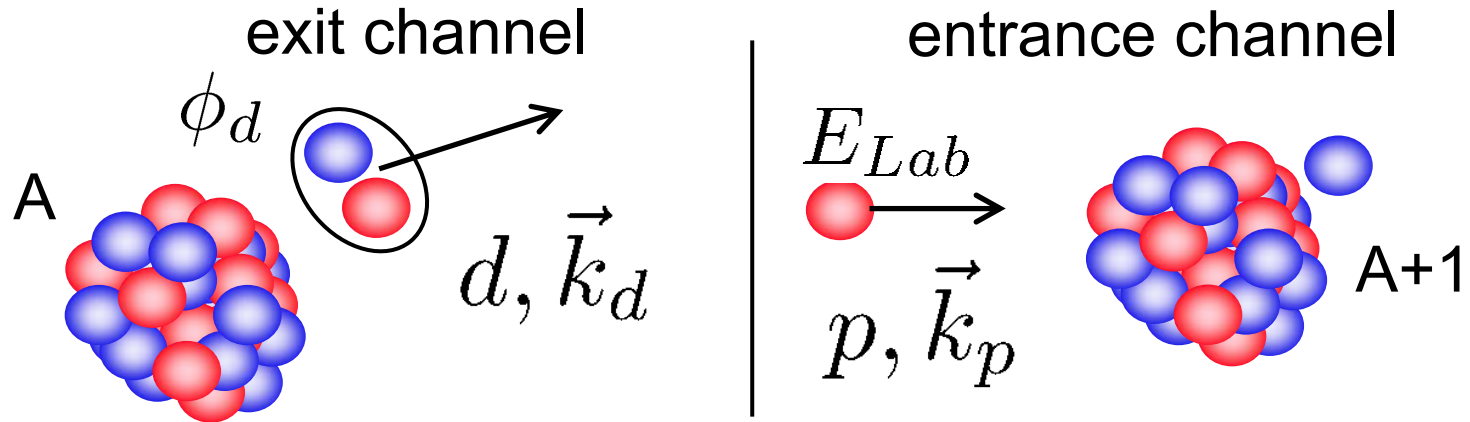
Data: M.S. Chowdhury and
H.M. Sen Gupta Nucl.
Phys. **A205**, 454 (2005)

Figure: Isotope Science
Facility (ISF) White Paper,
NSCL (2007)



Transfer reaction – beyond DWBA - breakup

$$T(p, d) = \underbrace{\langle \psi_{d, \vec{k}_d}^{(-)} \Phi(A, J_f) |}_{\text{exit channel}} V_{np} \underbrace{ \chi_{p, \vec{k}_p}^{(+)} \Phi(A+1, J_i) \rangle}_{\text{entrance channel}}$$



$$[T_R + \mathcal{H}_{np} + V_p(\vec{r}_p) + V_n(\vec{r}_n) - E] \psi_{d, \vec{k}_d}^{(+)} = 0$$

$$\psi_{d, \vec{k}_d}^{(+)} = \exp(i\vec{k}_d \cdot \vec{R}) \phi_d(r) + \text{outgoing waves}$$

$$\mathcal{H}_{np} \phi_d = \varepsilon_0 \phi_d, \quad \mathcal{H}_{np} \hat{\phi}_i = \hat{\varepsilon}_i \hat{\phi}_i$$

$$\psi_{d, \vec{k}_d}^{(+)}(\vec{r}, \vec{R}) = \chi_{d, \vec{k}_d}(\vec{R}) \phi_d(r) + \sum_i \chi_i(\vec{R}) \hat{\phi}_i(\vec{r})$$

