

Shell Model Calculations (with NuShellX@MSU)



TALENT Course 6
Theory for exploring nuclear reaction experiments

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Session outline

- A brief theoretical overview of the shell model
 - References
 - Central potential and residual interactions
 - General calculation procedure
- Examples of calculations
 - Levels and overlaps, what output is given
- A practical guide to shell model calculations
 - Interactions, models spaces
 - Truncations
 - Pitfalls...
- Overview of NuShellX@MSU calculations
 - Levels and overlaps

Tutorial

- Introduction and setup
- Proton spectroscopic factors: $^{26}\text{Ne}(-1\text{p})$ [or $^{26}\text{Ne}(\text{d}, ^3\text{He})^{25}\text{F}$]

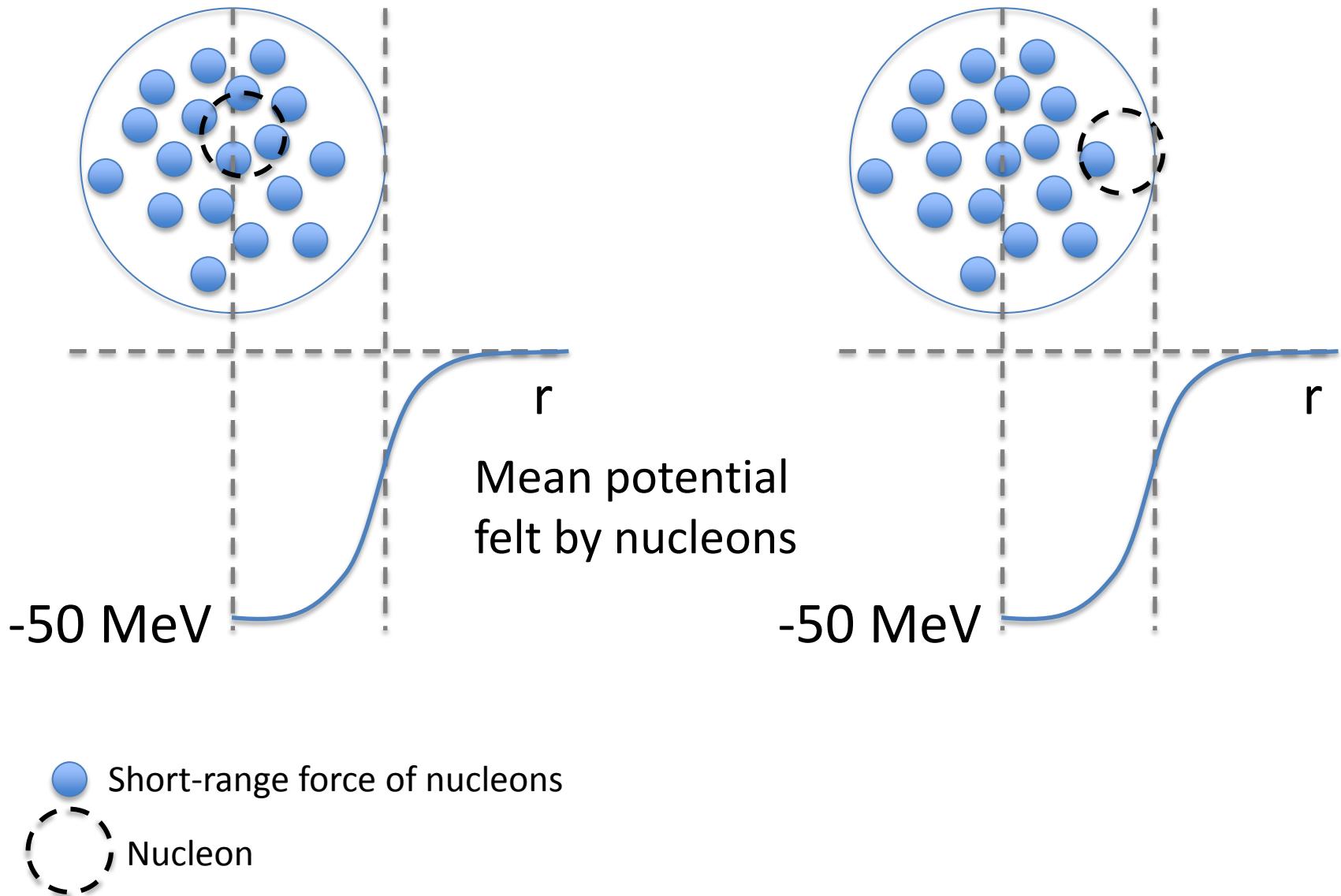
A brief overview of the shell model

Central potentials and residual interactions

References

1. H. Grawe, *Shell Model from a Practitioner's Point of View*, Euroschool Lectures Vol. 1
2. T. Otsuka, *Shell Structure of Exotic Nuclei*, Euroschool Lectures Vol. 2
3. J. B. McGrory and B. H. Wildenthal, Ann. Rev. Nucl. Part. Sci. 30, 383 (1980)
4. B. A. Brown, *Lecture Notes in Nuclear Structure Physics* (2010)
5. Amos de-Shalit and Igael Talmi, *Nuclear Shell Theory*
6. M. Vallieres and H. Wu, in *Computational Nuclear Physics I: Nuclear Structure*
7. E. Caurier and F. Nowacki, *Act. Phys. Pol. B* 30, 705 (1999)

Mean potential from NN interactions



In a Nu(t)shell

- Shell-model Hamiltonian written as:

$$H = T + V = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i>k=1}^A V_{ik}(r_1, r_2)$$

- Three-body interaction small enough at energies of interest such that effects are absorbed into effective two-body interaction

$$H = T + V = \sum_{i=1}^A \left[\frac{p_i^2}{2m} + U_i(\vec{r}) \right] + \sum_{i>k=1}^A V_{ik}(r_1, r_2) - \sum_{i=1}^A U_i(\vec{r})$$

$$H = H_0 + H_{12}$$

- Absorb most of the effects of the two-body interactions into a one-body central potential

The shell model

- We wish to solve find the eigenstates and eigenvalues (energies) of the Hamiltonian H

$$H \left| \Psi_{\alpha} \right\rangle = E \left| \Psi_{\alpha} \right\rangle$$

- Solutions of the central part H_0 are products of single particle wave functions

$$H_0 \left| \Phi_{\alpha} \right\rangle = E_{\alpha} \left| \Phi_{\alpha} \right\rangle \quad \left| \Phi_{\alpha} \right\rangle = \left| \phi_1 \right\rangle \left| \phi_2 \right\rangle \left| \phi_3 \right\rangle \dots \left| \phi_N \right\rangle$$

- The solution requires matrix elements of H :

$$\langle \Phi_{\alpha} | H | \Phi_{\beta} \rangle$$

- The matrix elements of the central part may be calculated using an assumed potential or related to single-particle energies

The shell model

- Any n -particle shell model matrix element can be reduced to a linear combination of two-particle matrix elements:

$$\left\langle \Phi_{\alpha} \left| H_{12} \right| \Phi_{\beta} \right\rangle = \sum_{i,j,k,l,J,T} C_{\alpha\beta}^{ijklJT} \left\langle ijJT \left| H_{12} \right| klJT \right\rangle$$

- Form a matrix of H and then diagonalize:

$$H = \begin{bmatrix} \left\langle \Phi_1 \left| H \right| \Phi_1 \right\rangle & \left\langle \Phi_1 \left| H \right| \Phi_2 \right\rangle & * & * & * & . & . \\ \left\langle \Phi_2 \left| H \right| \Phi_1 \right\rangle & \left\langle \Phi_2 \left| H \right| \Phi_2 \right\rangle & * & * & . & . & . \\ \left\langle \Phi_3 \left| H \right| \Phi_1 \right\rangle & * & * & . & . & . & . \\ * & * & . & . & . & . & . \\ * & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \end{bmatrix} \xrightarrow{\text{Diagonalise}} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ . \\ . \\ . \end{bmatrix}$$

