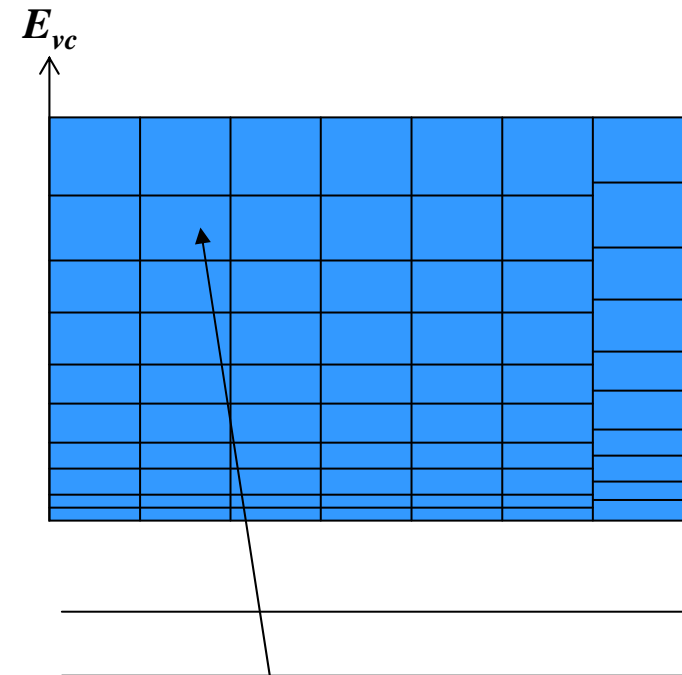
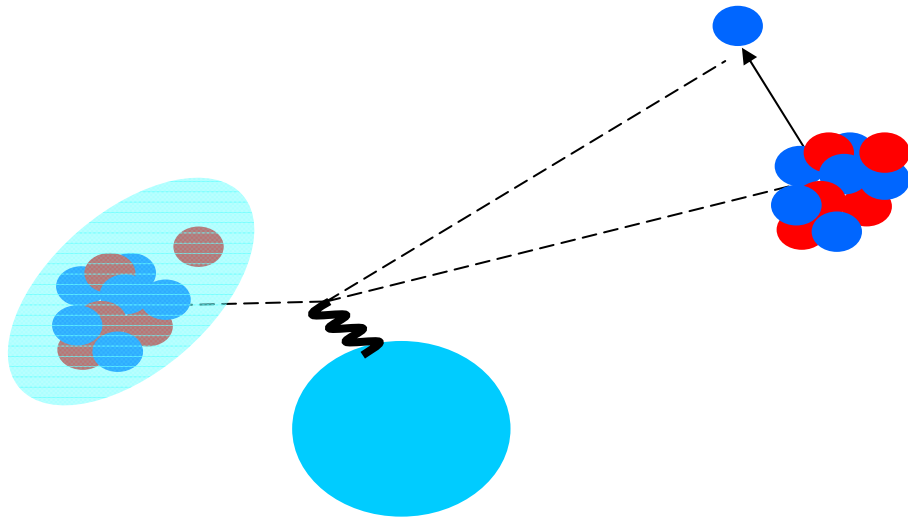


TALENT: theory for exploring reaction experiments

Breakup reactions with the Continuum discretized coupled channel method

Filomena Nunes

standard 3-body CDCC in one slide



- three-body Hamiltonian for reaction
- three-body wavefunction expanded in term of projectile states
- projectile described in terms of single particle states
- optical potentials from elastic scattering

How to do a breakup calculation with fresco?

cdc < short.in >fresco.in

```
8B+208Pb ; N+C breakup with q=0,1,2
CDCC
&CDCC
  hcm=0.01 rmatch=-60 rasym=1000 accrcy=0.001 absend=-50
  elab=656
  jbord= 0 200 300 1000 9000
  jump = 1 10 50 200
  thmax=20 thinc=0.05 cutr=-20 smats=2 xstabl=1
  ncoul=0 reor=0 q=2
/
&NUCLEUS part='Proj' name='8B' charge=5 mass=8
      spin=1.5 parity=-1 be = 0.137 n=1 l=1 j=1.5 /
&NUCLEUS part='Core' name='7Be' charge=4 mass=7 /
&NUCLEUS part='Valence' name='proton' charge=1 mass=1 spin=0.5/
&NUCLEUS part='Target' name='208Pb' charge=82 mass=208 spin=0 /

&BIN spin=0.5 parity=+1 start=0.001 step=0.50 end=10. energy=F l=0 j=0.5/
&BIN /

&POTENTIAL part='Proj' a1=1 rc=2.65 /
&POTENTIAL part='Core' a1=208 rc=1.3 v=114.2 vr0=1.286 a=0.853 w=9.44 wr0=1.739 aw=0.809 /
&POTENTIAL part='Valence' a1=208 rc=1.3 v=34.819 vr0=1.17 a=0.75 w=15.340 wr0=1.32 aw=0.601/
&POTENTIAL part='Gs' a1=1 rc=2.391 v=44.675 vr0=2.391 a=.48 vso=4.898 rso0=2.391 aso=0.48 /
```

**Box B.4 FRESKO input for the breakup of ^8B on ^{208}Pb at 82 MeV per nucleon
(short version)**

How to do a breakup calculation with fresco?

```
8B+208Pb ; N+C breakup with q=0,1,2
CDCC
&CDCC
  hcm=0.01 rmatch=-60 rasym=1000 accrcy=0.001 absend=-50
  elab=656
  jbord= 0 200 300 1000 9000
  jump = 1 10 50 200
  thmax=20 thinc=0.05 cutr=-20 smats=2 xstabl=1
  ncoul=0 reor=0 q=2
/
```

CDCC namelist: contains all the relevant parameters for the **fresco** namelist

- **rmatch**<0 and **rasym** (couplings have long range so match to coupled channel Coulomb function all the way to **rasym**)
- **Jmax**=9000! Need many partial waves for this high energy. But jumping...(all j's up to 200, then in 10's up to 300, then in 50's up to 1000, then in 200's up to 9000)
- Theta grid: angular distribution very forward focused – high energy
- **cutr** – avoids integration at small r due to the strong repulsive potentials (cutr=-20fm put the integration starting point 20 fm inside the coulomb turning radius)

How to do a breakup calculation with fresco?

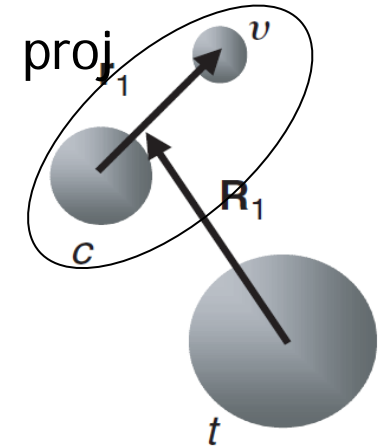
```
8B+208Pb ; N+C breakup with q=0,1,2
CDCC
&CDCC
  hcm=0.01 rmatch=-60 rasym=1000 accrcy=0.001 absend=-50
  elab=656
  jbord= 0 200 300 1000 9000
  jump = 1 10 50 200
  thmax=20 thinc=0.05 cutr=-20 smats=2 xstabl=1
  ncoul=0 reor=0 q=2
/
```

some key additional parameters:

- **ncoul** (0=nucl+Coul; 1=nucl only, 2=Coul only)
- **reor** (different choices for the $\langle \phi_i | V | \phi_i \rangle$ couplings)
- **q** (multipoles to include: all multipoles up to 2)

How to do a breakup calculation with fresco?

```
&NUCLEUS part='Proj' name='8B' charge=5 mass=8  
      spin=1.5 parity=-1 be = 0.137 n=1 l=1 j=1.5 /  
&NUCLEUS part='Core' name='7Be' charge=4 mass=7 /  
&NUCLEUS part='Valence' name='proton' charge=1 mass=1 spin=0.5/  
&NUCLEUS part='Target' name='208Pb' charge=82 mass=208 spin=0 /  
  
&BIN spin=0.5 parity=+1 start=0.001 step=0.50 end=10. energy=F l=0 j=0.5/  
&BIN /
```



NUCLEUS namelist contains info for **partition**, **overlap** and **states**

- part**= (the particle we wish to define including charge, mass, spin and parity, binding energy and the quantum numbers for the state)

BIN namelist contains info for **overlap part**

- energy**=F (grid in momentum) or T (grid in energy)

- start**, **step** and **end** define the details of the grid

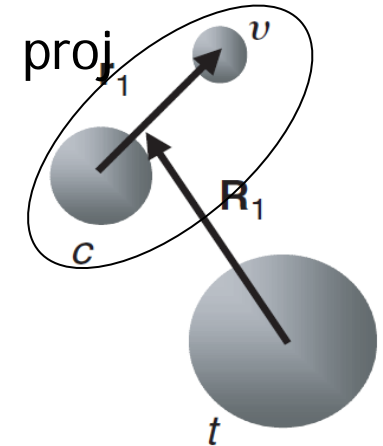
- spin**, **parity** are the total spin and parity of the projectile bin with quantum numbers l, j

How to do a breakup calculation with fresco?

```
&POTENTIAL part='Proj' a1=1 rc=2.65 /  
&POTENTIAL part='Core' a1=208 rc=1.3 v=114.2 vr0=1.286 a=0.853 w=9.44 wr0=1.739 aw=0.809 /  
&POTENTIAL part='Valence' a1=208 rc=1.3 v=34.819 vr0=1.17 a=0.75 w=15.340 wr0=1.32 aw=0.601 /  
&POTENTIAL part='Gs' a1=1 rc=2.391 v=44.675 vr0=2.391 a=.48 vso=4.898 rso0=2.391 aso=0.48 /
```

