Few-body models of direct reactions.

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1 Introduction

From the point of view of the practising theorist we want to have a manageable way of handling nuclear reactions involving complex nuclei. The quantum mechanical theory of 2- and 3-nucleon scattering with local or non-local inter-nucleon potentials is well understood and it is therefore natural to try and model nuclear reactions involving complex nuclei as 2- and 3-body sytems. In the case of reactions involving deuterons and single-nucleon halo nuclei (e.g., ^{11}Be) a 3-body model is particularly appropriate because the small neutron separation energies of these nuclei mean that the break-up degree of freedom is of great importance and needs to be treated explicitly in any realistic theoretical account of their reactions with other nuclei.

From the point of view of comparison with experiment, in addition to providing a theory which gives a good account of differential crosssections and polarization observables, the task of theory is to show how credible information about the structure of the nuclei involved can be extracted from experimental data. This means that we need to know how the few-body model is connected with the many-problem of real interest. We therefore have to develop approximation schemes which capture the most important features of the few-body dynamics in a way which generalises to the many-body case.

One of the most exciting scientific developments in recent years has been the advent of accelerated beams of radioactive nuclei with exotic combinations of neutron and proton numbers. The new techniques produce beams of nuclei which decay by the weak interaction but are stable against decay into their constituents. Nuclear reactions induced by beams incident on targets of ordinary stable nuclei are important sources of information about the structure of the exotic species. For example, experiments of this type led to the discovery of the important new class of nuclei known as halo nuclei.[1, 2, 3]

New experiments are extending our understanding of these novel systems. The mechanisms involved in reactions involving haloes and other nuclei far from the valley of stability present a special challenge to theorists. An im-

portant consideration is that exotic nuclei are often very weakly bound and easily broken up in the Coulomb and nuclear fields of the target nucleus. Halo nuclei are an extreme case with almost zero binding energy. As a consequence, theories which address the special features associated with strong coupling to excited states of the projectile which may be in the continuum are a prerequisite if reliable information on nuclear structure is to be deduced from reaction experiments. On the other hand, many of the relevant experiments, both current and planned, involve projectile energies which allow simplifying assumptions to be made which help to make the theory more transparent. I will present some of the insights obtained in this way with illustrations from recent experiments.

Earlier versions of these notes have been published in [4], [5] and [6]

2 Halo nuclei.

Key features of halo nuclei [7] remind us of some familiar features of the deuteron. For present purposes we can ignore the small deuteron D-state and consider the deuteron to be a $1S$ state in an attractive n-p potential of depth about 30MeV and range about 1fm. The deuteron has a binding energy of $\varepsilon_d = 2.2$MeV. Quantum mechanics tells us that in the classically forbidden region where the n-p separation r is bigger than 1fm the space part of the deuteron's wavefunction will have the form

$$\phi_0 = N_d \frac{\exp(-\lambda_d r)}{r}, \qquad r > 1\text{fm}, \tag{1}$$

where N_d is a normalisation constant. The constant λ_d is determined from the deuteron binding energy by $\lambda_d = \sqrt{2\mu\varepsilon_d/\hbar^2}$ where μ is the reduced mass of the n-p system. This is the functional form predicted by Yukawa for the interaction associated with the exchange of massive particles. Particle exchange also proceeds through classically forbidden regions and hence gives rise to the same functional dependence on distance.

The small deuteron binding energy results in the value $1/\lambda_d = 4.2$fm which is much larger than the range of the n-p interaction and means that in the deuteron the neutron and proton spend a significant part of the time in the classically forbidden region. Indeed, for many purposes its a good approximation to approximate the deuteron wavefunction by the form (1) for *all* values of r.

The one-neutron halo nucleus ^{11}Be can be described in a similar picture. It costs 0.503MeV to remove a neutron from ^{11}Be and leave ^{10}Be in its ground state. The simplest version of the halo model describes the corresponding component of the ^{11}Be wavefunction as a ^{10}Be core and a neutron in an S-state. For n-core separations r bigger than the core radius the neutron wavefunction is

$$\phi_0 = N_{11}\frac{\exp(-\lambda_{11}r)}{r}, \tag{2}$$

where $1/\lambda_{11} = 6.7$fm as detemined by the neutron separation energy.

The key point here is that $1/\lambda_{11}$ is much bigger than the size of the core so that the classically forbidden region outside the core plays a very important role, just as in the case of the deuteron.

There are however some important differences between the d and ^{11}Be cases. In the first place, unlike the deuteron, ^{11}Be has a bound excited state with a separation energy of 0.18MeV. Secondly the Pauli principle demands that the ground state of ^{11}Be be a $2S$ state with a node in the core region in contrast with the nodeless function which simple potential models give for the deuteron. This reflects the fact that underlying this 2-body picture of ^{11}Be is a many fermion system.

The qualitative features associated with very weak binding suggests that an approach based on the 2-body picture might be a good starting point for studying the structure of ^{11}Be, in contrast with mean field models which emphasise the identity of all nucleons. It is this possibility of an alternative good starting point for models of their structure which makes halo systems interesting from a theoretical point of view. For an excellent bibliography and a discussion of corrections to the basic few-body models see [16]

In these lectures I will concentrate on one-neutron halos. Much of what I have to say is also relevant to multi-neutron halos such as the famous two-neutron halo ^{11}Li [1, 7].

3 2-body scattering

To calculate the scattering of a particle from a target represented by a central potential $V(R)$ we have to solve the equation

$$(T_R + V(R))\chi(\vec{R}) = E\chi(\vec{R}). \tag{3}$$

The kinetic energy operator T_R is as defined by

$$T_R = -\frac{\hbar^2}{2\mu_{PT}}\nabla_R^2, \tag{4}$$

where μ_{PT} is the particle-target reduced mass.

We require solutions which are regular at $R = 0$ and which for $R \to \infty$ in the direction $\hat{\vec{R}}$ satisfy

$$\chi(\vec{R}) \to \exp(i\vec{K}.\vec{R}) + f(\hat{\vec{R}})\exp(iKR)/R, \tag{5}$$

where $\vec{K}$ is the incident momentum. For present purposes I have ignored spin dependence and Coulomb forces (their long range introduces well understood technical complications in the 2-body problem).

The elastic differential cross section is related to f by

$$\frac{d\sigma}{d\Omega_{\hat{\vec{R}}}} = \mid f(\hat{\vec{R}}) \mid^2 . \tag{6}$$

Calculating f for given V is a completely solvable problem. We use the fact that for a central V angular momentum is conserved just as in classical mechanics. This means that we can find solutions of (3) which are eigenfunctions of $\vec{L}^2$ and L_z where $\vec{L} = \vec{R} \wedge (i^{-1})\nabla_R$ is the angular momentum operator (in units of $\hbar$) and z is an arbitrary z-axis. Such solutions have the form

$$\chi(R, \theta, \phi) = \chi_{LM}(R)Y_{LM}(\theta, \phi), \tag{7}$$

where the angles (θ, ϕ) describe the direction of $\vec{R}$ in the chosen co-ordinate system. The spherical harmonics Y_{LM} are angular momentum eigenfunctions[8] and satisfy

$$\vec{L}^2 Y_{LM} = L(L+1)Y_{LM}, \tag{8}$$

$$L_z Y_{LM} = MY_{LM}. \tag{9}$$

The radial function $\chi_{LM}(R)$ satisfies the ordinary differential equation

$$-\frac{\hbar^2}{2\mu_{PT}}\frac{1}{R}\frac{d^2}{dR^2}(R\chi_{LM}(R)) + [V(R) + \frac{\hbar^2}{2\mu_{PT}}\frac{L(L+1)}{R^2}]\chi_{LM}(R) = E\chi_{LM}(R). \tag{10}$$

This equation has a unique solution within a constant of proportionality when the condition $\lim_{R\to 0} R\chi_{LM}(R) = 0$ is imposed.

For a finite range potential satisfying $V(R) = 0,\ R > R_0$ this unique solution has the form

$$\chi_{LM}(R) \stackrel{R \geq R_0}{=} \alpha(\cos\delta_L \, \mathrm{j}_L(KR) + \sin\delta_L \mathrm{n}_L(KR) \tag{11}$$

$$\stackrel{KR \gg L}{\rightarrow} \alpha \sin(KR - L\pi/2 + \delta_L)/KR, \tag{12}$$

where α is a constant independent of R and in the second line I have used the asymptotic forms of the regular and irregular spherical Bessel functions[8]:

$$\mathrm{j}_L(KR) \stackrel{KR \gg L}{\rightarrow} \sin(KR - L\pi/2)/KR \tag{13}$$

$$\mathrm{n}_L(KR) \stackrel{KR \gg L}{\rightarrow} \cos(KR - L\pi/2)/KR. \tag{14}$$

The spherical Bessel functions are 2 standard linearly independent solutions of eq.(10) in any region where $V(R)$ vanishes[8].

In these equations the phaseshifts δ_L appear. They are functions of energy and angular momentum and contain all the interesting dependence of the scattering on the potential V. The phaseshifts are complex if V is. The terminology is easily understood from eq.(12). If V is identically zero for all R the correct solution of eq.(10) is the spherical Bessel $\mathrm{j}_L(KR)$ with asymptotic form eqs.(13,14). Thus δ_L determines the phase difference at large distance between the free ($V = 0$) and the scattering wavefunction for a given L.

In order to relate the phase shifts to the scattering amplitude and hence to cross sections we must learn how to choose the c_{LM} so that the superposition

$$\chi(\vec{R}) = \sum_{LM} c_{LM} \chi_L^{(+)}(\vec{R}) Y_{LM}(\hat{R}), \tag{15}$$

has the asymptoic form (5). I have added a (+) superscript to χ_L to designate that this function is normalized so that it satisfies eq. (11) with $\alpha = 1$.

It is easy to find the correect c_{LM}'s because we already know that the incident plane wave has the expansion

$$\exp i\vec{K}.\vec{R} = 4\pi \sum_{LM} i^L Y^*_{LM}(\hat{K}) Y_{LM}(\hat{R}) \mathrm{j}_L(KR). \tag{16}$$

A simple calculation using eqs.(12)-(16) and (5) gives

$$c_{LM} = 4\pi \exp(i\delta_L) i^L Y^*_{LM}(\vec{K}), \tag{17}$$

and hence

$$f^{(+)}(\theta,\phi) = 4\pi \sum_{LM} Y^*_{LM}(\vec{K}) Y_{LM}(\theta,\phi) \exp(i\delta_L) \sin\delta_L / K, \quad (18)$$

$$\chi^{(+)}_{\vec{K}}(\vec{R}) = 4\pi \sum_{LM} i^L Y^*_{LM}(\vec{K}) Y_{LM}(\vec{R}) \exp(i\delta_L) \chi^{(+)}_L(R), \quad (19)$$

where (θ,ϕ) are the angles defining the direction of observation with respect to a chosen co-ordinate system.

