An introduction to the R-matrix method

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1 Principle

The R-matrix method is a powerful tool of quantum physics introduced by Wigner and Eisenbud [1]. Initially the theory was aimed at describing resonances in nuclear reactions. However it also contained the principle of an efficient technique for solving Schrödinger equations in the continuum.

The R-matrix theory was developed into two different directions. On one hand, the *phenomenological R matrix* offers an efficient way for accurately parametrizing low-energy cross sections with a small number of parameters [2]. On the other hand, the *calculable R matrix* provides a simple and elegant way for solving the Schrödinger equation. See [3] for a recent review.

Let us consider potential scattering in an arbitrary partial wave. One assumes that the potential V differs from the Coulomb potential V_C by a short-range term,

$$V(r) \underset{r\to\infty}{\longrightarrow} V_C(r) = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}. \tag{1}$$

The principle of the R-matrix method relies on a division of the configuration space into two regions: the internal and external regions (see Fig.1). The wave function can thus be approximated there by its asymptotic expression where only the phase shift is unknown. In the internal region, the full interaction is taken into account. The boundary between these regions is a parameter known as the *channel radius*. In the internal region, the system is considered as confined and the wave function is expanded over a finite square-integrable basis. The R matrix is calculated in the internal region. It is the inverse of the logarithmic derivative of the internal wave function at the boundary. Then a matching at the channel radius with the asymptotic expression provides the phase shift.

Two particles with reduced mass μ interact via a central potential V at positive energy

$$E = \frac{\hbar^2 k^2}{2\mu} \tag{2}$$

where k is the wavenumber. The Schrödinger equation for the relative motion reads

$$\left(-\frac{\hbar^2}{2\mu}\Delta + V(r)\right)\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}). \tag{3}$$

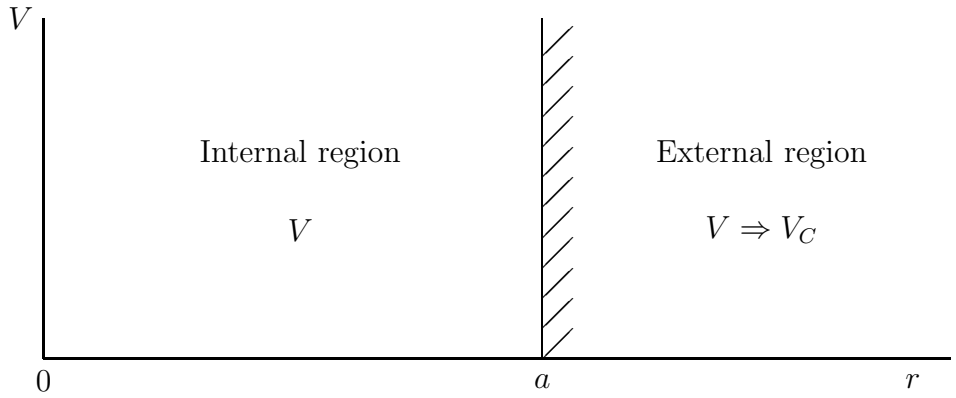

Figure 1: Principle of R matrix.

For a central potential, the wave function can be factorized in spherical coordinates $\boldsymbol{r} = (r, \theta, \varphi)$ as $\psi(\boldsymbol{r}) = r^{-1} u_l(r) Y_l^m(\theta, \varphi)$. The spherical harmonics Y_l^m depend on the orbital and magnetic quantum numbers l and m. After separation of the angular part, the radial Schrödinger equation in partial wave l can be written as

$$(H_l - E)u_l = 0. \tag{4}$$

In this expression, the radial Hamiltonian H_l is defined as

$$H_l = T_l + V(r), \tag{5}$$

where T_l is given by

$$T_l = -\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right). \tag{6}$$

Since the wave function ψ is bounded everywhere, we are interested in bounded radial solutions u_l of (4) vanishing at the origin

