The Joint Institute for Nuclear Astrophysics

JINA "Methods of Direct Nuclear Reactions" School

NSCL, East Lansing MI, 9-20th April 2007

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Session (learning) aims:

To introduce the (high energy) adiabatic approximation as a means of simplifying the description of the reaction dynamics in collisions of few-body systems and which is also essential to the eikonal / Glauber reaction methods

To discuss the adiabatic approximation as it affects transfer reactions, such as (d,p), (p,d), (d,n) etc., often referred to as the Johnson-Soper theory for treating the effects of breakup upon transfer – goes beyond DWBA.

To allow calculations of transfer reactions that include the adiabatic approximation and so to allow comparisons with subsequent (more exact) finite range and or coupled channels methods for the effects of breakup on transfers.

Adiabatic approximation for deuteron break-up

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Contribution of Deuteron Breakup Channels to Deuteron Stripping and Elastic Scattering

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AND

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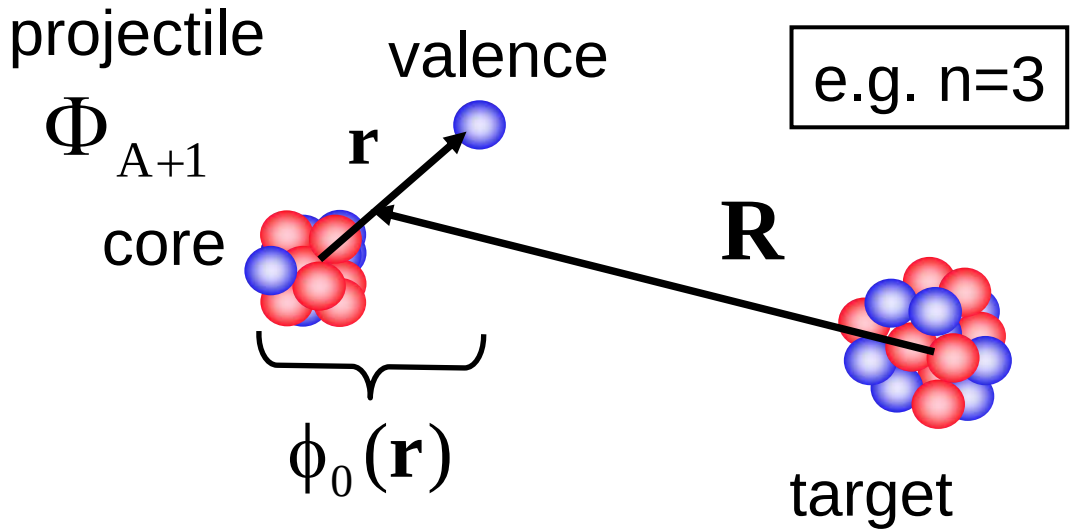
(Received 10 November 1969)

We present a model of deuteron stripping and elastic scattering which treats explicitly the contributions from channels in which the deuteron is broken up into a relative S state and the target is in its ground state. An adiabatic treatment of these channels leads to a description of deuteron stripping which resembles the distorted-wave Born approximation, although a deuteron optical potential plays no role. The adiabatic approximation is shown to give a good account of 21.6-MeV elastic deuteron scattering from Ni, at least for surface partial waves, and is expected to apply to other nuclei in this mass and energy region, as well as at higher energies. The calculations assume that the effective two-nucleon–nucleus interaction is the sum of the nucleon optical potentials evaluated at one-half the incident deuteron kinetic energy. Some possible corrections to this assumption are discussed.

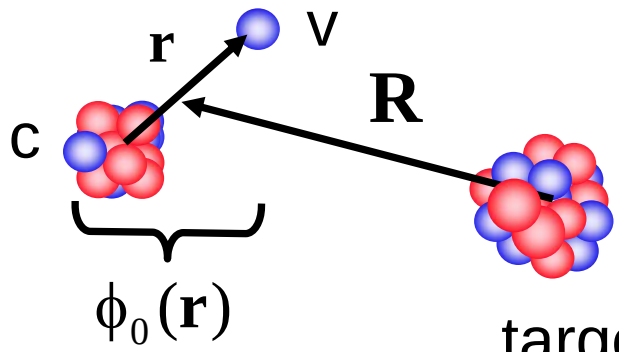
Few-body models of nuclear reactions

There are no practical many-body reaction theories - we construct model 'effective' few-body models (n=2,3,4 …)



Construct an effective Hamiltonian H and solve as best we can the few-body Schrödinger equation: $\mathrm{H}\Psi = \mathrm{E}\Psi$

Few-body reaction theory - definitions - notation



$$H = \underbrace{T_{\mathbf{r}} + V_{vc}}_{\text{projectile } H_p} + T_{\mathbf{R}} + \underbrace{V_{cT} + V_{vT}}_{U(\mathbf{r},\mathbf{R})}$$

effective interactions (tidal forces) of c and v with the target excite the projectile

excited continuum of c+v

$-\varepsilon_0$

$$H\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R}) = E\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R})$$

with boundary conditions:

$$\Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R}) \to e^{i\mathbf{K}\cdot\mathbf{R}}\phi_0(\mathbf{r}) + \text{outgoing waves}$$