Eq.(18) shows explicitly the relation between the scattering amplitude and the set of phase-shifts. The scattering amplitude is related to the elastic differential cross section by eq.(6). The phase shifts also determine the total reaction cross section, which is a measure of the flux going into all channels except the elastic channel and is given by

$$\sigma_R = \frac{\pi}{K^2} \sum_L (2L+1)(1-|S_L|^2), \quad (20)$$

where

$$S_L = \exp 2i\delta_L, \quad (21)$$

is the elastic S-matrix (a 1x1 matrix in this special case).

Eq.(19) gives the relation between the full scattering state wavefunction in 3 dimensional space and its components with definite angular momentum L. The radial wavefunctions, $\chi^{(+)}_L(R)$, have an asymptotic form which is determined by the phase shifts. At finite distances $\chi^{(+)}_L(R)$ is determined obtained by solving the radial equation (10) numerically or otherwise. The phase-shift is found by matching the numerical value of the logarithmic derivative at some $R > R_0$ to the formula (11). Knowing the phase-shift the computed numerical values of the radial wavefunction can now be re-normalized so that it satisfies eq.(11) with $\alpha = 1$, thus defining $\chi^{(+)}_L(R)$.

The function $\chi^{(+)}_{\vec{K}}(\vec{R})$ defined in this way is an example of a 'distorted wave'. The subscript tells us that it is associated with an incident plane wave with momentum $\vec{K}$. This label does *not* mean that $\chi^{(+)}_{\vec{K}}(\vec{R})$ is an eigenfunction of the momentum operator with eigenvalue $\vec{K}$! The superscript $(+)$ indicates that $\chi^{(+)}_{\vec{K}}(\vec{R})$ asymptotically has an *outgoing* spherical wave component. The complete asymptotic form is as in the RHS of eq.(5).

3.1 The mysterious $\chi^{(-)}_{\vec{K}}(\vec{R})$

In formulating theories of complex collisions we frequently come across other scattering states which asymptotically look like a plane wave plus an ingoing spherical wave. We can construct such a state using the same radial wavefunctions by making a different choice for the c_{LM} in eq.(15) (simply change the factor $\exp i\delta_L$ in eq.(17) to $\exp -i\delta_L$) .

In nuclear physics where the potentials V are frequently complex another case occurs. $\chi^{(-)}_{\vec{K}}(\vec{R})$ is defined to be a solution of eq.(3) with incoming spherical waves and with V replaced by its complex conjugate V^*:

$$(T_R + V^*(R))\chi^{(-)}_{\vec{K}}(\vec{R}) = E\chi^{(-)}_{\vec{K}}(\vec{R}). \tag{22}$$

It is easy to show that the radial part of this function must be the complex conjugate of $\chi^{(+)}_L$ within a multiplicative factor and hence does not require separate calculation and a new phase-shift. We find

$$\begin{aligned}
\chi^{(-)}_{\vec{K}}(\vec{R}) &\rightarrow \exp(i\vec{K}.\vec{R}) + f^{(-)}(\hat{\vec{R}})\exp(-iKR)/R, & (23)\\
f^{(-)}(\theta,\phi) &= 4\pi\sum_{LM}(-1)^L Y^*_{LM}(\hat{K})Y_{LM}(\theta,\phi)\exp -i\delta^*_L \sin\delta^*_L/K, & (24)\\
\chi^{(-)}_{\vec{K}}(\vec{R}) &= 4\pi\sum_{LM} i^L Y^*_{LM}(\hat{K})Y_{LM}(\hat{R})\exp -i\delta^*_L(\chi^{(+)}_L(R))^*. & (25)
\end{aligned}$$

With these definitions the precise relationship between $\chi^{(+)}$ and $\chi^{(-)}$ is

$$(\chi^{(-)}_{\vec{K}}(\vec{R}))^* = \chi^{(+)}_{-\vec{K}}(\vec{R}). \tag{26}$$

These 2 distorted waves and their multi-channel generalisations appear frequently in theories of nuclear reactions.

4 Formal methods

4.1 The 2-body case

We have seen that the problem of the scattering of 2 bodies interacting through a potential is completely solvable. We want to be able to recognise such solvable sub-problems when they appear in the formulation of many-body theories and to achieve this we must learn how to write the 2-body results in a more formal way.

We define the 2-body state $\mid \chi^{\varepsilon}_{\vec{K}} >$ as the solution of

$$(E + i\varepsilon - T_R - V) \mid \chi^{\varepsilon}_{\vec{K}} >= i\varepsilon \mid \vec{K} >, \tag{27}$$

where T_R is the kinetic energy operator and $\mid \vec{K} >$ is the incident plane wave state

$$< \vec{R} \mid \vec{K} >= \exp i\vec{K}.\vec{R} \tag{28}$$

Clearly for $\varepsilon \to 0$ the state $\mid \chi^{\varepsilon}_{\vec{K}} >$ satisfies the same equation as $\mid \chi^{(+)}_{\vec{K}} >$,eq.(3), but unlike the latter eq.(27) has the unique solution

$$\mid \chi^{\varepsilon}_{\vec{K}} >= \frac{i\varepsilon}{(E + i\varepsilon - T_R - V)} \mid \vec{K} >, \tag{29}$$

and incorporates the correct boundary conditions for $\varepsilon > 0$. To show this we re-write eq.(27) as

$$(E + i\varepsilon - T_R) \mid \chi^{\varepsilon}_{\vec{K}} >= i\varepsilon \mid \vec{K} > +V \mid \chi^{\varepsilon}_{\vec{K}} >, \tag{30}$$

and multiply both sides of this equation by the inverse of the operator $E + i\varepsilon - T_R$. We deduce

$$\begin{aligned} \mid \chi^{\varepsilon}_{\vec{K}} > &= \frac{i\varepsilon}{E + i\varepsilon - T_R} \mid \vec{K} > + \frac{1}{E + i\varepsilon - T_R} V \mid \chi^{\varepsilon}_{\vec{K}} > \\ &= \mid \vec{K} > + \frac{1}{E + i\varepsilon - T_R} V \mid \chi^{\varepsilon}_{\vec{K}} >, \end{aligned} \tag{31}$$

where we have used the result

$$(E - T_R) \mid \vec{K} >= 0. \tag{32}$$

When written out in configuration space eq.(31) is an integral equation for $\chi^{\varepsilon}_{\vec{K}}(\vec{R})$. In the same basis and for $\varepsilon \ll E$ and *positive* the matrix elements of $(E + i\varepsilon - T_R)^{-1}$ are

$$\begin{aligned} < \vec{R} \mid \frac{1}{E + i\varepsilon - T_R} \mid \vec{R}' > \quad &= \quad -\frac{2\mu}{4\pi\hbar^2} \frac{\exp iK|\vec{R} - \vec{R}'|}{|\vec{R} - \vec{R}'|} \exp -\frac{\varepsilon K|\vec{R} - \vec{R}'|}{2E} \\ &\overset{R \gg R'}{\to} \quad -\frac{2\mu}{4\pi\hbar^2} \frac{\exp iKR - i\vec{K}'.\vec{R}'}{R} \exp -\frac{\varepsilon KR}{2E} \end{aligned} \tag{33}$$

where $\vec{K}'$ has the same magnitude as $\vec{K}$ but points in the direction (θ, ϕ) of $\vec{R}$. The quantity μ is the reduced mass of the 2-body system.

Using (33) in (31) we find that for R well outside the range of V but not large compared with $\frac{2E}{\varepsilon K}$

$$\chi^{\varepsilon}_{\vec{K}}(\vec{R}) \rightarrow \exp i\vec{K}.\vec{R} + f^{\varepsilon}(\theta,\phi)\frac{\exp{(iK - \frac{\varepsilon K}{2E})R}}{R}, \tag{34}$$

where

$$f^{\varepsilon}(\theta,\phi) = -\frac{2\mu}{4\pi\hbar^2} < \vec{K}' \mid V \mid \chi^{\varepsilon}_{\vec{K}} > . \tag{35}$$

The scattering amplitude f introduced in (5) is given by

$$\begin{aligned} f &= \lim_{\varepsilon \rightarrow 0^+} f^{\varepsilon}, \\ &= -\frac{2\mu}{4\pi\hbar^2} < \vec{K}' \mid V \mid \chi^{(+)}_{\vec{K}} > . \end{aligned} \tag{36}$$

The amplitude f^{ε} also has a physical meaning. It can be thought of as the scattering amplitude for an incident wave packet with a spread in time of order $\hbar/\varepsilon$[12].

Eq.(36) expresses the scattering amplitude as a matrix element of the potential V between a plane wave describing the final observed state and the distorted wave $\mid \chi^{(+)}_{\vec{K}} >$ which, of course, also depends implicitly on V. This expression does not at first sight appear very useful as a way of determining V. However this type of expression, especially when generalised to systems with many degrees of freedom, lends itself well to generating useful insights and guiding the approximations which are essential when we are dealing with the reaction calculations invoving more than a few nucleons.

As a simple application, let us suppose that the potential V is very weak. Then one would expect that to first order in V the distorted wave in (36) might usefully approximated by the incident plane wave $\mid \vec{K} >$. The resulting formula is the famous Born approximation

$$\begin{aligned} f^{\mathrm{Born}} &= -\frac{2\mu}{4\pi\hbar^2} < \vec{K}' \mid V \mid \vec{K} > \\ &= -\frac{2\mu}{4\pi\hbar^2} \int d\vec{R} \exp{(i\vec{Q}.\vec{R})} V(R), \end{aligned} \tag{37}$$

where $\vec{Q}$ is the momentum transfer, $\vec{Q} = \vec{K} - \vec{K}'$.

Another way of looking at the matrix element in (36) it to relate it to a matrix element of a new operator $\mathcal{T}(E + i\varepsilon)$ defined by

$$\mathcal{T}(E + i\varepsilon) = V + V\frac{1}{E + i\varepsilon - T_R - V}V. \tag{38}$$

Acting on a plane wave with momentum $\vec{K}$ related to E by

$$K = \sqrt{\frac{2\mu E}{\hbar^2}}, \tag{39}$$

we find

$$\begin{aligned} \mathcal{T}(E+i\varepsilon) \mid \vec{K} > &= V\frac{1}{E+i\varepsilon-T_R-V}[(E+i\varepsilon-T_R-V)+V] \mid \vec{K} > \\ &= V\frac{i\varepsilon}{E+i\varepsilon-T_R-V} \mid \vec{K} > \\ &= V \mid \chi^{(+)}_{\vec{K}} >, \end{aligned} \tag{40}$$

where we have used the result (29) to identify $\mid \chi^{(+)}_{\vec{K}} >$ in the 2nd line in (40).

