# Transfer reactions: how to deal with different mass partitions

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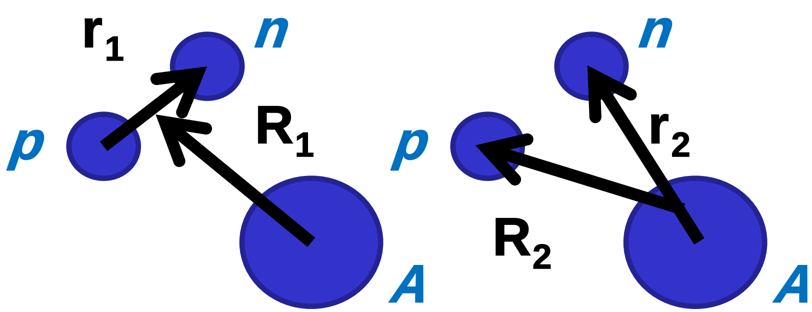
# Thinking about different mass partitions

Write the wavefunction as two components corresponding to each mass partitions



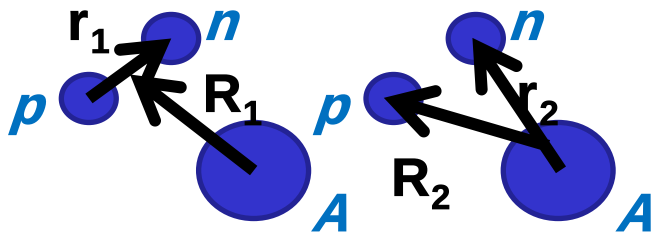

$$\Psi = \overbrace{\phi_{np}(\vec{r}_1)\,\chi_{dA}(\vec{R}_1)}^{1} + \overbrace{\phi_{nA}(\vec{r}_2)\,\chi_{pB}(\vec{R}_2)}^{2}$$

$$(H - E)\Psi = 0$$

$$\begin{cases} H_{np}\,\phi_{np} = +\varepsilon_d\,\phi_{np} \\ H_{nA}\,\phi_{nA} = +\varepsilon_B\,\phi_{nA} \end{cases}$$

# Thinking about different mass partitions

$$\left(T_{r_1} + T_{R_1} + V_{np} + V_{pA} + V_{nA} - E\right)\phi_{np}\,\chi_{dA} + \left(T_{r_2} + T_{R_2} + V_{np} + V_{pA} + V_{nA} - E\right)\phi_{nA}\,\chi_{pB} = 0$$




$$\left[T_{R_1} + V_{pA} + V_{nA} - (E - \varepsilon_d)\right]\phi_{np}\,\chi_{dA} + \left[T_{R_2} + V_{np} + V_{pA} - (E - \varepsilon_B)\right]\phi_{nA}\,\chi_{pB} = 0$$

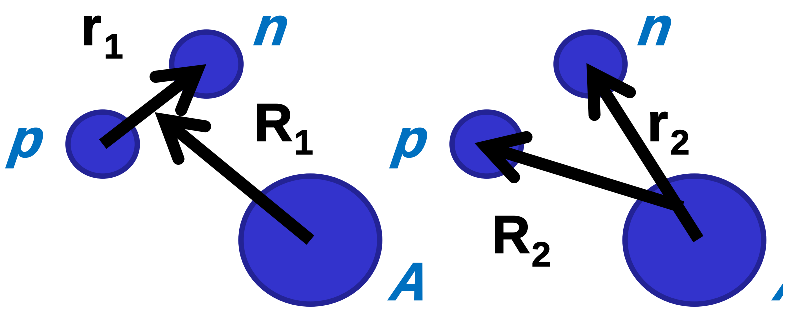
$$T_{r_1} + T_{R_1} = T_{r_2} + T_{R_2}$$

Jacobi coordinates!

