The Joint Institute for Nuclear Astrophysics

JINA "Methods of Direct Nuclear Reactions" School

NSCL, East Lansing MI, 9-20th April 2007

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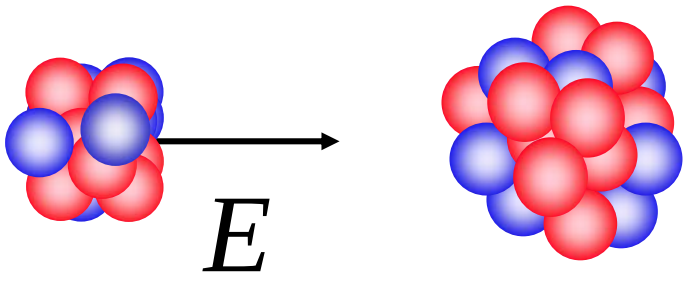
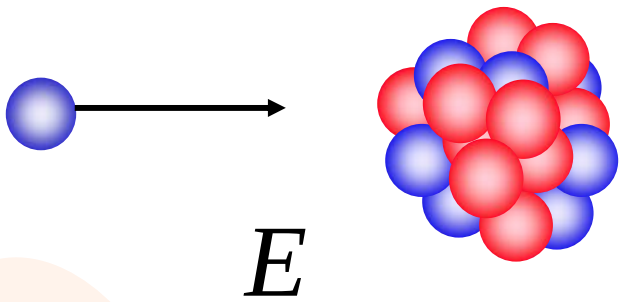
Interactions of composite systems

Reactions require us to make *educated* assumptions about the interactions between a wide variety of systems at different energies

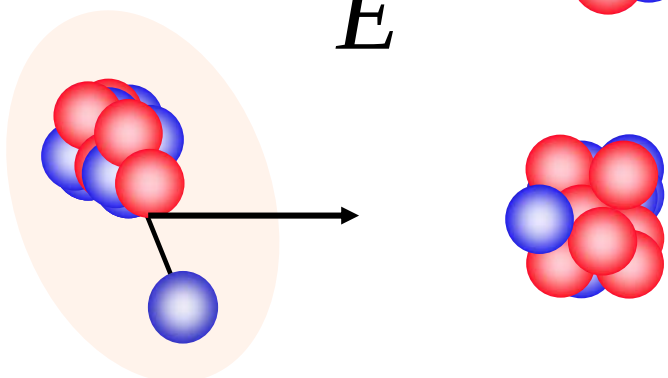
nucleus-nucleus folding



nucleon-nucleus folding



Cluster-nucleus folding



Can involve nucleus - nucleus or nucleon - nucleus depending on the clusters, e.g. our ^{11}Be example

Eikonal approach – few-body composites

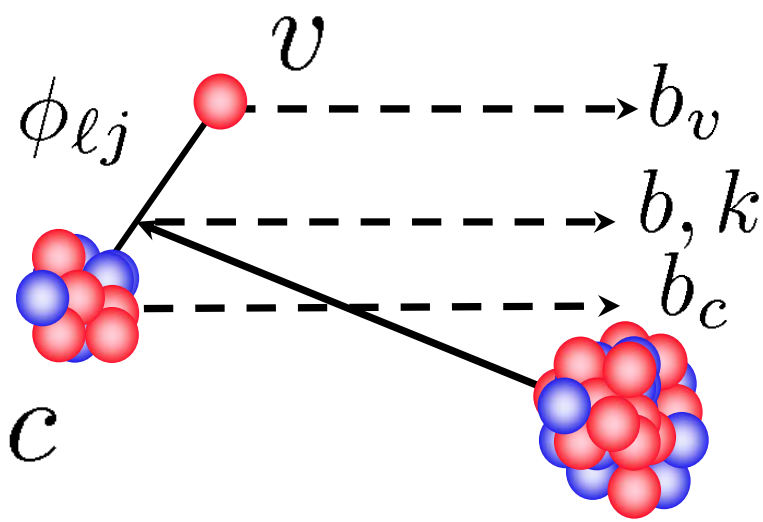
Total interaction energy

$$U(r_1,\ldots) = \sum_{i=c,v} U_i(r_i)$$

$$\exp[i\chi(b_1,\ldots)] = \prod_{i=c,v} S_i(b_i)$$

$$S_c(b_c) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_c(r)dz'\right]$$

$$S_v(b_v) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_v(r)dz'\right]$$



$$S_p(b) = \langle \phi_{\ell j} | S_c(b_c) S_v(b_v) | \phi_{\ell j} \rangle_{\vec{r}}$$

Few-body models deduce the dynamics and interaction of the composite from knowledge of the interactions of the constituent parts and their relative motion internal wave functions …

Phenomenological optical potentials – where?

C.M. Perey and F.G. Perey, At. Data Nucl. Data Tables 17, 1 (1976) Compilation for many systems

J.J.H. Menet, E.E. Gross, J.J. Malanify, and A. Zucker, Phys. Rev. C 4, 1114 (1971) – for nucleons

R.L. Varner, W.J. Thompson, T.L. McAbee, E.J. Ludwig, and T.B. Clegg, Phys. Rep. 201, 57 (1991) – Chapel Hill 89 potential – for nucleons

F.D. Becchetti, Jr. and G.W. Greenlees, Phys. Rev. 182, 1190 (1969) – 'old faithful' parameterisation – for nucleons

W.W. Daehnick, J.D. Childs, and Z. Vrcelj, Phys. Rev. C 21, 2253 (1980) – good parameter set – for deuterons

J.M. Lohr and W. Haeberli, Nucl. Phys. A232, 381 (1974) – for low energy deuterons

..... and many many more ... but many many gaps

http://www-nds.iaea.org/RIPL-2/optical.html

Session (learning) aims:

To introduce theoretical approaches to the interactions between composite nuclei or between a nucleon and a composite target nucleus based on a knowledge of the nuclear wave functions and / or their density distributions.

To look at (i) specific (commonly used) single and double folding model approaches and effective interactions for such problems and (ii) some general identities between the original and the folded potentials.

Through hands on exercises, to calculate nuclear density distributions, double and single folded potentials and use these to verify the folding model identities and apply these to nuclear reactions at higher energies.