Taking the innerproduct of both sides of (40) with a plane wave state $\mid \vec{K'} >$, where $\vec{K'}$ has the same magnitude as $\vec{K}$ and is given by (39), we obtain

$$f^{\varepsilon}(\theta,\phi) = -\frac{2\mu}{4\pi\hbar^2} < \vec{K'} \mid \mathcal{T}(E+i\varepsilon) \mid \vec{K} > . \tag{41}$$

The operator $\mathcal{T}(E+\varepsilon)$ has matrix elements between plane wave states with arbitrary momentum. It is only when the initial and final momenta are related to E by [39] that these so-called on-shell matrix elementa are related to the scattering amplitude. The half-on-shell matrix elements with $K$and E related by eq.(39) but K' arbitrary determine the distorted wave $\mid \chi^{(+)}_{\vec{K}} >$. To see this we use the last line in (31) and the result (40) to obtain

$$\begin{aligned} \mid \chi^{\varepsilon}_{\vec{K}} > &= \mid \vec{K} > + \frac{1}{E+i\varepsilon-T_R}\mathcal{T}(E+i\varepsilon) \mid \vec{K} > \\ &= \mid \vec{K} > + \int d\vec{K'} \frac{\mid \vec{K'} >< \vec{K'} \mid \mathcal{T}(E+i\varepsilon) \mid \vec{K} >}{E+i\varepsilon-E'}, \end{aligned} \tag{42}$$

where

$$E' = \frac{\hbar^2(K')^2}{2\mu}. \tag{43}$$

We see that all quantities of physical interest are determined by the $\mathcal{T}$-operator. Conversely, both the on-shell and half-off-shell matrix elements of $\mathcal{T}$ can be found by solving the Schrödinger equation for $\mid \chi^{+}_{\vec{K}} >$.

An important advantage of the formal methods we have introduced in this section is that they allow us to manipulate explicit expressions for wavefunctions and scattering amplitudes using the rules of operator algebra without having to deal with singular operators. For $\varepsilon \neq 0$ operators like $(E+i\varepsilon-H)^{-1}$ exist for any real E and for Hamiltonians H of physical interest. Boundary conditions have been taken care of once and for all by the ε prescription. The price we have to pay is that in carrying out these manipulations we have to remember thet the scattering states satisfy inhomogeneous equations such as eq.(27) rather than eigenvalue equations like eq.(3).

Of course at the end of the day we are interested in the limit $\varepsilon \to 0^+$. In the many-body case this limit has to be taken with care, but in practical nuclear reaction calculations this does not usually cause a problem. Difficulties with this limiting process can arise, for example, in the formulation of exact numerical solutions of the 3-body scattering problem.A re-formulation in terms of the Faddeev equations or their equivalents[9] is then an advantage, but for the purpose of exposing the structure of many body theories this step and its generalisations to more than 3 bodies is not necessary.

4.2 Target with internal degrees of freedom

For definiteness we consider the scattering of a neutron from a target A which has a set of bound states $\phi_0(\xi), \phi_2(\xi), \ldots, \phi_N(\xi)$ where ξ denotes the set of internal co-ordinates of A. By convention 0 labels the ground state, 1 the first excited state, and so on. The generalisation of the eq.(3) is

$$(T_R + H_A + V(\vec{R}, \xi))\chi(\vec{R}, \xi) = E\chi(\vec{R}, \xi), \tag{44}$$

where H_A is the Hamiltonian which describes the internal motion of A. T_R is the total kinetic energy operator of the neutron and the target in the overall centre of mass system and is defined in eq.(4) with a reduced mass μ_{nT}. $\vec{R}$ is the relative co-ordinate of the neutron and the centre of mass of A. The potential term $V(\vec{R}, \xi)$ depends on both $\vec{R}$ and ξ. It describes the interaction between the neutron and the target and at the most basic level can be expressed as the sum of the 2-body interactions between the neutron and the target nucleons. The ξ would then denote the co-ordinates of the target nucleons relative to its centre of mass.

One approach to solving eq.(44) is to expand χ as a superposition of the set of states ϕ_i which form a complete set if all possible states of A are

included. The coefficients in the expansion are functions of $\vec{R}$ which are the solution of a set of coupled differential equations.

The expansion is

$$\chi(\vec{R},\xi) = \chi_0(\vec{R})\phi_0(\xi) + \chi_1(\vec{R})\phi_1(\xi) + \ldots \tag{45}$$

Coupled equations are obtained by substituting the expansion (45) into (44), multiplying by ϕ_i and integrating over all ξ for $i = 0,\ 1\ldots, N$. The resulting equations for $i = 0,\ 1\ldots, N$ are

$$(E_i - T_R - V_{ii})\chi_i \;=\; \sum_{j\neq i} V_{ij}\chi_j, \tag{46}$$

where we have used the eigenvalue equations satisfied by the ϕ_i and their orthonormality relations

$$H_A\phi_i \;=\; \epsilon_i\phi_i, \tag{47}$$

$$\int d\xi \phi_i^*\phi_j \;=\; \delta_{ij}. \tag{48}$$

The energies E_i are defined by $E_i = E - \epsilon_i$.

The coupling potentials V_{ij} are functions of $\vec{R}$ defined by

$$V_{ij}(\vec{R}) = \int d\xi \phi_i^*(\xi) V(\vec{R},\xi)\phi_j(\xi). \tag{49}$$

The functions $\chi_i(\vec{R})$ have a definite physical meaning. They tell us the relative probability as a function of $\vec{R}$ for the target A to be in state i. The different possibilities for i are frequently referred to as 'channels' and the E_i are the corresponding channel energies. If the incident channel is $i = 0$ the boundary conditions to be satisfied by the χ_i for values of R outside the range of the coupling potentials are

$$\chi_0^{(+)} \;\rightarrow\; \exp(i\vec{K}_0.\vec{R}) + f_{00}^{(+)}(\hat{\vec{R}})\exp(iK_0R)/R, \tag{50}$$

$$\chi_i^{(+)} \;\rightarrow\; f_{i0}(\hat{\vec{R}})\exp(iK_iR)/R, \quad i \neq 0 \tag{51}$$

where the channel momenta K_i are defined by

$$K_i = \sqrt{\frac{2\mu_{nT}E_i}{\hbar^2}}, \tag{52}$$

provided that $E_i > 0$. If $E_i < 0$ the channel is said to be 'closed' and the corresponding χ_i vanishes exponentially at large distances. There is therefore

no outgoing flux in a closed channel and the crosssection for exciting the target into state i will vanish.

If the potentials $V_{ij} = 0$ vanish for $i \neq j$ there will be no coupling between channels. In this limit the regular solution of the coupled equations satisfying the conditions (89) is $\chi_i = 0$ for $i \neq 0$ and χ_0 satisfies

$$(E_0 - T_R - V_{00})\chi_0 = 0. \tag{53}$$

Only the cross section for the elastic scattering of the neutron by the target in its ground state is non zero.

According to (53) elastic scattering in the zero coupling case is generated by V_{00} which has the explicit expression (we assume the ground state has spin 0 for simplicity)

$$\begin{aligned} V_{00}(\vec{R}) &= \int d\xi \phi_0^*(\xi) V(\vec{R}, \xi) \phi_o(\xi). \\ &= \sum_i \int d\xi \mid \phi_0(\xi) \mid^2 v_{ni}(\vec{R} - \vec{r}_i) \end{aligned} \tag{54}$$

where the summation over i includes all the nucleons in the target and the $\vec{r}_i$ are the co-ordinates of the target nucleons relative to the target centre of mass. If the target consists of A identical nucleons (54) reduces to

$$V_{00}(\vec{R}) = A \int d\vec{r}_1 \rho(\vec{r}_1) v_{n1}(\vec{R} - \vec{r}_1), \tag{55}$$

where $\rho(\vec{r}_1)$ is the ground state one-body density of the target.

Eq.(55) is the simplest possible model for the optical potential. It relates the effective interaction between the projectile and the target to the fundamental 2-body interactions between the projectile and the target constituents and their density distribution in space. Eq(55) is often referred to as a folding model because of the way the co-ordinates appear in the integral in (55). Note that if the 2-body interaction v_{ni} is real so is V_{00}. This model cannot account for the imaginary part of the optical potential because it corresponds to a theoretical model where no flux is lost to non-elastic channels. We will see below by explicit calculation in a special case how open inelastic channels give rise to a complex effective interaction.

4.3 Formal theory of the multi-channel case

The formal approach developed in Subsection 4.1 is easily generalised to include the possibility that the target has internal degrees of freedom. Eq.(27) is replaced by

$$(E + i\varepsilon - T_R - H_A - V) \mid \chi^{\varepsilon}_{\vec{K}_0} >= i\varepsilon \mid \vec{K}_0, \phi_0 >, \tag{56}$$

where

$$< \vec{R}, \xi \mid \vec{K}_0, \phi_0 >= \exp{(i\vec{K}_0.\vec{R})}\phi_0(\xi). \tag{57}$$

is the wavefunction corresponding to a plane wave incident on the target in its ground state.

The presence of the $i\varepsilon$ term on the right hand side of (56) means that the coupled equations equivalent to (46) are

$$(E_i + i\varepsilon - T_R - V_{ii}) \mid \chi^{\varepsilon}_i > \;\; = \;\; \sum_{j \neq i} V_{ij} \mid \chi^{\varepsilon}_j > + \delta_{i0} i\varepsilon \mid \vec{K}_0 >, \tag{58}$$

where the channel components are related to $\mid \chi^{\varepsilon}_{\vec{K}_0} >$ by

$$\chi^{\varepsilon}_{\vec{K}_0}(\vec{R}, \xi) = \sum_i \chi^{\varepsilon}_i(\vec{R})\phi_i(\xi). \tag{59}$$

For $N = 1$ (2 channels) these equations reduce to

$$(E_0 + i\varepsilon - T_R - V_{00}) \mid \chi^{\varepsilon}_0 > \;\; = \;\; V_{01} \mid \chi^{\varepsilon}_1 > + i\varepsilon \mid \vec{K}_0 >, \tag{60}$$

$$(E_1 + i\varepsilon - T_R - V_{11}) \mid \chi^{\varepsilon}_1 > \;\; = \;\; V_{10} \mid \chi^{\varepsilon}_0 > . \tag{61}$$

The second equation allows us to express $\mid \chi^{\varepsilon}_1 >$ in terms of $\mid \chi^{\varepsilon}_0 >$ as

$$\mid \chi^{\varepsilon}_1 >= (E_1 + i\varepsilon - T_R - V_{11})^{-1} V_{10} \mid \chi^{\varepsilon}_0 > . \tag{62}$$

Substituting this result into the first of eqs.(61) gives an equation for $\mid \chi^{\varepsilon}_0 >$ which is

$$(E_0 + i\varepsilon - T_R - V_{00} - V_{01}(E_1 + i\varepsilon - T_R - V_{11})^{-1} V_{10} \mid \chi^{\varepsilon}_0 >= i\varepsilon \mid \vec{K}_0 > . \tag{63}$$

We recognise this as a 2-body elastic scattering equation for $\mid \chi^{\varepsilon}_0 >$ with the effective potential

$$\mathcal{V}_{opt} = V_{00} + V_{01}(E_1 + i\varepsilon - T_R - V_{11})^{-1} V_{10} \tag{64}$$

This analysis shows how an effective potential can always be found which generates the exact elastic scattering. Note that the second term in eq.(64) has an imaginary part in the limit $\varepsilon \to 0^+$, but only if $E_1 > 0$, *i.e.*. above the threshold for exciting the state ϕ_1. Furthermore this imaginary part is negative, indicating that it is associated with a loss of probability flux from the incident channel. Our treatment of the $N = 1$ problem is a special case of a general theory due to Feshbach[17]. A discussion of effective interactions including the effect of energy averaging and additional references can be found in Section 2.9 of ref.[11] and in Chapter 11 of [10].