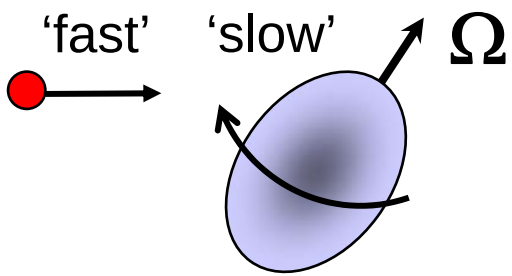
$$H_p\phi_0(\mathbf{r}) = -\varepsilon_0\phi_0(\mathbf{r})$$

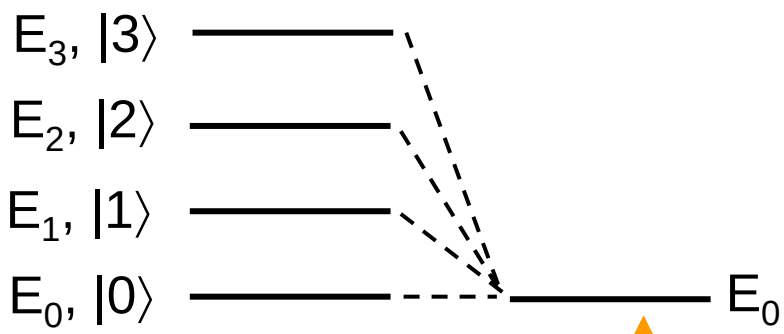
Projectile structure enters through the incident waves boundary condition

Adiabatic (sudden) approximations in physics

Identify high energy/fast and low energy/slow degrees of freedom

Fast neutron scattering from a rotational nucleus



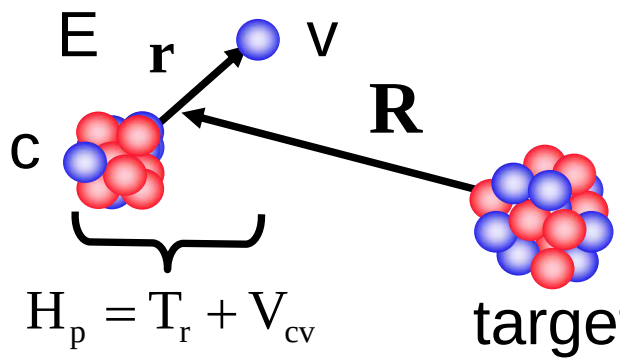


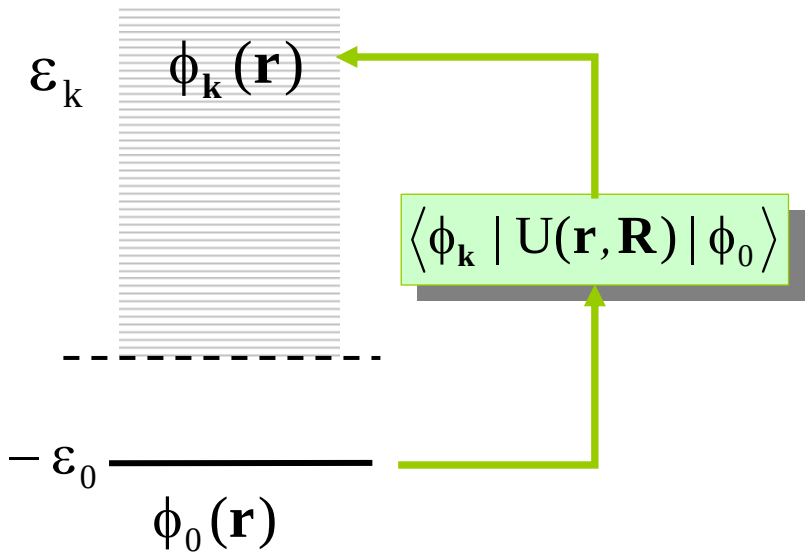
Fix Ω, calculate scattering amplitude $f(\theta, \Omega)$ for each (fixed) Ω.

→ moment of inertia $\rightarrow \infty$ and rotational spectrum is assumed degenerate

Transition amplitudes $f_{\alpha\beta}(\theta) = \langle \beta \,|\, f(\theta, \Omega) \,|\, \alpha \rangle_\Omega$

Energetics of few-body composite systems





spectrum of H_p

$$H = H_p + T_R + U(\mathbf{r},\mathbf{R})$$

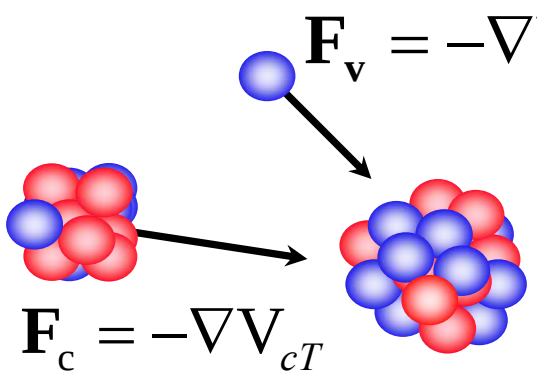
The tidal forces $U(\mathbf{r},\mathbf{R}) = V_{cT} + V_{vT}$ between c and v and the target cause excitation of the projectile to excited states of c+v and to the continuum states