$$u_l(0) = 0. \tag{7}$$

Solutions at positive energies have the asymptotic behaviour

$$u_l(r) \underset{r\to\infty}{\longrightarrow} \cos\delta_l \, F_l(\eta, kr) + \sin\delta_l \, G_l(\eta, kr), \tag{8}$$

where F_l and G_l are the regular and irregular Coulomb functions, respectively, and δ_l is the *additional phase shift*. The Sommerfeld parameter

$$\eta = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar v}, \tag{9}$$

where $Z_1 e$ and $Z_2 e$ are the charges of the colliding nuclei and v is their relative velocity, will be implied in the calculations to simplify the notation. Another possible choice is

$$u_l(r) \underset{r\to\infty}{\longrightarrow} I_l(\eta, kr) - U_l O_l(\eta, kr), \tag{10}$$

where $I_l = G_l - iF_l$ and $O_l = G_l + iF_l$ are the incoming and outgoing Coulomb functions, respectively, and U_l is the *scattering matrix* defined by

$$U_l = e^{2i\delta_l}. \tag{11}$$

For real potentials, the phase shifts are real and the scattering matrix is unitary.

In the external region, the radial wave function u_l is approximated by the exact asymptotic expression (8),

$$u_l^{\rm ext}(r) = \cos\delta_l\, F_l(kr) + \sin\delta_l\, G_l(kr) \tag{12}$$

or, alternatively,

$$u_l^{\rm ext}(r) = I_l(\eta, kr) - U_l O_l(\eta, kr). \tag{13}$$

In the internal region, the wave function $u_l^{\rm int}$ is expanded over some finite basis involving N linearly independent square integrable functions φ_j as

$$u_l^{\rm int}(r) = \sum_{j=1}^{N} c_j \varphi_j(r). \tag{14}$$

To satisfy (7), the functions φ_j vanish at the origin

$$\varphi_j(0) = 0 \tag{15}$$

but are not necessarily orthogonal. At $r = a$, we do not assume that they satisfy a specific boundary condition[1]. The internal and external pieces of the radial functions will be connected at the boundary a by the continuity of the wave function u_l and of its first derivative.

2 Bloch operator

The main advantage of the R-matrix method is that an expansion in square-integrable functions can be used in the internal region. However, the Hamiltonian H_l is not Hermitian over the internal region $(0, a)$,

$$\int_0^a f H_l\, g\, dr - \int_0^a g H_l\, f\, dr = \frac{\hbar^2}{2\mu}\left[f'(a)g(a) - f(a)g'(a)\right], \tag{16}$$

where appears the Wronskian of f and g calculated at a. This expression does not vanish in general. This property is not convenient for practical resolutions of the Schrödinger equation. This problem is elegantly solved with the help of a surface operator introduced by Bloch [4]

$$\mathcal{L} = \frac{\hbar^2}{2\mu}\,\delta(r-a)\frac{d}{dr}. \tag{17}$$

[1]Many papers impose such a condition but it has unfavourable effects on the convergence (see Ref. [3] for a discussion).

This operator vanishes everywhere except at $r = a$. The operator $H_l + \mathcal{L}$ is Hermitian over $(0, a)$,

$$\int_0^a f(H_l + \mathcal{L})g\, dr = \int_0^a g(H_l + \mathcal{L})f\, dr, \tag{18}$$

since

$$\int_0^a f\mathcal{L}g\, dr - \int_0^a g\mathcal{L}f\, dr = -\frac{\hbar^2}{2\mu}\left[f'(a)g(a) - f(a)g'(a)\right]. \tag{19}$$

Moreover $H_l + \mathcal{L}$ has a fully discrete spectrum as it defines a self-adjoint problem over a finite interval.