The shell model

- With the matrix diagonalised we have solved for the eigenvectors and values
- The wave functions are given as linear combinations of products of single-nucleon wave functions

$$H |\Psi\rangle = E |\Psi\rangle$$

- $$\Psi = c_1 \Phi_1 + c_2 \Phi_2 + c_3 \Phi_3 + c_4 \Phi_4 + \dots$$
- Obtain the decomposition of each state in terms of single-particle wave functions (protons and neutrons, and the angular momentum couplings)

$$S = \frac{|\langle \Psi^A \omega J | |a_k^+| | \Psi^{A-1} \omega' J' \rangle|^2}{(2J+1)}$$

Generalised procedure [McGrory (1980)]

1. Choice of the central potential H_0
2. Calculation of one-particle eigenstates in the desired model space
3. Construction of multi-nucleon eigenstates of H_0 from one-particle eigenstates
4. Specification of the residual interaction H_{12}
5. Evaluation of the matrix elements of H_{12} between the multi-nucleon eigenstates of H_0 , and the calculation of eigenvectors and eigenvalues of this matrix

Differences in approach

- Different codes use different approaches to basis construction:
 - m-scheme: basis states are eigenstates of J_z
 - jj-coupling: construction of antisymmetric N-particle states using coefficients of fractional parentage (CFPs):

$$\left[j^{N-1}(\alpha' J'), \ jJ \ | \} \ j^N \alpha J \right]$$

- Oxbash/NuShell/NuShellX use both m-scheme and jj-coupling
- NuShellX is pn-formalism: creates basis states for protons and neutrons separately
- Different procedures for diagonalization: very often the Lanczos algorithm used (finds lowest energy eigenvalues efficiently)

Examples of calculations

Protons in ^{26}Ne
Neutrons in Calcium isotopes

Example: ^{26}Ne calculated level scheme and .lpe files

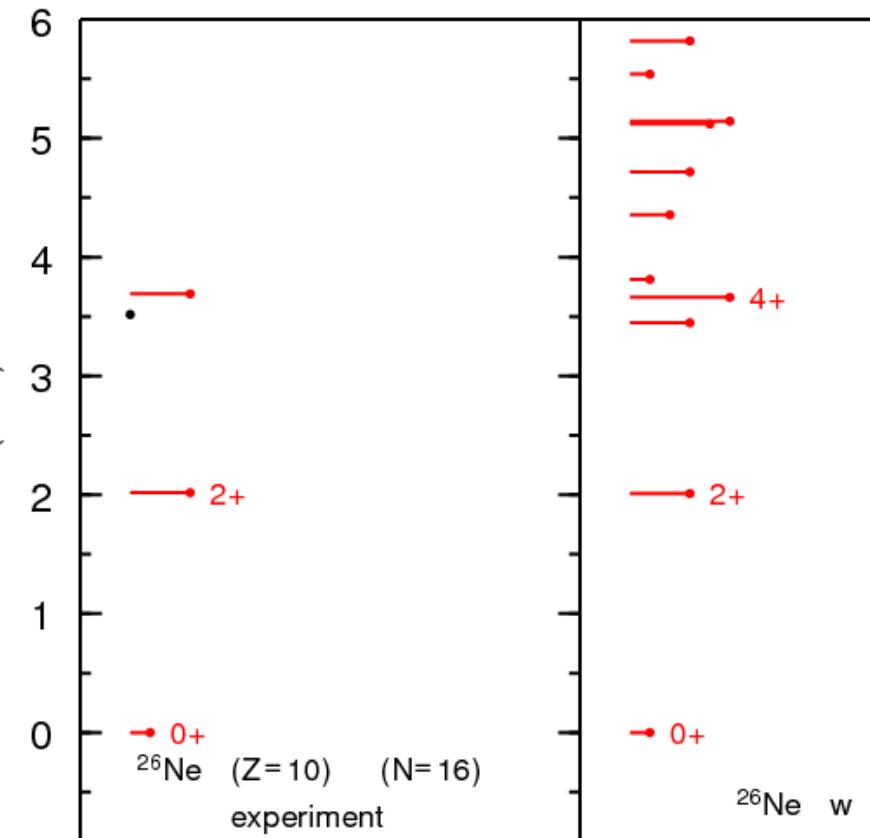
USD interaction in an sd-shell model space

a = 26 z = 10

Interaction file information from ne26w.mit

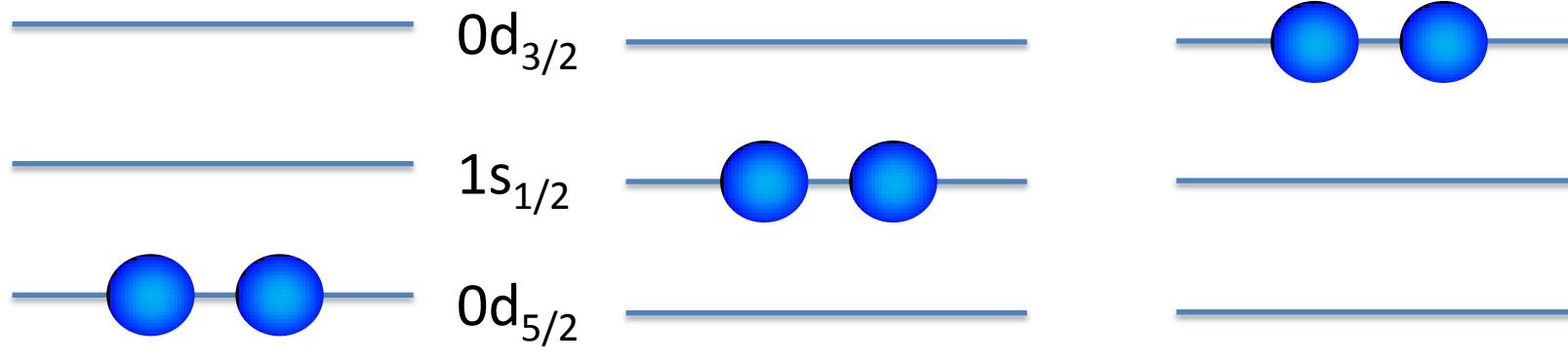
Interaction	spe-norm	tbme-norm	spe
w	1.00000	0.89555	1.6466 -3.9478 -3.1635

N	Njp	E (MeV)	Ex (MeV)	J	p	lowest Ex	name
1	1	-81.625	0.000	0	+	0.000	bw2a00.lpe
2	1	-79.614	2.011	2	+	2.011	bw2a04.lpe
3	2	-78.177	3.448	2	+		bw2a04.lpe
4	1	-77.963	3.662	4	+	3.662	bw2a08.lpe
5	2	-77.813	3.812	0	+		bw2a00.lpe
6	1	-77.269	4.356	1	+	4.356	bw2a02.lpe
7	3	-76.908	4.717	2	+		bw2a04.lpe
8	1	-76.505	5.120	3	+	5.120	bw2a06.lpe
9	2	-76.483	5.143	4	+		bw2a08.lpe
10	3	-76.086	5.539	0	+		bw2a00.lpe
11	4	-75.807	5.819	2	+		bw2a04.lpe
12	2	-75.569	6.056	3	+		bw2a06.lpe
13	5	-75.538	6.087	2	+		bw2a04.lpe
14	2	-75.415	6.210	1	+		bw2a02.lpe
15	1	-75.124	6.501	6	+	6.501	bw2a0c.lpe
16	6	-75.065	6.560	2	+		bw2a04.lpe
17	3	-74.876	6.749	3	+		bw2a06.lpe
18	7	-74.711	6.914	2	+		bw2a04.lpe
19	3	-74.602	7.023	4	+		bw2a08.lpe
20	4	-74.570	7.055	3	+		bw2a06.lpe
21	4	-74.368	7.257	0	+		bw2a00.lpe
22	3	-74.340	7.285	1	+		bw2a02.lpe
23	8	-74.197	7.428	2	+		bw2a04.lpe
24	1	-74.106	7.519	5	+	7.519	bw2a0a.lpe
25	4	-73.988	7.637	4	+		bw2a08.lpe
26	5	-73.949	7.676	3	+		bw2a06.lpe



