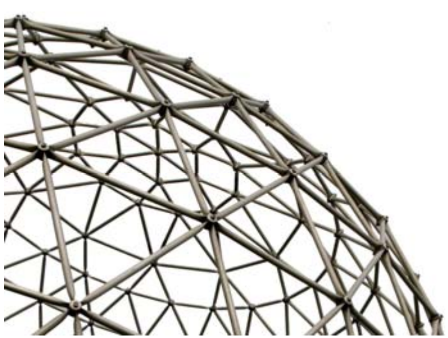
Welcome to GANIL - and the TALENT Course #6

TALENT Course 6: Theory for exploring nuclear reaction experiments

Reactions ingredients - Bound, continuum, resonance and bin states

GANIL, 1st-19th July 2013.

Jeff Tostevin, Department of Physics
Faculty of Engineering and Physical Sciences
University of Surrey, UK



100 years of nuclei – scattering was critical

[669]

LXXIX. *The Scattering of α and β Particles by Matter and the Structure of the Atom.* By Professor E. RUTHERFORD, *F.R.S., University of Manchester* *.

§ 1. IT is well known that the α and β particles suffer deflexions from their rectilinear paths by encounters with atoms of matter. This scattering is far more marked for the β than for the α particle on account of the much smaller momentum and energy of the form
There seems to be no doubt that such swiftly
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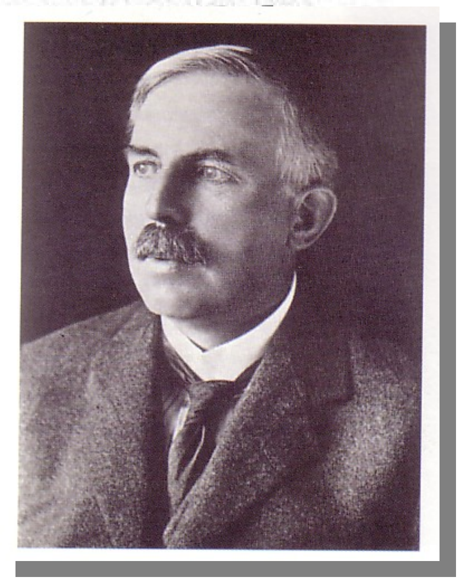
deflexions observed are due to the strong e
traversed within the atomic system. It has ge



Philosophical Magazine, volume **21** (1911), pages 669-688

Radioactive ion-beams: facilities – and the future

FAIR Facility for Antiproton and Ion Research

GSI

RIKEN RI BEAM FACTORY

RIKEN

RIKEN NiSHiNA CENTER

AVF

RRC

---A Dream Factory for Particle Beams---

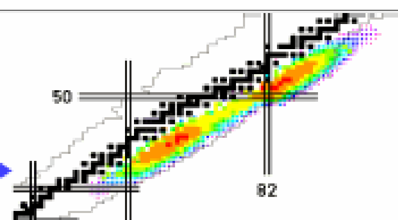
TRIUMF ARIEL

ADVANCED RARE ISOTOPE LABORATORY

ISOLDE

HIE ISOLDE

Spiral2



A&M

NSCL

FRIB

There are several good reaction theory texts:

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) Norman Austern

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages) G R Satchler

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) L S Rodberg, R M Thaler

Direct Nuclear Reactions (World Scientific Publishing, 396 pages) Norman K. Glendenning

Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages) C A Bertulani, P Danielewicz

Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages) Herman Feshbach

Introduction to Nuclear Reactions (Oxford University Press, 332 pages) G R Satchler

Nuclear Reactions for Astrophysics (Cambridge University Press, 2010) Ian Thompson and Filomena Nunes

Lectures, exercises, project information at:

This course TALENT Course #6

http://www.nucleartheory.net/NPG/Talent_material/index.htm/

Please let me know fast if there are problems.

Exotic Beams Summer School 2011 at NSCL:
Ian Thompson Introductory Lecture

http://www.nscl.msu.edu/~zegers/ebss2011/thompson.pdf

Homework 1: read intro to Thompson lectures 2011 …

… for a discussion of the characteristics of direct (fast) and compound (massive energy sharing) nuclear reactions.

Direct reactions: Reactions in which nuclei make glancing contact and then separate immediately. Projectile may exchange some energy and / or angular momentum, or have one or more nucleons transferred to it or removed from it.

Direct reactions: take place at/near the nuclear surface and at larger impact parameters

Direct reaction products tend to be strongly forward peaked as projectile continues to move in general forward direction

Direct reactions take place on a short timescale (we will quantify) – a timescale that reduces with increasing energy of the projectile beam (and that allows extra approximations)

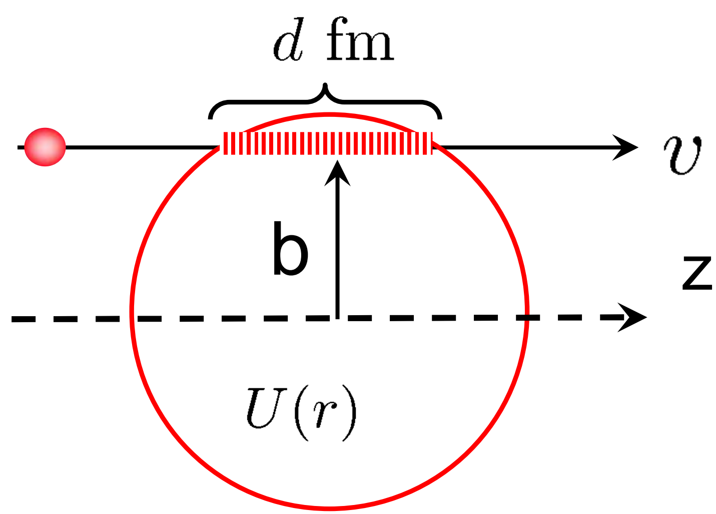
Direct reaction clock has ticks in units of $\sim 10^{-22}$ s – timescale for a nucleon's motion across in a typical nucleus

Reaction timescales – surface grazing collisions

For say 10 and 100 MeV/u incident energy:

$$\gamma = 1.01, \quad v/c = 0.14, \qquad \gamma = 1.1, \quad v/c = 0.42,$$

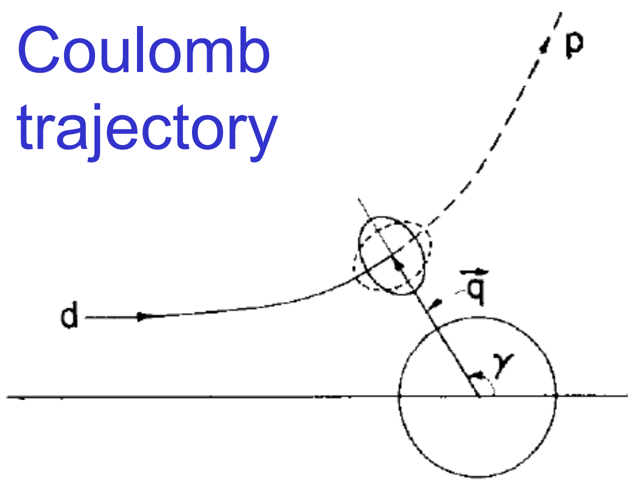
$$\Delta t = 2.4 \times d \times 10^{-23} s, \qquad \Delta t = 7.9 \times d \times 10^{-24} s$$



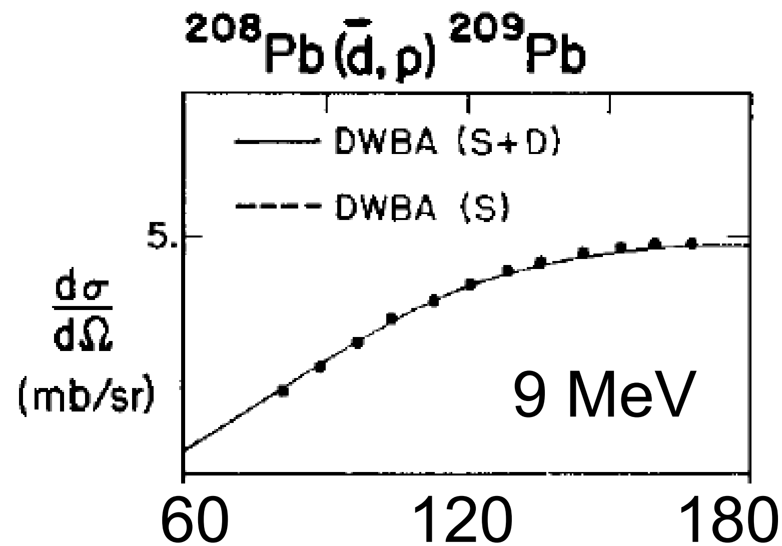
d<nuclear diameter (a few fm) for strong interactions: but if Coulomb effects are important or weakly bound systems with extended wave functions – extended interaction times and probable or likely <u>higher-order effects</u>

Direct reactions at low energies need extra care

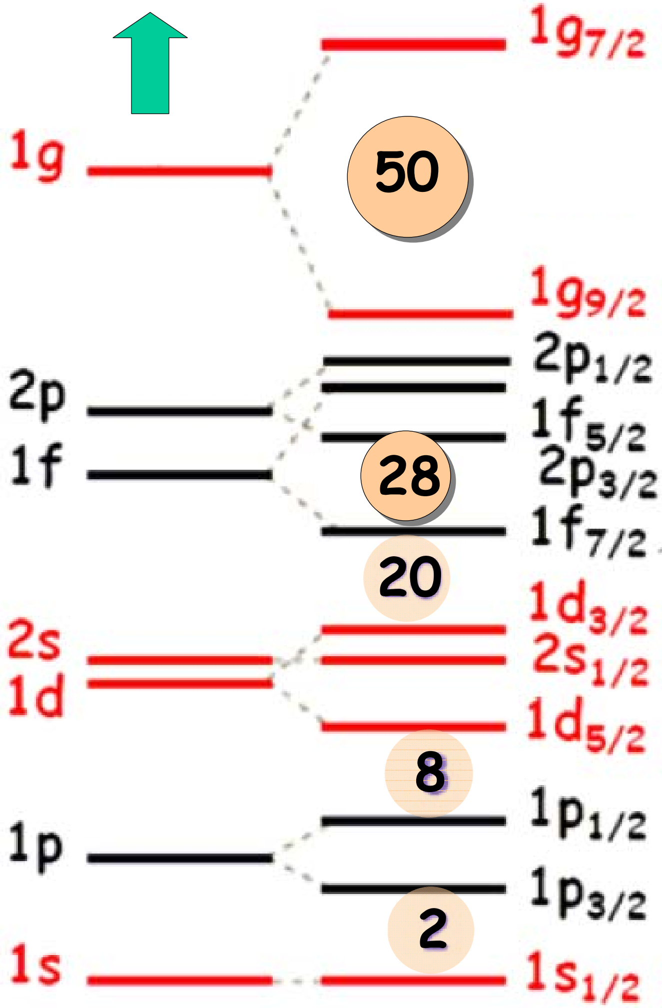
1. Energies near the Coulomb barriers of the reacting systems [$E_{cm} \sim Z_1 * Z_2 * e^2/(R_1+R_2)$]
2. So, incident energies of 1 to a few MeV per nucleon
3. Are such energies high enough and timescales short enough for reactions to be <u>fast</u> and <u>direct</u>?
4. Reaction products may not be forward peaked, e.g. (d,p) transfers near/below the Coulomb barrier

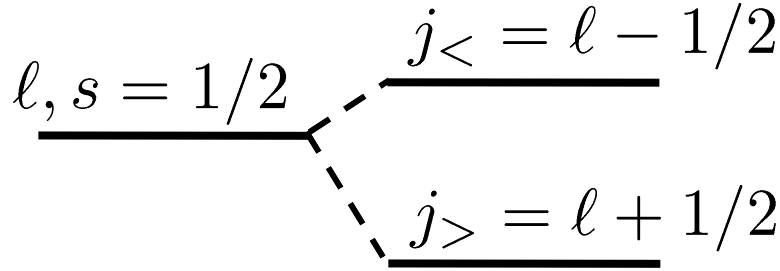


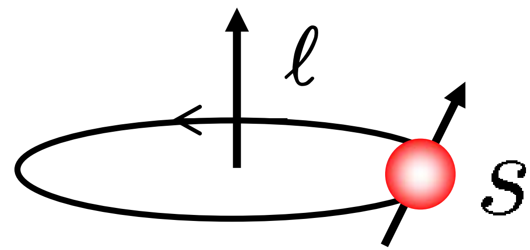
L.D. Knutson, Ann. Phys. **106**, 1 (1977)



Single-particle aspects of structure for reactions







$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V_{so}(r) < 0$$

First session aims:

To discuss: 1. solutions of the Schrodinger equation for states of **two** bodies with specific quantum numbers over a wide range of energies – the need for bound, resonant, continuum (and continuum bin) states.