$$H_p \phi_k(\mathbf{r}) = \varepsilon_k \phi_k(\mathbf{r})$$

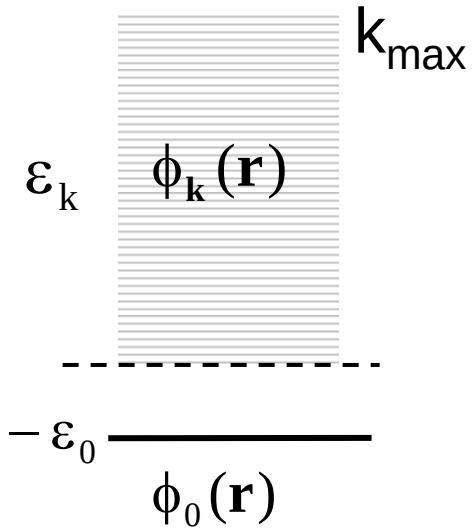
Which $\phi_k(\mathbf{r})$ are excited?

Continuum excitations and interactions

A major simplification to the reaction dynamics is possible if $\varepsilon_k << E$



Those states excited (to k_{max}) are dictated by the geometry of the interactions



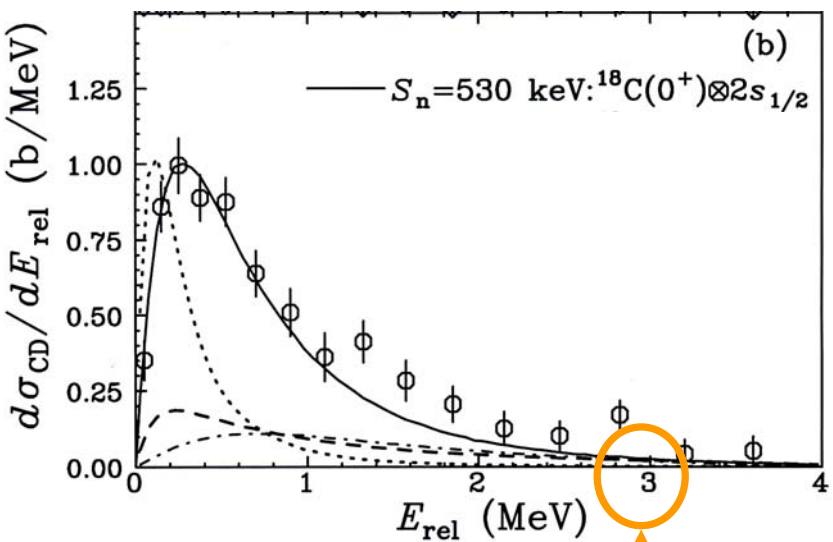
Nuclear forces, sharp surfaces, large $\mathbf{F}$, larger ε_k, universally, given surface diffuseness of nuclear potentials $\varepsilon_k \leq$ 20 MeV

Coulomb forces,slow spatial changes, small $\mathbf{F}$, typically $\varepsilon_k \leq$ 4 MeV (e.g. Nakamura et al, PRL **83** (1998) 1112)

In both cases, for the energies if RI beams from fragmentation facilities (50-100 MeV per nucleon), typical $\langle H_p \rangle << E$

