The Joint Institute for Nuclear Astrophysics

JINA "Methods of Direct Nuclear Reactions" School

NSCL, East Lansing MI, 9-20th April 2007

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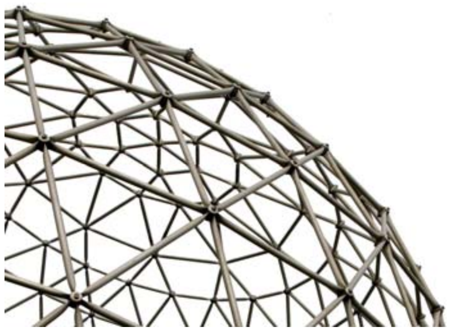


There is a **lot** of literature to refer to:

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) Norman Austern

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages) G R Satchler

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) L S Rodberg, R M Thaler

Direct Nuclear Reactions (World Scientific Publishing, 396 pages) Norman K. Glendenning

Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages) C A Bertulani, P Danielewicz

Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages) Herman Feshbach

Introduction to Nuclear Reactions (Oxford University Press, 332 pages) G R Satchler

Session (learning) aims:

To calculate solutions of the Schrodinger equation for states of **two** bodies with specific quantum numbers over a wide range of energies – the need for bound, resonant, continuum and continuum bin states.

The form of the two-body solutions at large separations and their relationships to absorption, reaction and scattering studies.

Constraints on two-body potentials and their parameters. Parameter conventions. The need to cross reference to known nuclear structures, resonances, nuclear sizes and experiment whenever possible in constraining parameter choices.

Session (learning) outcomes:

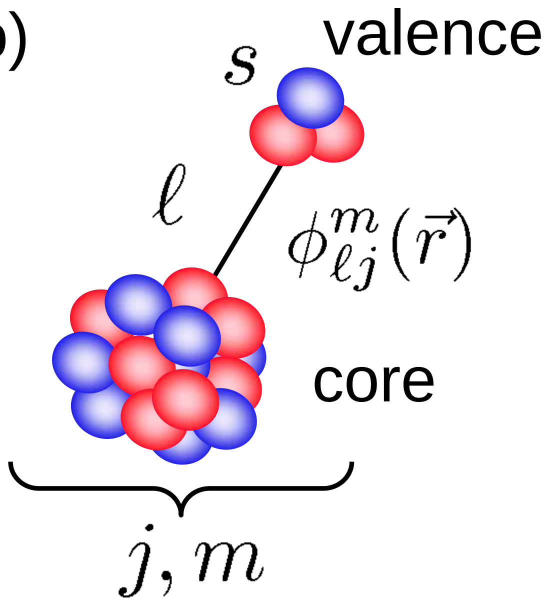
To gain familiarity in using a suite of small programs to solve the Schrodinger equation over a wide range of energies. To enable/encourage exploration of properties of bound, resonant, continuum and continuum bin states outside of a large code (such as FRESCO) to (a) tune and (b) understand the required input parameters outside of and prior to large scale (black-box) calculations.

To understand the role of the S-matrix in determining scattering and absorptive properties of the interaction.

To be critical of the use of 'standard' potential parameter sets and to be able explore the quality of these by making reference to experimental data and to systematics from theoretical models – such as Hartree-Fock. To perform examples at each important energy regime.

Part (a) – "The bits and pieces…"

Solution of Schrodinger's equation for (two) bodies interacting via a potential energy function of the form*



$$U(r) = \underbrace{V_C(r)}_{\text{Coulomb}} + \underbrace{V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}}_{\text{Nuclear}}$$

Need descriptions of wave functions of:

(1) Bound states of nucleons or clusters (valence particles) to a core (that is assumed for now to have spin zero).

(2) Unbound scattering or resonant states at low energy

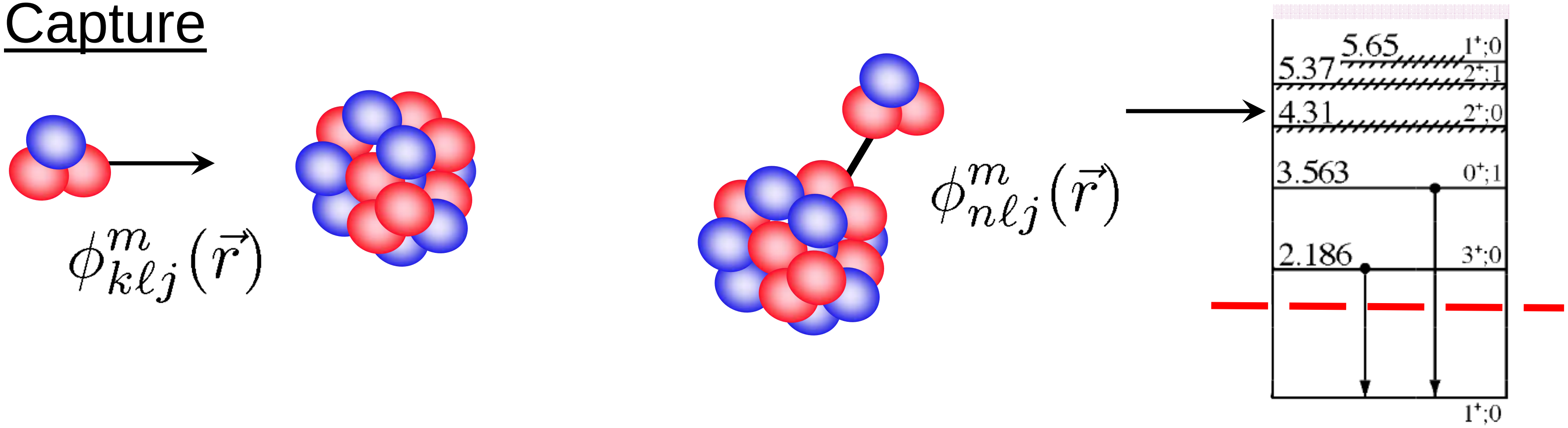
(3) Distorted waves of such bodies in complex potentials

$$U(r) = V_C(r) + V(r) + \boxed{iW(r)} + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

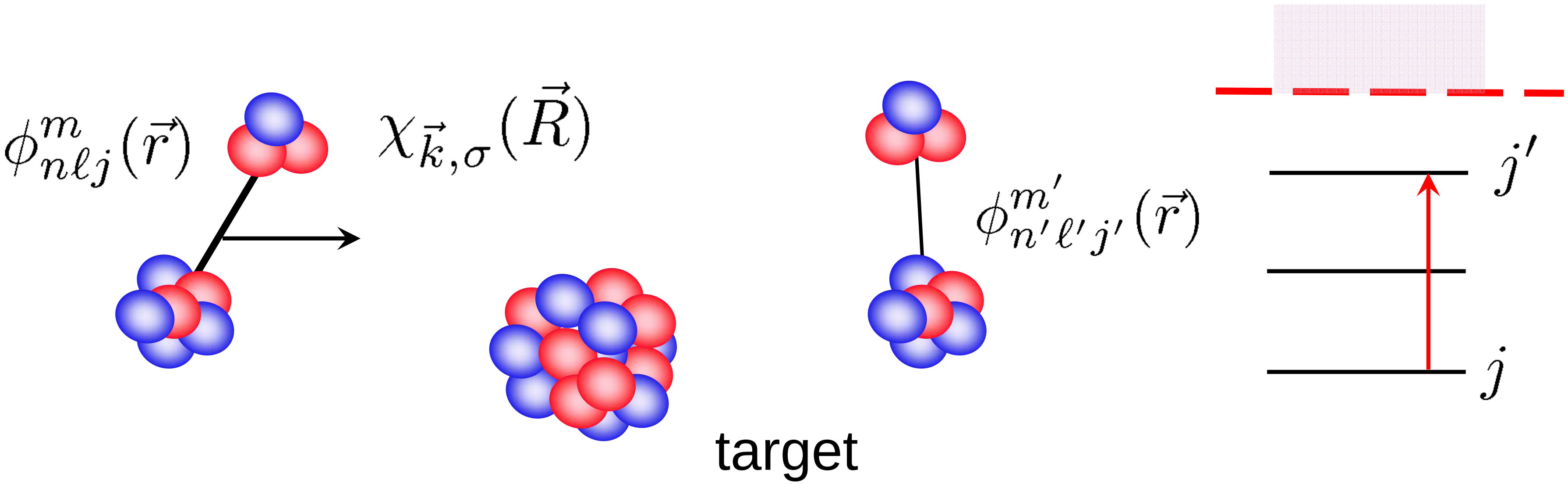
*Additional, e.g. tensor terms, when s=1 or greater neglected

