LECTURE I

Classification of Reactions

When two nuclei collide, two types of reactions can occur:

1. Nuclei can coalesce to form highly excited **Compound nucleus (CN)** that lives for relatively long time.

   [Long lifetime sufficient for excitation energy to be shared by all nucleons. If sufficient energy localised on one or more nucleons they can escape and CN decays]

   $$A + a \rightarrow C^* \rightarrow B^* + b$$

   B may further decay or de-excite.

   Independence hypothesis: C^* lives long enough that it loses its memory of how it was formed ($A + a$ channel). Probability of various decay modes ($B + b$) independent of entrance channel

2. Nuclei make 'glancing' contact and separate immediately, said to undergo **Direct reactions(DI)**.

   $$A + a \rightarrow B + b$$

   Projectile 'a' may lose some energy, or have one or more nucleons transferred to or from it.

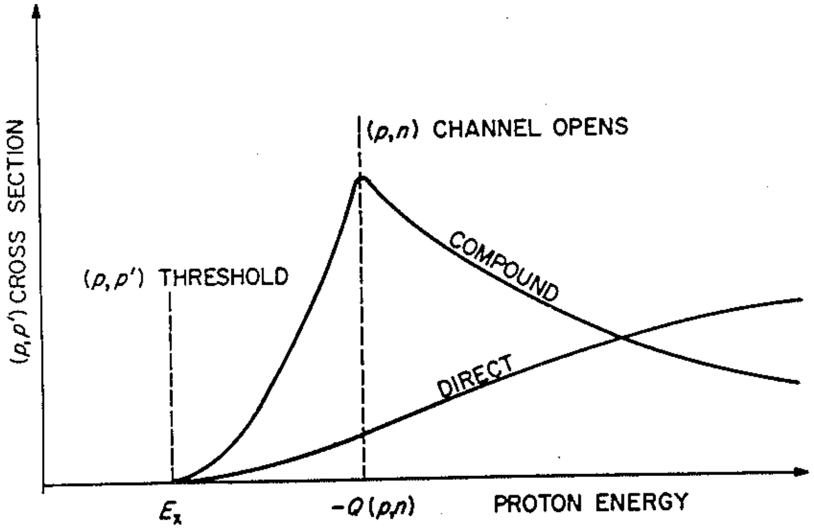
Both types of reaction are important. In general:

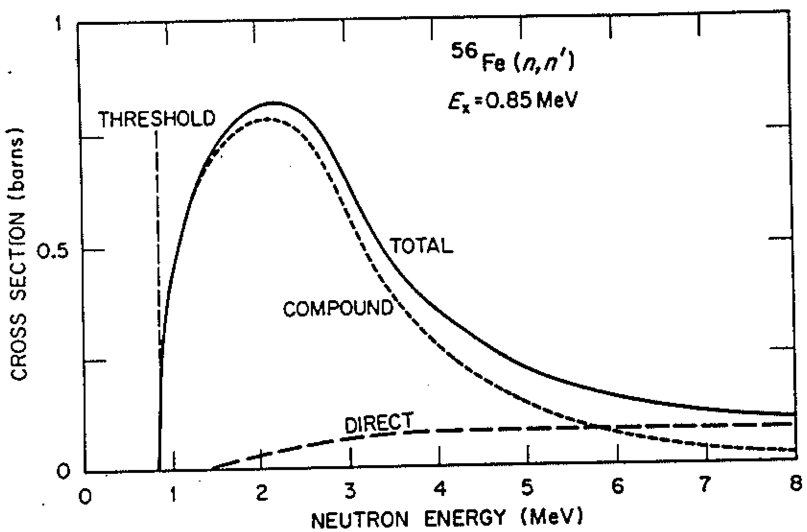
CN reactions at low energy, DI reactions at high energy.

[because formation of CN requires soft collision]

Sometimes both DI and CN may contribute to the same reaction. Some intermediate processes do not fall into either category, often referred to as **pre- equilibrium reactions.**

To see how DI and CN reactions compete at various energies, look at (p, p') and (n, n') reaction cross sections.





Duration of reactions:

Good way of distinguishing between DI and CN.

A typical nucleon orbits within a nucleus with a period of $\sim\ 10^{-22}$ sec. [corresponding to K.E. of $\sim$ 20 MeV].

If reaction complete within this time scale or less then no time for distribution of projectile energy around target $\Rightarrow$ DI reaction occurred.

CN reactions require $\gg\ 10^{-22}$ sec.

Angular distributions:

In DI reactions differential cross section strongly forward peaked as projectile continues to move in general forward direction.

Differential cross sections for CN reactions do not vary much with angle (not complete isotropy as still slight dependence on direction of incident beam).

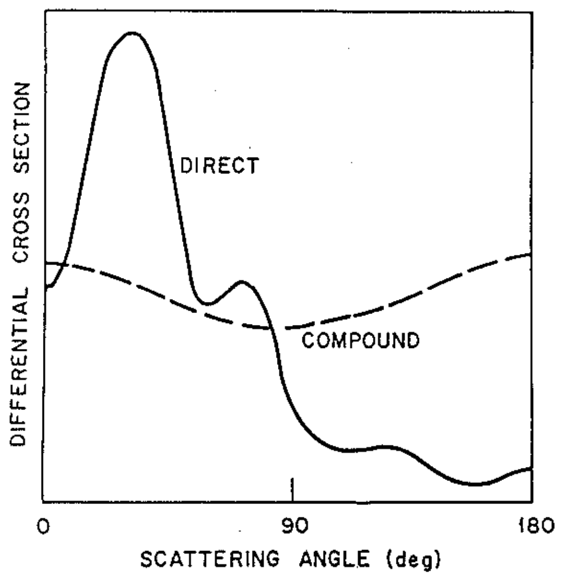


Figure 2.33 Typical angular distributions for direct and compound nucleus reactions induced by light ions with moderate bombarding energies (for example, some (d, p) reaction with 20-MeV deuterons)

Classification of direct reactions:

Can identify various types of DI processes that can occur, of which the following list by no means complete:

1. **Elastic scattering**: $A(a,a)A$ – zero Q-value — internal states unchanged.

2. **Inelastic scattering**: $A(a,a')A^*$ or $A(a,a^*)A^*$. Projectile a gives up some of its energy to excite target nucleus A. If nucleus a also complex nucleus, it can also be excited.

   [If energy resolution in detection of a not small enough to resolve g.s. of target from low-lying excited states then cross section will be sum of elastic and inelastic components. This is called **quasi-elastic scattering**].

3. **Breakup reactions**: Usually referring to breakup of projectile a into two or more fragments. This may

be **elastic** breakup or **inelastic** breakup depending on whether target remains in ground state. eg. $A(a, xy)A$ or $A(a, xy)A^*$ where $a = x + y$.

4. **Transfer reactions**: of which there are two types:-
   (a) **stripping reactions**: eg. deuteron stripping $A(d, p)B$.
   (b) **pickup reactions**: eg. deuteron pickup $A(p, d)B$.

5. **Knockout reactions**: nucleon or light nucleus ejected from target while projectile continues on. Rest of target acts as spectator (also know as **quasi-free scattering**) eg. $A(p, 2p)B$ and $A(e, e'p)B$.

6. **Charge exchange reactions**: mass numbers remain the same. Can be elastic or inelastic. eg. $^{14}C(p, n)^{14}N$ and $^7Li(^6Li, ^6He)^7Be$.

Reaction channels:

In nuclear reaction, each possible combination of nuclei is called a **partition**.

Each partition further distinguished by state of excitation of each nucleus and each such pair of states is known as a **reaction channel**.

The initial partition, $a + A$ (both in their ground states) is known as the incident, or entrance channel. The various possible outcomes are the possible exit channels.

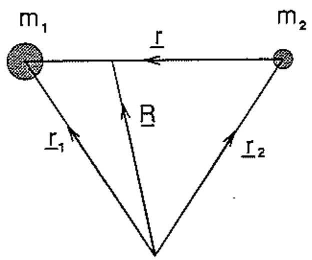
In a particular reaction, if not enough energy for a particular exit channel then it is said to be closed.

LECTURE II

Quantum Scattering Theory

Will be using non-relativistic, 2-body formalism of Schrödinger equation (SE).