Transfer reaction – three-body models - breakup

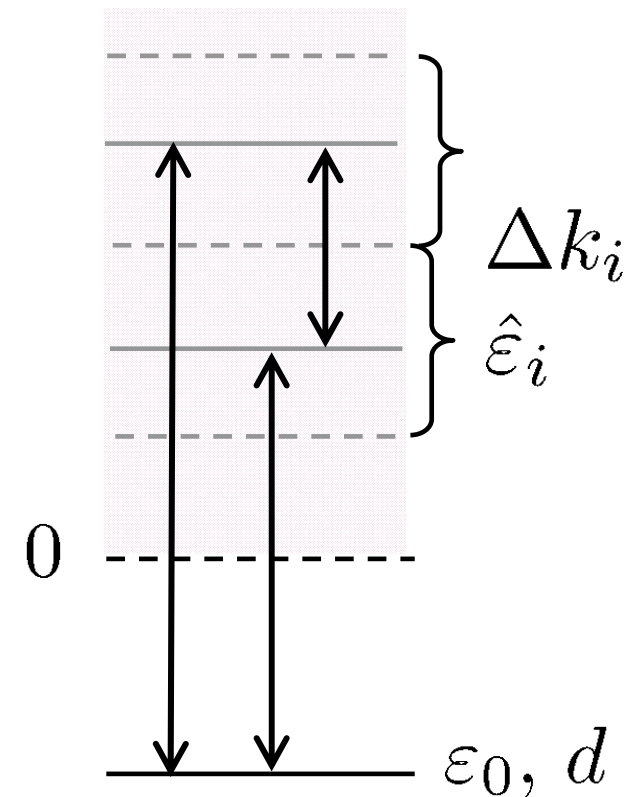
$$T(p, d) = \langle \psi_{d, \vec{k}_d}^{(-)} \Phi(A, J_f) | V_{np} | \chi_{p, \vec{k}_p}^{(+)} \Phi(A + 1, J_i) \rangle$$

$$\mathcal{H}_{np} \phi_d = \varepsilon_0 \phi_d, \quad \mathcal{H}_{np} \hat{\phi}_i = \hat{\varepsilon}_i \hat{\phi}_i$$

neutron is also transferred to unbound states (d^*) of the n-p system – represented by continuum bins – that are coupled to the deuteron g.s. for as long as the two nucleons remain within the range of

$$V_p(\vec{r}_{pA}), \quad V_n(\vec{r}_n)$$

These higher-order effects can be important in slower (lower-energy) reactions

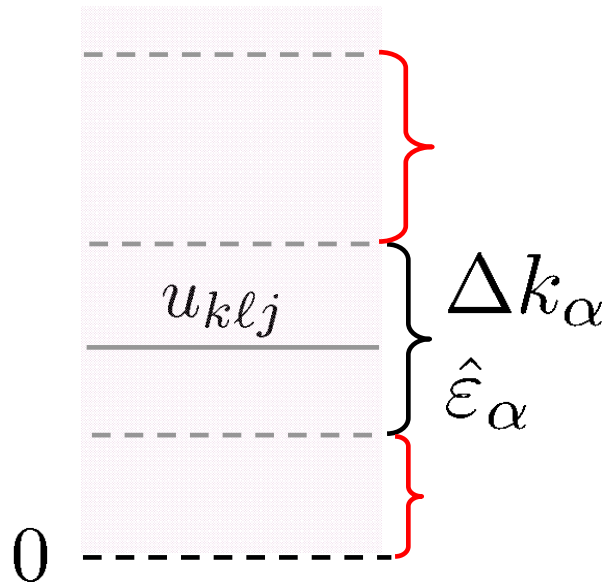


Treating breakup effects with continuum bins

Scattering states

$$u_{k\ell j}(r) \rightarrow e^{i\delta_{\ell j}} [\cos \delta_{\ell j} F_{\ell}(\eta, kr) + \sin \delta_{\ell j} G_{\ell}(\eta, kr)]$$

$$\int_0^{\infty} dr u_{k\ell j}(r) u_{k'\ell j}^*(r) = \frac{\pi}{2} \delta(k - k')$$



$$\hat{u}_{\alpha\ell j}(r) = \sqrt{\frac{2}{\pi N_{\alpha}}} \int_{\Delta k_{\alpha}} dk g(k) u_{k\ell j}(r)$$