POTENTIAL namelist is identical to POT namelist is standard input

- **part**=proj (projectile-target interaction) =core (core-target interaction)
- **part**=valence (valence-target interaction)
- **part**=gs/bin (core-valence interaction in g.s. or continuum)



How to do a breakup calculation with fresco?

```
CDCC 8B+208Pb ; nuclear and coulomb s-wave breakup
NAMELIST
&Fresco hcm= 0.01 rmatch= -60.000 rintp=0.15 rsp= 0.0 rasy= 1000.00 accrcy= 0.001
  jtm= 0.0 jtmx= 9000.0 absend= -50.0000
  jump = 1 10 50 200
  jbord= 0.0 200.0 300.0 1000.0 9000.0
  thmin= 0.00 thmax= 20.00 thinc= 0.05 cutr=-20.00
  ips= 0.0000 it0= 0 iter= 0 iblock= 21 nnu= 24 smallchan= 1.00E-12 smallcoup= 1.00E-12
  elab= 656.0000 pel=1 exl=1 lab=1 lin=1 lex=1 chans= 1 smats= 2 xstabl= 1 cdcc= 1/
&Partition namep='8B' massp= 8. zp= 5 nex= 21 pwf=T namet='208Pb' masst=208. zt= 82 qval= 0.1370/
&States jp= 1.5 ptyp= 1 ep= 0.0000 cpot= 1 jt= 0.0 ptyl= 1 et= 0.0000/
&States jp= 0.5 ptyp= 1 ep= 0.1583 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 0.2180 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 0.3260 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 0.4830 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 0.6889 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 0.9438 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 1.2478 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 1.6007 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 2.0027 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 2.4536 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 2.9536 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 3.5025 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 4.1005 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 4.7474 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 5.4434 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 6.1884 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 6.9824 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 7.8253 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 8.7173 cpot= 1 copyt= 1/
&States jp= 0.5 ptyp= 1 ep= 9.6583 cpot= 1 copyt= 1/
&Partition namep='7Be' massp= 7. zp= 4 nex= 4 pwf=T namet='209Pb' masst=209.0000 zt=83 qval= 0./
&States jp= 0.0 ptyp= 1 ep= 0.0000 cpot= 2 jt= 0.0 ptyl= 1 et= 0.0000/
&Partition /
&Pot kp= 1 type= 0 shape= 0 p(1:3)= 1.0000 0.0000 2.6500 /
&Pot kp= 2 type= 0 shape= 0 p(1:3)= 208.0000 0.0000 1.3000 /
&Pot kp= 2 type= 1 shape= 0 p(1:6)= 114.2000 1.2860 0.8530 9.4400 1.7390 0.8090 /
&Pot kp= 3 type= 0 shape= 0 p(1:3)= 208.0000 0.0000 1.3000 /
&Pot kp= 3 type= 1 shape= 0 p(1:6)= 34.8190 1.1700 0.7500 15.3400 1.3200 0.6010 /
&Pot kp= 4 type= 0 shape= 0 p(1:3)= 1.0000 0.0000 2.3910 /
&Pot kp= 4 type= 1 shape= 0 p(1:3)= 44.6750 2.3910 0.4800 /
&Pot kp= 4 type= 3 shape= 0 p(1:3)= 4.8980 2.3910 0.4800 /
&Pot /
&Overlap kn1= 1 ic1=1 ic2=2 in= 1 kind=0 nn= 1 l=1 sn=0.5 j= 1.5 nam=1 ampl= 1.00 kbpot= 4 be= 0.1370 isc=12 ipc=0 /
&Overlap kn1= 2 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -0.0182 isc=12 ipc=2 nk= 20 er= -0.0344 /
&Overlap kn1= 3 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -0.0771 isc=12 ipc=2 nk= 20 er= -0.0834 /
&Overlap kn1= 4 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -0.1850 isc=12 ipc=2 nk= 20 er= -0.1324 /
&Overlap kn1= 5 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -0.3419 isc=12 ipc=2 nk= 20 er= -0.1814 /
&Overlap kn1= 6 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -0.5479 isc=12 ipc=2 nk= 20 er= -0.2794 /
&Overlap kn1= 7 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -0.8028 isc=12 ipc=2 nk= 20 er= -0.3284 /
&Overlap kn1= 8 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -1.1067 isc=12 ipc=2 nk= 20 er= -0.3774 /
&Overlap kn1= 9 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -1.4596 isc=12 ipc=2 nk= 20 er= -0.4264 /
&Overlap kn1= 10 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -1.8616 isc=12 ipc=2 nk= 20 er= -0.4754 /
&Overlap kn1= 11 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -2.3125 isc=12 ipc=2 nk= 20 er= -0.5245 /
&Overlap kn1= 12 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -2.8125 isc=12 ipc=2 nk= 20 er= -0.5735 /
&Overlap kn1= 13 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -3.3614 isc=12 ipc=2 nk= 20 er= -0.6225 /
&Overlap kn1= 14 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -3.9594 isc=12 ipc=2 nk= 20 er= -0.6715 /
&Overlap kn1= 15 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -4.6064 isc=12 ipc=2 nk= 20 er= -0.7205 /
&Overlap kn1= 16 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -5.3023 isc=12 ipc=2 nk= 20 er= -0.7695 /
&Overlap kn1= 17 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -6.0473 isc=12 ipc=2 nk= 20 er= -0.8185 /
&Overlap kn1= 18 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -6.8413 isc=12 ipc=2 nk= 20 er= -0.8675 /
&Overlap kn1= 19 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -7.6843 isc=12 ipc=2 nk= 20 er= -0.9165 /
&Overlap kn1= 20 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -8.5763 isc=12 ipc=2 nk= 20 er= -0.9655 /
&Overlap kn1= 21 ic1=1 ic2=2 in= 1 kind=0 l=0 sn=0.5 j= 0.5 nam=1 ampl= 1.00 kbpot= 4 be= -9.5173 isc=12 ipc=2 nk= 20 er= -0.9655 /
&Overlap / !
&Coupling icto= 1 icfrom= 2 kind=3 ip1= 2 ip2= 0 ip3= 0 p1= 3.0000 p2= 2.0000 /
&Coupling /
```

cdc < short.in >fresco.in

This command produce the standard fresco input where all states/overlaps are explicitly defined

isc=12 standard normalization for momentum bins

kind=3 projectile single-particle coupling

How to do a breakup calculation with fresco?

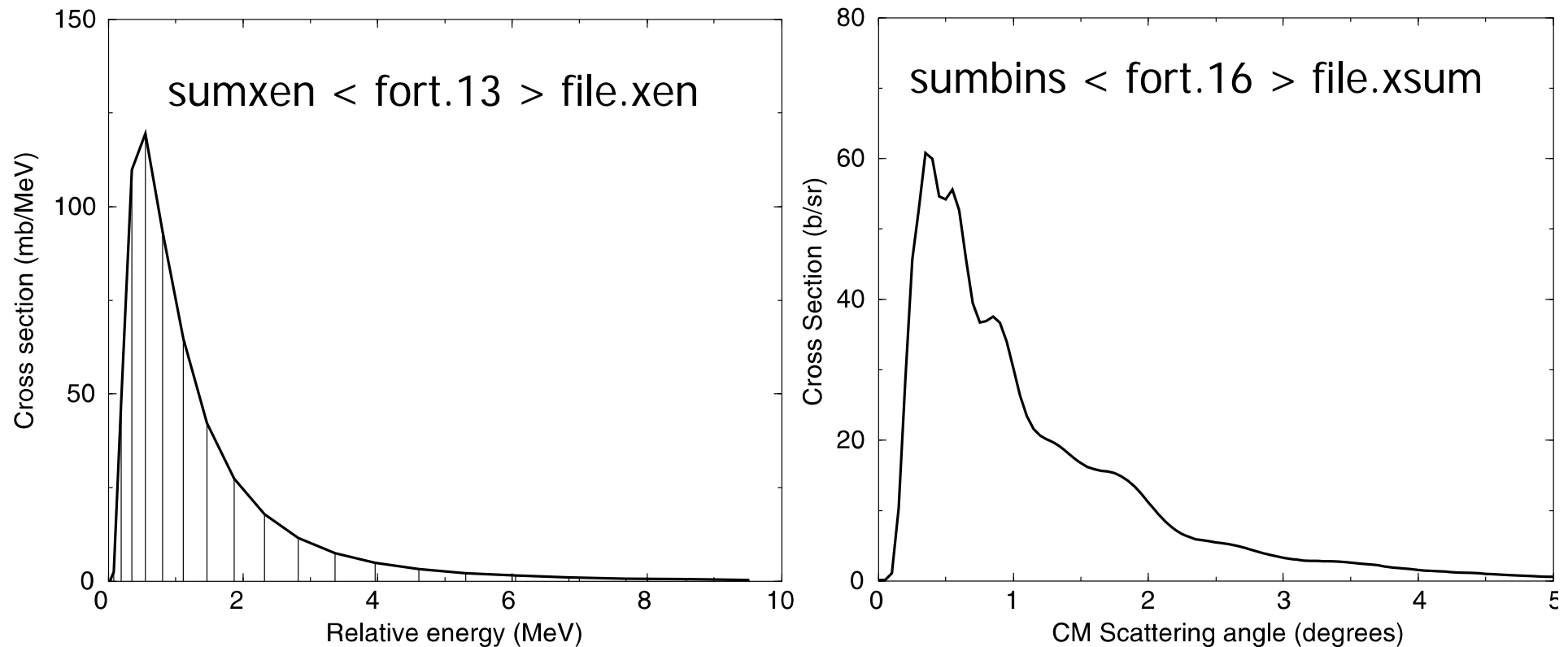


Fig. B.3. Breakup of ^8B on ^{208}Pb at 82 MeV/u. Left: p- ^7Be relative energy distribution. Right: center-of-mass angular distribution. Both are obtained with the input of Box B.3.