Look at 2-body system in potential $V(r)$



$$\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$$

$$\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$$

The T.I.S.E. is

$$\hat{H}\Psi = E\Psi \qquad (1)$$

The Hamiltonian for the system is

$$\begin{aligned}\hat{H} &= -\frac{\hbar^2}{2m_1}\nabla^2_{\mathbf{r}1} - \frac{\hbar^2}{2m_2}\nabla^2_{\mathbf{r}2} + V(\mathbf{r}) \\ &= -\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}} + V(\mathbf{r}) \qquad (2)\end{aligned}$$

[$m = m_1m_2/(m_1 + m_2)$ and $M = m_1 + m_2$]

Thus can look for separable solutions of the form

$$\Psi(\mathbf{R}, \mathbf{r}) = \phi(\mathbf{R})\psi(\mathbf{r}) \qquad (3)$$

Substituting for Ψ back in Eq.(1) means can rearrange SEso that LHS is function of **R** and RHS function of **r**. Thus both equal to common constant, E_{cm}. Hence

$$-\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}}\,\phi(\mathbf{R}) = E_{cm}\,\phi(\mathbf{R}) \qquad (4)$$

and

$$\left(-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}} + V(\mathbf{r})\right)\psi(\mathbf{r}) = E_{rel}\,\psi(\mathbf{r}) \qquad (5)$$

where $\quad E_{rel} = E - E_{cm}.$

In scattering, if m_1 is projectile incident on stationary target m_2 then

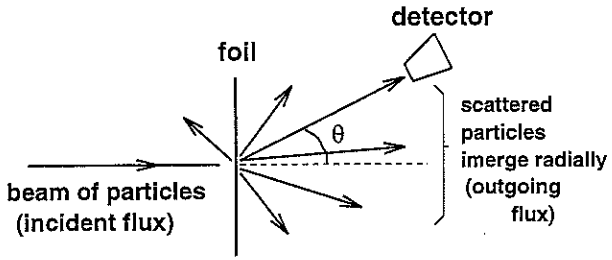
$$E_{cm} = \frac{m_1}{m_1 + m_2}\,E$$

$$E_{rel} = \frac{m_2}{m_1 + m_2}\,E$$

Solution to (4) is simple: $\phi(\mathbf{R}) = A\exp(i\mathbf{K}\cdot\mathbf{R})$ which is plane wave. Thus c.o.m. moves with constant momentum $\hbar\mathbf{K}$ and does not change after scattering.

(Note, $E_{cm} = \hbar^2K^2/2M$).

Boundary conditions on $\psi(\mathbf{r})$:



If incident beam $\sim$ 1 cm wide ($= 10^{12} \times$ nuclear size).

Thus beam can be represented by **plane wave**

As $|\mathbf{r}| \to \infty$ (i.e. moving away radially from scattering centre),

$$\psi(\mathbf{r}) \;\to\; N\left(\exp(i\mathbf{k}\cdot\mathbf{r}) \;+\; f(\theta,\varphi)\,\frac{\exp(ikr)}{r}\right) \qquad (6)$$

where k is defined as $E_{rel} = \hbar^2 k^2/2m$

(Take $N = 1$)

In QM, flux (probability current density) is given by

$$\mathbf{J} = Re\left[\psi^*\left(-\frac{i\hbar}{m}\nabla_{\mathbf{r}}\right)\psi\right]$$

For incident flux, $\psi_{inc} = \exp(i\mathbf{k}\cdot\mathbf{r})$ and

$$\begin{aligned}\mathbf{J}_{inc} &= Re\left[\exp(-i\mathbf{k}\cdot\mathbf{r})\left(-\frac{i\hbar}{m}\nabla_{\mathbf{r}}\right)\exp(i\mathbf{k}\cdot\mathbf{r})\right] \\ &= \frac{\hbar\mathbf{k}}{m}\;. \end{aligned} \qquad (7)$$

For scattered flux $\psi_{scat} = f(\theta,\varphi)\,\frac{\exp(ikr)}{r}$ and hence we obtain

$$\mathbf{J}_{scat} = J_{inc}\,\frac{|f(\theta,\varphi)|^2}{r^2}\,\hat{\mathbf{r}} \qquad (8)$$

i.e. $J_{scat} \propto J_{inc}$
and $\propto 1/r^2$ (traditional fall-off).

All scattering information contained in $f(\theta,\varphi)$ known as the **scattering amplitude**

During experiment, number of particles collected by detector in unit time

$$\begin{aligned} &= \mathbf{J}_{scat}\,dA \qquad (dA \text{ is cross-sectional area of detector}) \\ &= \mathbf{J}_{inc}\,|f(\theta,\varphi)|^2\,\frac{dA}{r^2} \\ &= \mathbf{J}_{inc}\,|f(\theta,\varphi)|^2\,d\Omega \qquad (d\Omega \text{ is solid angle subtended by detector}) \end{aligned}$$

Define the **differential cross-section** (in units of area) as

The number of particles scattered into unit solid angle per unit time, per unit incident flux ($J_{inc} = 1$).

Thus $$\frac{d\sigma}{d\Omega} = |f(\theta,\varphi)|^2 \quad . \tag{9}$$

What does $f(\theta,\varphi)$ look like?

Go back to SE in Eq.(5)

We know what the solution must look like asymptotically:

$$\psi(\mathbf{r}) \;\rightarrow\; N\left(\exp(i\mathbf{k}\cdot\mathbf{r}) \;+\; f(\theta,\varphi)\,\frac{\exp(ikr)}{r}\right) \tag{10}$$

Assume V central (i.e. function of $|\mathbf{r}|$) therefore can choose separable solution

$$\psi(\mathbf{r}) = \sum_\ell \frac{u_\ell(r)}{kr} Y_{\ell m}(\theta,\varphi) \tag{11}$$

Thus can extract radial SE as

$$\frac{d^2u_\ell}{dr^2} + \left[k^2 - \frac{2m}{\hbar^2}V(r) - \frac{\ell(\ell+1)}{r^2}\right]u_\ell = 0 \tag{12}$$

Choose $V(r) = 0$ for $r > r_0$

Beyond r_0 get **free solution**

$$u_\ell'' + \left[k^2 - \frac{\ell(\ell+1)}{r^2}\right]u_\ell = 0 \tag{13}$$

Solution related to spherical Bessel functions

$$r > r_0: \qquad u_\ell \;=\; A_\ell\,F_\ell(kr) \;+\; B_\ell\,G_\ell(kr) \tag{14}$$

$F_\ell(kr)$ ↑ $kr\,j_\ell(kr)$ (regular); $G_\ell(kr)$ ↑ $-kr\,n_\ell(kr)$ (irregular)

As $r \rightarrow \infty$

$$u_\ell \;\rightarrow\; A_\ell \sin(kr - \ell\pi/2) + B_\ell \cos(kr - \ell\pi/2)$$

$$= \; C_\ell \sin(kr - \ell\pi/2 + \delta_\ell) \tag{15}$$

where δ_ℓ is known as the **phase shift**.

If $V = 0$ then solution must be valid everywhere, even at origin where it has to be regular. Thus $B_\ell = 0$.

So, asymptotically (long way from scattering centre):

$$\text{For } V = 0 \qquad u_\ell = A_\ell \sin(kr - \ell\pi/2) \qquad (16)$$

$$\text{and for } V \neq 0 \qquad u_\ell = C_\ell \sin(kr - \ell\pi/2 + \delta_\ell) \qquad (17)$$

Thus, switching scattering potential 'on' produces a shift in the phase of the wave function at large distances from the scattering centre.

Now substituting for u_ℓ from Eq.(17) back into Eq.(11) for $\psi(\mathbf{r})$, and after some angular momentum algebra, we obtain a scattering wave function which, when equated with the required asymptotic form of Eq.(10) gives

$$f(\theta) \;=\; \frac{1}{k}\sum_\ell \,(2\ell+1)\; T_\ell \; P_\ell(\theta) \qquad (18)$$

where

$$T_\ell = \exp(i\delta_\ell)\, \sin\delta_\ell \qquad (19)$$

and is connected to the **T-matrix** (see later).

Note that there is no dependence on φ because central potentials lead to azimuthal symmetry.

Integral expressions:

Can write SE as

$$(E - H)\,\psi \;=\; 0 \qquad \text{or} \qquad (E - H_0)\,\psi \;=\; V\,\psi \quad (20)$$

where $H = H_0 + V$.