4.4 Inelastic scattering and the DWBA

From eq.(62) we obtain an explicit expression for the amplitude for exciting the state ϕ_1. We use the operator identity

$$\frac{1}{A} - \frac{1}{B} = \frac{1}{A}(B - A)\frac{1}{B}, \tag{65}$$

to deduce

$$\frac{1}{(E_1 + i\varepsilon - T_R - V_{11})} = \frac{1}{(E_1 + i\varepsilon - T_R)}\left[1 + V_{11}\frac{1}{(E_1 + i\varepsilon - T_R - V_{11})}\right], \tag{66}$$

and therefore

$$\mid \chi_1^{\varepsilon} >= \frac{1}{(E_1 + i\varepsilon - T_R)}\Omega_1^{(-\varepsilon)\dagger} V_{10} \mid \chi_0^{\varepsilon} >, \tag{67}$$

where

$$\Omega_1^{(-\varepsilon)} = \left[1 + \frac{1}{(E_1 - i\varepsilon - T_R - V_{11})}V_{11}\right]. \tag{68}$$

Following similar reasoning as in the analysis surrounding eq.(34) we find that for $R \to \infty$ and $\varepsilon \to 0^+$

$$< \vec{R} \mid \chi_1^+ > \to f_{10}^{(+)}\frac{\exp iK_1 R}{R}, \tag{69}$$

where

$$f_{10}^{(+)} = -\frac{2\mu}{4\pi\hbar^2} < \vec{K}_1 \mid \Omega_1^{(-)\dagger} V_{10} \mid \chi_0^{(+)} >, \tag{70}$$

and the wave number $\vec{K}_1$ is in the direction of observation and has magnitude

$$K_1 = \sqrt{\frac{2\mu}{\hbar^2} E_1}. \tag{71}$$

Note that in eq.(70), $\chi_0^{(+)}$ is the exact elastic scattering distorted wave as generated by $\mathcal{V}_{opt}$, or by solving the coupled equations (61).

It can be shown that for $\varepsilon \to 0$ through positive values

$$< \vec{K}_1 \mid \Omega_1^{(-)\dagger} =< \chi_{\vec{K}_1}^{(-)} \mid, \tag{72}$$

where $\mid \chi_{\vec{K}_1}^{(-)} >$ is a distorted wave of the type defined in Sub-section 3.1 and generated by the potential V_{11}. The inelastic scattering amplitude (70) can therefore be written

$$f_{10}^{(+)} = -\frac{2\mu}{4\pi\hbar^2} < \chi_{\vec{K}_1}^{(-)} \mid V_{10} \mid \chi_{\vec{K}_0}^{(+)} >, \tag{73}$$

where we have written χ_0^{ε} as $\chi_{\vec{K}_0}^{(+)}$ to conform to our earlier notation in the limit $\varepsilon \to 0$.

The DWBA as originally formulated can be obtained from this exact expression by replacing the initial distorted wave by the distorted wave generated by the potential V_{00} instead of $\mathcal{V}_{opt}$. This approximation includes the diagonal elements V_{11} and V_{00} to all orders but the potential V_{10} responsible for coupling the 2 channels is included in first order only. The usual Born approximation of inelastic scattering corresponds to replacing the 2 distortwaves by plane waves. The DWBA attempts to do better than that by recognising that the target-projectile interaction will scatter the 2 nuclei and convert the plane waves into distorted plane waves. The DWBA is a consistent way to take this physical effect into account when the channel coupling is weak enough to be treated to 1st order.

We emphasise that in the realistic multi-channel situation we frequently meet in nuclear physics the DWBA as just defined is rarely used. An expression with a structure similar to (73) is used but with the 2 distorted waves generated by complex potentials which fit elastic scattering data. This is called the 'Distorted Wave Method' by Satchler[11].

Modern computer methods emphasise exact solutions of coupled channels models although Distorted Wave ideas often still underlie much qualitative thinking about nuclear reactions.

4.5 Practical evaluation of DW matrix elements

It is useful to have some insight into how expressions such as (73) are actually evaluated. For central distorting potentials the 2 distorted waves have expansions of the form introduced in subsection 3. We can write

$$\chi^{(+)}_{\vec{K}_0}(\vec{R}) = 4\pi \sum_{L_0 M_0} i^{L_0} Y^*_{L_0 M_0}(\hat{K}_0) Y_{L_0 M_0}(\Omega_R) \exp(i\delta^0_{L_0}) \chi^{(+)}_{L_0}(R), \quad (74)$$

$$(\chi^{(-)}_{\vec{K}_1}(\vec{R}))^* = 4\pi \sum_{L_1 M_1} (-i)^{L_1} Y_{L_1 M_1}(\hat{K}_1) Y^*_{L_1 M_1}(\Omega_R) \exp(i\delta^1_{L_1}) \chi^{(+)}_{L_1}(R) \quad (75)$$

Inserting these expressions into (73) we obtain

$$f^{(+)}_{10} = -\frac{8\pi\mu}{\hbar^2} \sum_{L_0 M_0 L_1 M_1} i^{L_0 - L_1} Y^*_{L_0 M_0}(\hat{K}_0) Y_{L_1 M_1}(\hat{K}_1) \exp i(\delta^0_{L_0} + \delta^1_{L_1})$$
$$\times \int_0^\infty R^2 dR \chi^{(+)}_{L_1}(R) < L_1 M_1 \mid V_{10} \mid L_0 M_0 > \chi^{(+)}_{L_0}(R), \quad (76)$$

with summation over L_0, M_0, L_1, M_1. Integration over the direction of $\vec{R}$ and the internal co-ordinates of the target are contained in the matrix element $< L_1 M_1 \mid V_{10} \mid L_0 M_0 >$

$$< L_1 M_1 \mid V_{10} \mid L_0 M_0 >= \int d\Omega_R \, d\xi \, Y^*_{L_1 M_1}(\Omega_R) \phi^*_1(\xi) V_{10}(\vec{R}, \xi) \phi_0(\xi) Y_{L_0 M_0}(\Omega_R). \quad (77)$$

By making a multipole expansion of the interaction V_{10} this matrix element can be factorised into reduced multipole matrix elements, which contain all the dependence on the structure of the target states ϕ_0 and ϕ_1, Clebsch-Gordan coefficients which carry the implications of angular momentum conservation and form factors which depend on the radial co-ordinate R (see,*e.g.*, Section 5.6 of ref.[11]). We do not have space to expand further on these important ideas here.

Standard codes exist which evaluate DW amplitudes such as (76) and solve the coupled equations (46) exactly for given poentials and target wave functions as a routine matter[13].

We note that most of the formulae in this section have to be modified when the nuclei involved are charged. The formal expressions we have used are perfectly valid when all the Coulomb interactions are screened at large distances. The correct expressions to be used for partial wave expansions when the uninteresting dependence on the screening radius is extracted are given in many standard texts, *eg*, [11]. Reviews which include Coulomb and spin-dependent effects can be found in [14] and [15].

4.6 Many-body $\mathcal{T}$-operator

As in the 2-body case we introduce an operator $\mathcal{T}$ whose on-shell matrix elements are proportional to the inelastic and inelastic scattering amplitudes. The key difference is that this operator now acts in the space of the variables ξ as well as $\vec{R}$. The definition of $\mathcal{T}$ is

$$\mathcal{T}(E+i\varepsilon) = V + V\frac{1}{E+i\varepsilon - T_R - H_A - V}V. \tag{78}$$

This expression has exactly the same formal structure as (38) in the 2-body case, but the presence of the target Hamiltonian, H_A in the denominator in (78) makes this $\mathcal{T}$ a much more complicated operator.

In terms of $\mathcal{T}$ the elastic and inelastic scattering amplitudes are given by

$$f^{\varepsilon}(\theta,\phi)_{i0} = -\frac{2\mu}{4\pi\hbar^2} < \vec{K}_i, \phi_i \mid \mathcal{T}(E+i\varepsilon) \mid \vec{K}_0, \phi_0 >, \tag{79}$$

and the scattering state is expreessed in terms of the off-shell matrix elements of $\mathcal{T}$ through

$$\mid \chi^{\varepsilon}_{\vec{K}_0} >= \mid \vec{K}_0, \phi_0 > + \sum_i \int d\vec{K'} \frac{\mid \vec{K'}, \phi_i >< \vec{K'}, \phi_i \mid \mathcal{T}(E+i\varepsilon) \mid \vec{K}_0, \phi_0 >}{E_0 + \epsilon_0 + i\varepsilon - E' - \epsilon_i}. \tag{80}$$

The point about these formal expressions is that they help us to recognise quantities that are calculable using standard techniques when they are buried in a complicated theory of a nuclear reaction. To calculate a matrix element of a $\mathcal{T}$ operator in practice one usually, but not always re-expresses the calculation in terms of coupled differential equations.

5 Selected topics in the scattering and reactions of deuterons and other halo nuclei

5.1 The adiabatic approximation for loosely bound projectiles

The use of adiabatic approximations in nuclear reaction theories has a long history. The key idea is to separate the relevant degrees of freedom into 'slow' and 'fast' categories. The 'slow' variables are treated as fixed during the collision and the associated scattering problem for the 'fast' variables is treated

quantum mechanically or semiclassically depending on the masses and energies involved in the reaction. This is the analogue of the Bohr-Oppenheimer approximation for bound states of molecules, where the electronic motion for fixed nuclear positions is calculated quantum mechanically.

An early example is Barrett's treatment of neutron scattering by a nucleus regarded as a rigid rotor[18]. He calculated the scattering amplitude for the scattering of a neutron by a deformed potential as a function of the orientation of the nuclear body-fixed axes. Scattering amplitudes were then calculated by taking matrix elements of this amplitude between the nuclear states of interest. Here the 'slow' motion is obviously the nuclear collective rotation and the incident neutron motion is regarded as 'fast'. Other applications of the adiabatic approximation to theories of elastic and inelastic scattering in nuclear physics are reviewed in ref.[11], pages 83-84 and elsewhere in that book.

In the case studied by Barrett the approximation in the energy domain which complements this time picture is that the energy associated with relevant rotational excitations is assumed to be small compared to the translational energy of the neutron. In the adiabatic limit all excited rotational states are assumed to be effectively degenerate on a scale determined by the incident energy. We will see how this comes about formally below. This way of thinking tends to give too conservative an idea of the usefulness of the adiabatic approximation because it fails to take into account the crucial role played by absorption in nuclear reactions. In addition, we now have a better understanding of the spatial regions in which adiabatic solutions of the few-body Schrodinger equation are expected to be most valid and how to exploit this knowledge in applications to particular reaction channels. This insight has proved to be particularly useful in applications to (d, p) and $p, d)$ reactions[20, 21, 23, 25].

