TRIUMF Summer Institute

Nuclear Reactions: applications and examples

(http://departamento.us.es/famn/tsi08/)

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My plan for the talks...

- Lecture 1: Elastic scattering
  - Phenomenology: Rutherford, Fresnel and Fraunhofer scattering
  - The optical model
- Lecture 2: Inelastic scattering: coupled-channels (CC) method
  - Collective model
  - Cluster model
- Lecture 3: Transfer reactions: DWBA method
- Lecture 4:: Reaction involving continuum
  - Breakup: CDCC method
  - Spectroscopy to unbound states: the *transfer to the continuum* method

Lecture 1: Elastic scattering: optical model calculations

Direct versus compound reactions

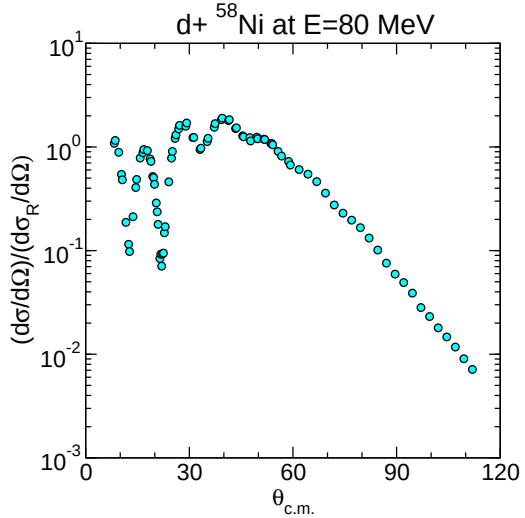
DIRECT: elastic, inelastic, transfer,. . .

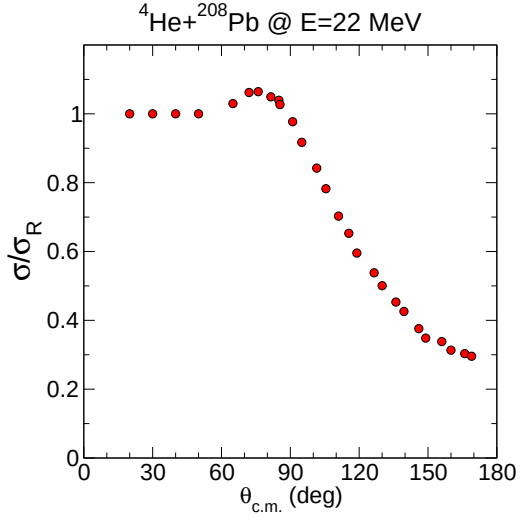
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

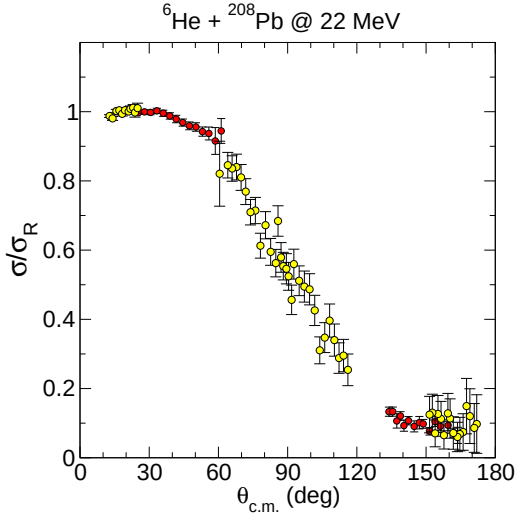
COMPOUND: complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- ”lose of memory” $\Rightarrow$ almost symmetric distributions forward/backward

Elastic scattering

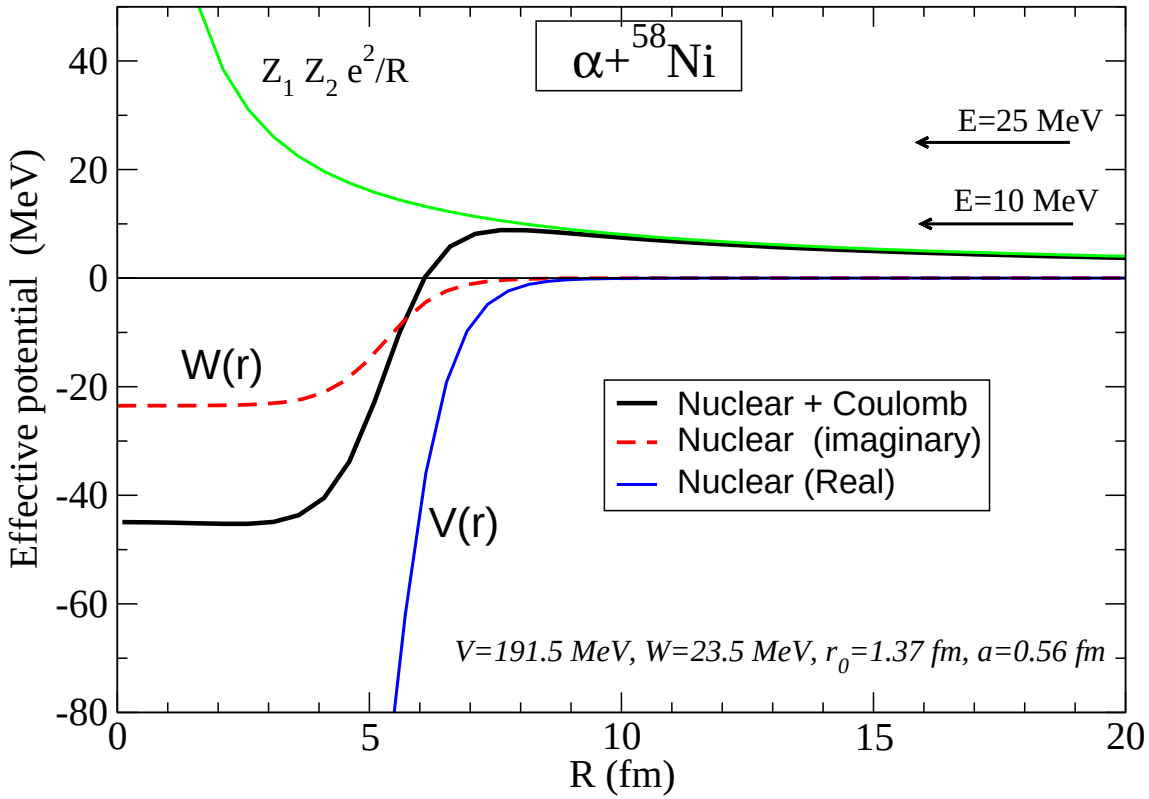






What can we learn from an optical model analysis of the elastic cross section?

EFFECTIVE PROJECTILE-TARGET INTERACTION:



Elastic scattering: phenomenology

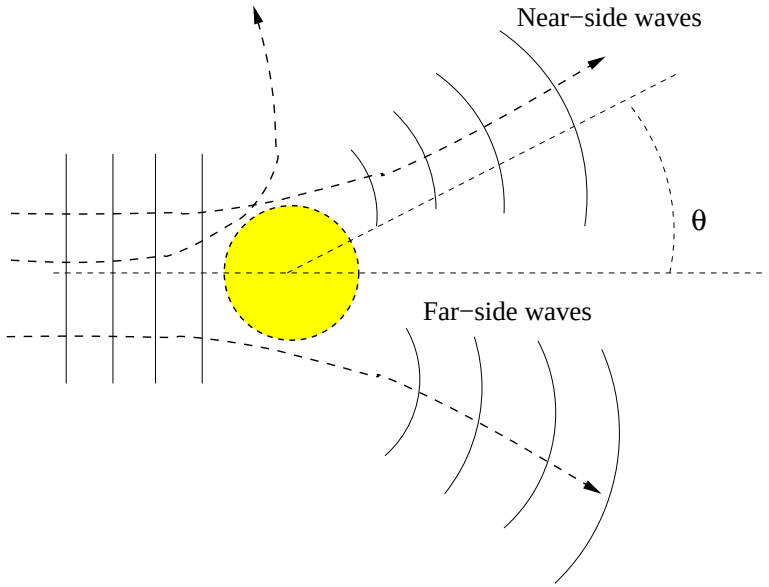
☞ Depending on the bombarding energy E and the charges of the interacting nuclei, we observe different types of elastic scattering.

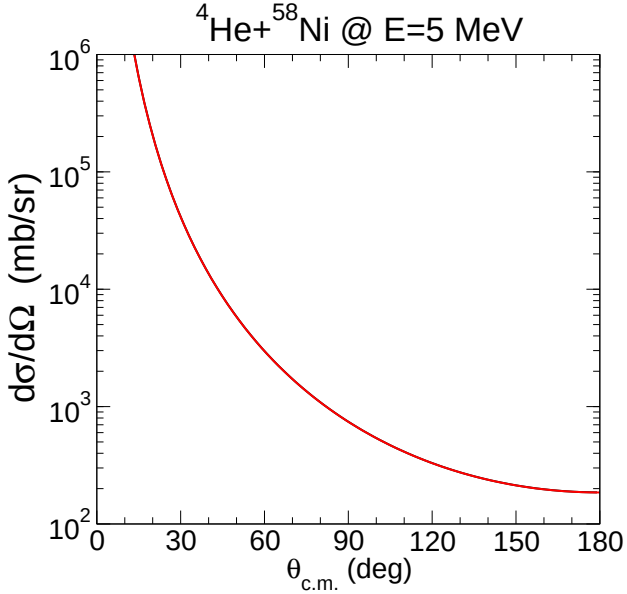
☞ This can be characterized in terms of the Coulomb (or Sommerfeld) parameter:

$$\eta = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar v}$$

- E well above the Coulomb barrier ($\eta \lesssim 1$) $\Rightarrow$ Fraunhofer scattering
- E around the Coulomb barrier ($\eta \gg 1$) $\Rightarrow$ Fresnel scattering
- E well below the Coulomb barrier ($\eta \ggg 1$) $\Rightarrow$ Rutherford scattering

RUTHERFORD SCATTERING



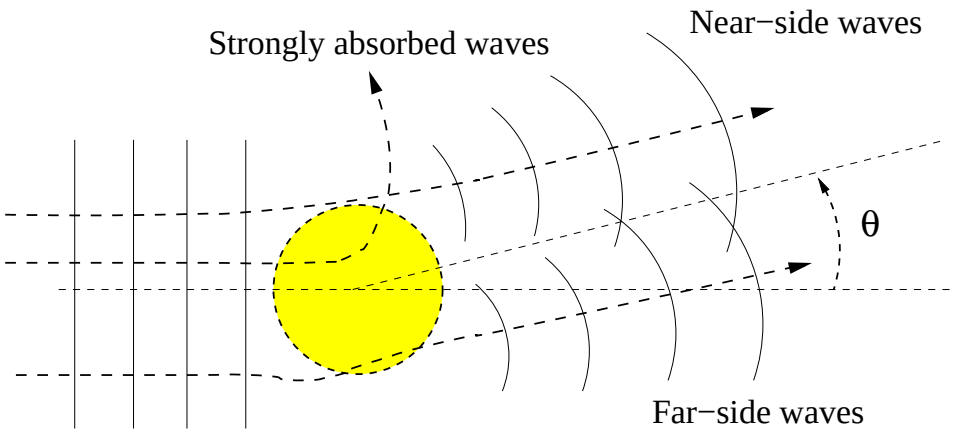


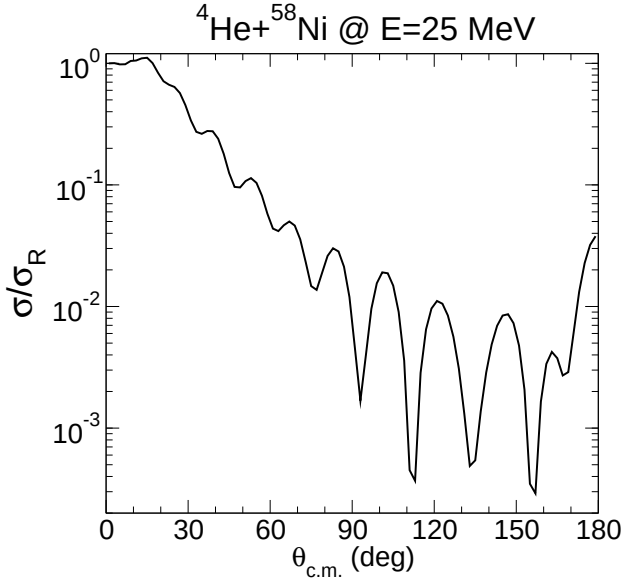
- Purely Coulomb potential ($\eta \ggg 1$)
- Bombarding energy well below the Coulomb barrier
- Obeys Rutherford law:

$$\frac{d\sigma}{d\Omega} = \frac{zZe^2}{4E} \frac{1}{\sin^4(\theta/2)}$$

.

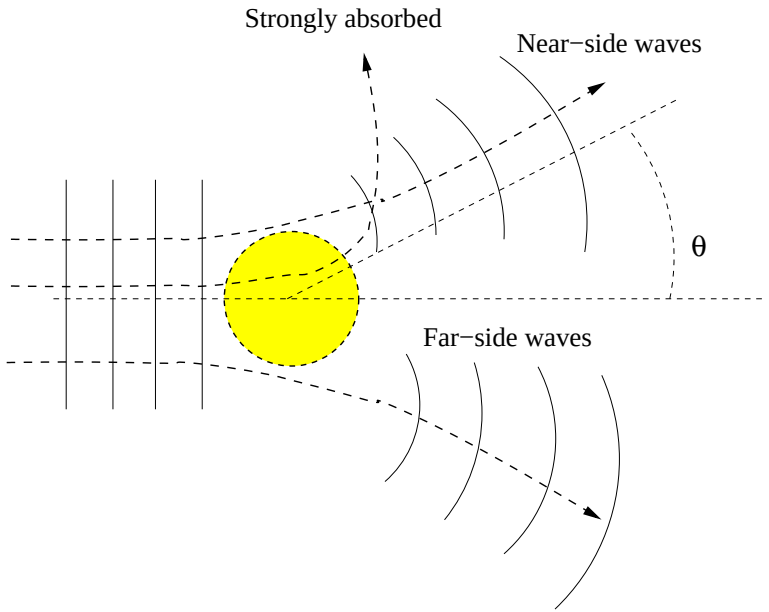
FRAUNHOFER SCATTERING:

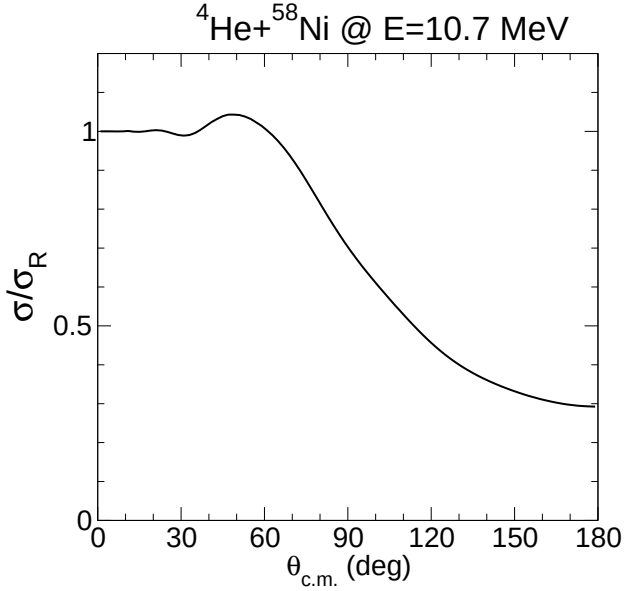




- Bombarding energy well above Coulomb barrier
- Coulomb weak ($\eta \lesssim 1$)
- Nearside/farside interference pattern (difracction)

FRESNEL SCATTERING:





- Bombarding energy around or near the Coulomb barrier
- Coulomb strong ($\eta \gg 1$)
- 'Illuminated' region $\Rightarrow$ interference pattern (near-side/far-side)

  'Shadow' region $\Rightarrow$ strong absorption

How does one describe the motion of a particle in quantum mechanics?

- Hamiltonian: $H = T_R + U(R)$

  $U(R)$: optical model $\Rightarrow$ effective projectile-target interaction

- Schrodinger equation: $[H - E]\Psi(\mathbf{R}) = 0$

- Partial wave expansion of the model wavefunction:

$$\Psi(\mathbf{R}) = \sum_{LM} C^{LM} \frac{f^L(R)}{R} Y_{LM}(\hat{R})$$

- $f^L(R)$ obtained as solution of:

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + U(R) - E \right] f^L(R) = 0$$

Numerical procedure:

1. Fix a *matching radius*, R_m, such that $V_{\rm nuc}(R_m) \ll$
2. Integrate $f(R)$ from $R = 0$ up to R_m, starting with the condition:

$$\lim_{R\to 0} f^L(R) = 0$$

3. At $R = R_m$ impose the boundary condition:

$$f^L(R) \to I_L(R) - S_L O_L(R)$$

☞ S_L=*scattering matrix*

☞ I_L *and* O_L *are the so called incoming and outgoing waves:*

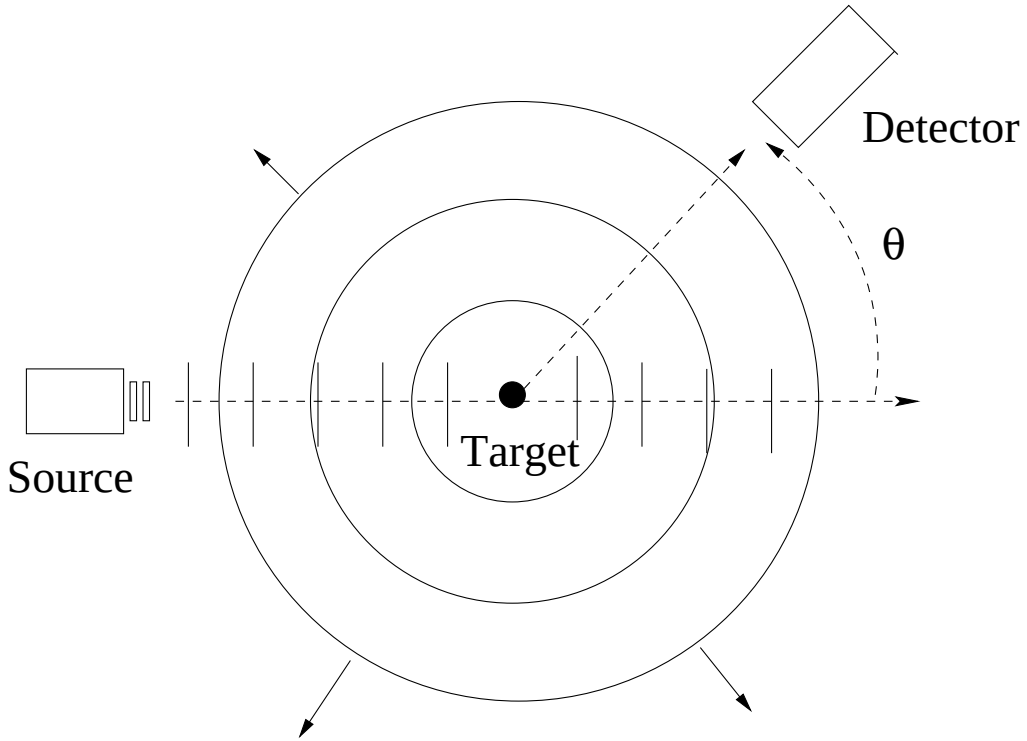
$$\begin{aligned} I_L(R) &= \frac{1}{\sqrt{v}}(KR)\, h_L^*(KR) \propto e^{-i(KR-\eta\log 2KR)} \\ O_L(R) &= \frac{1}{\sqrt{v}}(KR)\, h_L(KR) \propto e^{i(KR-\eta\log 2KR)} \end{aligned}$$

The S-matrix

- S_L =coefficient of the outgoing wave for partial wave L.
- Phase-shifts: $S_L = e^{2i\delta_L}$
- $U(R) = 0 \Rightarrow$ No scattering $\Rightarrow S_L = 1 \Rightarrow \delta_L = 0$
- U real $\Rightarrow |S_L| = 1 \Rightarrow \delta_L$ real

  U complex $\Rightarrow |S_L| < 1 \Rightarrow \delta_L$ complex
- For $L \gg \Rightarrow S_L \to 1$

Which one of the many solutions of Schrödinger equation is the one that correspond to a scattering experiment?



$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i\cdot\mathbf{R}} + \chi^{(+)}_{\mathbf{K}_i}(\mathbf{R})$$

- Scattering amplitude: $A(\theta)$

$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i\cdot\mathbf{R}} + \chi^{(+)}_{\mathbf{K}_i}(\mathbf{R}) \to e^{i\mathbf{K}_i\cdot\mathbf{R}} + A(\theta)\frac{e^{iK_iR}}{R}$$

- Partial wave decomposition:

$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = \frac{1}{R}\sum_{LM} C^{LM} f^L(R) Y_{LM}(\hat{R}) \to \frac{1}{R}\sum_{LM} C^{LM}\left[I_L(R) - S_L O_L(R)\right] Y_{LM}(\hat{R})$$

- Incident plane wave:

$$\begin{aligned} e^{i\mathbf{K}_i\cdot\mathbf{R}} &= \sum_{LM} 4\pi Y^*_{LM}(\hat{K}_i) i^L Y_{LM}(\hat{R}) j_L(K\ R) \\ &= \sum_{LM} \frac{2\pi i\sqrt{v}}{KR} Y^*_{LM}(\hat{K}_i) i^L Y_{LM}(\hat{R})\left[I_L(R) - O_L(R)\right] \end{aligned}$$

- Outgoing spherical waves:

$$\chi^{(+)}_{\mathbf{K}_i}(\mathbf{R}) \to \sum_{LM} \frac{2\pi i\sqrt{v}}{KR} Y^*_{LM}(\hat{K}_i)(1 - S^L) Y_{LM}(\hat{R}) O_L(R)$$

Scattering amplitude and cross sections

- Scattering amplitude:
  - Nuclear potential alone:

$$A(\theta) = \frac{i}{2K} \sum_L (2L+1) P_L(\cos\theta)(1 - S^L)$$

  - Nuclear+Coulomb: $A(\theta) = A_C(\theta) + A'(\theta)$

$$A_C(\theta) = \frac{i}{2K} \sum_L (2L+1)(1 - e^{2i\sigma_L}) P_L(\cos\theta)$$

$$A'(\theta) = \frac{i}{2K} \sum_L (2L+1) e^{2i\sigma_L}(1 - S^L) P_L(\cos\theta)$$

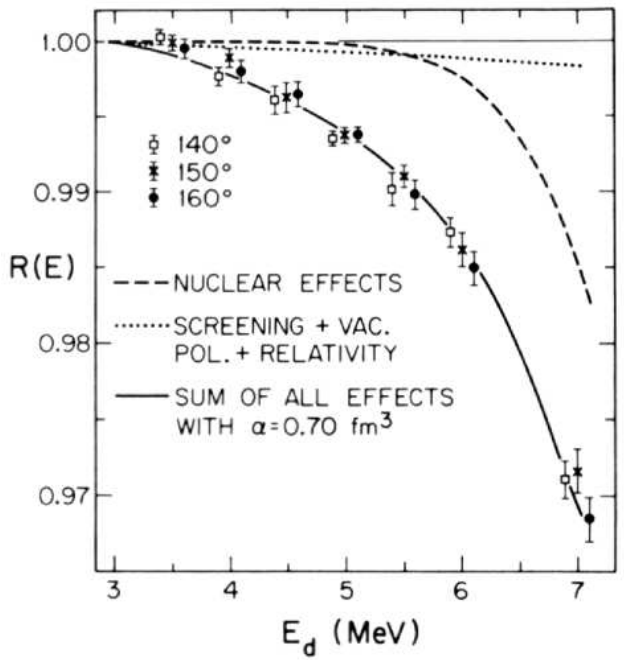
- Differential cross section:

$$\frac{d\sigma}{d\Omega} = |A(\theta)|^2$$

Extracting structure information from elastic scattering measurements

Eg: deuteron polarizability from d+^{208}Pb:

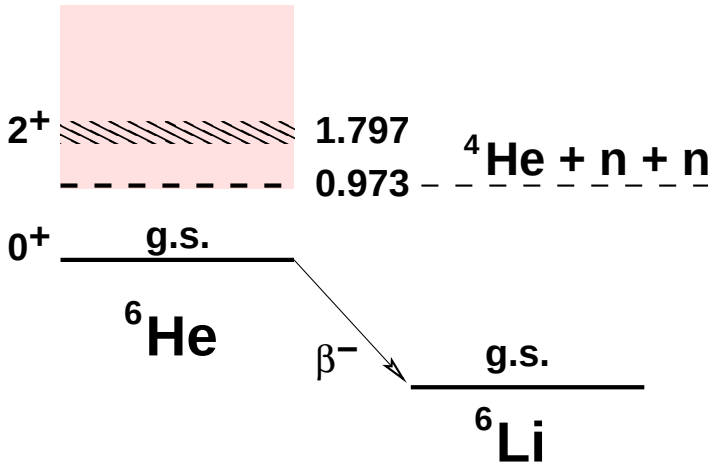
- Deuteron polarizability: $\mathbf{P} = \alpha \mathbf{E}$
- For $E < V_b$, the main deviation from Rutherford scattering comes from dipole polarizability.
- In the adiabatic limit ($E_x \gg$): $V_{\rm dip} = -\alpha \frac{Z_1 Z_2 e^2}{2R^4}$

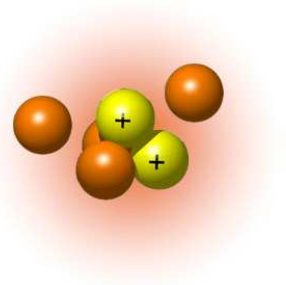


Rodning, Knutson, Lynch and Tsang, PRL49, 909 (1982)
$\alpha = 0.70 \pm 0.05$ fm^3

Halo and Borromean nuclei: the ^{6}He case

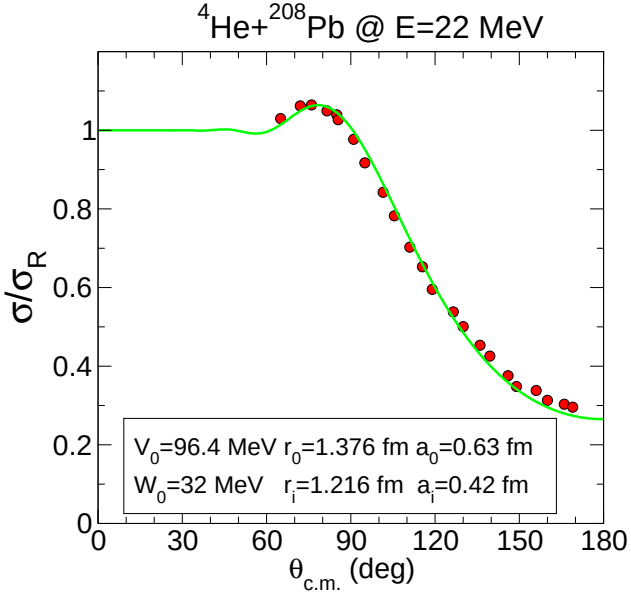
- Radioactive:
  $^6\text{He} \xrightarrow{\beta^-} {}^6\text{Li}$ ($t_{1/2} \simeq$807 ms)
- Weakly bound:
  $\epsilon_b = -0.973$ MeV
- Neutron halo
- Borromean system:
  n-n and α-n unbound
- $\sim$ 3 body system:
  α almost inert

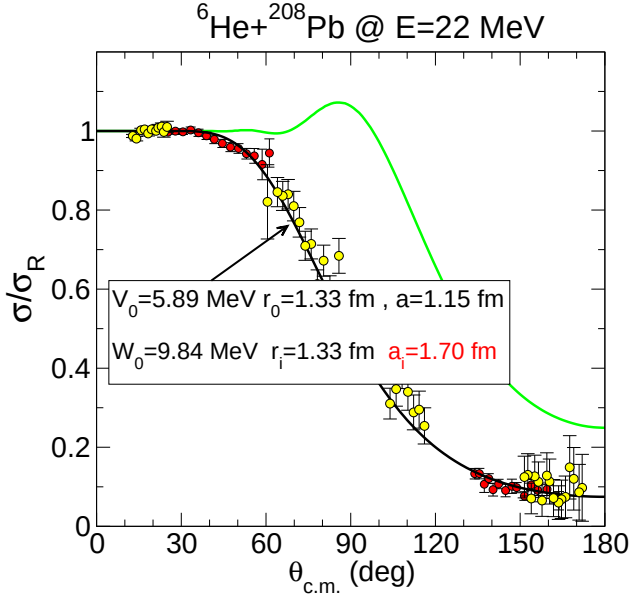




Normal versus halo nuclei

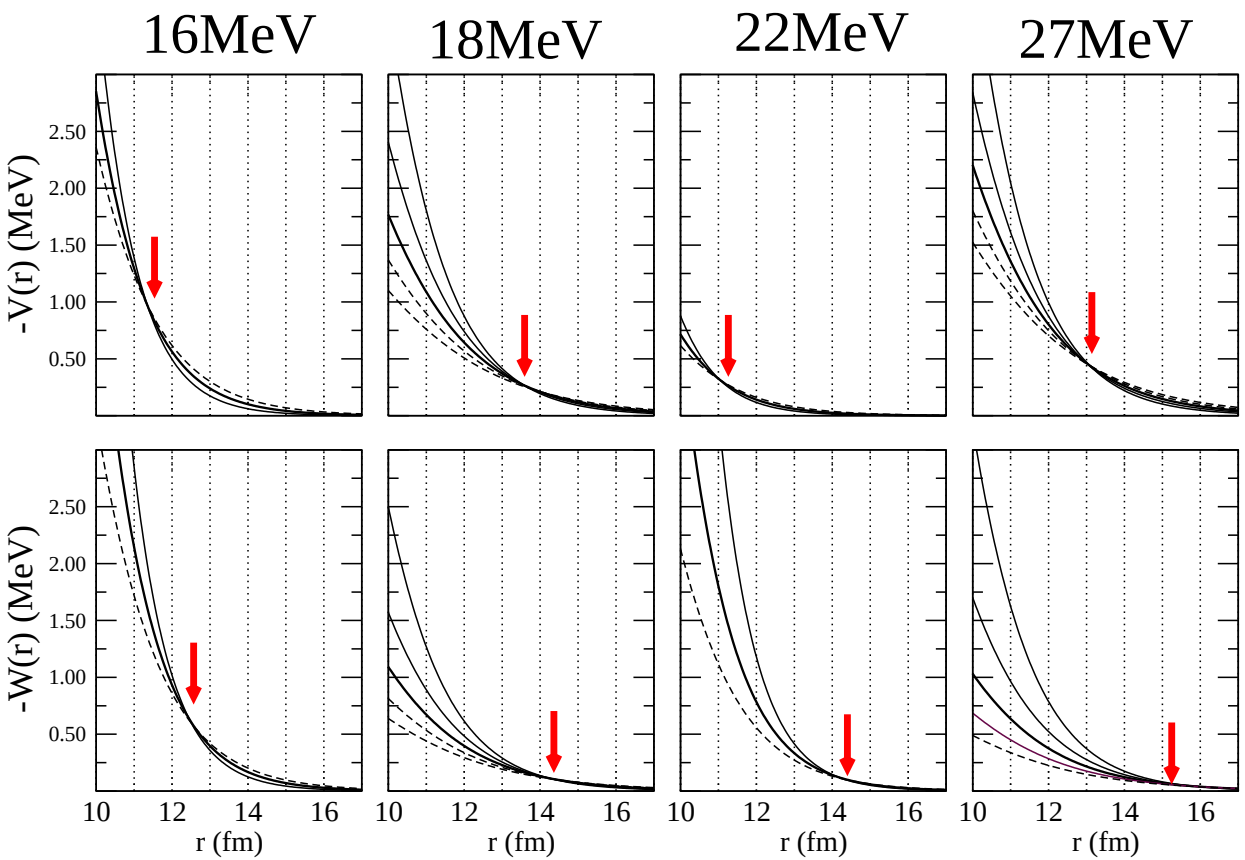
How does the halo structure affect the elastic scatterig?