Thus

$$\psi \;=\; (E - H_0)^{-1}\, V\,\psi \;=\; G_0(E)\, V\,\psi \qquad (21)$$

$G_0(E)$ is the **Green's operator.**

Eq.(21) is not general solution for ψ as can add on solution of homogeneous equation

$$(E \;-\; H_0)\,\phi \;=\; 0 \qquad (22)$$

General solution of Eq.(20) is

$$\psi \;=\; \phi \;+\; G_0(E)\, V\, \psi \qquad (23)$$

This is iterative

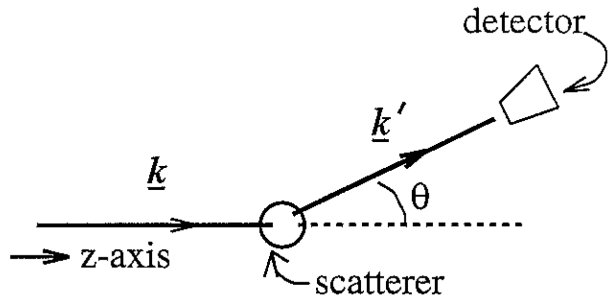
$$\psi \;=\; \phi \;+\; G_0 V \phi \;+\; G_0 V G_0 V \phi \;+\; .... \qquad (24)$$

Eq.(23) can be written in integral form as the **Lippmann-Schwinger** equation

$$\psi(\mathbf{r}) \;=\; \phi(\mathbf{r}) \;+\; \int d\mathbf{r}'\, G(\mathbf{r},\mathbf{r}')\, V(\mathbf{r}')\, \psi(\mathbf{r}') \qquad (25)$$

where $G(\mathbf{r},\mathbf{r}')$ is the **Green's function**.

If we choose z-axis to be along direction of incident beam



Then $\quad \phi(\mathbf{r}) \;=\; \exp(i\mathbf{k}\cdot\mathbf{r}) \;=\; \exp(ikz)$

and we use the notation $\psi^{(+)}_{\mathbf{k}}(\mathbf{r})$ for the scattering wave function.

(i.e. incident momentum $\mathbf{k}$ and $(+)$ for outgoing waves solution).

Comparing Eq.(25) with required asymptotic form for ψ we see that integral term must tend to

$$f(\theta,\varphi)\,\frac{\exp(ikr)}{r} \qquad \text{as} \quad |\mathbf{r}| \to \infty \;. \qquad (26)$$

Thus, using properties of Green's function we can obtain

$$f(\theta,\varphi) \;=\; -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\; \exp(-i\mathbf{k}'\cdot\mathbf{r})\, V(\mathbf{r})\, \psi^{(+)}_{\mathbf{k}}(\mathbf{r}) \quad .(27)$$

In Dirac (bra-ket) notation we write this

$$f(\theta) \; = \; -\frac{m}{2\pi\hbar^2} \langle \, \mathbf{k}' \mid V \mid \psi_{\mathbf{k}}^{(+)} \, \rangle \qquad (28)$$

$$= \; -\frac{m}{2\pi\hbar^2} \, T(\mathbf{k}', \mathbf{k}) \quad . \qquad (29)$$

$T(\mathbf{k}', \mathbf{k})$ is known as the **Transition matrix element**.

It can be shown, using some algebra (eg. plane wave expansion formula for $\exp(-i\mathbf{k}' \cdot \mathbf{r})$), that Eq.(27) leads to Eq.(18).

Internal states:

So far, we have dealt with 2-body system where colliding bodies do not have internal structure.

Thus, definition of the scattering wave function through the Lippmann-Schwinger equation (25) is incomplete for complex nuclei.

Assume projectile and target initially have intrinsic wave functions ϕ_a and ϕ_A, respectively.

If the entrance channel is denoted by α then the asymptotic form of the scattering wave function in Eq.(10) is written

$$\psi_{\mathbf{k},a,A}^{(+)}(\mathbf{r}) = N \left(\exp(i\mathbf{k}_\alpha \cdot \mathbf{r}_\alpha) \, \phi_a \, \phi_A \; + \; \sum_\beta \; \psi_\beta^{\text{scatt}} \right) \; , \quad (30)$$

where we must sum over all possible exit channels, β. For a particular channel, say for the reaction $A(a,b)B$, the scattered part of the wave function is therefore

$$\psi_\beta^{\text{scatt}} \; = \; f_{\alpha\beta}(\theta) \, \frac{\exp(ik_\beta r_\beta)}{r_\beta} \, \phi_b \, \phi_B \quad . \qquad (31)$$

The scattering amplitude of Eq.(28) is then written

$$f_{\alpha\beta}(\theta) \; = \; -\frac{m}{2\pi\hbar^2} \langle \, \mathbf{k}' \phi_b \, \phi_B \mid V \mid \psi_{\mathbf{k},a,A}^{(+)} \, \rangle \quad . \quad (32)$$

LECTURE III

Approximation methods

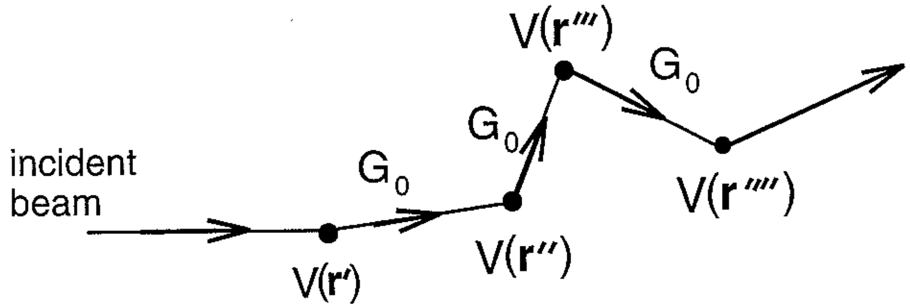
Born approximation

Substituting the iterative expression for the scattering wave function of Eq.(24) into the expression for the scattering amplitude of Eq(27) we obtain the multiple scattering series (or **Born series**)

$$f(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\Big[\int d\mathbf{r}\ \exp(-i\mathbf{k}'\cdot\mathbf{r})\,V(\mathbf{r})\ \exp(i\mathbf{k}\cdot\mathbf{r})$$
$$+\int d\mathbf{r}\int d\mathbf{r}'\ \exp(-i\mathbf{k}'\cdot\mathbf{r})\,V(\mathbf{r})\,G_0(\mathbf{r},\mathbf{r}')\,V(\mathbf{r}')\ \exp(i\mathbf{k}\cdot\mathbf{r}')$$
$$+\int d\mathbf{r}\int d\mathbf{r}'\int d\mathbf{r}''\ldots\ldots\Big]\ . \qquad (1)$$

The first term in above series is known as the first **Born approximation** and could have been obtained directly by replacing the scattering wave function in the the 2nd term of the L-S equation by a plane wave.

$$\psi(\mathbf{r}) = \phi(\mathbf{r}) = \exp(i\mathbf{k}\cdot\mathbf{r})\ . \qquad (2)$$



Thus can interpret Green's function as a **propagator** taking wave from one scattering point to the next.

In terms of the transferred momentum $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ the Born approximation scattering amplitude is

$$f_{BA}(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\ \exp(-i\mathbf{k}'\cdot\mathbf{r})\,V(\mathbf{r})\ \exp(i\mathbf{k}\cdot\mathbf{r})$$
$$= -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\ \exp(i\mathbf{q}\cdot\mathbf{r})\,V(\mathbf{r}) \qquad (3)$$

which is proportional to the Fourier transform of the potential. For the case of a central potential, this reduces to a 1-dimensional integral

$$f_{BA}(\theta) = -\frac{2m}{\hbar^2 q}\int_0^\infty dr\ \sin(qr)\,V(r)\ . \qquad (4)$$

Simple rule of thumb: approximation valid in the limit of weak potentials and/or high incident energies.

More careful estimate of validity yields the condition

$$\frac{|V_0|}{E} \ll \frac{1}{ka} \qquad (5)$$

where V_0 is the strength of the potential, and a is its diffuseness, which is a measure of the range over which it varies appreciably.

The Glauber (or eikonal) approximation:

This is a better approximation than Born for the case of elastic scattering. It is also known as the **high energy** or **semi-classical approximation**. There are many versions of it and many ways of defining it.

In general for the Glauber approximation to be valid, the following two conditions must hold

$$\frac{|V_0|}{E} \ll 1 , \qquad ka \gg 1 \ . \qquad (6)$$

In the Glauber approximation, scattering wavefunction written as

$$\psi(\mathbf{r}) = \rho(\mathbf{r}) \ \exp(i\mathbf{k}\cdot\mathbf{r}) \ , \qquad (7)$$

where $\rho(r)$ is the **modulating function** that distorts the plane wave and therefore depends on $V(r)$.