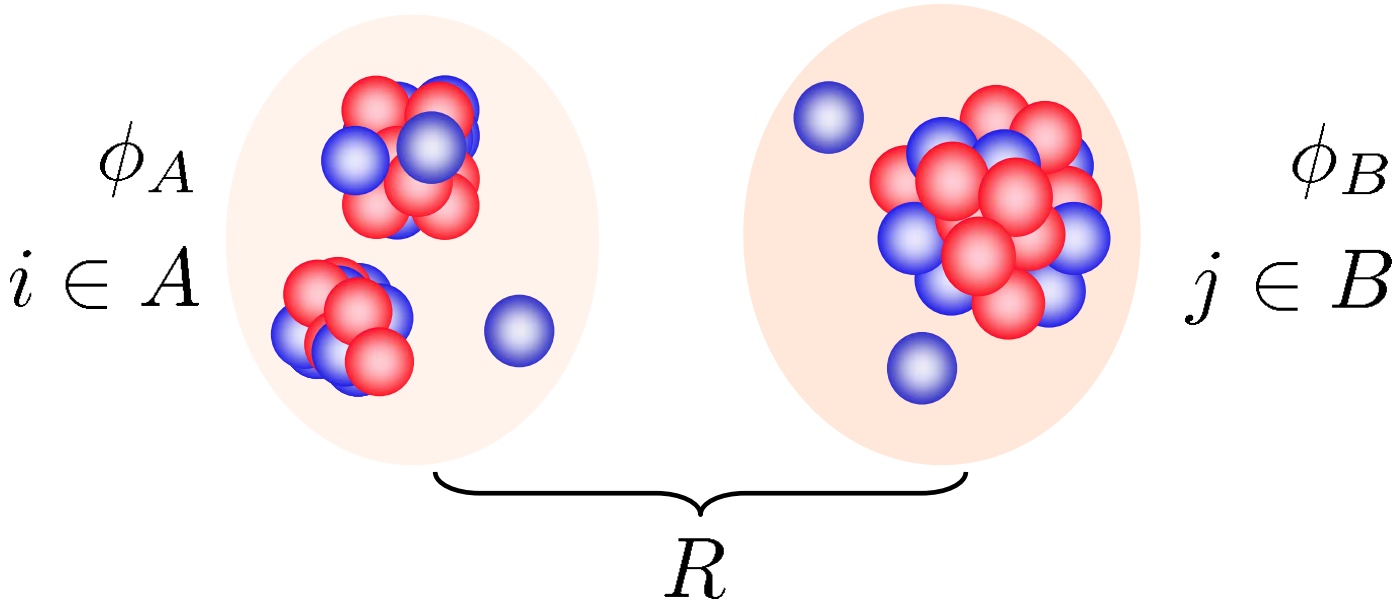
Session (learning) outcomes:

To be aware of what is involved in the computation of nucleus-nucleus and nucleon-nucleus interactions for composite projectiles in double and single folding models – and to calculate these in specific cases

To be aware of identities between properties of the original and the resulting folded potentials, for both checking purposes and to understand the qualitative features of the folded interactions.

To calculate examples of nuclear densities, nucleus-nucleus and nucleon-nucleus potentials based on an effective NN t-matrix and the JLM effective interaction and to subsequently use these for knockout reaction calculations.

Folding models are a general procedure



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} V_{ij}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

Pair-wise interactions integrated (averaged) over the internal motions of the two composites

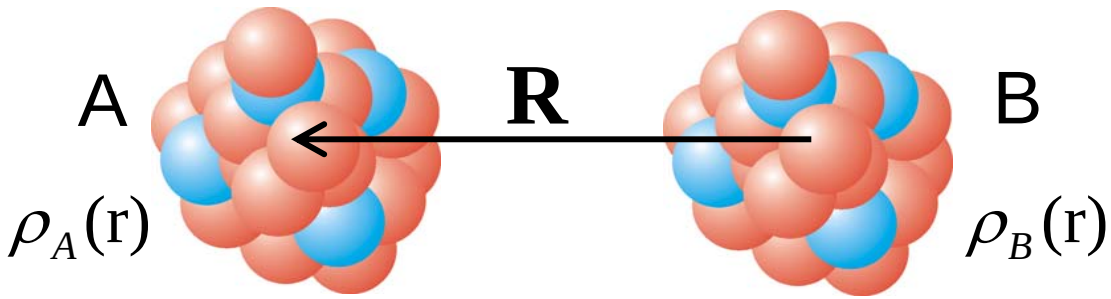
Folding models from NN effective interactions

$$V_F(R) = \langle \phi_A \phi_B | \sum V_{ij}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

Double folding

V_{AB}

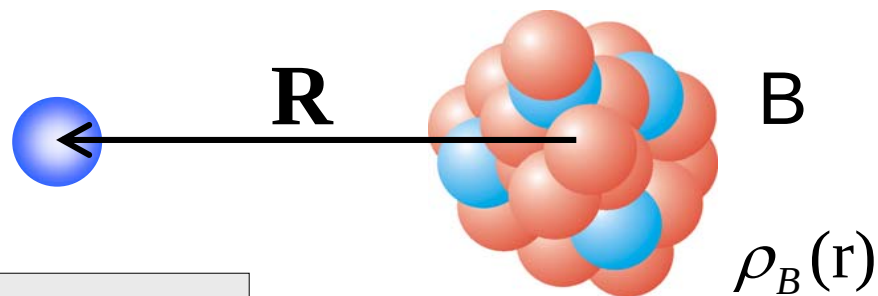
$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$



Single folding

V_B

$$V_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} - \mathbf{r}_2)$$



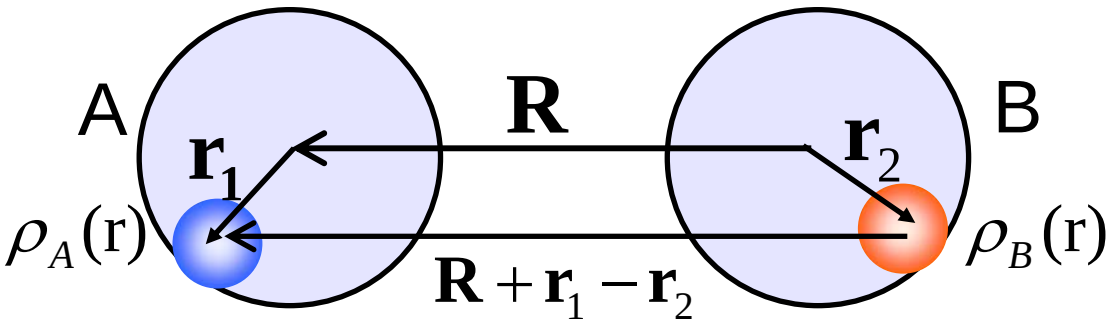
Only ground state densities appear

Effective interactions – Folding models

Double folding

V_{AB}

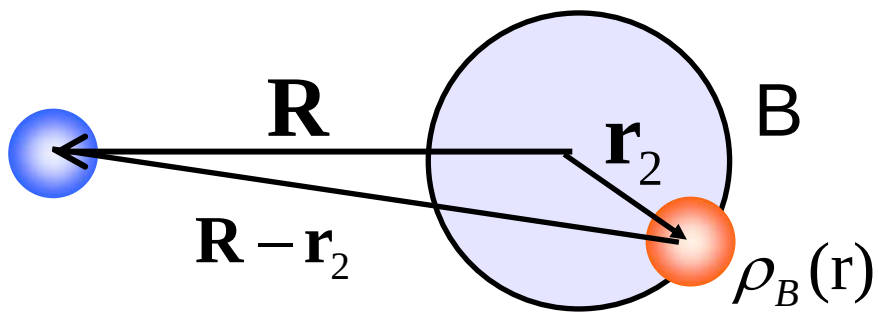
$$V_{AB}(\mathrm{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(\mathrm{r}_1)\, \rho_B(\mathrm{r}_2)\, v_{\mathrm{NN}}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$



Single folding

V_B

$$V_B(\mathrm{R}) = \int d\mathbf{r}_2 \; \rho_B(\mathrm{r}_2)\, v_{\mathrm{NN}}(\mathbf{R} - \mathbf{r}_2)$$