Direct reactions – types and characteristics

Capture

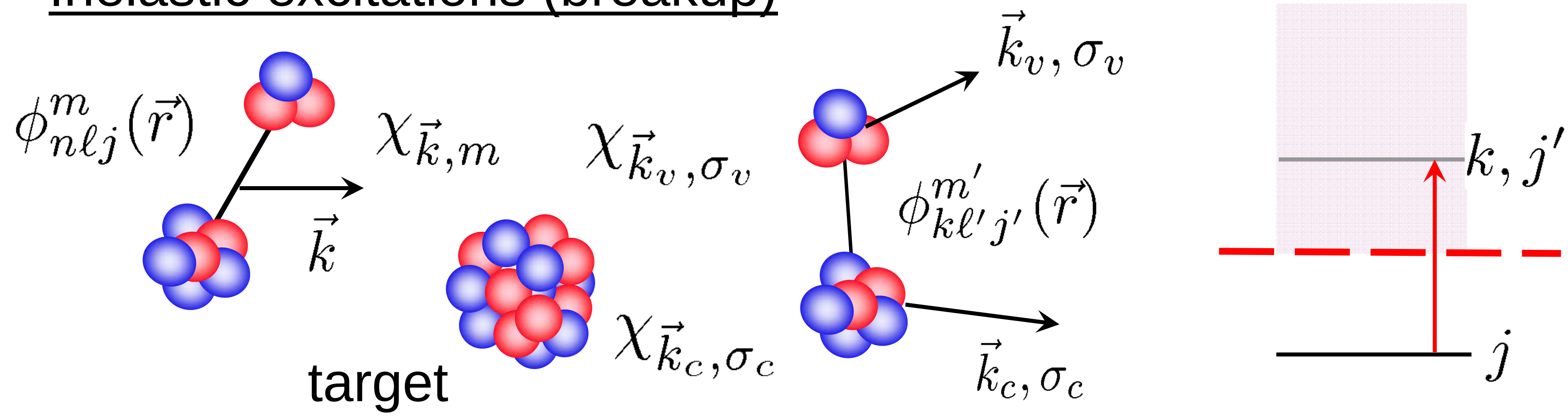


Inelastic excitations (bound to bound states) DWBA

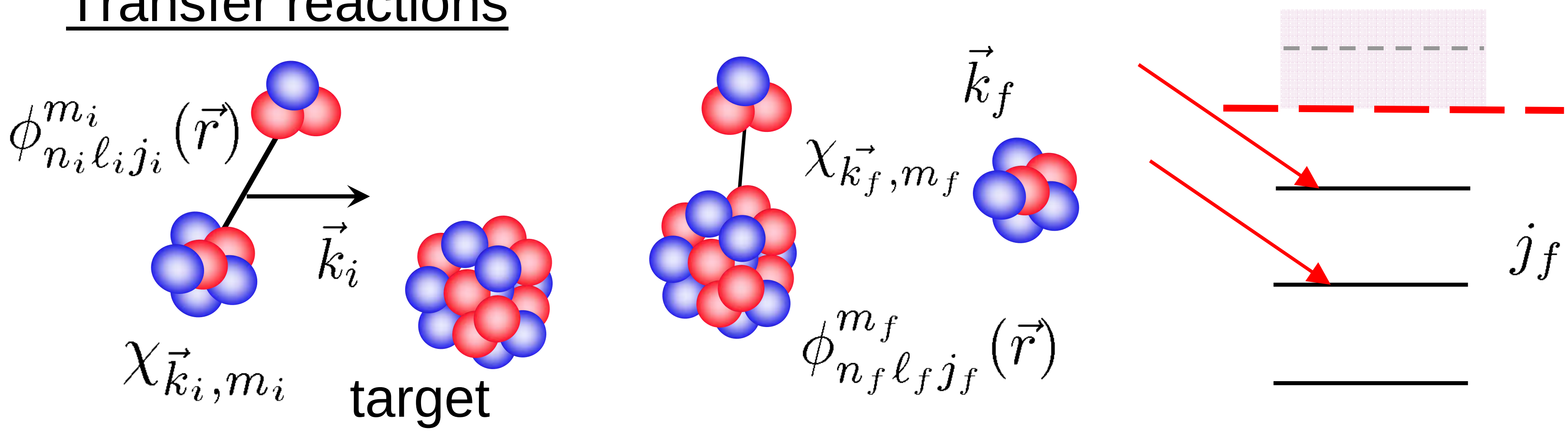


Direct reactions – types and characteristics

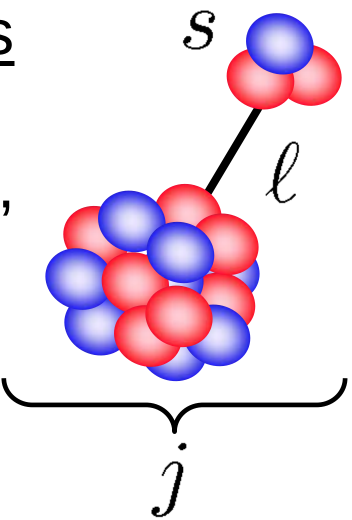
Inelastic excitations (breakup)



Transfer reactions



Direct reactions – requirements (1)



Description of wave functions of **bound** systems (both nucleons or clusters) – (a) can take from structure theory, if available or, (b) more usually, use a real potential model to bind system with the required experimental separation energy.

Refer to core and valence particles

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r}Y^\lambda_\ell(\hat{r})\chi^\sigma_s, \quad \int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$

Usually just one or a few such states are needed.

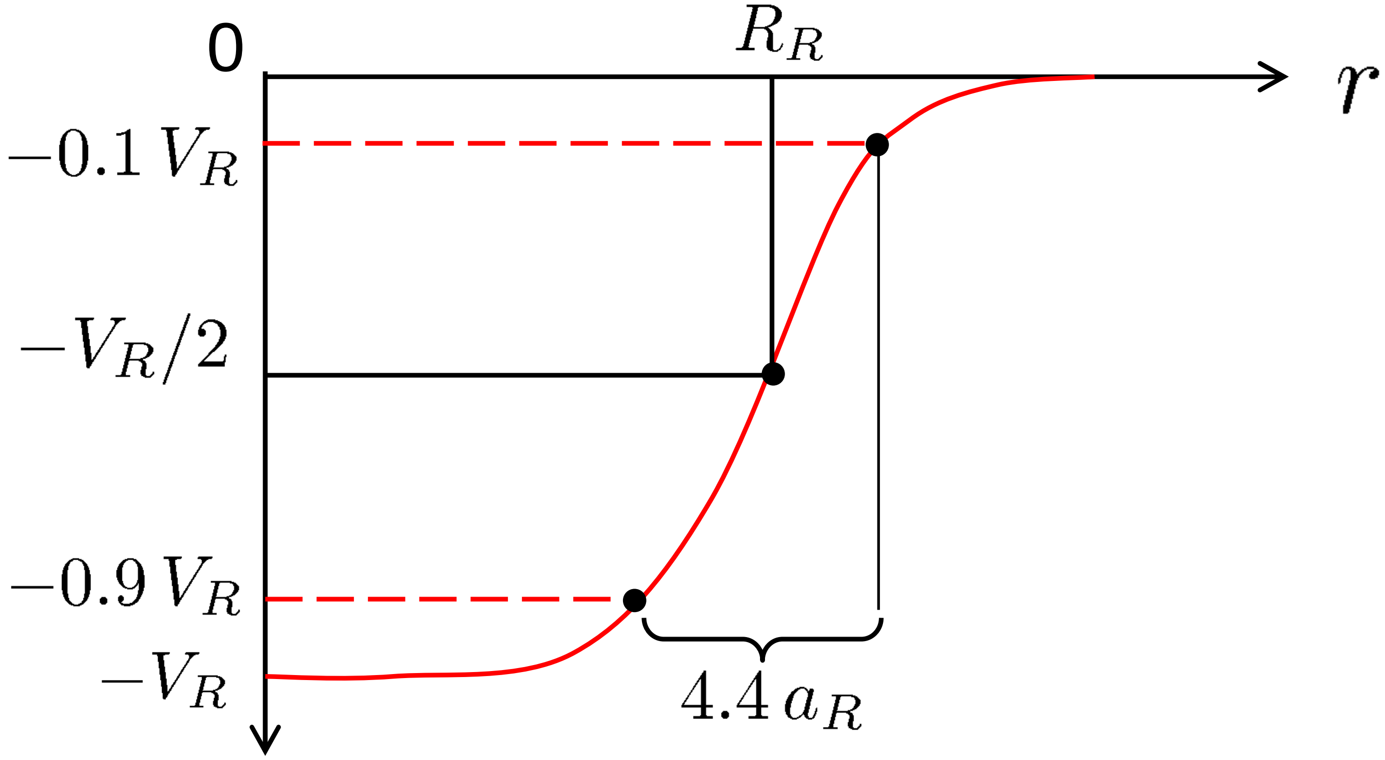
Separation energies/Q-values: many sites, e.g.

http://ie.lbl.gov/toi2003/MassSearch.asp

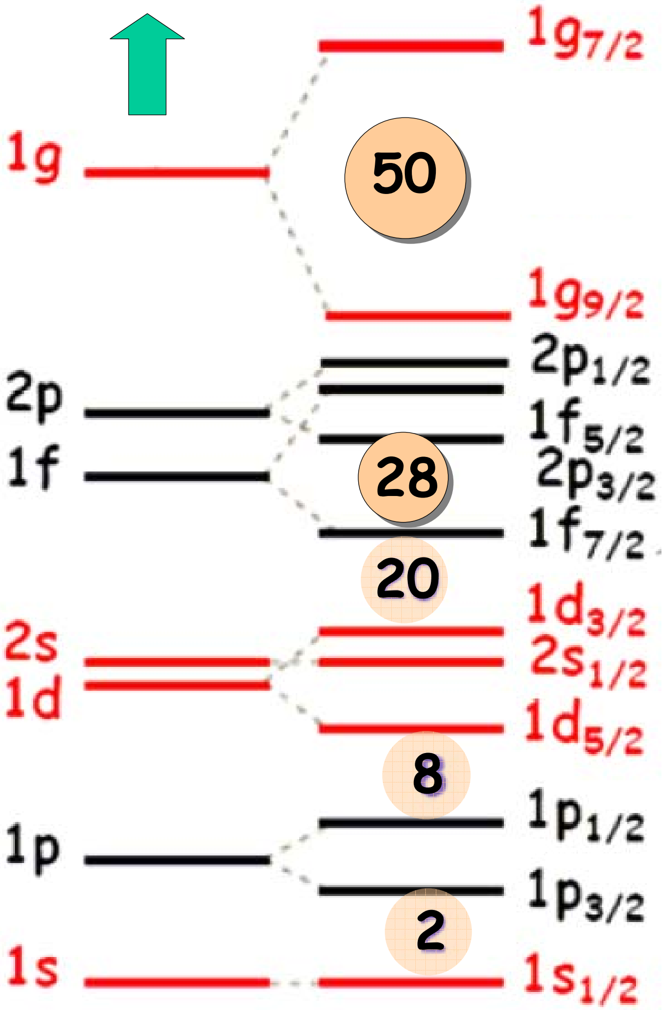
Bound states – real potentials

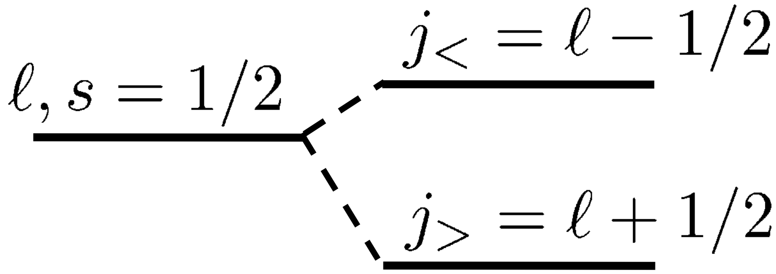
$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

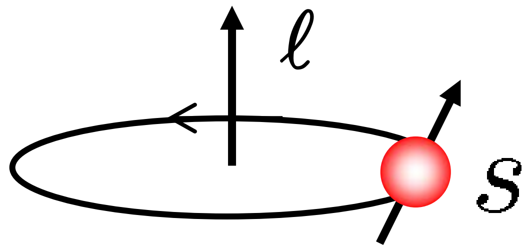
$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_R = \frac{r-R_R}{a_R}$$



Bound states – nuclear single particle structures







^{17}O

$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V_{so}(r) < 0$$

Bound states – potential conventions

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

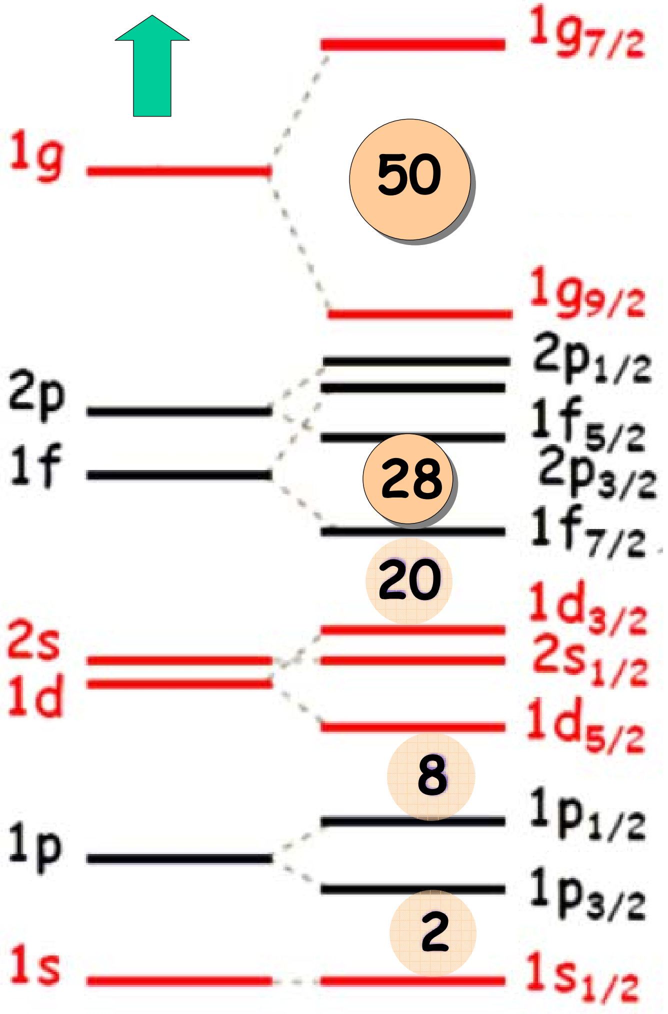
$$V_{so}(r) = -\frac{4\,V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