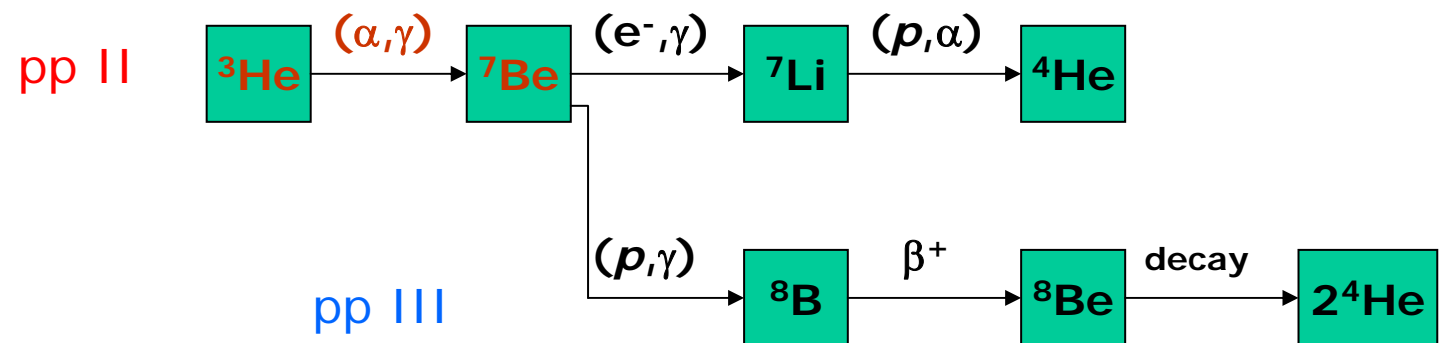
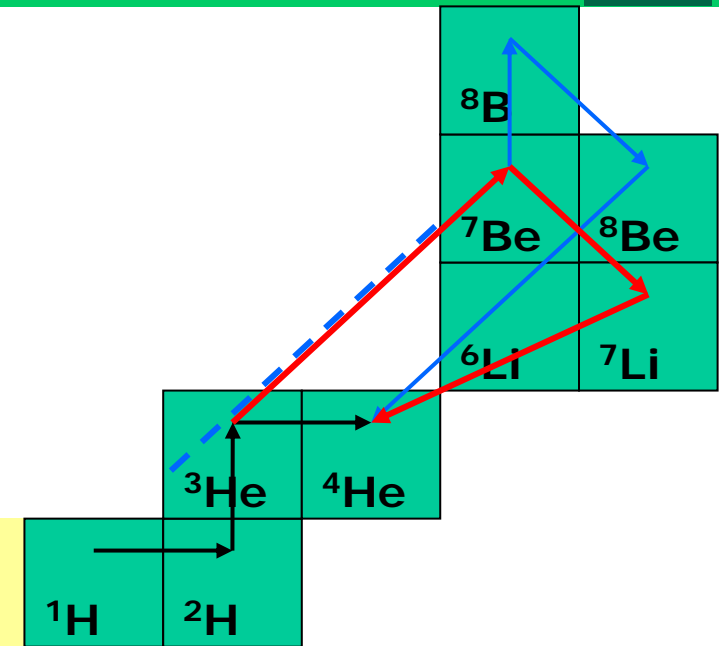
Breakup with CDCC: one case in detail

Capture rate ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$

Capture rates at astrophysical energies are hard to measure and thus rely on extrapolations to low energies

Motivates alternative methods using inverse reaction ${}^7\text{Be} + \gamma \rightarrow \alpha + {}^3\text{He}$

- (i) Coulomb Dissociation method
- (ii) Asymptotic Normalization Coefficient (ANC) method



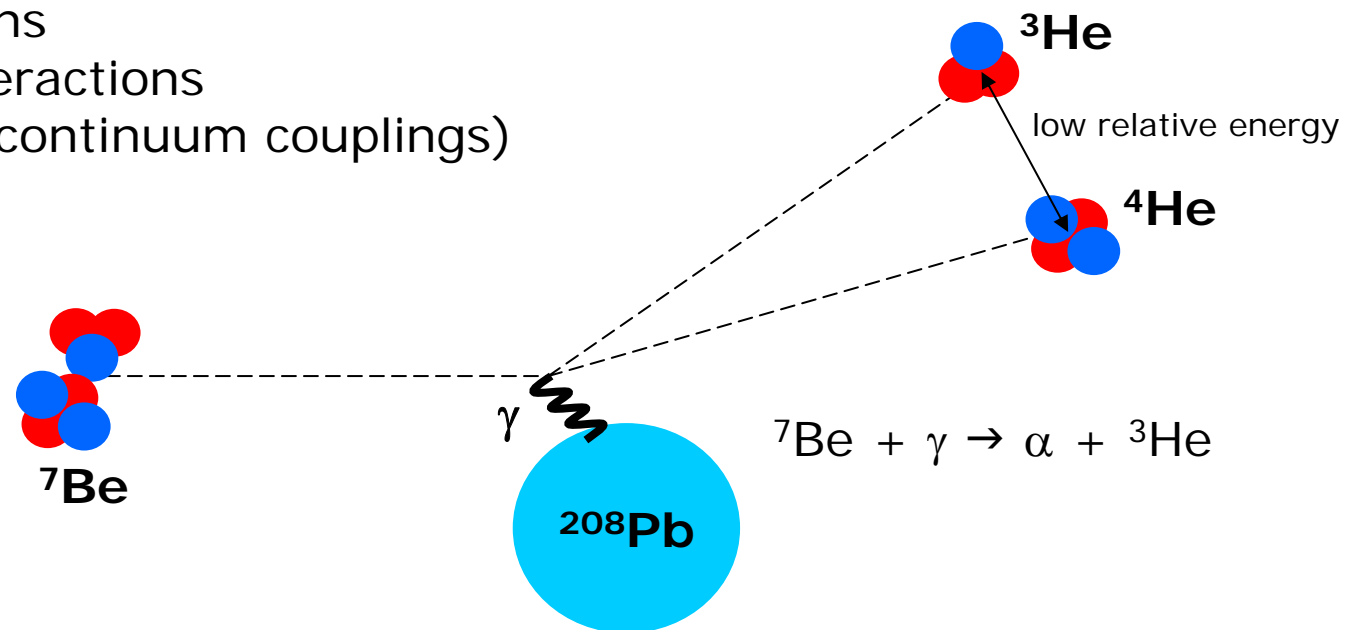
Coulomb Dissociation method

Cross sections larger for inverse breakup reaction

Can relate breakup cross section to capture rate using semi-classical reaction theory

complications

- **Nuclear breakup**
- E2 contributions
- Final state interactions
(continuum-continuum couplings)



Motivated experiment at NSCL : ${}^7\text{Be} + {}^{208}\text{Pb}$ @ 100 MeV/nucleon

ANC method (nuclear breakup)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \frac{C_{lj}^2}{b_{nlj}^2} \left(\frac{d\sigma}{d\Omega}\right)_{\text{theory}}^{nlj}$$

Outside range of nuclear force,
in asymptotic region

$$I_{c\nu l j}^P(r) \rightarrow S_{nlj}^{1/2} \phi_{nlj}(r) \rightarrow C_{c\nu l j}^P \frac{W_{-\eta, l+1/2}(2kr)}{r}$$

overlap integral

c = core ^4He

ν = valence ^3He

P = projectile ^7Be

ANC

g.s. single particle wavefunction

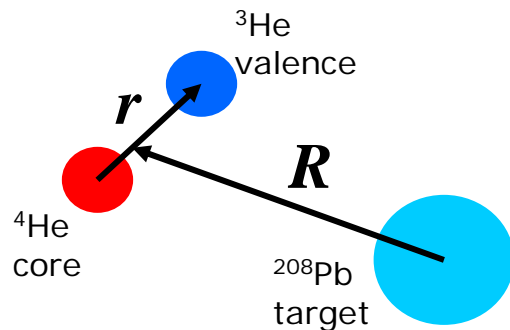
Spectroscopic factor

Whittaker function

$$C_{c\nu l j}^P = S_{nlj}^{1/2} b_{nlj}$$

Motivated experiment at Texas A&M : $^7\text{Be} + ^{12}\text{C}$ @ 25 MeV/nucleon

Breakup of ^7Be : one application in detail



- ^7Be treated as 2-body projectile
- 3-body Hamiltonian for reaction

$$H = T_R + U_{cT} + U_{vT} + h_{vc}$$

$$h_{vc} = T_r + V_{vc}$$

fix V_{cT} and V_{vT}
from elastic
scattering

ϕ is ^7Be ($\alpha + ^3\text{He}$) cluster wavefunction defined by potential V_{vc}

$$h_{vc}\phi_n(r) = \varepsilon_n\phi_n(r) \quad \varepsilon_n < 0$$

$$h_{vc}\phi_{lj}(k, r) = \varepsilon_k\phi_{lj}(k, r) \quad \varepsilon_k > 0$$

fixed by
binding energy for bound states

resonances and scattering phase
shifts for continuum

l = core-valence relative angular momentum
 j = projectile total angular momentum