Historically an important feature of the adiabatic approximation was that it is a cheap way of doing a complicated coupled channels calculation. For example, in Barrett's calculation, channels with a fast neutron and the rotor in any one of its excited states are taken into account coherently and non-perturbatively. With modern computing power this may not be a big advantage, but when the projectile is loosely bound and the relevant excitation spectrum is in the continuum the adiabatic approximation can be a powerful tool as well as frequently providing important insight and checks of more complete calculations. CDCC calculations, which were pioneered[20] and developed in [26],[27],[28],[29], discretise the continuum and are in prin-

ciple an improvement over the adiabatic approximation, but CDCC codes are generally available only for 2-body projectiles. See [30] for a recent review. CDCC calculations for a 3-body projectile have been published[31][32]. An adiabatic code which treats 3-body projectiles has been available for some time[33].

In this historical context we note that the adiabatic approximation in the sense we use here is the basis of Glauber's theory[34] of high energy composite particle scattering which has been widely used in the analysis of reaction experiments with halo and other light nuclei[30],[37]. In these calculations selected co-ordinates are treated adiabatically and the eikonal approximation is used to describe the scattering of the frozen object. In his development of a microscopic theory of the nucleon optical potential, Glauber goes further and treats *all* the internal co-ordinates of the target nucleus as frozen during the scattering. The resulting 2-body problem for a set of frozen internal nuclear co-ordinates is then solved using the eikonal approximation.

5.2 Deuteron-nucleus collisions

This section discusses a 3-body model of the system $n + p + A$, where A is a heavy nucleus in its ground state. The theory we describe can, and has been, applied to other systems and we shall mention several in passing. Some cases, for which the adiabatic approximation is potentially useful, have special problems, e.g., Coulomb and antisymmetrisation effects. These problems are best discussed separately and will be ignored here.

In a 3-body model of deuteron-nucleus collisions, for example, channels corresponding to elastic deuteron scattering and elastic deuteron break-up in which the target is left in its ground state are all included in a unified way. Excited states of the target A do not appear explicitly. The relation between this model and the underlying many-body problem will be discussed below.

In a widely used notation we use $\vec{r}$ for the position of the neutron relative to the proton and $\vec{R}$ for the position of the centre-of-mass of n and p relative to A. The Hamiltonian of the model is

$$H = T_R + H_{np} + V(\vec{R}, \vec{r}),\ V(\vec{R}, \vec{r}) \equiv V_{nA}(\vec{R} + \vec{r}/2) + V_{pA}(\vec{R} - \vec{r}/2) \quad (81)$$

where $H_{np} = T_r + V_{np}$ is the Hamitonian for relative motion of the $n - p$ system. The T's are kinetic energy operators. For the purpose of this talk we assume that all Coulomb interactions are screened at large distances. We use $\phi_0(\vec{r})$ for the ground state of the deuteron with energy $-\epsilon_0 < 0$, and

$\phi_{\vec{k}}^{(+)}(\vec{r})$ for the continuum of $n-p$ scattering states which are eigenstates of H_{np} with energy $\epsilon_k > 0$ and satisfy outgoing wave boundary conditions. It is the purpose of the adiabatic approximation to treat the coupling between the deuteron and the scattering state continuum in as accurate and transparent a way as possible. In the 3-body model this coupling comes from the tidal forces generated by the fact that, over the volume of the deuteron, $V_{nA}(\vec{R}+\vec{r}/2)$ and $V_{pA}(\vec{R}-\vec{r}/2)$ generate forces on the nucleons which differ in magnitude and direction.

5.3 Time dependent picture

Under the transformation

$$\Psi^{trans} = \exp(-\frac{iH_{np}t}{\hbar})\Psi. \tag{82}$$

the time dependent Schrodinger equation for $\Psi(\vec{R},\vec{r},t)$ becomes

$$(T_R + V(\vec{R},\vec{r}(t)))\Psi^{trans}(\vec{R},\vec{r},t) = i\hbar\frac{\partial\Psi^{trans}}{\partial t}. \tag{83}$$

In eq.(83) the $n-p$ relative co-ordinate $\vec{r}(t)$ has acquired a time dependence through the relation

$$\vec{r}(t) = \exp(\frac{iH_{np}t}{\hbar})\vec{r}\exp(-\frac{iH_{np}t}{\hbar}). \tag{84}$$

The adiabatic approximation assumes that the collision time T is so short that we can replace $\vec{r}(t)$ by $\vec{r}(0)=\vec{r}$. A sufficient condition for this step to be valid is that T satisfy

$$\left|\frac{< H_{np} > T}{\hbar}\right| \ll 1. \tag{85}$$

where $< H_{np} >$ is the maximum eigenvalue of H_{np} excited in the collision through the tidal forces. For the strong interaction this maximum is related to the shape of the nuclear surface and is insensitive to the incident deuteron energy. Hence for sufficiently high energy the collision time will always become small enough that the condition (85) is satisfied.

Implementation of the adiabatic approximation for a stationary state requires the solution of the adiabatic equation[20]

$$(T_R + V(\vec{R},\vec{r}) - E_d)\Psi^{ad}(\vec{R},\vec{r}) = 0, \tag{86}$$

which for fixed $\vec{r}$ is a 2-body problem. Note that even for central V_{nA} and V_{pA} the potential in eq. (86) is not central when considered as a function of $\vec{R}$ for fixed $\vec{r}$ so coupled equations still have to be solved in general.

5.4 Solution of the adiabatic equation

The solution of the adiabatic equation can be reduced to a manageable set of coupled equations by either of two methods which are based on different truncation schemes. Both methods assume the nucleon potentials V_{nA} and V_{pA} are central.

(i) In this method[26],[27],[35][36] the adiabatic wavefunction is expanded in the basis $[Y_l(\hat{r}) \times Y_L(\hat{R})]_{JM}$ and uses the fact that $V_{nA} + V_{pA}$ is diagonal in J, M, although not in l and L. Truncation in these angular momenta is required.

(ii) The method used by Barrett[18] uses the fact that $V_{nA} + V_{pA}$ is diagonal in $\vec{L}.\hat{r}$ and proceeds by making a truncated multipole expansion of the potentials.

Method (ii) is well adapted to the case of scattering by a deformed nucleus when a natural truncation of the multipole expansion occurs. In the present context, convergence in l is found to be very rapid and truncation is linked with the adiabatic assumption of low excitations in the $\vec{r}$ co-ordinate.

The adiabatic wavefunction corresponding to a deuteron incident with momentum $\vec{K}_d$ has the structure[20]

$$\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}), \tag{87}$$

where $\chi^{ad(+)}_{\vec{K}_d}$ satisfies

$$(T_R + V(\vec{R},\vec{r}) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = 0, \tag{88}$$

with the boundary condition ($\vec{r}$ fixed)

$$\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) \overset{R\to\infty}{\to} \exp(i\vec{K}_d.\vec{R}) + f(\hat{R},\vec{r})\frac{\exp(iK_dR)}{R}. \tag{89}$$

Note that the adiabatic equation (88) is exactly the equation one obtains by replacing H_{np} in the exact 3-body Hamiltonian (81) by a constant and choosing this constant to be the deuteron ground state energy[20].

The function $\chi^{ad(+)}_{\vec{K}_d}$ and the scattering amplitude $f(\hat{R},\vec{r})$ both depend on $\vec{r}$. The physical meaning of $f(\hat{R},\vec{r})$ is that it describes the elastic scattering in the direction $\hat{R}$ of an $n-p$ pair with fixed separation $\vec{r}$ by the potential $V(\vec{R},\vec{r})$. The factor ϕ_0 in (87) ensures that the coefficient of the plane wave in eq.(87) and the exact 3-body wave function $\Psi^{(+)}_{\vec{K}_d}(\vec{R},\vec{r})$ coincide. The formula (89) can also be derived by taking the adiabatic limit of formal expressions for the exact three-body wavefunction[19].

The adiabatic equation must be solved for as many values of $\vec{r}$ as is required for the application. For example, for elastic deuteron scattering we have to evaluate the scattering amplitude $f(\hat{R},\vec{r})$ for as many values of $\vec{r}$ as is required for the accurate evaluation of the integral

$$f_{elastic}(\hat{R}) = \int d\vec{r}\phi^*_o(\vec{r})f(\hat{R},\vec{r})\phi_o(\vec{r}), \tag{90}$$

i.e., for $0 < r < r_d$, where r_d is a measure of the size of the deuteron.

This formalism can be simplified considerably in several interesting cases.

5.5 Glauber theory

When the conditions are such that eq. (86) can be solved in the eikonal approximation an explicit formula for $f(\hat{R},\vec{r})$ in terms of a path integral of $V_{nA}+V_{pA}$ can be obtained. This is Glauber's theory of deuteron-nucleus scattering. The method has been extensively applied to the calculation of elastic scattering and reaction cross-sections of halo nuclei with 2 or more clusters[30],[38]. The integral in (90) is carried out numerically without further approximation of the expression using the best available models for the ground state wave function ϕ_0.

There are also considerable simplifications in the zero range limit of deuteron stripping or when one of the potentials V_{nA} and V_{pA} vanishes. These simplifications can be applied even when the eikonal approximation is not appropriate. They cases will be discussed below.

5.6 Application to Li scattering

The non-eikonal adiabatic method has been applied to the elastic scattering of ^{6}Li in an $\alpha+d$ model[39], ^{7}Li in an $\alpha+t$ model[40] and to the scattering of ^{11}Li in a $n+n$+^{9}Li 3-body model[33].

5.7 Validity of the adiabatic assumption

It is shown in[19] that estimates based on eq.(85) are too conservative. For short range forces the collision time decreases as the impact parameter increases. Hence for a given range of excitation energies the worst violations of the inequality (85) tend to occur at low impact parameters. Under conditions of strong absorption these are just the impact parameters whose contributions to the excitation are suppressed. A revised adiabatic condition which includes this effect is given in ref.[19]. Eq.(85) is replaced by an expression that is proportional to the derivative of the projectile-target optical potential i.e., the tidal force) at the strong absorption radius and inveresely proprtional to the cube of the relative velocity of the projectile and target. By comparison with CDCC calculations in a special case the revised condition was shown to give an excellent idea of the accuracy of the adiabatic approximation for a model of ^{11}Be elastic scattering. The new criterion shows that for elastic scattering of strongly absorbed particles the adiabatic approximation can be valid down to much lower energies than hitherto believed.

We show below that the criterion for validity of the adiabatic method to deuteron stripping is quite different from elastic scattering. It will be shown that the adiabatic approximation is a sufficient but not necessary condition for the validity of the Johnson-Soper method[20, 21, 23] instead of a deuteron optical potential for calculating the distorted wave in the deuteron channel.