# Thinking about different mass partitions

$\langle \phi_{np} |$

When projecting onto the first channel we obtain:




$$\left[ T_{R_1} + \hat{V}_{11}(\vec{R}_1) - (E - \varepsilon_d) \right] \chi_{dA}(\vec{R}_1) +$$

$$\left[ \hat{N}_{12}(\vec{R}_1, \vec{R}_2) T_{R_2} + \hat{V}_{12}(\vec{R}_2) - (E - \varepsilon_B) \right] \chi_{pB}(\vec{R}_2) = 0$$

where

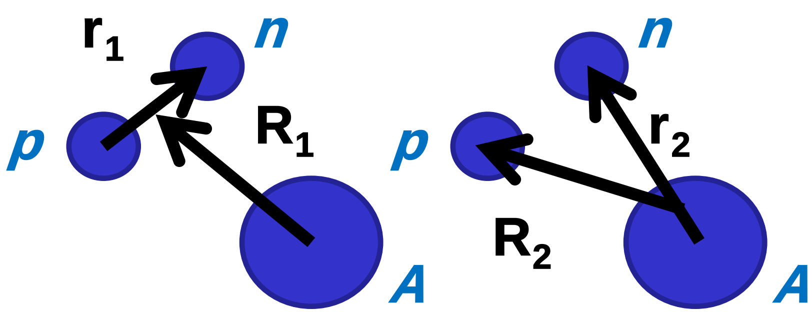
$$\hat{V}_{11}(\vec{R}_1) = \langle \phi_{np} | V_{nA} + V_{pA} | \phi_{np} \rangle_{\vec{r}_1}$$

$$\hat{V}_{12}(\vec{R}_2) = \langle \phi_{np} | V_{np} + V_{pA} | \phi_{nA} \rangle_{\vec{r}_2}$$

$$\hat{N}_{12}(\vec{R}_1, \vec{R}_2) = \langle \phi_{np} | \phi_{nA} \rangle$$

# Thinking about different mass partitions

Coupled channel equation has different features now!




Just use 1+2 channels

$$(T_1 - E_1)\chi_1(\vec{R}_1) + \left(\hat{V}_{11}\chi_1(\vec{R}_1) + \hat{V}_{12}\chi_2(\vec{R}_2)\right)$$

$$+ N_{12}(T_2 - E_2)\chi_2(\vec{R}_2) = 0$$

$\Downarrow$

Non local equation

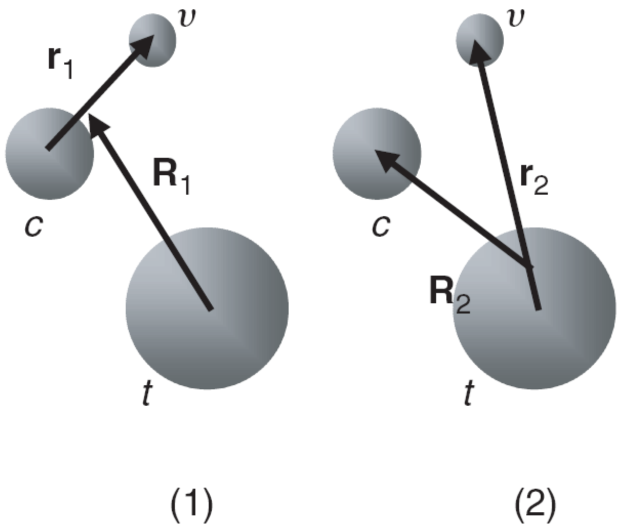
$\chi_1(\vec{R}_1)$ depends on $\vec{R}_2$!