$$N_{\alpha} = \int_{\Delta k_{\alpha}} dk [g(k)]^2 \quad \text{weight function}$$

orthonormal set

$$\int_0^{\infty} dr \hat{u}_{\alpha\ell j}^*(r) \hat{u}_{\beta\ell j}(r) = \delta_{\alpha\beta}$$

$$g(k) = 1 \quad g(k) = \sin \delta_{\ell j}$$

$\hat{\epsilon}_0$

Adiabatic three-body model – breakup made simpler

$$T(p, d) = \langle \psi_{d, \vec{k}_d}^{(-)} \Phi(A, J_f) | V_{np} | \chi_{p, \vec{k}_p}^{(+)} \Phi(A + 1, J_i) \rangle$$
$$[T_R + \mathcal{H}_{np} + V_p(\vec{r}_{pA}) + V_n(\vec{r}_n) - E] \psi_{d, \vec{k}_d}^{(+)}(\vec{r}, \vec{R}) = 0$$

Since to calculate the transfer amplitude we need the three body wave function only in regions where $V_{np} \neq 0$, $r \approx 0$

$$\mathcal{H}_{np} \rightarrow \varepsilon_0, \quad V_p(\vec{r}_{pA}) \rightarrow V_p(\vec{R}), \quad V_n(\vec{r}_n) \rightarrow V_n(\vec{R})$$

$$[T_R + \varepsilon_0 + V_p(\vec{R}) + V_n(\vec{R}) - E] \psi_{d, \vec{k}_d}^{(+)}(\vec{r} \approx 0, \vec{R}) = 0$$

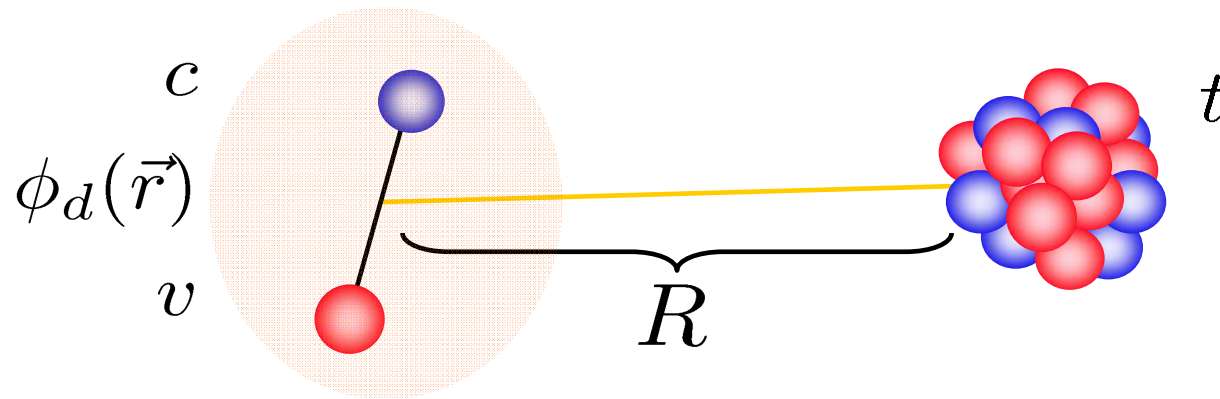
So, with $E_0 = E - \varepsilon_0$

$$\psi_{d, \vec{k}_d}^{(+)}(\vec{r} \approx 0, \vec{R}) \approx \chi_{d, \vec{k}_d}^{Ad}(\vec{R}) \phi_d(r),$$

$$[T_R + V_p(\vec{R}) + V_n(\vec{R}) - E_0] \chi_{d, \vec{k}_d}^{Ad}(\vec{R}) = 0$$

“ADWA”

The adiabatic deuteron distorting potential



$$V_{Ad}(\vec{R}) = \int d\vec{r} [V_n(\vec{r}_n) + V_p(\vec{r}_p)] |\phi_d(\vec{r})|^2$$