Conventions

$$R_i = r_i A_2^{1/3} \qquad \text{or} \qquad R_i = r_i\left[A_1^{1/3} + A_2^{1/3}\right]$$

$$\begin{aligned} V_C(r) &= Z_1 Z_2 e^2/r\ , r > R_C \\ &= \frac{Z_1 Z_2 e^2}{2R_C}\left[3 - \left(\frac{r}{R_C}\right)^2\right]\ , r \le R_C \end{aligned}$$

Bound states – for nucleons - conventions



Conventions $\phi^m_{n\ell j}(\vec{r})$

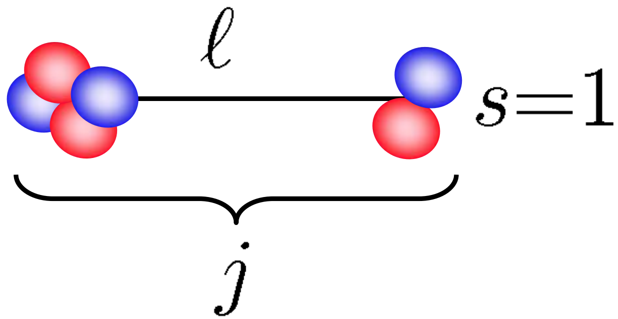
With this potential, and using sensible parameters, we will obtain the independent-particle shell model level orderings, shell closures with spin-orbit splitting.

NB: In diagram *2d*$_{5/2}$ means the second *d*$_{5/2}$ state. Defined this way, *n>0* and *n-1* is the number of nodes in the radial wave function. This is the convention used in FRESCO (used later) but many codes ask for the actual number of nodes. Care is needed.

Direct reactions – requirements (2)

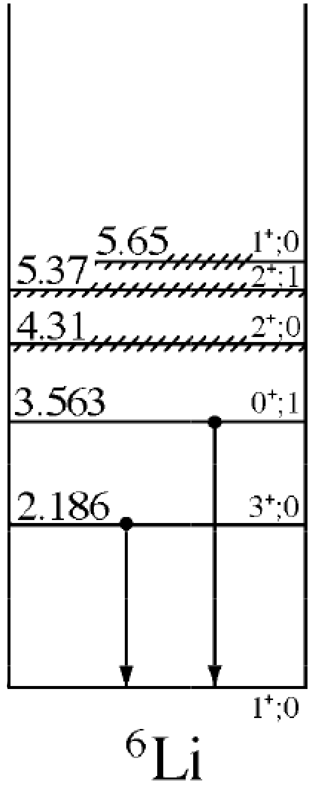
Description of wave functions for unbound (often light) systems (nucleons or clusters) with low relative energy: Usually have low nuclear level density of isolated resonances. Use the same real potential model as binds the system → scattering wave functions in this potential. Also ‘bin’ wave functions.

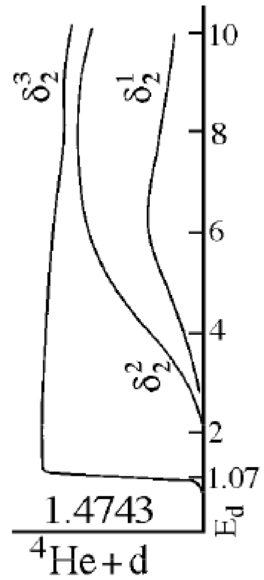
^{6}Li (α+d)



$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^m_{k\ell j}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm)\frac{u_{k\ell j}(r)}{kr} Y^\lambda_\ell(\hat{r})\chi^\sigma_s$$

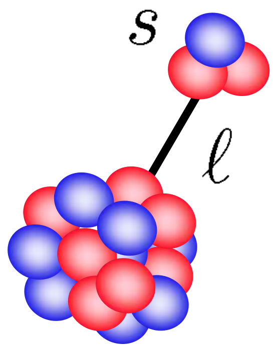




Completeness and orthogonality

Given a fixed two-body Hamiltonian

$$H = T + U(r) = T + V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$



the set of all of these bound and unbound wave functions

$$\{\phi^m_{n\ell j}(\vec{r}),\ \phi^m_{k\ell j}(\vec{r})\}$$

form a complete and orthogonal set, and specifically

$$\langle\phi^m_{n\ell j}(\vec{r})|\,\phi^m_{k\ell j}(\vec{r})\rangle = 0$$

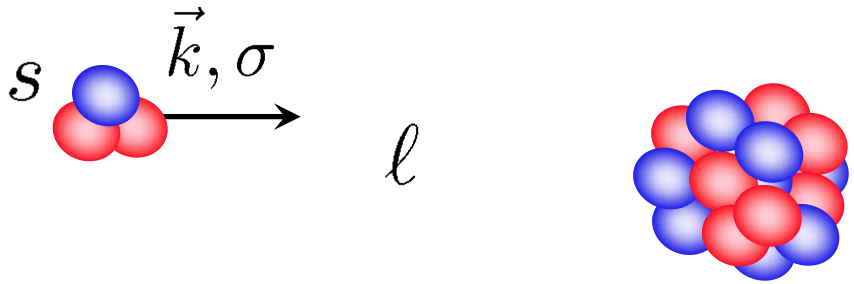
When including coupling of bound to unbound states it is essential we use a fixed Hamiltonian for both the bound and unbound states (in each $\ell\, j$ channel) else we lose the orthogonality and the states will couple even without a perturbation or interaction with the target.

Direct reactions – requirements (3)

Description of wave functions for scattering of nucleons or clusters from a heavier target and/or at higher energies: (a) high nuclear level density and broad overlapping resonances, (b) many open reaction channels, inelasticity and absorption. Use a complex (absorptive) optical model potential – from theory or simply fitted to elastic scattering data for the system and the energy of interest.

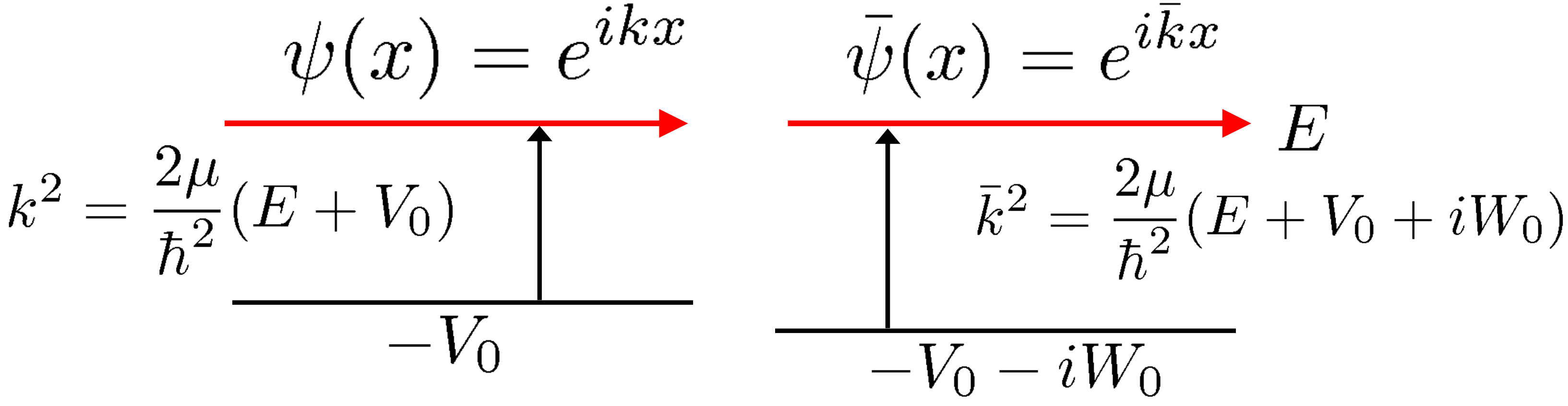
Distorted waves:

$$\chi_{\vec{k},\sigma}(\vec{r})$$



$$U(r) = V_C(r) + V(R) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

Optical potentials – the role of the imaginary part



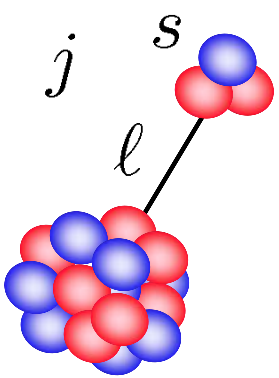
$$\bar{k}^2 = \frac{2\mu}{\hbar^2}(E + V_0 + iW_0) = \frac{2\mu}{\hbar^2}(E + V_0)\left[1 + \frac{iW_0}{E + V_0}\right]$$

$$\bar{k} = k\left[1 + \frac{iW_0}{E + V_0}\right]^{1/2} \approx k\left[1 + \frac{iW_0}{2(E + V_0)}\right], \quad W_0 \ll E, V_0$$

$$\text{So, for } W_0 > 0, \quad \bar{k} = k + ik_i/2, \quad k_i = kW_0/(E + V_0) > 0,$$

$$\bar{\psi}(x) = e^{i\bar{k}x} = e^{ikx}e^{-\frac{1}{2}k_i x}, \qquad |\bar{\psi}(x)|^2 = e^{-k_i x}$$

The Schrodinger equation (1)



So, using usual notation

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\phi_{\ell j}^m(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

and defining $\phi_{\ell j}^m(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{\ell j}(r)}{r}Y_\ell^\lambda(\hat{r})\chi_s^\sigma$

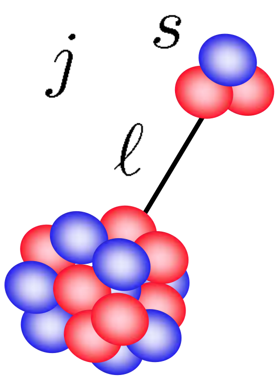
$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right)u_{\ell j}(r) = 0$$

bound states $E_{cm} < 0$ scattering states $E_{cm} > 0$

With $U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$

$$\begin{aligned} U_{\ell j}(r) &= V_C(r) + V(r) + iW(r) \\ &+ V_{so}(r)[j(j+1) - \ell(\ell+1) - s(s+1)]/2 \end{aligned}$$

The Schrodinger equation (2)



Must solve

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}\left[E_{cm} - U_{\ell j}(r)\right]\right) u_{\ell j}(r) = 0$$

<u>bound states</u> $E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) \boxed{u_{n\ell j}(r)} = 0$$

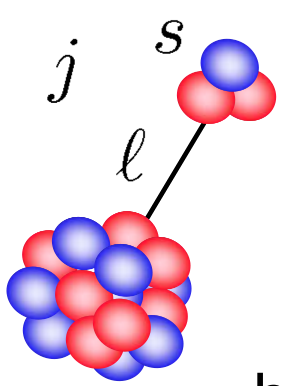
Discrete spectrum

<u>scattering states</u> $E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) \boxed{u_{k\ell j}(r)} = 0$$