The Schrödinger equation in the internal region is approximated by the inhomogeneous Bloch-Schrödinger equation

$$(H_l + \mathcal{L} - E)u_l^{\rm int} = \mathcal{L}u_l^{\rm ext}, \tag{20}$$

where the external solution is used in the right-hand member. This equation is equivalent to

$$\begin{cases} (H_l - E)u_l^{\rm int} = 0, \\ u_l^{\rm int\prime}(a) = u_l^{\rm ext\prime}(a). \end{cases} \tag{21}$$

Indeed, the equality

$$f(r) + C\,\delta(r - a) = 0 \tag{22}$$

is equivalent for a bounded function f to the equalities

$$\begin{cases} f(r) = 0 \qquad \forall r \neq a, \\ C = 0. \end{cases} \tag{23}$$

The equation (20) or (21) is complemented with the continuity condition

$$u_l^{\rm int}(a) = u_l^{\rm ext}(a) \tag{24}$$

at the boundary.

Because of the Dirac function in the Bloch operator, (20) is equivalent to the Schrödinger equation (4) restricted to the interval $(0, a)$ supplemented by the continuity condition at $r = a$ in (21) [4],

$$u_l^{\rm int\prime}(a) = u_l^{\rm ext\prime}(a). \tag{25}$$

Hence, beyond making $H_l + \mathcal{L}$ Hermitian, the Bloch operator enforces the continuity of the derivative of the wave function. The importance of this aspect of the Bloch operator has often been underestimated in the literature. No condition needs be imposed to the basis functions φ_j at $r = a$ since the Bloch operator imposes condition (25) to the physical solution u_l. For historical reasons, a lot of confusion about the R matrix arose from the misunderstanding of this property [3].

3 Definition and calculation of R matrix

The R matrix at energy E is defined through

$$u_l(a) = R_l(E) a u_l'(a). \tag{26}$$

The inverse of the R matrix is thus the dimensionless logarithmic derivative of the radial wave function at the boundary between both regions. This 'matrix' has dimension 1 in a single-channel case and is just a function of energy. It also depends on the channel radius. The method relies on the fact that the R matrix can be calculated from properties of the Hamiltonian in the internal region. Its knowledge allows determining the phase shift in the external region.

To obtain a practical expression, expansion (14) is introduced in (20) and the resulting equation is projected on $\varphi_i(r)$, giving for $i = 1$ to N,

$$\sum_{j=1}^{N} C_{ij}(E) c_j = \frac{\hbar^2}{2\mu} \varphi_i(a) u_l^{\text{ext}\prime}(a). \tag{27}$$

The elements of the symmetric matrix $\boldsymbol{C}$ are defined as

$$C_{ij}(E) = \langle \varphi_i | T_l + \mathcal{L} + V - E | \varphi_j \rangle. \tag{28}$$

Dirac brackets correspond here to one-dimensional integrals over the variable r from 0 to a. Because of the Bloch operator, the right-hand side of (27) only involves values at $r = a$.

Coefficients c_j are obtained by solving system (27),

$$c_j = \frac{\hbar^2}{2\mu} u_l^{\text{ext}\prime}(a) \sum_{i=1}^{N} (\boldsymbol{C}^{-1})_{ji} \varphi_i(a). \tag{29}$$

Introducing them in (14) at $r = a$ and comparing with (26) provides the calculable R matrix

$$R_l(E) = \frac{\hbar^2}{2\mu a} \sum_{i,j=1}^{N} \varphi_i(a) (\boldsymbol{C}^{-1})_{ij} \varphi_j(a). \tag{30}$$

The wave function (14) in the internal region is thus given by

$$u_l^{\text{int}}(r) = \frac{\hbar^2}{2\mu a R_l(E)} u_l^{\text{ext}}(a) \sum_{j=1}^{N} \varphi_j(r) \sum_{i=1}^{N} (\boldsymbol{C}^{-1})_{ij} \varphi_i(a). \tag{31}$$

4 Phase shift and collision matrix

Since the R matrix is known, definition (26) applied to the external function (12) leads to the simple equation

$$\cos\delta_l \, F_l(ka) + \sin\delta_l \, G_l(ka) = ka R_l(E) [\cos\delta_l \, F_l'(ka) + \sin\delta_l \, G_l'(ka)] \tag{32}$$

from which one extracts the phase shift for the lth partial wave

$$\tan\delta_l = -\frac{F_l(ka) - kaR_l(E)F_l'(ka)}{G_l(ka) - kaR_l(E)G_l'(ka)}. \tag{33}$$