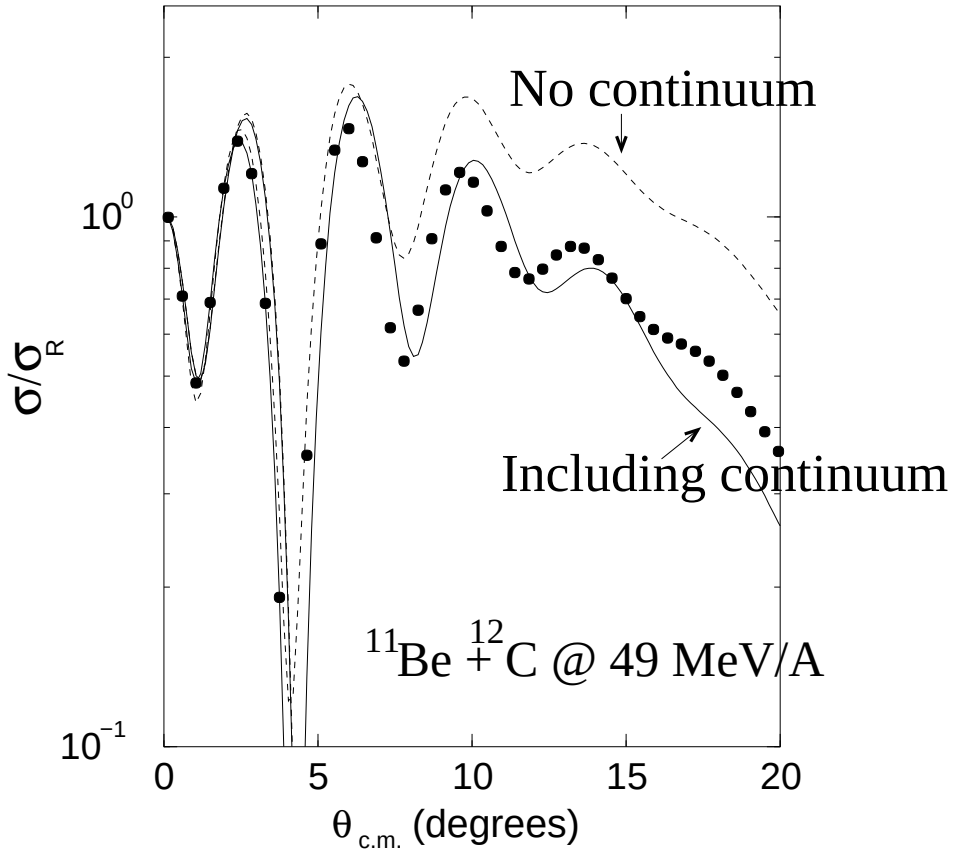
- ^{4}He+^{208}Pb shows typical Fresnel pattern → *strong absorption*
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- ^{6}He+^{208}Pb requires a large imaginary diffuseness → *long-range absorption*

RADIUS OF SENSITIVITY OF V(R) AND W(R)



Imaginary part $\Rightarrow$ long range compared to strong absorption radius

Normal versus halo nuclei: Fraunhofer



☞ In Fraunhofer scattering the presence of the continuum produces a reduction of the elastic cross section

- What is FRESCO?

  Program developed by Ian Thompson since 1983, to perform coupled-reaction channels calculations in nuclear physics.

- Some general features:
  - Multi-platform (Windows, Linux, Unix, VAX)
  - Treats many direct reaction models: elastic scattering (optical model), transfer, inelastic excitation to bound and unbound states, etc
  - Can be run in text mode and graphical mode (XFRESCO interface)
  - FRESCO and XFRESCO can be freely downloaded at `http://www.fresco.org.uk/`
  - SFRESCO: Extension of Fresco, to provide χ^2 searches of potential and coupling parameters.

Essential ingredients of an OM calculation:

- **Physical:**
  - Identify projectile and target (mass, spin, etc)
  - Incident energy
  - Parametrization of the optical potential
- **Numerical:**
  - Radial step for numerical integration (HCM in fresco)
  - Maximum radius R for integration (RMATCH)
  - Maximum angular momentum L. (JTMAX)

RMATCH and JTMAX are linked by: $kR_g\,(1 - 2\eta/kR_g) \approx L_g + 1/2$
(L_g=grazing angular momentum)

Effective potential: $U(R) = U_{\rm nuc}(R) + U_{\rm coul}(R)$

- Coulomb potential: charge sphere distribution

$$U_c(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c}\left(3 - \frac{R^2}{R_c^2}\right) & \text{if } R \le R_c \\ \frac{Z_1 Z_2 e^2}{R} & \text{if } R \ge R_c \end{cases}$$

- Nuclear potential (complex): Woods-Saxon parametrization

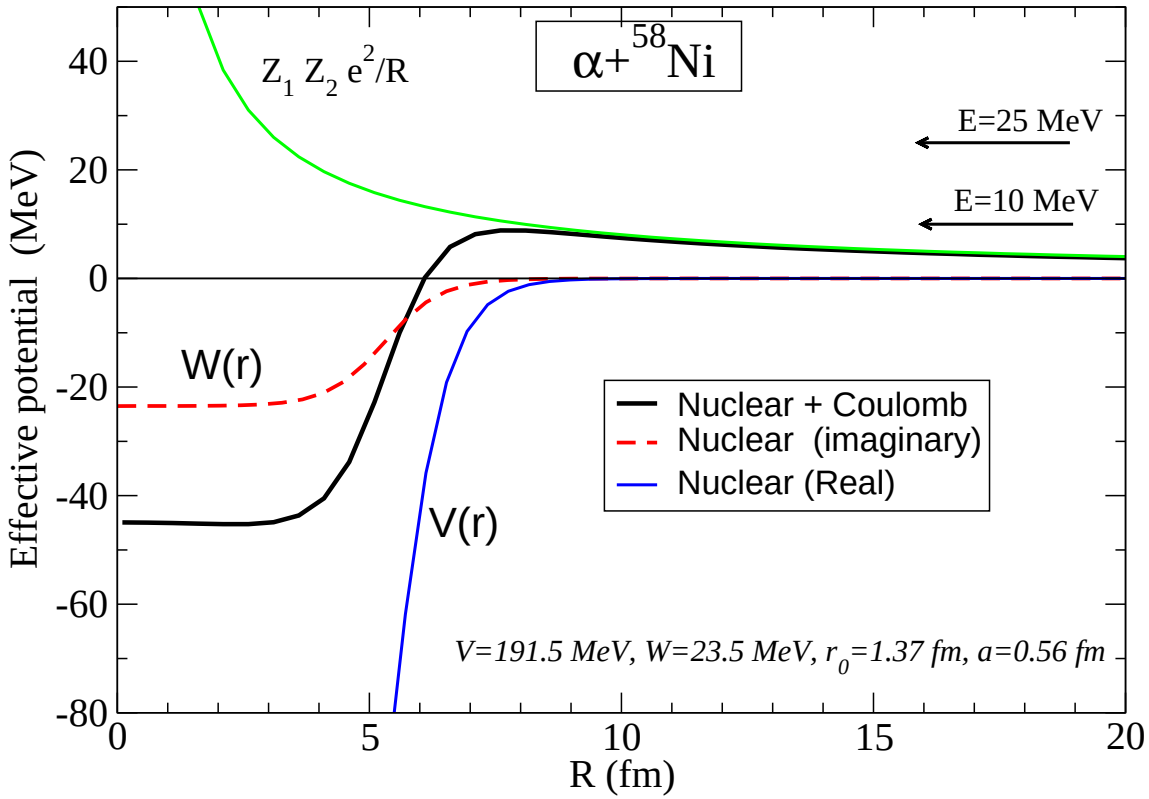
$$U_{\rm nuc}(R) = V(r) + iW(r) = -\frac{V_0}{1+\exp\left(\frac{R-R_0}{a_0}\right)} - i\,\frac{W_0}{1+\exp\left(\frac{R-R_i}{a_i}\right)}$$

Typically: $R_0 = r_0(A_p^{1/3} + A_t^{1/3})$

- r_0=reduced radius ($r_0 \sim 1.1 - 1.4$ fm)
- A_p,A_t: projectile, target masses (amu)

Elastic scattering: effective potential

Effective potential: $U(R) = U_{nuc}(R) + U_{coul}(R)$



OM example: $^{4}He+^{58}Ni$

Input example: `4he58ni_e10.in`

```
4he58ni_e10.in: 4He + 58Ni elastic scattering Ecm=10.0 MeV
NAMELIST
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
      thmin=1.0 thmax=180.0 thinc=2.0
      smats=2 xstabl=1
      elab=10.7 /

&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28 nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /

&POT kp=1 at=58 rc=1.4 /
&POT kp=1 type=1
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /

&overlap /
&coupling /
```

Elastic scattering example

General variables

```
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
      thmin=1.00 thmax=180.00 thinc=2.00
      smats=2 xstabl=1
      elab=10.7 /
```

Mass partitions & states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28
        nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /
```

Potentials

```
&POT kp=1 itt=F at=58 rc=1.4 /
&POT kp=1 type=1
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /
```

Essential input variables: FRESCO namelist

```
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
      thmin=1.00 thmax=180.00 thinc=2.00
      smats=2 xstabl=1
      elab=10.7 /
```

- `hcm`: step for integration of radial equations.
- `rmatch`: matching radius (for `R > RMATCH` asymptotic behaviour is assumed)
- `elab`: laboratory energy
- `jtmax`: maximum total angular momentum (projectile+target+relative)
- `smats`: trace variable
  `smats=2` → print elastic S-matrix
- `xstbl`: trace variable
  `xstbl=1` → print cross sections

Essential input variables: partitions and states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28
         nex=1 /
```

- `namep` / `namet`: projectile / target name
- `massp` / `masst`: projectile / target mass (amu)
- `zp` / `zt`: projectile / target charge
- `nex`: number of (pairs) of states in this partition

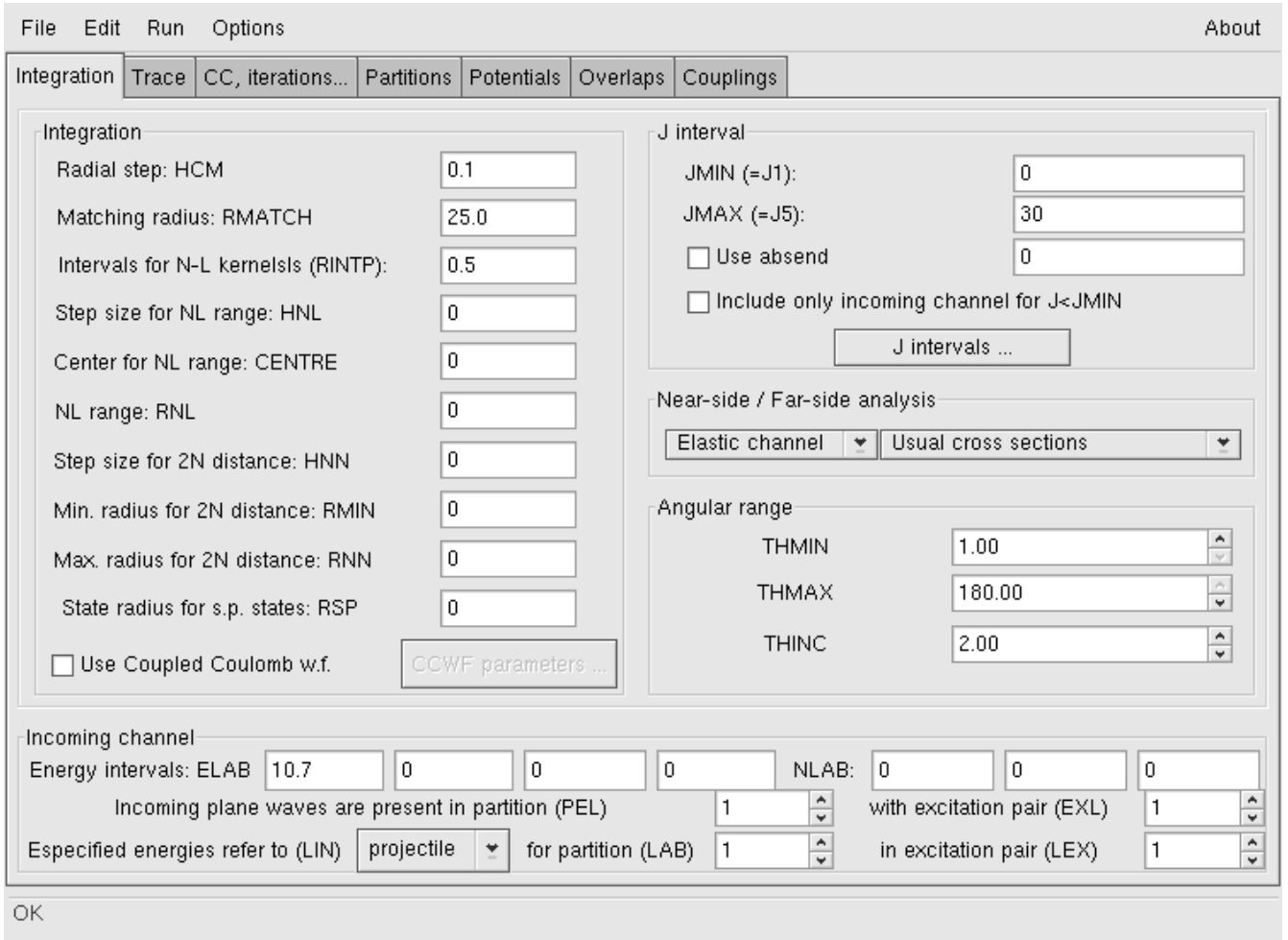
```
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- `jp` / `jt`: projectile / target spins
- `bandp` / `bandt`: projectile / target parities ($\pm$ 1)
- `cpot`: index of potential for this pair of states.

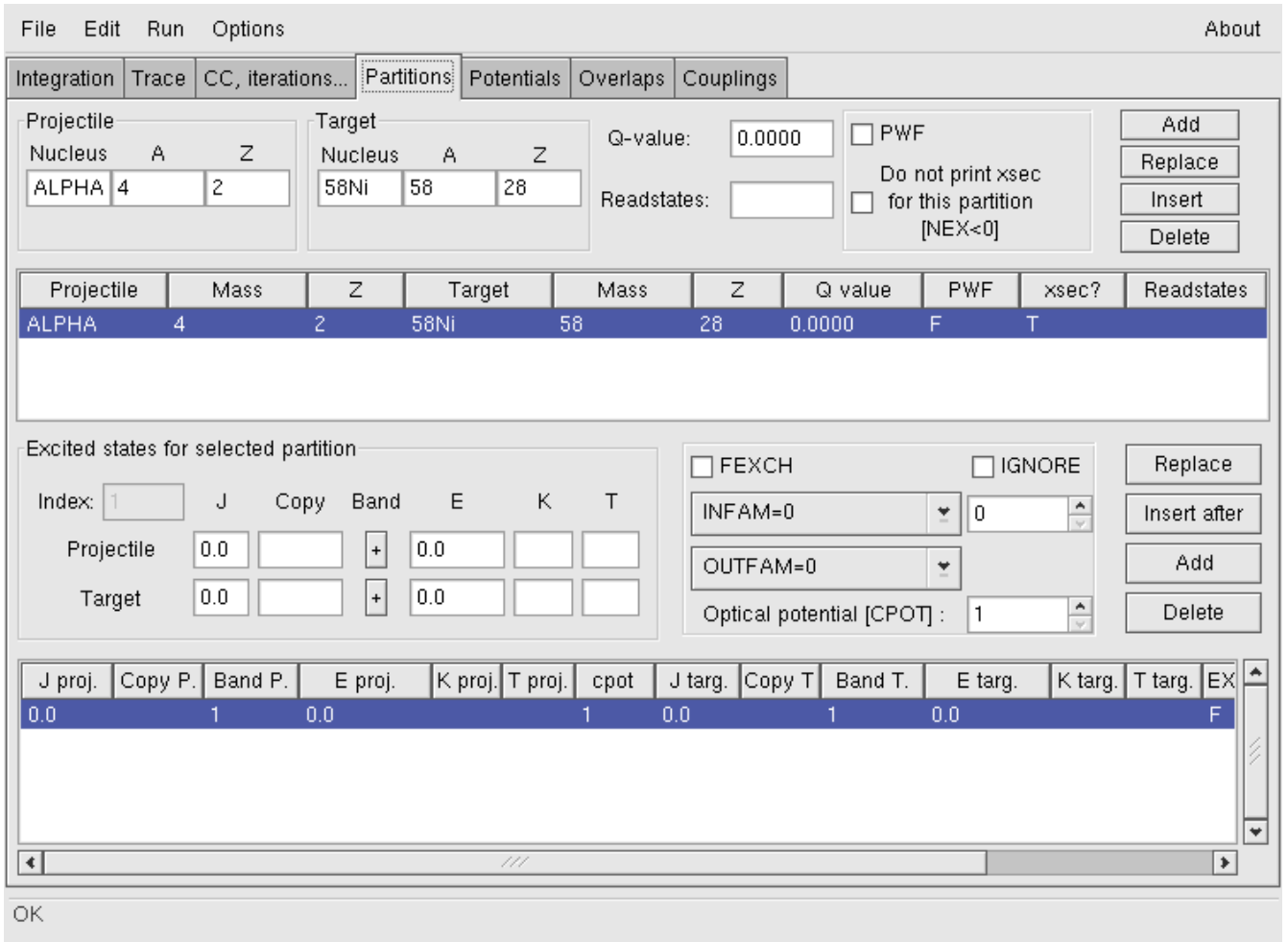
```
&POT kp=1 type=0 ap=0 at=58 rc=1.4 /
&POT kp=1 type=1 shape=0
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /
```

- `kp`: index to identify this potential
- `ap, at`: projectile and target mass, for conversion from reduced to physical radii: $R = r(\mathrm{ap}^{1/3} + \mathrm{at}^{1/3})$
- `type, shape`: potential cathegory and shape: $\Rightarrow$
  - `type=0`: Coulomb potential
    `shape=0`: uniform charge sphere
  - `type=1`: volume nuclear potential
    `shape=0`: Woods-Saxon shape
- `rc`: reduced radius for charge distribution
- `p1,p2,p3`: V_0, r_0, a_0 (real part)
- `p4,p5,p6`: W_0, r_i, a_i (imaginary part)

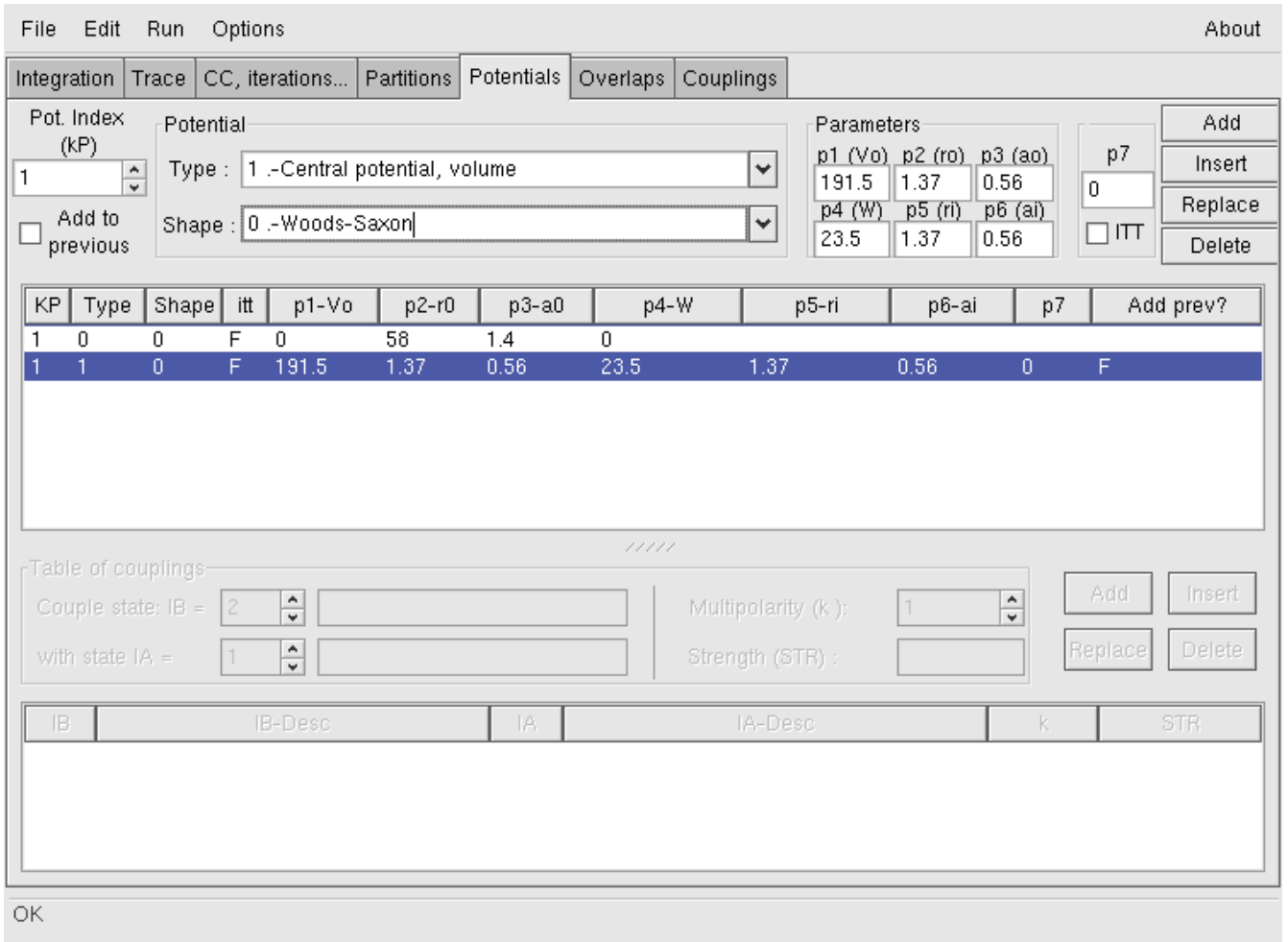
General variables:



Partitions & states:



Potentials:


File
Edit
Run
Options
About
Integration
Trace
CC, iterations...
Partitions
Potentials
Overlaps
Couplings
Pot. Index (kP)
1
Add to previous
Potential
Type : 1 .-Central potential, volume
Shape : 0 .-Woods-Saxon
Parameters
p1 (Vo) p2 (ro) p3 (ao)
191.5 1.37 0.56
p4 (W) p5 (ri) p6 (ai)
23.5 1.37 0.56
p7
0
ITT
Add
Insert
Replace
Delete
KP Type Shape itt p1-Vo p2-r0 p3-a0 p4-W p5-ri p6-ai p7 Add prev?
1 0 0 F 0 58 1.4 0
1 1 0 F 191.5 1.37 0.56 23.5 1.37 0.56 0 F
Table of couplings
Couple state: IB = 2
with state IA = 1
Multipolarity (k): 1
Strength (STR) :
Add
Insert
Replace
Delete
IB IB-Desc IA IA-Desc k STR
OK

Useful output files:

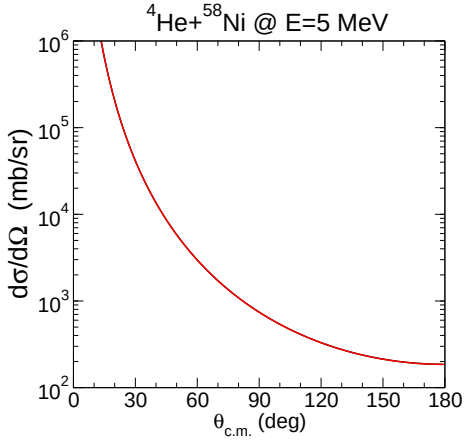
- Main output file (stdout)
- `fort.201` : Elastic scattering angular distribution
  - `thmax > 0`: relative to Rutherford.
  - `thmax < 0`: absolute units (mb/sr).
- `fort.7`: Elastic S-matrix (real part, imaginary part, angular momentum)
- `fort.56`: Fusion (absorption), reaction and inelastic cross section for each angular momentum

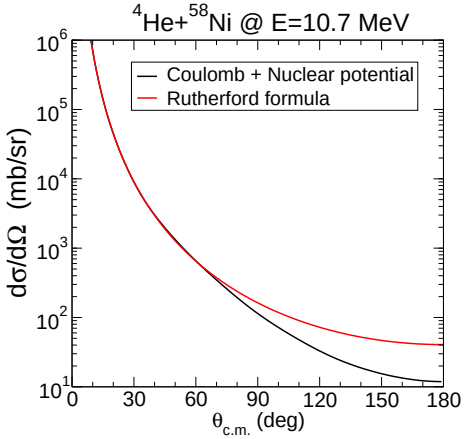
Dynamical effects: ^{4}He+^{58}Ni at E=5, 10.7, 25 and 50 MeV

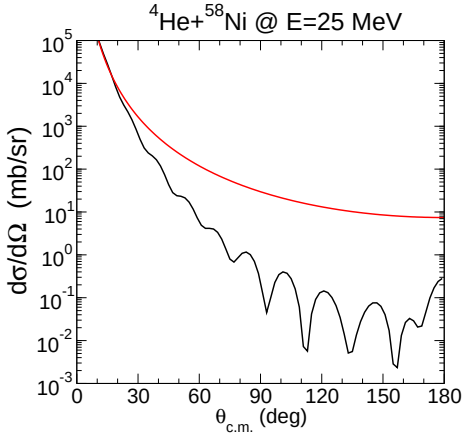
$E_{\rm lab}$ (MeV)	η	k (fm^{-1})	$\bar{\lambda} = 1/k$ (fm)	$2a_0$ (fm)
5	7.95	0.920	1.087	17.2
10.7	5.62	1.34	0.746	8.06
25	3.55	2.06	0.485	3.44
50	2.51	2.91	0.343	1.69

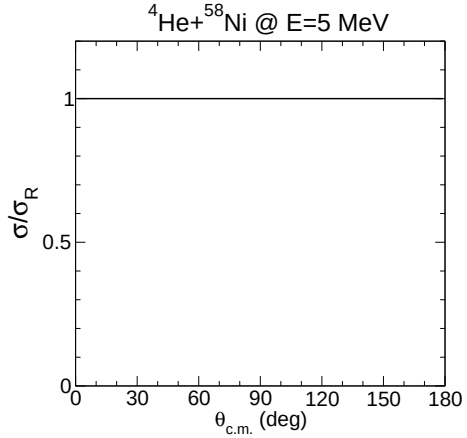
- $\eta \ggg 1$: Rutherford scattering: $\sigma(\theta) \propto 1/\sin^4(\theta/2)$
- $\eta \gg 1$: Fresnel scattering (rainbow)
- $\eta \leq 1$: Fraunhofer scattering (oscillatory behaviour):

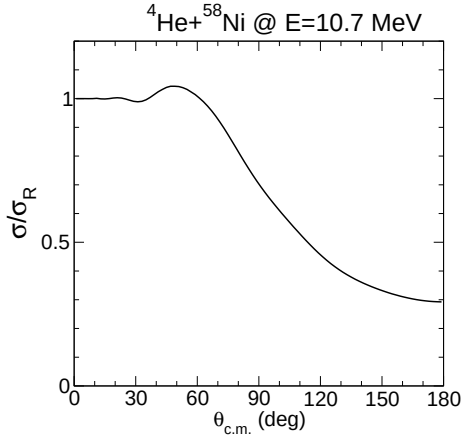
Elastic scattering: energy dependence

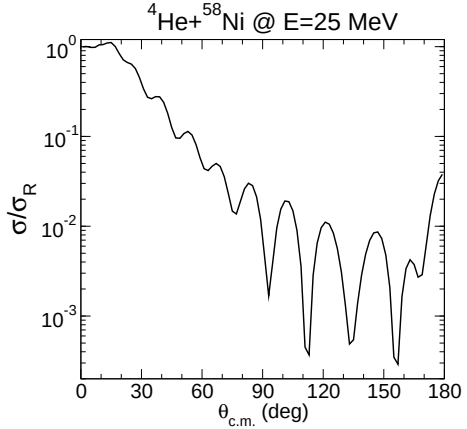










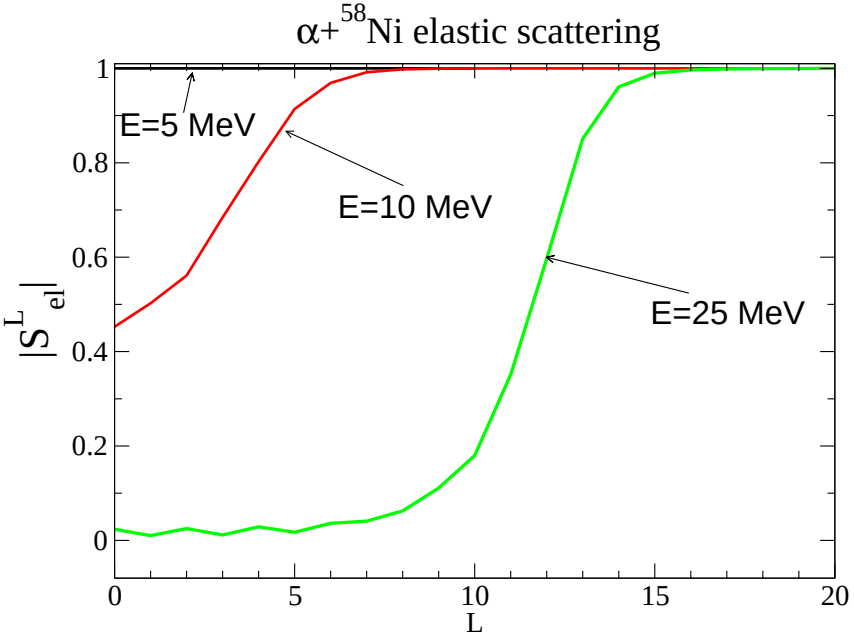


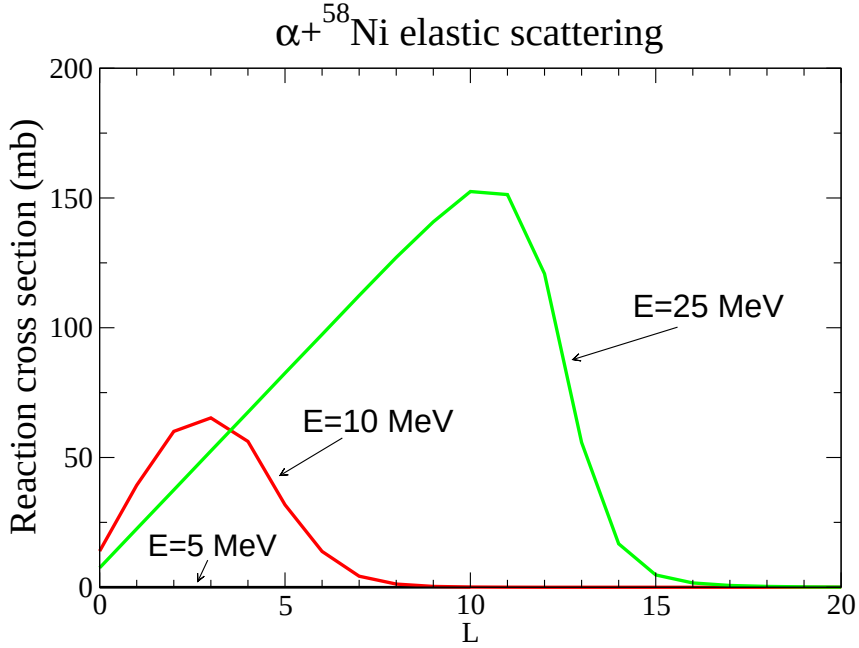
Rutherford scattering

Fresnel

Fraunhöfer

Elastic (nuclear) S-matrix (`fort.7`): $f_{el}^{L}(r) = I_L(r) - S_{el}^{L} O_L(r)$





$$kR_g\,(1 - 2\eta/kR_g) \approx L_g + 1/2$$

$\Rightarrow$ the number of partial waves required for convergence grows approximately as $\sqrt{E}$

Proposed exercise:

a) Calculate elastic angular distribution ($d\sigma/d\Omega$) for ^{7}Li+^{208}Pb OMP at 33 MeV.

b) Elastic distribution and reaction cross section for ^{8}Li+^{208}Pb at 33 MeV using the ^{7}Li potential.

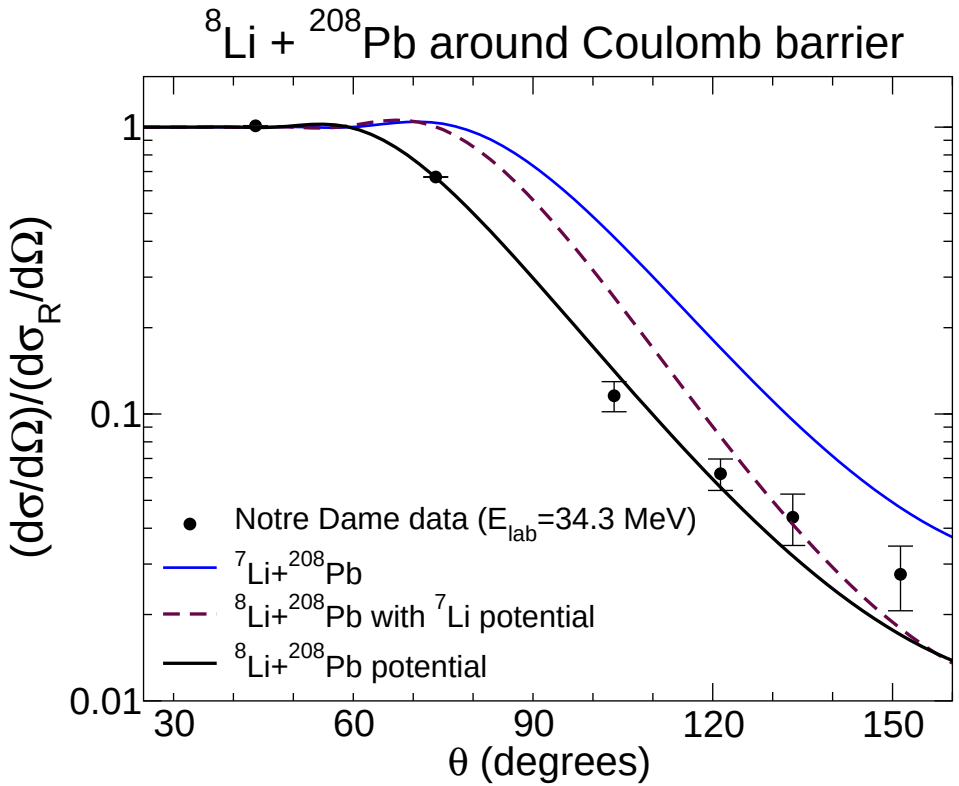
c) Elastic distribution and reaction cross section for ^{8}Li+^{208}Pb at 33 MeV using the ^{8}Li potential.

System	V_0	r_x	a_0	W_0	a_i
^{7}Li+^{208}Pb	15.4	1.3	0.65	13.2	0.65
^{8}Li+^{208}Pb	15.4	1.3	0.65	58.4	0.70

$$r_C = 1.25 \text{ fm}, R_x = r_x(A_p^{1/3} + A_t^{1/3})$$

FRESCO input file: `li8pb_e34.in`