Breakup of ${}^7\text{Be}$: continuum discretization

- Discretize continuum into bins
- average wavefunction over a bin with weight $w_i(k)$

$$\phi_{i,lj}(r) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} w_i(k) \phi_{lj}(k, r) dk$$

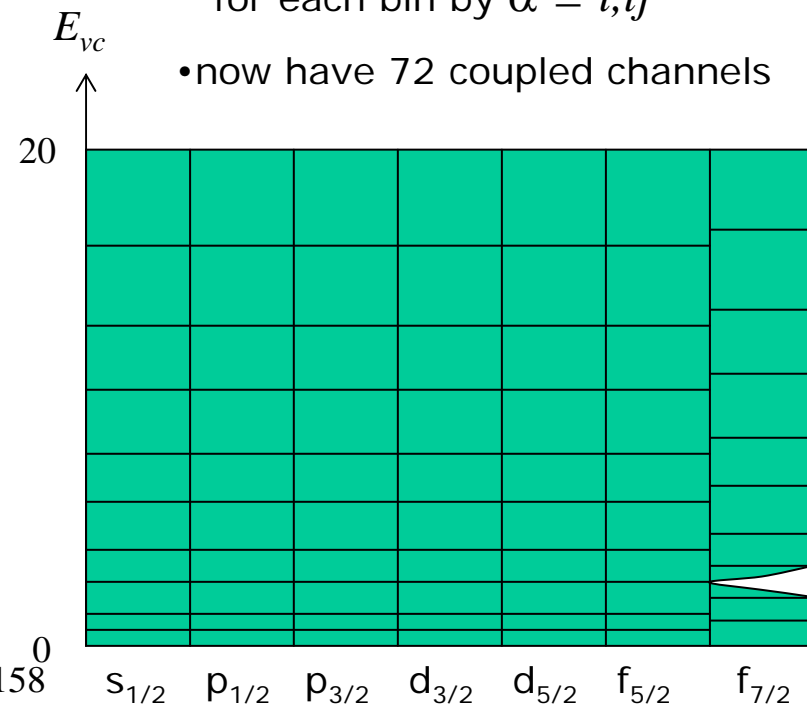
$w_i(k)$ chosen so that the bin wavefunctions are real and normalized correctly using

$$N_i = \int_{k_{i-1}}^{k_i} |w_i(k)|^2 dk$$

bound states \rightarrow

$p_{1/2}$ ——— 0
 ——— -1.158
 $p_{3/2}$ ——— -1.587

- label the quantum numbers for each bin by $\alpha \equiv i, lj$
- now have 72 coupled channels



Breakup of ^7Be : CDCC equations

- We have 72 coupled channels, each labeled by the set of quantum numbers

$$\alpha \equiv (i \ l \ j \ L)$$

- Solve set of radial coupled equations

$$\left[T_R^L + V_{\alpha\alpha}^J(R) - E_\alpha \right] u_\alpha^J(R) = - \sum_{\alpha' \neq \alpha} V_{\alpha\alpha'}^J(R) u_{\alpha'}^J(R) \quad (1)$$

- Where the coupling potential from state α to state α' is

$$V_{\alpha\alpha'}^J(R) = \left\langle \left[\phi_{\alpha'}(\vec{r}) \otimes Y_{L'}(\hat{R}) \right]_{JM} \left| V(\vec{r}, \vec{R}) \right| \left[\phi_\alpha(\vec{r}) \otimes Y_L(\hat{R}) \right]_{JM} \right\rangle$$

$$V(\vec{r}, \vec{R}) = U_{cT}(\vec{R}_{cT}) + U_{vT}(\vec{R}_{vT})$$

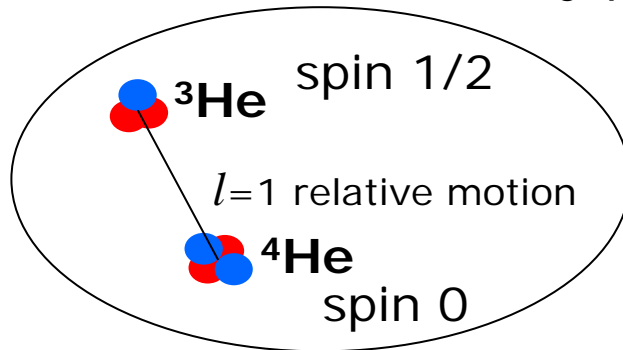
and the cluster target potentials include both Coulomb and Nuclear parts

Coulomb breakup \equiv neglect Nuclear in coupling potentials on rhs of Eq. (1)

Nuclear breakup \equiv neglect Coulomb in coupling potentials on rhs of Eq. (1)

Breakup of ^7Be : inputs

^7Be modeled as 2-body projectile



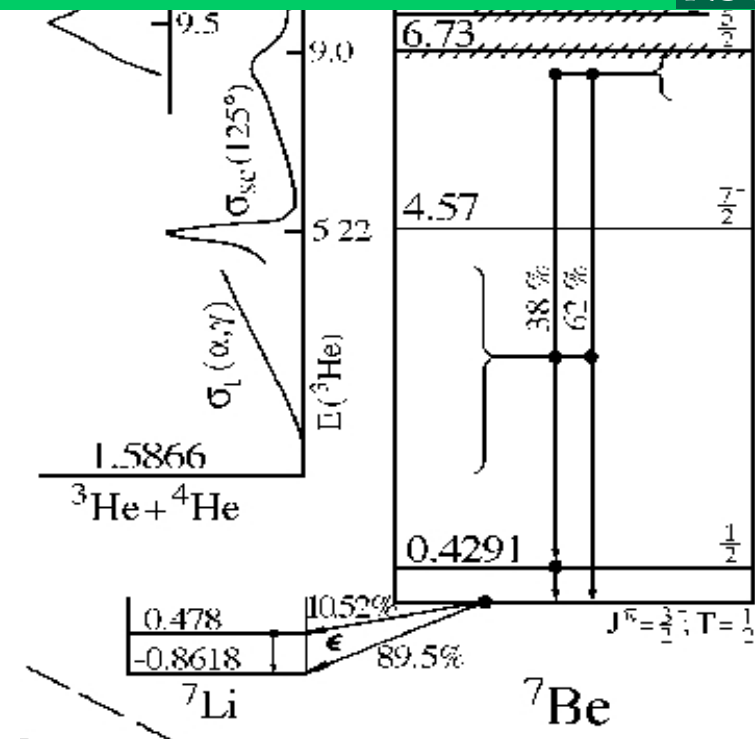
Buck *et al.*

potential fitted to :

- ground and excited state energies
- $7/2^-$ and $5/2^-$ resonances
- scattering phase shifts
- charge radius, quadrupole and octupole moments and $B(E2)$ for ^7Li