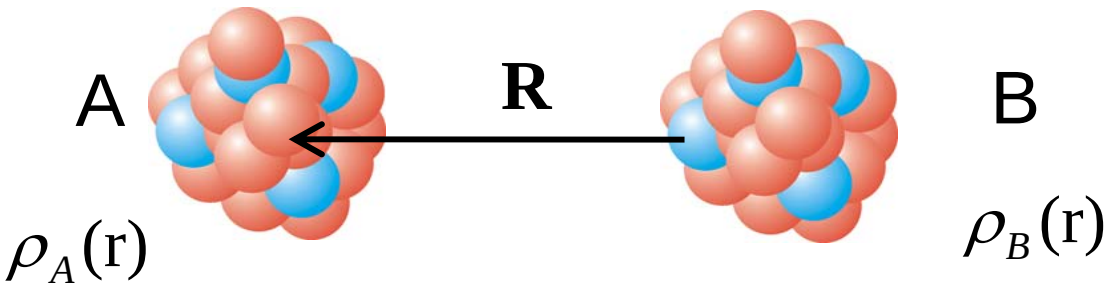


The M3Y interaction – nucleus-nucleus systems

Double folding

$$V_{AB}$$

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$



originating from a G-matrix calculation and the Reid NN force

$$v_{NN}(r) = \left[7999\frac{e^{-4r}}{4r} - 2134\frac{e^{-2.5r}}{2.5r}\right] + \hat{J}(E)\delta(\vec{r})$$

$$\hat{J}(E) = -276[1 - 0.005(E/A)] \text{ MeVfm}^3$$

resulting in a REAL nucleus-nucleus potential

M.E. Brandan and G.R. Satchler, The Interaction between Light Heavy-ions and what it tells us, Phys. Rep. **285** (1997) 143-243.

Making use of the elastic scattering data

Double folding model is often used to provide the radial shape and approximate strength of the real part of the potential, call it $F_E(R)$, Then, at each E

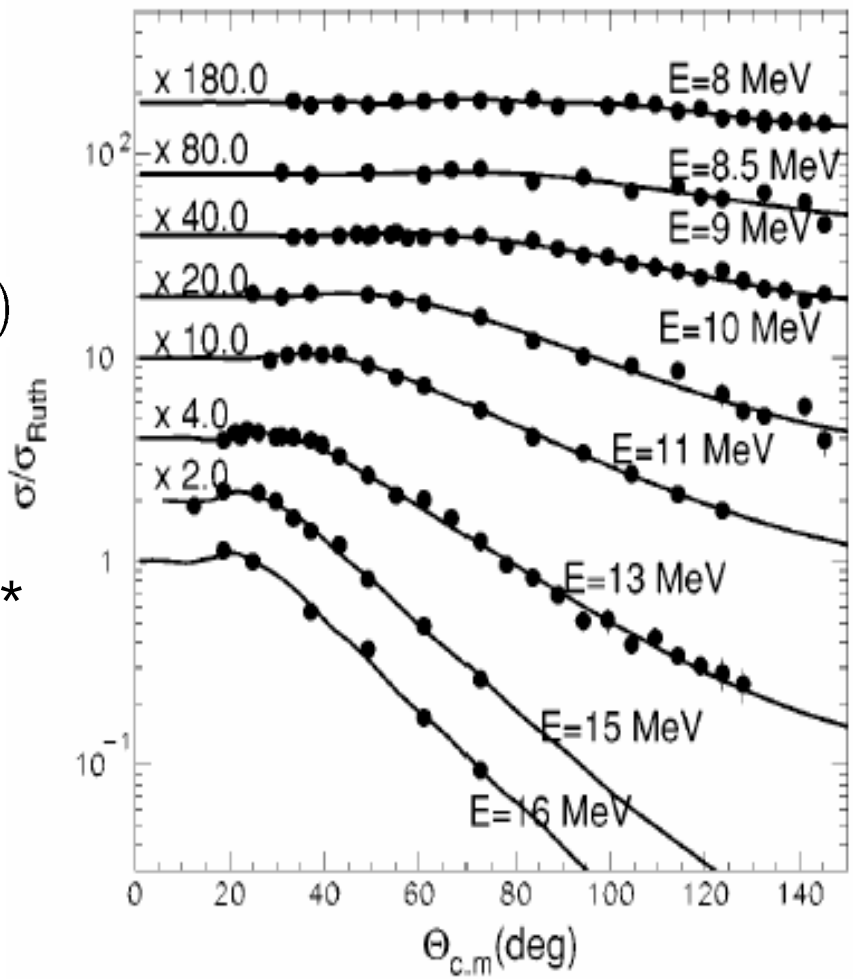
$$U_E(R) = [N_R(E) + iN_I(E)]\, F_E(R)$$

the N_R and N_I are fitted to data with N_R of order unity.

Quite generally, for many systems***

$$N_R(E) = 1.0 \pm 0.15$$
$$N_I(E) = 0.8 \pm 0.15$$

^{7}Li + ^{28}Si

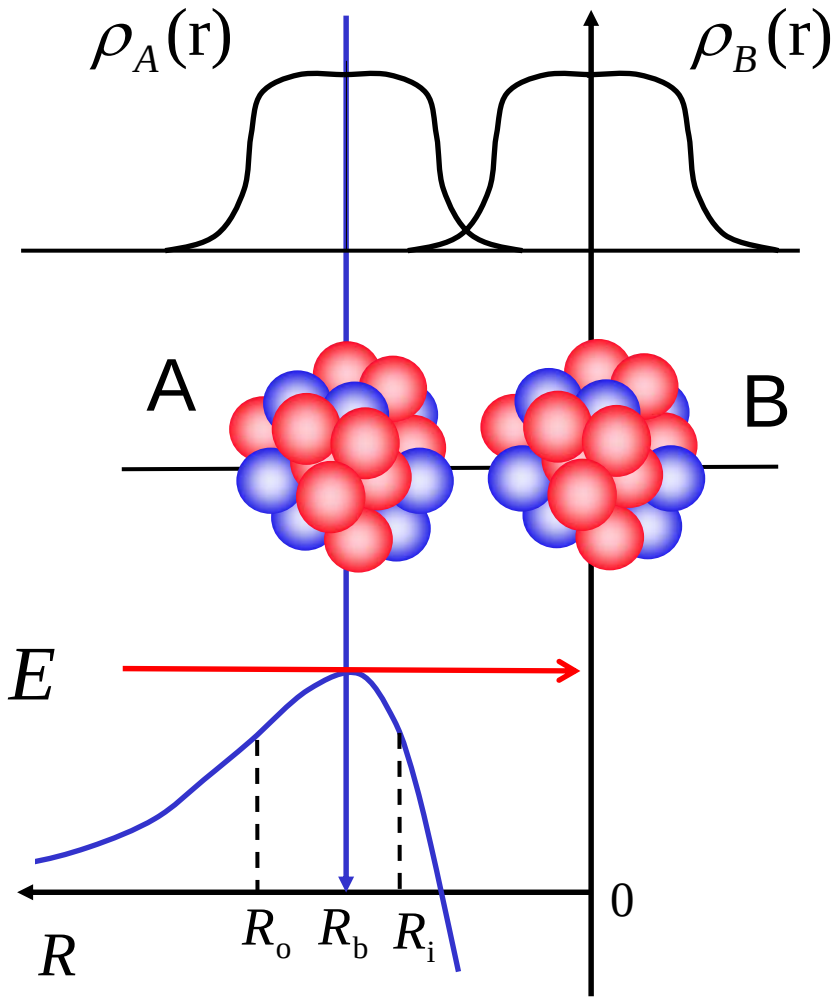


L.C. Chamon et al., PRC 66 (2002) 014610

*** G.R. Satchler and W.G. Love, Phys. Rep. **55** (1979) 183

A. Pakou et al., PRC 69 (2004) 054602

Fusion barrier radii and densities - surfaces



Fusion will be probe and is sensitive to:

nuclear binding (tails of the nuclear densities),

nuclear structure (tails of the single particle wave functions)