```
NAMELIST
&FRESCO hcm=0.05 rmatch=40.0 jtmax=60
      thmin=5.00
      thmax=160.00 thinc=2.00
      smats=2 xstabl=1
      elab= 34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
      masst=208 zt=82 qval=0.0000 pwf=T nex=1 /
&STATES jp=2.0 bandp=1 ep=0.000 cpot=1 jt=0.0
      bandt=1 et=0.000 fexch=F /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=58.3 p5=1.3 p6=0.7 /
&pot /
```


^{8}Li + ^{208}Pb around Coulomb barrier
(dσ/dΩ)/(dσ$_R$/dΩ)
1
0.1
0.01
30
60
90
120
150
θ (degrees)
Notre Dame data (E$_{lab}$=34.3 MeV)
^{7}Li+^{208}Pb
^{8}Li+^{208}Pb with ^{7}Li potential
^{8}Li+^{208}Pb potential

SFRESCO: Can be used together with FRESCO to do determine automatically optical model parameters by means of a χ^2 analysis of experimental angular distribution.

We need 3 input files:

1. FRESCO input file: `li8pb_e34.in`
2. MINUIT input file: `sfresco.in`
3. SEARCH input file: `search.in`

sfresco.in $\Longrightarrow$ search.in $\Longrightarrow$ `li8pb_e34.in`

Performing fits with SFRESCO:

1.- FRESCO input file: `li8pb_e34.in` (previous slide)

2.- MUNUIT input file: `sfresco.in`

```
search.in
min
fix
migrad
end
q
show
plot
```

Performing fits with SFRESCO (continued):

3.- SEARCH input file: `search.in`

```
'li8pb_e34.in' 'li8pb_e34.out' 2 1

&variable kind=1 name='V' kp=1 pline=2 col=1 valmin=5.0 valmax=150.0 step=0.2/
&variable kind=1 name='W' kp=1 pline=2 col=4 valmin=5.0 valmax=100.0 step=0.2 /

&data type=0 iscale=2 idir=1 lab=F abserr=T/
43.7    1.01026 0.014
73.76   0.67003 0.014
103.537 0.11577 0.01394
121.296 0.06194 0.00778
133.351 0.04369 0.00888
151.332 0.02763 0.00701
&
```

```
sfresco < sfresco.in > sfresco.out
```

Lecture 2: Inelastic scattering: DWBA and Coupled-Channels method

Coupled-channels method

- The Hamiltonian: $H = T_R + h(\xi) + \Delta(\mathbf{R}, \xi)$
- Internal states: $h(\xi)\phi_\alpha(\xi) = \epsilon_\alpha \phi_\alpha(\xi)$
- Model wavefunction: $\Psi(\mathbf{R}, \xi) = \phi_\alpha(\xi)\chi_\alpha(\mathbf{R}) + \phi_{\alpha'}(\xi)\chi_{\alpha'}(\mathbf{R}) + ...$
- Coupled equations: $[H - E]\Psi(\mathbf{R}, \xi)$

$$\left[E - \epsilon_\alpha - T_R - V_{\alpha,\alpha}(\mathbf{R})\right] \chi_\alpha(\mathbf{R}) = \sum_{\alpha' \neq \alpha} V_{\alpha,\alpha'}(\mathbf{R})\chi_{\alpha'}(\mathbf{R})$$

- Coupling potentials:

$$V_{\alpha,\alpha'}(\mathbf{R}) = \int d\xi \phi_{\alpha'}(\xi)^* \Delta(\mathbf{R}, \xi)\phi_\alpha(\xi)$$

☞ *$\phi_\alpha(\xi)$ will depend on the structure model (collective, single-particle,etc).*

Boundary conditions and scattering amplitude

- Boundary conditions:

$$\chi_0^{(+)}(\mathbf{R}) \rightarrow e^{i\mathbf{K}_0\cdot\mathbf{R}} + A_{0,0}(\theta)\frac{e^{iK_0R}}{R} \quad \text{(elastic)}$$

$$\chi_n^{(+)}(\mathbf{R}) \rightarrow A_{n,0}(\theta)\frac{e^{iK_nR}}{R}, \qquad n \neq 0 \quad \text{(non-elastic)}$$

☞ If Coulomb is present, then

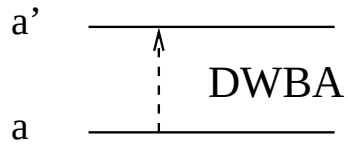
$$\frac{e^{iKR}}{R} \rightarrow \frac{1}{R}e^{i(KR-\eta 2KR)}$$

- Cross sections:

$$\frac{d\sigma_n(\theta)}{d\Omega} = \frac{K_n}{K_0}|A_{n,0}(\theta)|^2$$

- DWBA approximation:

$$\begin{aligned}[E-\epsilon_\alpha - T_\alpha - V_{\alpha:\alpha}(\mathbf{R})]\,\widetilde{\chi}_\alpha(\mathbf{K},\mathbf{R}) &= 0\\ [E-\epsilon_{\alpha'} - T_{\alpha'} - V_{\alpha':\alpha'}(\mathbf{R})]\,\widetilde{\chi}_{\alpha'}(\mathbf{K}',\mathbf{R}) &= 0\end{aligned}$$



- Scattering amplitude:

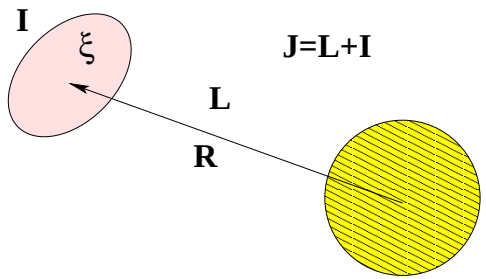
$$A(\mathbf{K}',\mathbf{K}) = -\frac{2\mu}{4\pi\hbar^2}\int d\mathbf{R}\widetilde{\chi}^{(-)}_{\alpha'}(\mathbf{K}',\mathbf{R})V_{\alpha':\alpha}(\mathbf{R})\widetilde{\chi}^{(+)}_{\alpha}(\mathbf{K},\mathbf{R})$$

☞ *The DWBA approximation amounts at solving the CC equations to first order (Born approximation)*

☞ *In practice, phenomenological optical potentials that fit the elastic cross section in the respective channels are used instead of* $V_{\alpha,\alpha}$ *and* $V_{\alpha',\alpha'}$

$$V_{\alpha,\alpha}(\mathbf{R}) \equiv (\alpha|\Delta(\mathbf{R},\xi)|\alpha) \rightarrow U_\alpha(\mathbf{R})$$

Partial wave decomposition: the channel basis



- The channel basis:

$$\Phi^{JM_J}_{nLI}(\hat{R},\xi) = \sum_{M_I M_L} i^L Y_{LM_L}(\hat{R}) |nIM_I\rangle \langle LM_L IM_I | JM_J\rangle$$

- Partial wave expansion of the total WF:

$$\Psi(\mathbf{R},\xi) = \sum_{nLIJM} C^{JM_J} \frac{f^J_{nLI}(R)}{R} \Phi^{JM}_{nLI}(\hat{R},\xi)$$

Coupled equations

- The coupled equations:

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{\hbar^2 L(L+1)}{2\mu R^2}+\epsilon_n-E\right)f^J_\beta(R)+\sum_{\beta'}V^J_{\beta,\beta'}(R)f^J_{\beta'}(R)=0$$

$\beta \equiv \{n, L, I\}$

- Coupling potentials:

$$V^J_{\beta,\beta'}(R)=\int d\hat{R}\, d\xi \Phi^{JM_J}_\beta(\hat{R},\xi)^*\Delta(\vec{R},\xi)\Phi^{JM_J}_{\beta'}(\hat{R},\xi)$$

Boundary conditions

Solution of the coupled equations:

1. Integrate the differential equation for $R \in [0, R_m]$ with the condition:

$$\lim_{R \to 0} f^J_{\beta;\beta_i}(R) = 0$$

2. Match the solution at R_m with the asymptotic form $\Rightarrow$ S-matrix:

$$f^J_{\beta;\beta_i}(R) \to \delta_{\beta,\beta_i} I_\beta(R) - S^J_{\beta,\beta_i} O_\beta(R)$$

$$\begin{aligned} I_\beta(R) &= (K_n R) h^*_L(K_n R)/\sqrt{v_n} \\ O_\beta(R) &= (K_n R) h_L(K_n R)/\sqrt{v_n} \end{aligned}$$

Scattering wavefunction

- Wavefunction that corresponds to the experimental condition:

$$\Psi_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi) = e^{i\mathbf{K}_i\cdot\mathbf{R}}|n_iI_iM_i\rangle + \chi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi)$$

- The outgoing wave:

$$\begin{aligned}\chi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi) &= \sum_{JM_JL_iN_i} \frac{2\pi i\sqrt{v_i}}{k_iR}\langle L_iN_iI_iM_i|JM_J\rangle Y^*_{L_iN_i}(\hat{K}_i) \\ &\times \sum_{nIL}\left(\delta_{n,n_i}\delta_{I,I_i}\delta_{n,n_i} - S^J_{nIL:n_iI_iL_i}\right)\Phi^{JM_J}_{nLI}(\hat{R},\xi)O_{nIL}(R)\end{aligned}$$

- The scattering amplitude:

$$\begin{aligned}A(\mathbf{K}_i,\mathbf{K})_{n_iI_iM_i;nIM} &= \frac{2\pi i}{\sqrt{KK_i}}\sum_{JM_JL_iN_i}\langle L_iN_iI_iM_i|JM_J\rangle Y^*_{L_iN_i}(\hat{K}_i) \\ &\times \sum_{nILN}\langle LNIM|JM_J\rangle Y_{LN}(\hat{K})\left(\delta_{n,n_i}\delta_{I,I_i}\delta_{L,L_i} - S^J_{nIL;n_iI_iL_i}\right)\end{aligned}$$

Scattering amplitude and cross sections

- Elastic and inelastic cross sections:

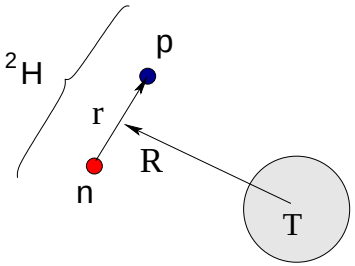
$$\frac{d\sigma}{d\Omega}_{i\to n} = \frac{1}{2I_i+1}\sum_{MM_i} |A(\mathbf{K}_i,\mathbf{K})_{n_i I_i M_i;nIM}|^2$$

☞ *Coupled channels calculations give elastic and inelastic cross sections, if the states are properly described, if the interactions are known, and if all "relevant" channels are included*

Inelastic scattering: cluster model

- Some nuclei permit a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
- Projectile-target interaction:

$$V(\mathbf{R},\mathbf{r}) = U_1(\mathbf{R}_1) + U_2(\mathbf{R}_2)$$

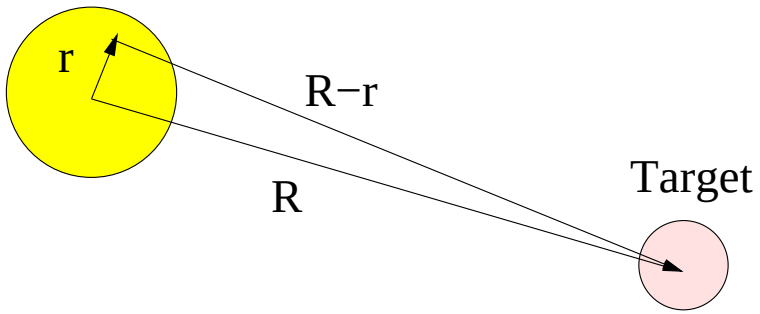


- Internal states: $[h - \epsilon_\alpha]\phi_\alpha(\mathbf{r}) = 0$
- Transition potentials:

$$V_{\alpha,\alpha'}(\mathbf{R}) = \int d\mathbf{r}\phi^*_\alpha(\mathbf{r})V(\mathbf{r},\mathbf{R})\phi_{\alpha'}(\mathbf{r})$$

- Projectile-target Coulomb interaction:

$$V(\mathbf{R},\xi) = \frac{Ze^2}{4\pi\epsilon_0}\sum_i \frac{1}{|\mathbf{R}-\mathbf{r}_i|}$$



- Multipolar expansion:

$$\frac{1}{|\mathbf{R}-\vec{r}_i|} = \sum_{\lambda\mu} \frac{r_i^\lambda}{R^{\lambda+1}} \frac{4\pi}{2\lambda+1} Y_{\lambda\mu}(\hat{r}_i) Y^*_{\lambda\mu}(\hat{R}) \qquad (R > r_i)$$

- Electric multipole operator: $M(E\lambda,\mu) = e\sum_i r_i^\lambda Y_{\lambda\mu}(\hat{r}_i)$
- Monopole and transition operator:

$$V(\mathbf{R},\xi) = V_0(R) + \Delta(\mathbf{R},\xi) = \frac{Zze^2}{4\pi\epsilon_0 R} + \frac{Ze}{\epsilon_0}\sum_{\lambda\neq 0,\mu}\frac{M(E\lambda,\mu)}{2\lambda+1}\frac{Y^*_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

- Transition potentials:

$$\Delta_{nm}(\mathbf{R}) = \frac{Ze}{\epsilon_0}\sum_{\lambda\neq 0,\mu}\frac{\langle nI_nM_n|M(E\lambda,\mu)|mI_mM_m\rangle}{2\lambda+1}\frac{Y^*_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

- Multipole expansion: $r(\theta,\phi) = R_0 + \sum_{\lambda\mu} \hat{\delta}_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)$

($\hat{\delta}_{\lambda\mu}$=deformation length operators)

- Transition operator:

$$V(\mathbf{R},\xi) = V_0(R-R_0) - \frac{dV_0(R-R_0)}{dR} \sum_{\lambda\mu} \hat{\delta}_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)$$

- Central and transition potential:

$$\Delta_{nm}(\mathbf{R}) = -\frac{dV_0(R-R_0)}{dR} \sum_{\lambda} \langle nI_nM_n|\hat{\delta}_{\lambda\mu}|mI_mM_m\rangle Y^*_{\lambda\mu}(\hat{R})$$

☞*The nuclear transition potentials are proportional to the matrix element of the deformation length operator*.

- Coulomb excitation $\longrightarrow$ electric reduced matrix elements

$$\Delta_{nm}(\mathbf{R}) = \frac{Ze}{\epsilon_0} \sum_{\lambda \neq 0, \mu} \frac{\langle n I_n M_n | M(E\lambda, \mu) | m I_m M_m \rangle}{2\lambda + 1} \frac{Y^*_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

$$\boxed{\langle n I_n || M(E\lambda) || m I_m \rangle = \sqrt{(2I_n + 1) B(E\lambda; I_n \rightarrow I_m)}}$$

- Nuclear excitation (collective model) $\longrightarrow$ deformation lengths

$$\Delta_{nm}(\mathbf{R}) = -\frac{dV_0(R - R_0)}{dR} \sum_{\lambda} \langle n I_n M_n | \widehat{\delta}_{\lambda\mu} | m I_m M_m \rangle Y^*_{\lambda\mu}(\hat{R})$$

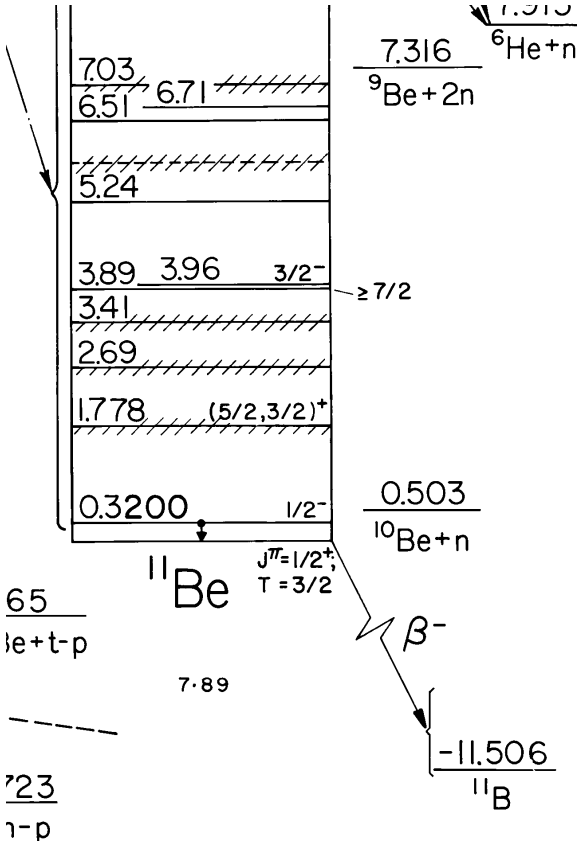
within the rotational model:

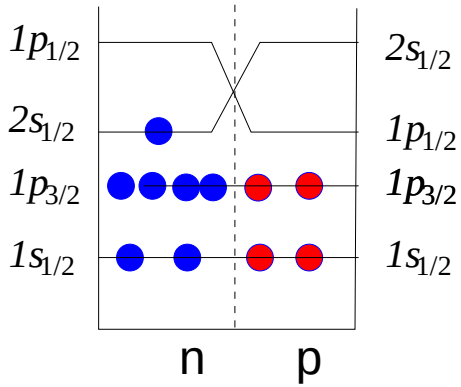
$$\boxed{\langle n I_n || \widehat{\delta}_{\lambda} || m I_m \rangle = \delta_\lambda \sqrt{2I_n + 1} \langle I_n K \lambda 0 | I_m K \rangle} \qquad \delta_\lambda = \beta_\lambda R$$

Inelastic scattering example: ^{11}Be+^{12}C

Example 1: ^{11}Be+ ^{12}C $\rightarrow$ ^{11}Be(1/2$^+$,1/2$^-$) + ^{12}C at 49.3 MeV/A
Phys. Rev. C 67, 037601 (2003)

Input file: `be11c12_inel.in`





^{11}Be+^{12}C inelastic scattering

General variables:

```
&FRESCO hcm=0.05 rmatch=60.0 jtmin=0.0
      jtmax=150.0 thmin=0.00 thmax=45.00 thinc=0.50
      iblock=2 nnu=24 chans=1 smats=2 xstabl=1
      elab=542.3 /
```

- `iblock=2`: number of channels coupled *exacly*.

Partitions & states:

```
&PARTITION namep='11Be' massp=11.0 zp=4 namet='12C' masst=12.0000 zt=6 nex=2 /
&STATES jp=0.5 bandp=1 cpot=1 jt=0.0 bandt=1 /
&STATES jp=0.5 bandp=-1 ep=0.3200 cpot=1 jt=0.0 copyt=1 /
```

- `nex=2`: This partition will contain two pairs of states.
- `copy=1`: The target of the second pair of states is just the same (a copy) of the first target stat.

```
&PARTITION namep='10Be' massp=10.0000 zp=4 namet='12C+n' masst=13.0000 zt=6 nex=1 /
&STATES jp=0.0 bandp=1 cpot=2 jt=0.0 bandt=1 /
```

^{11}Be+^{12}C inelastic scattering

Projectile-target Coulomb potential (monopole):

```
&POT kp=1 ap=11.000 at=12.000 rc=1.111 /
```

Neutron-target & core-target potentials:

```
&POT kp=3 ap=0.000 at=12.000 rc=1.111 /
&POT kp=3 type=1 p1=37.400 p2=1.200
      p3=0.750 p4=10.000 p5=1.300 p6=0.600 /
```

```
&POT kp=2 ap=10.000 at=12.000 rc=1.111 /
&POT kp=2 type=1 p1=123.000 p2=0.750
      p3=0.800 p4=65.000 p5=0.780 p6=0.800 /
```

Neutron binding potential:

```
&POT kp=4 ap=0 at=10.000 rc=1.0 /
&POT kp=4 type=1 p1=87.0 p2=1.0 p3=0.53 /
```

Bound state wave functions:

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=2 sn=0.5 l=0 j=0.5
      kbpot=4 be=0.500 isc=1 /
&OVERLAP kn1=2 ic1=1 ic2=2 in=1 nn=1 l=1 sn=0.5
      j=0.5 kbpot=4 be=0.180 isc=1 ipc=2 /
```

- `kn1=1,2`: Index for this WF
- `ic1/ic2`: Index of partition containing core (^{10}Be) / composite (^{11}Be)
- `in=1/2`: WF for projectile/target
- `nn,sn,l,j`: Quantum numbers for bound state
- `be`: separation energy.
- `kbpot=3`: Index KP of binding potential.

^{11}Be+^{12}C inelastic scattering

Couplings:

```
&COUPLING icto=1 icfrom=2 kind=3 ip1=4 ip2=1 p1=3.0 p2=2.0 /
```

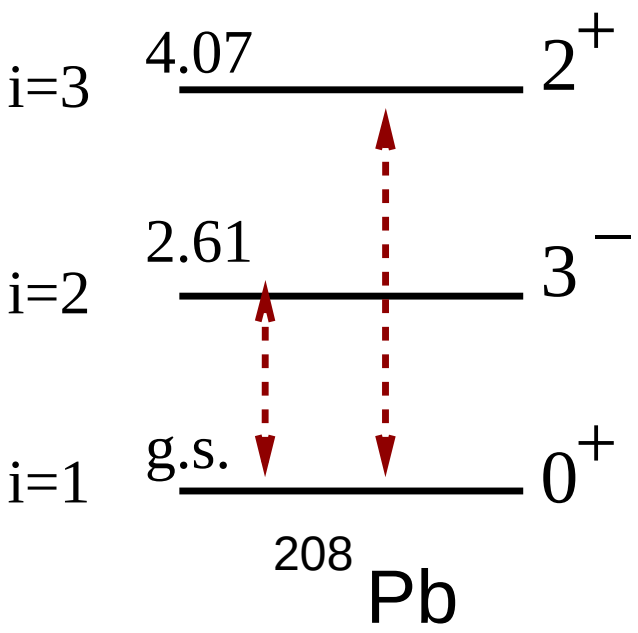
- `kind=3`: Single-particle excitations of projectile
- `icto=1`: Partition containing nucleus being excited (^{11}Be)
- `icfrom=2`: Partition containing core (^{10}Be)
- `ip1=4`: Maximum multipole for coupling potentials
- `p1/p2`: KP index for fragment-target / core-target potentials

Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.000 /
&CFP in=1 ib=2 ia=1 kn=2 a=1.000 /
```

- `in=1/2`: Projectile/target
- `ib/ia`: Index for composite/core state
- `a=1.0`: Spectroscopic amplitude

Physical example: $^{16}O + {}^{208}Pb \rightarrow {}^{16}O + {}^{208}Pb(3^-, 2^+)$



Input example 2: ^{208}Pb(^{16}O,^{16}O)^{208}Pb(3^-,2^+) (o16pb_cc1a.in)

```
o16pb_cc1a.in: 16O+208Pb 80 MeV
NAMELIST
&FRESCO hcm=0.05 rmatch=100.0
      jtmin=0.0 jtmax=300.0
      thmin=5.00 thmax=-180.00 thinc=2.50
      iblock=3
      smats=2 xstabl=1
      elab= 80.0 /

&PARTITION namep='16-O' massp=15.9949 zp=8
        namet='PB-208' masst=207.9770 zt=82
        nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0000 cpot=1
      jt=0.0 bandt=1 et=0.0000 /
&STATES jp=0.0 copyp=1 ep=0.0000 cpot=1
      jt=3.0 bandt=-1 et=2.6100 fexch=F /
&STATES jp=0.0 copyp=1 bandp=1 ep=0.0000 cpot=1
      jt=2.0 bandt=1 et=4.0700 /
&partition /
```

```
&POT kp=1 itt=F ap=208.000 at=16.000 rc=1.200 /
&POT kp=1 type=13 shape=10 itt=F p2=54.45 p3=815.0 /
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 p1=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=11 p2=0.400 p3=0.8 /
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
&step /

&pot /

&overlap /
&coupling /
```

^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

General variables:

```
&FRESCO hcm=0.05 rmatch=100.0
      jtmin=0.0 jtmax=300.0
      thmin=5.00 thmax=-180.00 thinc=2.50
      iblock=3
      smats=2 xstabl=1
      elab= 80.0 /
```

`iblock`: Number of states (including gs) that will be coupled to all orders.

- `iblock=1`: only elastic scattering
- `iblock=2`: elastic scattering + 1st inelastic channel (^{208}Pb(3^-))
- `iblock=3`: elastic scattering + ^{208}Pb(3^-) + ^{208}Pb(2^+)

Partitions and states:

```
&PARTITION namep='16-O' massp=15.9949 zp=8 namet='PB-208' masst=207.9770 zt=82
         nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=+1 et=0.00 /
&STATES    copyp=1       cpot=1 jt=3.0 bandt=-1 et=2.61 /
&STATES    copyp=1       cpot=1 jt=2.0 bandt=+1 et=4.07 /
&partition /
```

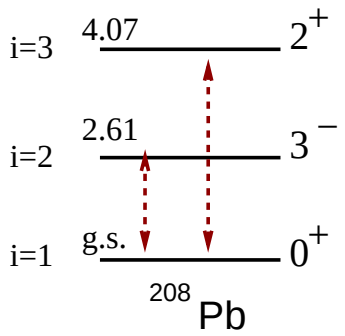
- `nex`: number of states within the partition
- `ep, et`: excitation energy for projectile / target
- `copyp=1` tells FRESCO that the 2nd and 3rd projectile states are just a copy of the ground state.

Coulomb excitation:

```
&POT kp=1 ap=208.000 at=16.000 rc=1.2 /
&POT kp=1 type=13 shape=10 p2=54.45 p3=815.0 p4=0 p5=0 p6=0 /
```

- `type=13`: couple target states by deforming previous potential
- `p1,...,p6`: consider couplings for multipolarities k with pk$\neq 0$
- `shape=10`: usual deformed charge sphere: $\Delta_{nm}(R) \propto M(Ek)/R^{k+1}$

```
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
```



- `ia, ib`: couple from state number `ia` to state `ib`
- `k`: multipolarity
- `str`=$\langle \mathrm{ib}||\mathrm{M(Ek}||\mathrm{ia}\rangle = \sqrt{(2\mathrm{I_a}+1)\mathrm{B(E}\lambda;\mathrm{ia}\rightarrow\mathrm{ib})}$

^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Nuclear excitation:

```
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 shape=0 p1=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=10 itt=F p2=0.400 p3=0.8 /
```

- `type=13`: couple target states by deforming preceding potential
- `shape=10`: usual deformed nuclear potential: $\Delta_{nm}(R) \propto \delta_k dU(R)/dR$

```
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
```

- `str`=$\langle ib||\delta_k||ia\rangle$ (reduced deformation length)

Useful output files:

- Main output file:

```
0CUMULATIVE REACTION cross section                =   11.22270  <L> =    47.07
0CUMULATIVE outgoing cross sections in partition 1 :    0.00000    7.67943    0.99138
0Cumulative ABSORBTION by Imaginary Potentials    =    2.55189  <L> =     6.99
```

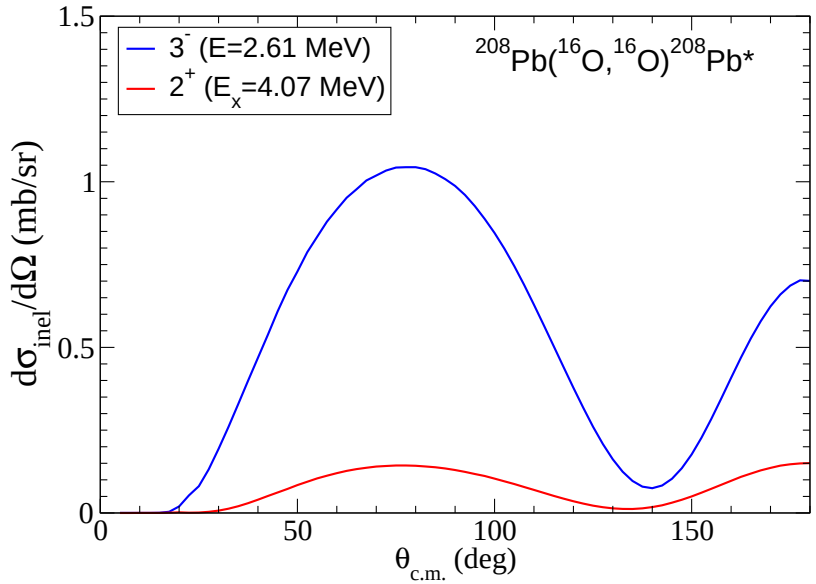
- Angular distributions:
  - `fort.201` : Elastic scattering angular distribution
  - `fort.202` : 1st state angular distribution
  - `fort.203` : 2nd excited state angular distribution

- `fort.56`: 3 columns: Fusion (absorption), reaction and inelastic cross section for each total angular momentum J.

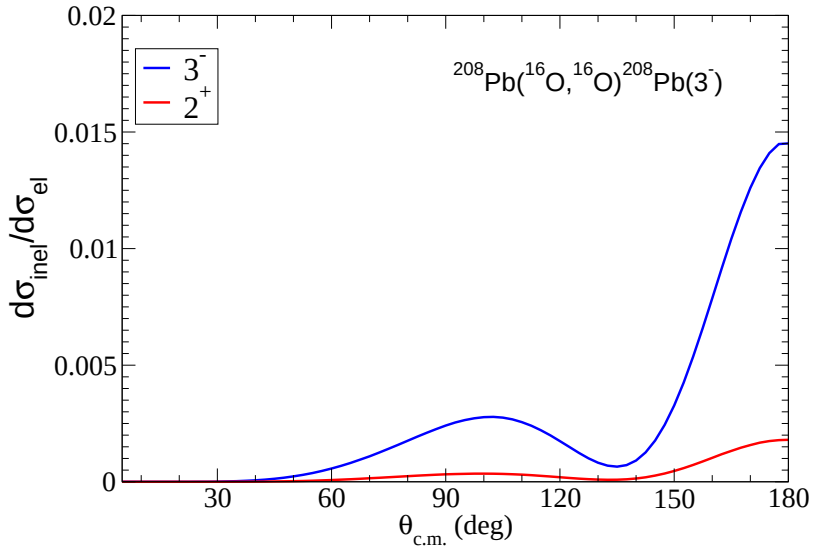
$$\sigma_{\rm reac} = \sigma_{\rm inel} + \sigma_{\rm abs}$$

Angular distribution of the ejectile in c.m. frame

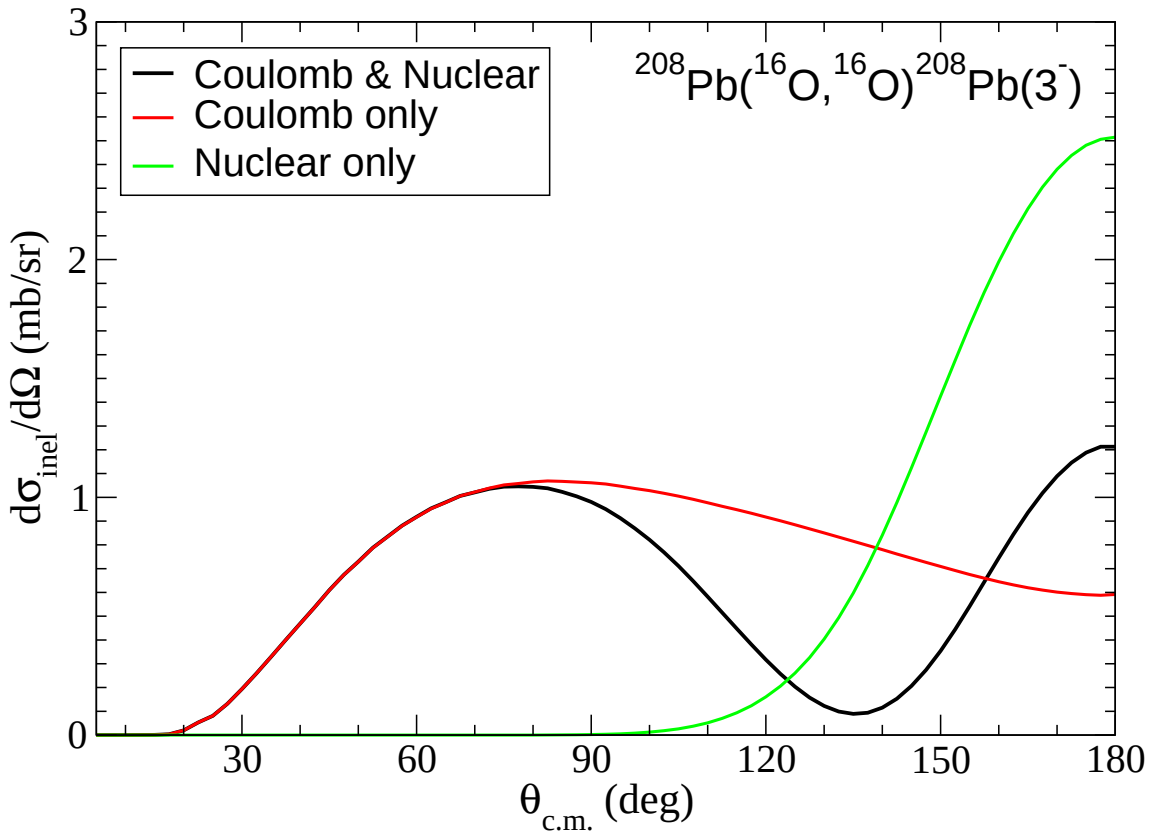
Absolute cross section



Ratio to elastic cross section

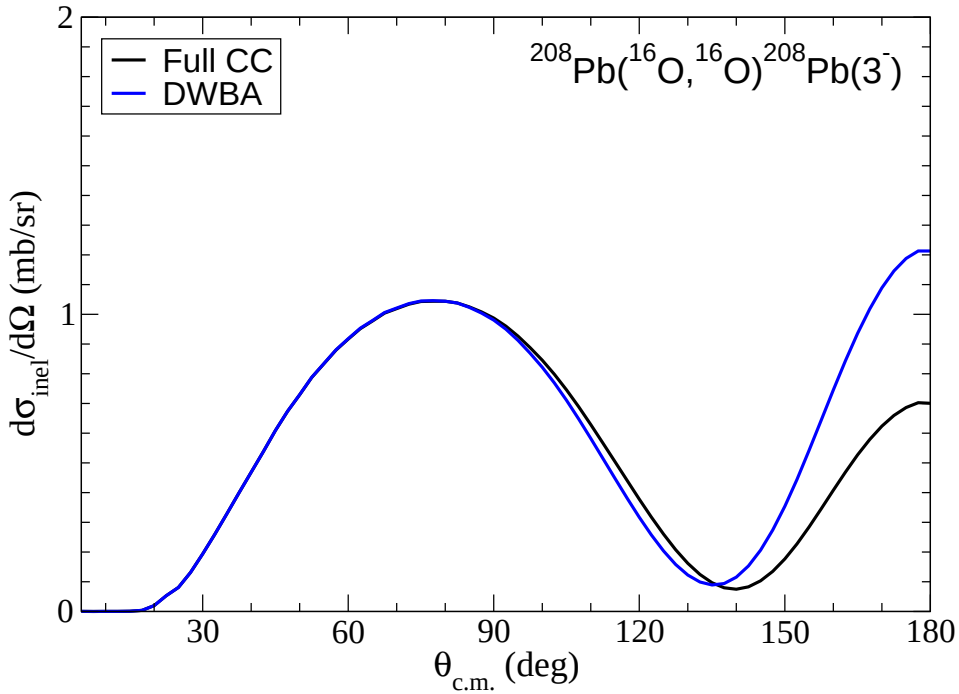


Coulomb and Nuclear excitations can produce constructive or destructive interference:



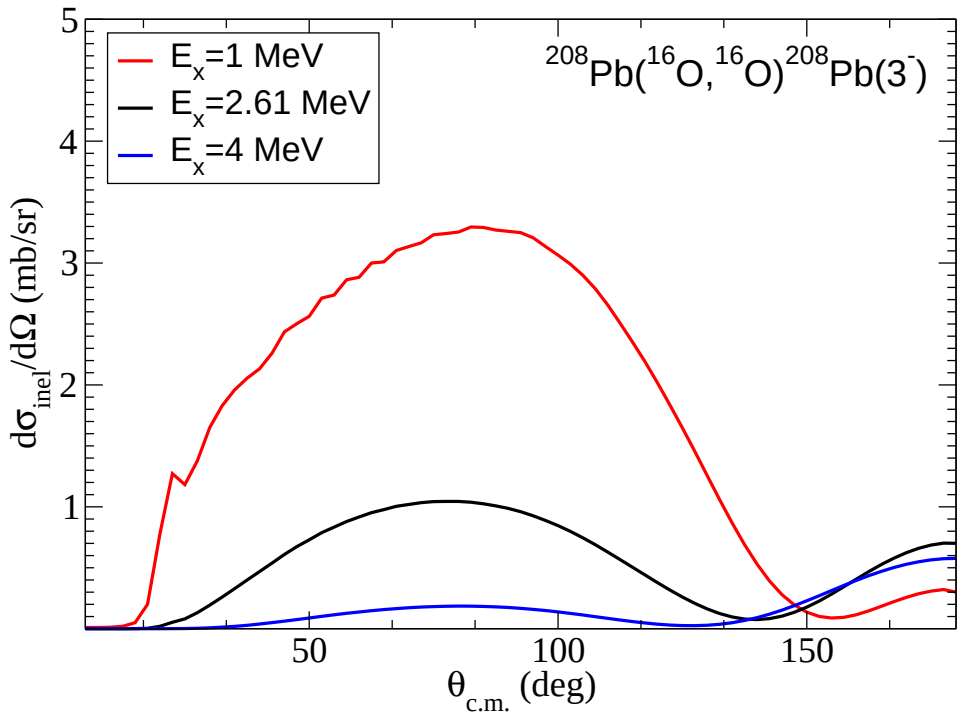
CC versus DWBA:

- Full coupled-channels: `iblock = 3`, `iter = 0`
- DWBA: `iblock = 0`, `iter = 1`



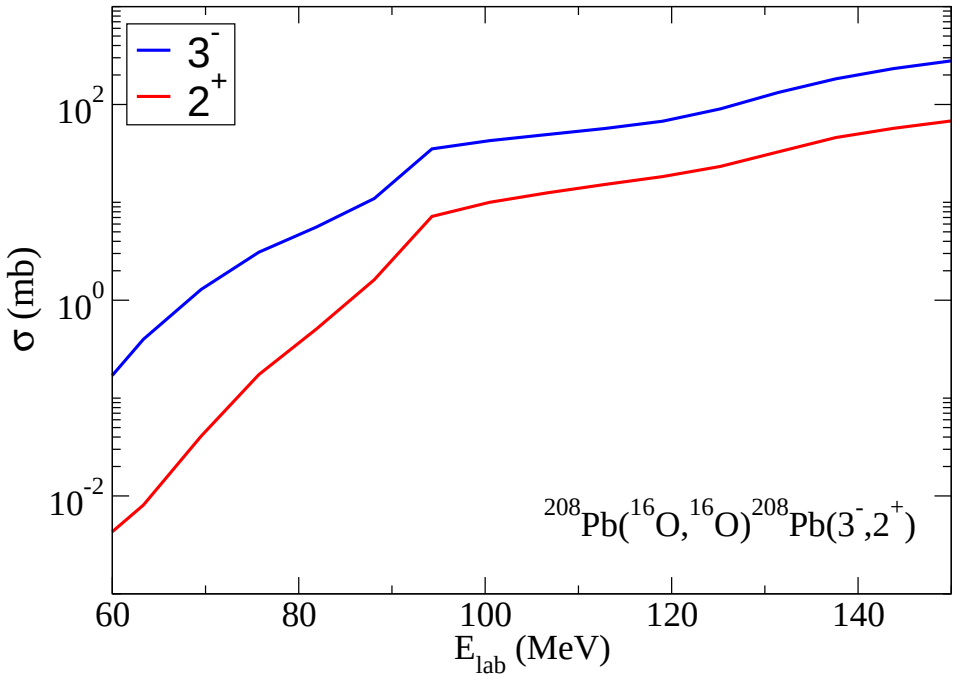
Effect of the excitation energy:

```
&STATES jp=0.0 copyp=1 ep=0.0000 cpot=1 jt=3.0 bandt=-1 et=2.6100 /
```



Effect of the incident energy: In FRESCO main output:

```
0CUMULATIVE REACTION cross section = 11.22270 <L> = 47.07 <L**2> = 3441.3
0CUMULATIVE outgoing cross sections in partition 1 : 0.00000 7.67943 0.99138
```



Example for inelastic scattering

Proposed exercise:

For the reaction ^{8}Li+^{208}Pb at $E_{\rm c.m.}$=24 MeV and 33 MeV evaluate the inelastic cross section to the first excited state of ^{8}Li ($E_x = 0.98$ MeV, 1^+) in the following situations:

- Coulomb excitation alone
- Only nuclear couplings
- Coulomb + nuclear excitation

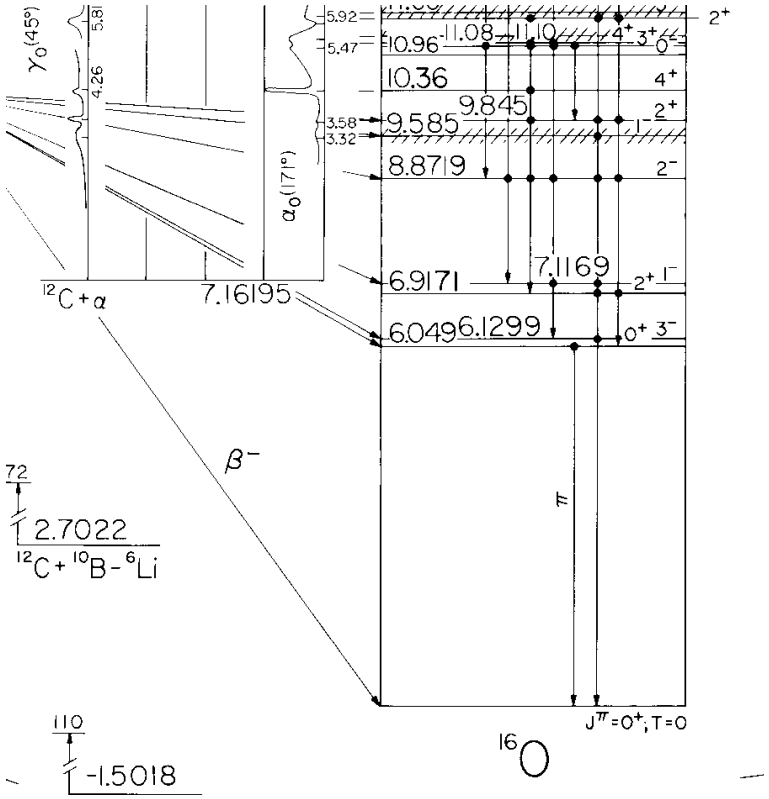
Data: $B(E2; 2^+ \rightarrow 1^+) = 30$ e^2 fm^4 and δ_2=1.75 fm.

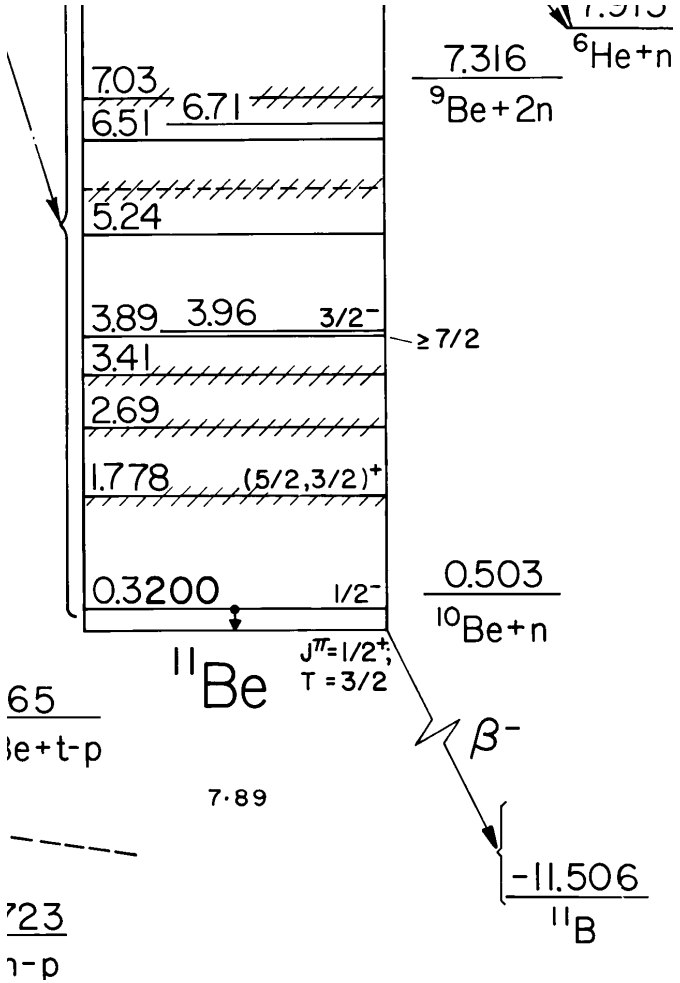
(*further details from Phys.Rev. C 68, 034614 (2003)*)

Input example: li8pb_inel.in

```
8Li+208Pb quasielastic
NAMELIST
&FRESCO hcm=0.1 rmatch=100 jtmax=150
      thmin=2.00 thmax=-180.00 thinc=1.00 iblock=2
      smats=2 xstabl=1 elab=34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
      masst=207.977 zt=82 qval=0.0000 pwf=F nex=2 /
&STATES jp=2.0 bandp=1 ep=0.000 kkp=1 cpot=1 jt=0.0 bandt=1 et=0.000 /
&STATES jp=1.0 bandp=1 ep=0.981 kkp=1 cpot=1 copyt=1 /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=10 shape=10 itt=F p2=10.00 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=80 p5=1.3 p6=0.70 /
&POT kp=1 type=10 shape=11 p2=1.75 /
&pot /

&overlap /
&coupling /
```

Inelastic scattering in exotic nuclei



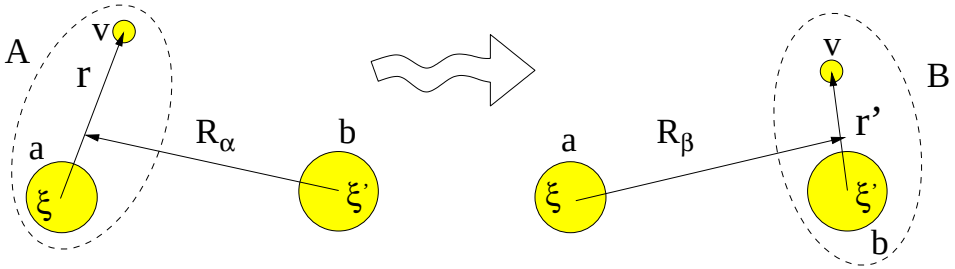


☞*Exotic nuclei are weakly bound ⇒ coupling to continuum states becomes an important reaction channel*

Lecture 3: Transfer reactions: the DWBA method

Transfer reactions: prior/post representation

- Transfer process:. $\underbrace{(a+v)}_{A} + b \rightarrow a + \underbrace{(b+v)}_{B}$



- Projectile–target interaction:
  - Prior form: $\mathbf{V}_{\rm prior} = V_{vb} + U_{ab} = U_\alpha + \underbrace{(V_{vb} + U_{ab} - U_\alpha)}_{\mathbf{\Delta}_{\rm prior}}$
  - Post form: $\mathbf{V}_{\rm post} = V_{av} + U_{ab} = U_\beta + \underbrace{(V_{av} + U_{ab} - U_\beta)}_{\mathbf{\Delta}_{\rm post}}$

☞ U_α, U_β: average proyectile-target interaction in entrance/exit channel

- Model wavefunction:

$$\Psi = \phi_A(\xi, \mathbf{r})\phi_b(\xi')\chi_\alpha(\mathbf{R}_\alpha) + \phi_a(\xi)\phi_B(\xi', \mathbf{r}')\chi_\beta(\mathbf{R}_\beta)$$

- Coupled-reaction channels (CRC) equations: $[H - E]\,\Psi = 0$

$$\begin{aligned}
[E - \epsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\,\chi_\alpha(\mathbf{R}_\alpha) &= \int d\mathbf{R}_\beta K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\beta(\mathbf{R}_\beta) \\
[E - \epsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)]\,\chi_\beta(\mathbf{R}_\beta) &= \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\alpha(\mathbf{R}_\alpha)
\end{aligned}$$

- Non-local kernels:

$$K_{\alpha,\beta}(\mathbf{R}_\beta, \mathbf{R}_\alpha) = \int d\xi d\xi' d\mathbf{r}\; \phi_a(\xi)\phi_B(\xi', \mathbf{r}')(H - E)\phi_A(\xi, \mathbf{r})\phi_b(\xi')$$

☞ *CRC equations have to be solved iteratively due to NL kernels.*

- Distorted wave Born approximation:

$$\begin{aligned}[E - \epsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\,\widetilde{\chi}_\alpha(\mathbf{R}_\alpha) &= 0 \\ [E - \epsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)]\,\widetilde{\chi}_\beta(\mathbf{R}_\beta) &= 0\end{aligned}$$

- DWBA amplitude (prior):

$$T_{\rm prior} = \int\int \widetilde{\chi}^{(-)}_\beta(\mathbf{R}_\beta)(\phi_a\phi_B|\Delta_{\rm prior}|\phi_A\phi_b)\widetilde{\chi}^{(+)}_\alpha(\mathbf{R}_\alpha)d\mathbf{R}_\alpha d\mathbf{r}$$

- Structure form-factor:

$$(\phi_a\phi_B|\Delta_{\rm prior}|\phi_A\phi_b) \equiv \int d\xi d\xi'\ \phi_a(\xi)\phi_B(\xi',\mathbf{r}')\Delta_{\rm prior}\phi_A(\xi,\mathbf{r})\phi_b(\xi')$$

Spectroscopic factors

- Parentage amplitudes:
  - Projectile: $\phi_A^{JM}(\xi, \mathbf{r}) = \frac{1}{\sqrt{n_A}} \sum_{I\ell j} A_{IJ;\ell sj} \left[\phi_a^I(\xi) \otimes \varphi_{\ell sj}(\mathbf{r})\right]_{JM}$
  - Target: $\phi_B^{J'M'}(\xi', \mathbf{r}') = \frac{1}{\sqrt{n_B}} \sum_{I\ell j} A_{IJ';\ell sj} \left[\phi_b^I(\xi') \otimes \varphi_{\ell sj}(\mathbf{r}')\right]_{J'M'}$

☞ $A_{IJ;\ell sj}$ = spectroscopic amplitudes

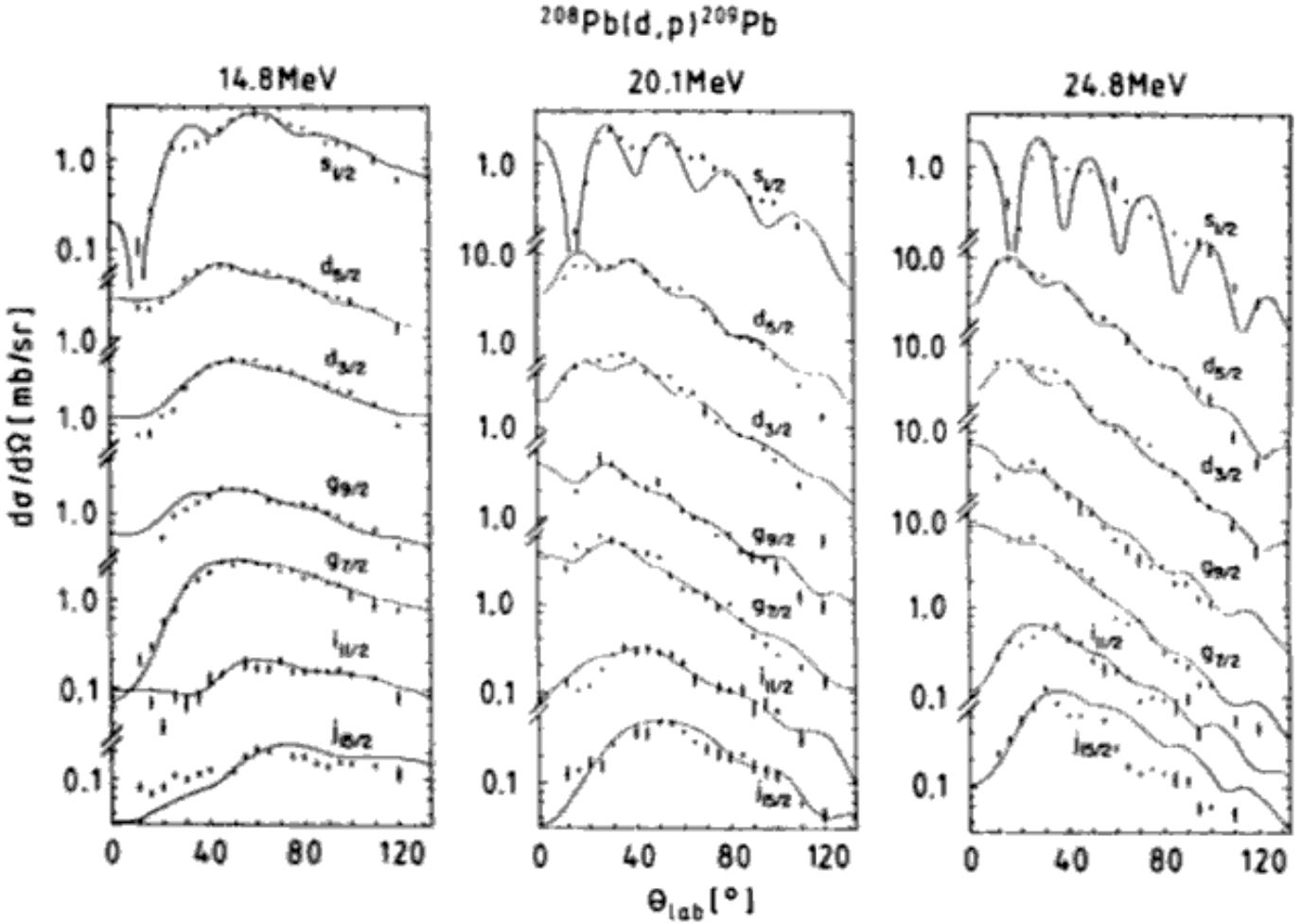
☞ $S_{IJ;\ell sj} = |A_{IJ;\ell sj}|^2$ = spectroscopic factors

- DWBA amplitude (prior)

$$T_{\rm prior} = A_{IJ;\ell sj} A_{I'J';\ell' sj'} \int\int \widetilde{\chi}_\beta^{(-)}(\mathbf{R}_\beta) \varphi_{\ell' sj'}(\mathbf{r}') \Delta_{\rm prior} \varphi_{\ell sj}(\mathbf{r}) \widetilde{\chi}_\alpha^{(+)}(\mathbf{R}_\alpha) d\mathbf{R}_\alpha d\mathbf{r}$$

☞ *In DWBA, the transfer cross section is proportional to the spectroscopic factors* $S_{IJ;\ell sj} S_{I'J';\ell' sj'}$

Extracting structure information from transfer reactions



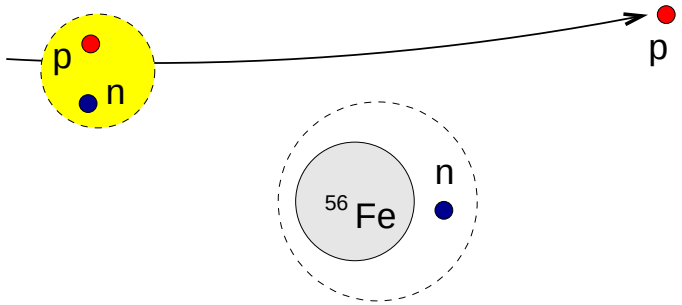
☞*Angular distributions of transfer cross sections is very sensitive to the single-particle configuration of the transferred nucleon/s.*

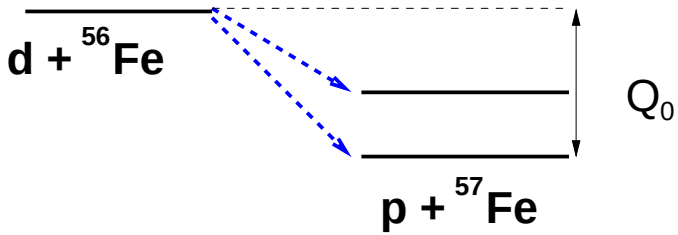
Brief summary on transfer reactions

- Inclusion of transfer couplings in the Schrodinger equation gives rise to a set of coupled equations with non-local kernels (Coupled Reactions Channels)
- If transfer couplings are weak, the CRC equations can be solved in Born approximation $\Rightarrow$ DWBA approximation
- The DWBA amplitude is proportional to the product of the projectile and target spectroscopic factors.
- The analysis of transfer reactions provide information on:
  - Spectroscopic factors
  - Quantum number for single-particle configurations (n, ℓ, j).

Transfer example

Physical example: ^{56}Fe(d,p)^{57}Fe at $E_d = 12$ MeV





Transfer example: ^{56}Fe(d,p)^{57}Fe

DWBA transfer amplitude:

$$T^{\mathrm{DWBA}} = A_i A_f \langle \chi^{(-)}_{\mathrm{p}-^{57}\mathrm{Fe}} \phi_{^{57}\mathrm{Fe}} | V_{\mathrm{prior/post}} | \chi^{(+)}_{\mathrm{d}-^{56}\mathrm{Fe}} \phi_d \rangle$$

Transfer example: ^{56}Fe(d,p)^{57}Fe

DWBA transfer amplitude:

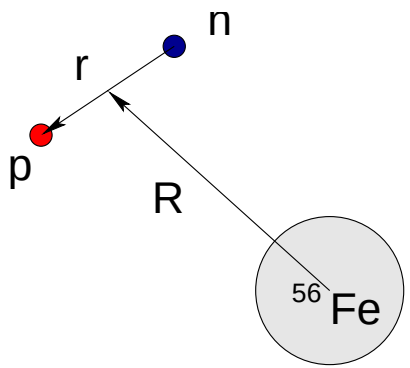
$$T^{\mathrm{DWBA}} = A_i A_f \langle \chi^{(-)}_{\mathrm{p}-^{57}\mathrm{Fe}} \phi_{^{57}\mathrm{Fe}} | V_{\mathrm{prior/post}} | \chi^{(+)}_{\mathrm{d}-^{56}\mathrm{Fe}} \phi_d \rangle$$

- $\chi_{\mathrm{d}-^{56}\mathrm{Fe}}, \chi_{\mathrm{p}-^{57}\mathrm{Fe}}$: initial and final distorted waves
- ϕ_d: projectile bound wavefunction($p-n$)
- $\phi_{^{57}\mathrm{Fe}}$: final (residual) wavefunction (n+^{56}Fe)
- A_i, A_f: initial / final spectroscopic amplitudes.
- $V_{\mathrm{prior/post}}$: transition potential in PRIOR or POST form

DWBA transfer amplitude:

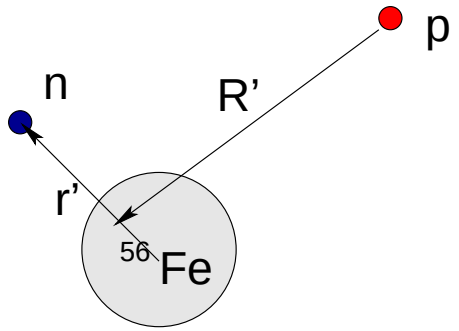
$$T^{\mathrm{DWBA}} = A_i A_f \langle \chi^{(-)}_{\mathrm{p}-^{57}\mathrm{Fe}} \phi_{^{57}\mathrm{Fe}} | V_{\mathrm{prior/post}} | \chi^{(+)}_{\mathrm{d}-^{56}\mathrm{Fe}} \phi_d \rangle$$

PRIOR



$$V_{\mathrm{prior}} = V_{\mathrm{n}-56\mathrm{Fe}} + \underbrace{U_{\mathrm{p}-56\mathrm{Fe}} - U_{\mathrm{d}-56\mathrm{Fe}}}_{\texttt{remnant}}$$

POST



$$V_{\mathrm{post}} = V_{\mathrm{p}-\mathrm{n}} + \underbrace{U_{\mathrm{p}-56\mathrm{Fe}} - U_{\mathrm{p}-57\mathrm{Fe}}}_{\texttt{remnant}}$$

Essential physical ingredients in a DWBA calculation:

- **Potentials (5):**
  - Distorted potential for entrance channel (complex): d+^{56}Fe
  - Distorted potential for exit channel (complex): p+^{57}Fe
  - Core-core interaction (complex): p+^{56}Fe
  - Binding potential for projectile (real): p+n
  - Binding potential for target (real): n+^{56}Fe
- **Spectroscopic amplitudes:** A_i, A_f

Transfer example: ^{56}Fe(d,p)^{57}Fe

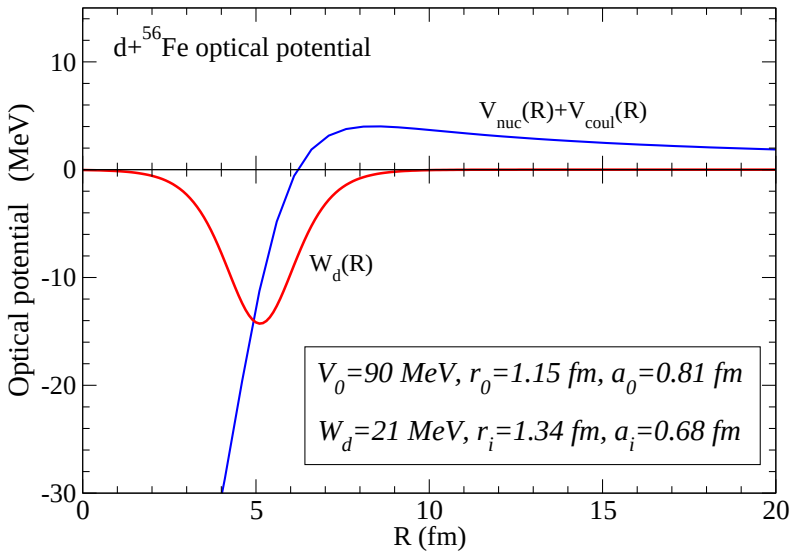
Physical ingredients: Optical and binding potentials (NPA(1971) 529)

System	$\mathbf{V}_0$ (MeV)	$\mathbf{r}_0$ (fm)	$\mathbf{a}_0$ (fm)	$\mathbf{W}_d$ (MeV)	$\mathbf{r}_i$ (fm)	$\mathbf{a}_i$ (fm)	$\mathbf{r}_C$ (fm)
d+^{56}Fe	90	1.15	0.81	21.0	1.34	0.68	1.15
p+^{56,57}Fe	47.9	1.25	0.65	11.5	1.25	0.47	1.15
$p+n$ [1]	72.15	0.00	1.484	-	-	-	
n+^{56}Fe	B.E.	1.25	0.65	-	-	-	

$$\Rightarrow U(R) = -V_0 f_{WS}(R) + 4\, i\, a\, W_d \frac{df_{WS}(R)}{dR}$$

$$f_{WS}(R) = \frac{1}{1+\exp\left(\frac{R-R_0}{a}\right)}$$

[1] Gaussian geometry: $V(r) = -V_0 \exp[-(r/a_0)^2]$.



Transfer example: ^{56}Fe(d,p)^{57}Fe

Input example for ^{56}Fe(d,p)^{57}Fe: `fe56dp_dwba.in`