wavefunction has node due to Pauli blocking

parity dependent potential



cluster-target potentials
from elastic scattering

$^3\text{He} + ^{208}\text{Pb}$ @ 300 MeV

$^4\text{He} + ^{208}\text{Pb}$ @ 400 MeV

$^3\text{He} + ^{12}\text{C}$ @ 75 MeV

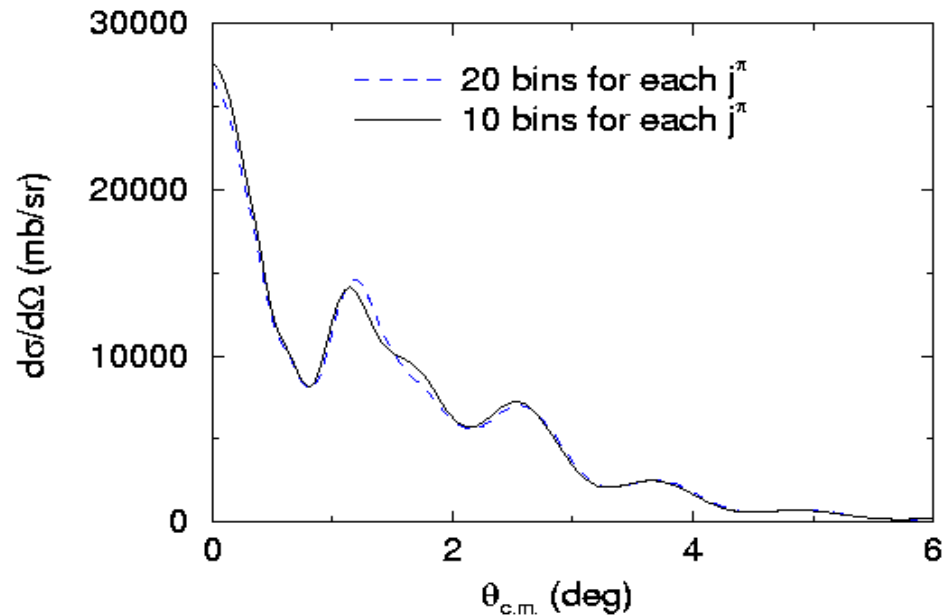
$^4\text{He} + ^{12}\text{C}$ @ 100 MeV

NSCL

Texas

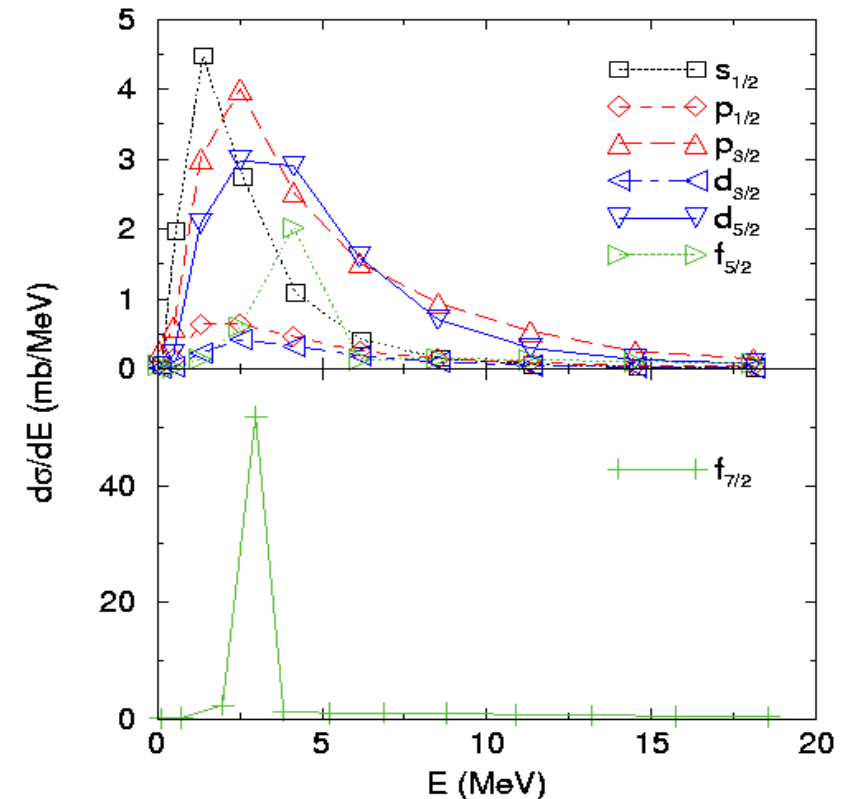
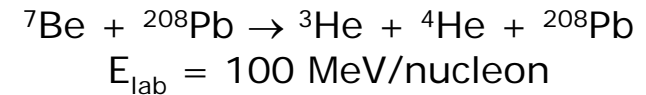
A & M

Breakup of ${}^7\text{Be}$: convergence

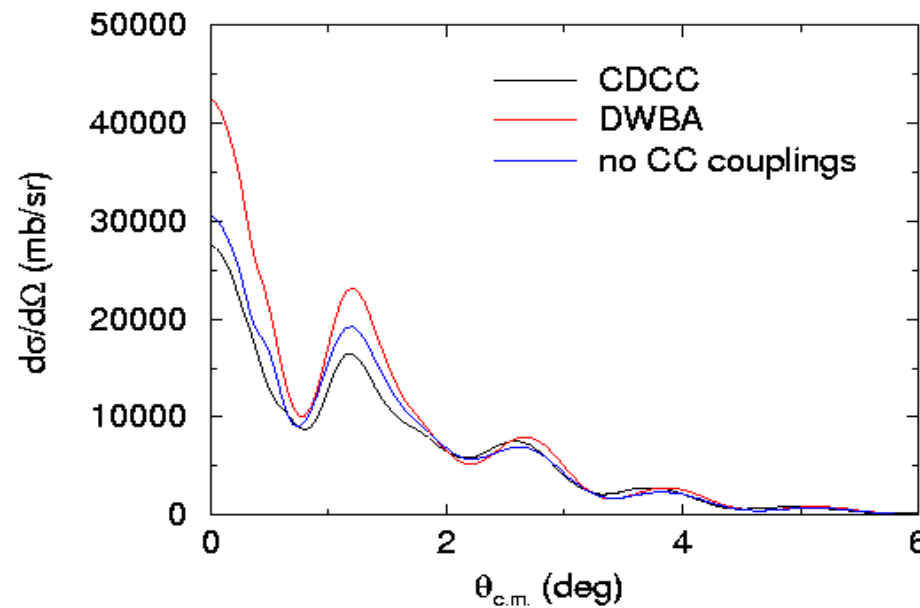


Convergence checked for many variables, such as:

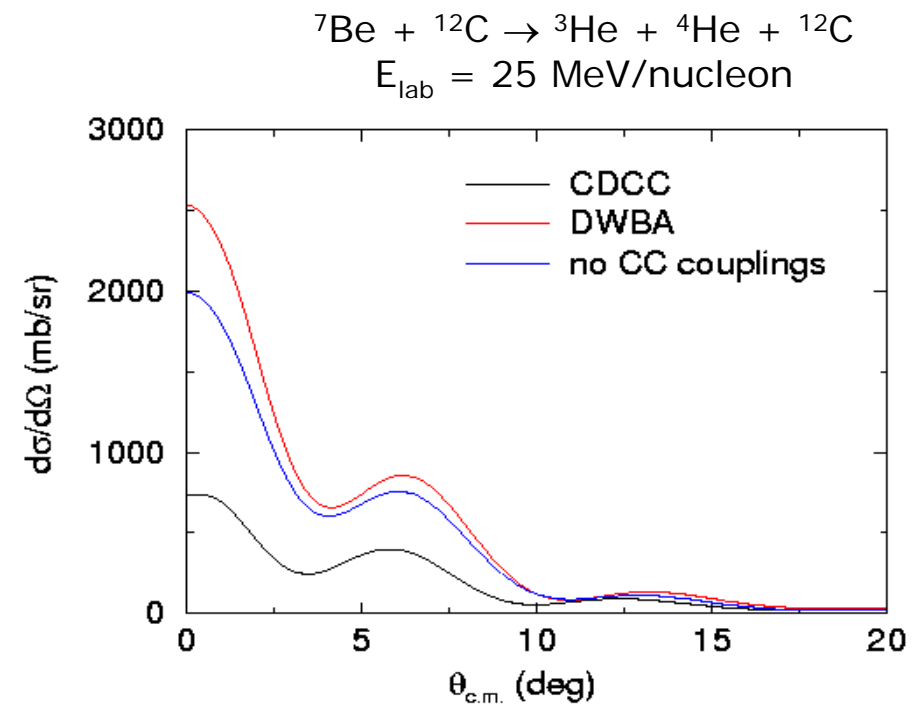
- Potential multipoles ($Q \leq 2$)
- Max radius for coupled equations ($R_{max} = 1000\text{fm}$)
- Maximum relative energy ($E_{max} = 20\text{ MeV}$)
- Number of partial waves in coupled equations ($L_{max} = 10000, 2000$ using interpolation)
- Bin radius and step length for coupling integrals ($r_{bin} = 50\text{fm}$, $h = 0.04\text{fm}$)
- Number of partial waves for breakup states ($l \leq 3$)
- Iterative solution of coupled equations versus solving set of equations exactly



Breakup of ^7Be : higher order couplings



$^7\text{Be} + ^{208}\text{Pb} \rightarrow ^3\text{He} + ^4\text{He} + ^{208}\text{Pb}$
 $E_{\text{lab}} = 100 \text{ MeV/nucleon}$

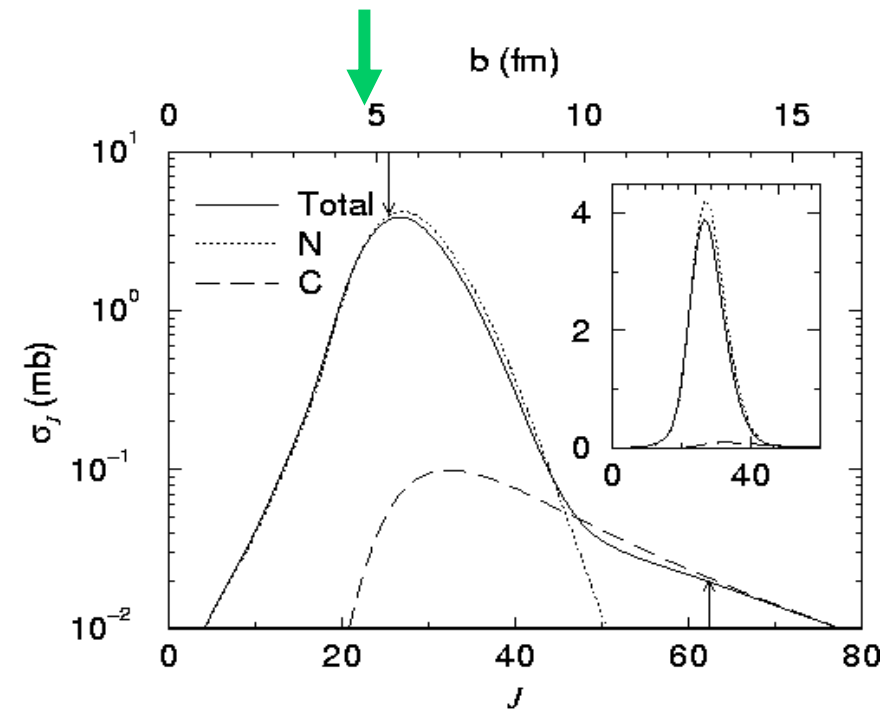
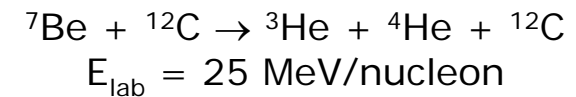


Breakup of ^7Be on C: peripherality

ANC method requires
that reaction is peripheral
and therefore only probes
the tail of the wavefunction