Break-up continua from nuclear and Coulomb

T. Nakamura et al, PRL **83** (1998) 1112



Experimental

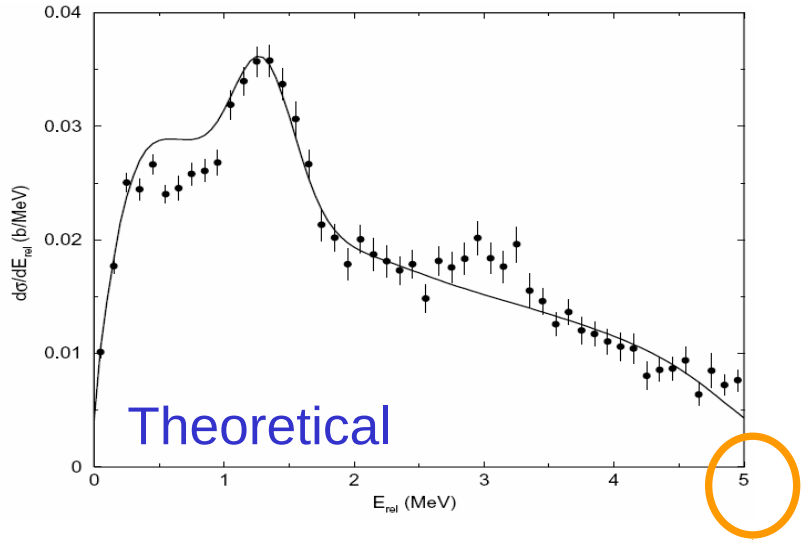
$^{19}C+Pb \rightarrow {}^{18}C+n+X$

E= 67A MeV = 1.33 GeV

Coulomb dominated

J.A. Tostevin et al, PRC **66** (2002) 02460

N. Fukuda et al., PRC **70** (2004) 054606



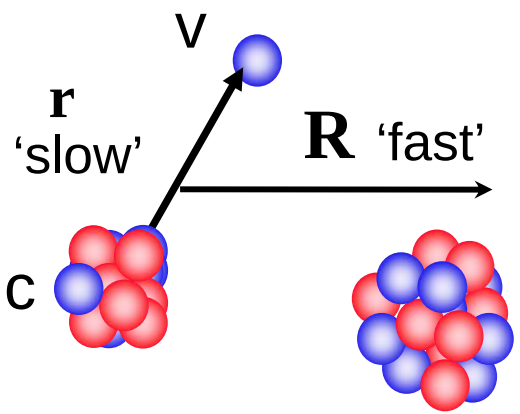
Experimental

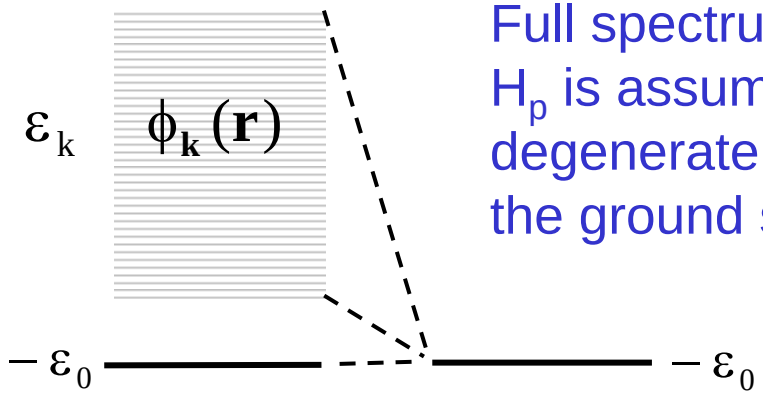
$^{11}Be + {}^{12}C \rightarrow {}^{10}Be+n+X$

E= 67A MeV = 737 MeV

Nuclear dominated

Adiabatic model for few-body projectiles





Full spectrum of H_p is assumed degenerate with the ground state

Freeze internal co-ordinate $\mathbf{r}$ then scatter c+v from target and compute $f(\theta,\mathbf{r})$ for all required fixed values of $\mathbf{r}$

Physical amplitude for breakup to state $\phi_k(\mathbf{r})$ is then,

$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_{\mathbf{r}}$$

Achieved by replacing $H_p \rightarrow -\varepsilon_0$ in Schrödinger equation

Adiabatic approximation - time perspective

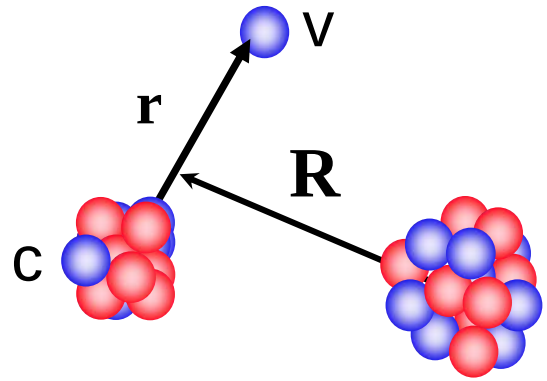
The time-dependent equation is

$$H\Psi(\mathbf{r},\mathbf{R},t) = i\hbar\frac{\partial\Psi}{\partial t}$$

and can be written

$$\Psi(\mathbf{r},\mathbf{R},t) = \Lambda\,\Phi(\mathbf{r}(t),\mathbf{R}), \quad \mathbf{r}(t) = \Lambda^{+}\,\mathbf{r}\,\Lambda$$

$$\Lambda = \exp\{-i(H_p + \varepsilon_0)t/\hbar\}$$ and where



$$[T_R + U(\mathbf{r}(t),\mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t),\mathbf{R}) = i\hbar\frac{\partial\Phi}{\partial t}$$

Adiabatic equation

$$[T_R + U(\mathbf{r},\mathbf{R})]\Phi(\mathbf{r},\mathbf{R}) = (E + \varepsilon_0)\Phi(\mathbf{r},\mathbf{R})$$

Adiabatic step assumes

$\mathbf{r}(t) \approx \mathbf{r}(0) = \mathbf{r}$=fixed or Λ=1 for the collision time t_{coll}

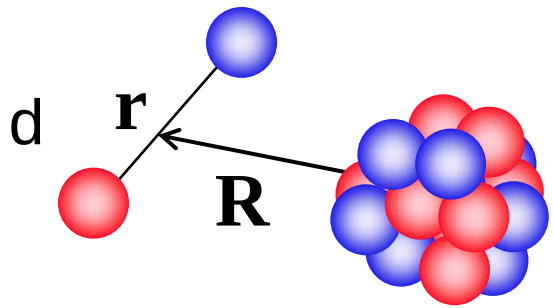
requires

$$(H_p + \varepsilon_0)t_{coll}/\hbar << 1$$

Adiabatic model for transfer reactions: e.g. (d,p)

$$T_{dp} = \left\langle \chi_p^{(-)}(\mathbf{r}_p)\phi_n(\mathbf{r}_n)\left| V_{np} \right| \Psi_{\mathbf{K}}^{(+)}(\mathbf{r},\mathbf{R}) \right\rangle$$

note $|\mathbf{r}| \leq$ range of V_{np}



$$[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) + H_d - E]\,\Psi_{\mathbf{K}}^{(+)} = 0$$

$$H_d \to -\varepsilon_0\,,\quad \Psi_{\mathbf{K}}^{(+)} \to \Psi_{\mathbf{K}}^{AD}$$

$$[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - E_0]\,\Psi_{\mathbf{K}}^{AD} = 0$$