As a fully equivalent variant, the external function (13) can also be introduced in relation (24) to determine the scattering matrix as

$$U_l = \frac{I_l(ka) - kaR_l(E)I_l'(ka)}{O_l(ka) - kaR_l(E)O_l'(ka)}. \tag{34}$$

It can also be written as

$$U_l = e^{2i\phi_l}\,\frac{1 - L_l^* R_l(E)}{1 - L_l R_l(E)}. \tag{35}$$

In this expression,

$$L_l = ka\frac{O_l'(ka)}{O_l(ka)} \tag{36}$$

is the dimensionless logarithmic derivative of O_l at the channel radius, L_l^* is the conjugate of L_l, and

$$\phi_l = \arg I_l(ka) = -\arctan[F_l(ka)/G_l(ka)] \tag{37}$$

is the hard-sphere phase shift. Note that the same notation ϕ_l in [2] represents the opposite of the hard-sphere phase shift.

A simple implementation of the calculable R-matrix method providing phase shifts for local and non-local potentials is presented in Ref. [5]. A simple extension to multichannel scattering is described in Ref. [6]. More general cases are discussed in Ref. [3].

5 Properties of the R matrix

Here, the basis functions $\varphi_i(r)$ are assumed to be orthonormal. Let us consider the eigenvalues E_{nl} and the corresponding normalized eigenvectors $\boldsymbol{v}_{nl}$ of matrix $\boldsymbol{C}(0)$,

$$\boldsymbol{C}(0)\boldsymbol{v}_{nl} = E_{nl}\boldsymbol{v}_{nl} \tag{38}$$

with the orthonormality property

$$\boldsymbol{v}_{nl}^{\mathrm{T}}\boldsymbol{v}_{n'l} = \delta_{nn'} \tag{39}$$

where T means transposition. With the spectral decomposition

$$[\boldsymbol{C}(E)]^{-1} = \sum_{n=1}^{N}\frac{\boldsymbol{v}_{nl}\boldsymbol{v}_{nl}^{\mathrm{T}}}{E_{nl} - E}, \tag{40}$$

the R function (30) can be written as

$$R_l(E) = \sum_{n=1}^{N}\frac{\gamma_{nl}^2}{E_{nl} - E}, \tag{41}$$

where γ_{nl}^2 are the *reduced widths* [2]. In this expression, the *reduced width amplitudes* are given by

$$\gamma_{nl} = \left(\frac{\hbar^2}{2\mu a}\right)^{1/2} \phi_{nl}(a) \tag{42}$$

and

$$\phi_{nl}(r) = \sum_{i=1}^{N} v_{nl,i}\varphi_i(r), \tag{43}$$

where $v_{nl,i}$ is the ith component of $\boldsymbol{v}_{nl}$. The reduced width amplitudes are proportional to the value at the channel radius of variational approximations ϕ_{nl} of the eigenfunctions of the Hermitian operator $H_l + \mathcal{L}$. Those corresponding to the lowest energies thus represent approximate eigenfunctions of the physical problem confined over the interval $(0, a)$ with vanishing logarithmic derivative at $r = a$.

The traditional expression for the theoretical R matrix is obtained when N tends towards infinity in a complete basis as

$$R_l(E) = \sum_{n=1}^{\infty} \frac{\gamma_{nl}^2}{E_{nl} - E}. \tag{44}$$

The energies E_{nl} are now the exact eigenvalues of the operator $H_l + \mathcal{L}$ and the reduced width amplitudes γ_{nl} are related to the values at $r = a$ of its exact eigenfunctions.

The R matrix is a real function when V is real. It has an infinity of real simple poles, bounded from below. Its derivative is always positive at regular points.