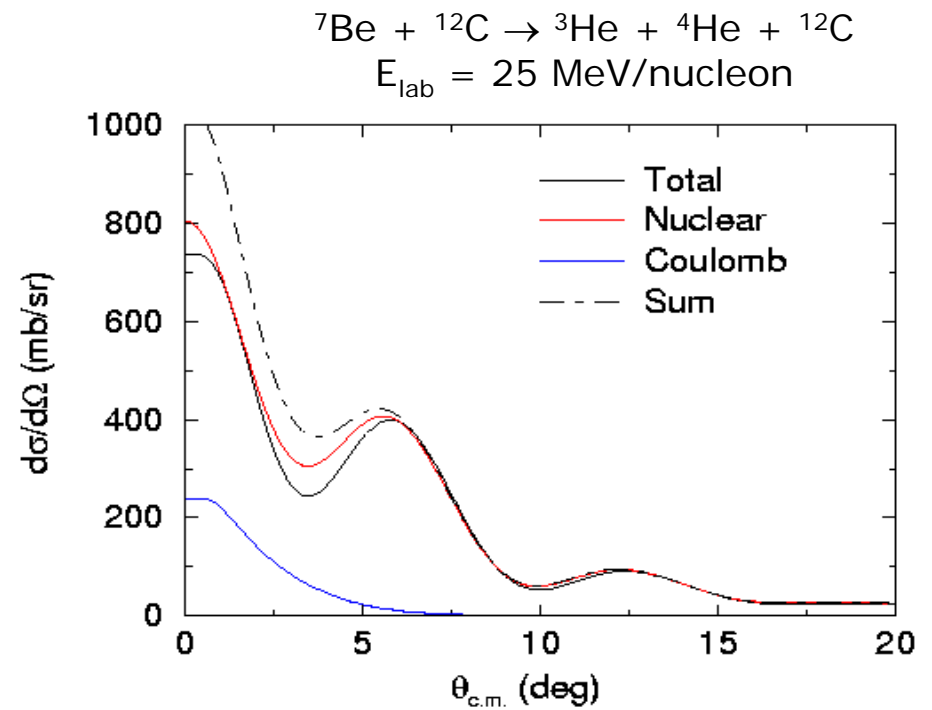
approx 28% of breakup
cross section comes from
impact parameters
less than sum of radii

impact parameter from
semi-classical relation $J=Kb$

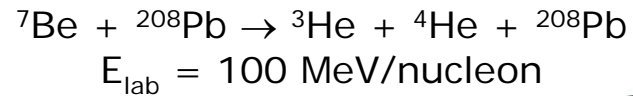


Breakup of ^7Be on C: Coulomb free?

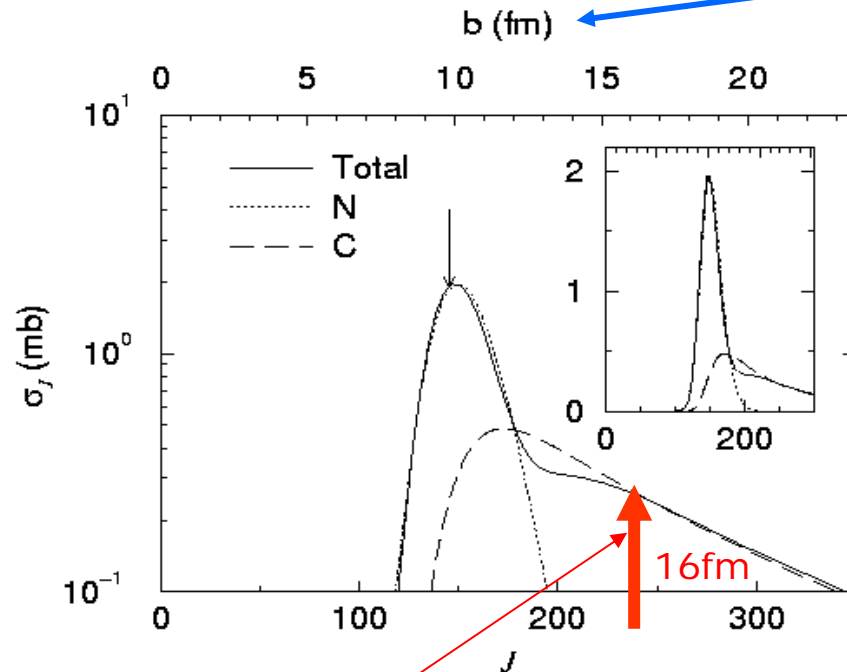
Note that Coulomb is not negligible



Breakup of ${}^7\text{Be}$: nuclear free?



impact parameter from
semi-classical relation $J=Kb$

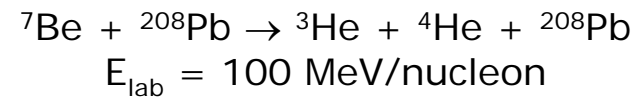
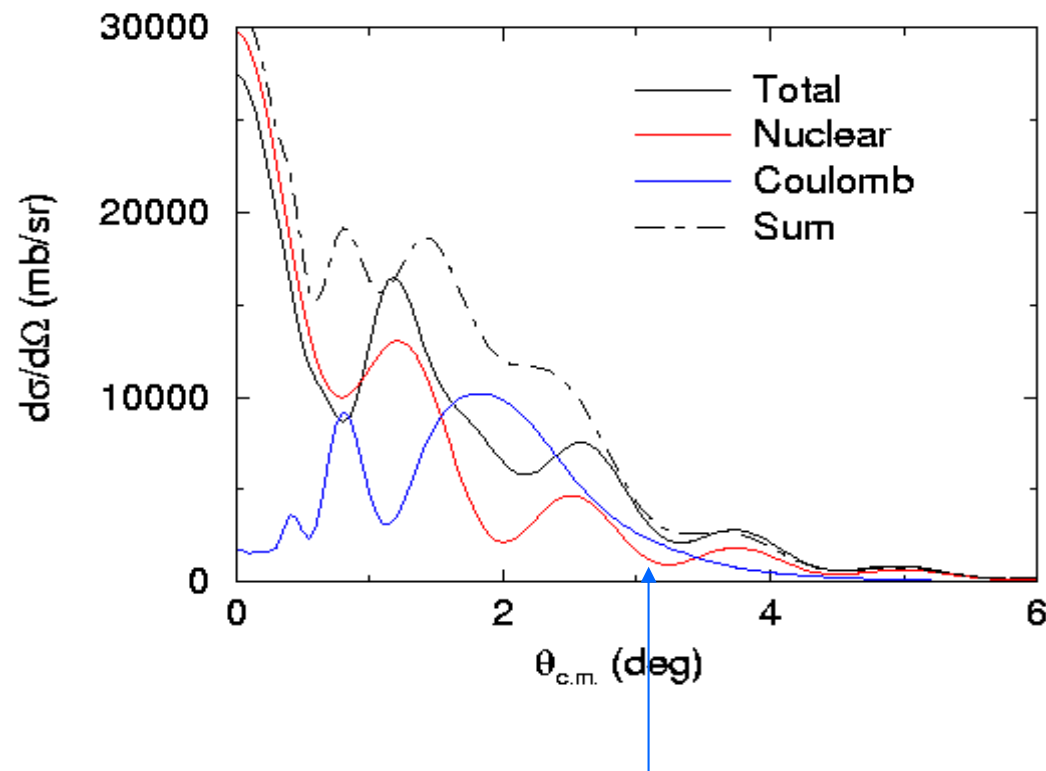


Coulomb dissociation method requires that the reaction is Coulomb dominated, outside the range of the nuclear force

maximum range of nuclear force

assuming Rutherford trajectories
this impact parameter relates to
a scattering angle of 2.5°

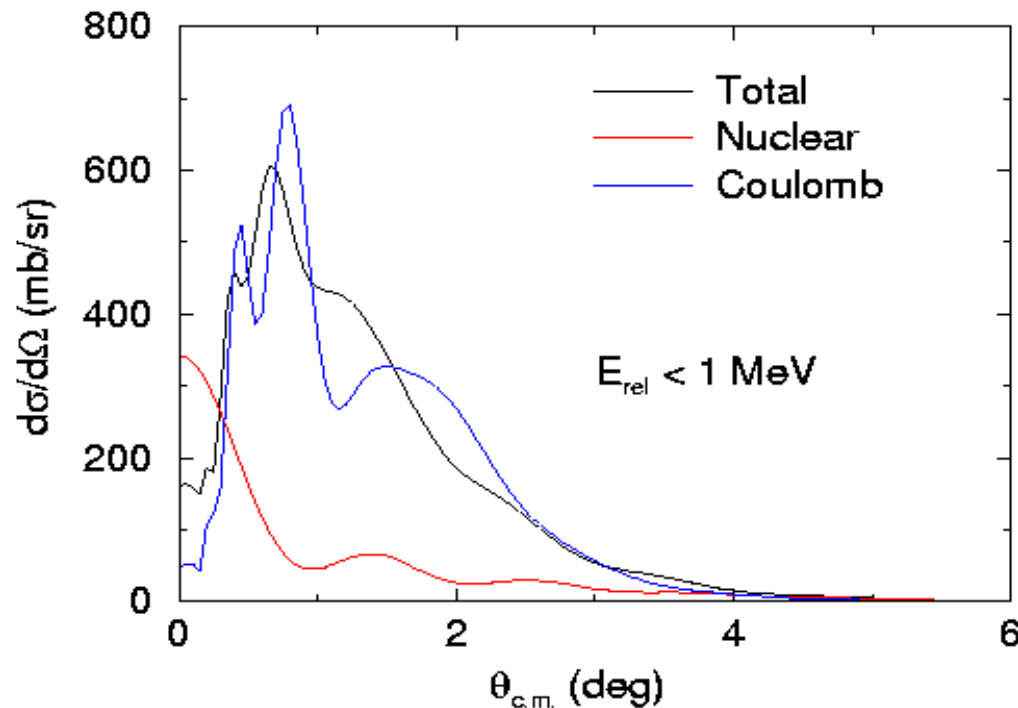
Breakup of ${}^7\text{Be}$: nuclear component