5.8 An instructive special case

We discuss the case of for deuteron scattering by a massive target as an example.

When one of the interactions V_{nA}, V_{pA} vanishes (or is a constant) the adiabatic equation can be solved exactly in a very simple way. We take $V_{nA} = 0$ for definiteness. This is obviously not a very realistic model of elastic deuteron scattering in general, but it is very relevant to Coulomb break-up of the deuteron[51]. Eq.(86) becomes

$$(T_R + V_{pA}(\vec{R} - \vec{r}/2) - E_d)\Psi^{ad}(\vec{R}, \vec{r}) = 0. \tag{91}$$

For a deuteron incident with momentum $\vec{K}_d$ this has the exact solution[41][42][43]

$$\Psi^{ad}_{\vec{K}_d}(\vec{R}, \vec{r}) = \phi_0(\vec{r}) \exp(i\vec{K}_d.\vec{r}/2)\chi^{(+)}_{\vec{K}_d}(\vec{R} - \vec{r}/2), \tag{92}$$

where $\chi^{(+)}_{\vec{K}_d}$ is the distorted wave for a particle with the mass of the deuteron by the potential V_{pA} and evaluated at the argument $\vec{R}-\vec{r}/2$ i.e., at the $p-A$ relative co-ordinate.

In this limit the elastic deuteron cross-section is simply expressed in terms of the deuteron ground state form factor and the deuteron elastic cross-section generated by V_{pA}. In the generalisation to the case of a projectile with unequal mass clusters the factors of $1/2$ are replaced by ratios involving the masses of the clusters [41]. The generalisation gives a good account of some features of ^{11}Be scattering[41].

The explicit form (92) also makes a deficiency of the adiabatic wavefunction very clear. It predicts that for any $\vec{R}$, $\Psi^{ad}_{\vec{K}_d} \to 0$ exponentially for $r \to \infty$, i.e, in regions of space where we look for outgoing waves in the stripping and break-up channels. We therefore cannot expect the adiabatic wavefunction to be accurate for large r even though it may be perfectly adequate for finite values of r. The reason for this shortcoming can be traced to the treatment of the break-up continuum as degenerate. It is then not possible for the 3-body wavefunction to carry the phase relations between the R and r dependence which are essential to generate the correct asymptotic form in rearrangement and break-up channels. We must use the adiabatic wavefunction in ways which respect this observation. We do this by using it as the basis for an iterative solution to the Schrödinger equation.

5.9 Iteration of the adiabatic solution. Implications for deuteron stripping calculations. The ADW method.

We re-write the 3-body Schrödinger equation in the form

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \;=\; V_{np}\Psi_{\vec{K}_d}, \tag{93}$$

where we have transferred the V_{np} term to the right-hand-side. This term requires $\Psi_{\vec{K}_d}$ only within the range of V_{np}, and for which we can therefore use, *e.g.*, the adiabatic wavefunction[20] or its generalisations [21, 23, 25, 22, 24].

The method proceeds by treating the equation

$$(E - T_R - T_{np} - V_{pA} - V_{nA})\Psi_{\vec{K}_d} \;=\; V_{np}\Psi^{ad}_{\vec{K}_d}, \tag{94}$$

as an inhomogeneous equation for $\Psi_{\vec{K}_d}$ with the right-hand-side given. In particular, by examining the outgoing Green's function for the operator

$E - T_R - T_{np} - V_{pA} - V_{nA}$ it is found that the iterated solution has the correct asymptotic form in the stripping and break-up channels with outgoing waves with momenta correctly given by the conservation of energy, i.e., without the assumption of degenerate break-up channels used in the adiabatic wavefunction. In the exact solution of the inhomogeneous equation the coefficients of the outgoing waves in the stripping and break-up channels (transition amplitudes) are given by

$$T_{d,p} = \langle \chi_p^{(-)} \phi_n \mid V_{np} \mid \Psi^{ad}_{\vec{K}_d} \rangle, \tag{95}$$

$$T_{d,np} = \langle \chi_p^{(-)} \chi_n^{(-)} \mid V_{np} \mid \Psi^{ad}_{\vec{K}_d} \rangle, \tag{96}$$

where the $\chi_p^{(-)}$ and $\chi_n^{(-)}$ are distorted waves generated by V_{pA} and V_{nA}, respectively, and ϕ_n is a neutron bound state, all evaluated at the correct energies predicted by energy conservation.

Strictly speaking, to deduce (95) and (96) from eq.(94) requires the additional assumption that the target has infinite mass, $A \to \infty$. It is only then that the kinetic energy separates into n and p terms and solutions of the homogeneous equation have a product form. Recoil terms of order $1/A$ can mix in terms in the final state in which the neutron is excited out of the state ϕ_n. These corrections (Recoil Excitation and Break-up (REB) effects) can be very significant for light nuclei[25].

All the quantities in (95) and (96) are solutions of 2-body problems and are readily calculated. The evaluation of the amplitudes requires techniques similar to those used in the evaluation of DWBA amplitudes. We emphasise that this iterated theory goes far beyond DWBA. No Born approximation is involved. Couplings between 3-body channels are included to all orders in $\Psi^{ad}_{\vec{K}_d}$.

It should be clear that if $\Psi^{ad}_{\vec{K}_d}$ were replaced in equations (95) and (96) by the exact three-body wavefunction $\Psi_{\vec{K}_d}$ then these expressions would give the exact reaction amplitudes. The approximations (95) and (96) assume that all coupling effects in the wavefunction for r less than the range of V_{np} can be adequately accounted for by the adiabatic wavefunction.

In principle this whole procedure could be iterated by calculating the complete solution of the inhomgeneous equation (93) (not just the asymptotic form as explained above) and then using the solution to re-calculate an improved inhomogeneous term (see [44] for an application of this idea to the (p, d^*) reaction).

In the zero range limit for V_{np} the evaluation of equations (95) and (96) becomes particularly simple because then we can use[20]

$$V_{np}\Psi^{ad}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},\vec{r}) = V_{np}\phi_0(\vec{r})\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0).. \tag{97}$$

From eq.(88) we see that $\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$ satisfies

$$(T_R + V(\vec{R},0) - E_d)\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0) = 0, \tag{98}$$

and is, therefore, simply a distorted wave generated by the central potential $V_{nA}(R) + V_{pA}(R)$.

We see that in this limit the adiabatic theory of stripping looks even more like a DWBA theory, but this is misleading because the function $\chi^{ad(+)}_{\vec{K}_d}(\vec{R},0)$ includes outgoing waves in break-up channels and the potential $V_{nA}(R) + V_{pA}(R)$ may have very little to do with elastic deuteron scattering. We would expect that a first approximation to the latter would be to average the nucleon optical potentials over the deuteron ground state density distribution $\phi_0(r)^2$ (see the Watanabe model[45]and its generalisation to non-local nucleon potentials [46]). This produces a much more diffuse potential than $V_{nA}(R) + V_{pA}(R)$. The nature of the differences produced by this smaller diffuseness in the (d,p) are analysed in detail in [21].

When s-wave break-up dominates at small r the formalism can be modified to correct for a finite range V_{np}[20]. We expand the three-body wavefunction in terms of the complete set of $n-p$ relative motion states $\{\phi_0, \phi^{(+)}_{\vec{k}}\}$ introduced earlier. Provided the continuum states which contribute do not have very high energy we can safely assume that only s-wave states will overlap V_{np} and we can write

$$\Psi_{\vec{K}_d}(\vec{R},\vec{r}) = \phi_0(r)\chi_o(\vec{R}) + \int d\vec{k}\phi^{(+)}_k(r)\chi_k(\vec{R}),\ r < r_{np}, \tag{99}$$

where r_{np} is the range of V_{np}. For continuum energies less than roughly 10 MeV the shape of the s-wave state $\phi^{(+)}_k(r)$ does not depend strongly on energy for $r < r_{np}$ and we can write

$$\phi_k(r) \simeq g(k)\phi_0(r),\ r < r_{np}, \tag{100}$$

where $g(k)$ is independent of r. Inserting this approximation into (99) gives

$$V_{np}\Psi_{\vec{K}_d}(\vec{R},\vec{r}) \simeq V_{np}\phi_0(r)\bar{\chi}_{\vec{K}_d}(\vec{R}), \tag{101}$$

where

$$\bar{\chi}_{\vec{K}_d}(\vec{R}) = \chi_o(\vec{R}) + \int d\vec{k} g(k) \chi_k(\vec{R}). \tag{102}$$

We emphasise that the form implied by (101) for the 3-body wavefunction is generally only valid inside the range of V_{np}. The r and R dependence of the adiabatic wavefunction does not factorise in this way in general (see, e.g., eq.(92)). It is this special nature of the assumptions involved in applying the ideas of ref.[20] to stripping which means that the Johnson-Soper distorted wave can have wider applicability than simple adiabatic criteria might suggest.

In the next subsection we will show how these qualitative ideas can be systematically exploited to give an equation for $\bar{\chi}_{\vec{K}_d}(\vec{R})$.

5.10 The method of Johnson and Tandy

A more general approach to obtaining $V_{np}\Psi$, the projection of the 3-body wavefunction which is most relevant to the transition amplitude for stripping, and break-up according to (95) and (96), is to expand in terms of a set of functions which are complete within the range of V_{np}.

A convenient set for this purpose is the set of Weinberg states[50], or Sturmians, $\bar{\phi}_i(r),\ i = 0 \ldots \infty$, used by Johnson and Tandy[22]. They satisfy

$$(T_r + \alpha_i V_{np})\bar{\phi}_i = -\epsilon_0 \bar{\phi}_i \tag{103}$$

where the α_i's are Sturmian eigenvalues. For $i = 0$, $\alpha_0 = 1$ and $\bar{\phi}_0$ is proportional to the deuteron ground state ϕ_0. These states all look like the deuteron asymptotically, but as i and α_i increase they oscillate more and more rapidly at short distances.

An expansion in terms of this set converges rapidly if the dependence on r of the three-body wave function inside the range of V_{np} is similar to ϕ_0. Coupled equations for the coefficients are readily derived using the orthogonality property

$$\langle \bar{\phi}_i \mid V_{np} \mid \bar{\phi}_j \rangle = -\delta_{i,j}. \tag{104}$$

The first term in the expansion has the form $\phi_0 \bar{\chi}$ with $\bar{\chi}$ defined by

$$(E_d - T_R - \bar{V}(R))\bar{\chi}_{\vec{K}_d}(\vec{R}) = 0, \tag{105}$$

where the potential $\bar{V}$ is given by

$$\bar{V}(R) = \frac{\langle \phi_0 \mid V_{np}(V_{nA} + V_{pA}) \mid \phi_0 \rangle}{\langle \phi_0 \mid V_{np} \mid \phi_0 \rangle}. \tag{106}$$

The bra and ket in this equation imply an integration over $\vec{r}$ with fixed $\vec{R}$. $\bar{V}$ reduces to the zero-range result $V(R,0)$ of eq.(98) if the variation of the nucleon optical potentials over a distance of the order of r_{np} can be neglected. For nucleon potentials with a Wood-Saxon shape the effect of the finite range correction in eq.(106) is to increase their diffuseness slightly. A simple way of incorporating these modifications, which can be important for light nuclei, can be found in refs.[21],[23].