but also expect sensitivity and complications due to the reaction dynamics –

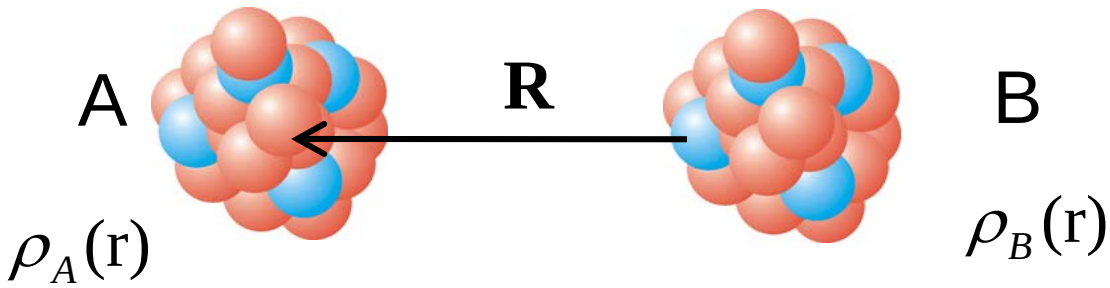
intrinsically surface dominated

t-matrix effective interactions – higher energies

Double folding

$$V_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, t_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

V_{AB}



At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2} \sigma_{NN} (i + \alpha_{NN}) \right] f(r), \quad \int d\vec{r} f(r) = 1$$

e.g. $f(r) = \delta(r)$

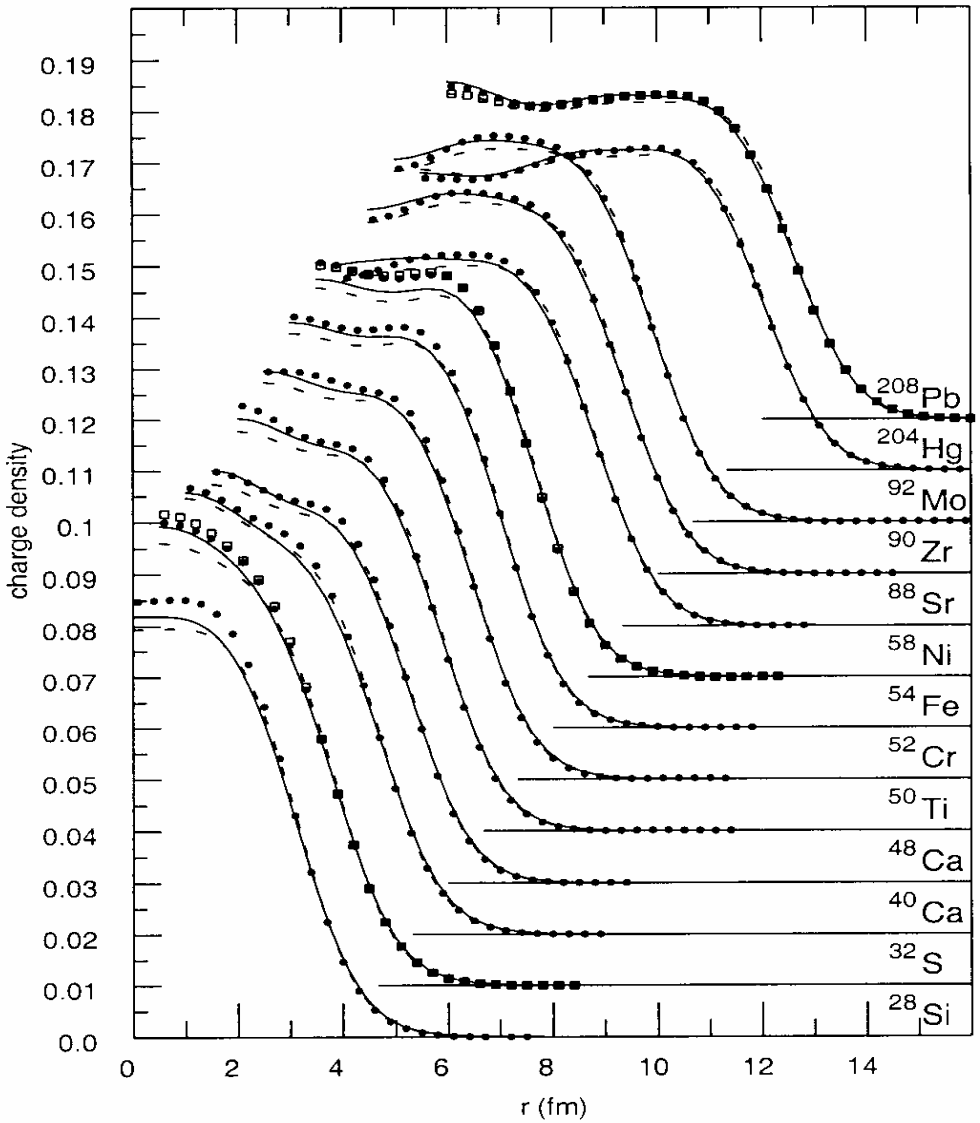
$f(r) = (\sqrt{\pi} t)^{-3} \exp(-r^2/t^2)$

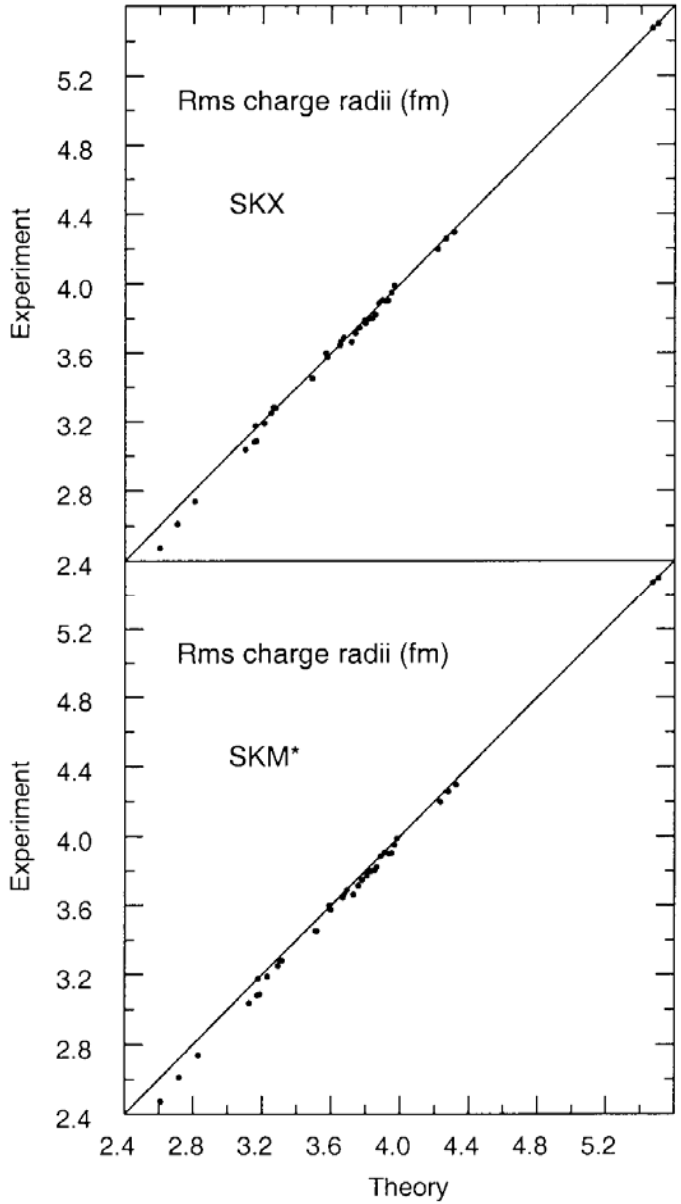
nucleon-nucleon cross section

resulting in a COMPLEX nucleus-nucleus potential

M.E. Brandan and G.R. Satchler, The Interaction between Light Heavy-ions and what it tells us, Phys. Rep. **285** (1997) 143-243.