```
6Fe(d,p)57Fe @ Ed=12 MeV;
AMELIST
FRESCO hcm=0.1 rmatch=20.000
     rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
     jtmax=15
     thmin=1.00 thmax=180.00 thinc=1.00
     it0=1 iter=1
     chans=1 smats=2 xstabl=1
     elab= 12 /

PARTITION namep='d' massp=2.014 zp=1 namet='56Fe'
     masst=55.934 zt=26 nex=1 /
STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0
     bandt=1 et=0.0 /

PARTITION namep='p' massp=1.0078 zp=1 namet='57Fe'
     masst=56.935 zt=26 qval=5.421 pwf=F nex=1 /
STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5
     bandt=-1 et=0.0 /
partition /

POT kp=1 itt=F at=56 rc=1.15 /
POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /

POT kp=2 itt=F at=57 rc=1.15 /
POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

```
POT kp=3 itt=F at=56 rc=1.00 /
POT kp=3 type=1 itt=F p1=65.0 p2=1.25 p3=0.65 /
POT kp=4 itt=F ap=1.0000 at=0.0000 rc=1.0000 /
POT kp=4 type=1 shape=2 itt=F p1=72.1500
     p2=0.0000 p3=1.4840 /
POT kp=5 itt=F at=56 rc=1.15 /
POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
pot /

OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 sn=0.5
     j=0.5 kbpot=4 be=2.2250 isc=1 /
OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5
     j=0.5 kbpot=3 be=7.646 isc=1 /
overlap /

COUPLING icto=2 icfrom=1 kind=7 ip2=-1 ip3=5 /
CFP in=1 ib=1 ia=1 kn=1 a=1.0000 /
CFP in=2 ib=1 ia=1 kn=2 a=1.0000 /
cfp /

coupling /
```

Transfer example: ^{56}Fe(d,p)^{57}Fe

General variables:

```
56Fe(d,p)57Fe @ Ed=12 MeV;
NAMELIST
&FRESCO hcm=0.1 rmatch=20.000
      rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
      jtmax=15
      thmin=1.00 thmax=180.00 thinc=1.00
      it0=1 iter=1
      chans=1 smats=2 xstabl=1
      elab= 12 /
```

- `rnl`: range of non-locality
- `centre,rintp,hnl`: parameters for numerical integration (see `fresco` manual)
- `iter`: Number of iterations so, for DWBA, `iter=1`

Transfer example: ^{56}Fe(d,p)^{57}Fe

Partitions and states:

- Incoming (initial) partition: d+^{56}Fe

```
&PARTITION namep='d' massp=2.014 zp=1
        namet='56Fe' masst=55.934 zt=26 nex=1 /
&STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- Outgoing (final) partition: p+^{57}Fe

```
&PARTITION namep='p' massp=1.0078 zp=1
        namet='57Fe'masst=56.935 zt=26
        qval=5.421 nex=1 /
&STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5 bandt=-1 et=0.0 /
```

- `qval`: Q-value for gs-gs transfer

Transfer example: ^{56}Fe(d,p)^{57}Fe

Interactions:

- Entrance channel distorted potential: d+^{56}Fe

```
&POT kp=1 itt=F at=56 rc=1.15 /
&POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
&POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /
```

- Exit channel distorted potential: p+^{57}Fe

```
&POT kp=2 itt=F at=57 rc=1.15 /
&POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
&POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

- Core-core potential: p+^{56}Fe

```
&POT kp=5 itt=F at=56 rc=1.15 /
&POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
&POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

Interactions (continued ...)

Binding potentials:

- n+ ^{56}Fe: Woods-Saxon

```
&POT kp=3 at=56 rc=1.0 /
&POT kp=3 type=1 p1=65.0 p2=1.25 p3=0.65 /
```

- n+p: Gaussian

```
&POT kp=4 ap=1.0 at=0.0 /
&POT kp=4 type=1 shape=2 p1=72.15 p2=0.00 p3=1.484 /
```

Bound wavefunctions (overlaps):

- d=p+n: simple 1S model

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 l=0 sn=0.5 j=0.5
      kbpot=4 be=2.2250 isc=1 /
```

- ^{57}Fe=^{56}Fe+n: assume $2p_{1/2}$ configuration

```
&OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5 j=0.5
      kbpot=3 be=7.646 isc=1 /
```

  - `in=1`: projectile
    `in=2`: target
  - `nn,l,sn,j`: quantum numbers: $\vec{l} + \vec{sn} = \vec{j}$
  - `be`: binding (separation) energy
  - `kbpot`: potential index

Transfer example: ^{56}Fe(d,p)^{57}Fe

Transfer coupling between the two partitions:

```
&COUPLING icfrom=1 icto=2 kind=7 ip1=0 ip2=-1 ip3=5 /
```

- `icfrom`: index for partition of initial state
- `icto`: index for partition of final state
- `kind`: kind of coupling. kind=7 means finite-range transfer.
- `ip1=0`: post representation
  `ip1=1`: prior
- `ip2=-1`: include full remnant
- `ip3`: index for core-core potential (p+^{56}Fe)

Transfer example: ^{56}Fe(d,p)^{57}Fe

Spectroscopic factors:

$$\phi_B^{JM}(\xi, \mathbf{r}) = \sum_{I\ell j} A_{\ell sj}^{IJ} \left[\phi_b^I(\xi) \otimes \varphi_{\ell sj}(\mathbf{r})\right]_{JM}$$

So, for example:

$$|^{57}\mathrm{Fe}; 1/2^-\rangle = \alpha \left[|^{56}\mathrm{Fe}; \mathrm{gs}\rangle \otimes |\nu 2p_{1/2}\rangle\right]_{1/2^-} + \beta \left[|^{56}\mathrm{Fe}; 2^+\rangle \otimes |\nu 2p_{3/2}\rangle\right]_{1/2^-} + \ldots$$

- α, β, $\ldots$: spectroscopic amplitudes
- $|\alpha|^2$, $|\beta|^2$, $\ldots$: spectroscopic factors

Transfer example: ^{56}Fe(d,p)^{57}Fe

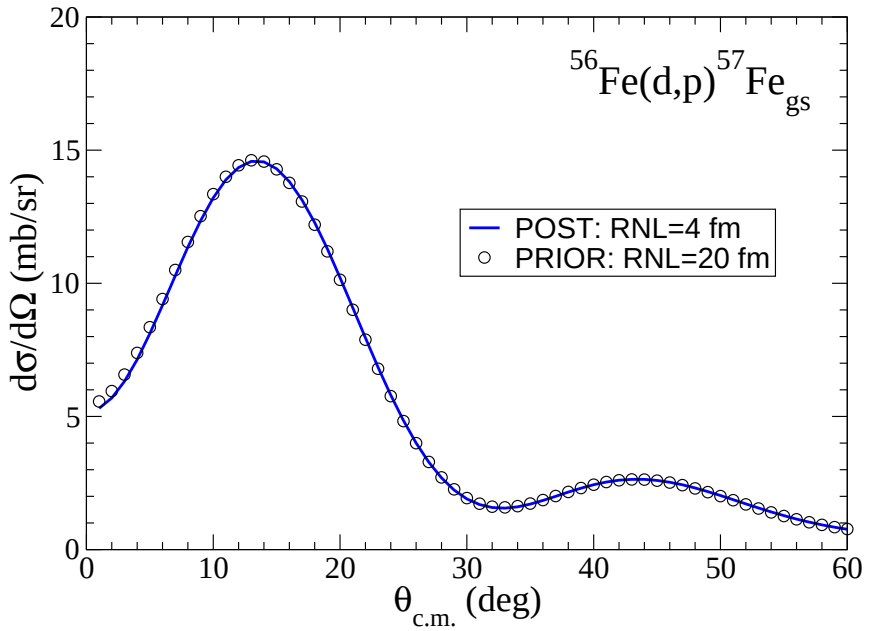
Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.0 /
&CFP in=2 ib=1 ia=1 kn=2 a=1.0 /
```

- `in=1`: projectile state
  `in=2`: target state
- `ib`: index for state of composite
  `ia`: index for state of core
- `a`: spectroscopic amplitude

Transfer example: ^{56}Fe(d,p)^{57}Fe

Post and prior equivalence:

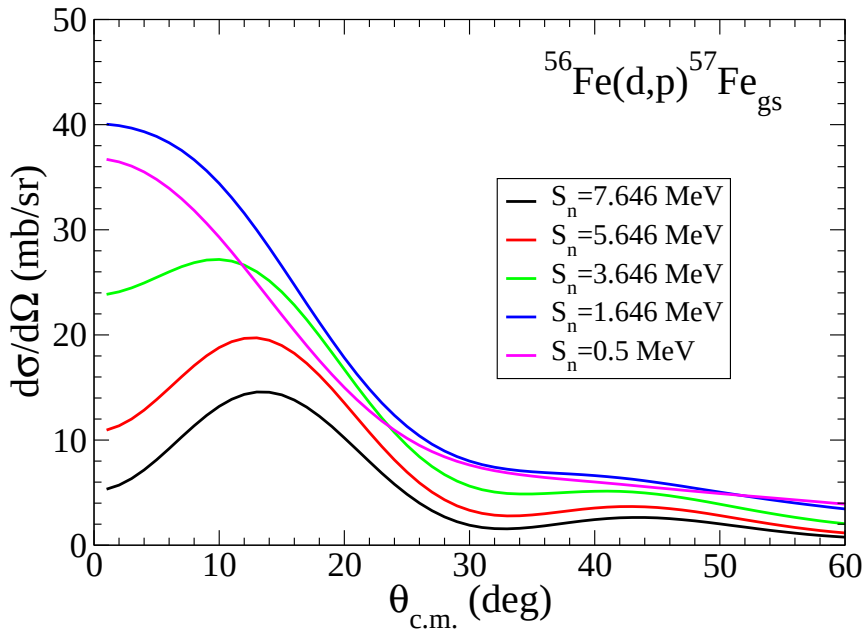


☞ *Post and prior give identical results, provide that the parameters are adequate for convergence.*

Transfer example: $^{56}Fe(d,p)^{57}Fe$

Dependence with binding energy:

```
&OVERLAP kn1=2 (...) be=7.646 /
&OVERLAP kn1=2 (...) be=5.646 /
&OVERLAP kn1=2 (...) be=5.646 /
&OVERLAP kn1=2 (...) be=3.646 /
&OVERLAP kn1=2 (...) be=1.646 /
&OVERLAP kn1=2 (...) be=0.100 /
```



Transfer example: ^{56}Fe(d,p)^{57}Fe

Dependence with beam energy:

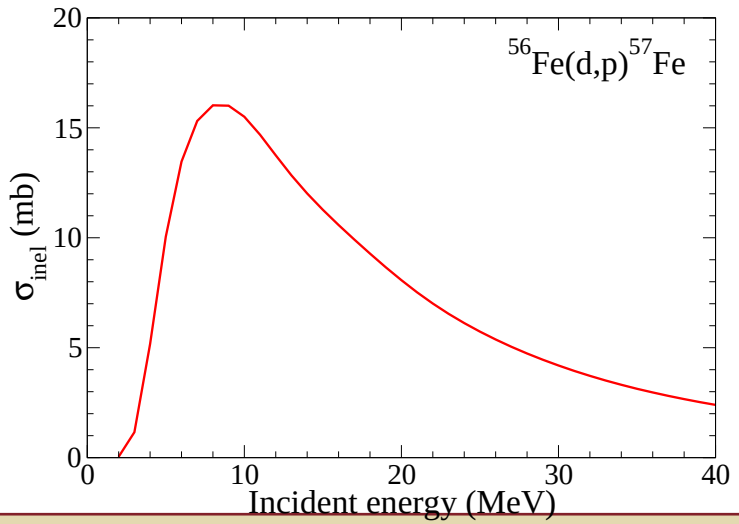
- Calculating several energies in one single run. In `fe56dp_ebeam.in` :

```
&FRESCO (...) elab(1:4)= 2 40 0 0 nlab(1:3)= 38 0 0 /
```

  will run FRESCO for $E_{\rm lab}$=2, 3, 4, . . . , 40 MeV.

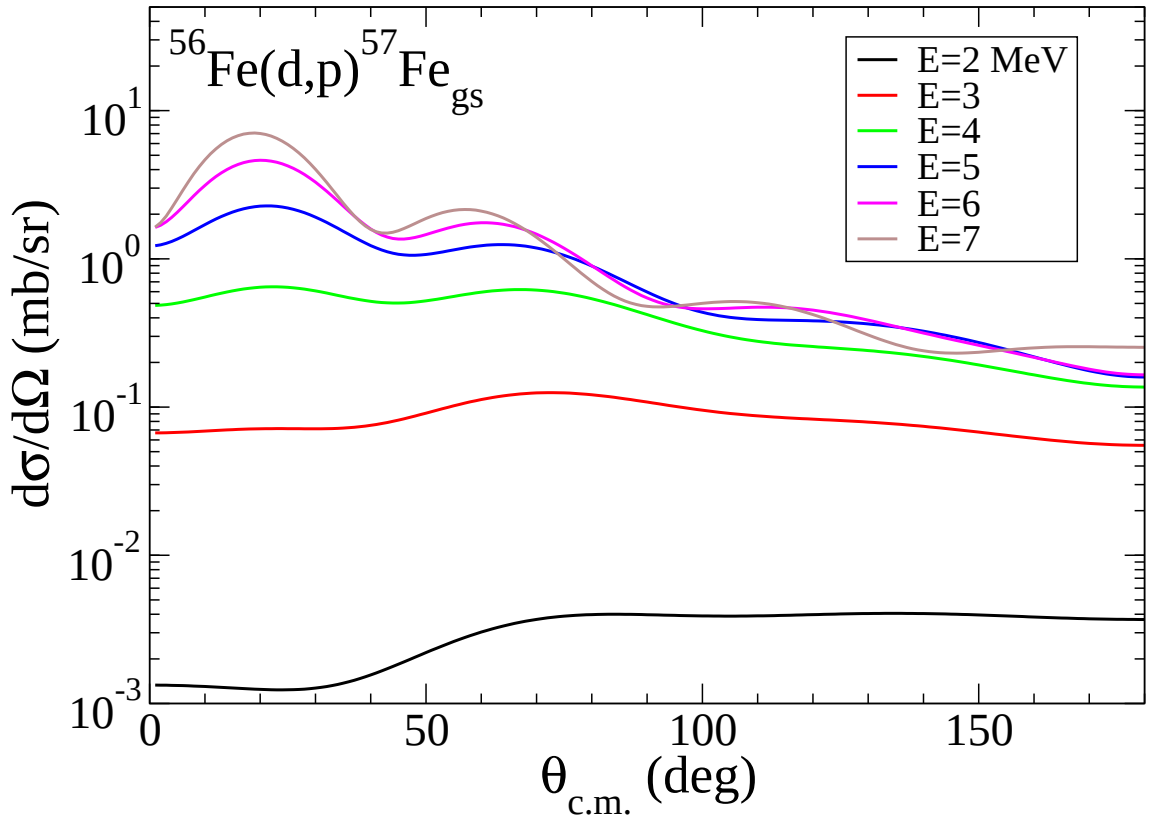
- Extracting total inelastic cross sections from main output:

```
grep "partition 2" fe56dp_ebeam.out | awk '{print NR+1, $9}' > xsec_vs_elab.dat
```



Dependence with beam energy

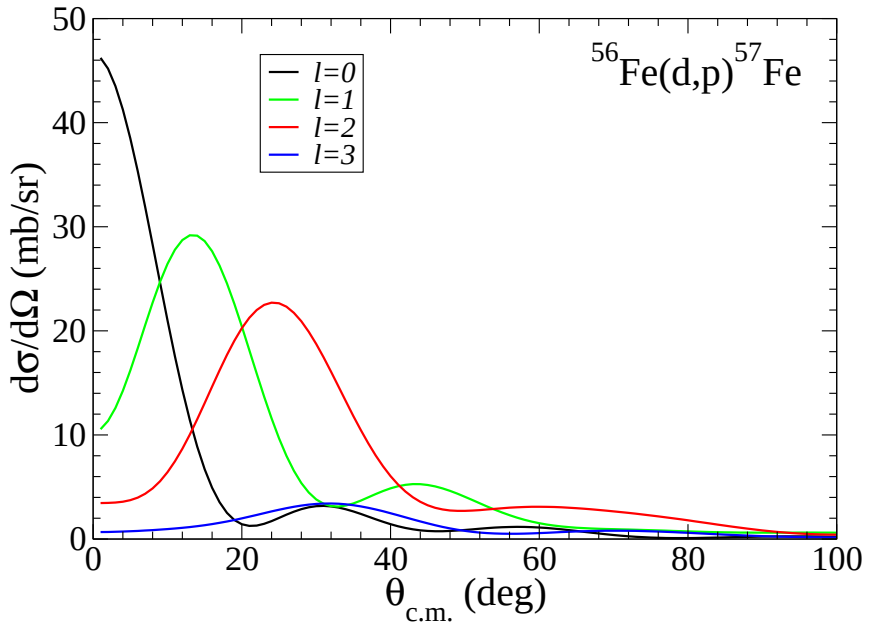
- $E \gg V_b$: diffractive structure, forward peaked.
- $E \ll V_b$: smooth dependence with θ, backward peaked.



Transfer example: ^{56}Fe(d,p)^{57}Fe

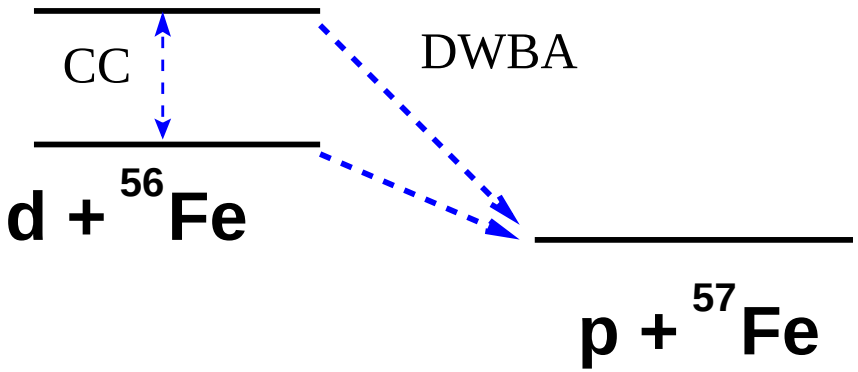
Selectivity of ℓ:

```
&OVERLAP (...) nn=2 l=0 sn=0.5 j=0.5 kbpot=3 be=7.646 /
&OVERLAP (...) nn=2 l=1 sn=0.5 j=1.5 kbpot=3 be=7.646 /
&OVERLAP (...) nn=2 l=2 sn=0.5 j=2.5 kbpot=3 be=7.646 /
&OVERLAP (...) nn=1 l=3 sn=0.5 j=2.5 kbpot=3 be=7.646 /
&OVERLAP (...) nn=1 l=4 sn=0.5 j=4.5 kbpot=3 be=7.646 /
```



Beyond DWBA: CCBA formalism

When there are strongly coupled excited states in the initial or final partition, the CC and DWBA formalisms can be combined → CCBA



Proposed exercise: ^{208}Pb(^{8}Li,^{7}Li)^{209}Pb

Proposed exercise: Calculate the transfer differential cross section angular distribution for 1 neutron transfer reaction ^{208}Pb(^{8}Li,^{7}Li)^{209}Pb, leading to the ground state of the ^{209}Pb nucleus ($J^{\pi} = 9/2^{+}$).
Ingredients:

- for the optical potentials, you may use those of the previous exercises.
- for the bound wavefunctions, use a WS potential with $r_0 = 1.25$ fm and $a_0 = 0.65$ fm.
- assume unit spectroscopic factors for the initial and final states.

Input example for ^{208}Pb(^{8}Li,^{7}Li)^{209}Pb: `li8pb_trans.in`

```
208Pb(8Li,7Li)209Pb(gs) input example
NAMELIST
&FRESCO hcm=0.05 rmatch=60.000 rintp=0.2
      hnl=0.025 rnl=3 centre=0.00 jtmin=0.0 jtmax=80.
      thmin=2.00 thmax=180.00 thinc=2.00 it0=1 iter=1 smats=2
      xstabl=1 elab=34.404 /

&PARTITION namep='8Li' massp=8.0225 zp=3
      namet='208Pb' masst=207.9766 zt=82 qval=0.0000 pwf=T
      nex=1 /
&STATES jp=2.0 bandp=1 ep=0.0000 kkp=1.0 cpot=1
      jt=0.0 bandt=1 et=0.0000 kkt=0.0 fexch=F /

&PARTITION namep='7Li' massp=7.0160 zp=3
      namet='209Pb' masst=208.9810 zt=82 qval=1.9040 pwf=T
      nex=1 /
&STATES jp=1.5 bandp=-1 ep=0.0000 cpot=2 jt=4.5
      bandt=1 et=0.0000 fexch=F /

&partition /
```

(continued ...)

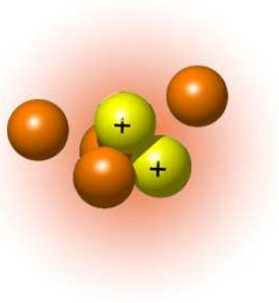
```
&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=58.3 p5=1.3 p6=0.7 /

 &POT kp=2 itt=F at=209.000 ap=7.000 rc=1.250 /
&POT kp=2 type=1 itt=F p1=15.4 p2=1.300 p3=0.650
      p4=13.2 p5=1.3 p6=0.65 p7=0.000 /

&POT kp=3 type=0 shape=0 itt=F ap=7 at=0.000 /
&POT kp=3 type=1 shape=0 itt=F p1=44.675
      p2=1.25 p3=0.65 /

&POT kp=4 itt=F ap=208.000 at=0.000 rc=1.250 /
&POT kp=4 type=1 itt=F p1=60.000 p2=1.250
      p3=0.650 /

&POT kp=5 itt=F at=208.000 ap=7.000 rc=1.250 /
&POT kp=5 type=1 itt=F p1=15.4 p2=1.300 p3=0.650
      p4=13.2 p5=1.300 p6=0.65 p7=0.000 /
&pot /
```

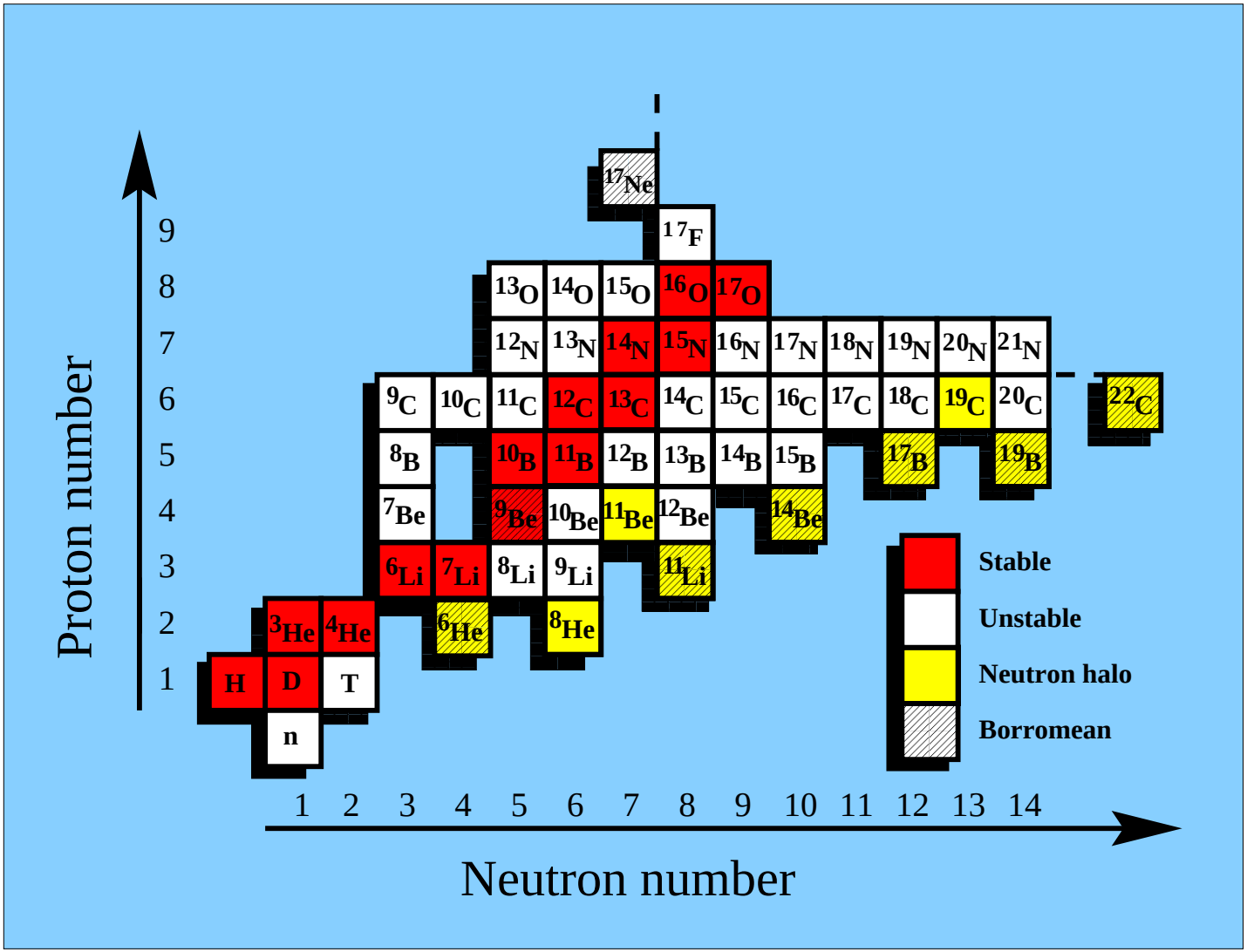
(continued ...)