Thus comparing with Eq.(3), the scattering amplitude can now be written

$$f_{GL}(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r} \ \exp(i\mathbf{q}\cdot\mathbf{r})\, V(\mathbf{r})\,\rho(\mathbf{r}) \ . \qquad (8)$$

There are various ways of deriving the Glauber amplitude. Will start here from S.E.

Need to solve

$$(k^2+\nabla^2)\psi(\mathbf{r}) = U(\mathbf{r})\,\psi(\mathbf{r}) \ , \qquad (9)$$

where U $(= 2mV/\hbar^2)$ is the reduced potential.

Substituting for ψ from Eq.(7) the LHS of (9) becomes

$$(k^2-k^2)\exp(i\mathbf{k}\cdot\mathbf{r}) + 2i\mathbf{k}.\nabla\left(\rho \ \exp(i\mathbf{k}\cdot\mathbf{r})\right) + \exp(i\mathbf{k}\cdot\mathbf{r})\nabla^2\rho$$

and choosing the z-axis to be along the incident beam

$$\text{LHS} \ = \ \exp(ikz)\left(2ik\,\partial\rho/\partial z + \partial^2\rho/\partial z^2\right) \ .$$

It can be shown from the Glauber conditions that the 2nd term above is much smaller than the first. Thus the S.E. is reduced to a first order differential equation:

$$\frac{\partial\rho}{\partial z} \ \simeq \ \frac{1}{2ik}U(\mathbf{r})\,\rho(\mathbf{r}) \ , \qquad (10)$$

and the solution is of the form

$$\rho(\mathbf{r}) = \exp\left\{\frac{1}{2ik}\int_{-\infty}^{z} U(\mathbf{r}')\,dz'\right\} \quad . \tag{11}$$

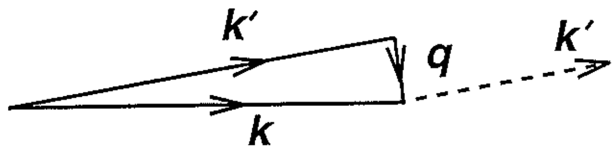
[Note this assumes that $\rho(-\infty) = 1$ so that $\psi(-\infty)$ is a plane wave].

Now introduce the **impact parameter**, **b**, such that

$$\mathbf{r} = \mathbf{b} + \hat{\mathbf{k}}z \quad , \qquad \mathbf{b}\cdot\hat{\mathbf{k}} = 0 \quad . \tag{12}$$

Glauber approximation is valid at small forward angles where **q** almost $\perp$ to **k**. That is

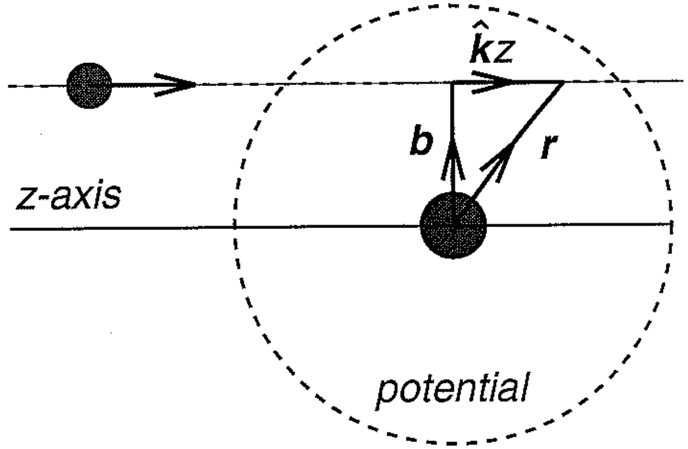
$$\mathbf{q}\cdot\mathbf{r} = \mathbf{q}\cdot\mathbf{b} + \mathbf{q}\cdot\hat{\mathbf{k}}z \approx \mathbf{q}\cdot\mathbf{b} \quad . \tag{13}$$



this is **eikonal** part of approximation and assumes straight line trajectories.

Thus, the z integration in Eq.(8) can be done first

$$f_{GL}(\theta,\varphi) = -\frac{1}{4\pi}\int d\mathbf{b}\,\exp(i\mathbf{q}\cdot\mathbf{b})\int_{-\infty}^{\infty} dz\,V(\mathbf{r})\,\rho(\mathbf{r}) \quad . \tag{14}$$



Using Eq.(10) this can be written

$$f_{GL}(\theta,\varphi) = -\frac{2ik}{4\pi}\int d\mathbf{b}\,\exp(i\mathbf{q}\cdot\mathbf{b})\,(\exp(i\chi(\mathbf{b})) - 1) \tag{15}$$

The function $\chi(\mathbf{b})$ is known as the **Glauber phase shift**

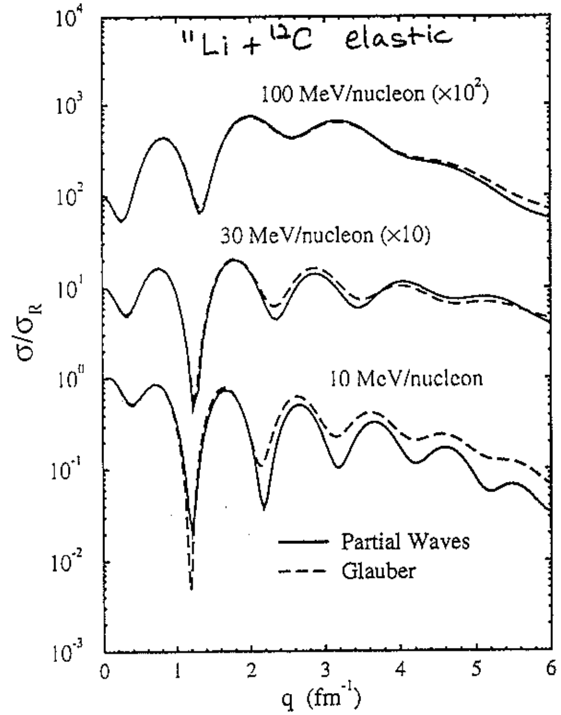
$$\chi(\mathbf{b}) = -\frac{m}{\hbar^2 k}\int_{-\infty}^{\infty} V(\mathbf{r})dz \quad . \tag{16}$$

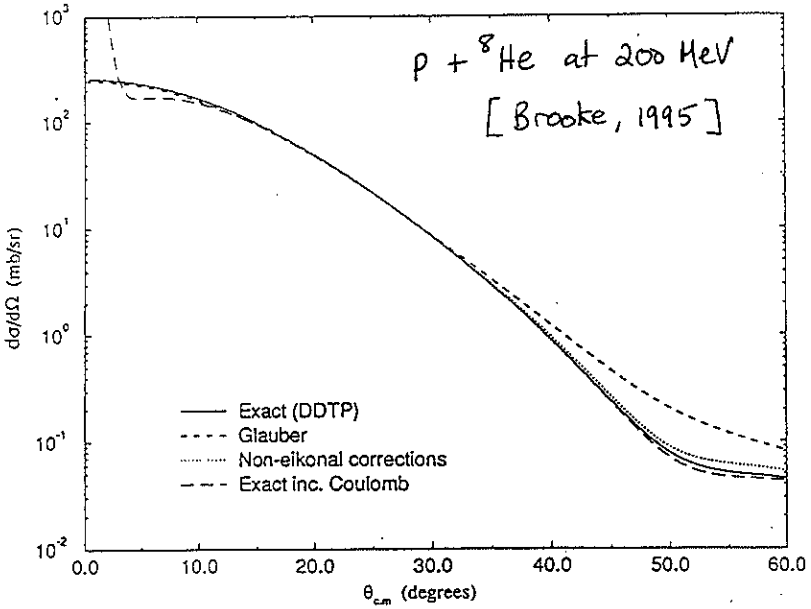
If we are dealing with central potentials (not neccesary just simpler) then azimuthal symmetry means

$$f_{GL}(\theta) = -ik\int_0^{\infty} b\,db\,J_0(qb)\,(\exp(i\chi(b)) - 1) \quad , \tag{17}$$

where $q = 2k\sin\theta/2$.

Note that the small angle approximation of Eq.(13) is only true for elastic scattering (where $|\mathbf{k}| = |\mathbf{k}'|$). For inelastic scattering other approximations are more appropriate.





Distorted Wave Born Approximation (DWBA):

More sophisticated version of the Born approximation. Very widely used for inelastic scattering and reactions.