6 Penetration and shift factors

For a better physical interpretation of the results, L_l is separated into its real and imaginary parts as

$$L_l = S_l + iP_l. \tag{45}$$

The real part S_l and imaginary part P_l of L_l are called the *shift* and *penetration* factors, respectively. They depend on energy and on the channel radius. The penetration factor can be written with the Wronskian relation $I_l O_l' - I_l' O_l = 2i$ as

$$P_l(E) = \frac{ka}{|O_l(ka)|^2} = \frac{ka}{F_l(ka)^2 + G_l(ka)^2}. \tag{46}$$

It is always positive and increasing [2]. The shift factor reads

$$S_l(E) = P_l(E)[F_l(ka)F_l'(ka) + G_l(ka)G_l'(ka)]. \tag{47}$$

It is always negative for $\eta \geq 0$ [2].

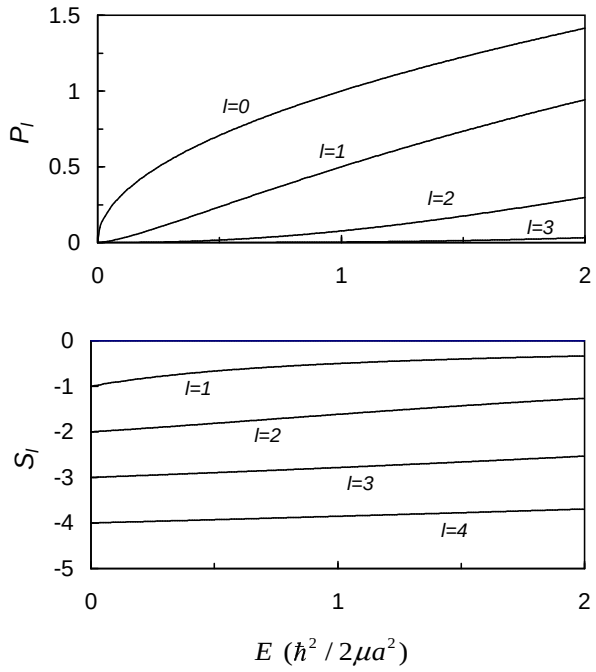

Figure 2: Penetration factors $P_l(E)$ (upper panel) and shift factors $S_l(E)$ (lower panel) in the neutral case ($\eta = 0$) as a function of E in units of $\hbar^2/2\mu a^2$.

These factors have simple analytical expressions in the neutral case ($\eta = 0$). For the s and p waves, one has

$$F_0(x) = xj_0(x) = \sin x, \qquad F_1(x) = xj_1(x) = \frac{\sin x}{x} - \cos x, \tag{48}$$

$$G_0(x) = xn_0(x) = \cos x, \qquad G_1(x) = xn_1(x) = \frac{\cos x}{x} + \sin x, \tag{49}$$

and thus

$$O_0(x) = e^{ix} = I_0^*(x), \qquad O_1(x) = e^{ix}\left(\frac{1}{x} - i\right) = I_1^*(x). \tag{50}$$

The hard-sphere phase shifts are

$$\phi_0 = -ka, \qquad \phi_1 = -ka + \arctan ka. \tag{51}$$

The logarithmic derivatives read

$$L_0 = ika, \qquad L_1 = \frac{-1 + i(ka)^3}{1 + (ka)^2}. \tag{52}$$

The s and p penetration factors have simple analytical expressions,

$$P_0(E) = ka, \qquad P_1(E) = \frac{(ka)^3}{1 + (ka)^2}. \tag{53}$$

Penetration factors do not vary very fast with energy (see Fig. 2). Fig. 2 is universal, i.e. independent of the collision. Notice that the derivative of P_0 with respect to energy is