DWBA ($|\mathbf{r}| \leq$ range of ϕ_0)

$$\Psi_{\mathbf{K}}^{(+)} \to \phi_0(\mathbf{r})\left\langle \phi_0(\mathbf{r}) \left| \Psi_{\mathbf{K}}^{(+)} \right\rangle_{\mathbf{r}} \right.$$

$$= \phi_0(\mathbf{r})\chi_{\mathbf{K}}^{(+)}(\mathbf{R})$$

elastic scattering

ADIABATIC ($|\mathbf{r}| \leq$ range of V_{np})

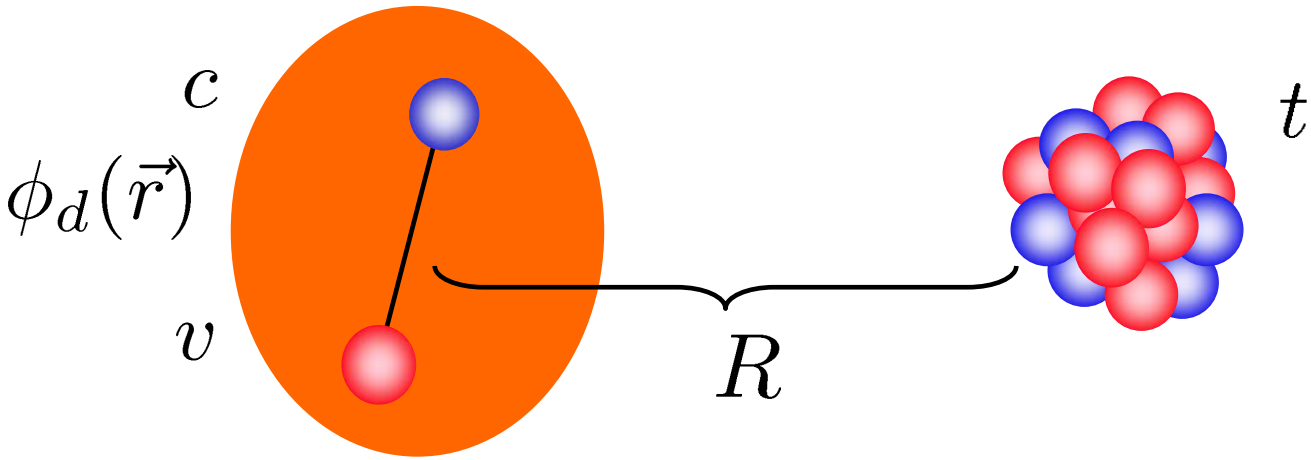
$$\Psi_{\mathbf{K}}^{AD} \approx \phi_0(\mathbf{r})\,\tilde{\chi}_{\mathbf{K}}^{AD}(\mathbf{R})$$

$$[T_{\mathbf{R}} + \tilde{V}(\mathbf{R}) - E_0]\,\tilde{\chi}_{\mathbf{K}}^{AD} = 0$$

$$\tilde{V}(\mathbf{R}) = \frac{\left\langle \phi_0 \left| V_{np} U(\mathbf{r},\mathbf{R}) \right| \phi_0 \right\rangle}{\left\langle \phi_0 \left| V_{np} \right| \phi_0 \right\rangle} \approx U(\mathbf{r}=0,\mathbf{R})$$