Suppose that the potential V can be written as a sum of terms $V = V_1 + V_2$, where V_2 weaker than V_1 and that we know the scattering solution for V_1

$$(E - H_0)\,\chi_1(\mathbf{k},\mathbf{r}) = V_1\,\chi_1(\mathbf{k},\mathbf{r}) \tag{18}$$

which leads to a scattering amplitude

$$f_1(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\,\exp(-i\mathbf{k}'\cdot\mathbf{r})\,V_1(\mathbf{r})\,\chi_1^{(+)}(\mathbf{k},\mathbf{r}) \tag{19}$$

Using standand derivation one can obtain expression (also known as the two potential formula) for the scatting amplitude

$$f(\theta,\varphi) = f_1(\theta,\varphi) + f_2(\theta,\varphi)\ , \tag{20}$$

where the second term is defined as

$$f_2(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\,\chi_1^{(-)}(\mathbf{k}',\mathbf{r})\,V_2(\mathbf{r})\,\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) \tag{21}$$

Above expression is EXACT.

The DWBA involves replacing $\psi_{\mathbf{k}}^{(+)}(\mathbf{r})$ (the scattering solution in presence of $V_1 + V_2$) with $\chi_1^{(+)}(\mathbf{k},\mathbf{r})$ (scattering solution in presence of V_1 only).

Note that $\chi_1^{(+)}(\mathbf{k},\mathbf{r})$ and $\chi_1^{(-)}(\mathbf{k},\mathbf{r})$ are both solutions of Eq.(18). But one is time reverse of other.

$$\begin{aligned}\chi_1^{(+)}(\mathbf{k},\mathbf{r}) &= \text{plane wave} + \textbf{outgoing} \text{ spherical wave}\\ \chi_1^{(-)}(\mathbf{k},\mathbf{r}) &= \text{plane wave} + \textbf{incoming} \text{ spherical wave}\\ \chi_1^{(-)}(\mathbf{k},\mathbf{r}) &= \left(\chi_1^{(+)}(-\mathbf{k},\mathbf{r})\right)^* \quad .\end{aligned}$$

Thus full amplitude (for V) is sum of exact amplitude for V_1 plus DWBA amplitude for V_2

$$f_{\rm DWBA}(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\,\chi_1^{(-)}(\mathbf{k}',\mathbf{r})\,V_2(\mathbf{r})\,\chi_1^{(+)}(\mathbf{k},\mathbf{r}) \quad (22)$$

Why **DWBA**?

Born because first order in V_2

Distorted wave because instead of using plane waves (as in Eq.(3), use $\chi_1^{(+)}$ and $\chi_1^{(-)}$ (plane waves **distorted** in the presence of V_1).

Typically, V_1 is optical potential describing elastic scattering and V_2 describes some inelastic process.

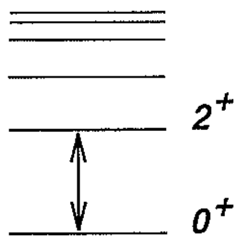
(i.e elastic scattering most important event. Other processes are perturbations.)

Coupled channels method

In some cases, first order perturbation theory not good.

Example: collective excitations of 1-phonon state by inelastic scattering i.e. $A(p,p')A^*(2^+)$.

Assume cross section large. Thus coupling between 0^+ (g.s.) and 2^+ (excited state) strong.



Thus, DWBA not good (one step).

Multistep processes important.

Assume internal wave functions of nucleus A for 0^+ and 2^+ known. Call them $\phi_0(\xi)$, $\phi_1(\xi)$

$$H_A\,\phi_0(\xi) = \epsilon_0\,\phi_0(\xi) \quad , \qquad H_A\,\phi_1(\xi) = \epsilon_1\,\phi_1(\xi) \quad . \quad (23)$$

Thus approximate scattering wave function for system is

$$\Psi(\mathbf{R},\xi) = \phi_0(\xi)\,\chi_0(\mathbf{R}) + \phi_1(\xi)\,\chi_1(\mathbf{R}) \quad . \qquad (24)$$

The SE for the system is written

$$(T_R + H_A + U(\mathbf{R},\xi) - E)\,\Psi(\mathbf{R},\xi) = 0 \qquad (25)$$

Substituting for $\Psi(\mathbf{R},\xi)$ then multiplying from the left by $\phi_0^*(\xi)$ and $\phi_1^*(\xi)$, respectively and integrating over ξ gives two **coupled** equations:

$$(T_R + \langle\phi_0|U|\phi_0\rangle - E_0)\,\chi_0(\mathbf{R}) = -\,\langle\phi_0|U|\phi_1\rangle\,\chi_1(\mathbf{R})$$

$$(T_R + \langle\phi_1|U|\phi_1\rangle - E_1)\,\chi_1(\mathbf{R}) = -\,\langle\phi_1|U|\phi_0\rangle\,\chi_0(\mathbf{R}) \quad (26)$$

where $E_0 = E - \epsilon_0$ and $E_1 = E - \epsilon_1$ and the matrix elements

$$\langle\phi_n|U|\phi_m\rangle = \int d\xi\ \phi_n^*(\xi)\ U(\mathbf{R},\xi)\ \phi_m(\xi) \ = \ U_{nm}(\mathbf{R}) \quad (27)$$

The matrix elements on the LHS of Eq.(26) are called the **coupling potentials.**

LECTURE IV

Direct reaction models

The optical model for elastic scattering

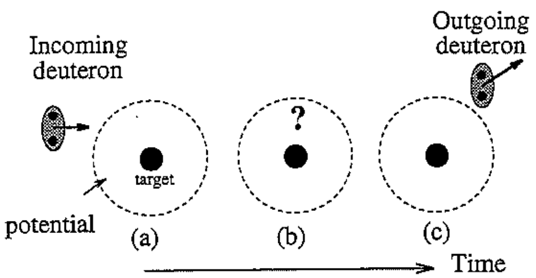
(Optical: due to similarity with scattering of light).

When a nuclear beam interacts with a nuclear target, part of the flux may be lost (absorbed by target). The interaction is therefore described by a complex potential called the **optical potential** whose imaginary part describes absorption of flux from elastic channel due to various competing reactions.

Thus optical potential describes all processes that contribute in elastic scattering $A(a,a)A$.

Thus for $A(p,p)A$, optical potential takes into account possibility of A being excited in intermediate states.

Composite projectile may also be excited eg. $A(d,d)A$



Optical potential used in DWBA to calculate distorted waves, and in coupled channels calculations.

Optical potentials usually chosen to have simple analytical form with parameters chosen so as to reproduce experimental cross section.

Most popular form is the **Woods-Saxon** (W-S) potential.

$$V_{cent}(r) = -V_0 f_V(r) - iW_0 f_W(r) \quad , \tag{1}$$

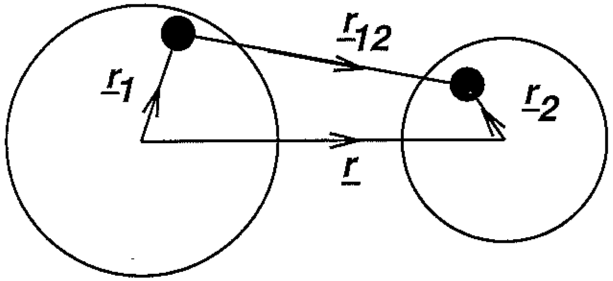
where the W-S formfactors are defined as

$$f(r) = \left[1 + \exp(r - r_0 A^{1/3})/a\right]^{-1} \quad . \tag{2}$$

Often, for **nucleus-nucleus** scattering, the formfactor for the real part is obtained microscopically using the **double folding model**. Here it is assumed that potential is the sum of all pairs of nucleons avaraged over the ground state densities of the two nuclei.

$$f_{DF}(r) = \int d\mathbf{r}_1 \int d\mathbf{r}_2\, \rho_1(\mathbf{r}_1)\, \rho_2(\mathbf{r}_2)\, v_{NN}(\mathbf{r}_{12}) \quad . \tag{3}$$

Note this 6-dimensional integral is in practice reduced by taking the Fourier transforms of the quantities in the integrand.



Then it is assumed that the imaginary part of the optical potential has the same formfactor

$$V_{cent}(r) = -(N_R + iN_I)\, f_{DF}(r) \quad , \tag{4}$$

and the two strength parameters are varied to fit the data.