# non-orthogonality

$$H = H_{xp}(\xi_p) + H_{xt}(\xi_t) + \hat{T}_x(R_x) + \mathcal{V}_x(R_x, \xi_p, \xi_t)$$

$$H_{prior} = H_{1p}(r_1) + H_{1t} + T_1(R_1) + \upsilon(R_1)$$

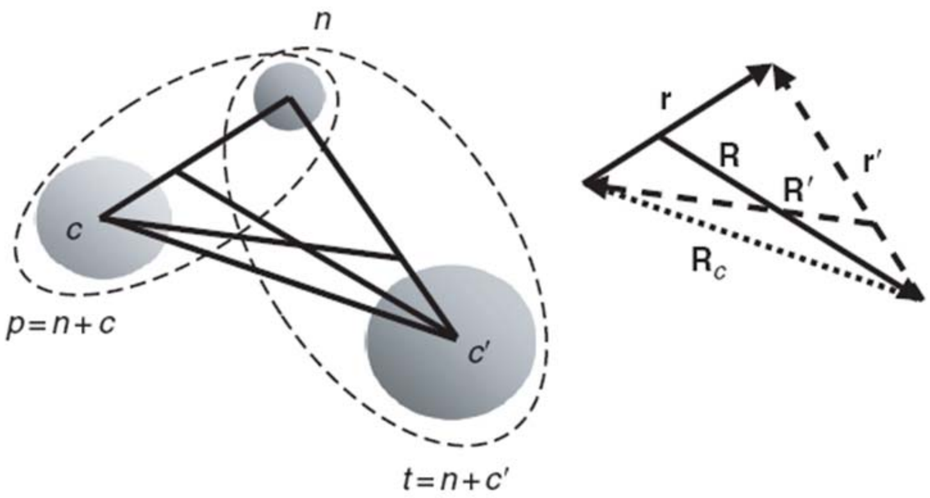
$$H_{post} = H_{2p} + H_{2t}(r_2) + T_2(R_2) + \upsilon(R_2)$$




$$[\hat{T}_{xL}(R_x) - E_{xpt}]\psi_\alpha(R_x) + \sum_{\alpha'} \hat{V}^{\text{prior}}_{\alpha\alpha'}\psi_{\alpha'}(R_{x'})$$

$$+ \sum_{\alpha',x'\neq x} \hat{N}_{\alpha\alpha'}[\hat{T}_{x'L'} - E_{x'p't'}]\psi_{\alpha'}(R_{x'}) = 0.$$

# one nucleon transfer:coordinates




$$[H_p - \varepsilon_p]\phi_p(\mathbf{r}) = 0 \quad \text{where} \quad H_p = T_{\mathbf{r}} + V_p(\mathbf{r})$$

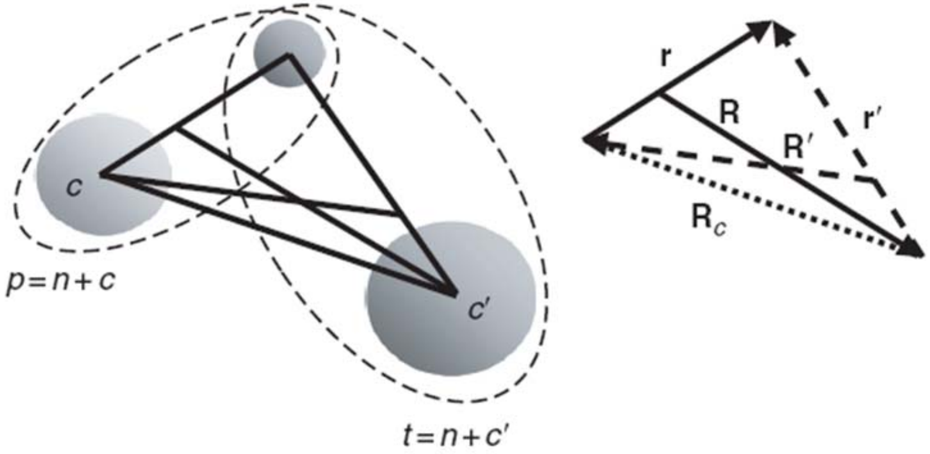
$$[H_t - \varepsilon_t]\phi_t(\mathbf{r}') = 0 \quad \text{where} \quad H_t = T_{\mathbf{r}'} + V_t(\mathbf{r}').$$