$$\tilde{V}(\mathbf{R}) \approx V_n(R) + V_p(R)$$

So, for a deuteron for example …fold



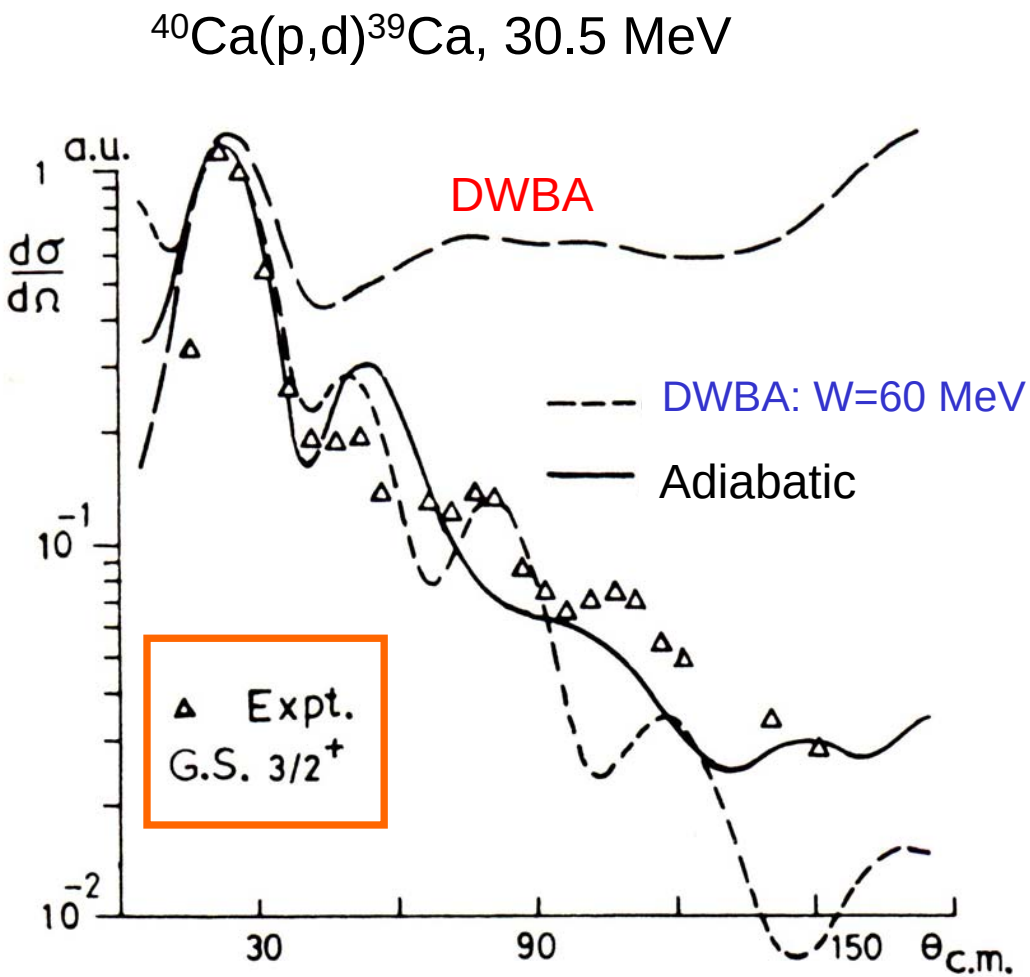
$$V_d(R) = \langle \phi_d | V_{nt}(\vec{r}_{nt}) + V_{pt}(\vec{r}_{pt}) | \phi_d \rangle$$

$$J_{V_d} = J_{V_{nt}} + J_{V_{pt}} = 2J_{V_N}$$

$$\langle r^2 \rangle_{V_F} = \langle r^2 \rangle_{V_N} + \frac{1}{4}\langle r^2 \rangle_{\phi_d}$$

$$J_f = \int d\vec{r}\, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r}\, r^2 f(r)/J_f$$

Key features for transfer reactions - spectroscopy



Increased reflection at nuclear surface - less diffuse 'deuteron' channel potential

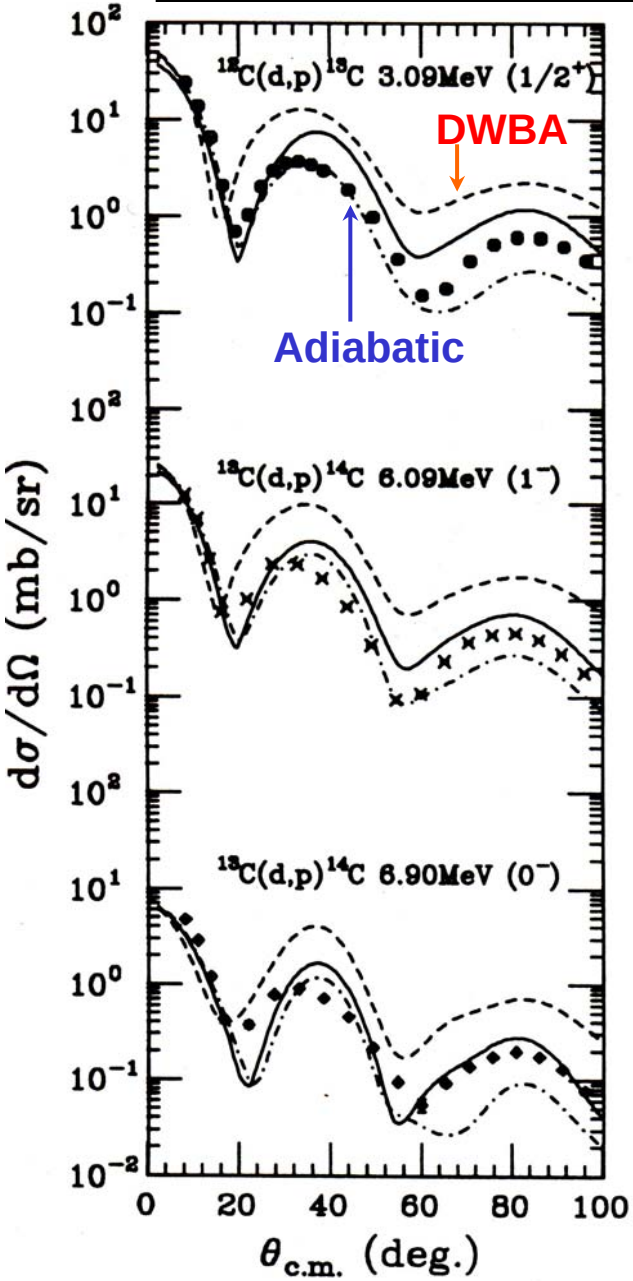
Greater surface localisation - L-space localisation

Less nuclear volume contribution and less sensitivity to optical model parameters

More consistent sets of deduced spectroscopic factors

J.D. Harvey and R.C. Johnson, Phys. Rev.C **3** (1971) 636

Spectroscopic factors in the adiabatic limit



Excitation energy (MeV)	J^π	Transferred nl_j	Spectroscopic factor DWBA	ADBA	CCBA	Shell Model
0.00	$1/2^-$	$0p_{1/2}$	1.0	0.7	0.8	0.61
3.09	$1/2^+$	$1s_{1/2}$	1.8	0.8	0.9	—
3.68	$3/2^-$	$0p_{3/2}$	0.14	0.14	0.14	0.19
3.85	$5/2^+$	$0d_{5/2}$	0.7	0.6	0.6	—

Table 16. Spectroscopic factors obtained from the $^{12}C(d,p)^{13}C$ reactions and the shell model calculations.