Partitions: distribution particles amongst orbits

Two-protons in an sd-shell model space



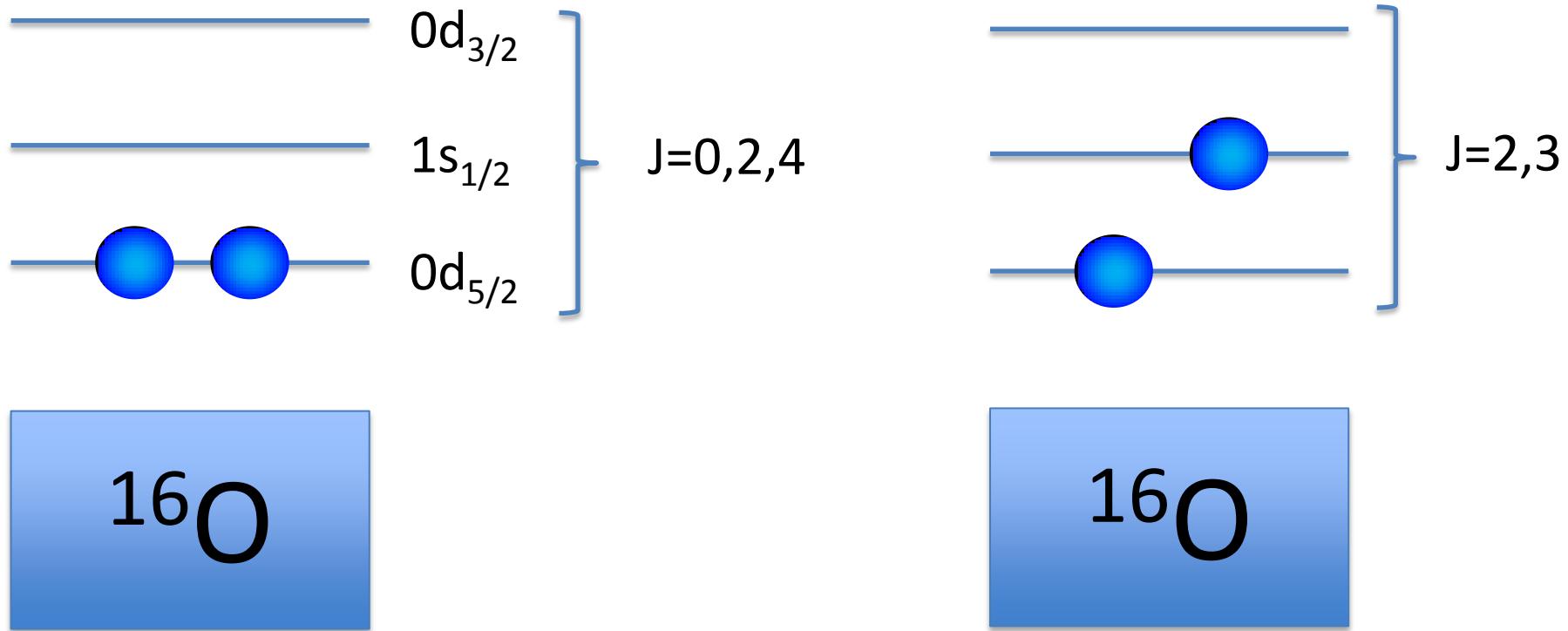
16 O

16 O

16 O

Angular momentum couplings of partitions

Two-protons in an sd-shell model space



Plus neutron configurations!

Example: ^{26}Ne levels (bw2a00.lpe)

Header Information

```
! ----- shell input for bw2a00.lpe
sd           input *.sp file name
sdpn         proton-neutron *.sp file
name
w           interaction file name
6           number of pn orbitals
```

NuShellX V4.0 R2.003

W.D.M. Rae, Garsington, Oxford 2008

Uses NuShell code by W.D.M. Rae, Garsington,
Oxford, UK, 2006/7

ne260s

NuShellX is a nuclear shellmodel program
written in Fortran95 for Windows/Linux.
It is based on NuShell by W D M Rae 2007

2*J, 2*T 0 6

Lowest Energy Levels

List of Partitions

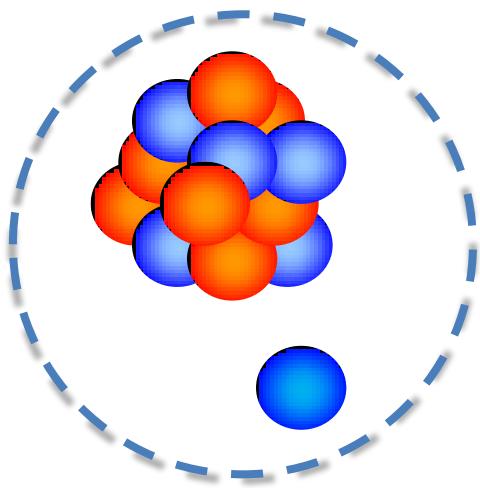
	Type a = n	Type b = p					
A partitions							
-1	0	0	0	4	4	4	0
-2	0	0	0	3	5	0	0
-3	0	0	0	2	6	0	0
-4	0	0	0	4	3	1	0
-5	0	0	0	3	4	1	0
-6	0	0	0	2	5	1	0
-7	0	0	0	1	6	1	0
-8	0	0	0	4	2	2	0
-9	0	0	0	3	3	2	0
-10	0	0	0	2	4	2	0
-11	0	0	0	1	5	2	0
-12	0	0	0	0	6	2	0
B partitions							
-1	2	0	0	0	0	0	0
-2	1	1	0	0	0	0	0
-3	0	2	0	0	0	0	0
-4	1	0	1	0	0	0	0
-5	0	1	1	0	0	0	0
-6	0	0	2	0	0	0	0

Example: ^{26}Ne levels (bw2a00.lpe)

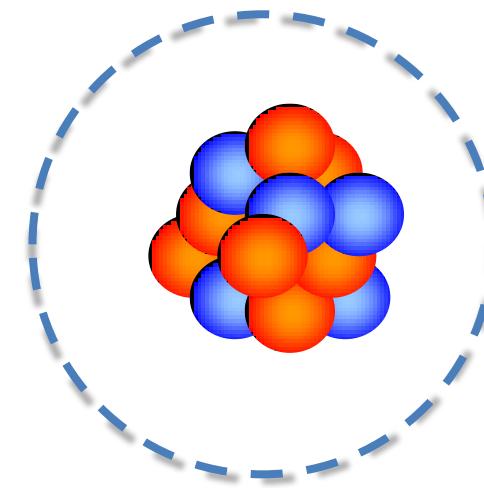
Decomposition of each state

State	1	-81.62506	+			
Average A type nucleons						
0.0000	0.0000	0.0000	0.7201	5.6681	1.6118	
Average B type nucleons						
0.1341	1.6118	0.2540	0.0000	0.0000	0.0000	
Positive and Negative Parity						
+ve	100.00 %	-ve	0.00%			
2*JA, 2*JB, amplitude	0	0	81.35678			
B partition, amplitude	1	4.801839				
A partitions, amplitudes						
(1, 0.03) (2, 0.00) (3, 0.49) (4, 0.00) (5, 0.01) (6, 0.02)						
(7, 0.00) (8, 0.02) (9, 0.00) (10, 0.54) (11, 0.00) (12, 3.68)						
B partition, amplitude	3	67.79593				
A partitions, amplitudes						
(1, 0.58) (2, 0.00) (3, 9.52) (4, 0.00) (5, 0.13) (6, 0.17)						
(7, 0.00) (8, 0.30) (9, 0.02) (10, 8.00) (11, 0.00) (12, 49.08)						
B partition, amplitude	6	8.759010				
A partitions, amplitudes						
(1, 0.07) (2, 0.00) (3, 1.11) (4, 0.00) (5, 0.02) (6, 0.07)						
(7, 0.00) (8, 0.03) (9, 0.00) (10, 0.88) (11, 0.00) (12, 6.58)						
2*JA, 2*JB, amplitude	4	4	16.04640			
B partition, amplitude	2	1.509106				
A partitions, amplitudes						
(1, 0.00) (2, 0.09) (3, 0.04) (4, 0.01) (5, 0.02) (6, 0.13)						
(7, 0.44) (8, 0.00) (9, 0.03) (10, 0.03) (11, 0.71) (12, 0.00)						
B partition, amplitude	3	6.580017				