Continuous spectrum

Large r: The Asymptotic Normalisation Coefficient



Bound states

$$E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

but beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta_b\kappa_b}{r} - \kappa_b^2\right) u_{n\ell j}(r) = 0, \qquad \eta_b = \frac{\mu Z_c Z_v e^2}{\hbar\kappa_b}$$

$$u_{n\ell j}(r) \rightarrow C_{\ell j} W_{-\eta_b,\ell+1/2}(2\kappa_b r) \xrightarrow[r\rightarrow\infty]{} C_{\ell j}\exp(-\kappa_b r)$$

Whittaker function

$C_{\ell j}$ ANC completely determines the wave function outside of the range of the nuclear potential – only requirement if a reaction probes only these radii

Large r: The phase shift and partial wave S-matrix

Scattering states $$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr),\ G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\begin{aligned} u_{k\ell j}(r) &\to e^{i\delta_{\ell j}}[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)] \\ &\to (i/2)[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)] \end{aligned}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm i F_\ell(\eta, kr)$$

Phase shift and partial wave S-matrix: Recall

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}}\underbrace{[\cos\delta_{\ell j}F_\ell(\eta,kr)+\sin\delta_{\ell j}G_\ell(\eta,kr)]}$$

If *U(r)* is real, the phase shifts $\delta_{\ell j}$ are real, and […] also

$$u_{k\ell j}(r) \quad \rightarrow \quad (i/2)[\underline{H_\ell^{(-)}(\eta,kr)} - S_{\ell j}\underline{H_\ell^{(+)}(\eta,kr)}]$$

Ingoing waves — outgoing waves

$$S_{\ell j} = e^{2i\delta_{\ell j}}$$

$|S_{\ell j}|^2$ survival probability in the scattering

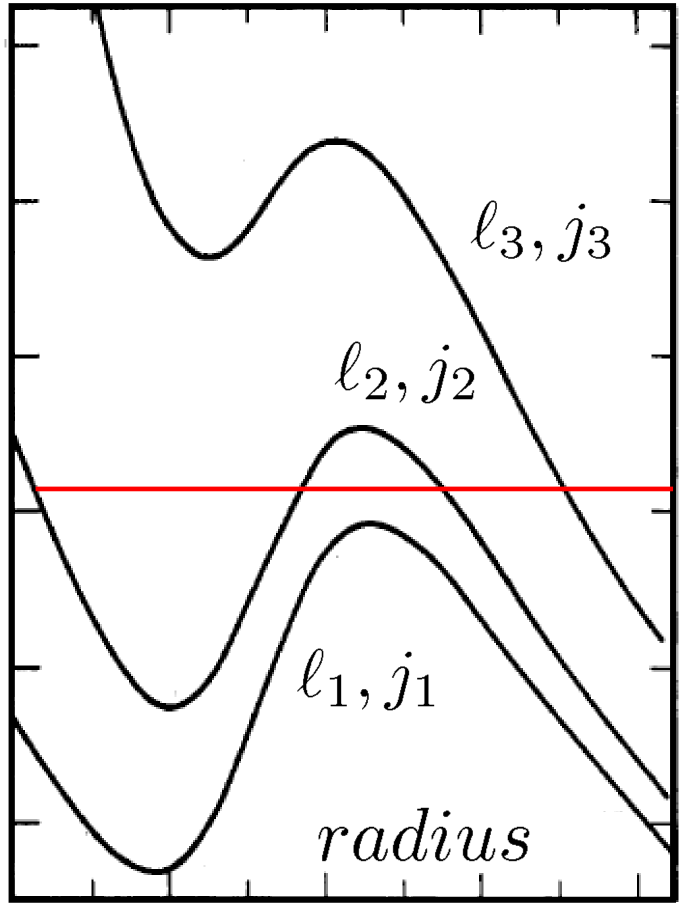
$(1-|S_{\ell j}|^2)$ absorption probability in the scattering

Having calculate the phase shifts and the partial wave S-matrix elements we can then compute all scattering observables for this energy and potential (but later).

Phase shifts and S-matrix: Resonant behaviour

In real potentials, at low energies, the combination of an attractive nuclear, repulsive Coulomb and centrifugal terms can lead to potential pockets and resonant behaviour – the system being able to trapped in the pocket for some (life)time τ.

$$\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} + U_{\ell j}(r)$$

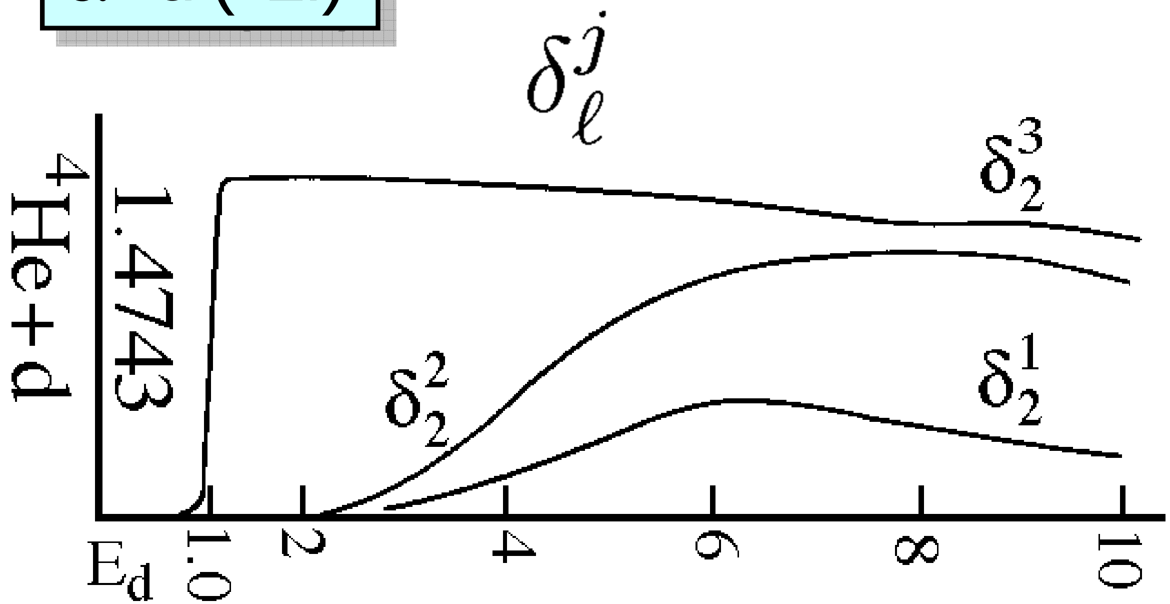


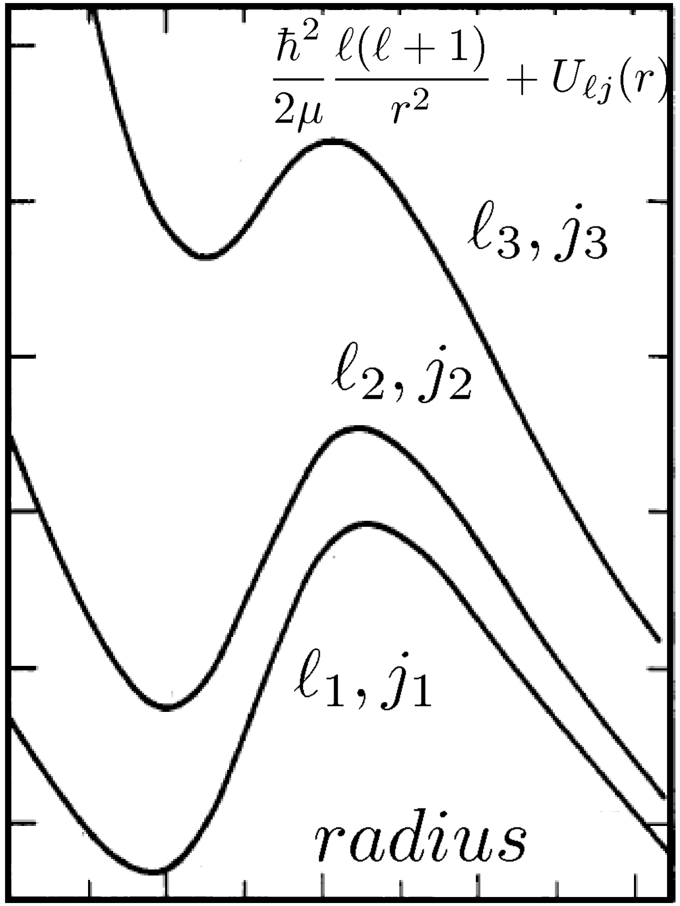
Phase shifts and S-matrix: Resonant behaviour

Potential pockets can lead to resonant behaviour – the system being able to trapped in the pocket for some (life)time τ.

A signal is the rise of the phase shift through 90 degrees.

α+d (^{6}Li)





Potential parameters should describe any known resonances

Bound states – potential conventions

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r - R_i}{a_i}$$

$$V_{so}(r) = -\frac{4\,V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

Conventions

$$R_i = r_i A_2^{1/3} \qquad \text{or} \qquad R_i = r_i\left[A_1^{1/3} + A_2^{1/3}\right]$$

$$\begin{aligned} V_C(r) &= Z_1 Z_2 e^2/r\ , r > R_C \\ &= \frac{Z_1 Z_2 e^2}{2R_C}\left[3 - \left(\frac{r}{R_C}\right)^2\right]\ , r \le R_C \end{aligned}$$

Bound states potential parameters - nucleons

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell} \cdot \vec{s}$$

$$V(r) = -\frac{V_R}{[1 + \exp(X_R)]}, \qquad X_i = \frac{r - R_i}{a_i}$$

$$V_{so}(r) = -\frac{4\,V_{so}}{r\,a_{so}} \frac{\exp(X_{so})}{[1 + \exp(X_{so})]^2},$$