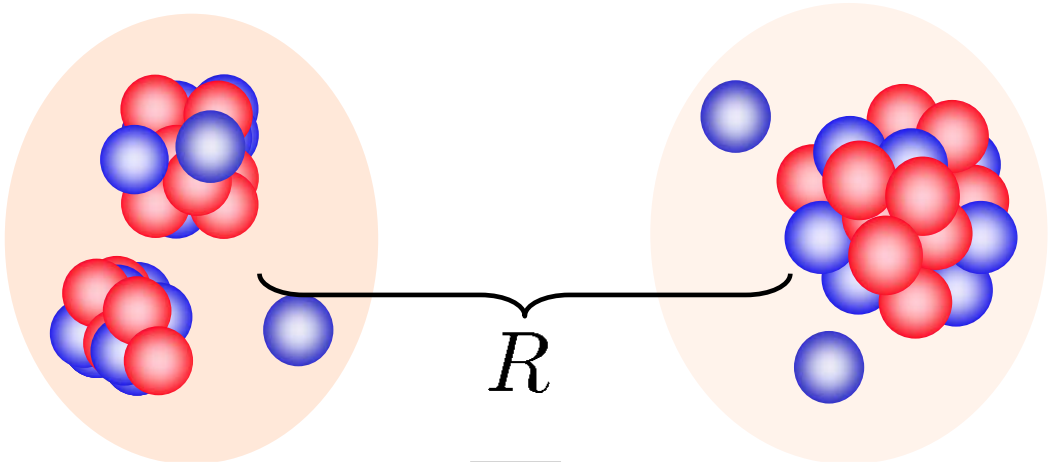
Skyrme Hartree-Fock radii and densities





W.A. Richter and B.A. Brown, Phys. Rev. **C67** (2003) 034317

Double folding models – useful identities



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} v_{NN}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

$$J_{V_F} = A_1 \, A_2 \, J_{v_{NN}}$$

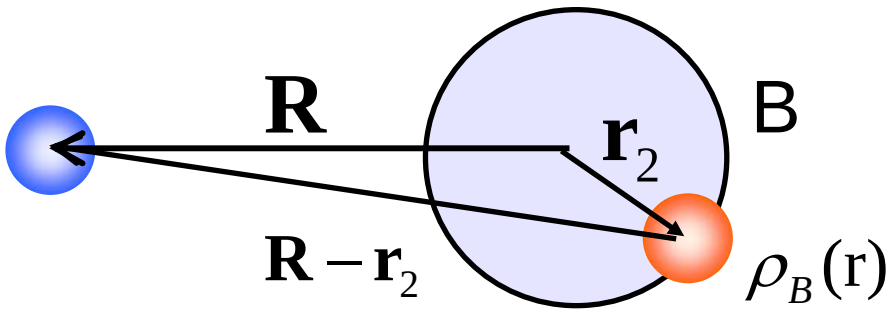
$$\langle r^2 \rangle_{V_F} = \langle r^2 \rangle_A + \langle r^2 \rangle_B + \langle r^2 \rangle_{v_{NN}}$$

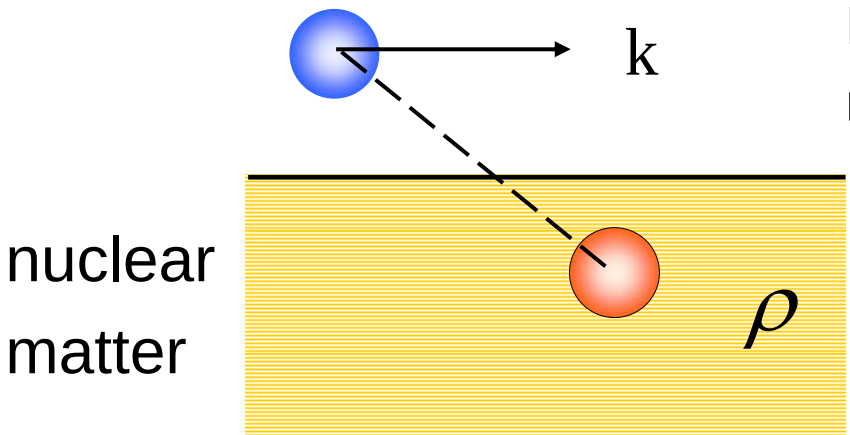
$$J_f = \int d\vec{r} \, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r} \, r^2 f(r) / J_f$$

proofs by taking Fourier transforms of each element

Effective NN interactions – not free interactions

$$V_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(|\mathbf{R} - \mathbf{r}_2|)$$





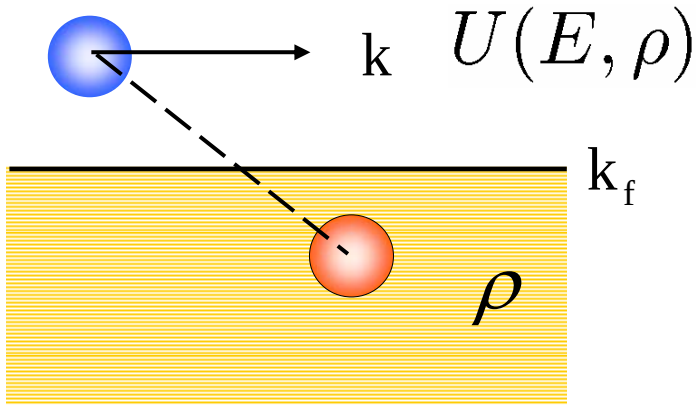
include the effect of NN interaction in the "nuclear medium" – Pauli blocking of pair scattering into occupied states

$$\rightarrow v_{NN}(\rho, \mathbf{r})$$

But as E → high

$$v_{NN} \rightarrow v_{NN}^{free}$$

JLM interaction – local density approximation



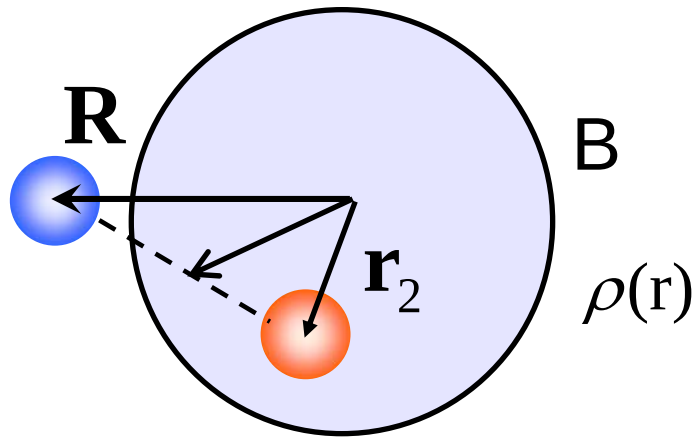
nuclear matter

complex and density dependent interaction

$$v_{NN}(r) = \frac{U(E,\rho)}{\rho} f(r)$$

$$f(r) = (\sqrt{\pi}t)^{-3} \exp(-r^2/t^2)$$

For finite nuclei, what value of density should be used in calculation of nucleon-nucleus potential? Usually the local density at the mid-point of the two nucleon positions $\mathbf{r}_x$



$$U_B(R) = V_B(R) + iW_B(R) = \int d\vec{r}_2 \, \rho_B(r_2) \frac{U(E,\rho(r_x))}{\rho(r_x)} f(r)$$