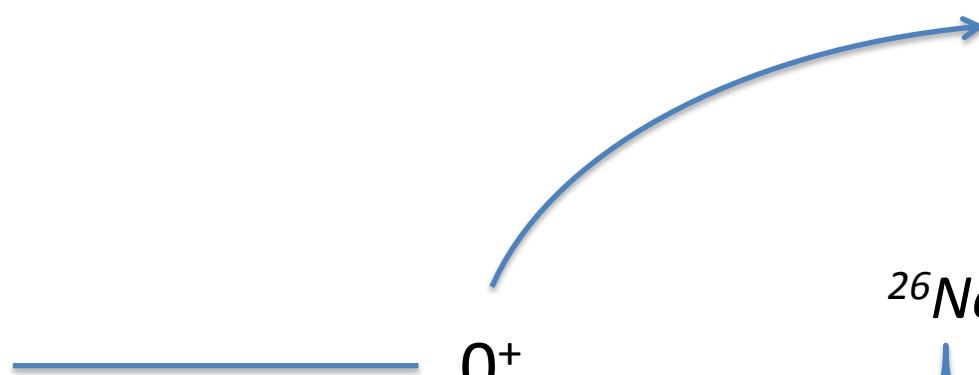
Spectroscopic factors for $^{26}\text{Ne}(-1\text{p}) \rightarrow ^{25}\text{F}$



$(J_F M_F \ j m \ | J_{\text{Ne}} M_{\text{Ne}})$



$1/2^+, 3/2^+, 5/2^+$



$$S = \frac{|\langle \Psi^A \omega J | |a_k^+| | \Psi^{A-1} \omega' J' \rangle|^2}{(2J + 1)}$$

^{26}Ne j ^{25}F

Examples: $^{26}\text{Ne}(-1\text{p})$ SF Output (ne26w.lsf)

```
! model space = sd
! interaction = w
```

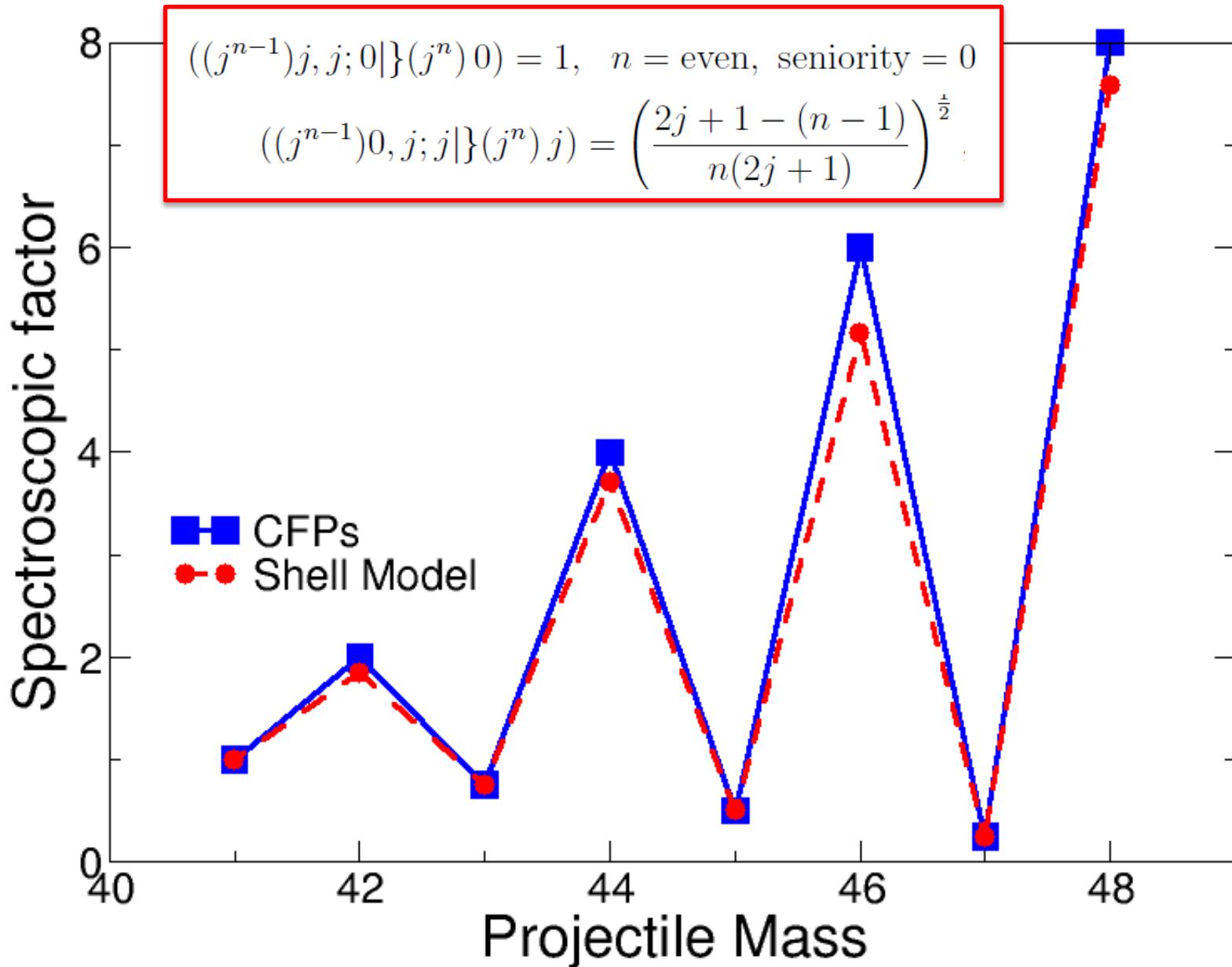
Initial=lighter, final=heavier

(Ai	Tzi)	(Af	Tzf)	(type	n,l,2j)	Ji	Jf	ni	nf	C^2S	Ei	Ef	C^2S*(Ef-Ei)	Exi	Exf
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	1	1	0.2427	-58.503	-81.625	-5.612	0.911	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	2	1	0.0001	-55.345	-81.625	-0.003	4.069	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	3	1	0.0081	-53.397	-81.625	-0.229	6.017	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	4	1	0.0004	-52.331	-81.625	-0.012	7.083	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	5	1	0.0004	-51.589	-81.625	-0.012	7.825	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	6	1	0.0006	-51.408	-81.625	-0.018	8.006	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	7	1	0.0000	-50.856	-81.625	0.000	8.558	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	8	1	0.0002	-49.672	-81.625	-0.006	9.742	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	9	1	0.0004	-49.475	-81.625	-0.013	9.939	0.000
(25	3.5)	(26	3.0)	(p	2 0 1)	0.5+	0.0+	10	1	0.0002	-48.821	-81.625	-0.007	10.593	0.000
								sum	0.2531				-5.911		

(Ai	Tzi)	(Af	Tzf)	(type	n,l,2j)	Ji	Jf	ni	nf	C^2S	Ei	Ef	C^2S*(Ef-Ei)	Exi	Exf
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	1	1	0.0300	-56.341	-81.625	-0.759	3.073	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	2	1	0.0507	-55.427	-81.625	-1.328	3.987	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	3	1	0.0355	-54.365	-81.625	-0.968	5.049	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	4	1	0.0034	-53.953	-81.625	-0.094	5.461	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	5	1	0.0003	-52.833	-81.625	-0.009	6.581	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	6	1	0.0003	-52.477	-81.625	-0.009	6.937	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	7	1	0.0054	-51.591	-81.625	-0.162	7.823	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	8	1	0.0038	-51.007	-81.625	-0.116	8.407	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	9	1	0.0002	-50.891	-81.625	-0.006	8.523	0.000
(25	3.5)	(26	3.0)	(p	1 2 3)	1.5+	0.0+	10	1	0.0000	-50.363	-81.625	0.000	9.051	0.000
								sum	0.1296				-3.451		