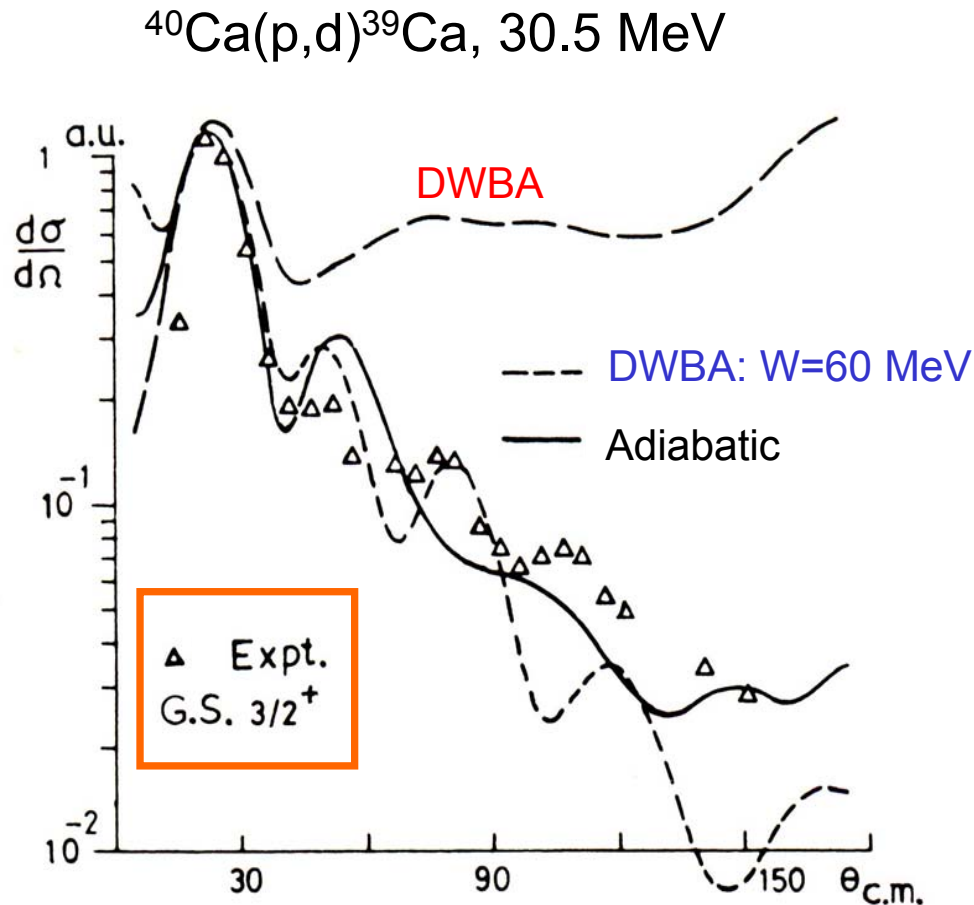
$$[T_R + \underbrace{V_p(\vec{R}) + V_n(\vec{R})}_{V_{Ad}(\vec{R})} - E_0] \chi_{d, \vec{k}_d}^{Ad}(\vec{R}) = 0$$

$$V_{Ad}(\vec{R}) = V_n(\vec{R}) + V_p(\vec{R})$$

$$V_{Ad}(\vec{R}) = \int d\vec{r} [V_n(\vec{r}_n) + V_p(\vec{r}_{pA})] \delta(\vec{r})$$

this is NOT an optical potential and is not meant to and
DOES NOT describe deuteron elastic scattering