JLM interaction – fine tuning

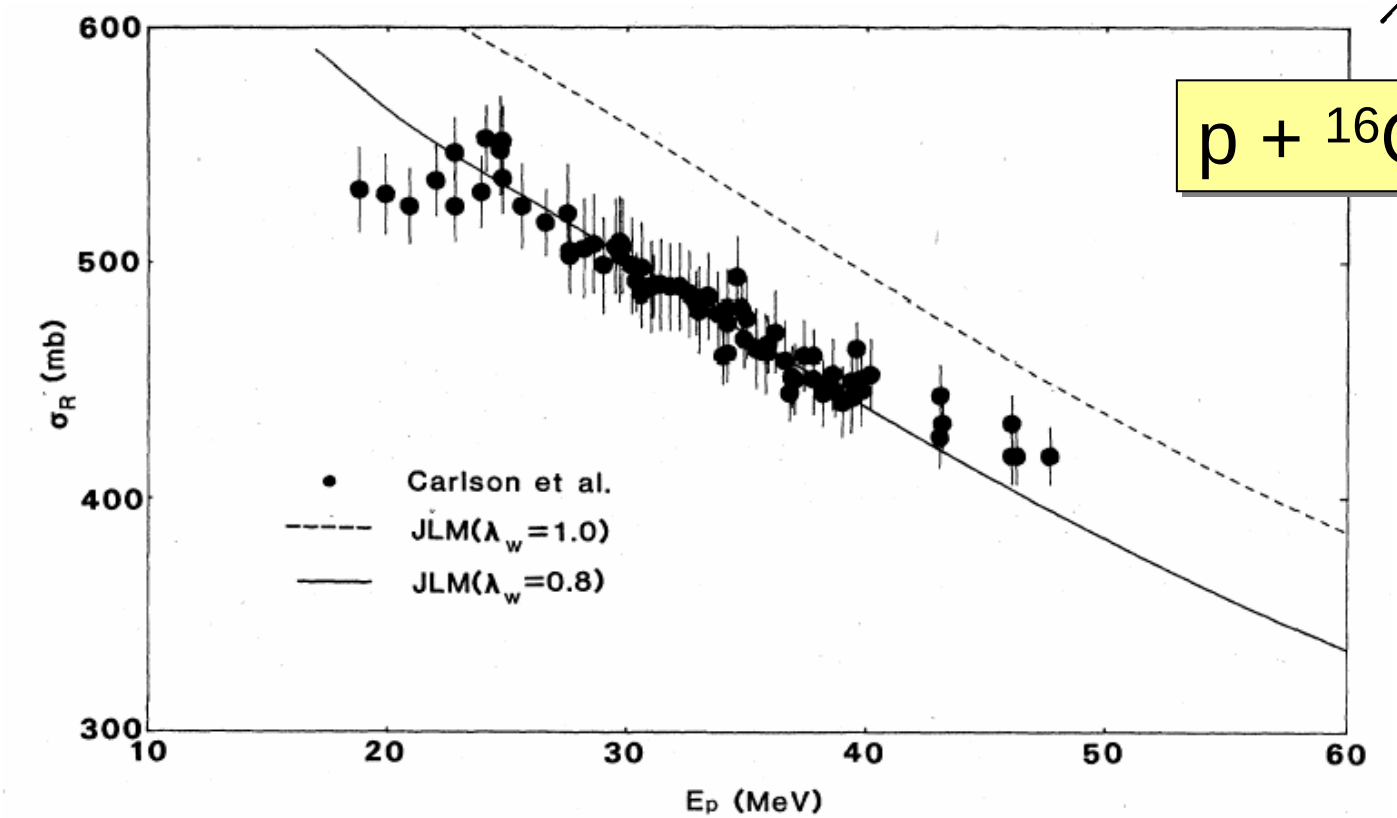
Strengths of the real and imaginary parts of the potential can be adjusted based on experience of fitting data.