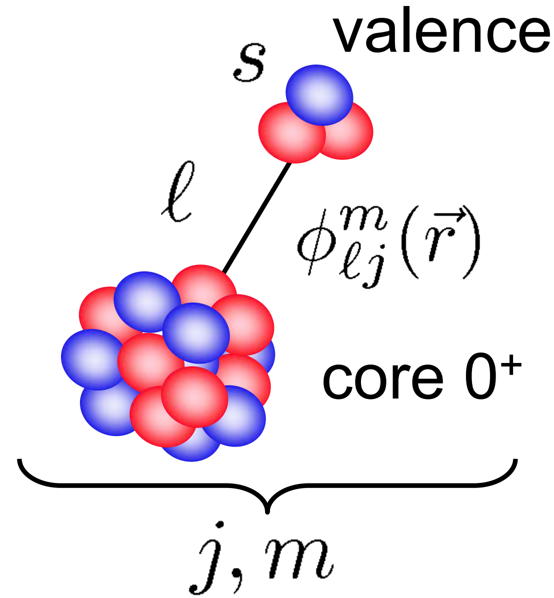
2. The form of these two-body problem solutions at large separations and their relationships to nuclear structure, absorption, reaction and scattering observables.

3. Stress constraints on two-body interactions and their parameters. Parameter conventions. The need to cross reference to known nuclear structures, resonances, nuclear sizes and experiment whenever possible in constraining parameter choices for realistic calculations.

Underpinnings of direct reaction methods

Solutions of Schrodinger's equation for (pairs of) nuclei interacting via a potential energy function of the form*

$$U(r) = \underbrace{V_C(r)}_{\text{Coulomb}} + \underbrace{V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}}_{\text{Nuclear}}$$



Need descriptions of wave functions of:

(1) Bound states of nucleons or clusters (valence particles) to a core (that is assumed for now to have spin zero).

(2) Unbound scattering or resonant states at low energy

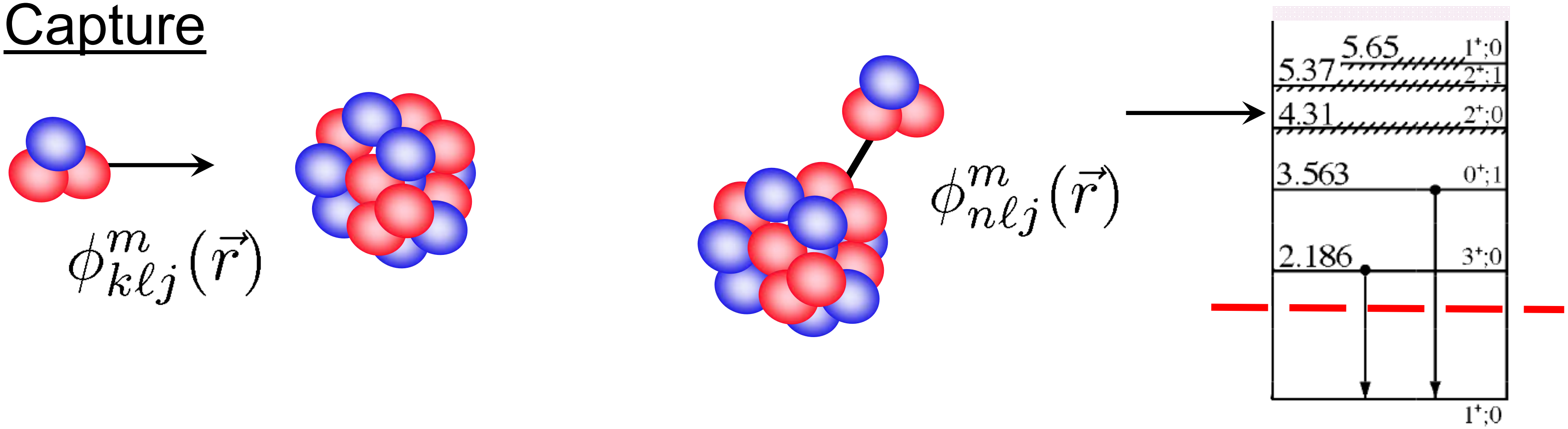
(3) Distorted waves for such bodies in complex potentials

$$U(r) = V_C(r) + V(r) + \boxed{iW(r)} + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

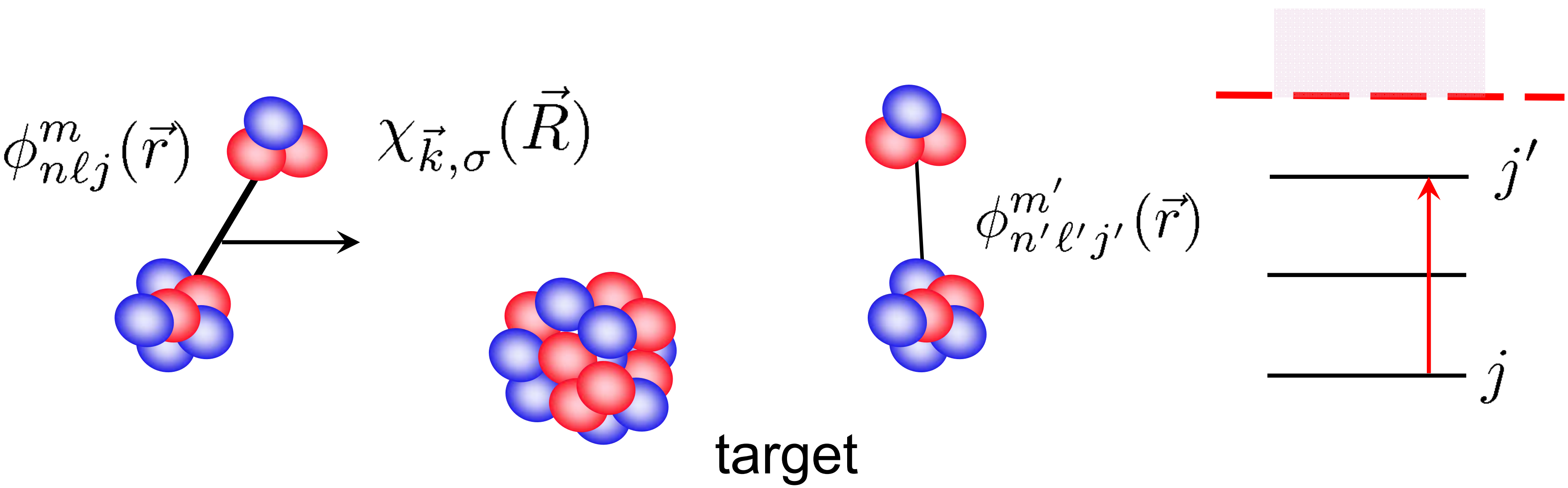
*Additional, e.g. tensor terms, when s=1 or greater neglected

Direct reactions – types and characteristics

Capture

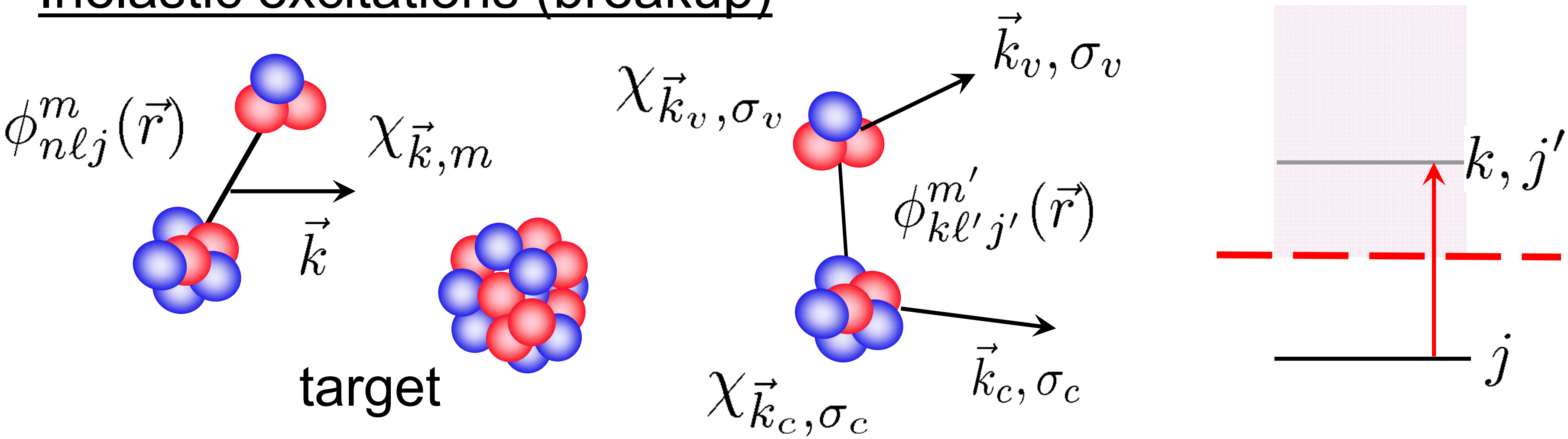


Inelastic excitations (bound to bound states) e.g. DWBA

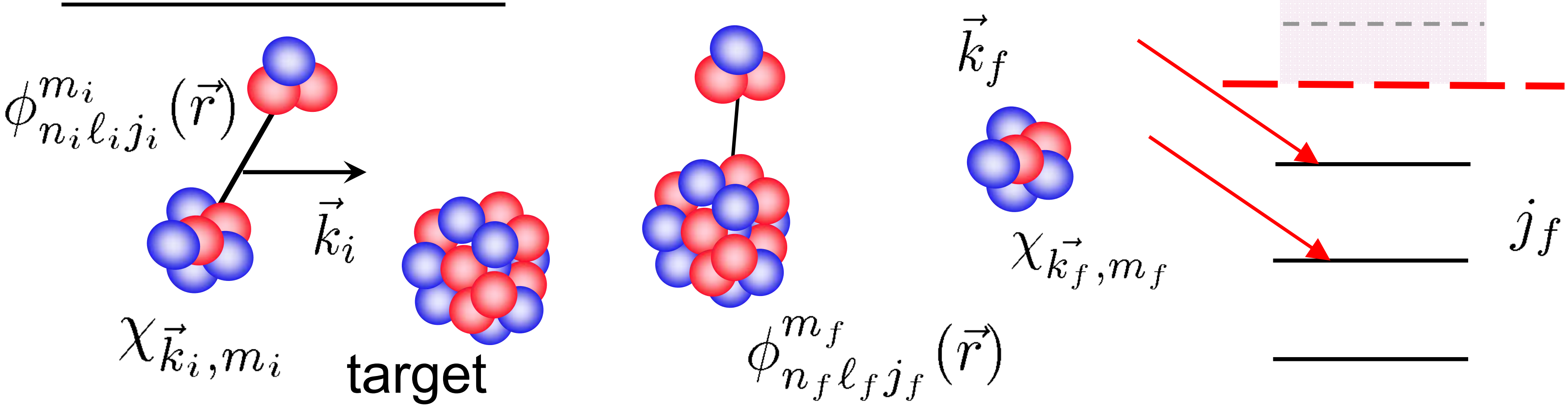


Direct reactions – types and characteristics

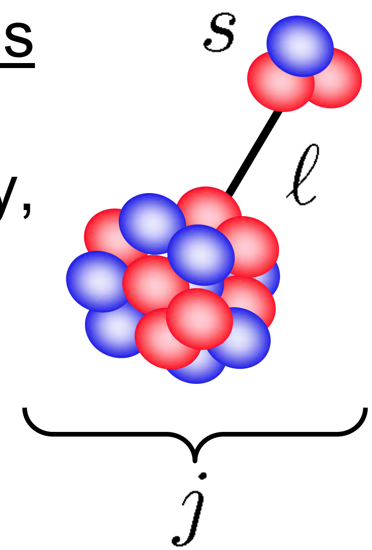
Inelastic excitations (breakup)



Transfer reactions



Direct reactions – requirements (1)



Description of wave functions of **bound** systems (both nucleons or clusters) – (a) can take from structure theory, if available or, (b) more usually, use a real potential model to bind system with the required experimental separation energy.