Key features for transfer reactions - spectroscopy



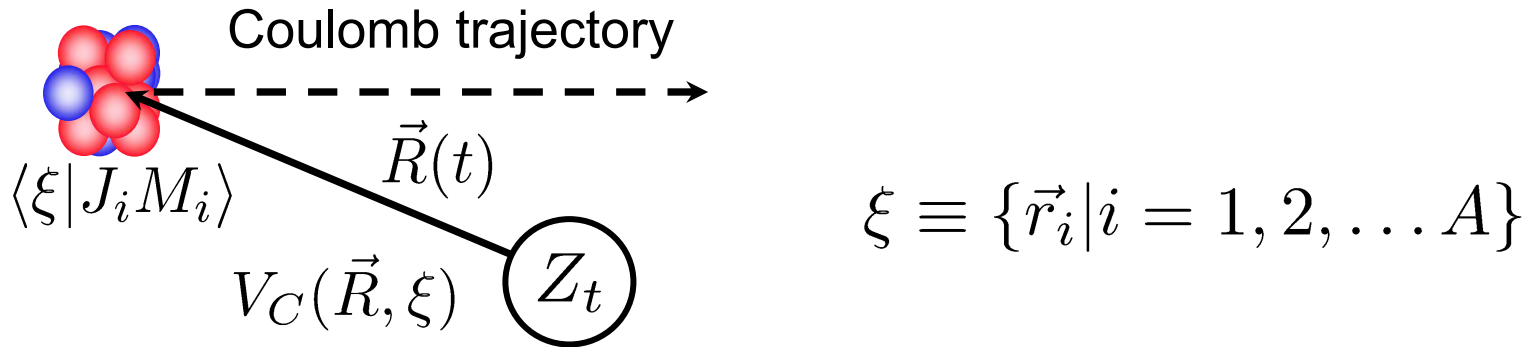
Increased reflection at nuclear surface - less diffuse deuteron channel potential

Greater surface localisation - L-space localisation

Less nuclear volume contribution and less sensitivity to optical model parameters

More consistent sets of deduced spectroscopic factors

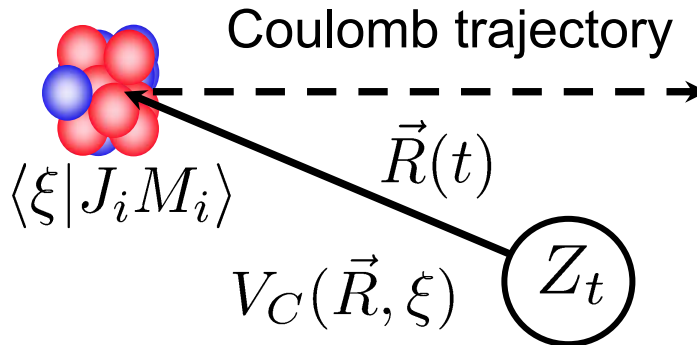
Coulomb interaction – electromagnetic probe



$$\begin{aligned}
 V_C(\vec{R}, \xi) &= \sum_{\text{protons}} \frac{Z_t e^2}{|\vec{R} + \vec{r}_i|}, \quad (R > r_i), \\
 &= \sum_{LM, i}^Z \frac{4\pi Z_t e^2}{2L + 1} \frac{(-1)^L}{R^{L+1}} r_i^L Y_{LM}(\hat{r}_i) Y_{LM}^*(\hat{R})
 \end{aligned}$$

$$Q_{LM} = \sum_{\text{protons}}^Z e r_i^L Y_{LM}(\hat{r}_i) = \sum_{i=1}^Z Q_{LM}(i)$$

Coulomb excitation - transition strengths

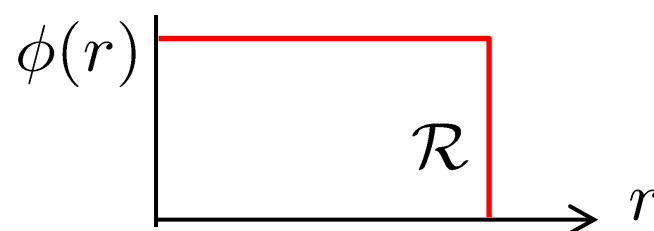
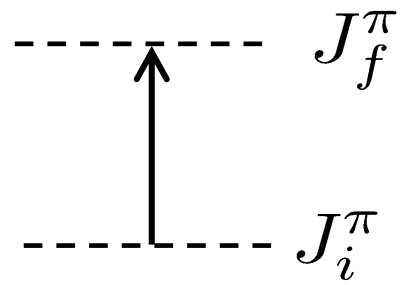