```
 &OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 l=1 sn=0.5
      ia=1 j=1.5 ib=1 kbpot=3 be=2.0330 isc=1
      ampl=0.0000 /
&OVERLAP kn1=5 ic1=1 ic2=2 in=2 nn=2 l=4 sn=0.5
      j=4.5 kbpot=4 be=3.9440 isc=1 nam=1 ampl=0.0000 /
&overlap /

&COUPLING icto=-2 icfrom=1 kind=7 ip2=-1 ip3=5 /
&CFP in=1 ib=1 ia=1 kn=1 a=1 /
&CFP in=2 ib=1 ia=1 kn=5 a=0.8450 /
&cfp /

&coupling /
```

Lecture 4: Calculations including continuum states

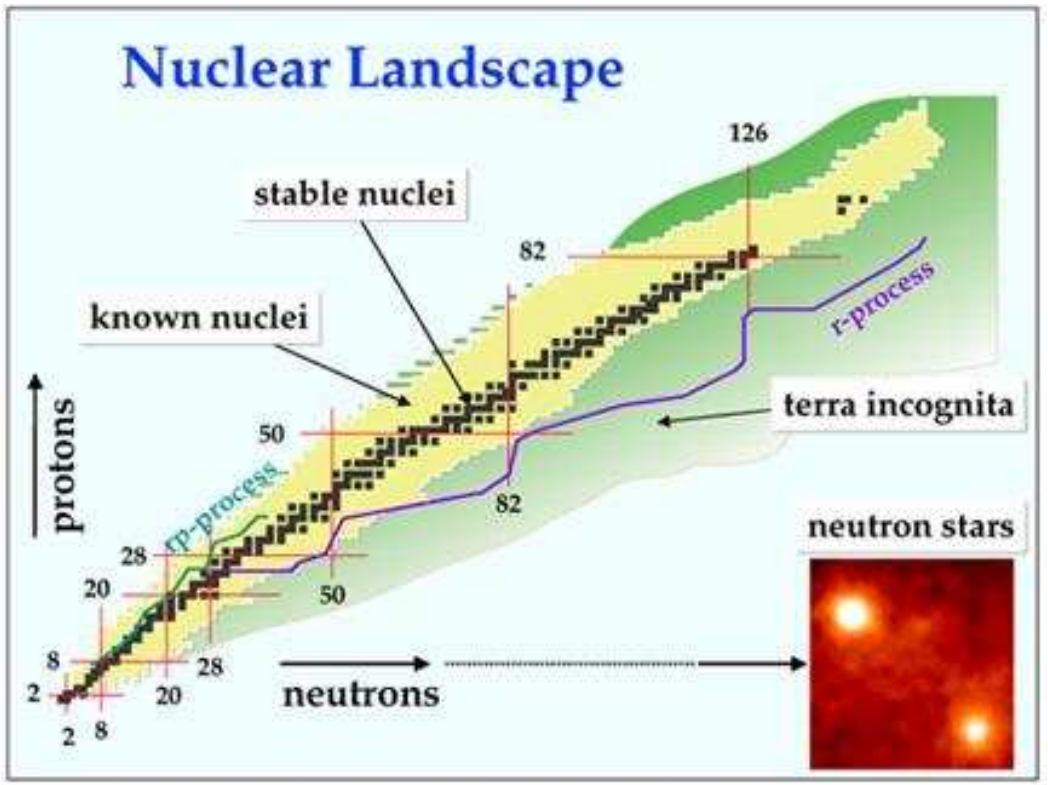
- Radioactive nuclei: they typically decay by β emission.
  E.g.: ^{6}He $\xrightarrow{\beta^-}$ ^{6}Li ($t_{1/2} \simeq$807 ms)
- Weakly bound: typical separation energies are around 1 MeV or less.
- Spatially extended
- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.
- Borromean nuclei: Three-body systems with no bound binary sub-systems.




Proton number
Neutron number
1
2
3
4
5
6
7
8
9
1 2 3 4 5 6 7 8 9 10 11 12 13 14
H
D
T
n
3He
4He
6He
6Li
7Li
8Li
9Li
8He
7Be
9Be
10Be
11Be
12Be
11Li
14Be
8B
10B
11B
12B
13B
14B
15B
17B
19B
9C
10C
11C
12C
13C
14C
15C
16C
17C
18C
19C
20C
22C
12N
13N
14N
15N
16N
17N
18N
19N
20N
21N
13O
14O
15O
16O
17O
17F
17Ne
Stable
Unstable
Neutron halo
Borromean

Why study reactions with exotic nuclei?

- Many properties of nuclear structure (level spacing, magic numbers,etc) could be different from normal nuclei.
- Many reactions of astrophysical interest are known to involve nuclei far from the stability valley.



Some difficulties inherent to the study of reactions with exotic beams

Experimentally:

- Exotic nuclei are short-lived and difficult to produce. Beam intensities are typically small.

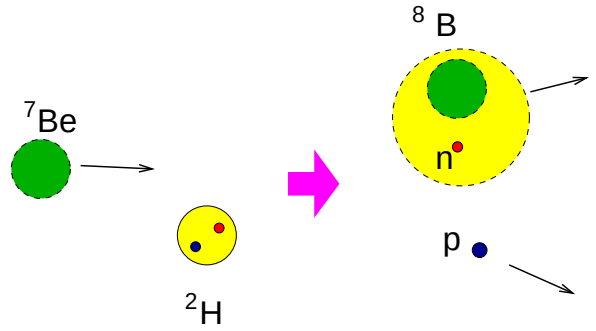
Theoretically:

- Exotic nuclei are easily broken up in nuclear collisions $\Rightarrow$ coupling to the continuum plays an important role.
- Effective NN interactions, level schemes, etc are different from stable nuclei.
- Many exotic nuclei exhibit complicated cluster (few-body) structure.

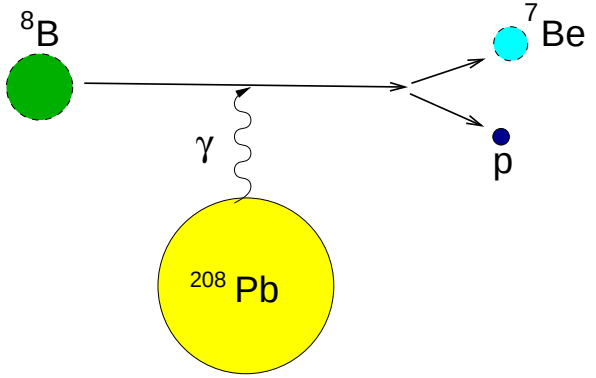
- Many reactions of astrophysical interest at energies too low to be measured e at present experimental facilities.
- Coulomb breakup and transfer reactions provide indirect information for these processes.

Example: The rate for the ^{7}Be+ p $\rightarrow$ ^{8}B + γ reaction depends mainly on the overlap (^{8}B$|^{7}$Be), which can be investigated by means of other direct reactions:

Transfer reactions:
E.g.: d(^{7}Be, ^{8}B)n

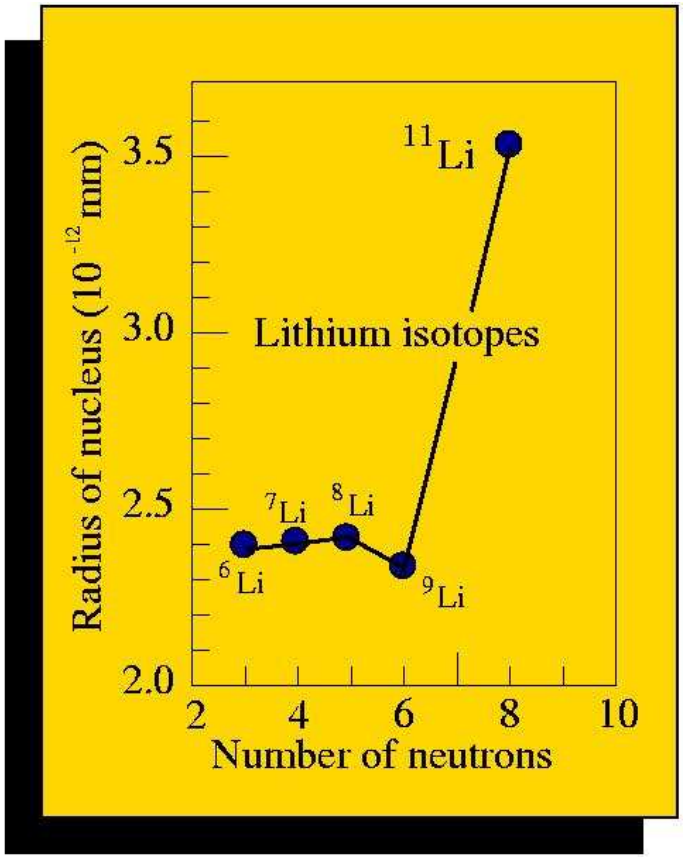


Coulomb breakup reactions:
E.g.: ^{8}B+^{208}Pb $\rightarrow$ ^{7}Be+ p + ^{208}Pb



Some difficulties inherent to the study of reactions with exotic beams

☞ First evidences of the existence of halo nuclei came from reaction cross sections measurements.

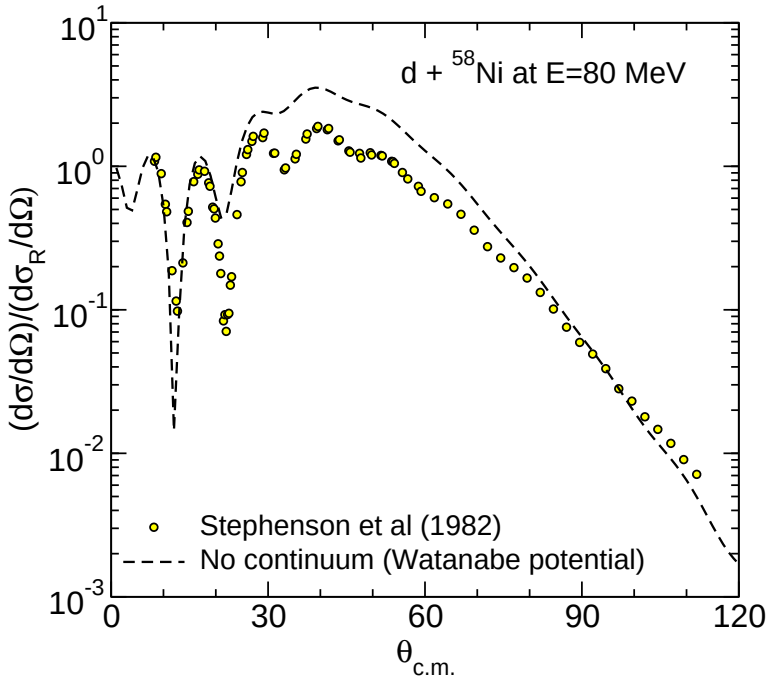


Tanihata *et al*, PRL55, 2676 (1985)

Failure of the calculations without continuum

Three-body calculation (p+n+^{58}Ni) with Watanabe potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\phi_{\rm gs}(\mathbf{r})\left(V_{pt}+V_{nt}\right)\phi_{\rm gs}(\mathbf{r})$$



☞*Three-body calculations omitting breakup channels fail to describe the experimental data.*

The origins of the CDCC method:

- Pioneering work of Johnson & Soper for deuteron scattering: PRC1,976(1970) $\Rightarrow$ p-n continuum represented by a single s-wave state.

PHYSICAL REVIEW C VOLUME 1, NUMBER 3 MARCH 1970

Contribution of Deuteron Breakup Channels to Deuteron Stripping and Elastic Scattering

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AND

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International Centre for Theoretical Physics, Trieste, Italy

(Received 10 November 1969)

We present a model of deuteron stripping and elastic scattering which treats explicitly the contributions from channels in which the deuteron is broken up into a relative S state and the target is in its ground state. An adiabatic treatment of these channels leads to a description of deuteron stripping which resembles

- More realistic formulation by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)].

PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

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Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139,†
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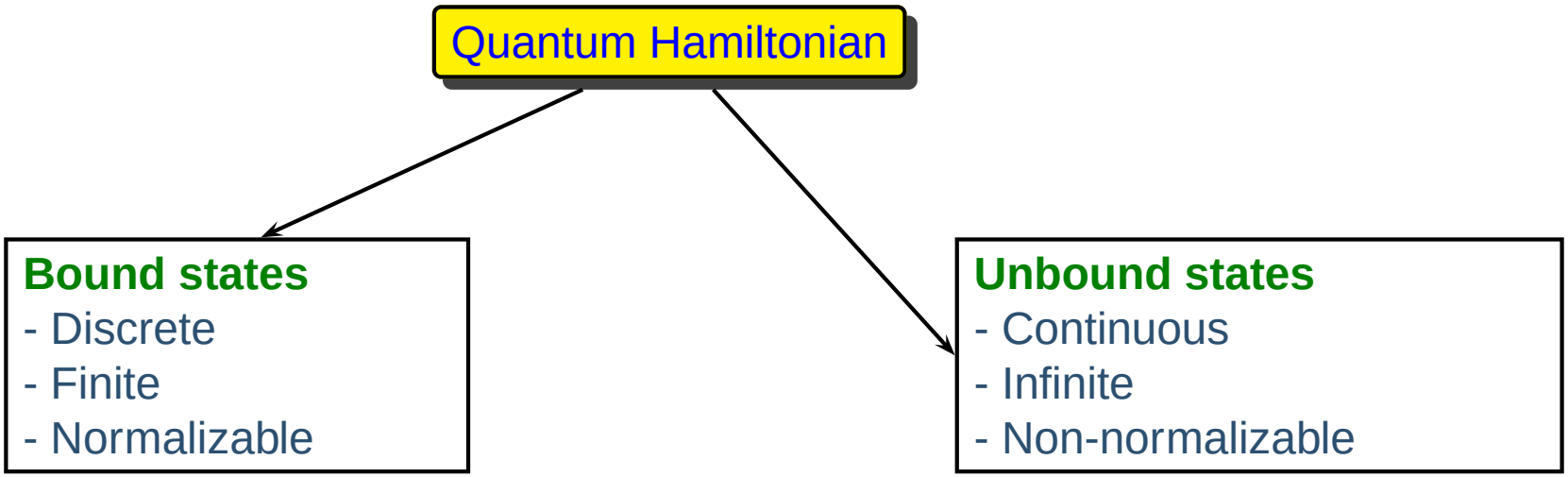
(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transi-

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Inclusion of the continuum in CC calculations: continuum discretization



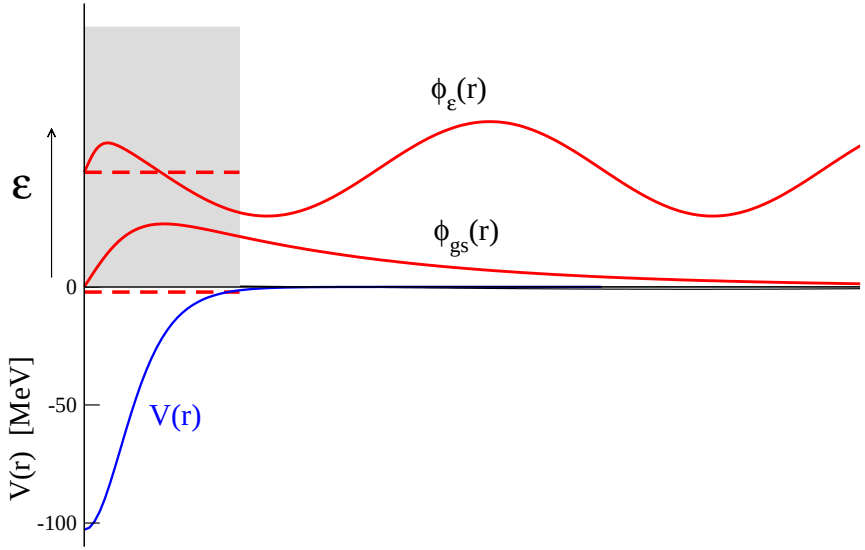
Continuum discretization: represent the continuum by a finite set of square-integrable states

***True* continuum**	$\longrightarrow$	**Discretized continuum**
Non normalizable	$\longrightarrow$	Normalizable
Continuous	$\longrightarrow$	Discrete

SOME POPULAR METHODS OF CONTINUUM DISCRETIZATION:

- Box method:
  Eigenstates of the H in a large box.
- Sturmian basis
- Gamow states: complex-energy eigenstates of the Schrödinger equation.
- Pseudostate method:
  Expand continuum states in a complete basis of square-integrable states (eg. HO)
- Bin method:
  Square-integrable states constructed from scattering states.

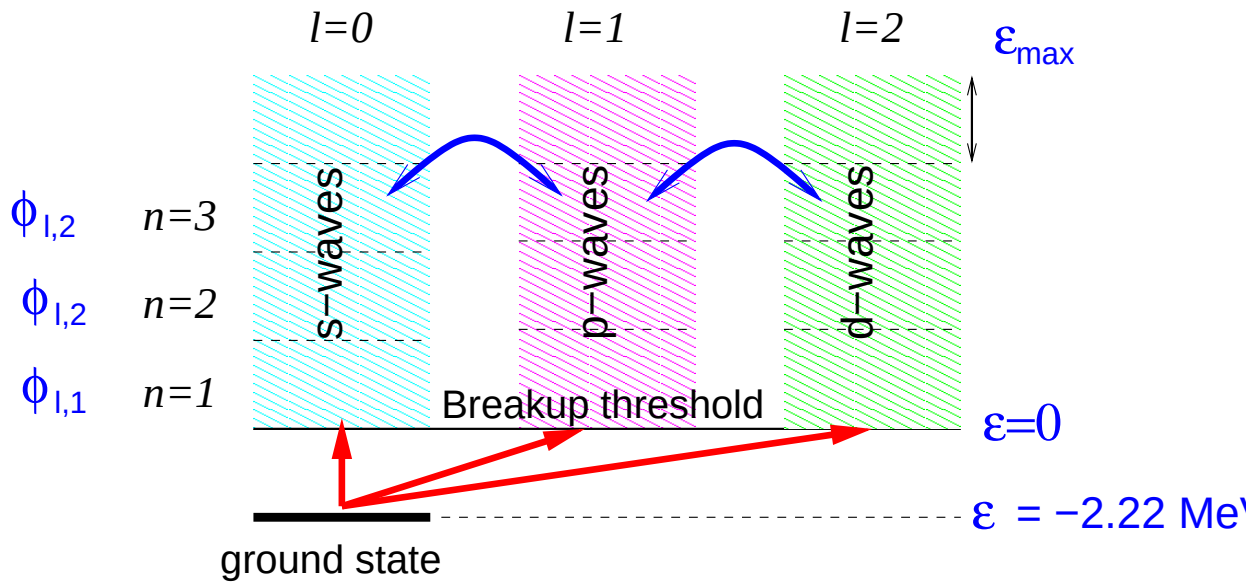
Bound versus scattering states



Unbound states are not suitable for CC calculations:

- Continuous (infinite) distribution in energy.
- Non-normalizable: $\langle \phi_k(r)^* | \phi_{k'}(r) \rangle \propto \delta(k - k')$

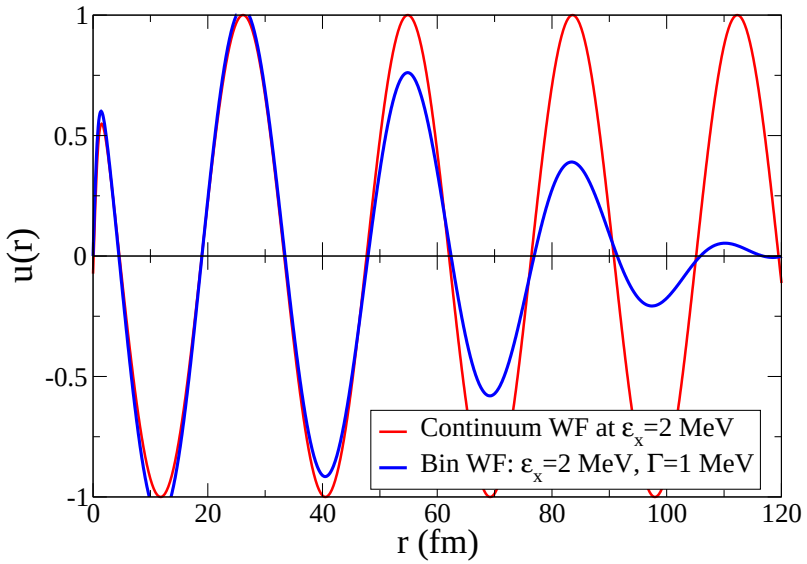
Example: discretization of the deuteron continuum in terms of energy bins.



Bin wavefunction:

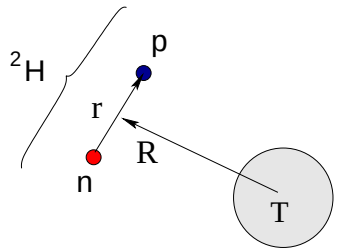
$$u_{\ell sj,n}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{\ell sj,k}(r) dk$$

- k: linear momentum
- $u_{\ell sj,k}$: scattering states (radial part)
- $w(k)$: weight function



- Hamiltonian: $H = T_R + h_r + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:

$$\Psi(\mathbf{R},\mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$



- Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r})$

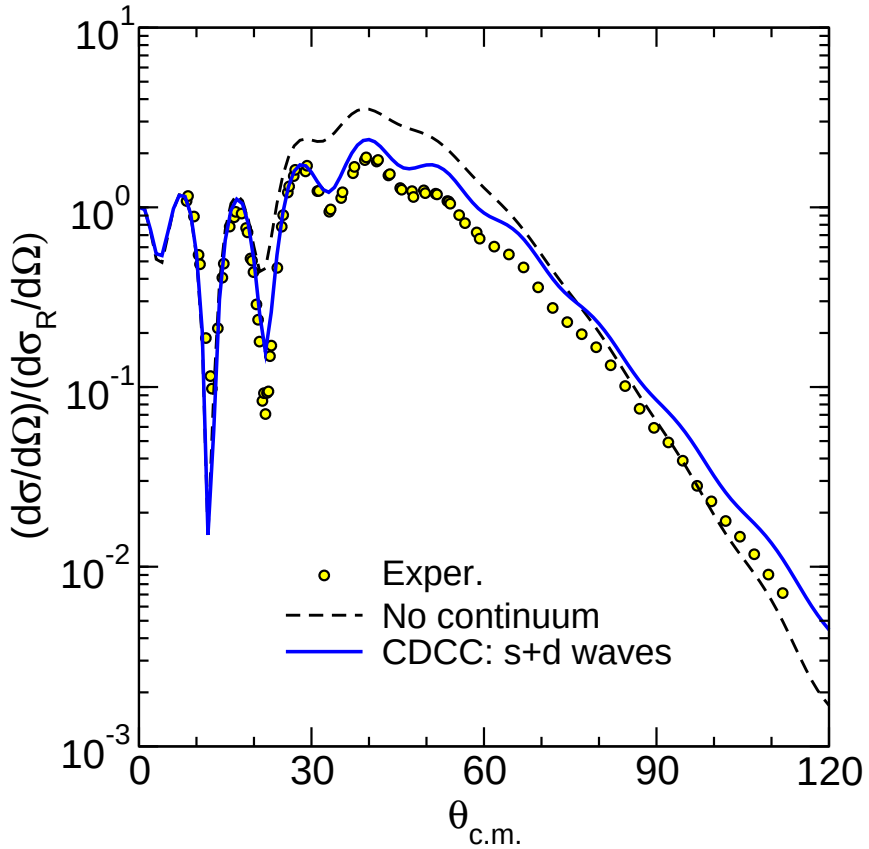
$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2}\right) + \epsilon_n - E\right] f_{\alpha J}(R) + \sum_{\alpha'} i^{L'-L}\, V^J_{\alpha:\alpha'}(R) f_{\alpha' J}(R) = 0$$

$\alpha = \{L, \ell, s, j, n\}$

- Transition potentials:

$$V^J_{\alpha;\alpha'}(\mathbf{R}) = \int d\mathbf{r}\phi_\alpha(\mathbf{r})^* \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2})\right] \phi_{\alpha'}(\mathbf{r})$$

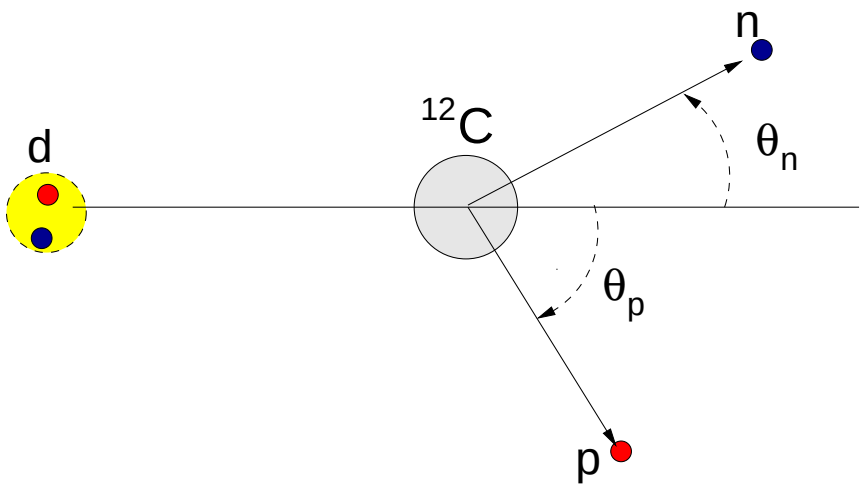
Application of the CDCC formalism: d+ ^{58}Ni



☞Inclusion of the continuum is important to describe the data

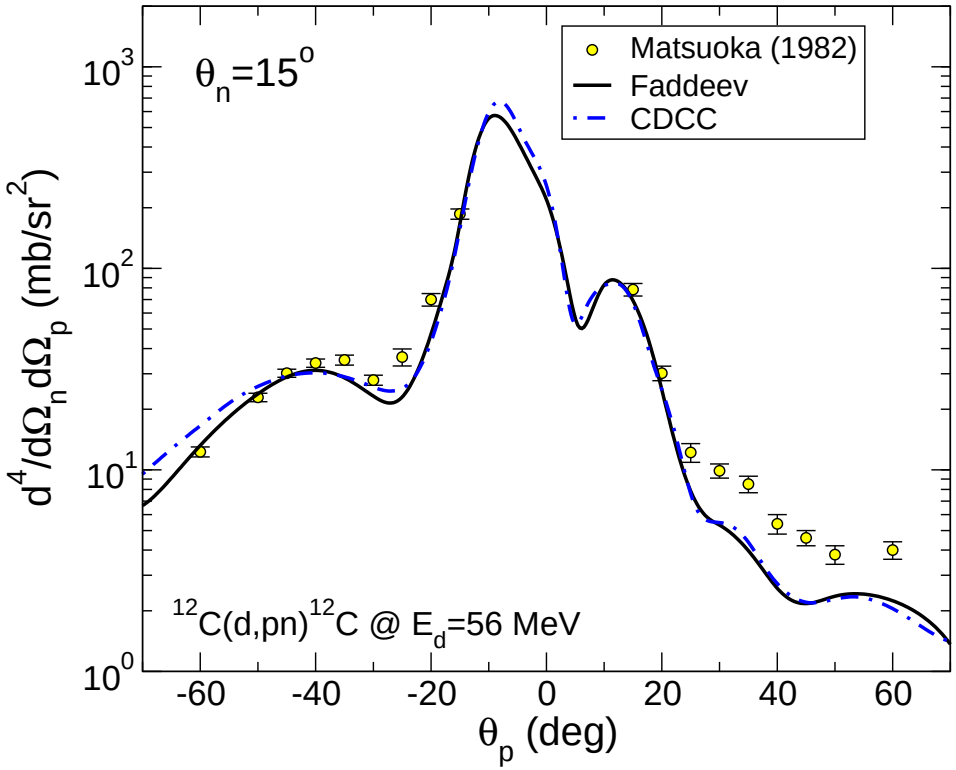
Observables for exclusive breakup can be obtained from the S-matrix elements:

$$\Psi^{\mathrm{CDCC}}(\mathbf{r},\mathbf{R}) \rightarrow S_{\mathrm{gs},n} \rightarrow \frac{d\sigma^3}{d\Omega_p d\Omega_v dE_p}$$



N. Matsuoka *et al.*, Nucl. Phys. **A 391**, 357 (1986).

Observables for exclusive breakup: proton angular distribution