Way to systematically improve the adiabatic approximation to transfer reactions (Weinberg states)

R.C. Johnson and P.C. Tandy, Nucl. Phys. A 235 (1974) 56

implemented for practical calculations

A. Laid, J.A. Tostevin and R.C. Johnson, Phys. Rev. C **48** (1993), 1307

H. Toyokawa, PhD Thesis, RCNP, Osaka University 1995

Eikonal approximation: point particles (1)

Approximate (semi-classical) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0, \qquad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2}U(r) + k^2\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0$$

valid when $|U|/E \ll 1, \quad ka \gg 1$ → small wavelength → high energy

Key steps are: (1) the distorted wave function is written

$$\chi_{\vec{k}}^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r})$$

all effects due to *U(r)*, modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

Eikonal approximation: point neutral particles (2)

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

The conditions $|U|/E \ll 1, \quad ka \gg 1$ → imply that

$$2\vec{k}\cdot\nabla\omega(\vec{r}) \gg \nabla^2\omega(\vec{r})$$

Slow spatial variation cf. k

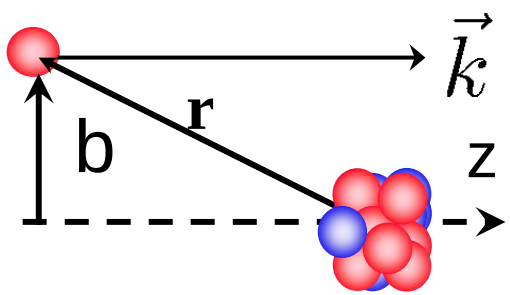
and choosing the z-axis in the beam direction $\vec{k}$

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r})$$

with solution

phase that develops with z

$$\omega(\vec{r}) = \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point neutral particles (3)

$$\chi^+_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \omega(\vec{r}) \approx \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$

So, after the interaction and as $z\to\infty$

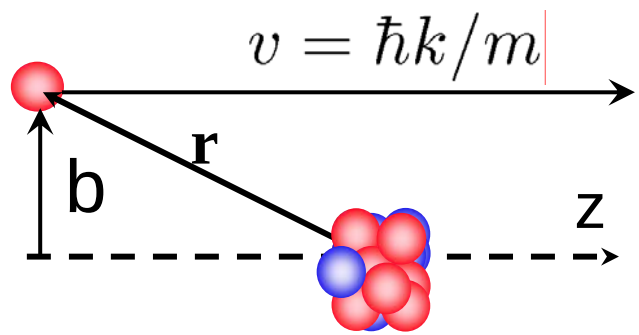
$$\chi^+_{\vec{k}}(\vec{r}) \to \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{\infty} U(r)dz'\right] = S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\chi^+_{\vec{k}}(\vec{r}) \to S(b)\exp(i\vec{k}\cdot\vec{r})$$

Eikonal approximation to the S-matrix S(b)

S(b) is amplitude of the forward going outgoing waves from the scattering at impact parameter b

$$S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]$$

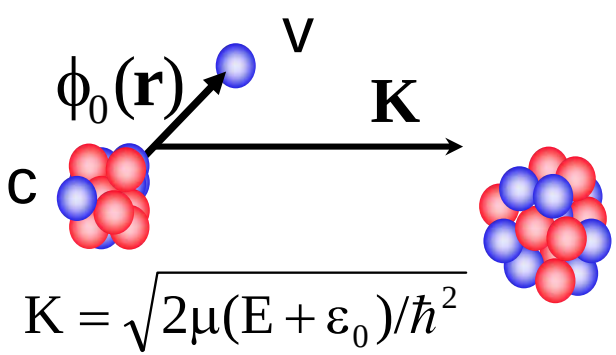


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal solution of the few-body model

Practical application of adiabatic approximation: $H_p \rightarrow -\varepsilon_0$

$$H = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) + H_p \quad \rightarrow \quad H^{AD} = T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - \varepsilon_0$$



substituting the eikonal form solution

$$\Psi^{AD}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) = \underbrace{e^{i\mathbf{K}\cdot\mathbf{R}}\phi_0(\mathbf{r})}_{\text{incident wave}}\ \underbrace{\omega(\mathbf{r},\mathbf{R})}_{\text{modulating function}}$$

$$[T_{\mathbf{R}} + U(\mathbf{r},\mathbf{R}) - (E+\varepsilon_0)]\,\Psi^{AD}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) = 0$$

and neglecting the curvature term $\nabla^2_{\mathrm{R}}\omega(\mathbf{r},\mathbf{R}) << 2\nabla_{\mathrm{R}}\omega\cdot\mathbf{K}$