Refer to core and valence particles

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\boxed{\frac{u_{n\ell j}(r)}{r}}Y^\lambda_\ell(\hat{r})\chi^\sigma_s, \quad \int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$

Usually just one or a few such states are needed.

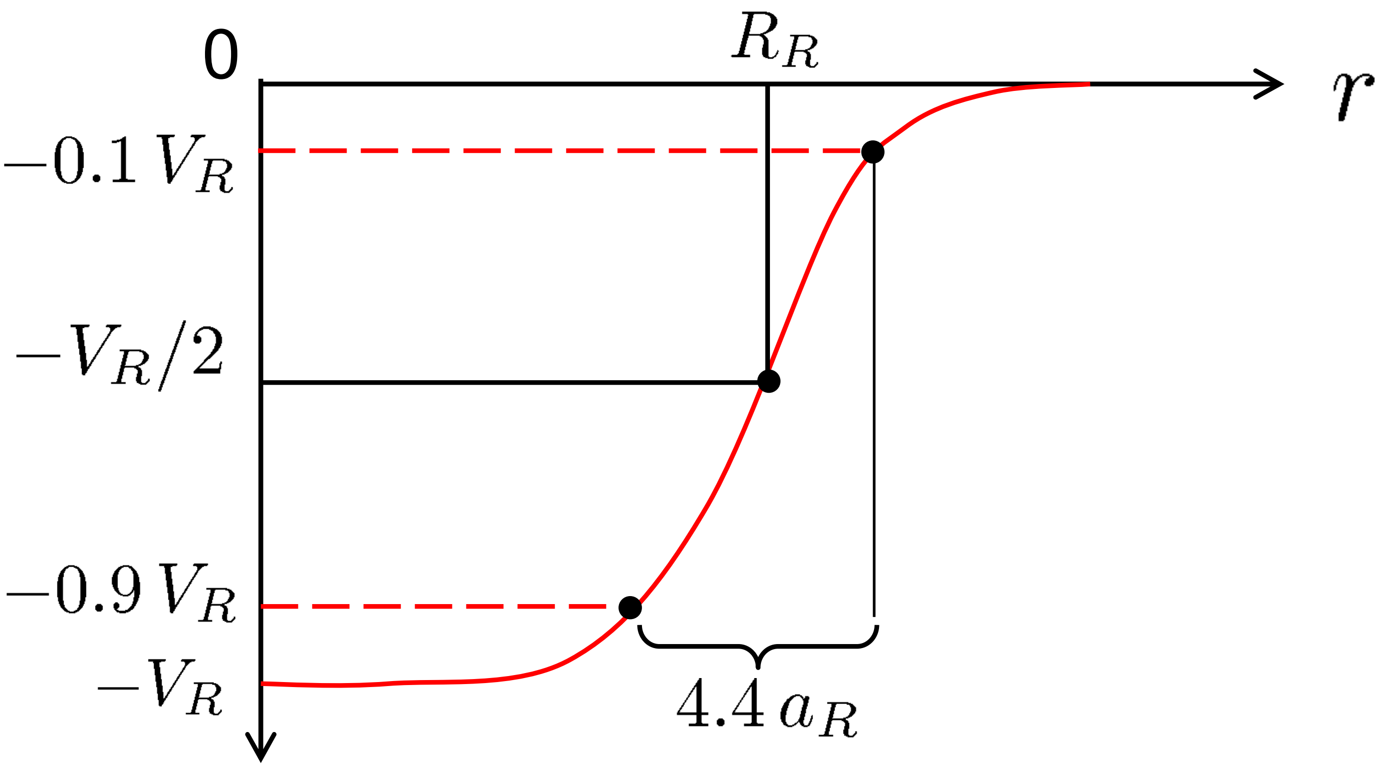
Separation energies/Q-values: many sites, e.g.

http://ie.lbl.gov/toi2003/MassSearch.asp

Bound states – real potentials

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\,, \qquad X_R = \frac{r-R_R}{a_R}$$



Bound states potential parameters - nucleons

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\,, \qquad X_i = \frac{r-R_i}{a_i}$$

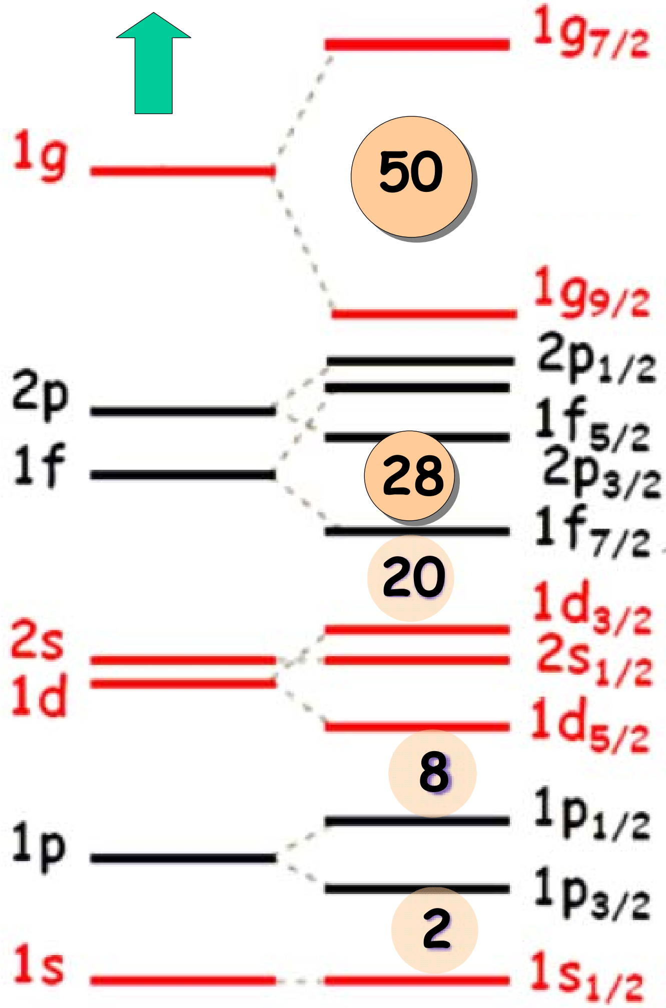
$$V_{so}(r) = -\frac{4\,V_{so}}{r\,a_{so}}\,\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\,,$$

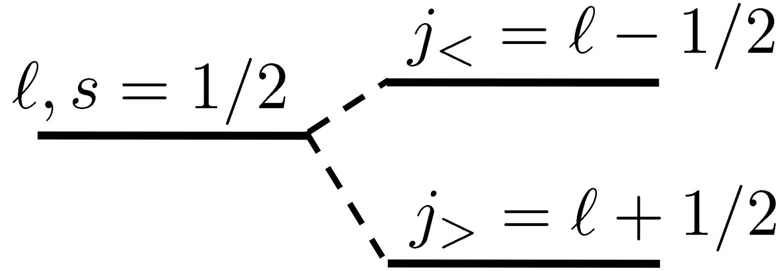
$$R_i = r_i\,A_c^{1/3}$$

$$r_R = r_C = r_{so} \approx 1.25\text{fm}$$

$$a_R = a_{so} \approx 0.7\text{fm} \quad V_{so} = 6\text{MeV}$$

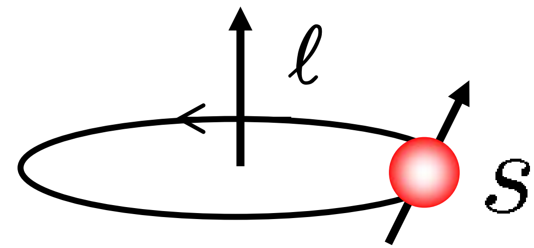
Bound states – single particle quantum numbers





^{24}O

N=16

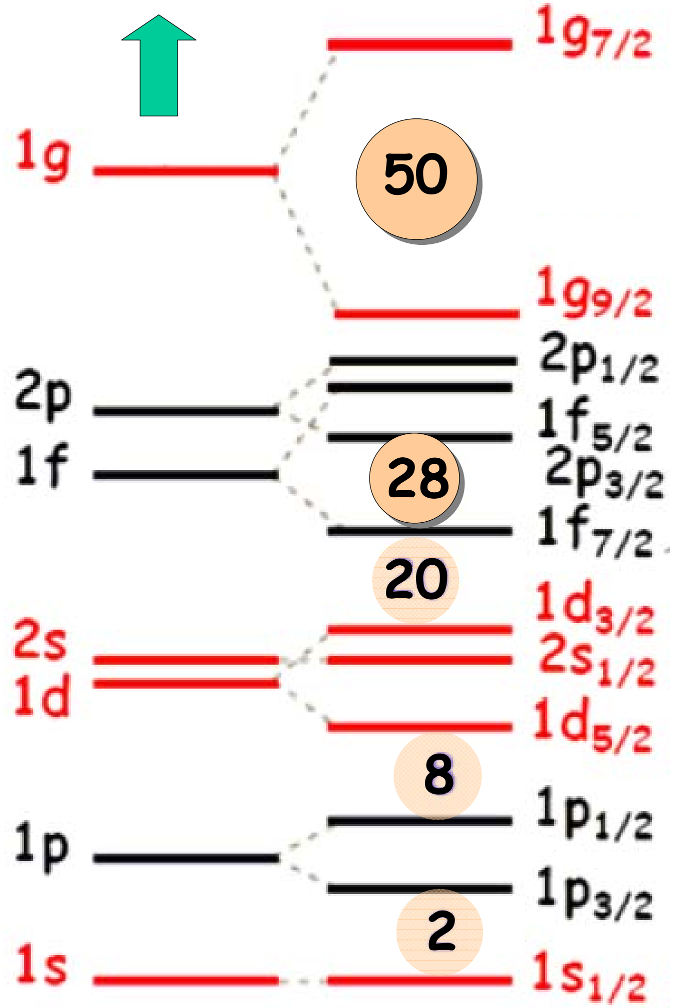


$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V_{so}(r) < 0$$

Bound states – for nucleons - conventions



Conventions $\phi^{m}_{n\ell j}(\vec{r})$

With this potential, and using sensible parameters, we will obtain the independent-particle shell model level orderings, shell closures with spin-orbit splitting.

NB: In diagram $2d_{5/2}$ means the second $d_{5/2}$ state. Defined this way, $n>0$ and $n-1$ is the number of nodes in the radial wave function. Reaction codes can ask for n, or n-1 (the actual number of nodes). Care is needed.

Bound states – can also use mean field information

```
                         ***********************
                         *                     *
INPUT VALUES             * IA,IZ =    24     8 *
                         *                     *
                         ***********************
```

----- Neutron bound state results -----

k	n	l	j	e	IE	OCC		
1	1	s	1/2	-26.757	1	2.00	36.70	35.28
2	1	p	3/2	-16.883	1	4.00	36.70	35.80
3	1	p	1/2	-12.396	1	2.00	36.70	36.04
4	1	d	5/2	-6.166	1	6.00	36.70	36.37
5	1	d	3/2	-0.109	1	0.00	36.70	36.69
6	2	s	1/2	-3.360	1	2.00	36.70	36.52
7	1	f	7/2	-0.200	3	0.00	46.02	46.01
8	1	f	5/2	-0.200	3	0.00	60.56	60.55
9	2	p	3/2	-0.200	3	0.00	48.10	48.09

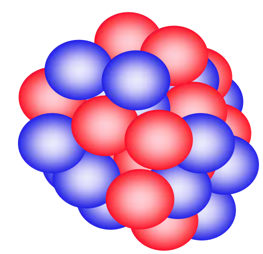
----- Neutron single-particle radii -----

				R(2)	R(4)	OCC	rho(8.9)	rho(9.9)	rho(10.9)
1	1	s	1/2	2.274	2.575	2.000	0.848E-09	0.706E-10	0.600E-11
2	1	p	3/2	2.863	3.133	4.000	0.188E-07	0.244E-08	0.325E-09
3	1	p	1/2	2.954	3.268	2.000	0.727E-07	0.122E-07	0.210E-08
4	1	d	5/2	3.434	3.757	6.000	0.524E-06	0.129E-06	0.327E-07
5	1	d	3/2	4.662	6.063	0.000	0.131E-04	0.675E-05	0.371E-05
6	2	s	1/2	4.172	4.895	2.000	0.769E-05	0.278E-05	0.102E-05
7	1	f	7/2	3.865	4.440	0.000	0.324E-05	0.134E-05	0.600E-06
8	1	f	5/2	3.890	4.477	0.000	0.341E-05	0.141E-05	0.631E-06
9	2	p	3/2	6.815	8.635	0.000	0.451E-04	0.270E-04	0.167E-04