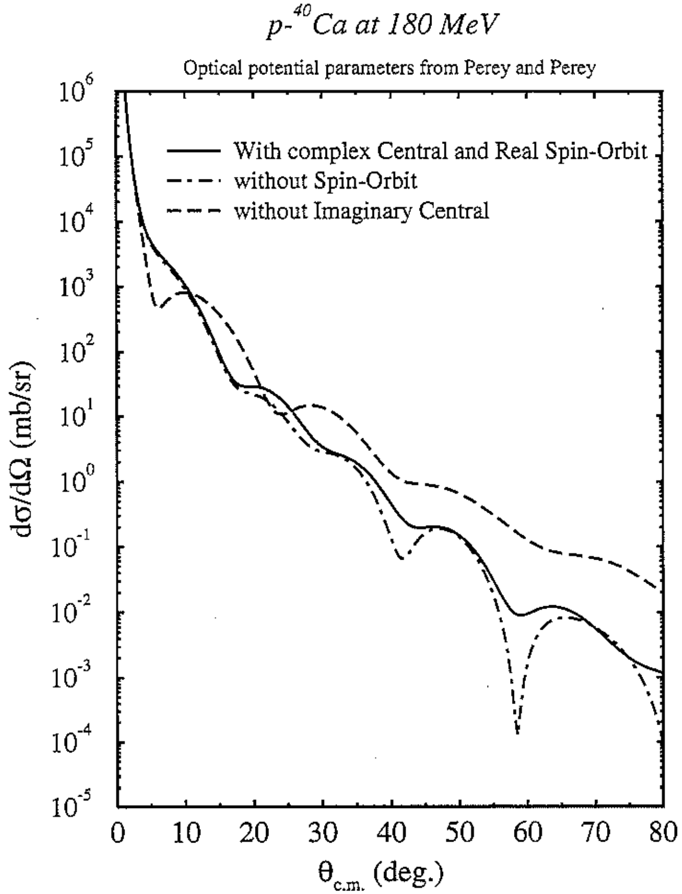
Often (eg. for proton scattering) a **spin-orbit** potential is required. Often taken to have a form factor of **Thomas** form (i.e. derivative of a Wood-Saxon)

$$V_{opt}(r) = V_{cent} + V_{SO}(r)\mathbf{l}\cdot\mathbf{s} \quad , \tag{5}$$

where

$$V_{SO}(r) = \lambda_\pi^2 V_0^{so} \frac{1}{r}\frac{d}{dr} f(r) \quad , \tag{6}$$

$\lambda_\pi = (h/m_\pi c) = \sqrt{2}$.



Dirac optical model

For nucleon-nucleus scattering at intermediate energies (several hundred MeV) standard W-S model no longer adequate. Therefore use optical model based on the **Dirac equation** [Clark, 1983]

$$[\alpha\cdot\mathbf{p} + \beta(m + U_S(r)) + U_V(r)]\,\Psi(\mathbf{r}) = E\,\Psi(\mathbf{r}) \quad . \tag{7}$$

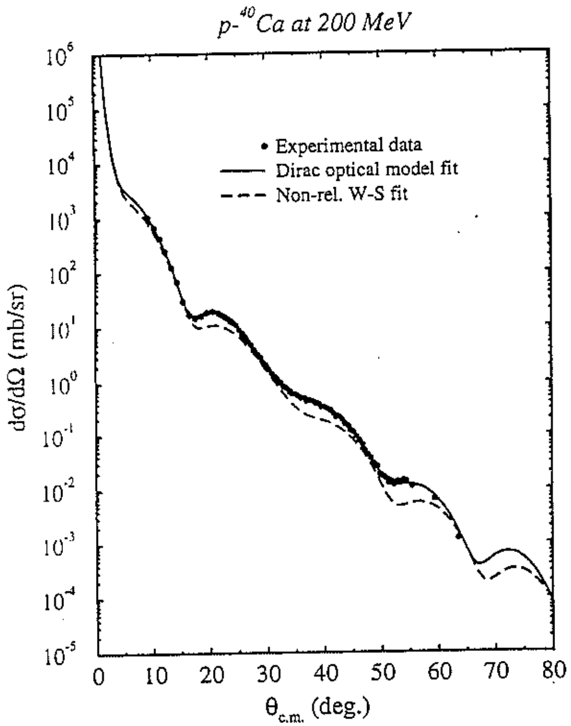
U_S and U_V are Lorentz scalar and vector (time-like comp.)

potentials, respectively. Each is parameterised as complex W-S.

Dirac Eq. can be reduced exactly to an equivalent S.E. with effective Central and S.O. potential terms that depend on U_S and U_V.

$$V_{cent}^{Dirac} = \tilde{U}_V + \frac{m}{E} U_S - \frac{1}{2E}\left(\tilde{U}_V^2 - U_S^2\right) + U_{Darwin} \quad , \qquad (8)$$

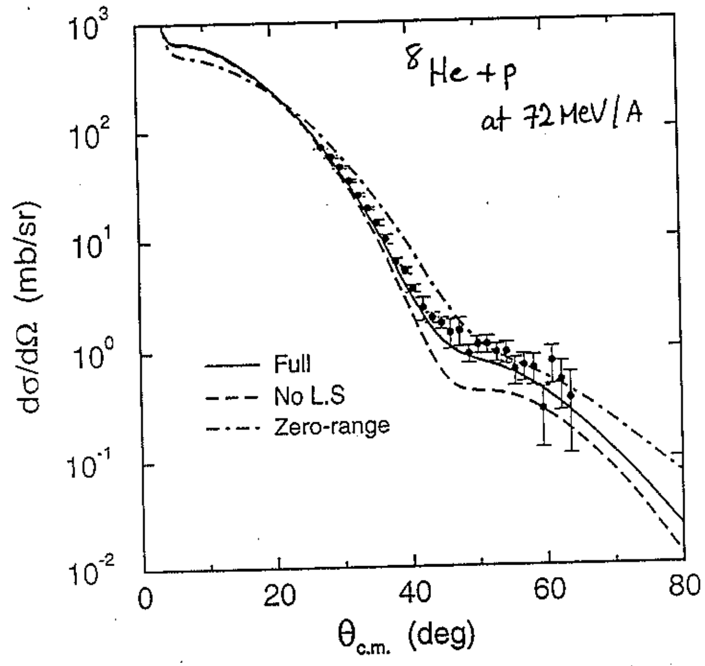
where $\tilde{U}_V = U_V + V_{Coul}$, and Darwin term responsible for non-local effects.



Proton-nucleus scattering now interesting with advent of RNB by studying them in inverse kinematics. eg. study of **halo** nuclei such as ^{8}He and ^{11}Li.

Thus to study the reaction $p(^8He,^8He)p$, theoretical cross sections can be calculated for the inverse reaction $^8He(p,p)^8He$.

[eg. Crespo, 1995]



Will now look at **scattering of composite projectiles**

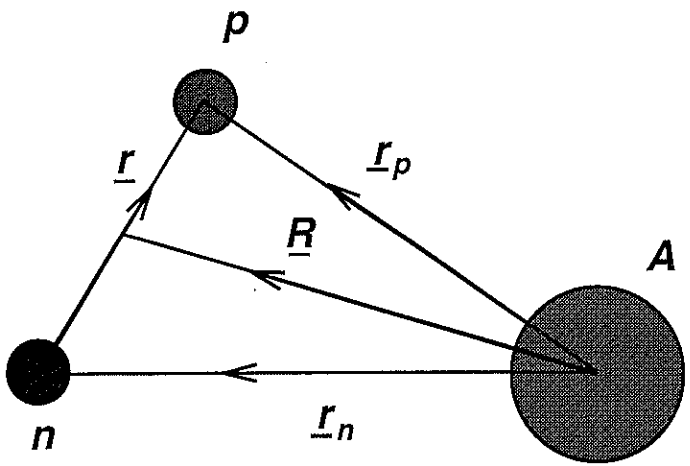
The deuteron is **simplest composite nucleus** and many reaction mechanisms understood by studying deuteron case.

The 3-body model

Exact Hamiltonian for $A+2$ system too complicated. Therefore suppress internal degrees of freedom of target (treat as structureless body) $\Rightarrow$ Problem reduced to 3-body $(p+n+A)$ one [Austern, 1987].

$$(T+p+T_n+V_{pA}+V_{nA}+V_{pn})\,\Psi(\mathbf{r}_p,\mathbf{r}_n)=E\,\Psi(\mathbf{r}_p,\mathbf{r}_n)\quad .(9)$$

Single folding model (SFM) [Watanabe, 1958]



In c.o.m. and relative coordinates, SE becomes

$$(T_R+V(\mathbf{R},\mathbf{r})+H_{pn})\,\Psi(\mathbf{R},\mathbf{r})=E\,\Psi(\mathbf{R},\mathbf{r})\quad , \qquad (10)$$

where $V(\mathbf{R},\mathbf{r})=V_{pA}(\mathbf{R}+\mathbf{r}/2)+V_{nA}(\mathbf{R}-\mathbf{r}/2)$

and $H_{pn}=H_r+V_{pn}$ is the internal Hamiltonian of the deuteron.