Results that take into account terms in the expansion in $\bar{\phi}_i$'s beyond $i = 0$ are given in refs.[22],[24] in specific cases. They show that this is a promising approach to the calculation of break-up effects in stripping which go beyond the adiabatic approximation.

An interesting feature of this derivation is that it makes no reference to the incident energy but only the assumption that the break-up states excited have low enough excitation that the 3-body wave function is well approximated by the form $\phi_0(r)\bar{\chi}(\vec{R})$ inside the range of V_{np}. This suggests that a stripping theory which takes into account break-up effects can be based on the use of $\bar{\chi}$ as a distorted wave even at low energies where the adiabatic condition is not well satisfied.

The situation for elastic deuteron scattering is quite different because there the adiabatic wave function is needed out to distances of the order of the size of the deuteron where the form $\phi_0(r)\bar{\chi}(\vec{R})$ has no justification. The use of Sturmians is not then appropriate.

There have been many comparisons between theory based on eqs. (95) with $V_{np}\Psi^{ad}$ given by (101), (105), (106) and stripping experiments. We call this the Adiabatic Distorted Wave (ADW) method. Over a wide range of energies ADW has gives angular distributions for differential crosssections and polarization observables which agree with stripping and pick-up experiments more convincingly and consistently than the DWBA and without the extra ambiguities associated with the use of a deuteron optical potential in the DWBA. Everything in an ADW calculation is determined by *nucleon* optical potentials for the appropriate energy and target. Some early examples are given in ref.[11], pages 732-734 but there have been many others since, *e.g.*[55]. The method has also been successfully used for (p,d*) [44] and (d,^{2}He)[56] charge exchange reactions.

The study of Cadmus and Haeberli[47] is particularly noteworthy. These authors measured a large number of deuteron and proton elastic scattering observables to pin down optical model parameters so that the DWBA could be applied unambiguously. It was found to fail badly. They were able to use their measurements of deuteron and proton polarization parameters to identify the source of the failures and how these were remedied by the ADWA method.

A more recent example of how ADW method can be used to give an improved account of the systematics of a particular (d,p) transition as function of energy is ref.[48].

Although the ADWA method goes well beyond the DWBA and includes effects due to coupling between the elastic deuteron channel and other 3-body channels to all orders it is nevertheless an approximate theory which is expected to need correction at some level. An important example of a clear indication from experiment of the need to go beyond ADW theory and the nature of those corrections is the work of the Indiana-Surrey collaboration[49].

One way of going beyond the ADW method for stripping and pick-up is to use the Sturmian expansion method of refs.[22],[24]. An alternative is to use the CDCC wavefunction in (95). For deuteron stripping this is done in refs.[52],[53]. Ref.[54] reports a surprisingly large discrepancy between measured proton polarisation and CDCC predictions for ^{208}Pb(d,p)^{209}Pb at 20 MeV incident energy. Other observables are well reproduced.

5.11 Link with the CDCC method

In the CDCC method the 3-body wave function for deuteron-nucleus scattering, $\Psi(\vec{R},\vec{r})$, is expanded in a set of orthonormal functions $\phi_s(\vec{r})$, $s = 0,1,2.....$, which diagonalise H_{np} with eigenvalues ϵ_s and discretise the $n-p$ continuum. The set are usually defined so that $\phi_{s=0}$ is the deuteron ground state. Coupled equations are then derived as a technique for solving the 3-body Schrödinger equation.

We can expand the adiabatic wavefunction $\Psi^{ad}(\vec{R},\vec{r})$ in a volume $\mathcal{V}$ in $\vec{r}$ space in terms of an orthonormal set $\psi_s(\vec{r})$, $s = 0,1,2.....$ which is complete in $\mathcal{V}$:

$$\Psi^{ad}(\vec{R},\vec{r}) = \sum_{s=0}^{\infty} \psi_s(\vec{r})\chi_s(\vec{R}), \tag{107}$$

and derive coupled equations for the χ_s's of the form

$$(E_d - T_R)\chi_s(\vec{R}) = \sum_{s'} \langle \psi_s \mid V \mid \psi_{s'} \rangle \chi_{s'}(\vec{R}), \tag{108}$$

where V is defined in eq.(81) and the coupling matrix elements involve an integration over $\mathcal{V}$.

If we identify the ψ_i's and ϕ_i's (to obtain the CDCC equations the ϕ_i's must diagonalise H_{np}) these equations are similar to the CDCC equations with all the channel energies set equal to $-\epsilon_0$. We can put this another way. If the set $\psi_s(\vec{r}),\ s = 0, 1, 2....., $ is complete in $\mathcal{V}$, and the functions $\chi_s(\vec{R})$ satisfy the coupled equations (108) then these equations show that, for $\vec{r}$ in $\mathcal{V}$, $\sum_s \psi_s(\vec{r})\chi_s(\vec{R})$ satisfies

$$\begin{aligned}(E_d - T_R)\sum_s \psi_s(\vec{r})\chi_s(\vec{R}) &= \sum_s \psi_s(\vec{r}) \int d\vec{r}' \psi_s^*(\vec{r}') \sum_{s'} V(\vec{R}, \vec{r}')\psi_{s'}(\vec{r}')\chi_{s'}(\vec{R}) \\ &= V(\vec{R}, \vec{r}) \sum_{s'} \psi_{s'}(\vec{r})\chi_{s'}(\vec{R}), \end{aligned} \tag{109}$$

where the completeness of the ψ_s's has been used. Eq.(109) is just the adiabatic equation. Hence $\sum_s \psi_s(\vec{r})\chi_s(\vec{R})$ is the adiabatic solution $\Psi^{ad}(\vec{R}, \vec{r})$.

We see that the Adiabatic approach can be regarded as an approximation to the CDCC method. Note that the adiabatic method does not take into account any restrictions imposed by the Pauli Principle on the states which should be included in the set ϕ_s. For example, in the case of ^{11}Be scattering the adiabatic calculations include transitions into a state in which the neutron is in a nodeless s-state with respect to the ^{10}Be core. Such contributions are easily excluded in the CDCC calculation or in the Johnson-Tandy approach[22], but it is not obvious how to do this in an adiabatic calculation without introducing non-local projection operators with the consequential loss of some of the characteristic simplicity of the adiabatic equation.

At first sight it is puzzling that the adiabatic calculation can take into account effects due to excited deuteron states when only the deuteron ground state wave function appears explicitly. In CDCC calculations the wave functions of all excited states deemed to be important must be inserted explicitly into the calculation of the coupling matrix elements. Our derivation above shows how this puzzle can be resolved but it is also helpful to note that the ground state wavefunction ϕ_0 determines the Hamiltonian through the identity (see the Appendix to [19])

$$H_{np} = -\epsilon_0 - \frac{\hbar^2}{2\mu_{np}} \phi_0^{-1} \nabla_r \phi_0^2 . \nabla_r \phi_0^{-1}, \tag{110}$$

where the ∇_r operators act on everything to the right of them.

We note that the CDCC method uses a basis which is complete in large volumes of $\vec{r}$ space. As we have seen the stripping and break-up matrix elements (95) and (96) explore a very restricted part of this space, i.e., within the range of V_{np}. For this purpose the complete set used by Johnson and Tandy[22] and its generalisations may be more efficient.

In their exploration of the adiabatic approximation Johnson and Soper[**?**] proposed an approximation to the CDCC method which replaced the deuteron continuum by a single pseudo state. In their method the component $V_{np}\Psi$ of the 3-body wave function is still governed by equations (101), (105) and (106), but the deuteron elastic scattering wavefunction and the pseudo break-up state satisfy a pair of coupled equations. This method was critically examined in great detail by Rawitscher[26],[27] within the CDCC framework. A more sophisticated version of the single pseudostate method was developed by Amakawa, Austern and Vincent[57] and is known as the quasi-adiabatic method.

Some of the clearest evidence for the importance of deuteron break-up effects and the failure of the DWBA for (d,p) and (p,d) reactions has been obtained by using the adiabatic approximation as implemented in the ADWA. However, we have seen that the adiabatic approximation can be regarded as an approximation to the CDCC, so it might be argued that the adiabatic approximation no longer has a role. It is only recently, however, that the CDCC method has become available for projectiles with more than 2 clusters and, when coupled with the eikonal approximation where applicable, the adiabatic approximation is a powerful tool for the analysis of reactions with composite projectiles.

An attractive feature of the adiabatic approach which it shares with CDCC is that it provides a framework for inserting the systematics of the interaction of the constituents of the projectile with the target into reaction analyses. This means that the need for optical potentials for unstable projectiles can often be avoided, but it still requires reliable information about the constituents' optical potentials and hence good elastic scattering data for appropriate energies and targets.

An advantage of the adiabatic method over CDCC is that its implementation does not need detailed wavefunctions of strongly coupled excited bound and continuum states of the projectile. The construction of these states may introduce considerable uncertainties into a CDCC calculation. It is important therefore to understand the limitations of the adiabatic approximation.

Perhaps the most important feature of the adiabatic approximation is its ability to provide insights into the mechanism of complex reactions. It can be used to provide checks of CDCC and other theories as well as being a relatively easy and transparent way to take into account complicated effects of channel coupling in some important special cases.

5.12 Link between CDCC, ADW and Faddeev methods for 3-body models of the deuteron-nucleus system

One of the attractive features of studying the $d+A$ system on the basis of a 3-body Hamiltonian is that the Faddeev formulation of the scattering problem or its practical equivalents[9] can be used to perform calculations that are not based on the expansions used in the CDCC or Johnson-Tandy methods and can be carried through to arbitrary precision in terms of truncation parameters that are well under control.

In the theory of Alt, Grassberger and Sandhas[9] transfer amplitudes appear as the matrix elements between plane wave channel states of a set of operators $U_{\alpha,\beta}$, where α and β label all possible channels in the 3-body system. These operators satisfy coupled integral equations that are particularly convenient when an exact solution of the 3-body Schrödinger equation is required. A major recent advance has made it possible to include Coulomb break-up accurately[58, 59].

Three-body models of nuclear reactions involving deuterons and massive nuclei have proved invaluable in increasing our understanding of the key role of deuteron break-up effects, but we are usually interested in extracting nuclear structure information from data in a credible fashion and therefore a key issue is the link between the 3-body models and the underlying many-body problem. Three-body model Hamiltonians are always going to be an approximate image of the many-body system. From the point of view of the nuclear structure practitioner, therefore, we must learn how to use the exact solutions of 3-body models to provide guidance for how deuteron break-up effects can be included in a way that can be generalized to the many-body case. This means, for example that emphasis is placed on determining an adequate representation of the 3-body scattering wavefunction in restricted regions of the 6-dimensonal configuration space rather than calculating an accurate version everywhere, for which latter purpose the coupled equations

of Alt, Grassberger and Sandhas[9] are well adapted.