But must make <u>small</u> corrections as HF is a fixed centre calculation

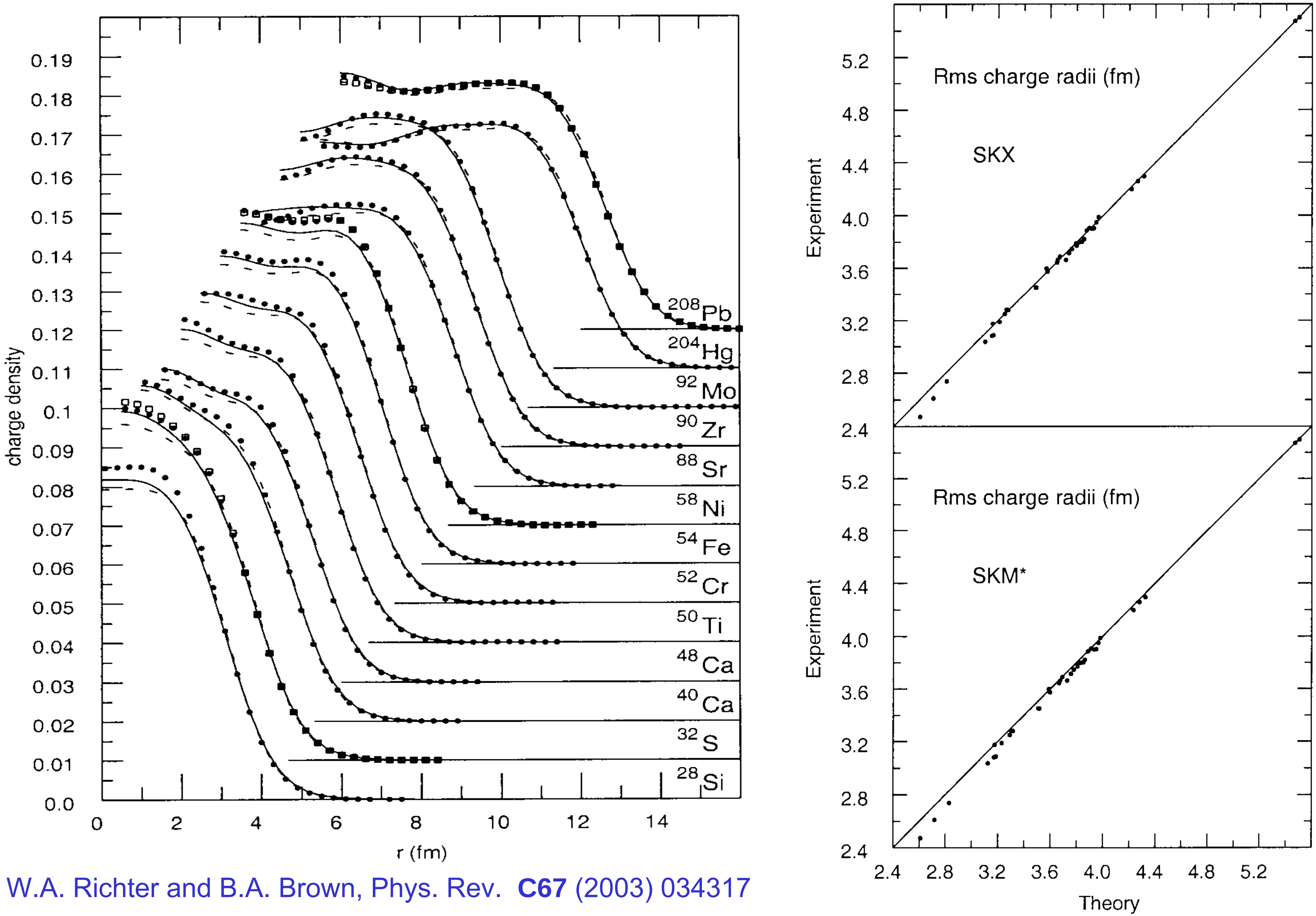
^{24}O

$$\langle r^2 \rangle = \frac{A}{A-1} \langle r^2 \rangle_{HF}$$



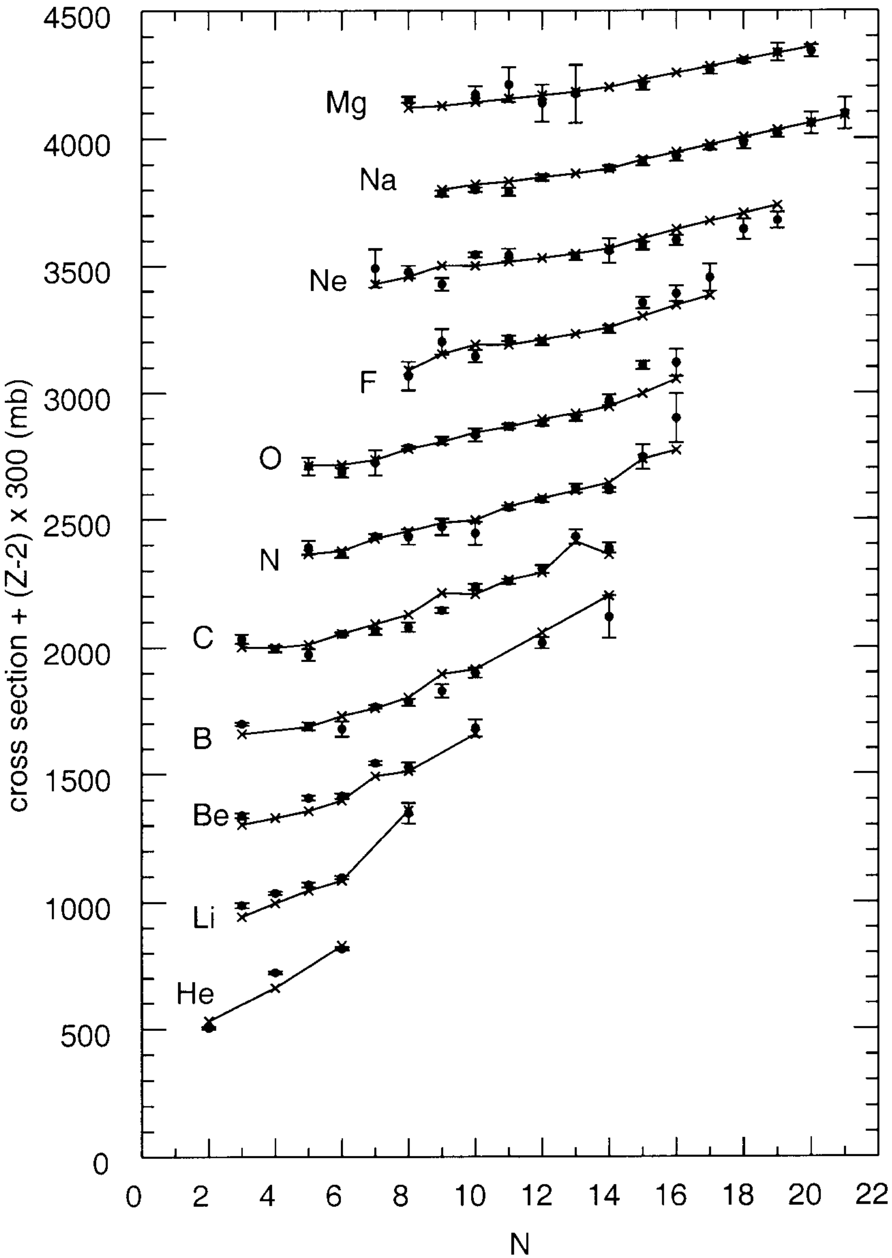
$^{24}O(g.s.)$

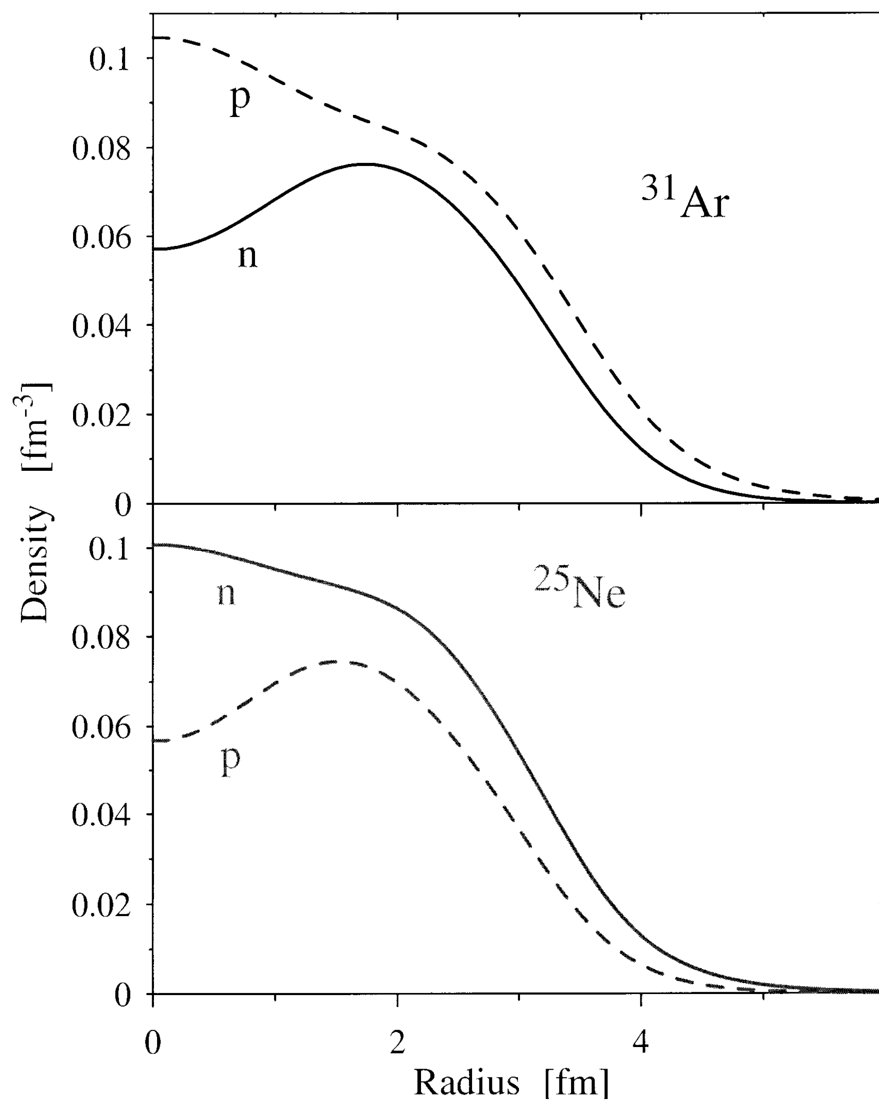
Sizes - Skyrme Hartree-Fock radii and densities



W.A. Richter and B.A. Brown, Phys. Rev. **C67** (2003) 034317

Sizes - Skyrme Hartree-Fock radii and densities





B.A. Brown, S. Typel, and W.A. Richter, Phys. Rev. **C65** (2002) 014612

The 'menu' for **dens** …

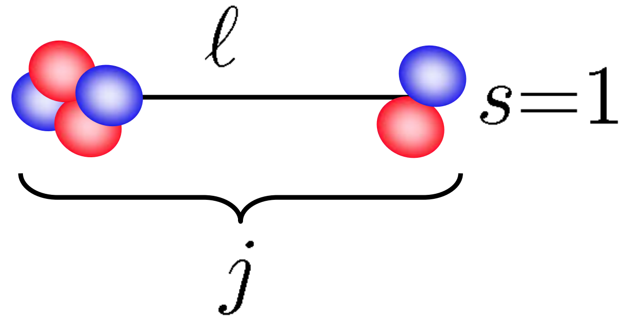
```
dens
az          (signal mass and charge input, a,z)
24,8        (the mass and charge, choice is yours)
cp          (signal choice of potential)
sk20        (is Skyrme SkX – stable and exotics)
gd          (go work out the density)
wr          (signal  writing the density out)
1,,,,O24    (density file is O24.RHO)
st          (stop – we are done – enough!)
```

Other useful stuff is written to **dens.dao**

Direct reactions – requirements (2)