(Ai	Tzi)	(Af	Tzf)	(type	n,l,2j)	Ji	Jf	ni	nf	C^2S	Ei	Ef	C^2S*(Ef-Ei)	Exi	Exf
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	1	1	1.5665	-59.414	-81.625	-34.794	0.000	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	2	1	0.0203	-55.658	-81.625	-0.527	3.756	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	3	1	0.0095	-54.615	-81.625	-0.257	4.799	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	4	1	0.0001	-53.783	-81.625	-0.003	5.631	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	5	1	0.0051	-53.392	-81.625	-0.144	6.022	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	6	1	0.0004	-52.909	-81.625	-0.011	6.505	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	7	1	0.0000	-52.618	-81.625	0.000	6.796	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	8	1	0.0001	-51.380	-81.625	-0.003	8.034	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	9	1	0.0008	-51.228	-81.625	-0.024	8.186	0.000
(25	3.5)	(26	3.0)	(p	1 2 5)	2.5+	0.0+	10	1	0.0001	-51.016	-81.625	-0.003	8.398	0.000
								sum	1.6029				-35.766		
								total	sum	1.9856					

Examples: Calcium Isotopes (full fp-shell SM)

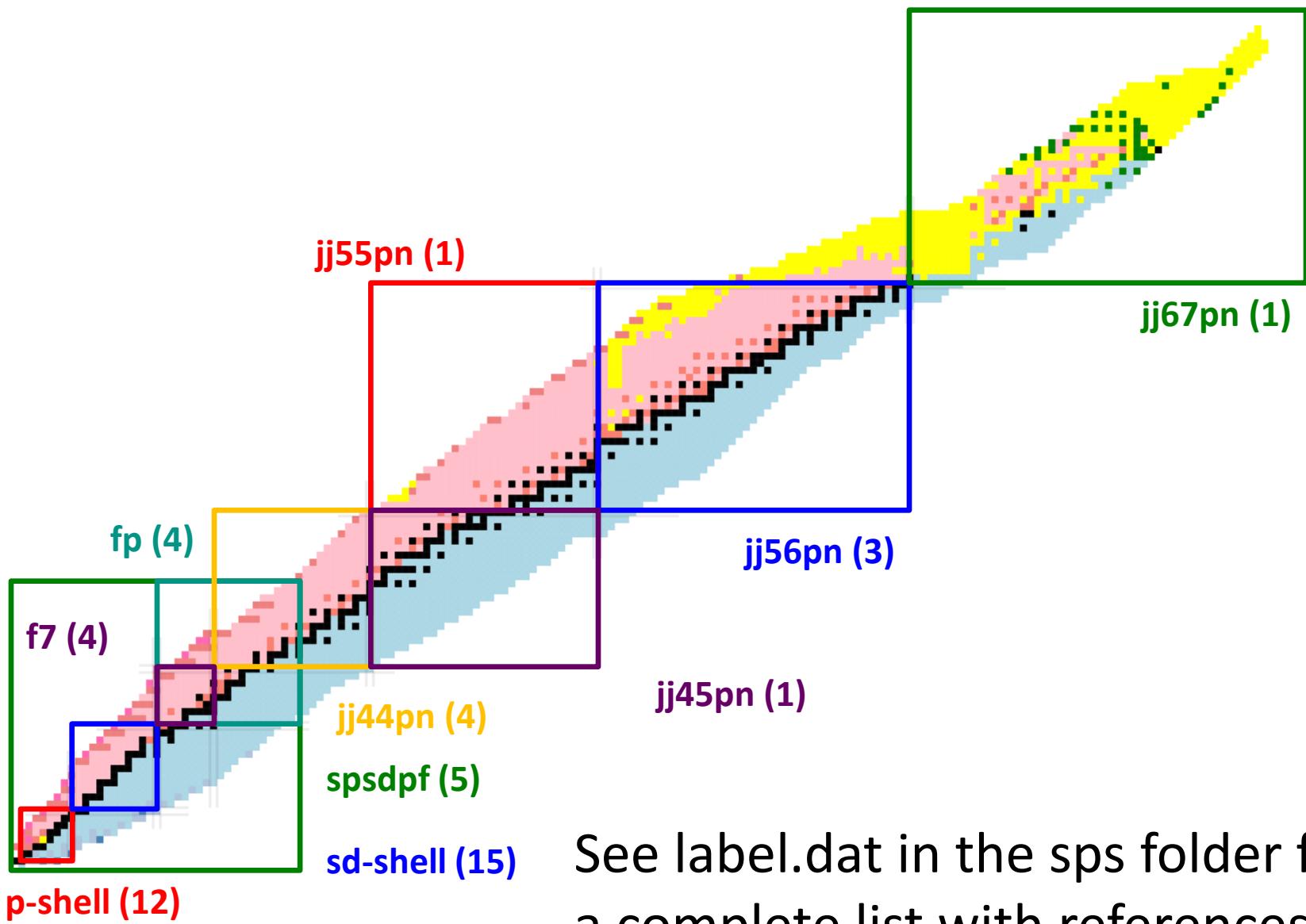


Model spaces, truncations and interactions

Model spaces

- Calculations in an infinite space are not possible – some truncation is required
- Model space defines the active valence nucleon orbits (two truncations, top and bottom)
- Generally, the best and most complete results are found with the largest model spaces
- ... But the computation time increases exponentially with model space size...
- ... And the interaction used must be appropriate for the model space ...
- ... And empirical interactions are (usually) better determined for smaller model spaces (fewer TBME)

(Some) Model Spaces



Model spaces in NuShellX: .sp files

```
! spsdpf.sp          ← Comment (model space name)
t                  ← Interaction format isospin/pn
0 0                ← Model space core
10                ← Number of subshells
4 1 2 3 4          ← Number of major shells,
1 1 0 1           ← numbers of subshells in
2 1 1 3           ← each major shell
3 1 1 1           ← (s-p-sd-fp)
4 1 2 5
5 1 2 3
6 2 0 1
7 1 3 7
8 1 3 5
9 2 1 3
10 2 1 1
```

Number of major shells,
numbers of subshells in
each major shell
(s-p-sd-fp)

Definitions of single-particle
orbits ($i, n, l, 2j$)

Residual Interactions

- Consist of a set of single particle energies and two-body matrix elements (TBME)
- Designed for a particular model space
 - BUT they may only be appropriate/feasible for a part of that model space
 - AND in model spaces crossing major shells the interaction may only be appropriate for a truncation of the (apparent) full model space

Origins of interactions

- Interactions are defined for a particular model space and consist of single-particle energies (SPE) and two-body matrix elements (TBME)
- Single-particle energies often taken from experiment, from the core + 1 nucleon nucleus
- The TBME are more complicated. Two methods are used:
 1. Empirical: either from phenomenological potentials or unconstrained fitting of TBME (but which states are included in the fit, is the solution unique, is it still predictive?)
 2. Realistic: TBME calculated using free nucleon-nucleon interaction
- Often a realistic interaction is used as a best first guess

Interactions in Oxbash/NuShellX

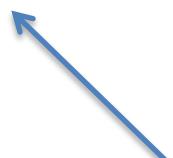
Comment (interaction details)



```
! The "USD" interaction of B. H. Wildenthal for A=18
! ORDER: 1 = 1D3/2      2 = 1D5/2      3 = 2S1/2
! The following spe give values of 15.63, 21.75 and 18.13 relative to
! 40Ca (1.612 -2.684 -2.967)
```

63	1.6465800	-3.9477999	-3.1635399			
1	1	1	1	0	1	-2.1845000
1	1	1	1	1	0	-1.4151000
1	1	1	1	2	1	-0.0665000
1	1	1	1	3	0	-2.8842001
2	1	1	1	1	0	0.5647000
2	1	1	1	2	1	-0.6149000
2	1	1	1	3	0	2.0337000
2	1	2	1	1	0	-6.5057998
2	1	2	1	1	1	1.0334001