$$\xi \equiv \{\vec{r}_i | i = 1, 2, \dots A\}$$

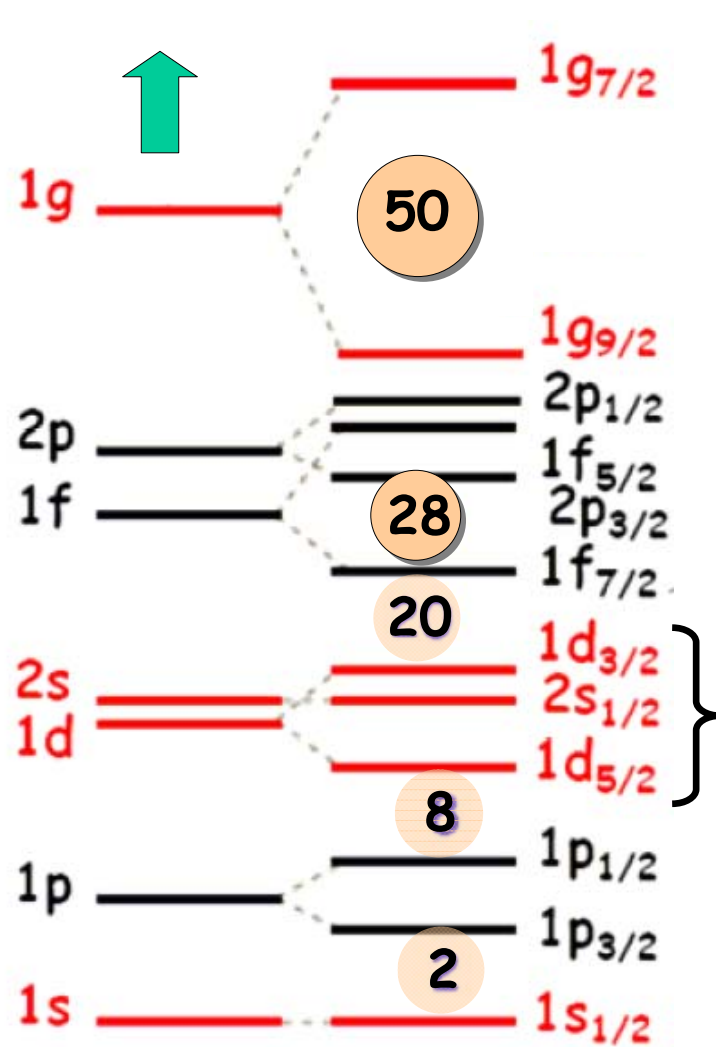
$$Q_{LM} = \sum_{\text{protons}}^Z e r_i^L Y_{LM}(\hat{r}_i) = \sum_{i=1}^Z Q_{LM}(i)$$

$$B(EL; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \sum_{M_i M_f} |\langle J_f M_f | Q_{LM} | J_i M_i \rangle|^2$$

Weisskopf units – single proton expectation

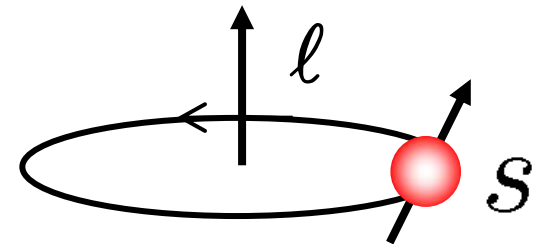
$$B(EL)_{Wu} = \frac{2L + 1}{4\pi} \left(\frac{3\mathcal{R}^L}{L + 3} \right)^2 e^2 \text{ fm}^{2L}$$



Halo configurations – use of Coulomb dissociation



^{19}C
 $Z=6$
 $N=13$

$$\ell, s = 1/2 \begin{cases} j_{<} = \ell - 1/2 \\ j_{>} = \ell + 1/2 \end{cases}$$