$$Q = \varepsilon_p - \varepsilon_t$$

# one nucleon transfer: operator

$$H = T_{\mathbf{r}} + T_{\mathbf{R}} + V_p(\mathbf{r}) + V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c),$$

$$T_{\mathbf{r}} + T_{\mathbf{R}} = T_{\mathbf{r}'} + T_{\mathbf{R}'}$$




$$H = H_{\text{prior}} = T_{\mathbf{R}} + U_i(R) + H_p(\mathbf{r}) + \mathcal{V}_i(\mathbf{R},\mathbf{r})$$

$$= H_{\text{post}} = T_{\mathbf{R}'} + U_f(R') + H_t(\mathbf{r}') + \mathcal{V}_f(\mathbf{R}',\mathbf{r}'),$$

$$\mathcal{V}_i(\mathbf{R},\mathbf{r}) = V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c) - U_i(R)$$

$$\text{or}\quad \mathcal{V}_f(\mathbf{R}',\mathbf{r}') = V_p(\mathbf{r}) + U_{c'c}(\mathbf{R}_c) - U_f(R').$$

# REMINDER: two potential formula: result

| | | | |
|---|---|---|---|
| Free: | $[E-T]\phi = 0$ | $\hat{G}_0^+ = [E-T]^{-1}$ | $\phi = F$ |
| Distorted: | $[E-T-U_1]\chi = 0$ | $\chi = \phi + \hat{G}_0^+ U_1 \chi$ | $\chi \to \phi + \mathbf{T}^{(1)} H^+$ |
| Full: | $[E-T-U_1-U_2]\psi = 0$ | $\psi = \phi + \hat{G}_0^+ (U_1+U_2)\psi$ | $\psi \to \phi + \mathbf{T}^{(1+2)} H^+$ |

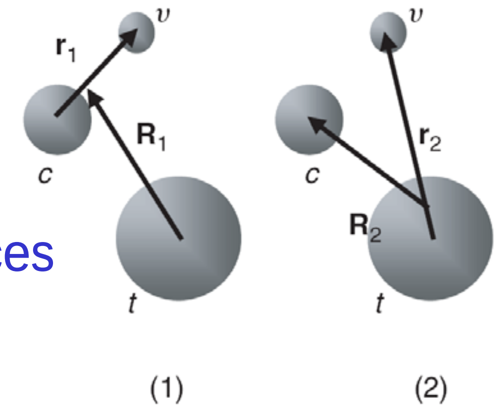
$$\mathbf{T}^{(1+2)} = \mathbf{T}^{(1)} + \mathbf{T}^{2(1)}$$

$$\mathbf{T}^{2(1)} = -\frac{2\mu}{\hbar^2 k} \int \chi U_2 \psi \, \mathrm{d}R$$

If $U_1$ cannot produce the desired transition, how much is $T^{(1)}$?

# Thinking about different mass partitions

$$\mathbf{T}^{2(1)} = -\frac{2\mu}{\hbar^2 k}\int \chi\, U_2 \psi \; \mathrm{d}R$$




Connecting to our problem we obtain the following T-matrices

POST: Hamiltonian written in final coordinates

PRIOR: Hamiltonian written in initial coordinates

$$T^{POST} = \langle \chi_f^{(-)}(\vec{R}_f)\, \phi_{vt}(\vec{r}_f) \,|\, V_f \,|\, \Psi(\vec{r}_i, \vec{R}_i) \rangle$$

$$T^{PRIOR} = \langle \Psi^{(-)}(\vec{r}_f, \vec{R}_f) \,|\, V_i \,|\, \phi_{cv}(\vec{r}_i)\, \chi_i(\vec{R}_i) \rangle$$

# REMINDER: distorted wave Born approximation (DWBA)

Born series is truncated after the first term

$$\mathbf{T}^{\text{DWBA}} = \mathbf{T}^{(1)} - \frac{2\mu}{\hbar^2 k} \langle \chi^{(-)} | U_2 | \chi \rangle$$