Number of TBME and
single particle energies



Two-body matrix elements
 $o_1, o_2, o_3, o_4, J, T, \text{TBME}$

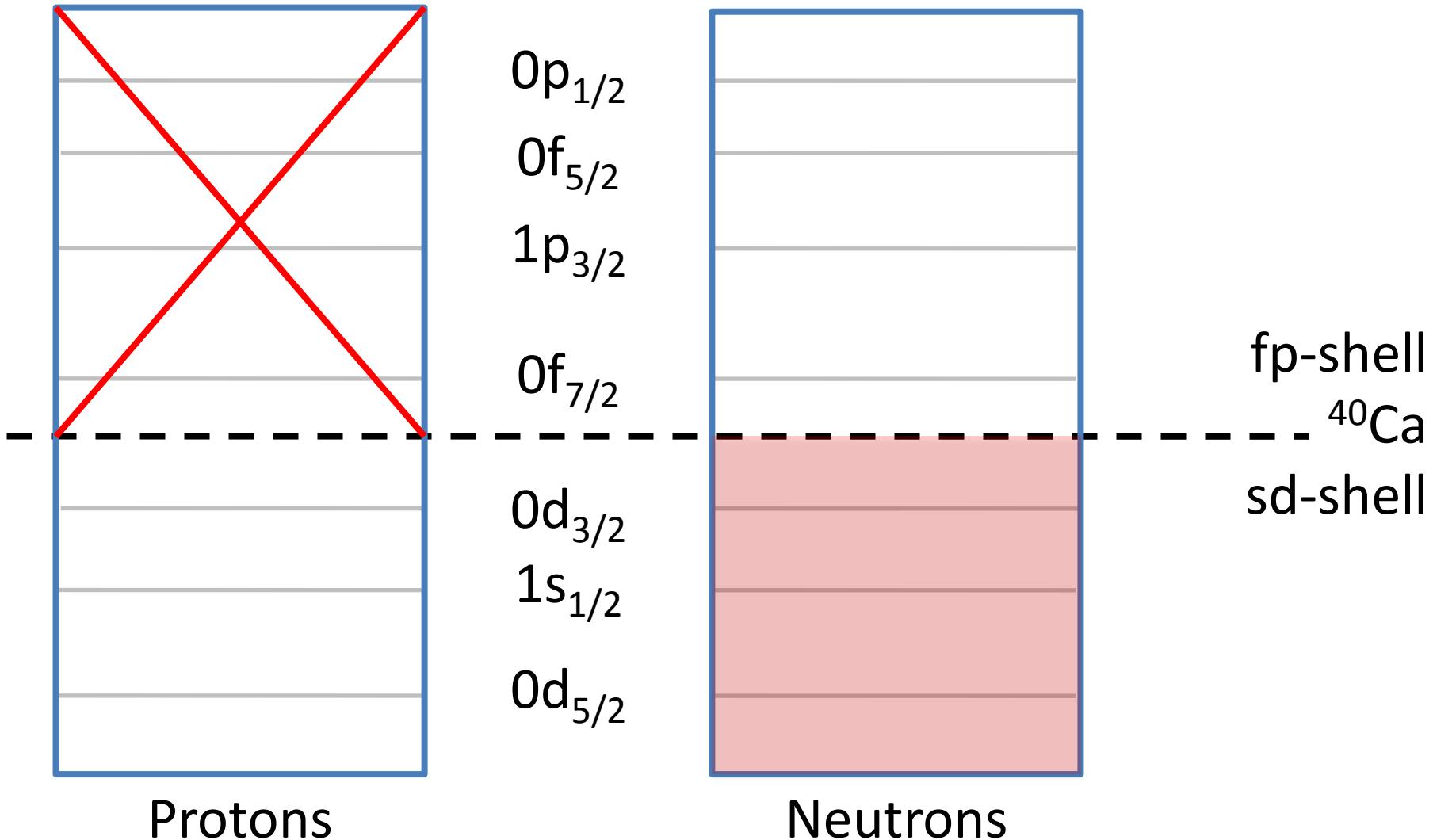
Isospin-formalism interactions automatically converted to pn-format

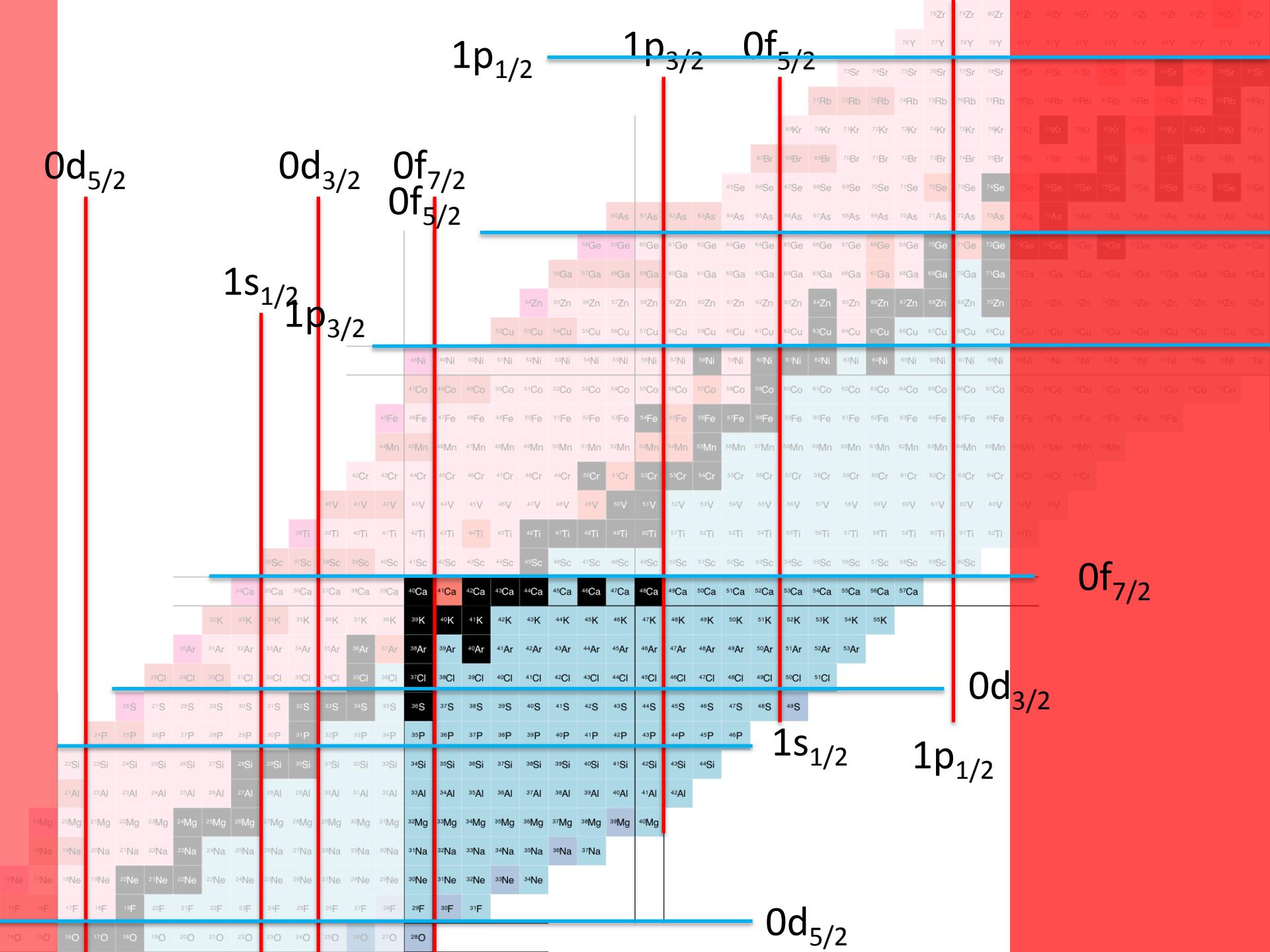
Model space truncations

- Calculations in the full model space may be too computationally intensive OR inappropriate for the interaction
- Truncate the model space to make calculations feasible
- Only subshell truncations are possible in NuShellX
 - limit numbers of protons (neutrons) in a given orbital
- Cannot restrict the total number of nucleons in a given orbital (proton and neutrons are treated separately)
- Truncations must be appropriate for interaction