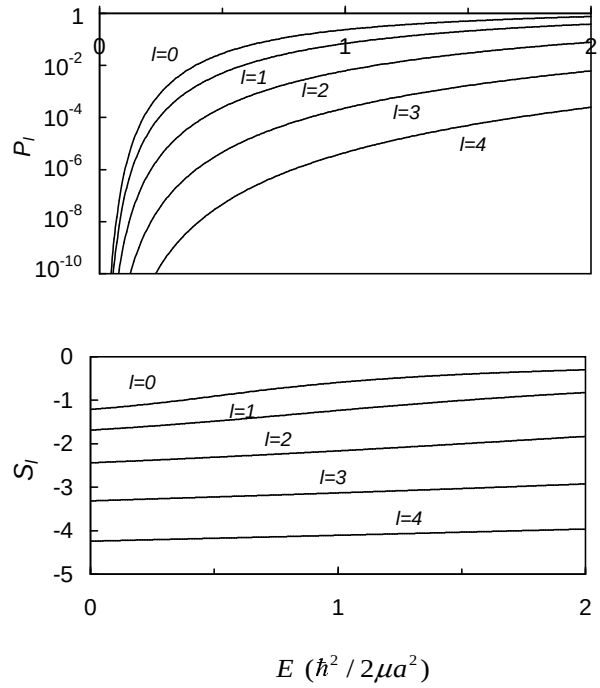

Figure 3: Penetration factors $P_l(E)$ (upper panel) and shift factors $S_l(E)$ (lower panel) in the repulsive charged case for $a = a_B$ as a function of E in units of $\hbar^2/2\mu a^2$.

infinite at the origin. This property leads to the special behaviour of neutron scattering in the s wave. Penetration factors decrease with the orbital momentum l as expected from the occurrence of an increasing centrifugal barrier. The s and p shift factors read

$$S_0(E) = 0, \qquad S_1(E) = -\frac{1}{1+(ka)^2}. \tag{54}$$

The shift factors vary smoothly with energy, starting from the integer values $-l$ (see Fig. 2). This weak energy dependence is the origin of the Thomas approximation [2] where the shift factor is assumed to vary linearly in a limited energy range.

In the repulsive charged case, the shift factors vary also weakly with the energy (see Fig. 3). However, the energy dependence of the penetration factors is much stronger (notice the logarithmic scale in Fig. 3). Figure 3 corresponds to a channel radius a equal to the nuclear Bohr radius $a_B = 1/k\eta$. The strong dependence at low energies is due to the difficulty of penetrating a Coulomb barrier when the scattering energy becomes much smaller than the height of the Coulomb barrier. Beyond $l = 1$, increasing from l to $l + 1$ decreases the penetration factors by more than an order of magnitude.

With definition (45), the collision matrix (35) becomes

$$e^{2i\delta_l} = e^{2i\phi_l}\frac{1 - S_lR_l + iP_lR_l}{1 - S_lR_l - iP_lR_l} \tag{55}$$

and provides an explicit expression for the phase shift,

$$\delta_l = \phi_l + \arctan\frac{P_lR_l}{1 - S_lR_l}. \tag{56}$$

This expression is useful to study resonances.

7 Resonances

To study a resonance, let us consider an energy very close to a pole E_{nl} of the R matrix. If all terms with $n' \neq n$ can be neglected, the R matrix is approximated as $R_l(E) \approx \gamma_{nl}^2/(E_{nl} - E)$. A simple calculation provides

$$\delta_l \approx \phi_l + \arctan \frac{\gamma_{nl}^2 P_l(E)}{E_{nl} - \gamma_{nl}^2 S_l(E) - E}. \tag{57}$$

This expression resembles the Breit-Wigner form of a resonant phase shift

$$\delta_l^{\mathrm{BW}} \approx \phi_l + \arctan \frac{\frac{1}{2}\Gamma(E)}{E_R - E}. \tag{58}$$

By comparison, one defines the *resonance energy*

$$E_R = E_{nl} - \gamma_{nl}^2 S_l(E_R) \tag{59}$$

and the *formal width*

$$\Gamma(E) = 2\gamma_{nl}^2 P_l(E). \tag{60}$$

Let us remark that the resonance energy is shifted with respect to the pole energy. The factor $S_l(E_R)$ in (59) is calculated at the resonance energy E_R. Hence, E_R is defined by an implicit equation which can be solved by iteration. While the reduced width and penetration factor depend on a, the width may not depend on the channel radius. It is an energy-dependent quantity whose asymmetric shape depends on the behaviour of P_l. Roughly, one can interpret γ_{nl}^2 as the nuclear component of the width and P_l as its Coulomb component.