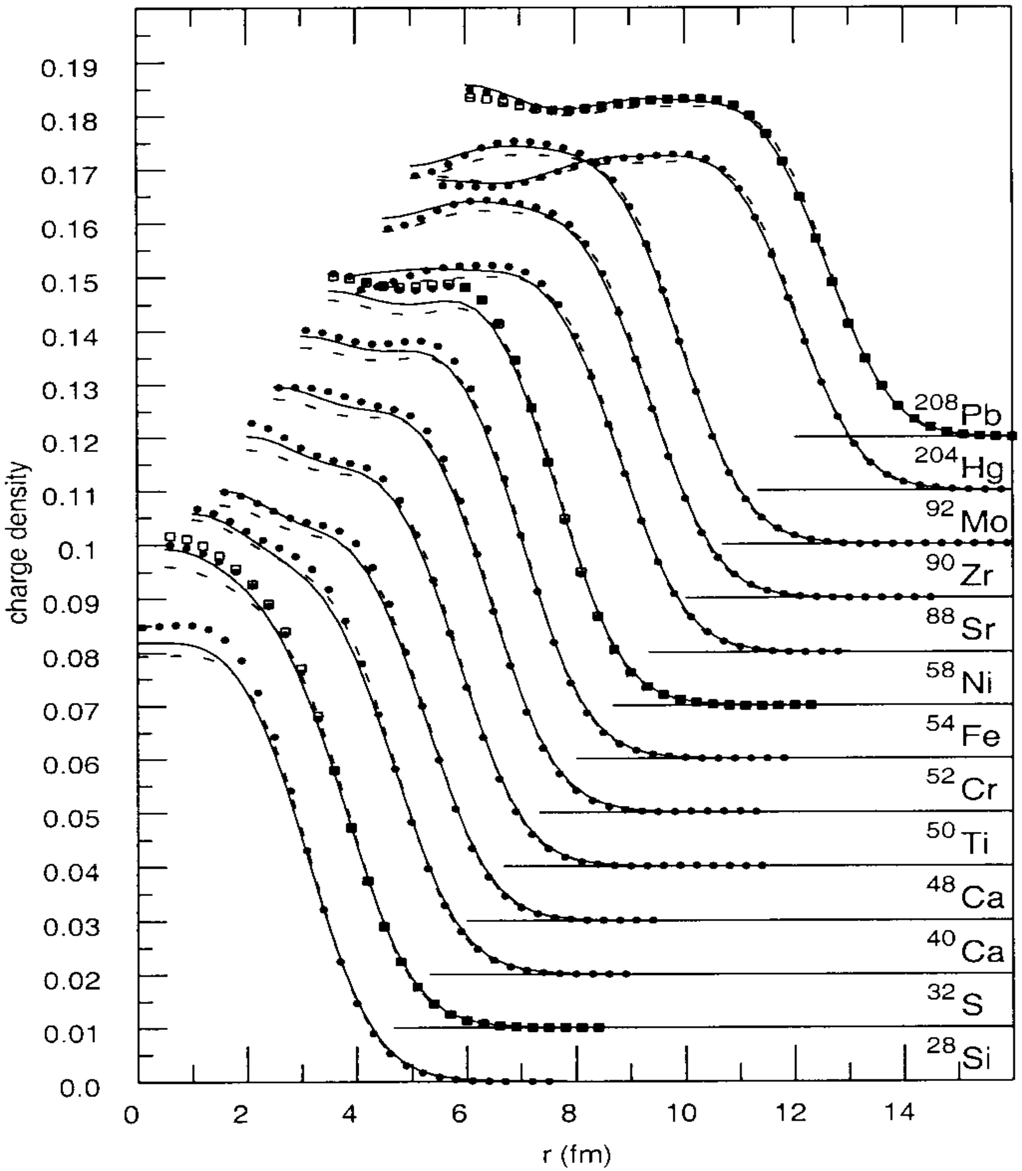
$$R_i = r_i\,A_2^{1/3}$$

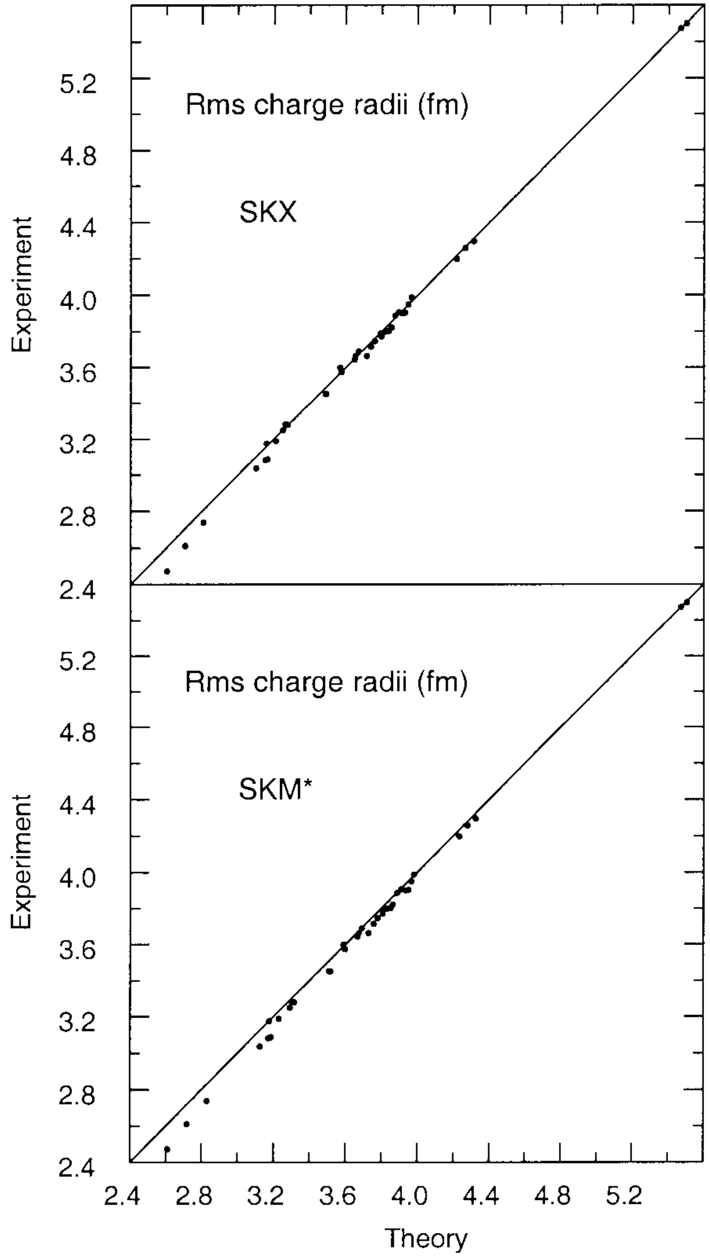
^{17}O

$$r_R = r_C = r_{so} \approx 1.25\text{fm}$$

$$a_R = a_{so} \approx 0.65 - 0.7\text{fm}$$

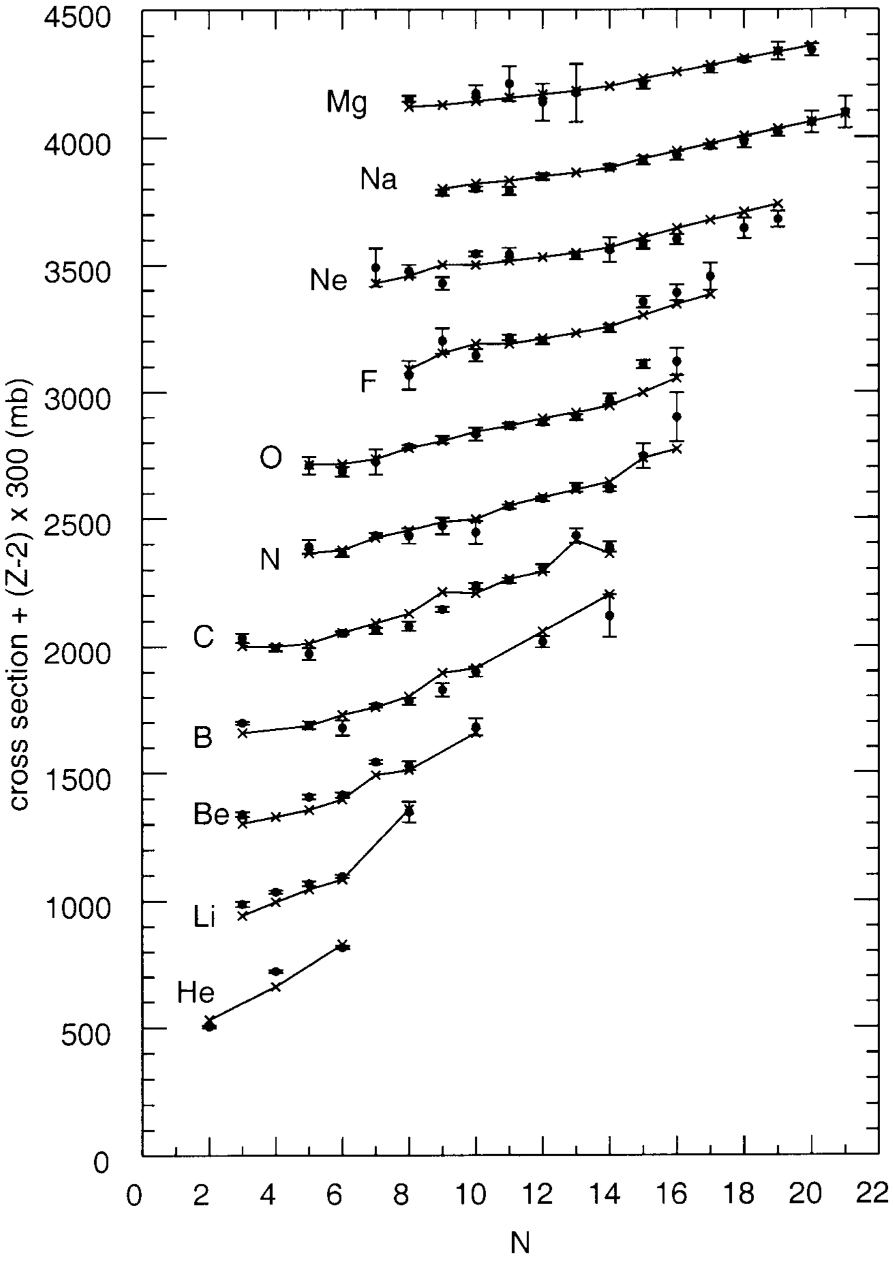
Sizes - Skyrme Hartree-Fock radii and densities