Truncations – sdpf space

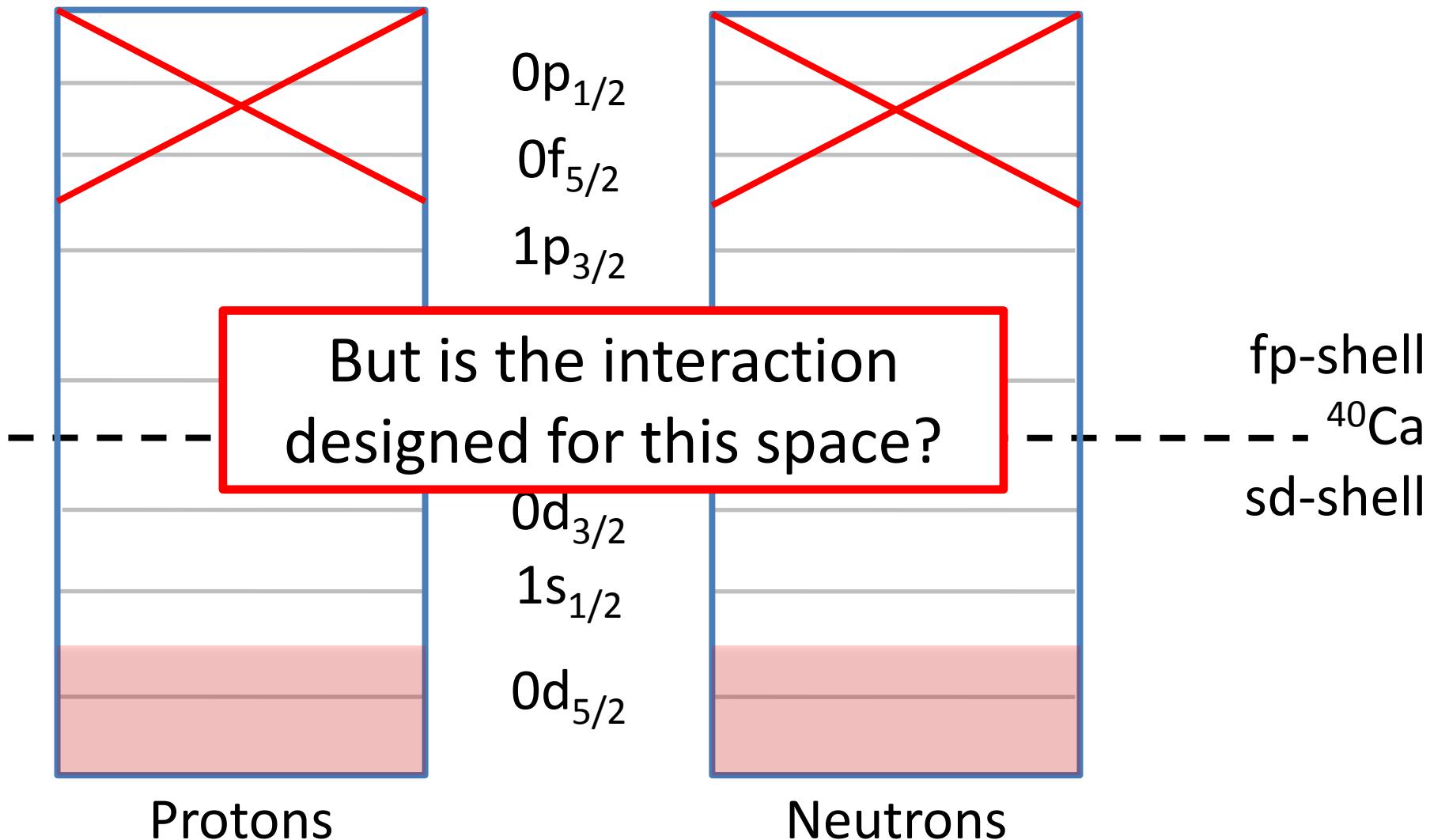
South-east of ^{40}Ca





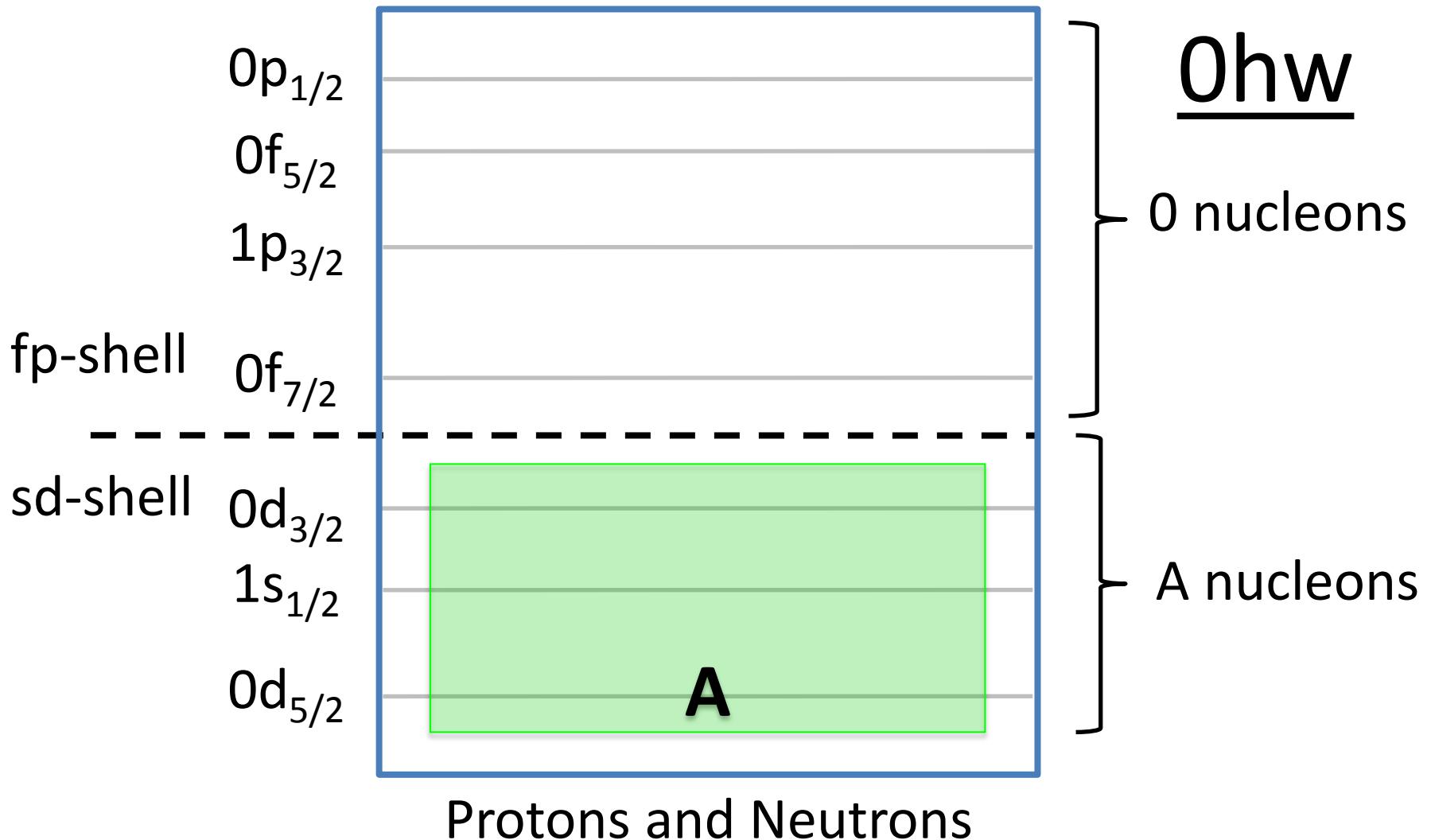
Truncations – sdpf space

Isotope shifts in $^{40-48}\text{Ca}$, $^{42-46}\text{Sc}$



Truncations – hw ($\hbar\omega$ -omega)