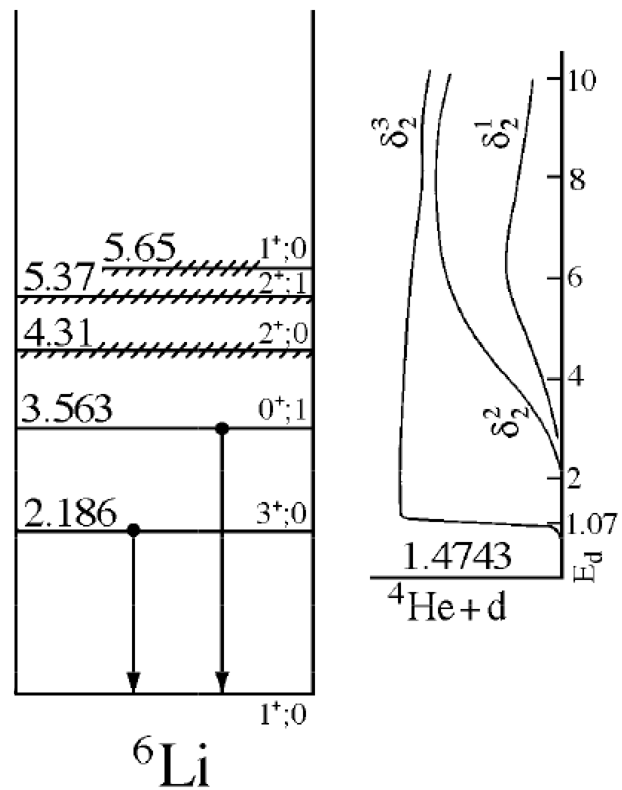
Description of wave functions for unbound (often light) systems (nucleons or clusters) with low relative energy: Usually have low nuclear level density of isolated resonances. Use the same real potential model as binds the system → scattering wave functions in this potential. (Also 'bin' wave functions)

^{6}Li (α+d)



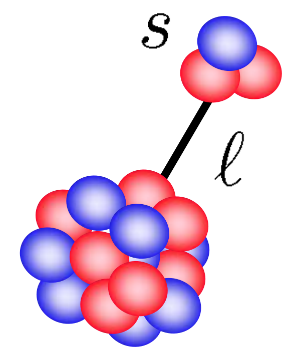
$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$\phi^m_{k\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{k\ell j}(r)}{kr}Y^\lambda_\ell(\hat{r})\chi^\sigma_s$$



Completeness and orthogonality - technical piont

Given a fixed two-body Hamiltonian



$$H = T + U(r) = T + V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

the set of all of the bound and unbound wave functions

$$\{\phi^m_{n\ell j}(\vec{r}),\ \phi^m_{k\ell j}(\vec{r})\}$$

form a complete and orthogonal set, and specifically

$$\boxed{\langle\phi^m_{n\ell j}(\vec{r})|\,\phi^m_{k\ell j}(\vec{r})\rangle = 0}$$

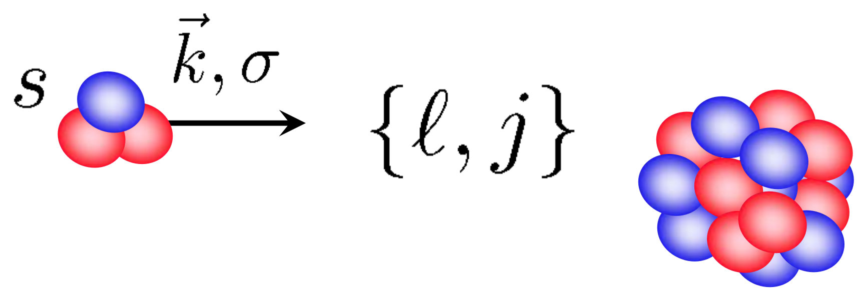
When including both bound to unbound states it is essential to use a fixed Hamiltonian for both the bound and unbound states (in each ℓj channel) else we lose the orthogonality and the states will couple even without any perturbation or interactions with a reaction target.

Direct reactions – requirements (3)

Description of wave functions for scattering of nucleons or clusters from a heavier target and/or at higher energies: (a) high nuclear level density and broad overlapping resonances, (b) many open reaction channels, inelasticity and absorption. Use a complex (absorptive) optical model potential – from theory or ‘simply’ fitted to a body of elastic scattering data for a system and energy near that of interest.

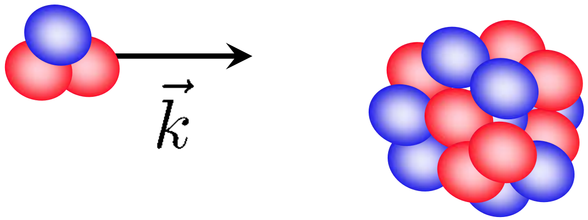
Distorted waves:

$$\chi_{\vec{k},\sigma}(\vec{r})$$



$$U(r) = V_C(r) + V(R) + iW(r) + V_{so}(r)\vec{\ell} \cdot \vec{s}$$

Optical potential – formal– Feshbach P's and Q's



Elastic channel $|\Psi_0\rangle$ describes motion when both projectile and target in their ground states

$$|\Psi\rangle = |\Psi_0\rangle + |\Psi_1\rangle + |\Psi_2\rangle \ldots = |\Psi_0\rangle + |\Psi_{in}\rangle$$

$$H|\Psi\rangle = E|\Psi\rangle \qquad P|\Psi\rangle = |\Psi_0\rangle \qquad Q|\Psi\rangle = |\Psi_{in}\rangle$$

Orthogonality of states of H: $PQ = QP = 0, \; P + Q = 1$

P and Q are projection operators: $PP = P, \; QQ = Q$

$$H(P+Q)|\Psi\rangle = E(P+Q)|\Psi\rangle$$

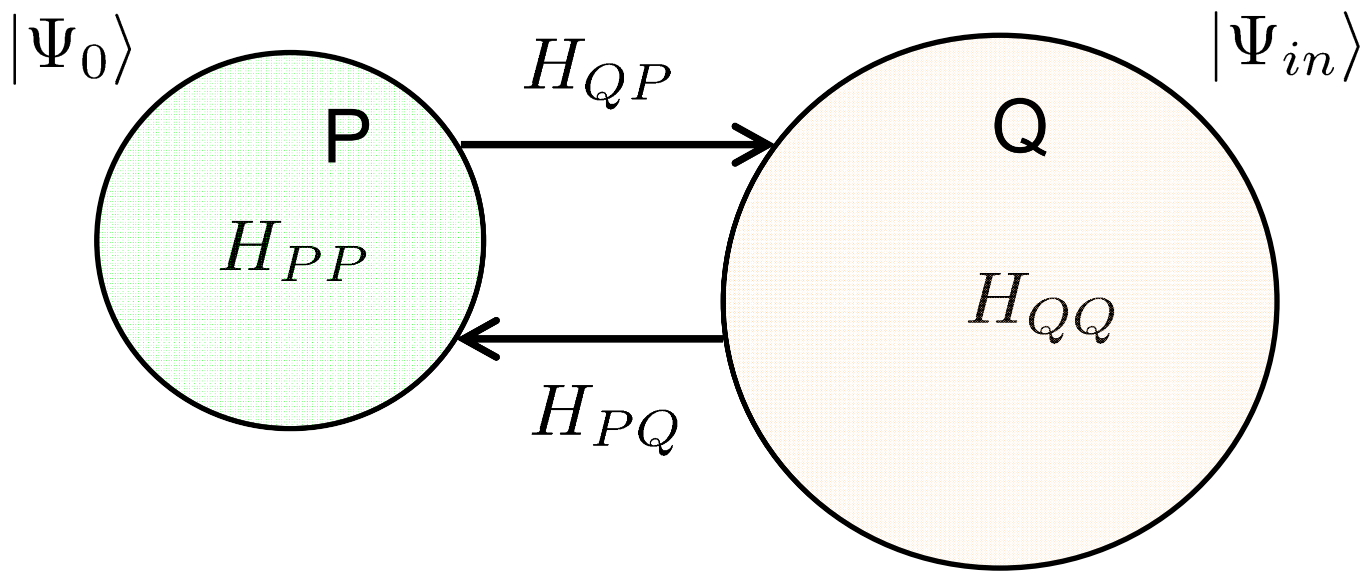
$$PH(P+Q)|\Psi\rangle = PE(P+Q)|\Psi\rangle = EP|\Psi\rangle = E|\Psi_0\rangle$$

$$QH(P+Q)|\Psi\rangle = QE(P+Q)|\Psi\rangle = EQ|\Psi\rangle = E|\Psi_{in}\rangle$$

Optical potential – formal – Feshbach P's and Q's

$$[E - PHP]|\Psi_0\rangle = PHQ|\Psi_{in}\rangle$$

$$[E - QHQ]|\Psi_{in}\rangle = QHP|\Psi_0\rangle$$

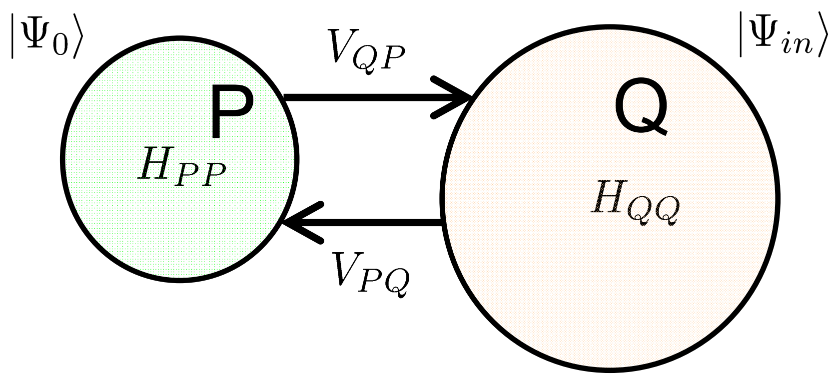


$$H_{PP} = PHP = T + PVP = T + V_{PP}, \text{ etc.}$$

Optical potential – formal – Feshbach P's and Q's

$$[E - T - V_{PP}]|\Psi_0\rangle = V_{PQ}|\Psi_{in}\rangle$$

$$[E^{(+)} - T - V_{QQ}]|\Psi_{in}\rangle = V_{QP}|\Psi_0\rangle$$

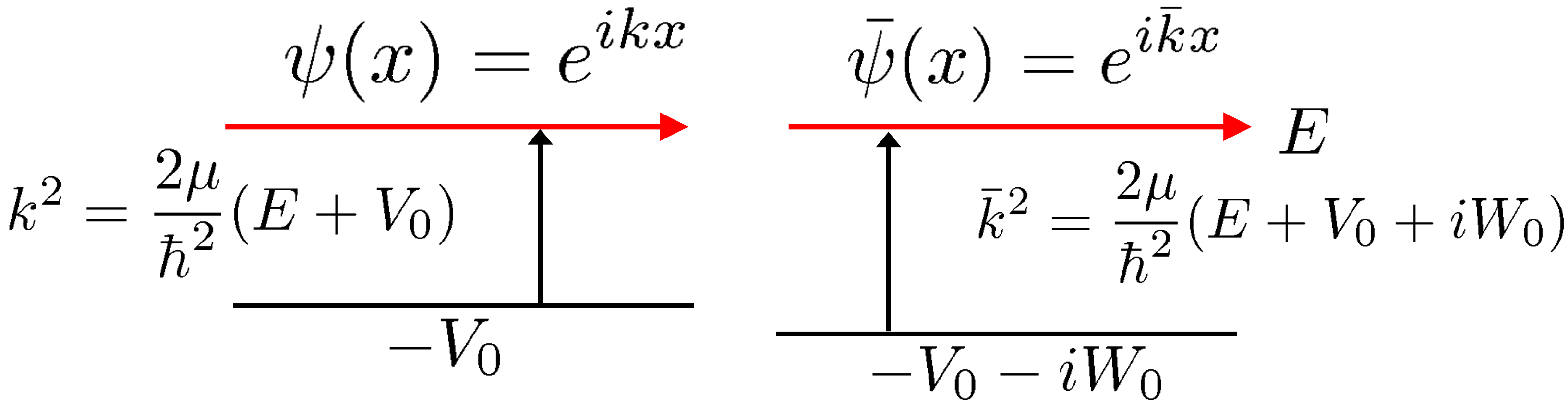


$$|\Psi_{in}\rangle = [E^{(+)} - T - V_{QQ}]^{-1} V_{QP}|\Psi_0\rangle$$

$$[E - T - V_{PP}^{opt}]|\Psi_0\rangle = 0$$

$$V_{PP}^{opt} = V_{PP} + V_{PQ}[E^{(+)} - T - V_{QQ}]^{-1} V_{QP}$$

Optical potentials – the role of the imaginary part



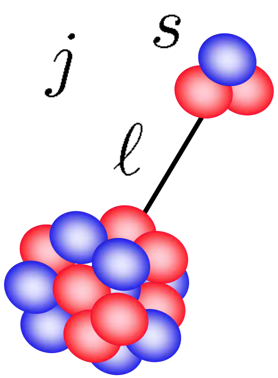
$$\bar{k}^2 = \frac{2\mu}{\hbar^2}(E + V_0 + iW_0) = \frac{2\mu}{\hbar^2}(E + V_0)\left[1 + \frac{iW_0}{E + V_0}\right]$$

$$\bar{k} = k\left[1 + \frac{iW_0}{E + V_0}\right]^{1/2} \approx k\left[1 + \frac{iW_0}{2(E + V_0)}\right], \quad W_0 \ll E, V_0$$