At high enough energies, p and n feel target potential independently. Can make approximation

$$\Psi(\mathbf{R},\mathbf{r})\simeq\phi_d(\mathbf{r})\,\psi(\mathbf{R})\quad . \qquad (11)$$

Then multiplying Eq.(10) from left by $\phi_d^*(\mathbf{r})$ and integrating over $\mathbf{r}$ gives

$$[T_R+U_{Wat}(\mathbf{R})-(E+|\epsilon_d|)]\;\psi(\mathbf{R})=0\quad , \qquad (12)$$

where ϵ_d is B.E. of deuteron, $\quad H_{pn}\phi_d(\mathbf{r})=\epsilon_d\phi_d(\mathbf{r})$

and U_{Wat} is the **single folded** (or **Watanabe**) potential

$$U_{Wat}(\mathbf{R})=\int d\mathbf{r}\,\phi_d^*(\mathbf{r})\,[V_{pA}(\mathbf{R},\mathbf{r})+V_{nA}(\mathbf{R},\mathbf{r})]\,\phi_d(\mathbf{r})\quad .(13)$$

Thus comparing (12) with (10) see problem has been reduced to 2-body one.

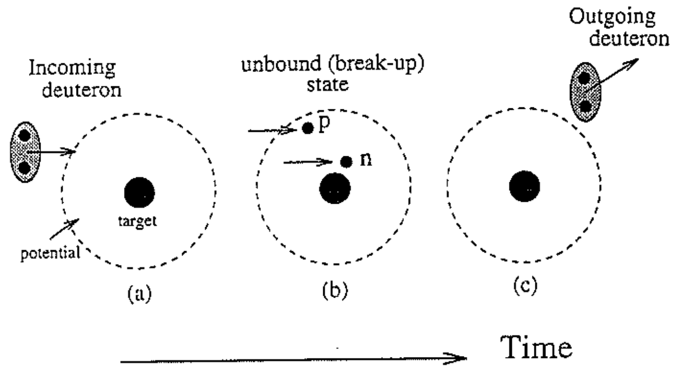
Virtual breakup

(i.e. effect of breakup on elastic channel)

The reactions $A(d,pn)A$ and $A(^6Li,\alpha d)A$ are examples of **real** breakup.

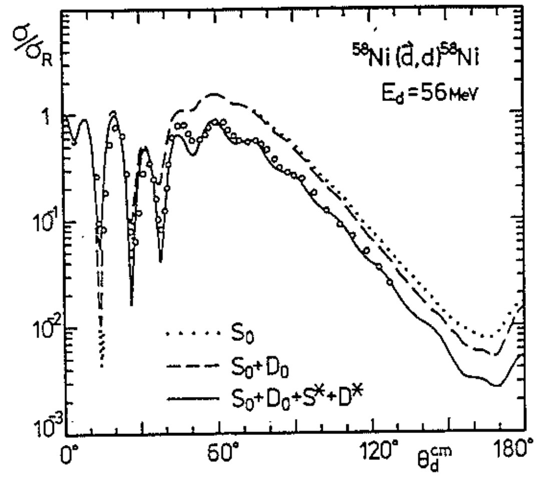
But if projectile detected in exit channel then it may still have been excited (or broken up) in interaction region (intermediate state).

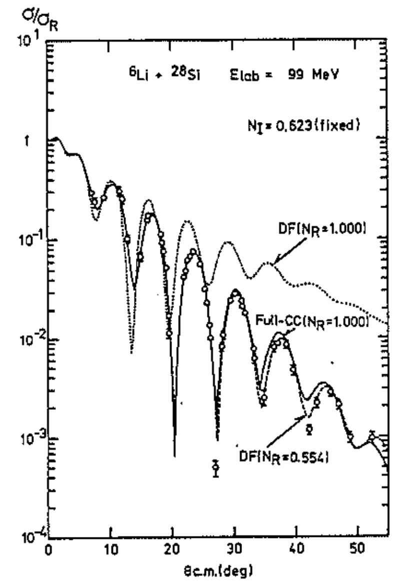
QM: as long as projectile in g.s. in initial and final states then anything is possible in between.



In SFM, deuteron was not allowed to breakup in intermediate states. But d should be allowed to breakup (couple to continuum) in intermediate states

Figures below are from [Yahiro, 1986] and [Sakuragi, 1986].





Adiabatic approximation

[Johnson, 1970]

If incident deuteron energy high enough then assume that internal motion of deuteron frozen for duration of interaction (i.e. **r** varies slowly wrt **R**).

[cf. low energy adiabatic approx. as used in fusion reactions.]

H_{pn} in Eq.(10) replaced by ϵ_0. Thus no energy transfer between c.o.m. and relative motion ⇒ **r** no longer dynamical variable but simply parameter.

Now, S.E. to be solved is

$$[T_R + V(\mathbf{R},\mathbf{r}) - (E + |\epsilon_d|)]\,\chi^{AD}(\mathbf{R},\mathbf{r}) = 0 \quad , \qquad (14)$$

where we have replaced $\Psi(\mathbf{R},\mathbf{r})$ with $\Psi^{AD}(\mathbf{R},\mathbf{r})$ and

$$\Psi^{AD}(\mathbf{R},\mathbf{r}) = \phi_d(\mathbf{r})\,\chi^{AD}(\mathbf{R},\mathbf{r}) \quad . \qquad (15)$$

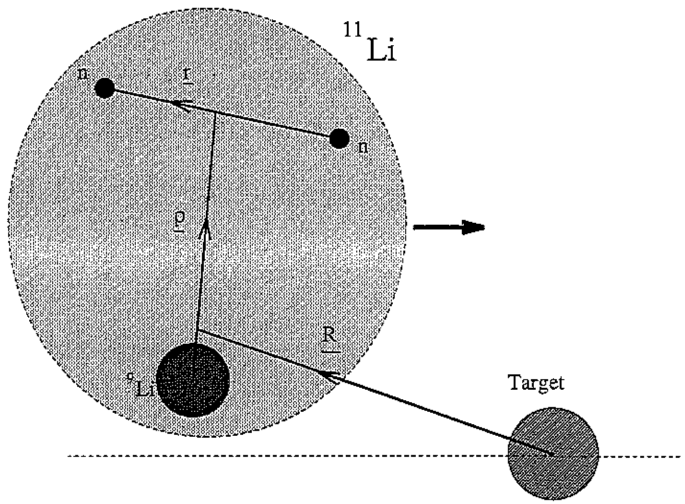
Eq.(14) solved as a scattering problem in **R** for each value of **r**.

Elastic scattering obtained from overlap of $\Psi^{AD}(\mathbf{R},\mathbf{r})$ with $\phi_d(\mathbf{r})$, and breakup amplitude from overlap of $\Psi^{AD}(\mathbf{R},\mathbf{r})$ with $\phi_k(\mathbf{r})$ (the $n-p$ continuum wave function).

4-body models

If projectile can be considered as 3-body object (eg. ^{3}He, ^{6}He, ^{11}Li) and target not of interest ⇒ structureless. Then require 4-body scattering model [Al-Khalili, 1995a].