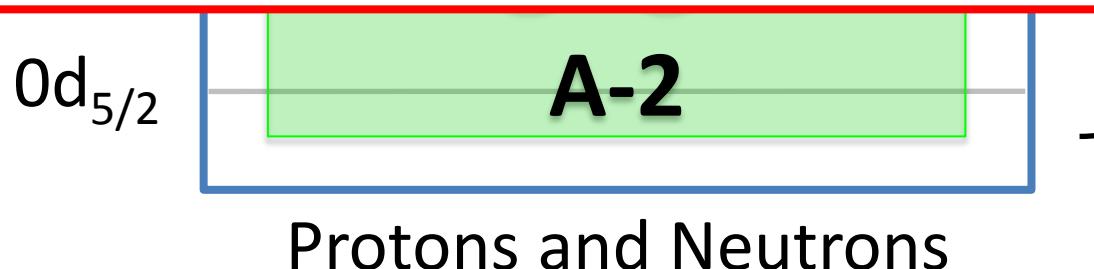
Number of excitations across a major shell gap



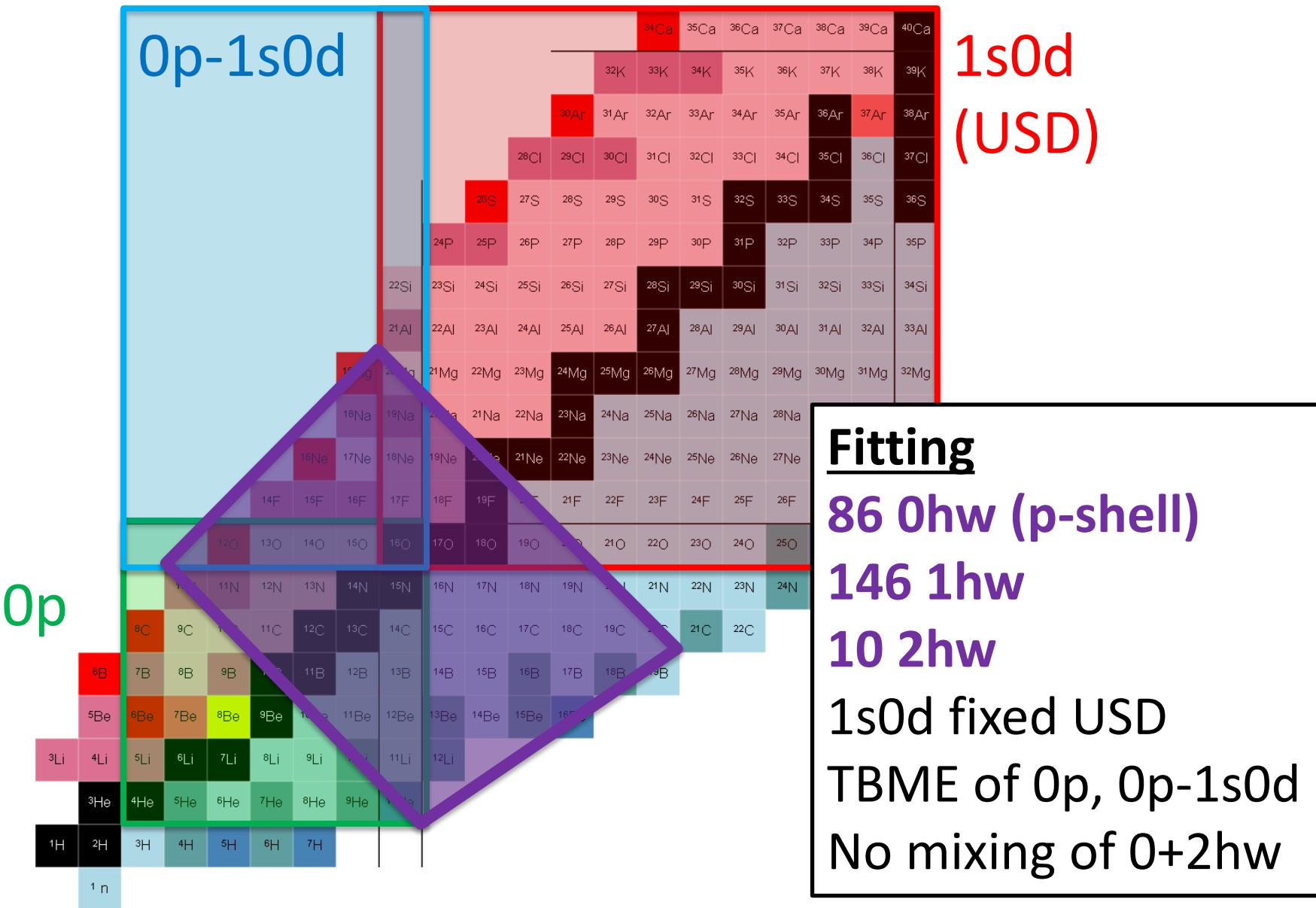
Truncations – hw (hbar-omega)

Number of excitations across a major shell gap

This sort of truncation is **NOT** possible with NuShellX as it involves truncating protons and neutrons together. Some interactions are designed with these truncations in mind, e.g. with no mixing of 0hw and 2hw components. In general, it is not possible to recreate these truncations with NuShellX due to the separation of protons and neutrons. Interactions may not be appropriate for an unrestricted model space!



Example: WBP/WBT interactions (p-sd)



Doing shell model calculations with NuShellX

History, setup, levels and overlaps

A brief history of Oxbash/NuShell/NuShellX

- Oxbash, W. D. M. Rae, B. A. Brown *et al.* (1976)
- NuShell ,W. D. M. Rae (2007)
- NuShellX, W. D. M. Rae (2008)
- NuShellX@MSU, B. A. Brown (2008)
- Provides an incredibly powerful tool for structure calculations, with enormously flexibility and freedom...
- BUT it is very easy to produce unrealistic or inappropriate calculations

More detail can be found in the NuShellX@MSU help.pdf file, located in
X:/aaa/nushellx/help/

Doing shell model calculations

- Use (non-standard) command **lab** to open the file `label.dat`, located in `c:\TalentShare\aaa\nushellx\sps\`
- Choose appropriate model space
 - Several model spaces cover any given nuclide
 - How big does the model space need to be?
- Choose interaction
 - Can the calculations be done on your computer in the model space the interaction was designed for?
 - Similar published example for comparison/checking?
 - Recent
- Decide if model space truncations are required
 - Truncate appropriately for interaction
 - To begin, truncate severely – how good are simple calculations?
 - Relax truncations
- When publishing, give sufficient detail such that the calculations can be easily reproduced

Calculation of levels

- All calculations require levels
- Run **shell** and choose option **lpe**
- Enter the information requested
- Output of individual levels in **.lpe** files
- A summary of levels (calculated and experimental) in **.lpt** files

Calculating overlaps

- First calculate the levels (.lpe) of both the initial and final nuclei using **shell** option **lpe**
 - This must be done using the same model space and interaction
- Then calculate the overlaps of the form:

$$S = \frac{|\langle \Psi^A \omega J || a_k^+ || \Psi^{A-1} \omega' J' \rangle|^2}{(2J + 1)}$$

- All overlaps of the same general form
- Some [E(L λ), M(L λ), B(GT)] then require calculations using **dens** – more info in the manual

Conclusions

- NuShellX@MSU is easy to use, but model spaces and interactions must be appropriately chosen
- Further information in help/help.pdf

Tutorial

- Run through sample calculation...
- Then tackle any problem you like