$U_2$ appears to first order

There is similarly a second-order DWBA expression

$$\mathbf{T}^{\text{2nd}-\text{DWBA}}_{\alpha\alpha_i} = -\frac{2\mu_\alpha}{\hbar^2 k_\alpha} \left[ \langle \chi^{(-)}_\alpha | U_2 | \chi_{\alpha_i} \rangle + \langle \chi^{(-)}_\alpha | U_2 \hat{G}^{+}_1 U_2 | \chi_{\alpha_i} \rangle \right].$$

$U_2$ appears to second order

# one nucleon transfer: amplitude

$$\mathcal{V}_i(\mathbf{R},\mathbf{r}) = V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c) - U_i(R)$$

or $$\mathcal{V}_f(\mathbf{R}',\mathbf{r}') = V_p(\mathbf{r}) + U_{c'c}(\mathbf{R}_c) - U_f(R').$$

binding potential

remnant term

$$\mathbf{T}_{fi}^{\mathrm{DWBA}} = \langle \chi_f^{(-)}(\mathbf{R}_f)\Phi_{I_A:I_B}(\mathbf{r}_f)|\mathcal{V}|\Phi_{I_b:I_a}(\mathbf{r}_i)\chi_i(\mathbf{R}_i)\rangle$$

# one nucleon transfer: auxiliary potential

$$\mathcal{V}_i(\mathbf{R},\mathbf{r}) = V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c) - U_i(R)$$

$$\text{or} \quad \mathcal{V}_f(\mathbf{R}',\mathbf{r}') = V_p(\mathbf{r}) + U_{c'c}(\mathbf{R}_c) - U_f(R').$$

binding potential

remnant term

$U_i$ is an auxiliary potential and therefore the solution is independent of that choice!

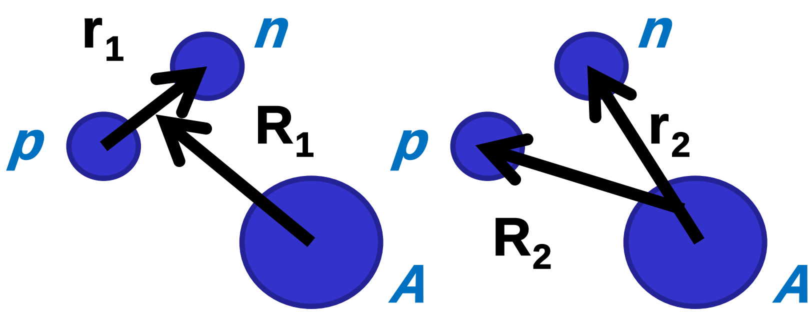
- ❑ standard choice in DWBA $U_i$ is optical potential reproducing elastic scattering
- ❑ standard choice in CDCC $U_i$ is $U_{ct}$ + $U_{xt}$ folded over the bound state c+x
- ❑ other possible choise $U_i$ = $U_{cc'}$ ($R_{cc'}$) to cancel the remnant term
- ❑ etc...

# Think specifically about A(d,p)B

PRIOR $\mathcal{V}_i(\vec{R}_1,\vec{r}_1) = V_{nA} + V_{pA} - U_{dA}$.

POST $\mathcal{V}_f(\vec{R}_2,\vec{r}_2) = V_{np} + V_{pA} - U_{pB}$

$$\mathcal{V}_f(\vec{R}_2,\vec{r}_2) \simeq V_{np}$$




POST $T^{exact}_{fi\equiv d,p} = \langle \chi_{pB}\, \Phi_B \,|\, \mathcal{V}_f \,|\, \Phi_A\, \psi^i_{3b} \rangle$

DWBA $\psi^i_{3b} \to \phi_d\, \chi_{dA}$

$$T^{DWBA}_{dp} = \langle \chi_{pB}\, \Phi_B \,|\, \mathcal{V}_f \,|\, \Phi_A\, \phi_d\, \chi_{dA} \rangle$$