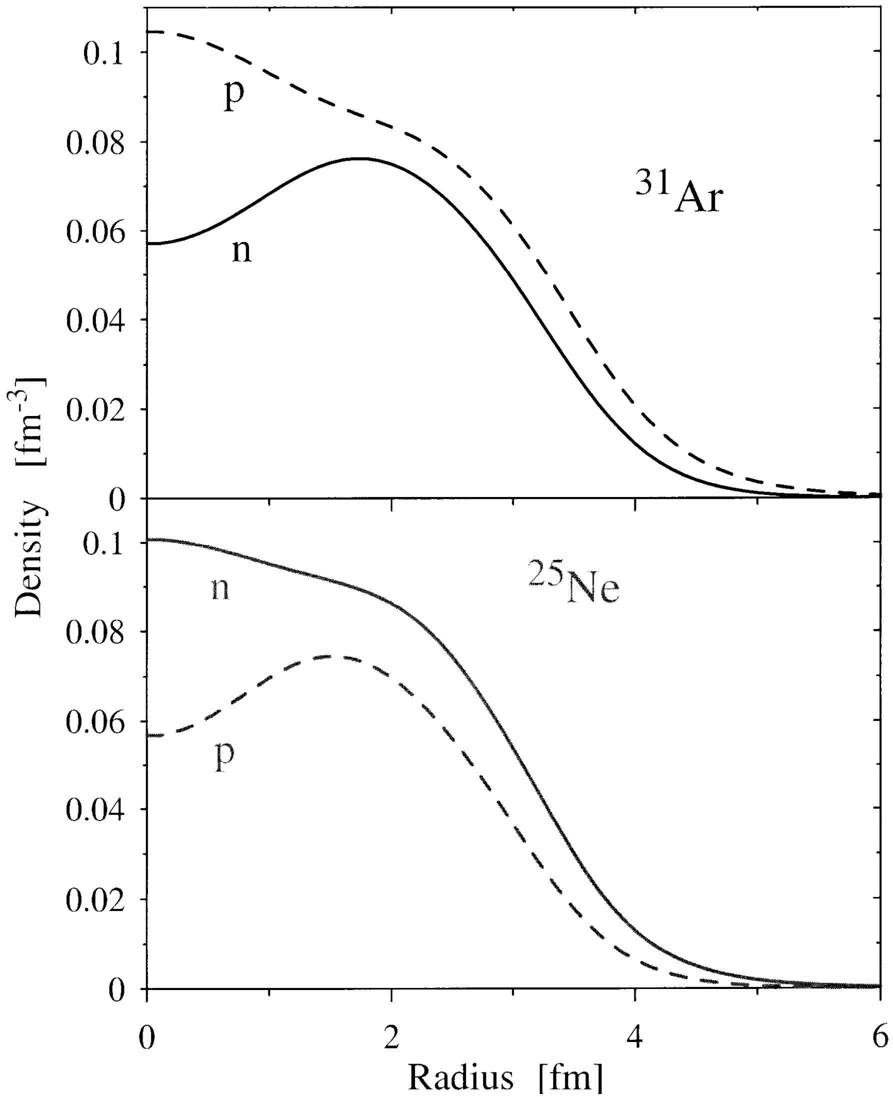




W.A. Richter and B.A. Brown, Phys. Rev. **C67** (2003) 034317

Sizes - Skyrme Hartree-Fock radii and densities





B.A. Brown, S. Typel, and W.A. Richter,
Phys. Rev. **C65** (2002) 014612

Bound states – use mean field information

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                  **********************
                  *                    *
INPUT VALUES      * IA,IZ =    24    8 *
                  *                    *
                  **********************
```

----- Neutron bound state results -----

k	n	l	j	e	IE	OCC		
1	1	s	1/2	-26.757	1	2.00	36.70	35.28
2	1	p	3/2	-16.883	1	4.00	36.70	35.80
3	1	p	1/2	-12.396	1	2.00	36.70	36.04
4	1	d	5/2	-6.166	1	6.00	36.70	36.37
5	1	d	3/2	-0.109	1	0.00	36.70	36.69
6	2	s	1/2	-3.360	1	2.00	36.70	36.52
7	1	f	7/2	-0.200	3	0.00	46.02	46.01
8	1	f	5/2	-0.200	3	0.00	60.56	60.55
9	2	p	3/2	-0.200	3	0.00	48.10	48.09

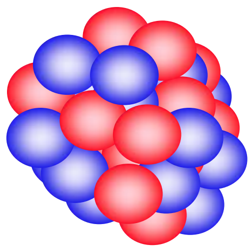
----- Neutron single-particle radii -----

				R(2)	R(4)	OCC	rho(8.9)	rho(9.9)	rho(10.9)
1	1	s	1/2	2.274	2.575	2.000	0.848E-09	0.706E-10	0.600E-11
2	1	p	3/2	2.863	3.133	4.000	0.188E-07	0.244E-08	0.325E-09
3	1	p	1/2	2.954	3.268	2.000	0.727E-07	0.122E-07	0.210E-08
4	1	d	5/2	3.434	3.757	6.000	0.524E-06	0.129E-06	0.327E-07
5	1	d	3/2	4.662	6.063	0.000	0.131E-04	0.675E-05	0.371E-05
6	2	s	1/2	4.172	4.895	2.000	0.769E-05	0.278E-05	0.102E-05
7	1	f	7/2	3.865	4.440	0.000	0.324E-05	0.134E-05	0.600E-06
8	1	f	5/2	3.890	4.477	0.000	0.341E-05	0.141E-05	0.631E-06
9	2	p	3/2	6.815	8.635	0.000	0.451E-04	0.270E-04	0.167E-04

But must make <u>small</u> correction as HF is a fixed centre calculation

^{17}O

$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



$^{24}\mathrm{O}(g.s.)$

Bound states – for clusters – conventions (1)

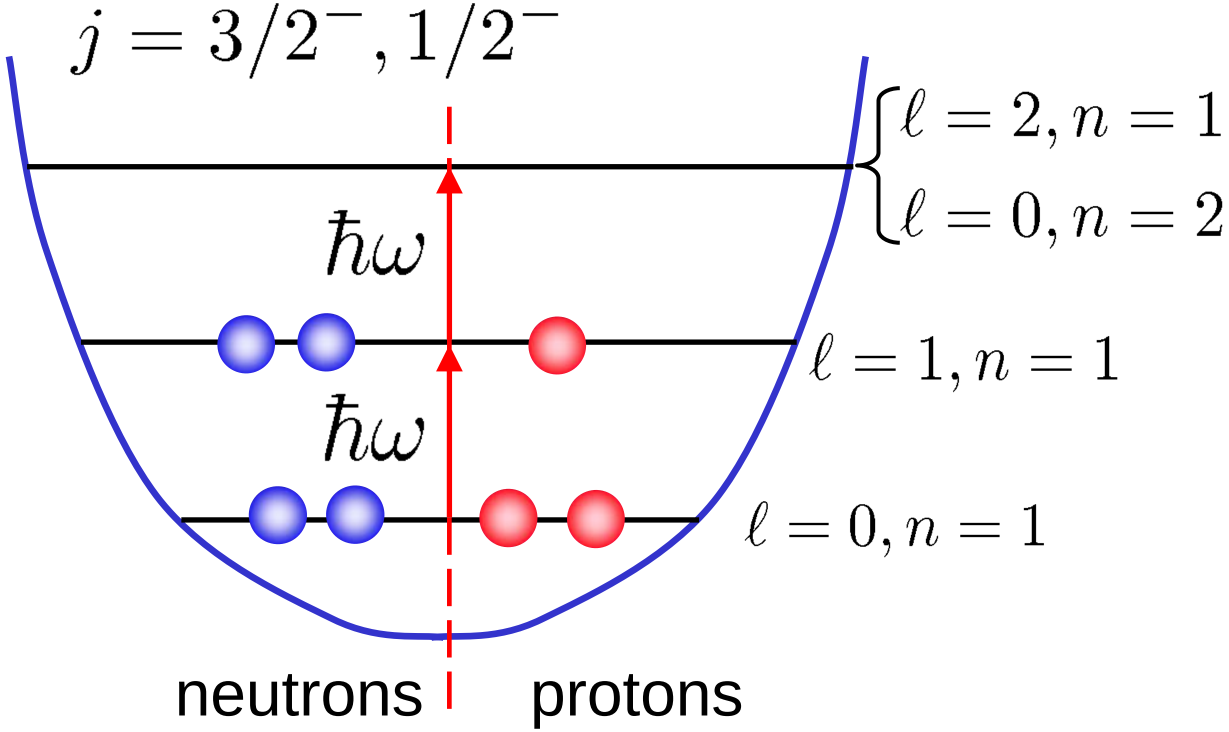
How many nodes for cluster states ?

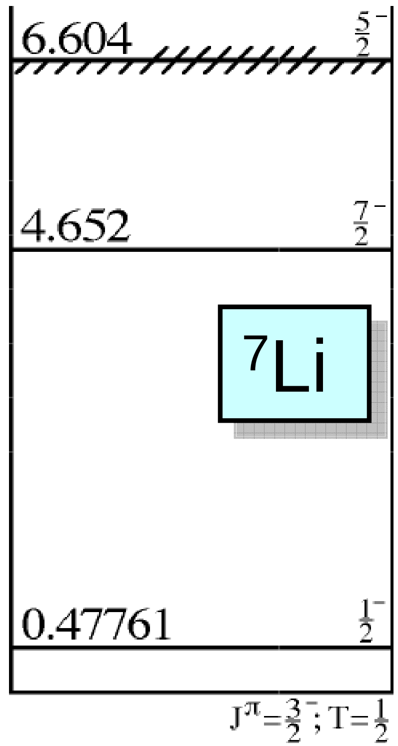
$$\phi^{m}_{n\ell j}(\vec{r})$$

Usually guided by what the 3D harmonic oscillator potential requires - so as not to violate the Pauli Principle.

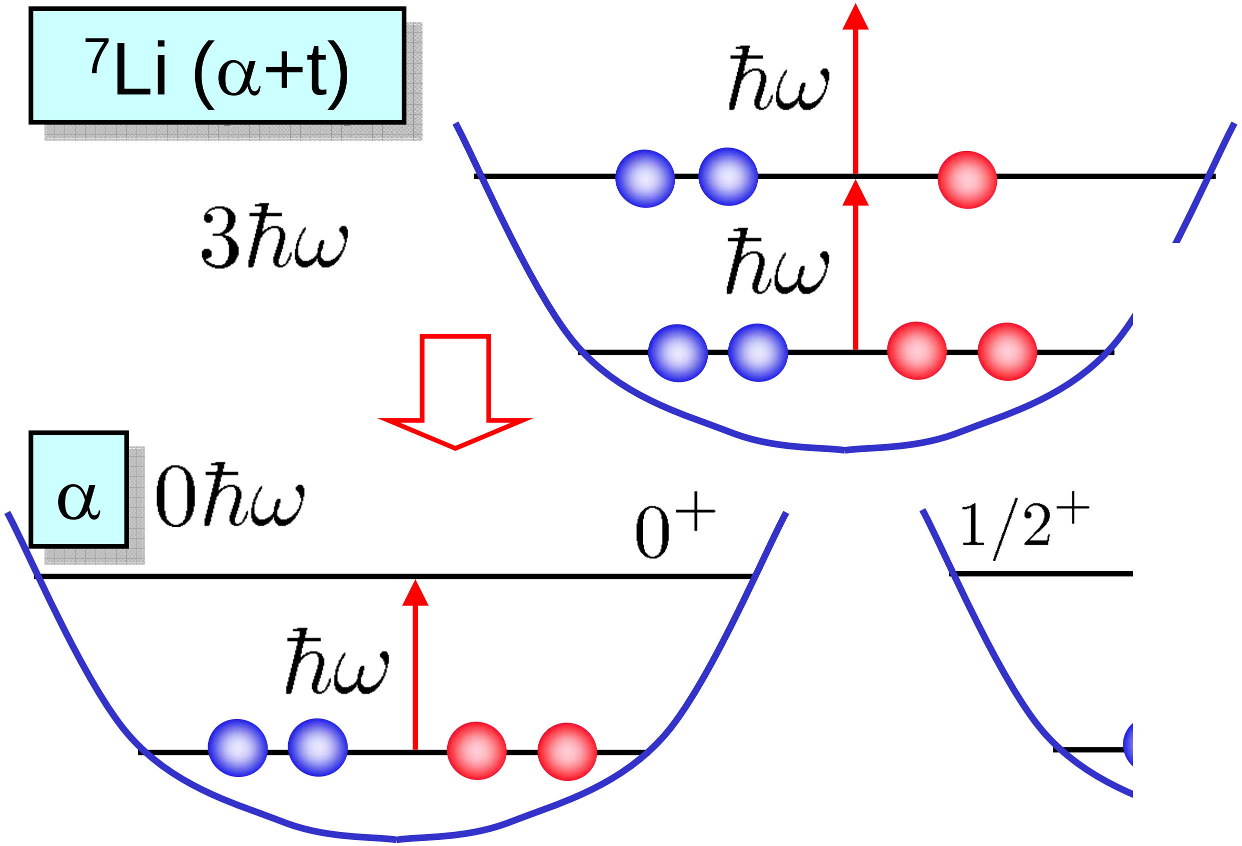
^{7}Li (α+t)