So, for $W_0 > 0$, $\bar{k} = k + ik_i/2$, $k_i = kW_0/(E + V_0) > 0$,

$$\bar{\psi}(x) = e^{i\bar{k}x} = e^{ikx}e^{-\frac{1}{2}k_i x}, \qquad |\bar{\psi}(x)|^2 = e^{-k_i x}$$

The Schrodinger equation (1)



So, using usual notation

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\phi_{\ell j}^m(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

and defining $\phi_{\ell j}^m(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{\ell j}(r)}{r}Y_\ell^\lambda(\hat{r})\chi_s^\sigma$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right)u_{\ell j}(r) = 0$$

bound states $E_{cm} < 0$ scattering states $E_{cm} > 0$

With $U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$

$$\begin{aligned} U_{\ell j}(r) &= V_C(r) + V(r) + iW(r) \\ &+ V_{so}(r)[j(j+1) - \ell(\ell+1) - s(s+1)]/2 \end{aligned}$$

Optical potentials - parameter conventions

$$U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r - R_i}{a_i}$$

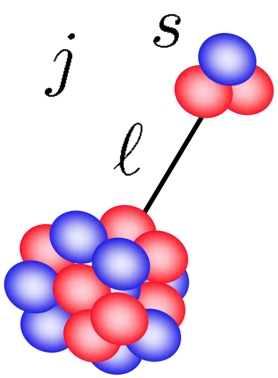
$$V_{so}(r) = -\frac{4V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

usual conventions

$$W(r) = -\frac{W_V}{[1+\exp(X_V)]} - \frac{4W_S\exp(X_S)}{[1+\exp(X_S)]^2}\ ,$$

$$R_i = r_i\,A_2^{1/3} \quad \text{or} \quad R_i = r_i\left[A_1^{1/3} + A_2^{1/3}\right]$$

The Schrodinger equation (2)



Must solve

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right) u_{\ell j}(r) = 0$$

<u>bound states</u> $E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

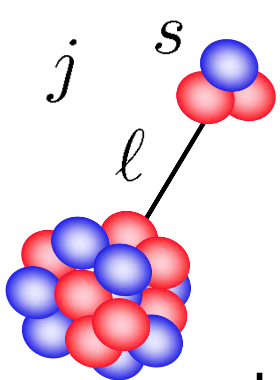
Discrete spectrum

<u>scattering states</u> $E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

Continuous spectrum

Large r: The Asymptotic Normalisation Coefficient



Bound states

$$E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

but beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta_b\kappa_b}{r} - \kappa_b^2\right) u_{n\ell j}(r) = 0, \qquad \eta_b = \frac{\mu Z_c Z_v e^2}{\hbar\kappa_b}$$

$$u_{n\ell j}(r) \to C_{\ell j} W_{-\eta_b,\ell+1/2}(2\kappa_b r) \underset{r\to\infty}{\longrightarrow} C_{\ell j}\exp(-\kappa_b r)$$

Whittaker function

$C_{\ell j}$ ANC completely determines the wave function outside of the range of the nuclear potential – only requirement if a reaction probes only these radii

Large r: The phase shift and partial wave S-matrix

Scattering states

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr),\ G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\begin{aligned} u_{k\ell j}(r) &\rightarrow e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right] \\ &\rightarrow (i/2)\left[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)\right] \end{aligned}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm i F_\ell(\eta, kr)$$

Phase shift and partial wave S-matrix

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}} \underbrace{[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)]}$$

If *U(r)* is real, the phase shifts $\delta_{\ell j}$ are real, and [...] also

$$u_{k\ell j}(r) \quad \rightarrow \quad (i/2)[\underline{H_\ell^{(-)}(\eta, kr)} - S_{\ell j} \underline{H_\ell^{(+)}(\eta, kr)}]$$

$$S_{\ell j} = e^{2i\delta_{\ell j}}$$

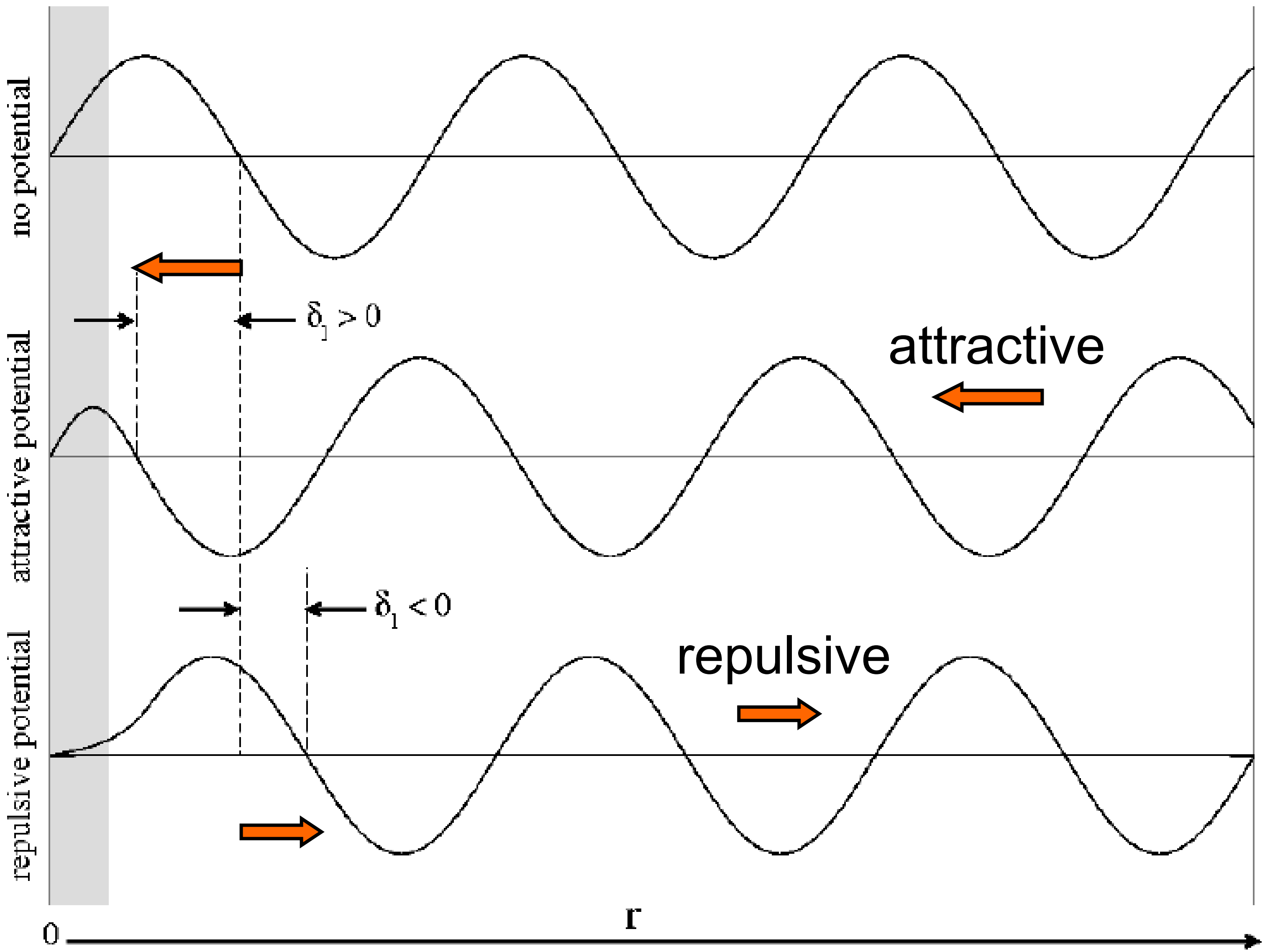
Ingoing waves — outgoing waves

$|S_{\ell j}|^2$ survival probability in the scattering

$(1 - |S_{\ell j}|^2)$ absorption probability in the scattering

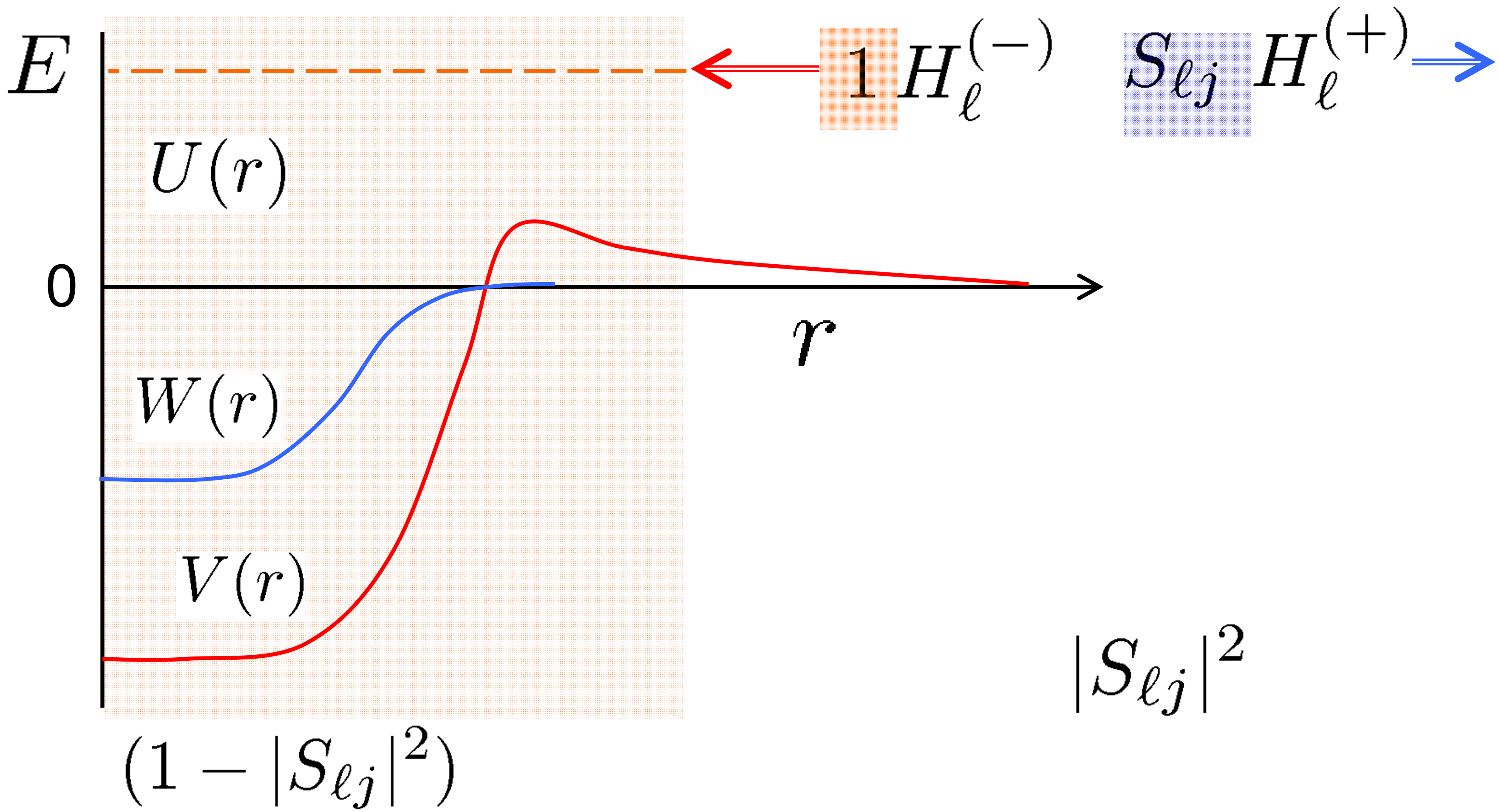
Having calculate the phase shifts and the partial wave S-matrix elements we can then compute all scattering observables for this energy and potential (but later).