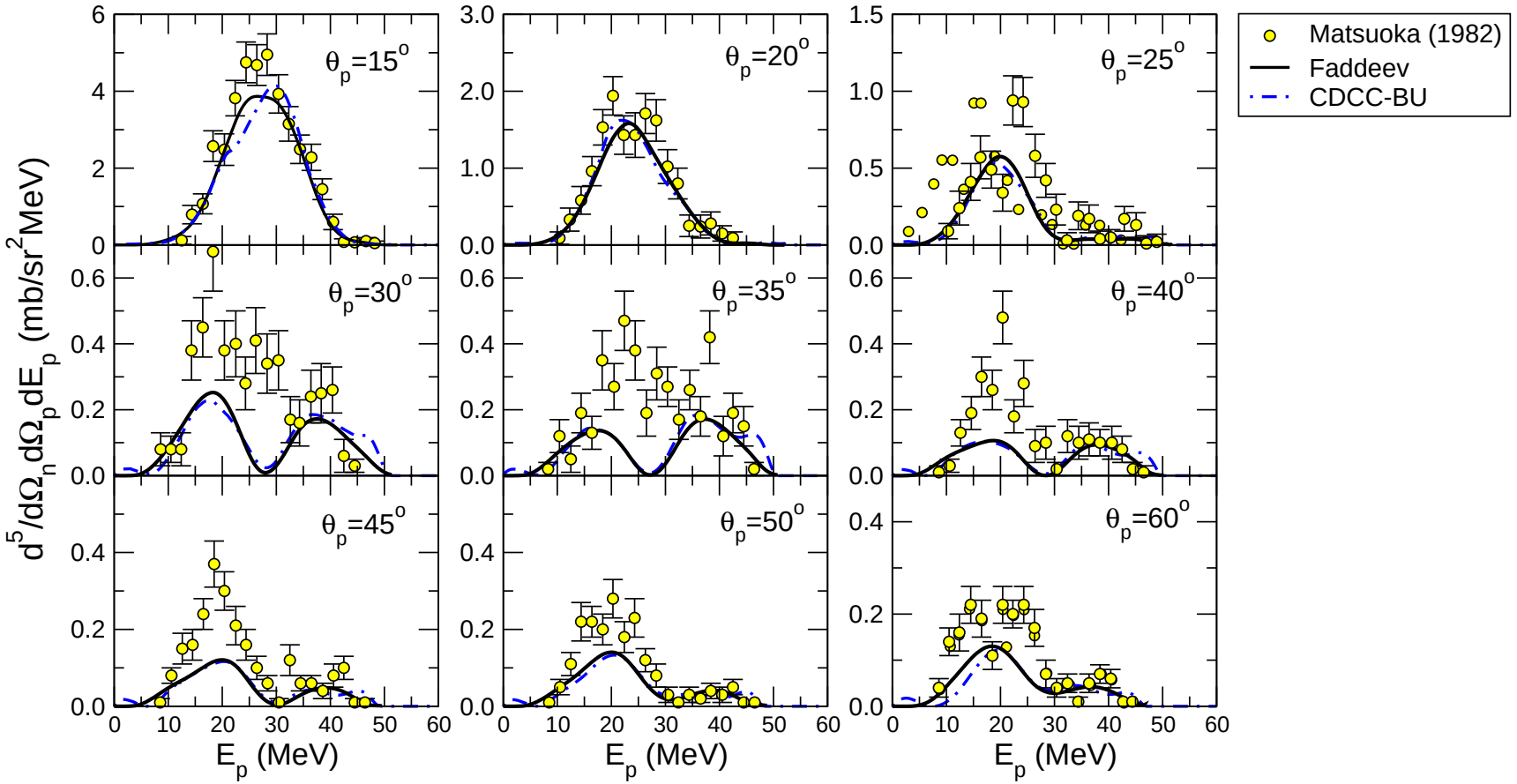


A.Deltuva, A.M.M., E.Cravo, F.M.Nunes, A.C.Fonseca, Phys.Rev. C 76, 064602 (2007)

☞ Very good agreement with Faddeev calculations!

Application of the CDCC formalism: d+ ^{58}Ni

Observables for exclusive breakup: proton energy distribution for fixed θ_n and θ_p

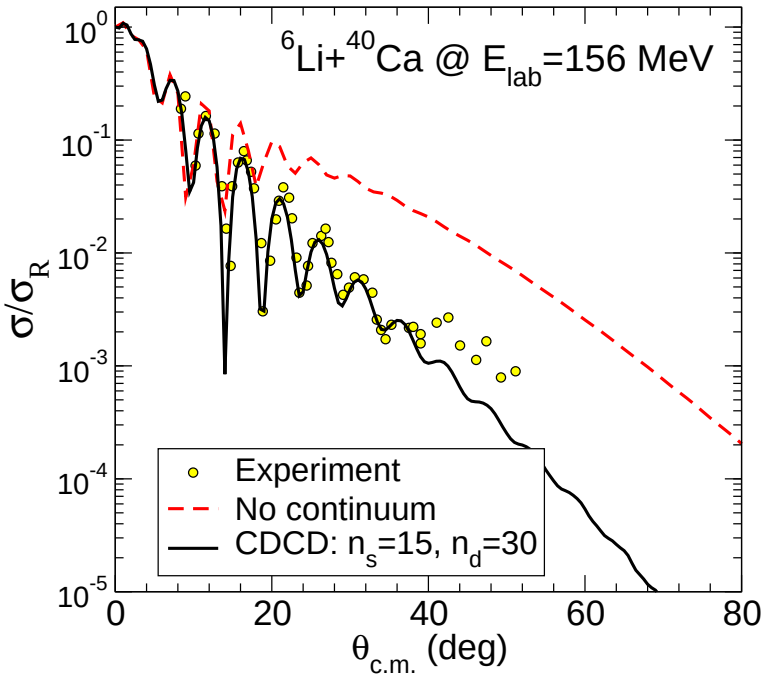


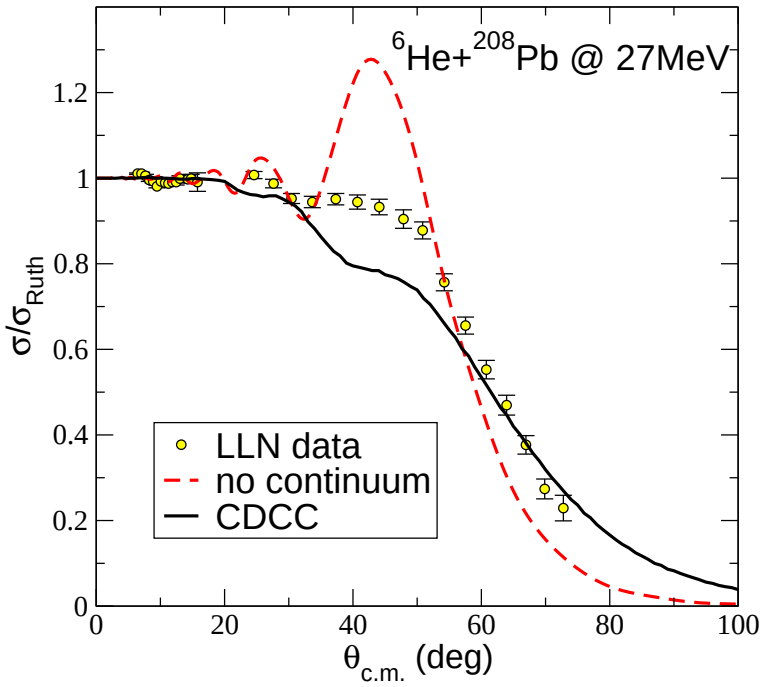
A.Deltuva et al, Phys.Rev. C 76, 064602 (2007)

Application of the CDCC method: ^{6}Li and ^{6}He scattering

☞ The CDCC has been also applied to nuclei with a cluster structure:

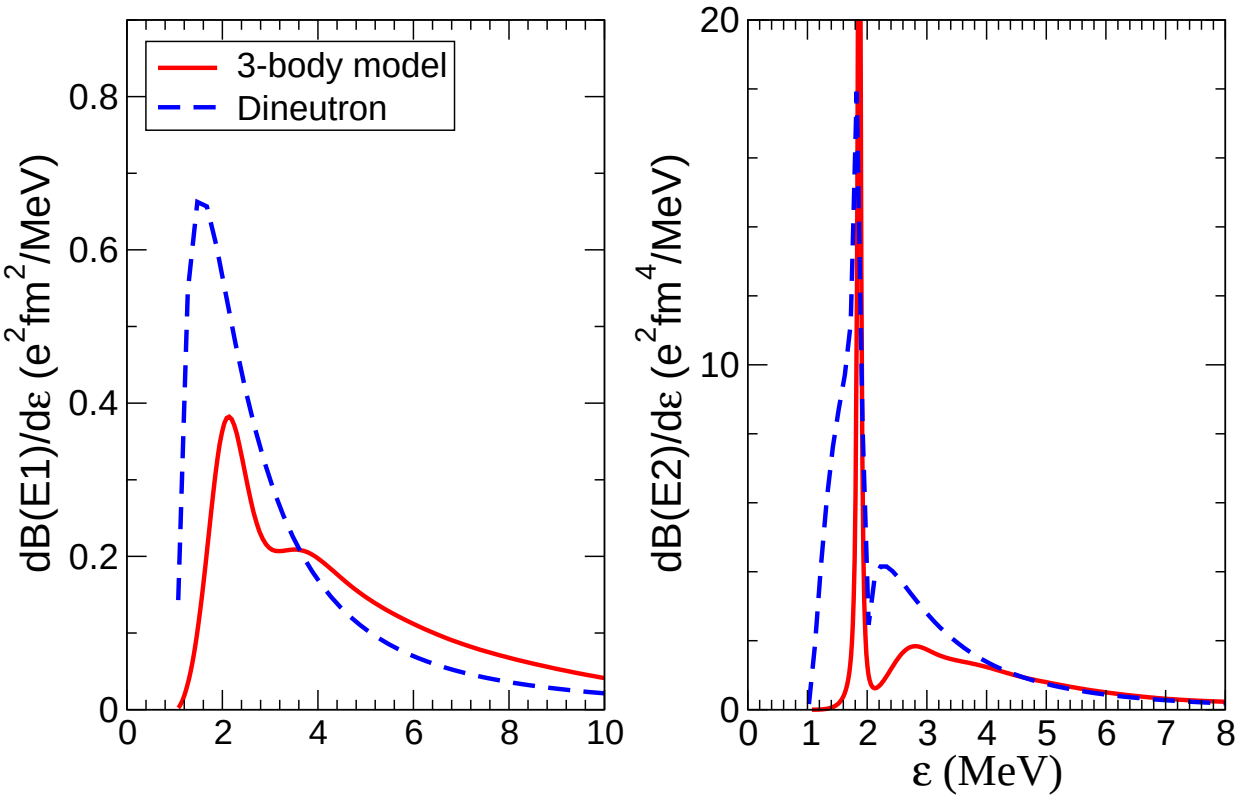
- ^{6}Li=α + d
- ^{6}He=α + ^{2}n





☞ *CDCC works for ^{6}Li but fails for ^{6}He!*

^{6}He+^{208}Pb within a di-neutron model

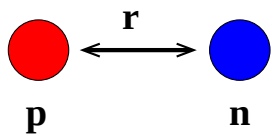


➪ *The dineutron model tends to overestimate the coupling to the continuum: we need a more sophisticated model for ^{6}He $\Rightarrow \alpha$ +n +n*

Extension of the CDCC method to 3-body projectiles: four-body CDCC calculations

2-body case: 1 single degree of freedom (inter-cluster relative coordinate)

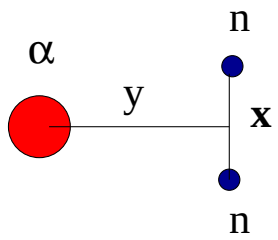
$$\phi_{n,\ell m}(\vec{r}) = R_{n\ell}(r) Y_{\ell,m}(\Omega),$$



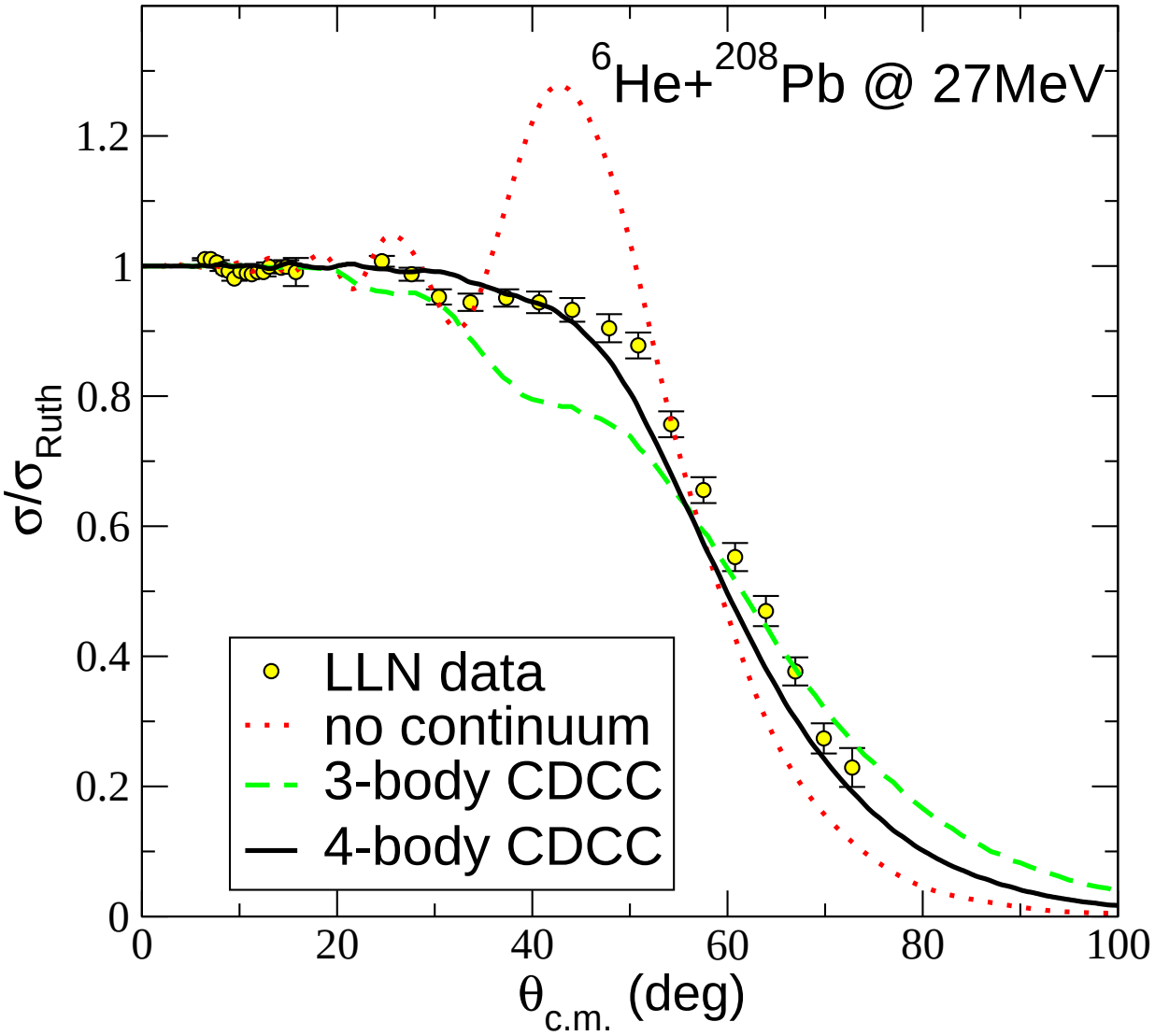
3-body case:

- 2 degrees of freedom (6 coordinates)
- 3-body Hamiltonian: $H = T + V_{nn} + V_{n\alpha} + V_{n\alpha} + V_{nn\alpha}$
- The 3-body wavefunction can be expressed in different coordinate systems:
  - Jacobi coordinates: $\{\mathbf{x}, \mathbf{y}\}$
  - Hypersherical coordinates: $\{\rho, \Omega_x, \Omega_y, \alpha\}$ $\quad \rho^2 \equiv x^2 + y^2 \quad \tan\alpha = \frac{x}{y}$

$$\Psi_{jm}(\rho, \Omega) = \frac{1}{\rho^{5/2}} \sum_{\beta=1}^{N_\beta} R_{n\beta}(\rho) \left[\Upsilon_{Kl}^{l_x l_y}(\Omega_x, \Omega_y, \alpha) \otimes X_S \right]_{jm}$$



Four-body CDCC calculations: ^{6}He+^{208}Pb



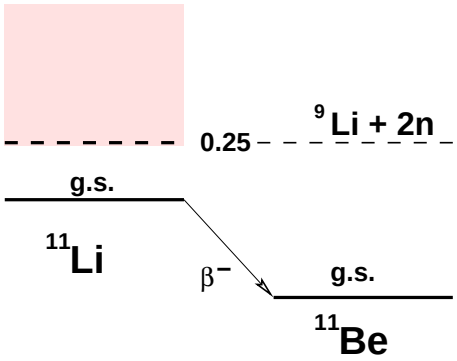
Present and future developments

- Calculation of nucleon-nucleon and nucleon-core correlations from three-body breakup
- Application to more complicated Borromean nuclei: ^{11}Li, ^{14}Be, etc
- Core excitation: ^{11}Be=^{10}Be* + n
  Summers, Nunes and Thompson, Phys.Rev. C 74, 014606 (2006)

Spectroscopy to unbound states: the ^{9}Li(d,p)^{10}Li case

The ^{10}Li and ^{11}Li systems

- ^{11}Li radioactive:
  $^{11}\text{Li} \xrightarrow{\beta^-} {}^{11}\text{Be}$ ($t_{1/2} \simeq$8.5 ms)



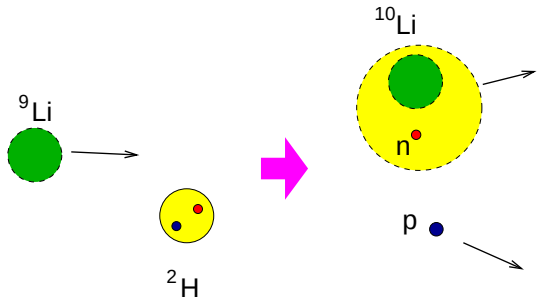
- ^{11}Li key example of Borromean nucleus:
  - n+n and n+^{9}Li unbound but,
  - n+n+^{9}Li has a (weakly) bound state.

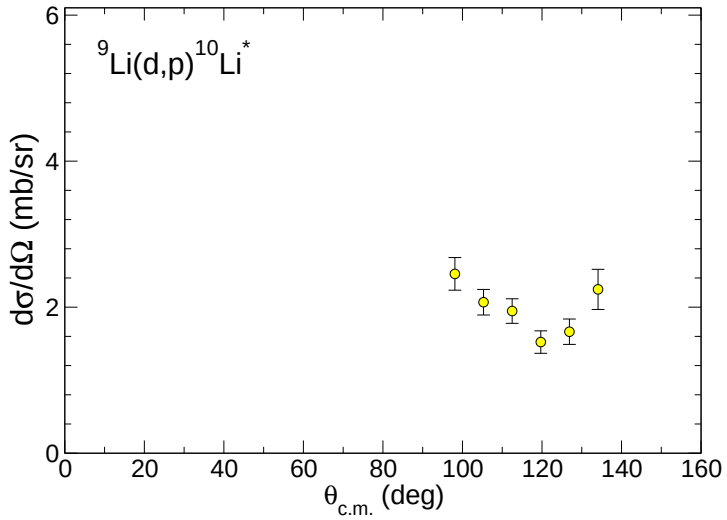
- The structure of ^{11}Li depends critically on:
  - n+n $\rightarrow$ well understood.
  - n+^{9}Li $\rightarrow$ dominated by: $\left\{ \begin{array}{l} \text{p-wave resonance} \\ \text{s-wave virtual state} \end{array} \right\} \rightarrow$ not well understood

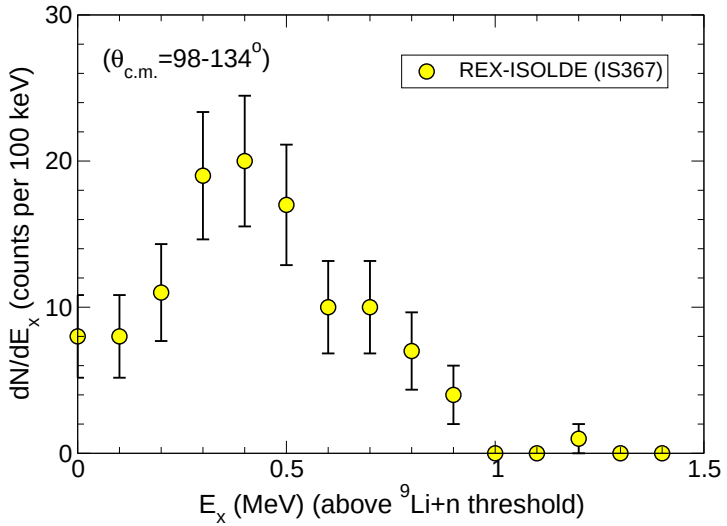
Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

The Experiment:

- REX-ISOLDE (2002)
- ^{9}Li beam on D target at $E = 2.75$ MeV/u
- The experiment provided angular and energy distributions for protons $\Rightarrow$ ^{10}Li.







Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

MOTIVATION: *What spectroscopic information can be obtained from the ^{10}Li distributions?*

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

MOTIVATION: *What spectroscopic information can be obtained from the ^{10}Li distributions?*

IMPORTANT QUESTIONS TO ADDRESS:

- Q: What is the mechanism producing protons?

  A: Direct (compound estimated to be very small)

- Q: How does the reaction mechanism affect the ^{10}Li energy spectrum? (eg. relationship between observed bump and continuum structures)

  A: Requires a proper reaction calculation; not just a fit to data

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

STRUCTURE + REACTION

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

STRUCTURE + REACTION

^{10}Li

^{9}Li	+	n	= ^{10}Li
$3/2^-$	+	$s_{1/2}$	$\Rightarrow 1^-/2^-$
$3/2^-$	+	$p_{1/2}$	$\Rightarrow 1^+/2^+$

$p_{3/2}$: bound state ($\epsilon_b \simeq 4.1$ MeV)

$s_{1/2}$: deeply 0s bound state
virtual 1s virtual state

$p_{1/2}$: low energy resonance

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

STRUCTURE + REACTION

^{10}Li

^{9}Li	+	n	= ^{10}Li
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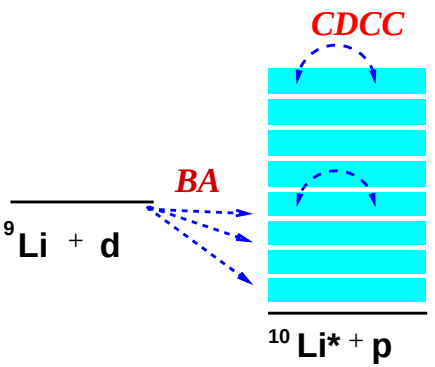
$p_{3/2}$: bound state ($\epsilon_b \simeq 4.1$ MeV)

$s_{1/2}$: deeply 0s bound state
virtual 1s virtual state

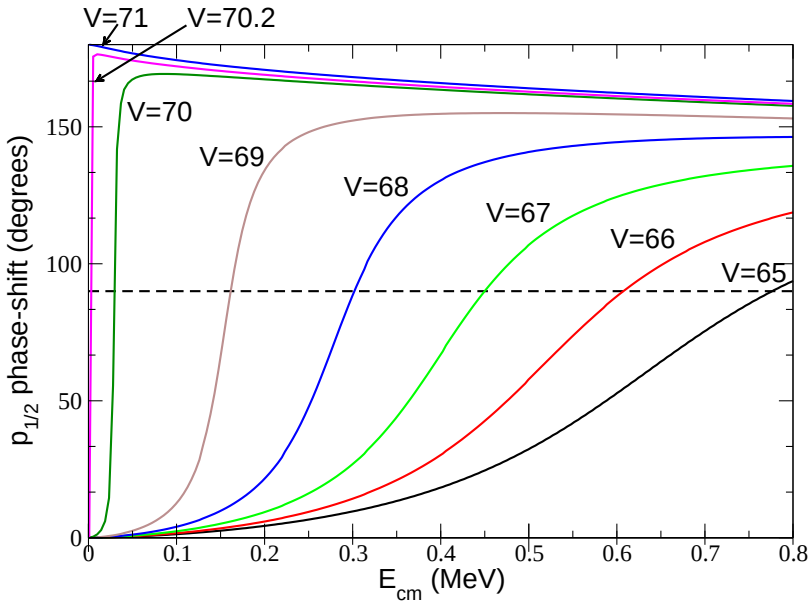
$p_{1/2}$: low energy resonance

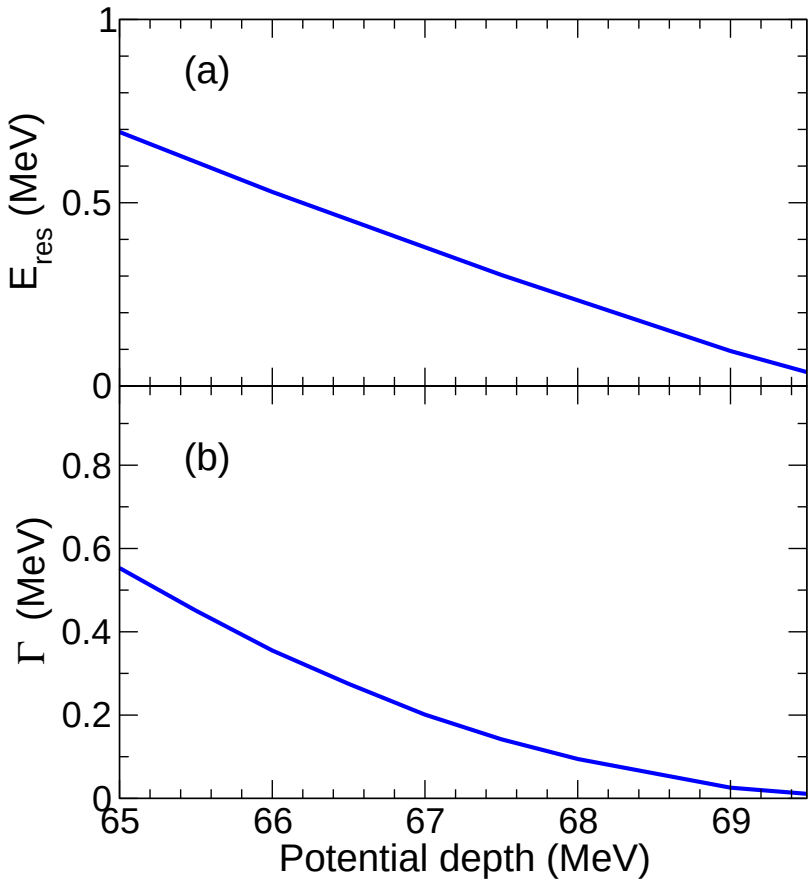
Transfer to the continuum
(direct mechanism)



Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

$p_{1/2}$ resonance

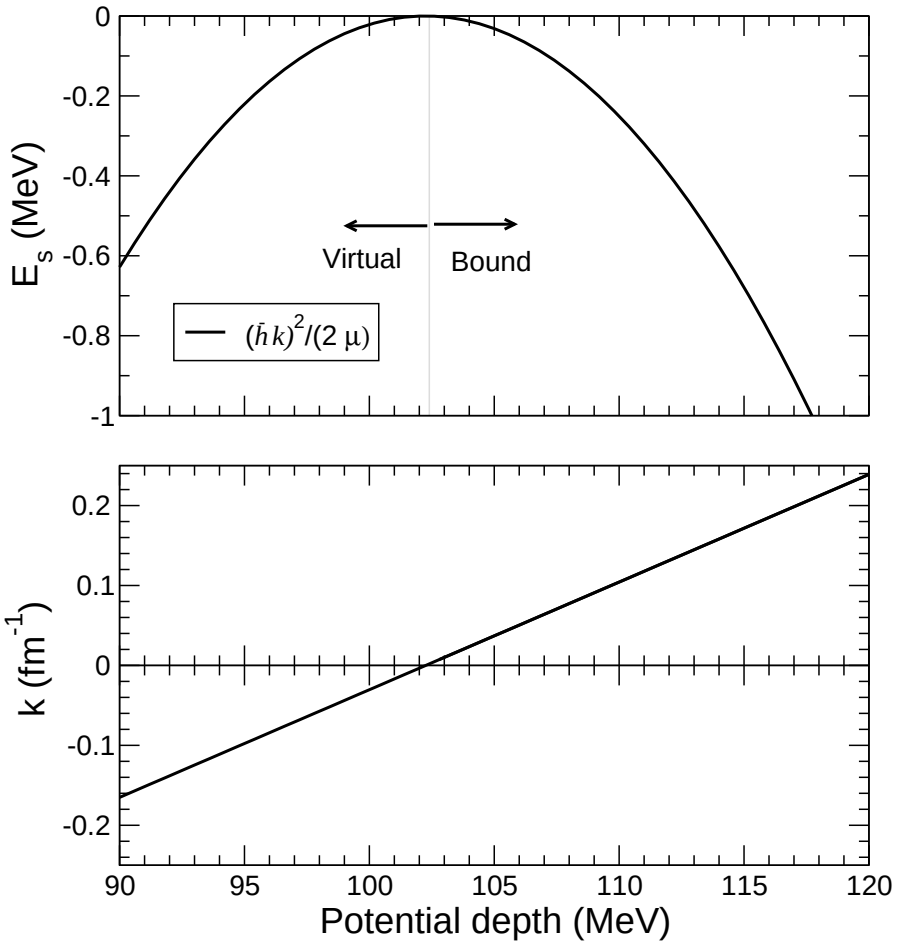


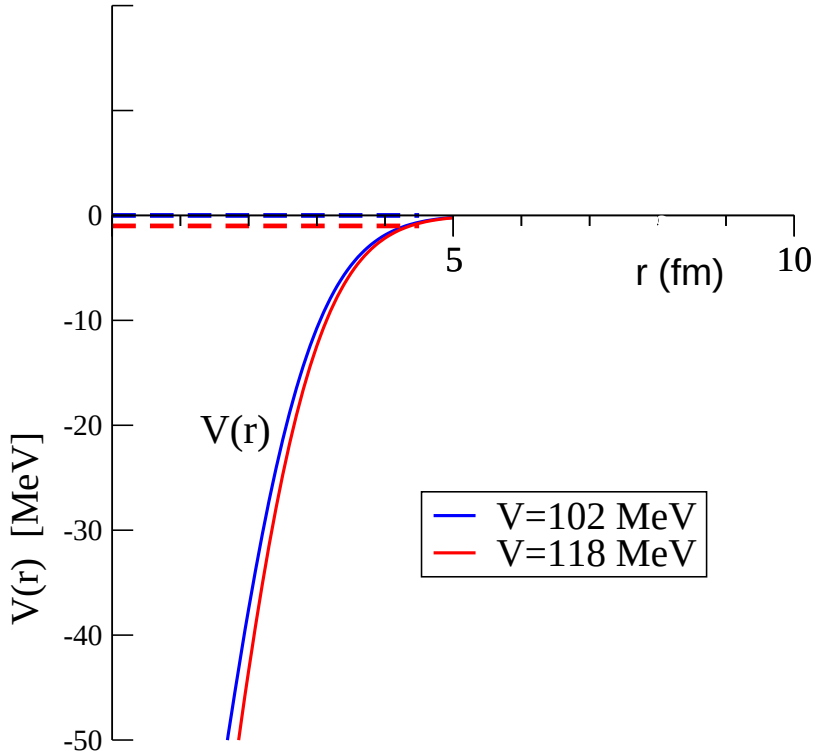


- $E_{\rm res} \rightarrow \delta(E_{\rm res}) = \frac{\pi}{2}$
- $\frac{2}{\Gamma} = \frac{d\delta}{dE}$

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

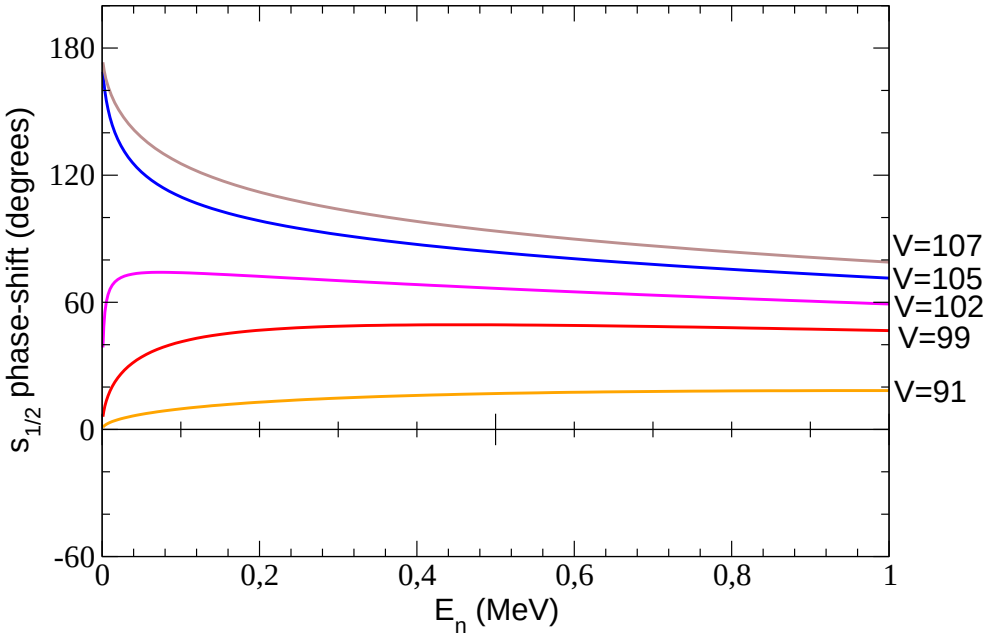
Appearance of a virtual state in ^{10}Li=^{9}Li+n:

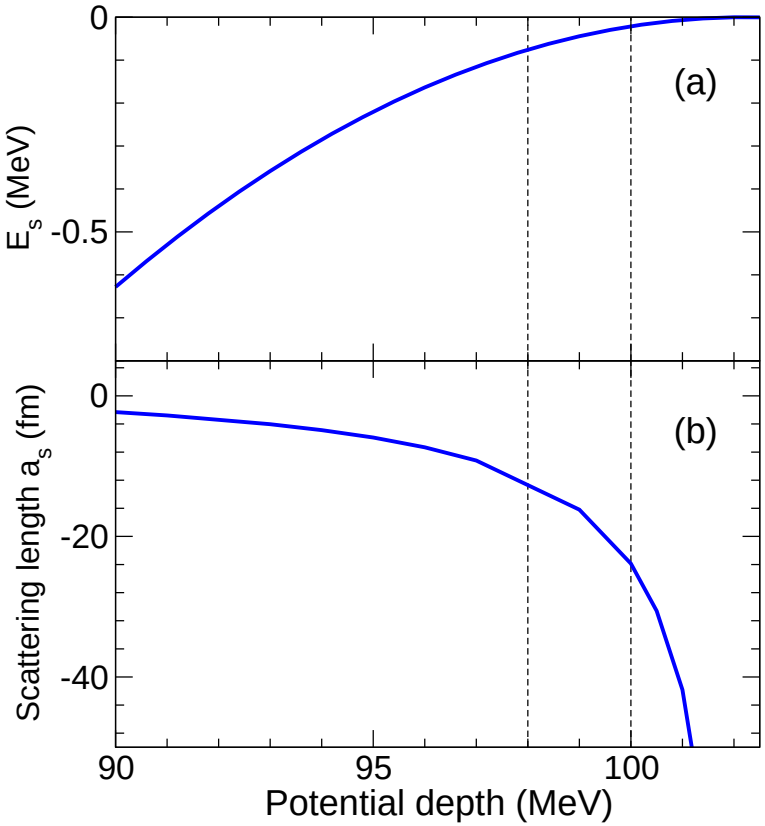




Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

V_s (virtual state)





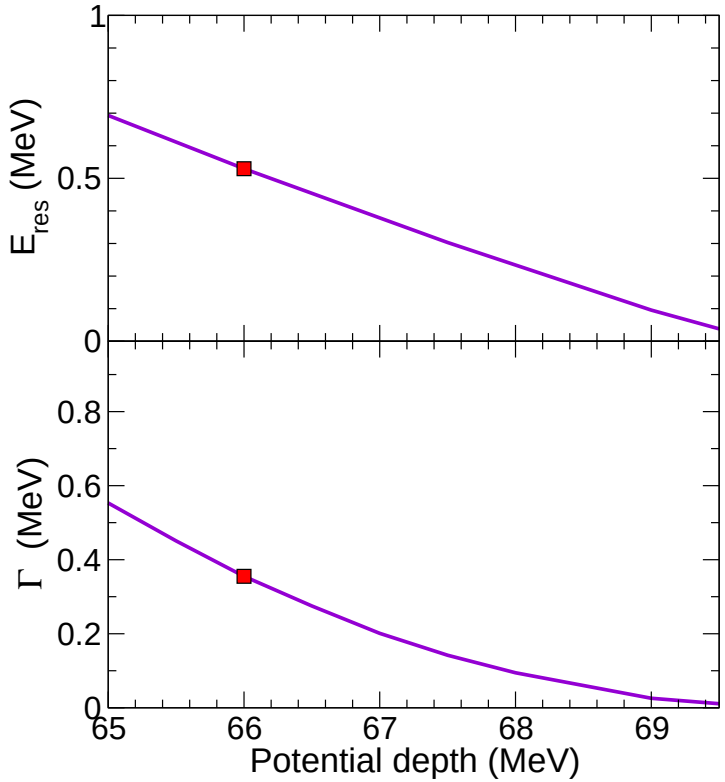
Scattering length:

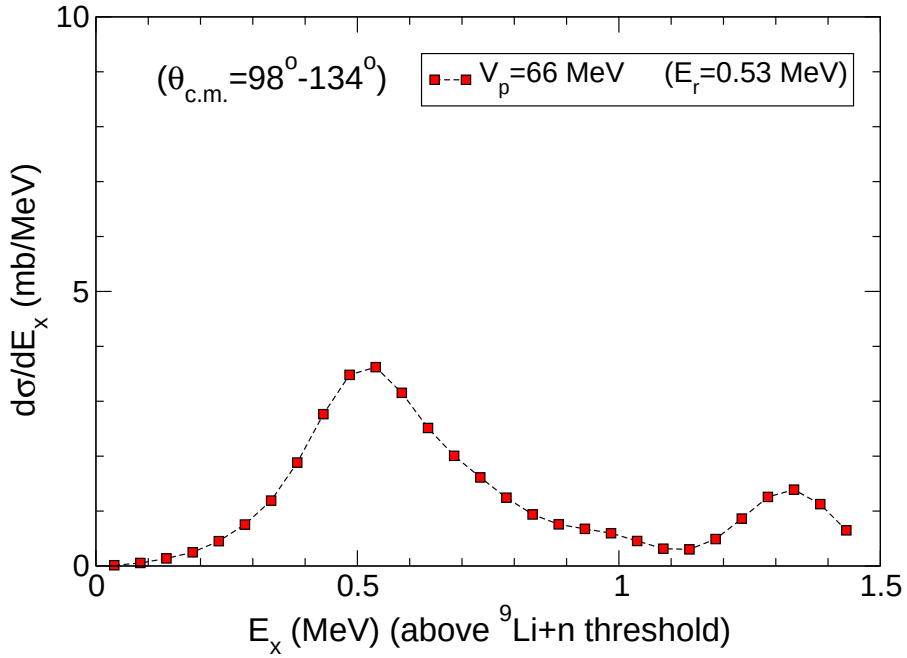
$$a_s = -\lim_{k\to 0} \tan\frac{\delta(k)}{k}$$

.

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

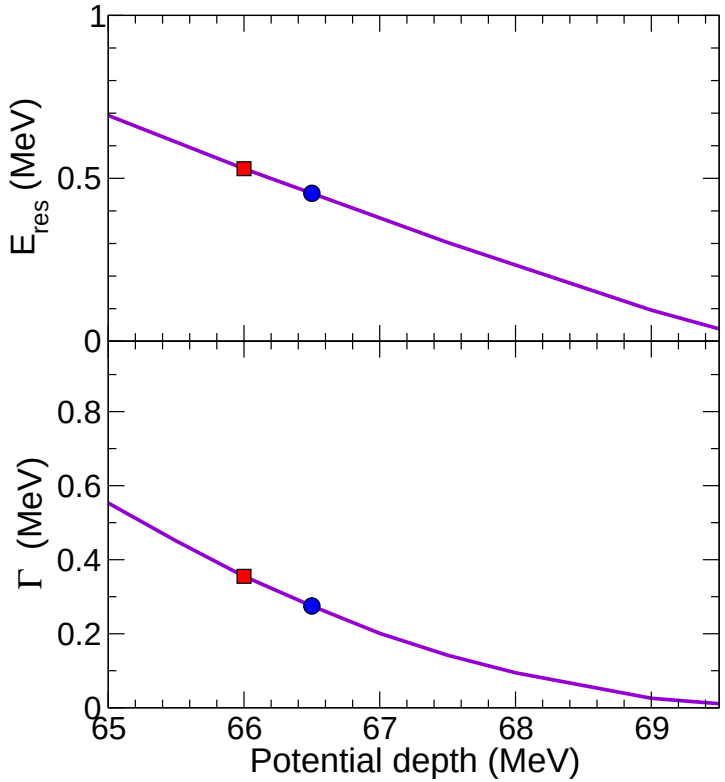
Structure: $p_{1/2}$ resonance → Reaction

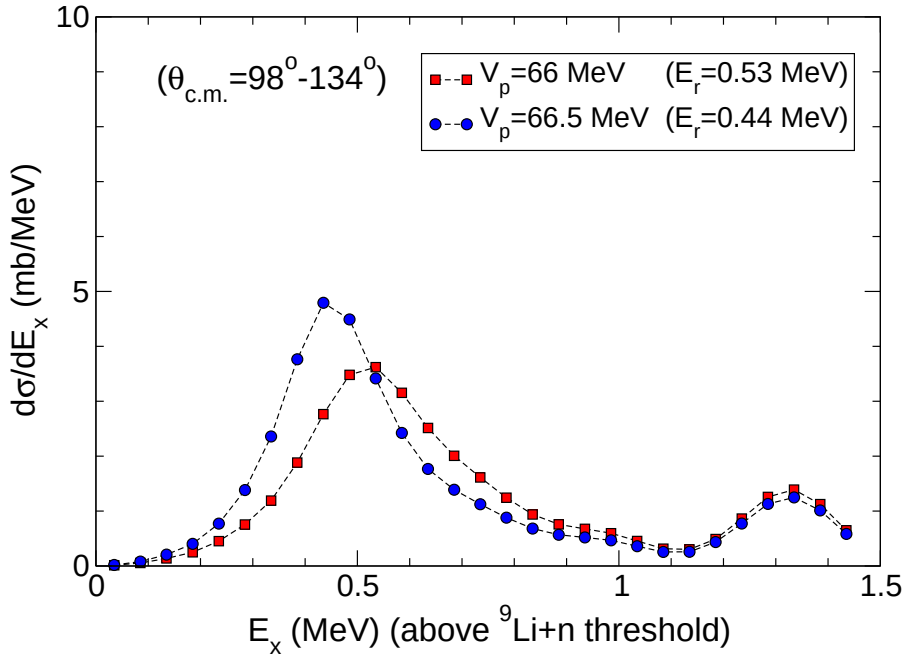




Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

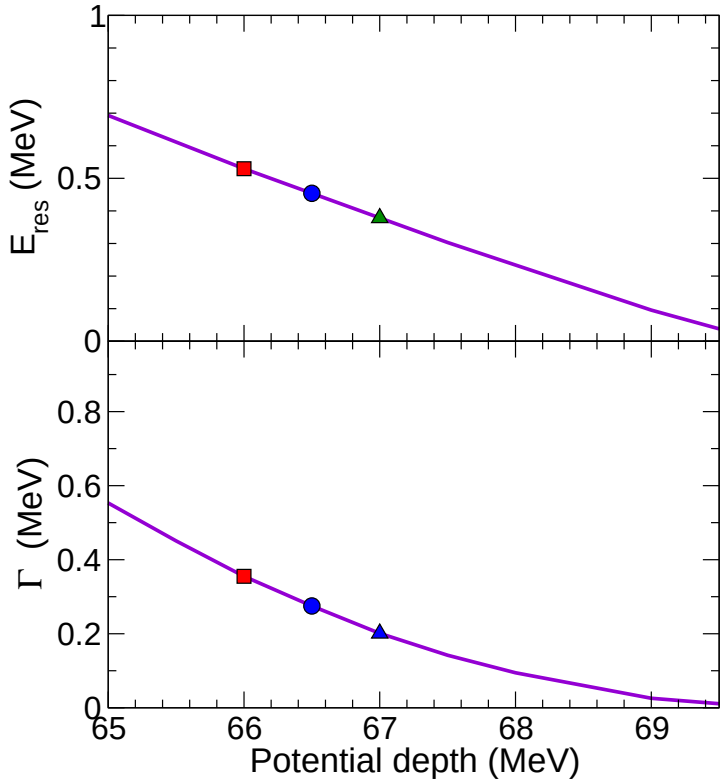
Structure: $p_{1/2}$ resonance → Reaction

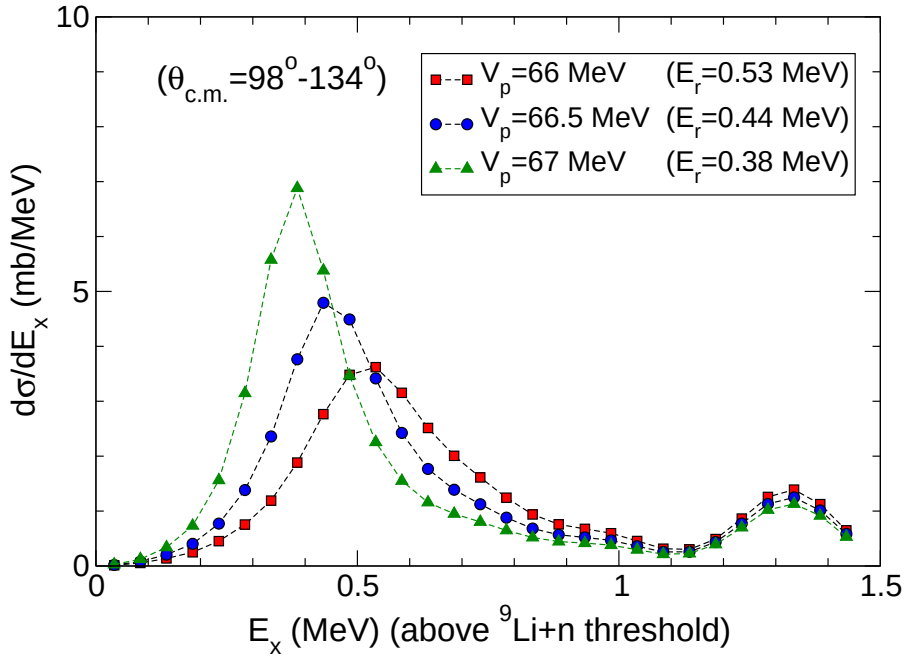




Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

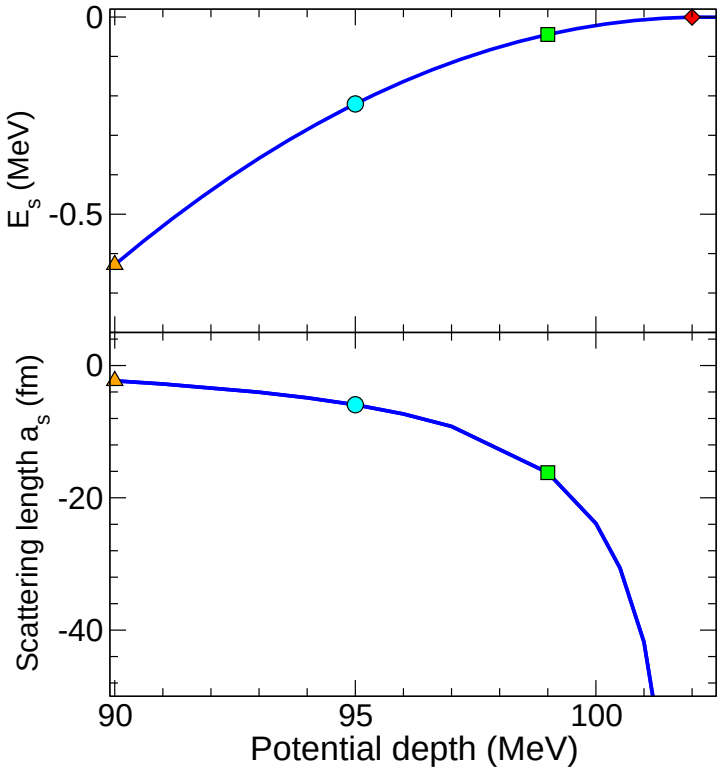
Structure: $p_{1/2}$ resonance → Reaction

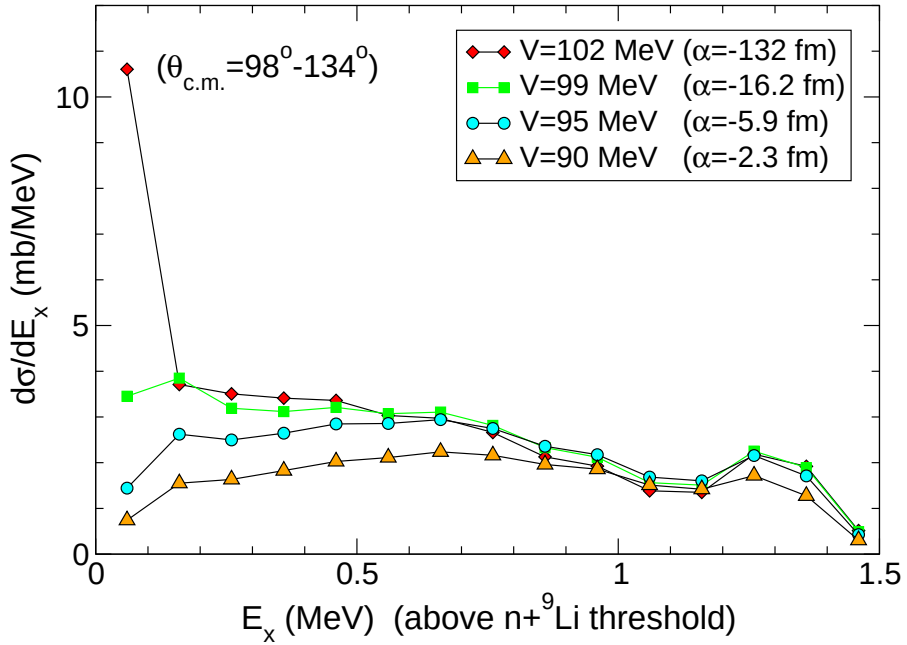




Spectroscopy to unbound states: $^{9}Li(d,p)^{10}Li$ case

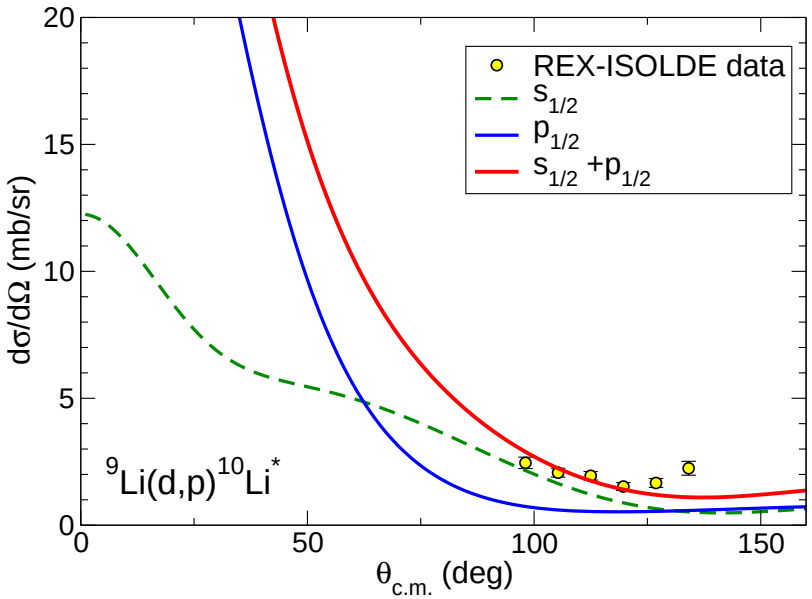
Structure: $s_{1/2}$ v.s. → Reaction

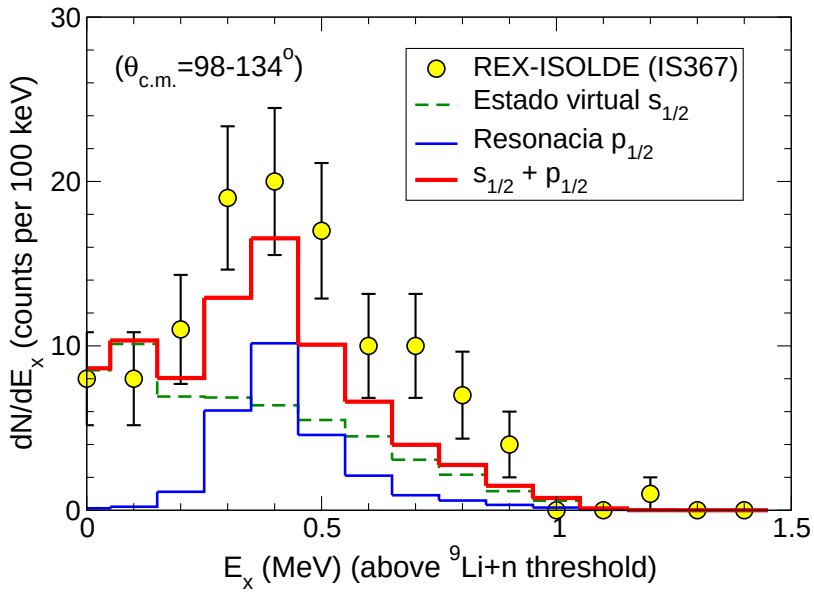




BEST FIT RESULTS: HP.Jeppesen et al, PLB642 (2006) 449

- $p_{1/2}$ resonance ($1^+/2^+$ doublet): $E_r \simeq 0.38$ MeV, $\Gamma = 0.2$ MeV
- $s_{1/2}$ virtual state ($1^-/2^-$ doublet): $a_s \simeq -24$ fm






Thanks!

Appendix: commented example for CDCC calculations with FRESCO

CDCC formalism: physical ingredients

- Physical example: **d+^{58}Ni** at $E = 80$ MeV
- We need to provide FRESCO with the following ingredients:
  - Participants: projectile (core+valence) and target: p, n, ^{58}Ni
  - Potentials:
    - Valence-target and core-target (complex)
    - p-n potential (for gs wavefunction and continuum states)
  - Multipolarities (Q) for coupling potentials
  - Binning squeme for each spin/parity combination:
    - Average bin energy: $\bar{\epsilon}_n$
    - Width for each bin: $\Delta\epsilon_n$

CDCC input example for d+^{58}Ni: `dni_e80_cdc.in`