Cut-off angle from Rutherford orbit = 2.5°

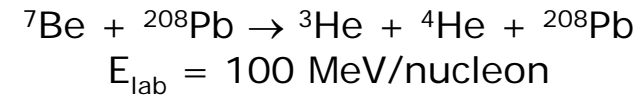
cross section dominated by
nuclear breakup
 even for small scattering
 angles below the **cut-off angle**

Breakup of ${}^7\text{Be}$: small relative energies

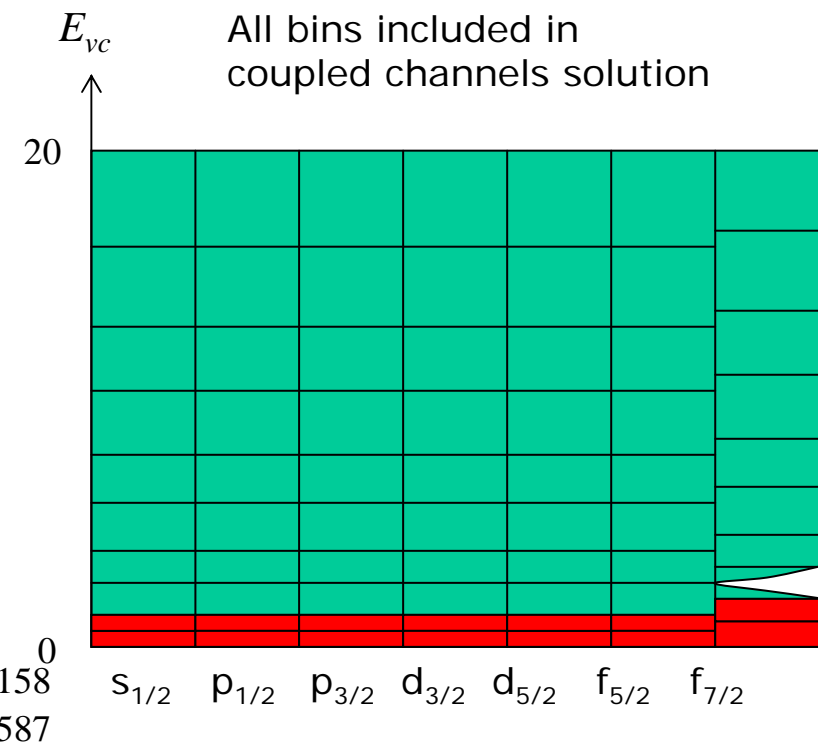


Angular cross section summed for
only first 2 bins in each j^π set

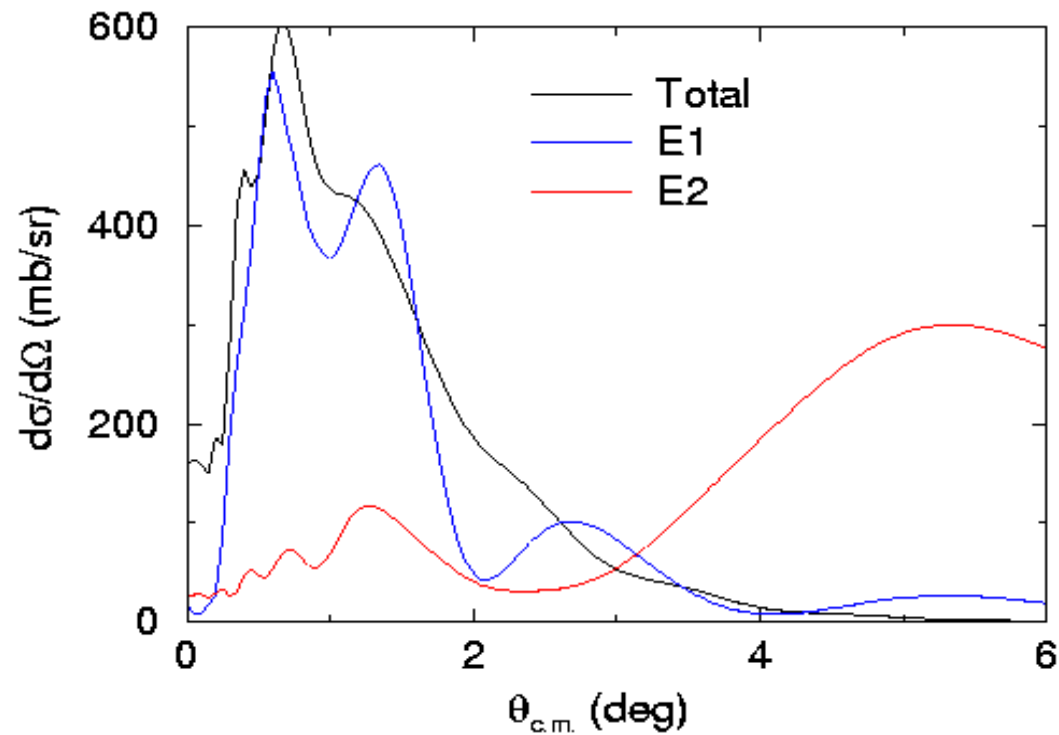
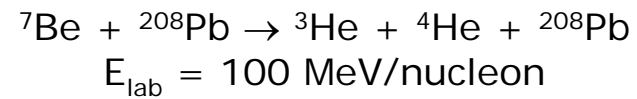
This dramatically reduces the amount of nuclear breakup in the forward angles



restricted maximum
relative energy between
 ${}^3\text{He}$ and ${}^4\text{He}$ fragments
in final state

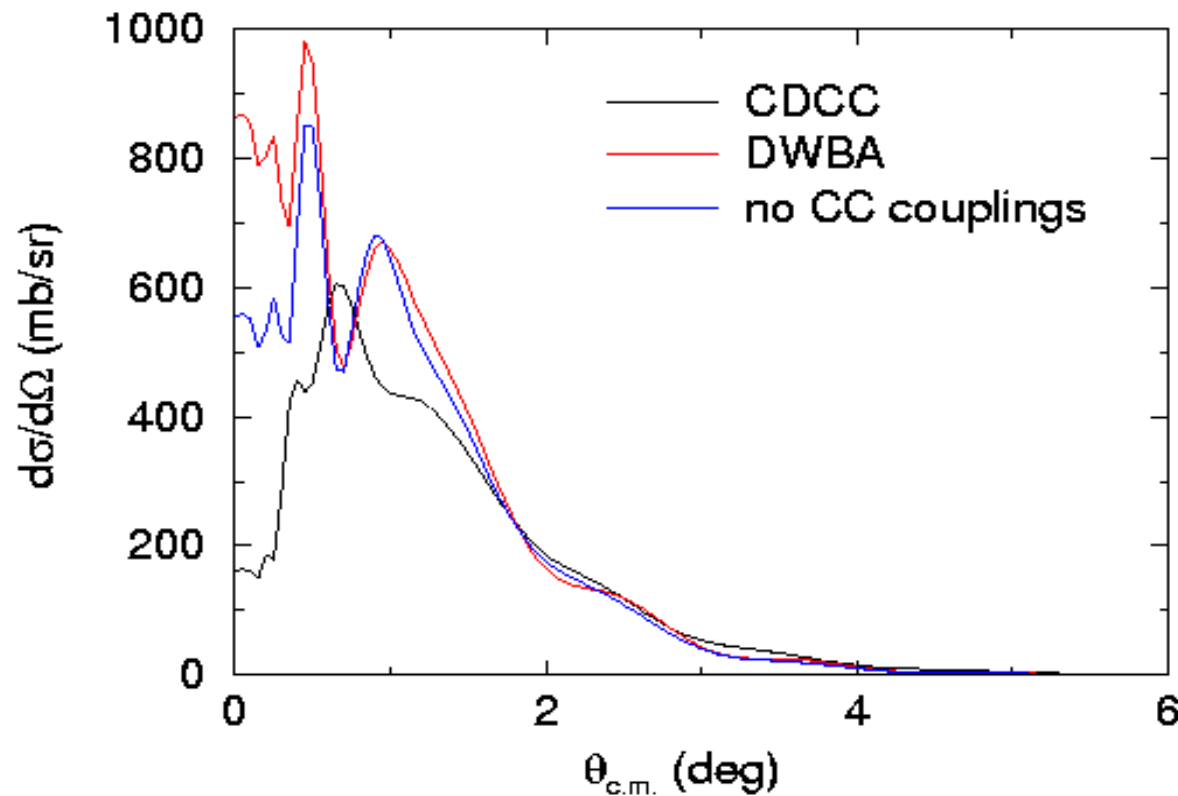


Breakup of ${}^7\text{Be}$: E1 versus E2



restricted maximum
relative energy between
 ${}^3\text{He}$ and ${}^4\text{He}$ fragments
in final state