Phase shifts – a preview (or reminder)



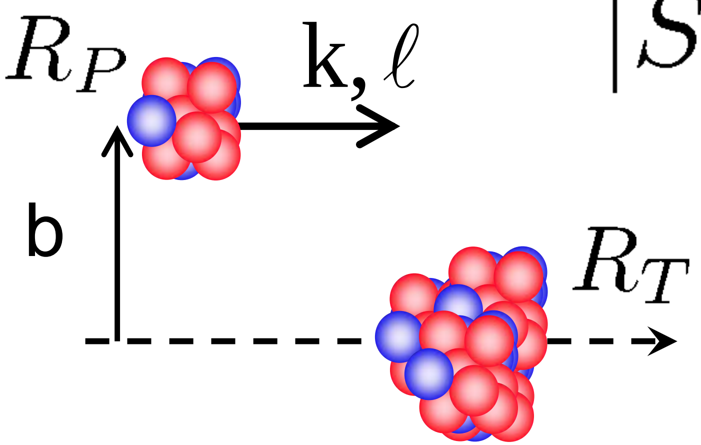
Ingoing and outgoing waves amplitudes

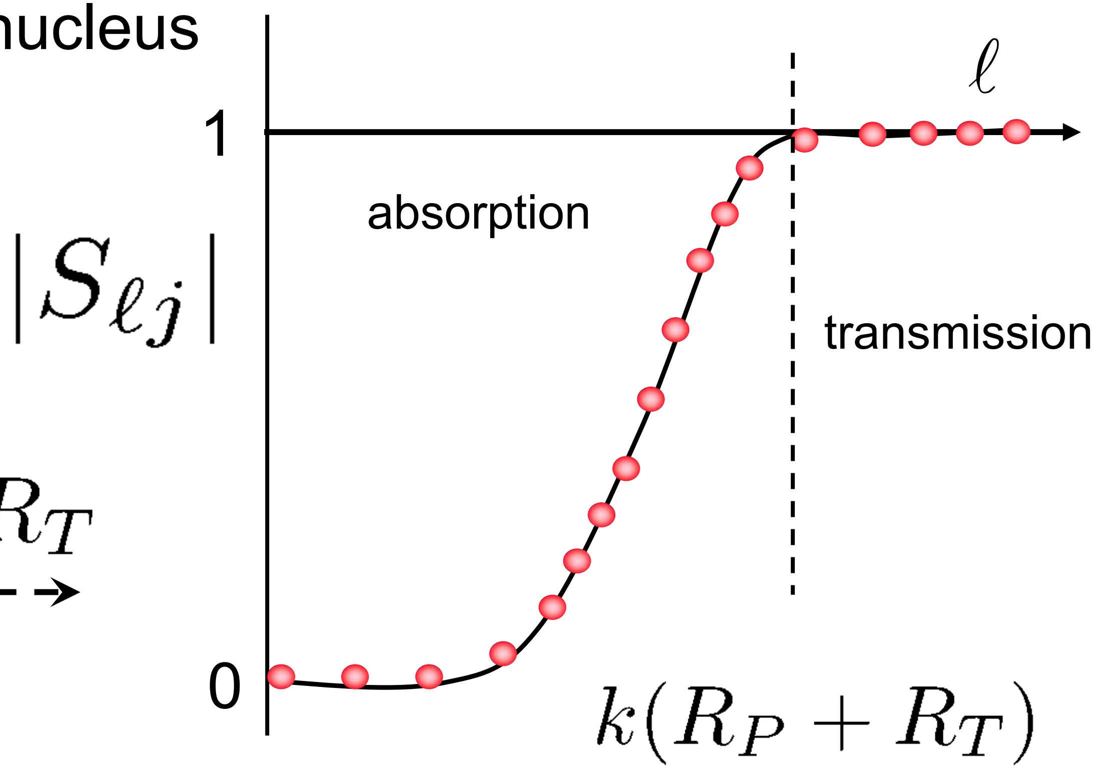
$$u_{k\ell j}(r) \rightarrow (i/2)[1\, H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$



Semi-classical approximations – many ℓ-values

large k – e.g. nucleus-nucleus





$$\text{semi} - \text{classical} : \; S(b), \quad \ell = kb$$

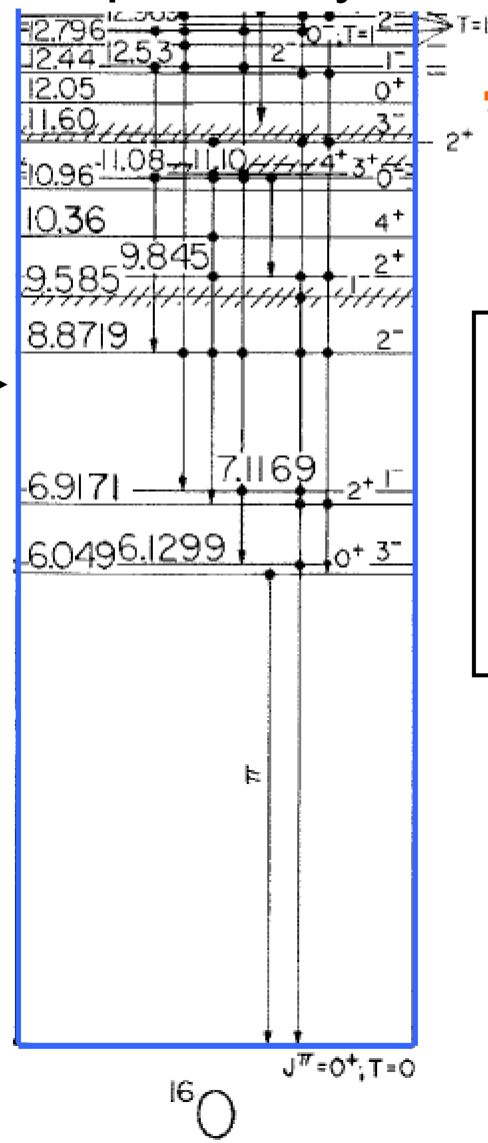
Low energy collisions and low level densities

Entrance channel | Compound system

$\{J^{\pi}\}$

E_{cm}

$^{12}\mathrm{C}+\alpha \quad 7.162$



$^{15}\mathrm{N}+p \quad 12.128$

A common feature is *resonant* behaviour in measured cross sections/observables

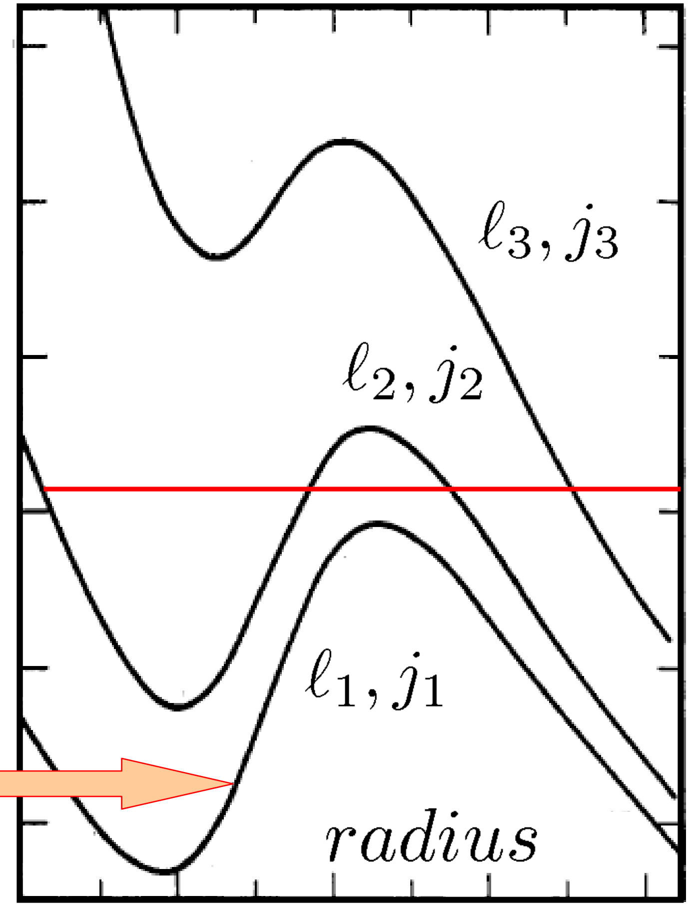
$$\alpha: \quad J = L$$
$$\pi = (-1)^{L}$$

Example for later in course

Phase shifts and S-matrix: Resonant behaviour

In real potentials, at low energies, the combination of an attractive nuclear, repulsive Coulomb and centrifugal terms can lead to potential pockets and resonant behaviour – the system being able to trapped in the pocket for some (life)time τ.

$$\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} + U_{\ell j}(r)$$

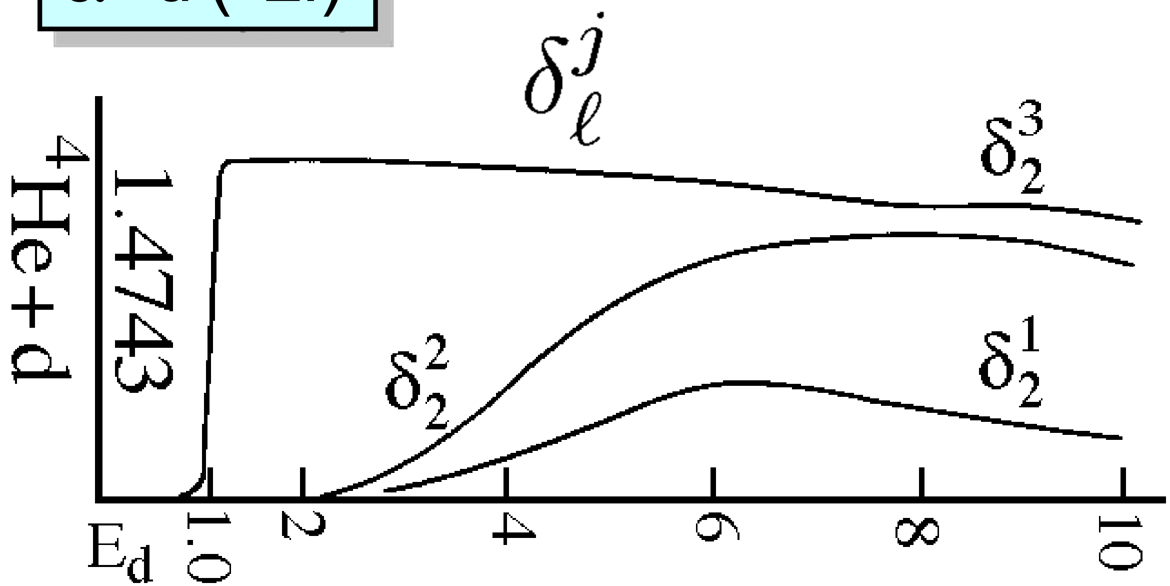


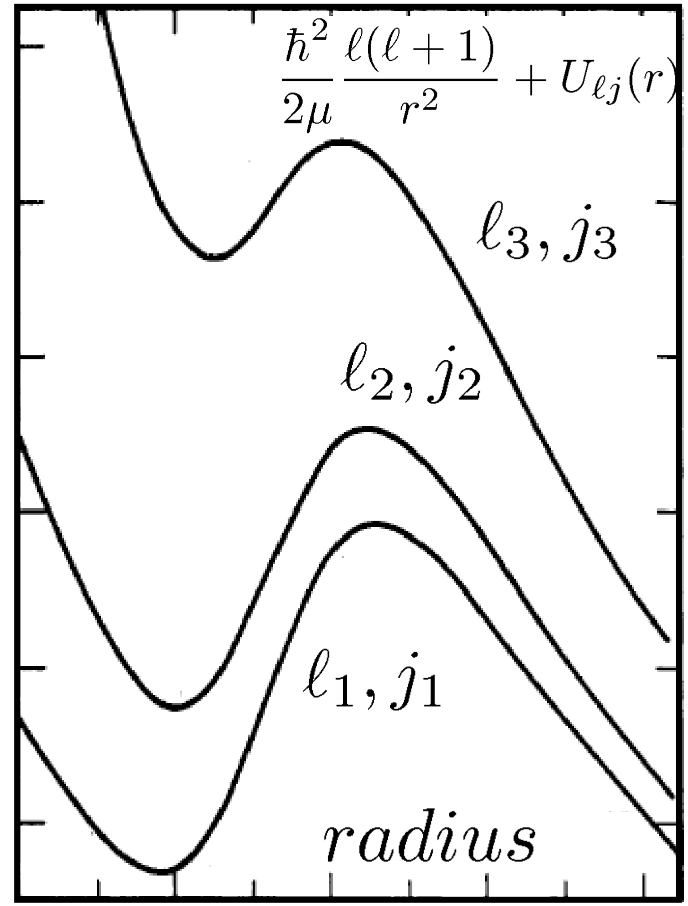
Phase shifts and S-matrix: Resonant behaviour

Potential pockets can lead to resonant behaviour – the system being able to trapped in the pocket for some (life)time τ.