$$U_B(R) = \lambda_v V_B(R) + i\lambda_w W_B(R)$$

$\lambda_v \approx 1.0$

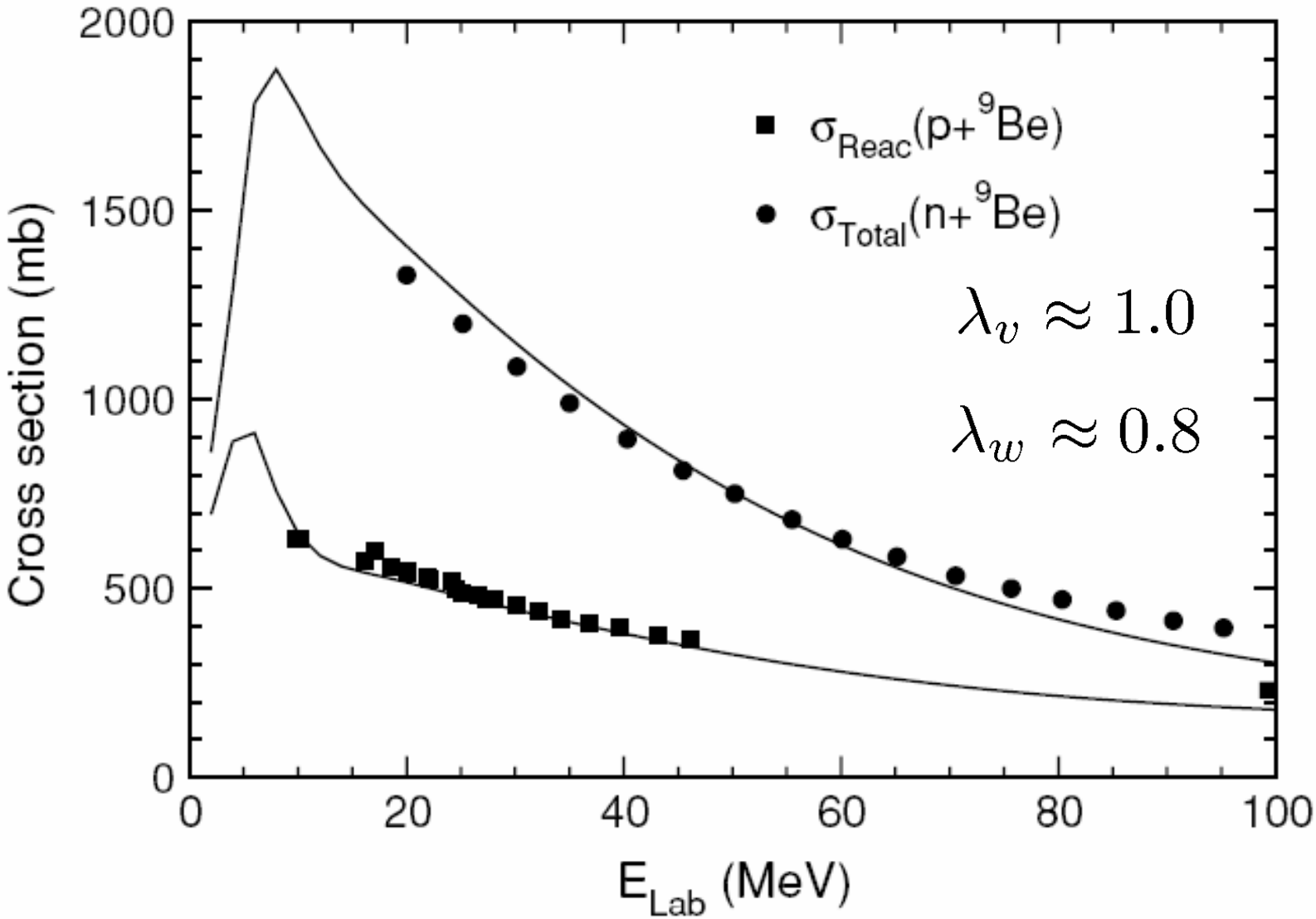
$\lambda_w \approx 0.8$

p + ^{16}O



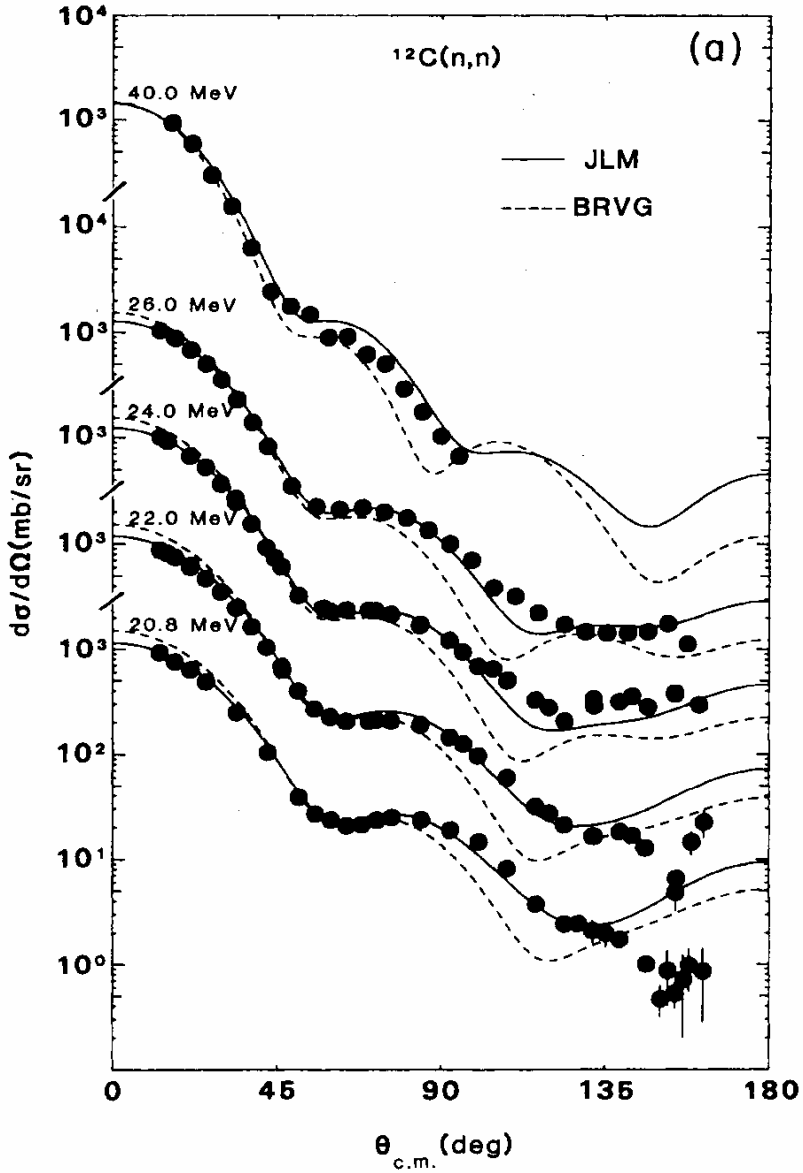
J.S. Petler et al. Phys. Rev. C **32** (1985), 673

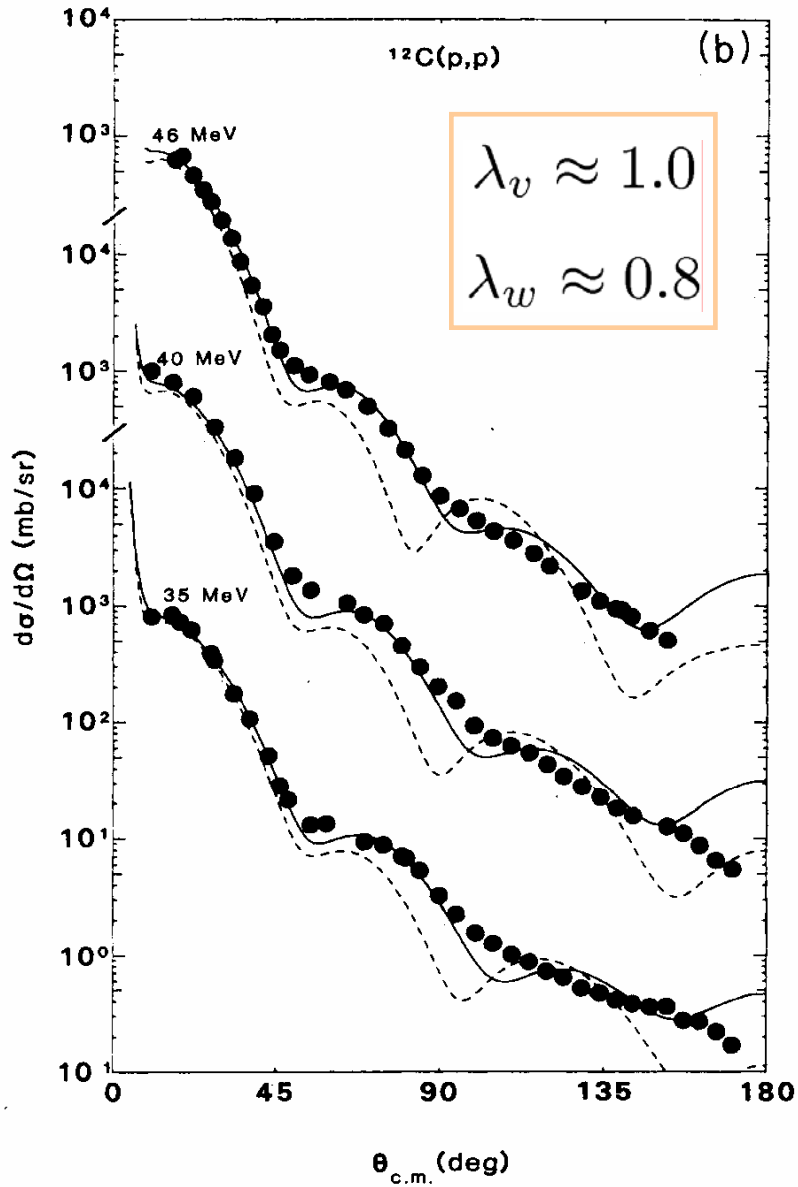
JLM predictions for N+^{9}Be cross sections