$$[2(n-1)+\ell\,]\hbar\omega$$ { excitation due to a nucleon each level

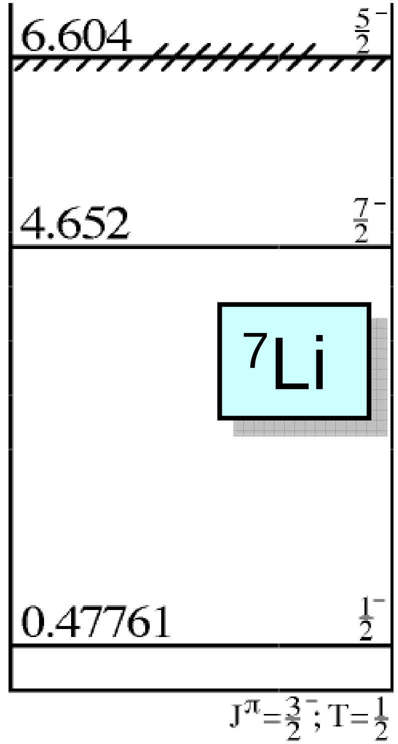




Bound states – for clusters - conventions (2)



$$j = 3/2^-$$



$$[2(N-1)+L\,]\hbar\omega = 3\hbar\omega \Rightarrow$$

$$L=1, N=2 \qquad L=3, N=1$$

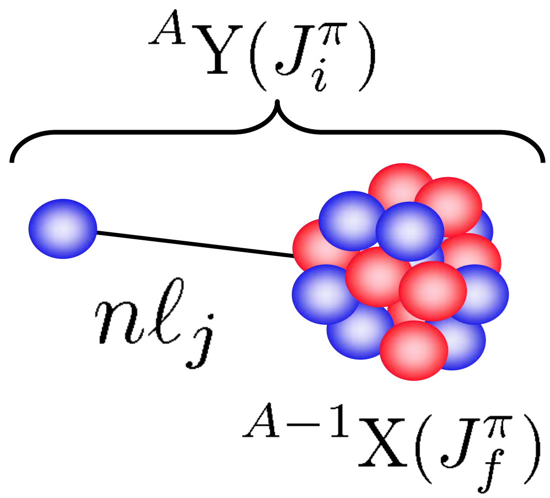
$$j = 3/2^-, 1/2^-, \quad j = 7/2^-, 5/2^-$$

Bound states – spectroscopic factors

In a potential model it is natural to define <u>normalised</u> bound state wave functions.

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r} Y^\lambda_\ell(\hat{r})\chi^\sigma_s,$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$



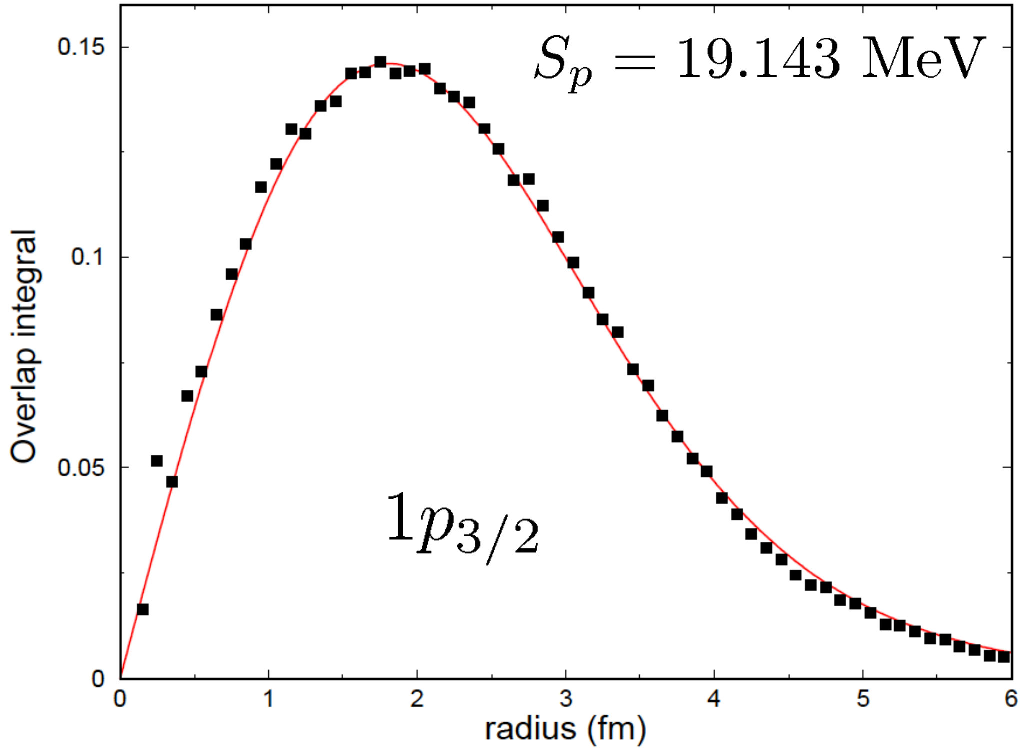
The potential model wave function approximates the <u>overlap function</u> of the A and A–1 body wave functions (A and A–n in the case of an n-body cluster) i.e. the overlap

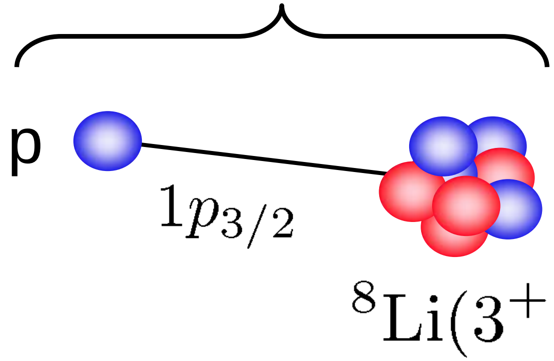
$$\langle \ell j, \vec{r}, {}^{A-1}\mathrm{X}(J_f^\pi) | {}^A\mathrm{Y}(J_i^\pi)\rangle \rightarrow I_{\ell j}(r), \quad \int_0^\infty [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

S(…) is the <u>spectroscopic factor</u> ← a structure calculation

Bound states – microscopic overlaps

$$\langle \vec{r}, {}^8\mathrm{Li}(3^+) | {}^9\mathrm{Be}(3/2^-, \mathrm{g.s.}) \rangle$$





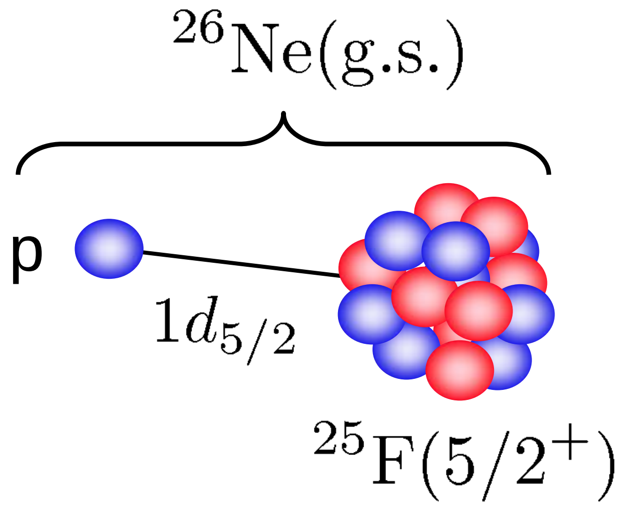
- Microscopic overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a few cases

Normalised bound state in Woods-Saxon potential well x $(0.23)^{1/2}$ Spectroscopic factor

$$r_V = r_{so} = \mathrm{fitted},\ a_V = a_{so} = \mathrm{fitted},\ V_{so} = 6.0$$

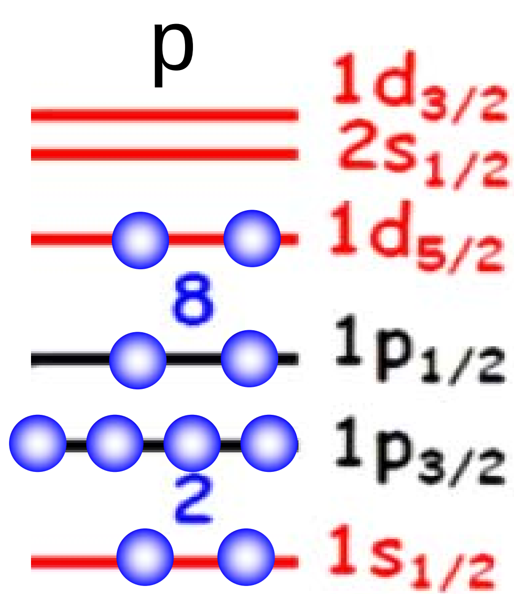
Bound states – shell model overlaps

$$\langle \vec{r}, {}^{25}\mathrm{Ne}(5/2^+, E^*) | {}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.}) \rangle$$



USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides spectroscopic factors but **not** the bound state radial wave function.

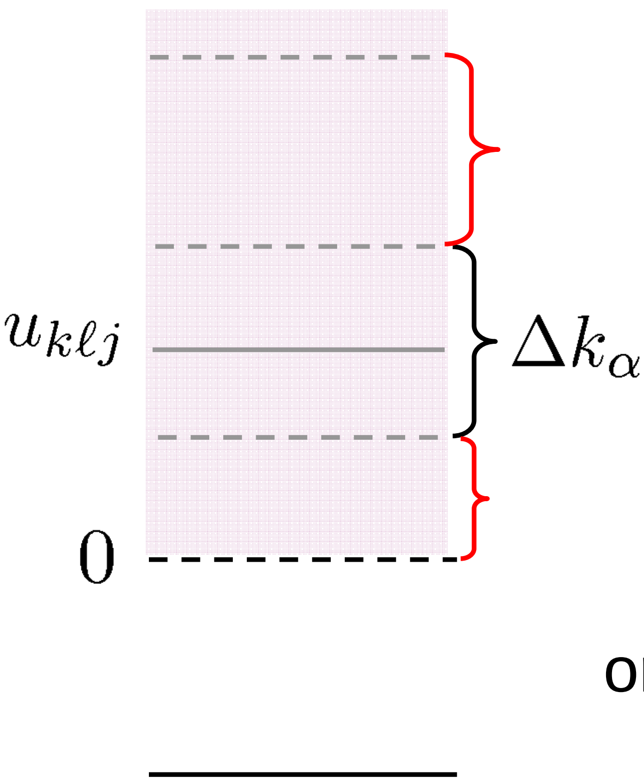
-- core state ---					- overlap state -					(1 2 5)(
2j2t		p	n	e	2j2t		p	n	e	s
				-59.414					-81.625	
5	7	+	1	0.000	0	6	+	1	0.000	1.79039
5	7	+	2	3.756	0	6	+	1	0.000	0.02316