```
d+ 58Ni at E=80 MeV (CDCC) s+d waves
&CDCC
 hcm=0.1 rmatch=60
 elab=80 pel=1 lab=1 jbord=100
 thmax=180 thinc=1 chans=1 smats=2 xstabl=1
 nk=200 ncoul=0 reor=0 q=2 hat=F /

&NUCLEUS part='Proj' name='d' charge=1 mass=2.0 spin=0. parity=1 be=2.225 n=1 l=0 j=0. /
&NUCLEUS part='Core' name='n' charge=0 mass=1.0 spin=0 parity=+1 /
&NUCLEUS part='Valence' name='p' charge=1 mass=1 spin=0.0 /
&NUCLEUS part='Target' name='58Ni' charge=28 mass=58.0 spin=0. parity=1 /

&BIN spin=0.0 parity=+1 start=0.01 step=5.0 end=30 energy=F l=0 j=0.0 /
&BIN spin=2.0 parity=+1 start=0.01 step=5.0 end=50 energy=F l=2 j=2.0 /
&BIN /

&POTENTIAL part='Proj' a1=58. rc=1.25 /

# Becchetti-Greenless potential for p +58Ni
&POTENTIAL part='Valence' a1=58 rc=1.
   V=44.9 vr0=1.17 a=0.75 w=6.1 wr0=1.32 aw=0.53
   wd=2.21 wdr0=1.320 awd=0.53 /

# Becchetti-Greenless potential for p +58Ni
&POTENTIAL part='Core' a1=58 rc=1.25
   V=42.6 vr0=1.17 a=0.75 w=7.24 wr0=1.26 aw=0.58
   wd=2.590 wdr0=1.26 awd=0.58 /

# Gaussian potential for p-n
&POTENTIAL part='Gs' shape= 2 V=72.150 a=1.484 /
```

General variables: CDCC namelist

```
&CDCC hcm=0.1 rmatch=60
      elab=80 jbord=100
      thmin=0 thmax=180 thinc=1
      smats=2 xstabl=1
      nk=200 ncoul=0 reor=0 q=2 hat=F /
```

- `ncoul=0`: Coulomb + nuclear couplings
  `ncoul=1`: only nuclear couplings
  `ncoul=2`: only Coulomb couplings

- `q`: number of multipoles

- `nk`: number of continuum states to construct the bin.

NUCLEUS namelist:

```
&NUCLEUS part='Proj' name='d' charge=1 mass=2.0 spin=0. parity=1
       be=2.225 n=1 l=0 j=0. /
&NUCLEUS part='Valence' name='n' charge=0 mass=1.0 spin=0 parity=+1 /
&NUCLEUS part='Core' name='p' charge=1 mass=1 spin=0.0 /
&NUCLEUS part='Target' name='58Ni' charge=28 mass=58.0 spin=0. parity=1 /
```

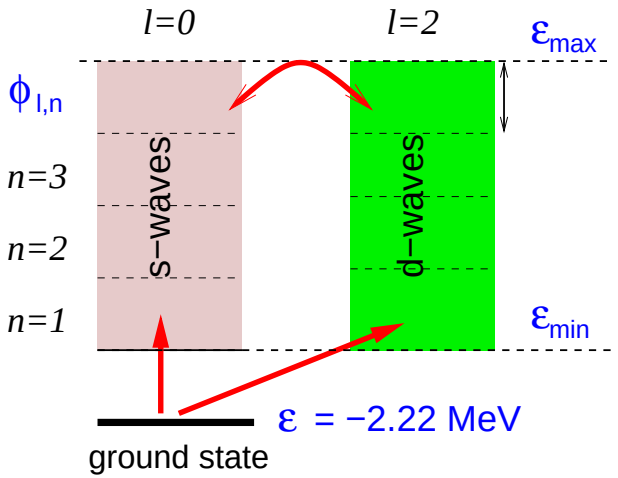
- `part`: specifies each cluster :
  `part='proj'` = projectile $\longrightarrow$ d
  `part='valence'` $\longrightarrow$ n
  `part='core'` $\longrightarrow$ n
  `part='target'` $\longrightarrow$ ^{58}Ni
- `name`, `charge`, `mass`, `spin`, `parity`
- `be`: binding energy
- `n,l,j`: quantum numbers

(*for simplicity, the neutron and proton spins are ignored*)

BIN namelist:

```
&BIN spin=0.0 parity=+1 start=0.0 step=5.0 end=30 energy=F l=0 j=0.0 /
&BIN spin=2.0 parity=+1 start=0.0 step=5.0 end=50 energy=F l=2 j=2.0 /
```

- `spin, parity`: total angular momentum and parity
- `start`: minimum excitation energy
  `end`: maximum excitation energy
  `step`: make N=`(end-start)/step` bins
- `energy`=T: bins evenly spaced in energy
  `energy`=F: bins evenly spaced in k
- `l,j`: orbital and total angular momentum of valence particle



Potentials:

- projectile-target potential (only Coulomb potential is needed here)

```
&POTENTIAL part='Proj' a1=58. rc=1.25 /
```

- valence-target and core-target potentials:

```
&POTENTIAL part='Valence' a2=58
   V=44.9 vr0=1.17 a=0.75 w=6.1 wr0=1.32 aw=0.53
   wd=2.21 wdr0=1.320 awd=0.53 /

&POTENTIAL part='Core' a2=58 rc=1.25
   V=42.6 vr0=1.17 a=0.75 w=7.24 wr0=1.26 aw=0.58
   wd=2.590 wdr0=1.26 awd=0.58 /
```

- p-n potential (for gs and bins):

```
&POTENTIAL part='Gs' shape= 2 V=72.150 a=1.484 /
```

CDCC formalism: d+ ^{58}Ni

- Generate the FRESCO input (`dni_e80_cdcc.in`) from the CDC input (`dni_e80_cdc.in`):

```
cdc < dni_e80_cdc.in > dni_e80_cdcc.in
```

- Run fresco input:

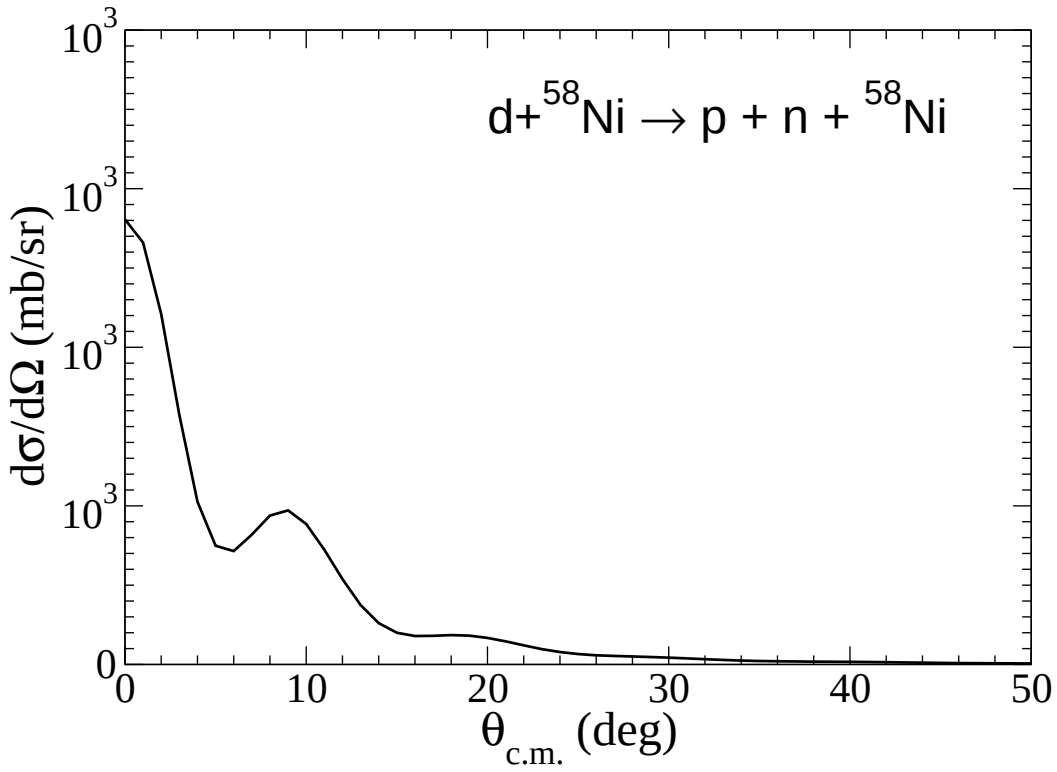
```
fresco < dni_e80_cdcc.in > dni_e80_cdcc.out
```

Useful output files:

- `fort.16`: Angular distributions. Also separately in:
  - `fort.201`: elastic angular distribution
  - `fort.202`: breakup angular distribution for 1st bin
  - `fort.203`: breakup angular distribution for 2nd bin
  - ...
- `fort.13`: total (angle integrated) cross section for each bin.
- `fort.56`: Four columns:J , Fusion , Reaction , Breakup

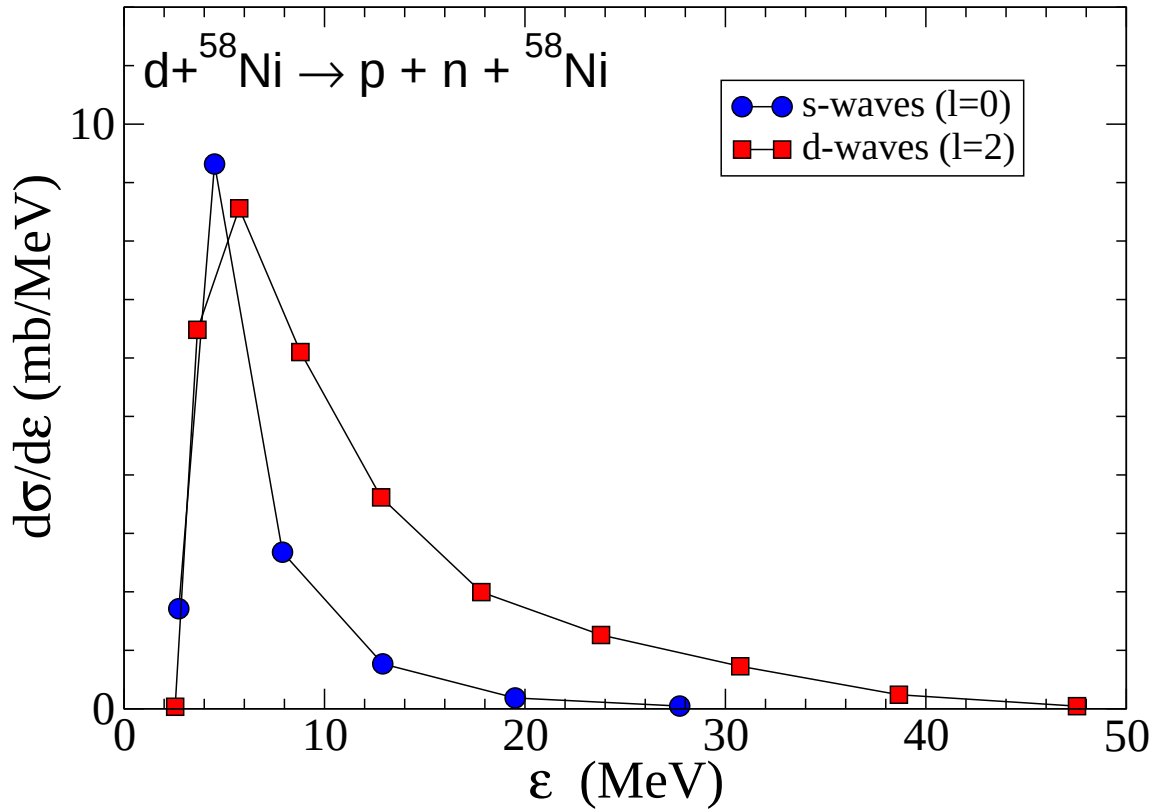
Total breakup angular distribution:

```
sumbins < fort.16 > sumbins.out
```



Breakup energy distribution for each spin configuration

```
sumxen < fort.13 > sumxen.out
```



Useful scripts and post-processing codes:

- `cdc.f`: Application to create FRESCO inputs for CDCC calculations.
- `getsmat.sh`: Extract S-matrix from fort.7 file
  `getsmat < fort.7 > smat.out`
- `sumbins.f`: Adds all non-elastic angular distributions from `fort.16`
  `sumbins < fort.16> sumbins.out`
- `sumxen.f`: Energy distribution for each spin/parity from `fort.13`
  `sumxen < fort.13> sumxen.out`
- `frto2col.f`: Converts overlaps written by fresco in a two column format.