8 Phenomenological R matrix

The goal of the phenomenological R-matrix method is to use a parametrization based on expression (44) with a small number of poles. The properties of these poles are adjusted to some data, in place of being derived from some Hamiltonian, as in the calculable approach.

A drawback of the phenomenological R-matrix formalism is that, though the pole energies and reduced widths are associated with physical properties, they cannot be directly compared with experiment. Indeed, experimental data are usually parametrized with the Breit-Wigner expression (57) simplified as

$$\delta_l^{\mathrm{BW}}(E) \approx \phi_l(E) + \arctan \frac{\frac{1}{2}\Gamma_R}{E_R - E}, \tag{61}$$

where Γ_R is the experimental width of the resonance. The *observed reduced width* γ_{obs}^2 is defined from Γ_R with a relation similar to (60),

$$\Gamma_R = 2\gamma_{obs}^2 P_l(E_R). \tag{62}$$

The corresponding *formal pole location* E_{nl} and *formal reduced width* γ_{nl}^2 must then be deduced for a given channel radius a but their determination is not immediate because of the shift factor S_l and its energy dependence.

Let us approximate R matrix (44) by a single pole with energy E_1 and reduced width γ_1^2 (index l is dropped for the sake of clarity),

$$R_l(E) = \frac{\gamma_1^2}{E_1 - E}. \tag{63}$$

This approximation is frequently used at low energy, where single isolated resonances are present. The phase shift associated with (63) is given by (57) as

$$\delta_l(E) \approx \phi_l(E) + \arctan \frac{\gamma_1^2 P_l(E)}{E_1 - \gamma_1^2 S_l(E) - E}. \tag{64}$$

Let us now use the Thomas approximation [2], which consists in a linearization of the shift function $S_l(E)$ near the pole energy E_1,

$$S_l(E) \approx S_l(E_1) + (E - E_1) S_l'(E_1), \tag{65}$$

where S_l' is the derivative of the shift factor with respect to energy (which appears both in the wave number k and in the Sommerfeld parameter η). The validity of this approximation is supported by Figs. 2 and 3, where it is clear that, in a limited energy range, the linearization of the shift function is quite appropriate. Equation (64) becomes

$$\delta_l(E) \approx \phi_l(E) + \arctan \frac{\gamma_1^2 P_l(E)}{(E_R - E)(1 + \gamma_1^2 S_l'(E_1))}, \tag{66}$$

where the observed resonance energy E_R reads

$$E_R = E_1 - \frac{\gamma_1^2 S_l(E_1)}{1 + \gamma_1^2 S_l'(E_1)}. \tag{67}$$

The shift between energies E_R and E_1 is proportional to the shift factor and depends on E_1 and on a. It is in general non-negligible, unless γ_1^2 is very small. The observed reduced width reads

$$\gamma_{obs}^2 = \frac{\gamma_1^2}{1 + \gamma_1^2 S_l'(E_1)}. \tag{68}$$

It also depends on E_1 and a.

By comparing expression (66) of the phase shift at the Thomas approximation with the Breit-Wigner expression (61), the observed properties (E_R, γ_{obs}) have been derived from the formal parameters (E_1, γ_1) of an isolated pole. In practice, however, the reversed relationships are needed. Indeed, in many cases, observed values (E_R, γ_{obs}) are known from experiment and one wants to derive the corresponding R-matrix parameters (E_1, γ_1). The inverses of (67) and (68) are obtained by linearizing the shift factor $S_l(E)$ around E_R as

$$S_l(E) \approx S_l(E_R) + (E - E_R) S_l'(E_R), \tag{69}$$

which provides

$$\gamma_1^2 = \frac{\gamma_{obs}^2}{1 - \gamma_{obs}^2 S_l'(E_R)}, \tag{70}$$

$$E_1 = E_R + \gamma_1^2 S_l(E_R). \tag{71}$$

They can thus easily be obtained if E_R and Γ_R are known. They also allow using directly E_R and Γ_R as adjustable parameters. The problem is more complicated if several resonances or several channels must be taken into account [2, 3].