5	7	+	3	4.799	0	6	+	1	0.000	0.01084
5	7	+	4	5.631	0	6	+	1	0.000	0.00012
5	7	+	5	6.022	0	6	+	1	0.000	0.00589
5	7	+	6	6.504	0	6	+	1	0.000	0.00044
5	7	+	7	6.796	0	6	+	1	0.000	0.00002
5	7	+	8	8.034	0	6	+	1	0.000	0.00006
5	7	+	9	8.186	0	6	+	1	0.000	0.00097
5	7	+	10	8.398	0	6	+	1	0.000	0.00006
total =										1.83196
centroid =				0.102	centroids =					0.000
centroid*=				-22.313	centroids =					-22.211



Neither bound nor scattering – continuum bins

Scattering states

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right]$$



$$\int_0^\infty dr\, u_{k\ell j}(r)\, u^*_{k'\ell j}(r) = \frac{\pi}{2}\delta(k-k')$$

$$\hat{u}_{\alpha\ell j}(r) = \sqrt{\frac{2}{\pi N_\alpha}}\int_{\Delta k_\alpha} dk\, g(k)\, u_{k\ell j}(r)$$

$$N_\alpha = \int_{\Delta k_\alpha} dk\, [g(k)]^2$$

weight function

orthonormal set

$$\int_0^\infty dr\, \hat{u}^*_{\alpha\ell j}(r)\, \hat{u}_{\beta\ell j}(r) = \delta_{\alpha\beta}$$

$$g(k) = 1 \qquad g(k) = \sin\delta_{\ell j}$$

Optical potentials - parameter conventions

$$U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

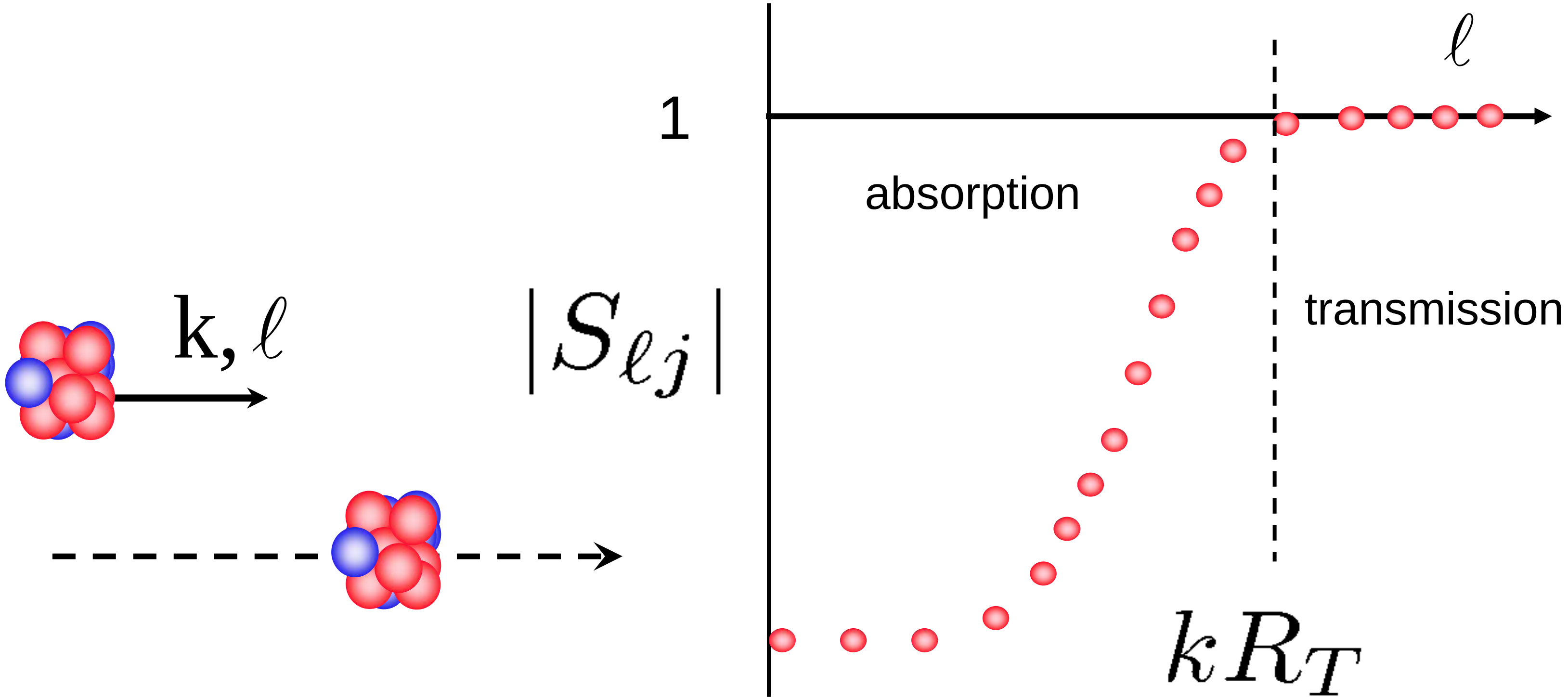
$$V_{so}(r) = -\frac{4V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

FRESCO conventions

$$W(r) = -\frac{W_V}{[1+\exp(X_V)]} - \frac{4W_S\exp(X_S)}{[1+\exp(X_S)]^2}\ ,$$

$$R_i = r_i\,A_2^{1/3} \quad \text{or} \quad R_i = r_i\left[A_1^{1/3} + A_2^{1/3}\right]$$

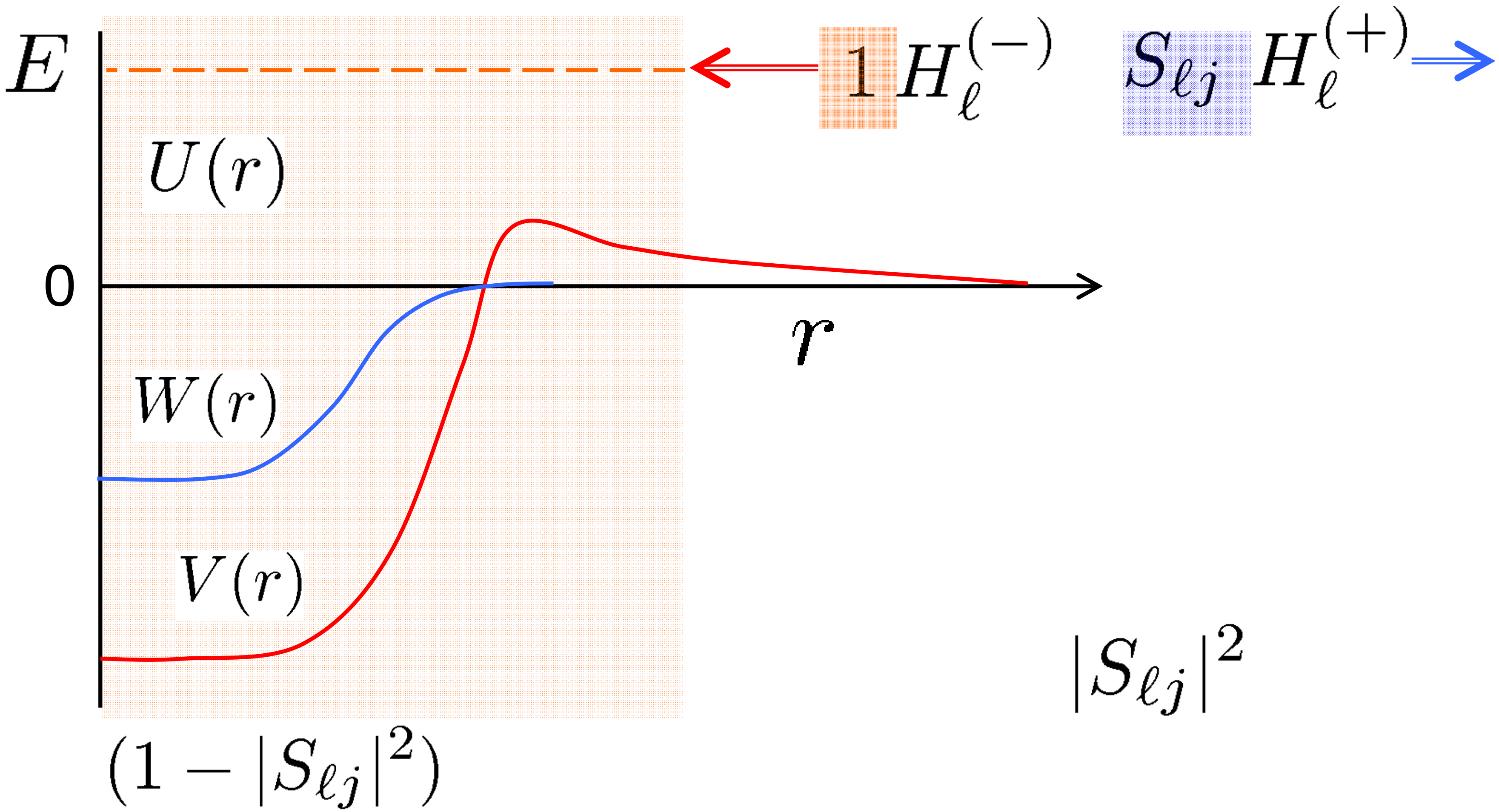
S-matrix with absorption



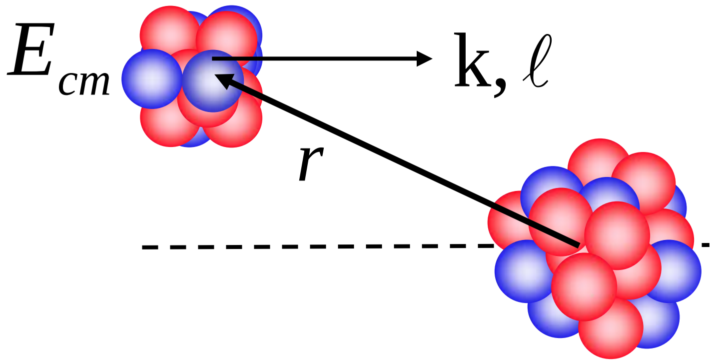
$$u_{k\ell j}(r) \rightarrow (i/2)[H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$

Ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[1\, H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$

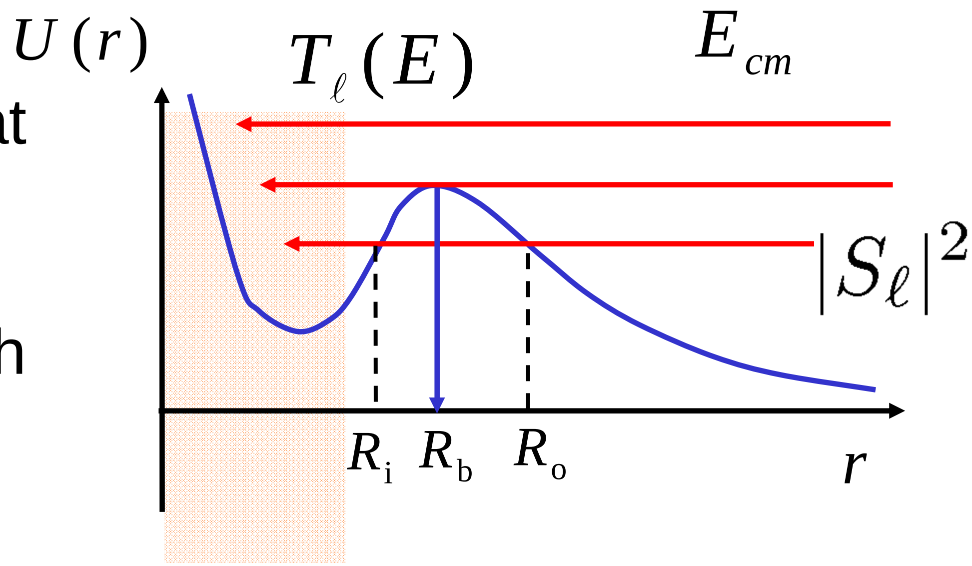


Barrier passing models of fusion



Gives basis also for simple (barrier passing) models of nucleus-nucleus fusion reactions

an imaginary part in *U(r)*, at short distances, can be included to absorb all flux that passes over or through the barrier – assumed to result in fusion



$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)(1-|S_\ell|^2)$$