Example: scattering of ^{11}Li from ^{12}C

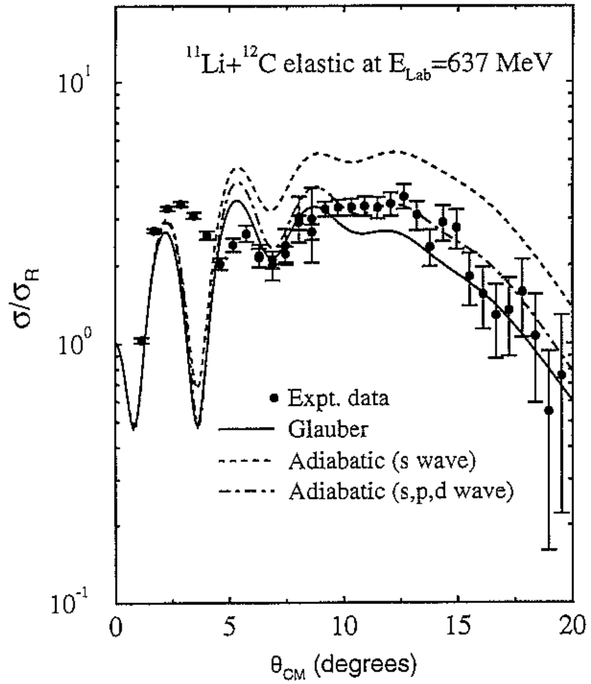


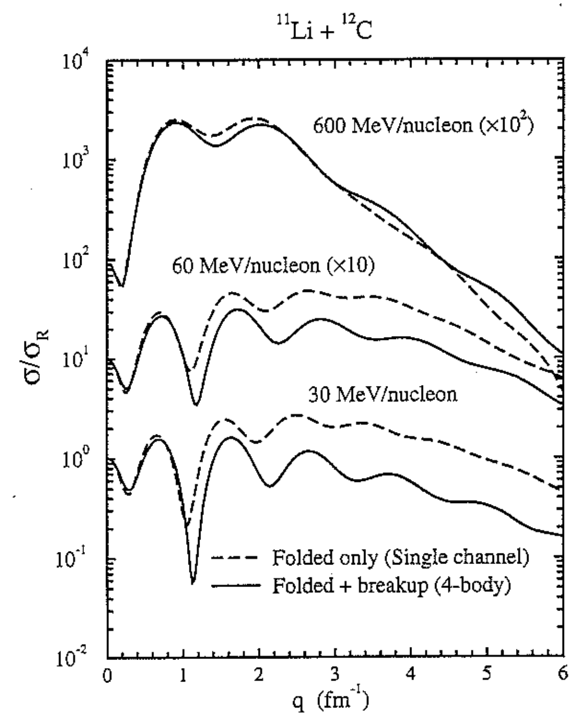
S.E. equation for system is

$$(T_R + H_3 + V(\mathbf{R},\rho,\mathbf{r}))\,\Psi(\mathbf{R},\rho,\mathbf{r}) = E\,\Psi(\mathbf{R},\rho,\mathbf{r}) \quad (16)$$

where H_3 is the internal 3-body Hamiltonian of the ^{11}Li.

Can also make adiabatic approximation [Christley, 1995] and replace H_3 by the ^{11}Li B.E. Then both **r** and ρ become parameters.





Breakup reactions

(eg. $A(d,pn)A$). Usual approach is DWBA.

Transition amplitude is

$$T = \langle \chi_p^{(-)}(\mathbf{r}_p)\, \chi_n^{(-)}(\mathbf{r}_n) \mid V_{pn} \mid \Psi^{(+)}(\mathbf{r}, \mathbf{R}) \rangle \qquad (17)$$

where $\Psi^{(+)}$ is solution of Eq.(10).

This is known as the **Post** form. Can also write amplitude in **Prior** form

$$T = \langle \Psi^{(-)}(\mathbf{r}, \mathbf{R}) \mid V_p + V_n \mid \phi_d(\mathbf{r})\, \chi_d^{(+)}(\mathbf{R}) \rangle \qquad (18)$$

In the DWBA approach, we replace $\Psi^{(\pm)}$ by $\chi^{(\pm)}$. eg

$$T_{DWBA}^{Post} = \langle \chi_p^{(-)}(\mathbf{r}_p)\, \chi_n^{(-)}(\mathbf{r}_n) \mid V_{pn} \mid \phi_d(\mathbf{r})\, \chi^{(+)}(\mathbf{R}) \rangle \quad , (19)$$

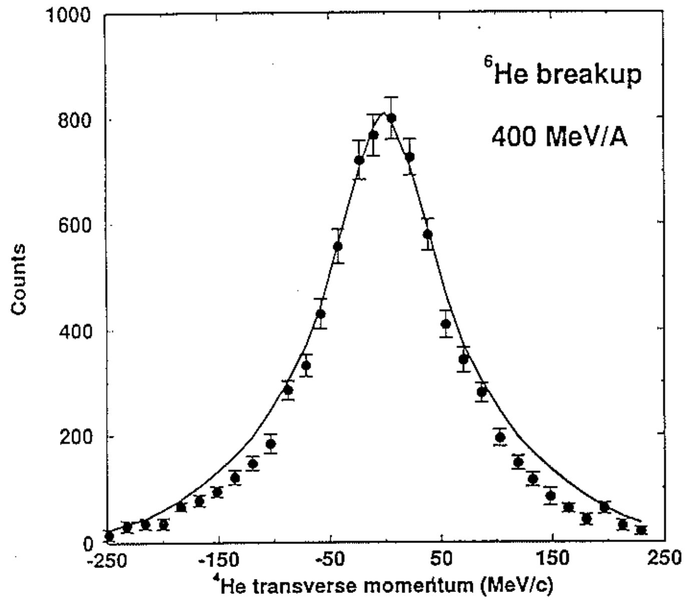
where $\chi_p^{(-)}$, $\chi_n^{(-)}$ are distorted waves due to V_p and V_n
$\chi_d^{(+)}$ is distorted wave due to V_d.

For discussion of Post and Prior forms of DWBA see [Rybicki, 1972]

Breakup reactions can tell us about structure of projectiles if loosely bound (eg. halo nuclei).

One method is to study momentum distrubution of constituents of projectile in exit channel. For example, in breakup of ^{6}He (into $\alpha + n + n$), narrow transverse momentum distribution of detected α's led to verification of large spatial extent of ^{6}He.

Explanation: narrow momentum-space wave function $\Rightarrow$ extended configuration-space wave function (by Fourier transforming or via the Uncertainty principle). [Zhukov, 1993].



Transfer reactions

Simplest example is one-nucleon transfer, and best studied reactions are those involving deuterons (i.e (d,p), (p,d)).

(a) Stripping $A(d,p)B$

$$T_{dp}^{Post} = \langle \chi_p^{(-)}(\mathbf{r}_p)\,\phi_n(\mathbf{r}_n) \mid V_{pn} \mid \phi_d(\mathbf{r})\,\chi_d^{(+)}(\mathbf{R})\rangle \quad . \qquad (20)$$

(b) Pickup $A(p,d)B$

$$T_{pd}^{Prior} = \langle \Psi^{(-)}(\mathbf{r},\mathbf{R}) \mid V_{pn} \mid \chi_p^{(+)}(\mathbf{r}_p)\,\phi_n(\mathbf{r}_n)\rangle \qquad (21)$$

Can calculate above amplitude in different ways.

a) replace $\Psi^{(\pm)}$ by $\Psi^{(\pm)}_{AD}$ (3-body)

b) replace $\Psi^{(\pm)}$ by $\phi_d\,\chi_d^{(\pm)}$ (2-body).

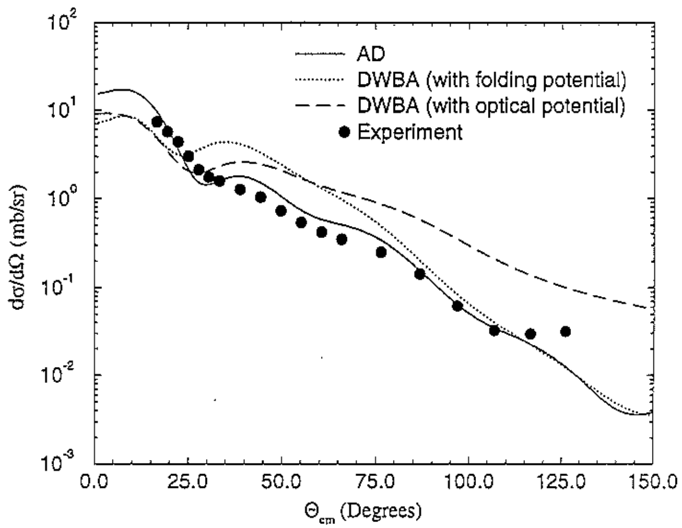


Figure from [Gönül, 1994].

Charge exchange reactions

eg. $A(p,n)B$, $A(n,p)B$, $A(^3He,t)B$

Study of c.e. reactions useful in providing information about structure of nuclear states (eg. low-lying resonances, isobaric analogue states).

Usual theoretical model is DWBA. For the reaction $A(a,b)B$

$$T = \langle \chi^{(-)}\, \phi_b\, \phi_B \mid V \mid \phi_a\, \phi_A\, \chi^{(+)} \rangle \quad , \tag{22}$$

where V is sum of all NN interactions.