As an example, let us consider the elastic scattering of protons by ^{12}C. Three resonances are known in the energy range covered by the data: $1/2^+$ at 0.424 MeV, $3/2^-$ at 1.558 MeV and $5/2^+$ at 1.604 MeV. Data sets are available at the c.m. angles $\theta = 89.1°$ and $146.9°$. They are fitted simultaneously by using E_R and Γ_R of the resonant partial waves as adjustable parameters in the single-pole approximation (63). For other partial waves, the hard-sphere phase shift is used.

The fitted resonance properties are given in Table 1 for different channel radii [3]. The results are almost independent of a. The corresponding cross sections are shown in Fig. 4. The three channel radii provide fits which are indistinguishable at the scale of the figure. Notice that the width of the $1/2^+$ resonance depends a little on the channel radius. The R-matrix parametrization reproduces the data very well, not only in the vicinity of the resonances, but also between them, where the process is mostly non-resonant. This 6-parameter fit provides accurate cross sections at all angles for all energies below 2 MeV.

Table 1: R-matrix parameters from a simultaneous fit of ^{12}C+p scattering data [7] at $\theta = 89.1°$ and $146.9°$. Resonance energies E_R are expressed in MeV and widths Γ_R in keV.

	$J^\pi = 1/2^+$		$J^\pi = 3/2^-$		$J^\pi = 5/2^+$	
	E_R	Γ_R	E_R	Γ_R	E_R	Γ_R
$a = 4$ fm	0.427	33.8	1.560	51.4	1.603	48.1
$a = 5$ fm	0.427	32.9	1.559	51.4	1.604	48.1
$a = 6$ fm	0.427	30.9	1.558	51.3	1.606	47.8
Exp. [7]	0.424	33	1.558	55	1.604	50

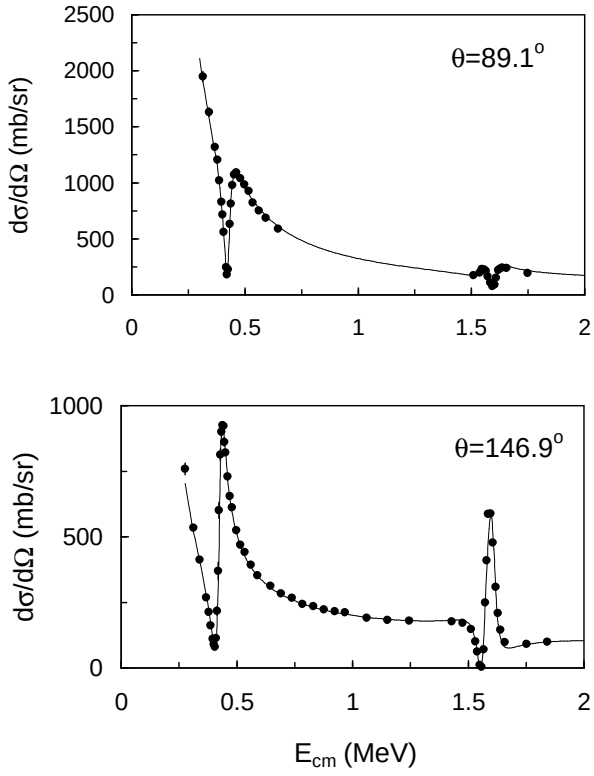

Figure 4: R-matrix fits of ^{12}C+p experimental excitation functions at two c.m. angles [7] with the parameters of Table 1 [3].

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