The 1999 paper by Timofeyuk and Johnson[25] was motivated along these lines. In doing so they used a particular formula for the (d,p) and (p,d) transition amplitude, namely eq.(95) generalised to include recoil (REB) effects. Deuteron breakup effects were taken into account using the ADW method as described above. At the same time, because their formula did not contain the "remnant term" that appears in standard many-body theories of deuteron stripping and pick-up (see,e.g., [10], page 151) there was a transparent link to many-body concepts such as overlap functions, spectroscopic factors and asymptotic normalization factors[6],[60]. In a recent paper[**?**] it was shown that this particular formula can be derived from the coupled equations of Alt, Grassberger and Sandhas[9].

5.13 The underlying many-body theory

Our presentation so far is based on the 3-body Hamiltonian (81) in which V_{nA} and V_{pA} are optical potentials. These are usually taken at $\frac{1}{2}$ of the incident deuteron kinetic energy E_d. This is reasonable if any break-up components in the 3-body wave function have a small fraction of E_d and is certainly consistent with the adiabatic assumption. More generally the $\frac{1}{2}E_d$ prescription can be justified[46] by detailed calculation if the energy dependence arises purely from non-locality and break-up effects are negligible. A deeper question is why the effective interaction in the 3-body model should have anything to do with the nucleon optical potential.

To study this further we recall that the 3-body wavefunction in eq.(83) is the projection of the full many-body $A+2$ wavefunction onto the target ground state. In a standard fashion[17] we can show that the effective Hamiltonian which governs this component when the target is in its ground state in the incident channel is

$$H_{eff} = T_R + H_{np} + \langle \phi_A \mid U \mid \phi_A \rangle, \tag{111}$$

where the bra-ket notation implies integration over the target nucleus co-ordinates to leave an operator in n and p co-ordinates only. The complicated many-body operator U satisfies

$$U = (v_{nA} + v_{pA}) + (v_{nA} + v_{pA})\frac{Q_A}{e}U, \quad v_{NA} = \sum_{i=1}^{A} v(N,i). \tag{112}$$

The v_{NA}'s, $N = p, n$, are the sums of the 2-body interactions between the incident p and n and the target nucleons $1...A$. The operator Q_A projects on to excited states of the target. U sums up all processes via excited states which begin and end on the target ground state.

U can be separated into its p and n contributions by using manipulations from multiple scattering theory. We obtain

$$U = (U_{nA} + U_{pA}) + U_{nA}\frac{Q_A}{e}U_{pA} + U_{pA}\frac{Q_A}{e}U_{nA} + ..., \tag{113}$$

where

$$U_{nA} = v_{nA} + v_{nA}\frac{Q_A}{e}U_{nA}, \quad U_{pA} = v_{pA} + v_{pA}\frac{Q_A}{e}U_{pA}, \tag{114}$$

and the dots in (113) are terms of 3rd or higher order in U_{nA} and/or U_{pA}, always with an excited target (though not necessarily excited deuteron) as intermediate state.

The above expressions for U_{nA} and U_{pA} are strongly reminiscent of Feshbach's[17] expressions for the operator which gives the nucleon optical potential when sandwiched between the target ground state. Note however that the energy denominator e which appears everywhere is not the denominator one expects to see in the nucleon operator. It is given (for an infinitely massive target) by $e = E + i0 - T_n - T_p - V_{np} - H_A$ where H_A is the target Hamiltonian. It is plausible that if low energy weakly correlated break-up states dominate, then in U_{nA}, for example, we can neglect V_{np} and replace $E - T_p$ by $\frac{1}{2}E_d$ on the average. $\langle \phi_A \mid U_{nA} \mid \phi_A \rangle$ then reduces to a formal expression for the neutron optical potential at energy $\frac{1}{2}E_d$.

The higher order terms in (113) still have to be dealt with, however. The second order terms describe a process in which the neutron excites the nucleus and the proton subsequently de-excites it, and vice versa. The magnitude of such effects will be small for weakly correlated $n - p$ configurations such as in the deuteron or low energy break-up configurations. Their neglect is consistent with approximations already made.

We learn from this analysis that the validity of the 3-body model as usually assumed is intimately bound up with the assumption of dominance of low energy break-up configurations. However, all the arguments given above are very qualitative. Very little work has been done to give substance to them and estimate any corrections to the usual model. It would seem to be hardly worth while to go much beyond the adiabatic or CDCC treatments

of three-body effects, both of which assume that break-up excitations can be truncated, without investigating many-body corrections to the 3-body Hamiltonian, eq.(81), more thoroughly.

Finally in this section, note that even in the lowest order version of the effective interaction one expects to see corrections arising from the identity of n and p and the target nucleons. There are essentially two distinct approaches to these antisymmetrization effects. The RGM methods starts from a many-body Hamiltonian with an assumed N-N interaction and puts in antisymmetrization right from the start, treating the nucleons in the deuteron and in the target on the same footing. On the other hand it is difficult in practice to treat all possible open channels and absorption has to be inserted by hand. RGM calculations have been published[63] which include deuteron break-up effects using discretisation methods similar to the CDCC method. Antisymmetrisation effects are very important in this approach[63].

The alternative approach of refs.[20],[64],[65] is based on the idea that because of the loosely bound extended spatial nature of the deuteron, the nucleons in the deuteron see the target nucleus much as if they were completely independent and so much of the effects of antisymmetry and coupling to excited target states are contained in the complex optical potentials of the 3-body model.In this way one automatically generates a total deuteron reaction cross section which is close to that observed even when deuteron elastic break-up is neglected. New effects arise for deuteron collisions only because the nucleons in the deuteron may scatter off each other into occupied target states (Pauli blocking). These effects are included through a generalisation of the Bethe-Goldstone equation. The role of break-up channels is to re-adjust the flow of flux into inelastic channels involving excited target states as well as transfering flux into break-up channels. For some impact parameters the effect of the break-up channel may even be to *decrease* the partial reaction crosssection because in the break-up configuration the nucleons may overlap spatially less well with the imaginary parts of the nucleon optical potentials. Tostevin, et al[65] found that absorption tends to suppress Pauli blocking and no new major qualitative effects were found. Aoki[52], using a formula for these effects due to Pong and Austern[64], reported that Pauli blocking effects gave a 10% repulsive correction to the deuteron optical potential and improved CDCC fits to elastic deuteron scattering on ^{208}Pb at 20 MeV. The effect on (d,p) crosssections was negligible.

5.14 Elastic Coulomb break-up

An interesting application of the expression (96) is the case of Coulomb break-up of a 2-body projectile where one body is uncharged and we can neglect its interaction with the target. In the deuteron case, for example, we can then use eq.(92) and the matrix element factorises[51] as

$$T^{ADWA}_{d,np} = \left(\int d\vec{r} \exp(i(\vec{k}_n - \frac{1}{2}\vec{K}_d).\vec{r}) V_{np}\phi_0(\vec{r}) \right) \int d\vec{r}_p \chi^{(-)*}_{\vec{k}_p}(\vec{r}_p) \exp(-i\vec{k}_n.\vec{r}_p) \chi^{(+)}_{\vec{K}_d}(\vec{r}_p) \quad (115)$$

where $\chi^{(+)}_{\vec{K}_d}$ and $\chi^{(-)}_{\vec{k}_p}$ are distorted waves describing the scattering of a point deuteron and a proton by the Coulomb field of the target. For a very large screening radius the second factor has the form of an unobservable phase factor which goes to infinity with the screening radius, multiplied by an integral which is similar to that which occurs in the theory of Bremsstrahlung and can be evaluated analytically for a point target. The first factor is easily evaluated for any V_{np}. The restriction to $A \rightarrow \infty$ is easily lifted[51].

This theory has been applied successfully to Coulomb break-up of the deuteron[51], ^{11}Be[66], ^{6}He [67] and ^{19}C[66],[68]

We emphasise that the theory which leads to (115) is not perturbation theory. Terms of all orders in V_{pA} and V_{np} are included. The effects of coupling between Coulomb break-up channels are included in all orders with the 2 assumptions that the wavefunction eq.(92) adequately represents the 3-b0dy scattering wavefunction inside the range of V_{np} and that the nuclear interaction between the neutron and the target can be neglected.

A DWBA theory which is often used for Coulomb break-up starts from the expression

$$T^{DWBA}_{d,np} = \langle \chi^{(-)}_p \vec{k}_n \mid V_{np} \mid \Psi^{elast}_{\vec{K}_d} \rangle, \quad (116)$$

where the elastic deuteron wavefunction $\Psi^{elast}_{\vec{K}_d}$ has the form

$$\Psi^{elast}_{\vec{K}_d}(\vec{R}, \vec{r}) = \phi_o(\vec{r}) \chi^{(+)}_{\vec{K}_d}(\vec{R}). \quad (117)$$

For the case of Coulomb break-up $\chi^{(+)}_{\vec{K}_d}(\vec{R})$ is a Coulomb wavefunction describing the scattering of a point deuteron in the Coulomb field of the target.

The input data required for the 2 expressions (115) and (116) are identical, i.e., V_{np}, and the Coulomb potential of the target, although they are based on very different physical assumptions. The DWBA expression assumes that the

coupling between deuteron elastic and break-up channels is small and can be treated in first order. The expression (115) makes no such approximation but instead makes the assumption[1] that any break-up channels that are relevant for the 3-body scattering wavefunction inside the range of V_{np} have low energy compared with the incident deuteron energy. It should be noted that this calculation does *not* assume that that wavefunction (92) is valid for large values of r_{np}. The corect relation between the outgoing nucleon momenta and the incident deuteron energy is used in calculating the outgoing distorted waves in(115), i.e, no approximation involving neglect of the relative energy of the 2 nucleons is made. The 3-body Schr$\ddot{}$odinger equation guarantees that the expression (96) is correct if the 3-body wavefunction used adequately represents the 3-body scattering wavefunction inside the range of V_{np}.

The DWBA amplitude for Coulomb break-up involves a 6-dimensional integration. Various approximations, including the zero-range approximation for V_{np}, have invariably been made to simplify its evaluation. Zadro[68],[69] has published a momentum-space technique for the exact evaluation of the DWBA amplitude with a finite range V_{np}. This has enabled a meaningful comparison to be made with the ADWA theory. He studied ^{11}Be$\rightarrow^{10}$Be$+n$ and ^{19}C$\rightarrow^{18}$C$+n$ elastic break-up on a ^{208}Pb target at energies near 70 MeV/nucleon. Both theories give very similar projectile-fragment relative energy distributions in quite good agreement with experiment[70] but the predicted DWBA crosssection magnitudes for a 2-body projectile model are up to 50% bigger. Crosssections for break-up into states of more than a few MeV are very small. The adiabatic approximation therefore ought to be excellent at 70MeV/nucleon[19]. This suggests that spectroscopic factors obtained by comparison of predicted DWBA crosssections with break-up data may be significantly underestimated.

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