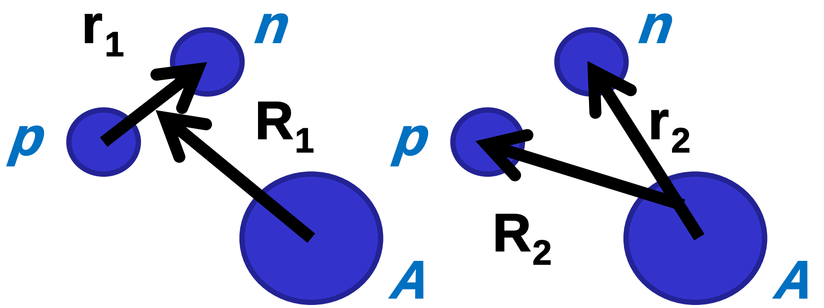
$$= \langle \chi_{pB}\, \Phi_{B:A} \,|\, \mathcal{V}_f \,|\, \phi_d\, \chi_{dA} \rangle$$

# Think specifically about A(d,p)B

PRIOR $\mathcal{V}_i(\vec{R}_1,\vec{r}_1) = V_{nA} + V_{pA} - U_{dA}$.

POST $\mathcal{V}_f(\vec{R}_2,\vec{r}_2) = V_{np} + V_{pA} - U_{pB}$

$$\mathcal{V}_f(\vec{R}_2,\vec{r}_2) \simeq V_{np}$$




PRIOR $T^{EXACT}_{fi\equiv d,p} = \langle \Psi_{3b}\,\Phi_A \,|\, \mathcal{V}_i \,|\, \Phi_d\,\Phi_A\,\chi_{dA} \rangle$

DWBA $\Psi_{3b}\Phi_A \rightarrow \chi_{pB}\,\Phi_B$

$$T^{DWBA}_{fi\equiv dp} = \langle \chi_{pB}\,\Phi_B \,|\, \mathcal{V}_i \,|\, \Phi_A\,\phi_d\,\chi_{dA} \rangle$$

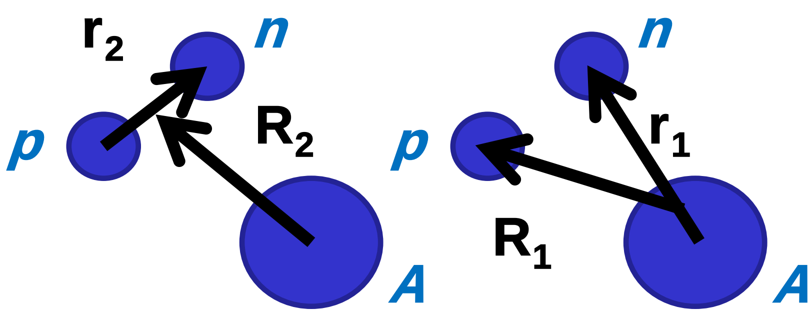
$$= \langle \chi_{pB}\,\Phi_{B:A} \,|\, \mathcal{V}_i \,|\, \phi_d\,\chi_{dA} \rangle$$

# Now think specifically about B(p,d)A

PRIOR $\quad \mathcal{V}_i(\vec{R}_1, \vec{r}_1) = V_{np} + V_{pA} - U_{pB}$

POST $\quad \mathcal{V}_f(\vec{R}_2, \vec{r}_2) = V_{nA} + V_{pA} - U_{dA}$

makes sense to use PRIOR $\quad \mathcal{V}_i \simeq V_{np}$




PRIOR

$$T^{exact}_{pd} = \langle \psi^{f}_{3b}\, \Phi_A \,|\, \mathcal{V}_i \,|\, \Phi_B\, \chi_{pB} \rangle$$

$$T^{DWBA}_{pd} = \langle \chi_{dA}\, \phi_d \,|\, \mathcal{V}_i \,|\, \Phi_{A:B}\, \chi_{pB} \rangle$$

# questions