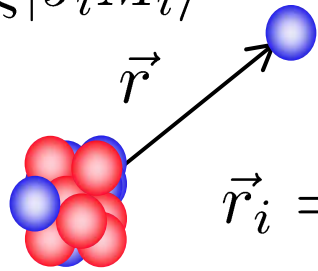
$$V_{\ell s}(r) \vec{\ell} \cdot \vec{s}$$

$$V(r) + V_{so}(r) \vec{\ell} \cdot \vec{s}$$

$$V_{so}(r) < 0$$

Coulomb breakup – n-halo systems: weak binding

$\langle \xi | J_i M_i \rangle$

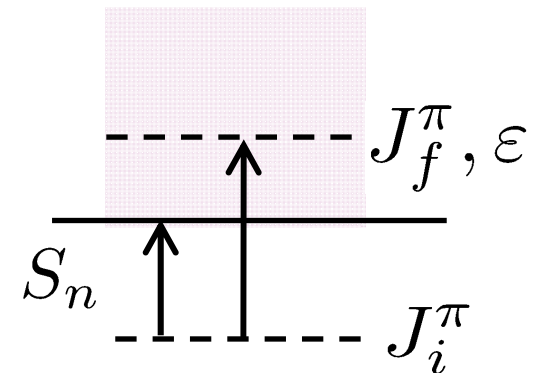
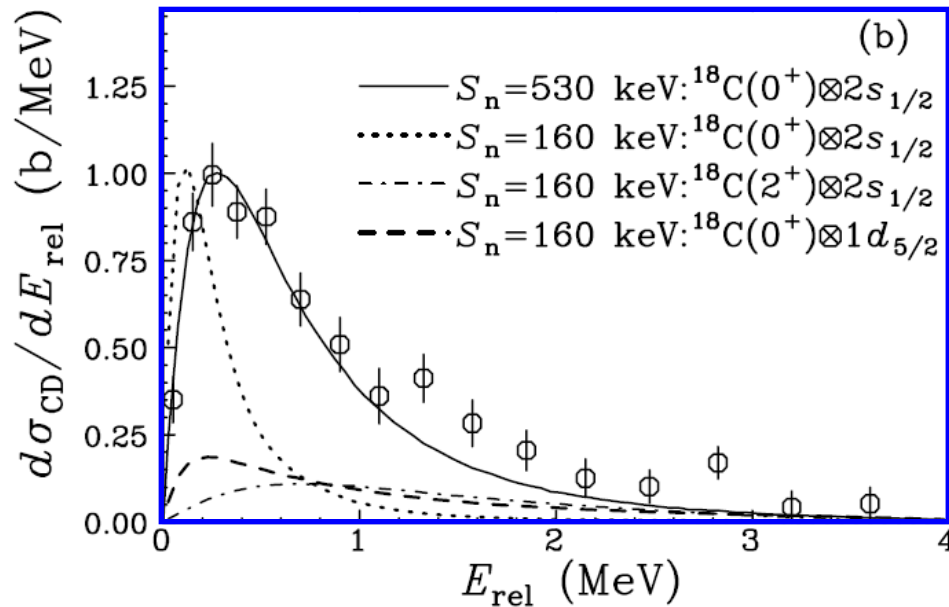


$$\vec{r}_i = \vec{x}_i - \frac{\vec{r}}{(A_c + 1)}$$

$$Q_{1M} = \sum_{\text{protons}} e r_i Y_{LM}(\hat{r}_i)$$

$$= Q_{1M}^c - \frac{Z_c}{A_c + 1} e r Y_{1M}(\hat{r})$$

$$B(E1; J_i \rightarrow J_f, \varepsilon) = \frac{1}{2J_i + 1} \sum_{M_i M_f} |\langle J_f M_f, \varepsilon | Q_{1M} | J_i M_i \rangle|^2$$



T. Nakamura et al., Phys. Rev. Lett. **83**, 1112–1115 (1999)