A. Garcıa-Camacho, et al. Phys. Rev. C **71**, 044606(2005)

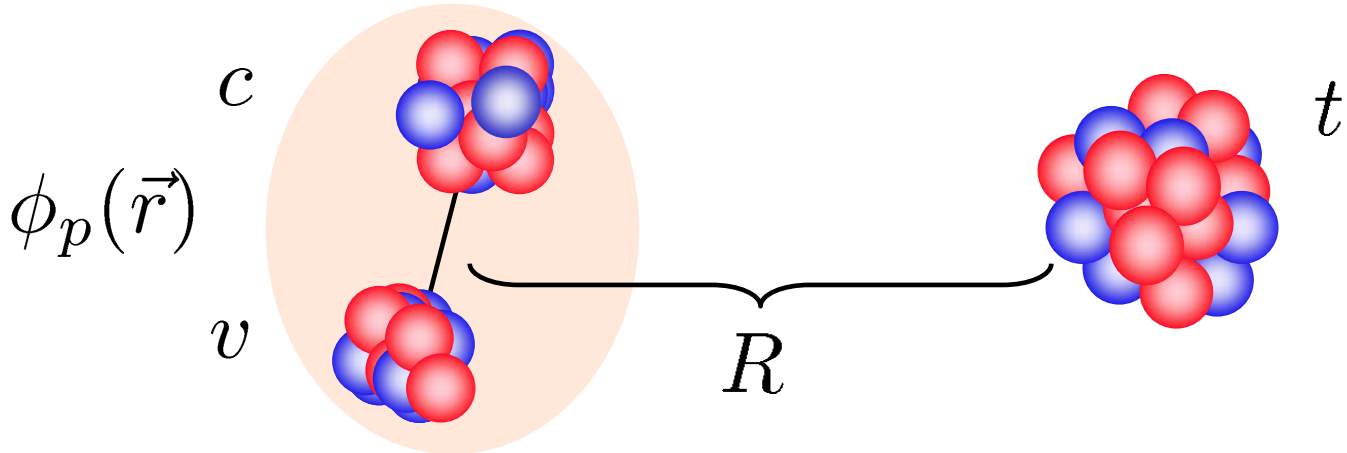
JLM folded nucleon-nucleus optical potentials





J.S. Petler et al. Phys. Rev. C **32** (1985), 673

Cluster folding models – the halfway house

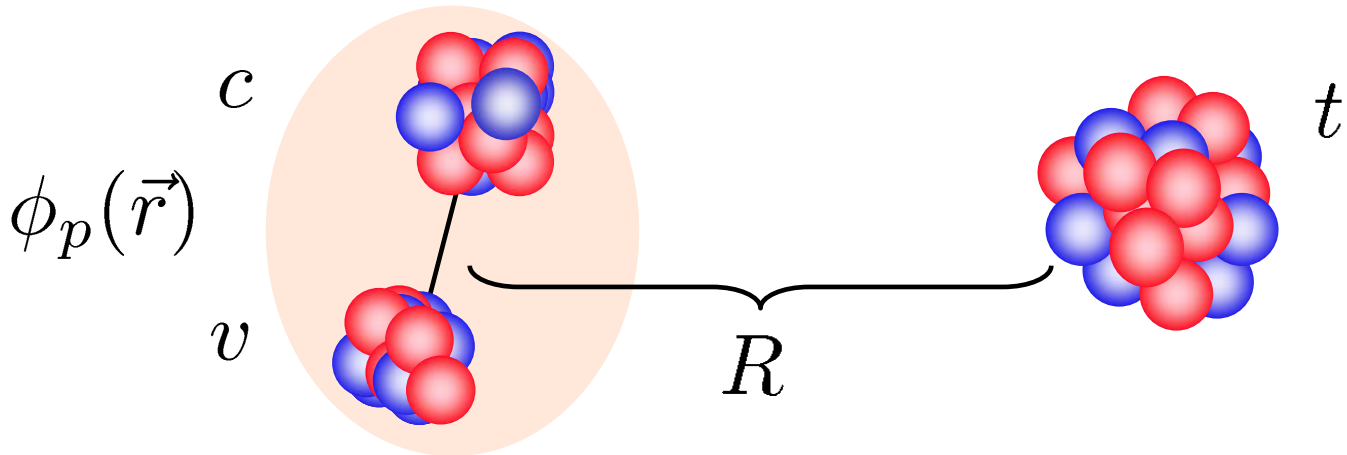


for a two-cluster projectile (core +valence particles) as drawn

$$V_F(R) = \langle \phi_p | V_{ct}(\vec{r}_{ct}) + V_{vt}(\vec{r}_{vt}) | \phi_p \rangle$$

can use fragment-target interactions from phenomenological fits to experimental data or the nucleus-nucleus or nucleon-nucleus interactions just discussed to build the interaction of the composite from that of the individual components.

Cluster folding models – useful identities



$$V_F(R) = \langle \phi_p | V_{ct}(\vec{r}_{ct}) + V_{vt}(\vec{r}_{vt}) | \phi_p \rangle$$

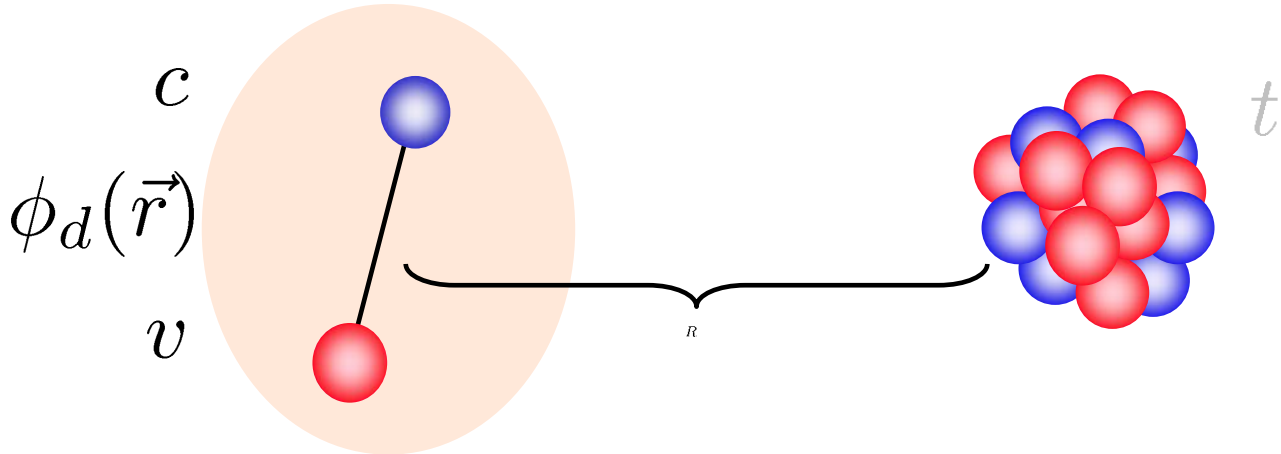
$$J_{V_F} = J_{V_{ct}} + J_{V_{vt}}$$

$$\langle r^2 \rangle_{V_F} = \frac{A_c}{A_p}\langle r^2 \rangle_{V_{ct}} + \frac{A_v}{A_p}\langle r^2 \rangle_{V_{vt}} + \frac{A_c A_v}{A_p^2}\langle r^2 \rangle_{\phi_p}$$

$$J_f = \int d\vec{r}\, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r}\, r^2 f(r)/J_f$$

proofs by taking Fourier transforms of each element

So, for a deuteron for example …



$$V_d(R) = \langle \phi_d | V_{nt}(\vec{r}_{nt}) + V_{pt}(\vec{r}_{pt}) | \phi_d \rangle$$

$$J_{V_d} = J_{V_{nt}} + J_{V_{pt}} = 2J_{V_N}$$

$$\langle r^2 \rangle_{V_F} = \langle r^2 \rangle_{V_N} + \frac{1}{4}\langle r^2 \rangle_{\phi_d}$$

$$J_f = \int d\vec{r}\, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r}\, r^2 f(r)/J_f$$