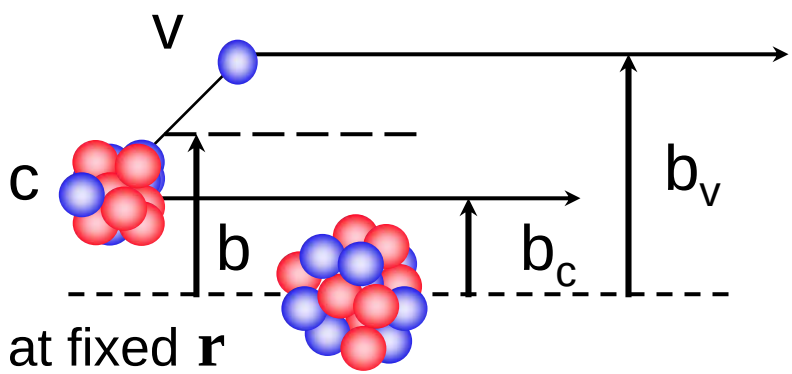
$$\omega(\mathbf{r},\mathbf{R}) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{Z} dZ'\, U(\mathbf{r},\mathbf{R}')\right\} \quad \rightarrow \quad V_{cT} + V_{vT}$$

Few-body eikonal model amplitudes

So, after the collision, as $Z \to \infty$ $\omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\, S_v(b_v)$

$$\Psi^{\text{Eik}}_{\mathbf{K}}(\mathbf{r},\mathbf{R}) \to e^{i\mathbf{K}\cdot\mathbf{R}}\, S_c(b_c)\, S_v(b_v)\, \phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



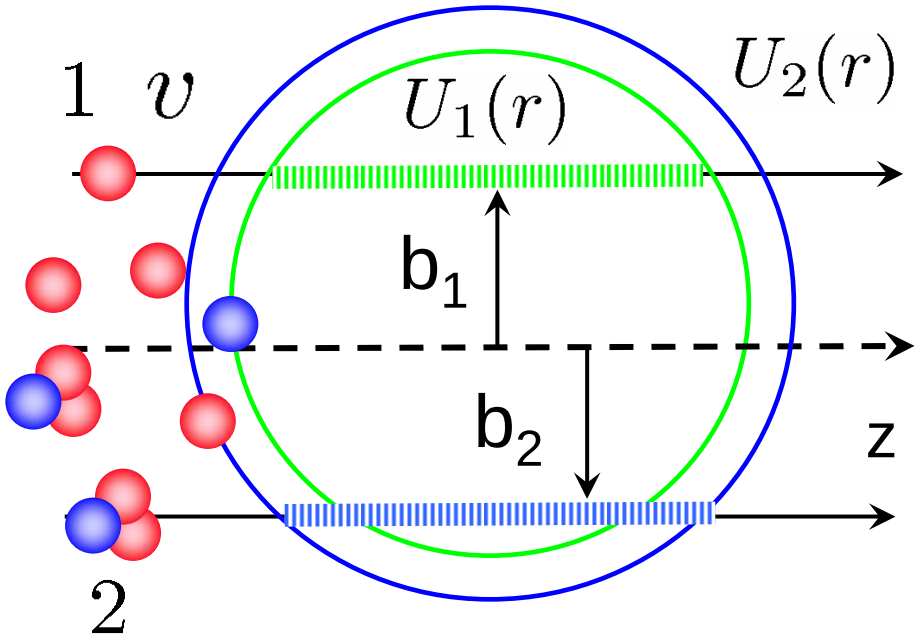
So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)} \,|\, \phi_0 \rangle_{\mathbf{r}}$$

amplitude that c,v survive interaction with b_c and b_v

averaged over position probabilities of c and v

Eikonal approximation: several particles (preview)



$$\chi_i(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_i(r)dz$$

Total interaction energy

$$U(r_1,\ldots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp\left[i\chi_i(b_i)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_i(r_i)dz'\right]$$

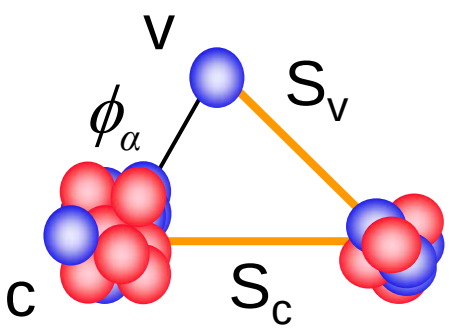
$$\chi(b_1,\ldots) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}\sum_i U_i(r_i)dz$$

with composite objects we will get products of the S-matrices

$$\exp[i\chi(b_1,\ldots)] = \prod_i S_i(b_i)$$

Eikonal theory - dynamics and structure

Independent scattering information of c and v from target



$$S_{\alpha\beta}(b) = \langle \phi_\beta \mid \overbrace{S_c(b_c)\, S_v(b_v)}^{\text{dynamics}} \mid \phi_\alpha \rangle$$

$\longleftrightarrow$ structure

Use the best available few- or many-body wave functions

More generally,

$$S_{\alpha\beta}(b) = \langle \varphi_\beta \mid S_1(b_1)\, S_2(b_2) \ldots\ldots\ S_n(b_n) \mid \varphi_\alpha \rangle$$

for any choice of 1,2 ,3, ….. n clusters for which a most realistic wave function φ is available