Breakup of ^7Be : cont-cont couplings

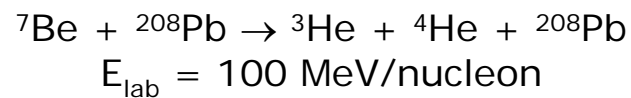
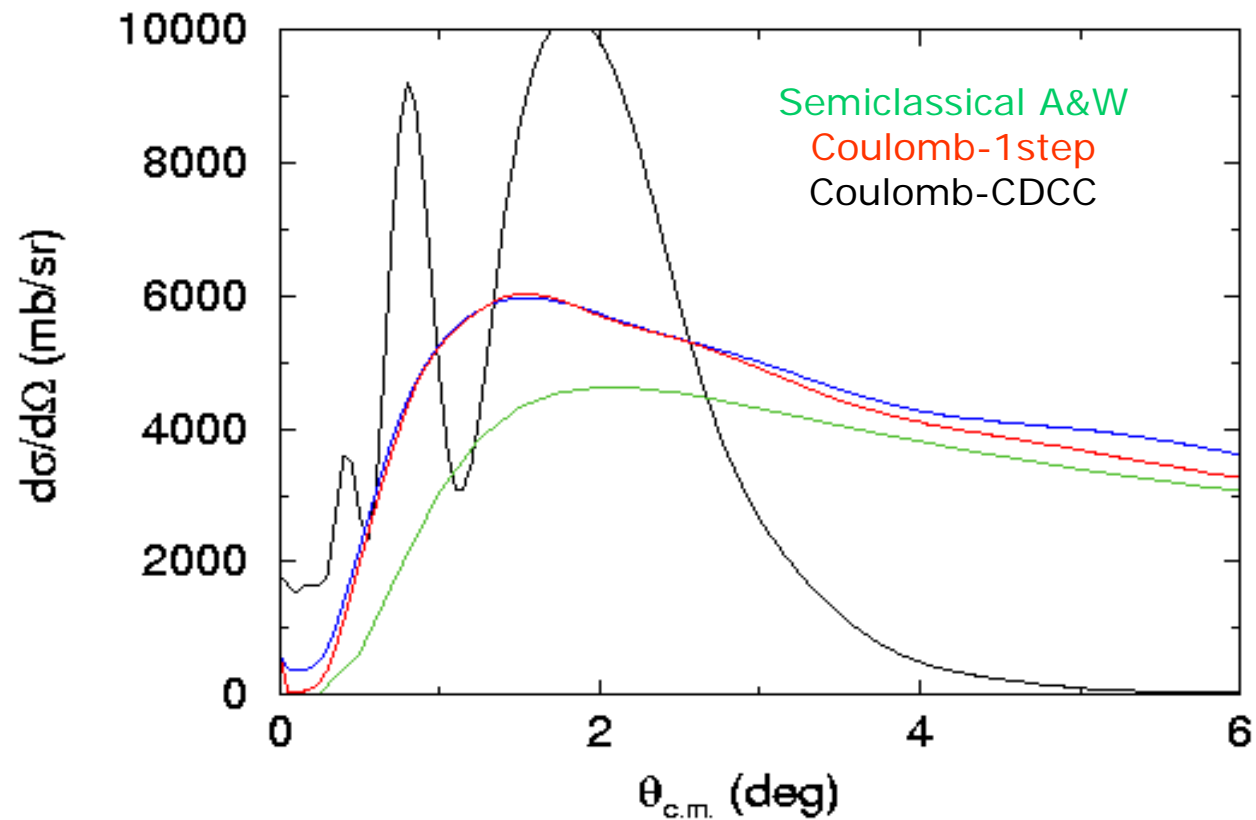


restricted maximum
relative energy between
 ^3He and ^4He fragments
in final state

DWBA \rightarrow first order

No CC couplings
includes couplings
to all orders to/from
ground state only

Breakup of ^7Be : comparison with other models



CDCC cross sections



- 1) Calculate:
 - projectile wfns: bound and scattering states
 - bins wfns
 - coupling potentials
- 2) Solve coupled channel equations and match to obtain S-matrix
- 3) Construct observables.

Scattering amplitude

$$\begin{aligned} \tilde{\mathcal{F}}_{M'M}(\mathbf{K}_{p'}) &= \frac{4\pi}{K_0} \sqrt{\frac{K_{p'}}{K_0}} \sum_{LL'J} \langle L0, I_p M | JM \rangle \langle L'M - M', I_{p'} M' | JM \rangle \\ &\times \exp(i[\sigma_L + \sigma_{L'}]) \frac{1}{2i} \mathbf{S}_{\alpha'\alpha}^J(p') Y_L^0(\hat{\mathbf{K}}_0) Y_{L'}^{M-M'}(\hat{\mathbf{K}}_{p'}). \end{aligned}$$

$$\frac{d\sigma(p')}{d\Omega_K} = \frac{1}{2I_p + 1} \sum_{MM'} \left| \tilde{\mathcal{F}}_{M'M}(\mathbf{K}_{p'}) \right|^2$$

Three-body observables: T-matrix



T matrix can be constructed from the CDCC solution

$$\mathbf{T}_{\mu\sigma:M}(\mathbf{k}, \mathbf{K}) = \langle \phi_{\mathbf{k}\mu\sigma}^{(-)}(\mathbf{r}) e^{i\mathbf{K}\cdot\mathbf{R}} | U(\mathbf{r}, \mathbf{R}) | \Psi_{\mathbf{K}_0 M}^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) \rangle$$

$$\mathbf{T}_{\mu\sigma:M}(\mathbf{k}, \mathbf{K}) = \sum_{p, M'} \langle \phi_{\mathbf{k}\mu\sigma}^{(-)} | \tilde{\phi}_p^{M'} \rangle \langle \tilde{\phi}_p^{M'} e^{i\mathbf{K}\cdot\mathbf{R}} | U(\mathbf{r}, \mathbf{R}) | \Psi_M(\mathbf{r}, \mathbf{R}) \rangle$$

$$\langle \hat{\phi}_{\mathbf{k}\mu\sigma}^{(-)} | \phi_p^{M'} \rangle = \frac{(2\pi)^{3/2}}{k\sqrt{N_\alpha}} \sum_v (-i)^\ell \langle \ell v, s\sigma | jm \rangle \langle jm, I_c \mu | I_p' M' \rangle e^{i\bar{\delta}_p(k)} \tilde{u}_p(k) Y_{\ell v}(\hat{\mathbf{k}}).$$

$$\hat{\mathcal{T}}_{M'M}^p(\mathbf{K}_p) = \langle \hat{\phi}_p^{M'} e^{i\mathbf{K}_p\cdot\mathbf{R}} | U(\mathbf{r}, \mathbf{R}) | \Psi_{\mathbf{K}_0 M}^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) \rangle = -\frac{2\pi\hbar^2}{\mu_{(vc)t}} \sqrt{\frac{K_0}{K_p}} \tilde{\mathcal{F}}_{M'M}(\mathbf{K}_p)$$

CDCC three-body observables



Let's say we measure the energy of one fragment c and the angles of both:

$$\frac{d^3\sigma}{d\Omega_c d\Omega_v dE_c} = \frac{2\pi \mu_{(vc)t}}{\hbar^2 K_0} \frac{1}{(2I_p + 1)} \sum_{\mu\sigma M} |\mathbf{T}_{\mu\sigma:M}(\mathbf{k}, \mathbf{K})|^2 \rho_{ps}(E_c, \Omega_c, \Omega_v),$$

Phase space factor

$$\rho_{ps}(E_c, \Omega_c, \Omega_v) dE_c d\Omega_c d\Omega_v =$$

$$\int d^3\mathbf{k}_c d^3\mathbf{k}_v d^3\mathbf{k}_t \delta(\mathbf{P} - \hbar\mathbf{k}_c - \hbar\mathbf{k}_v - \hbar\mathbf{k}_t) \delta(E - E_c - E_v - E_t),$$

CDCC three-body observables

Let's say we only measure fragment c both energy and angles, with a certain solid angle and an efficiency dependent on angle:

$$\left\langle \frac{d^2\sigma}{d\Omega_c dE_c} \right\rangle = \frac{1}{\Delta\Omega_c} \int_{\Delta\Omega_c} d\Omega_c \left\{ e(\Omega_c) \int d\Omega_v \frac{d^3\sigma}{d\Omega_c d\Omega_v dE_c} \right\}$$

Other typical observables:
momentum distributions

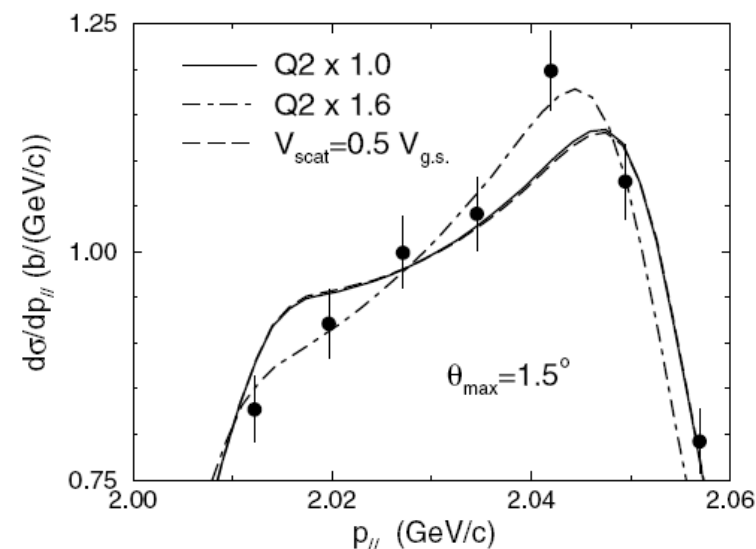


FIG. 4: Momentum distribution of ${}^7\text{Be}$ following the breakup of 44 MeV/A ${}^8\text{B}$ on a ${}^{208}\text{Pb}$ target integrated up to $\theta_{\max} = 1.5^\circ$: the previous results [8] with no renormalisation of the quadrupole excitation (solid line) and with a 1.6 renormalization (dot-dashed line) and the result using $0.5 \cdot V(\text{p-Be})$ for the final state (dashed line).