A signal is the rise of the phase shift through 90 degrees.

α+d (^{6}Li)





Potential parameters should describe any known resonances

Resonance forms of the phase shift /S-matrix (i)

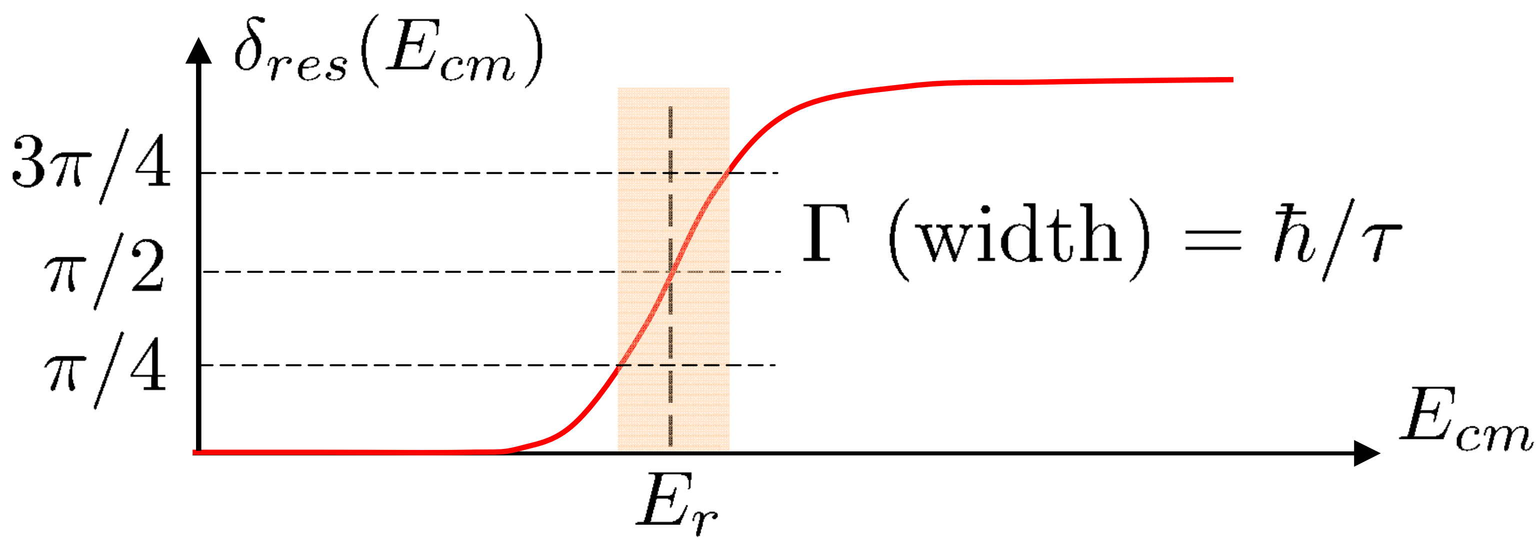
In the vicinity of an isolated (separated) resonance (with L)

$$\delta_L(E_{cm}) = \delta_{res}(E_{cm}) + \delta_{bg}(E_{cm})$$

with rapidly varying $\delta_{res}(E_{cm})$ over a small range of E_{cm}

$$\delta_{res}(E_{cm}) = \arctan\left(\frac{\Gamma/2}{E_r - E_{cm}}\right) \quad (\text{modulo } \pi)$$

and a slowly varying background phase $\delta_{bg}(E_{cm})$



Resonance forms of the phase shift /S-matrix (ii)

If $\delta_{res}(E_{cm})$ was the only phase present, $\delta_{bg} \approx 0$

$$\tan \delta_{res}(E_{cm}) = \frac{\Gamma/2}{E_r - E_{cm}}$$

and

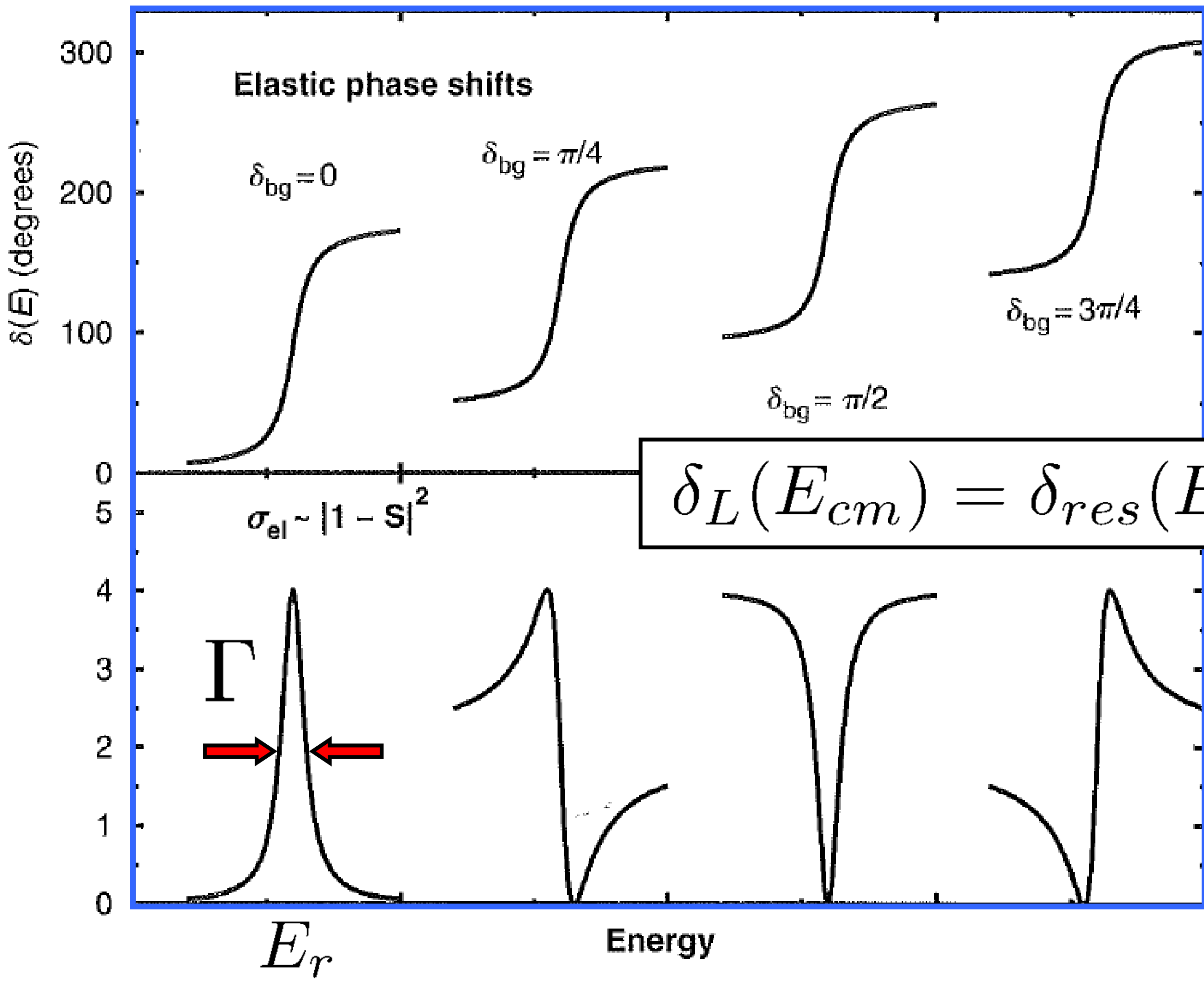
$$\sin^2 \delta_{res}(E_{cm}) = \frac{(\Gamma/2)^2}{(E_{cm} - E_r)^2 + (\Gamma/2)^2}$$

The (elastic channel) cross section resulting from this partial wave resonance is thus of Breit-Wigner form

$$\sigma_{res}(E_{cm}) = \frac{4\pi}{k^2}(2L+1)\sin^2 \delta_{res}(E_{cm})$$

$$= \frac{4\pi}{k^2}(2L+1)\frac{(\Gamma/2)^2}{(E_{cm} - E_r)^2 + (\Gamma/2)^2}$$

Resonance forms of the phase shift /S-matrix (iii)



phase shifts
$\delta_L(E_{cm})$

$$\delta_L(E_{cm}) = \delta_{res}(E_{cm}) + \delta_{bg}(E_{cm})$$

cross sections
$\sigma_{res}(E_{cm})$

From: Thompson and Nunes, Nuclear Reactions for Astrophysics, Cambridge University Press, 2009

Resonance forms of the phase shift /S-matrix (iv)

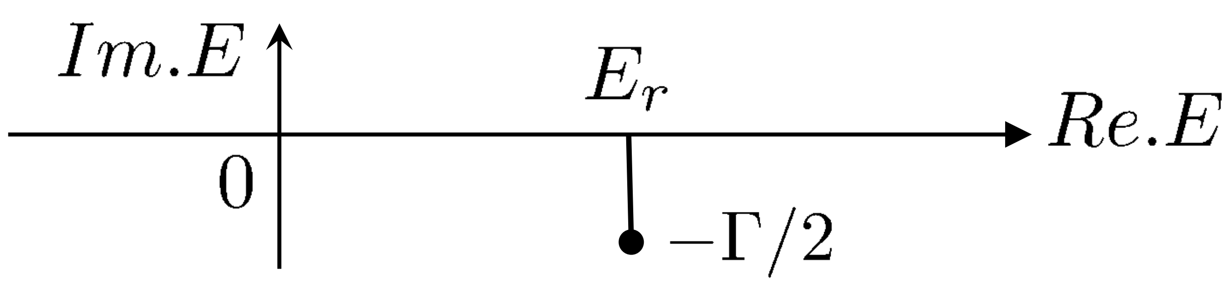
Additionally, if

$$\tan \delta_{res}(E_{cm}) = \frac{\Gamma/2}{E_r - E_{cm}}$$

$$S_L(E_{cm}) = e^{2i\delta_{bg}} \frac{E_{cm} - E_r - i\Gamma/2}{E_{cm} - E_r + i\Gamma/2}$$

So, cross section fitting means fitting these complex S as functions of the centre-of-mass energy: the E_r, Γ, δ_{bg}

Resonances in the physical system are, mathematically, poles of the complex collision matrix $S_L(E_{cm})$ in the complex energy plane, at $E_{cm} = E_r - i\Gamma/2$

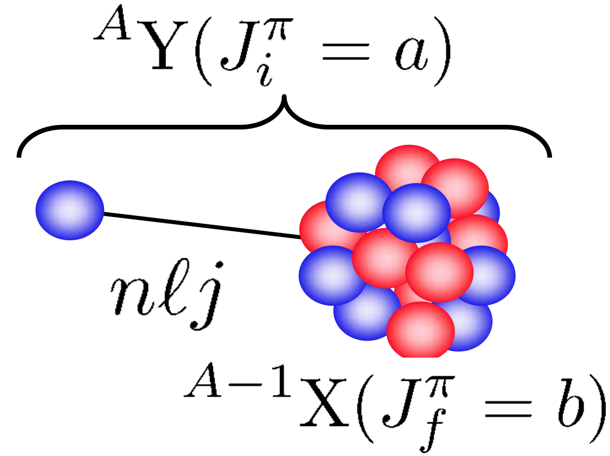


Bound states – Overlaps and spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.

$$\phi^m_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r}Y^\lambda_\ell(\hat{r})\chi^\sigma_s$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$



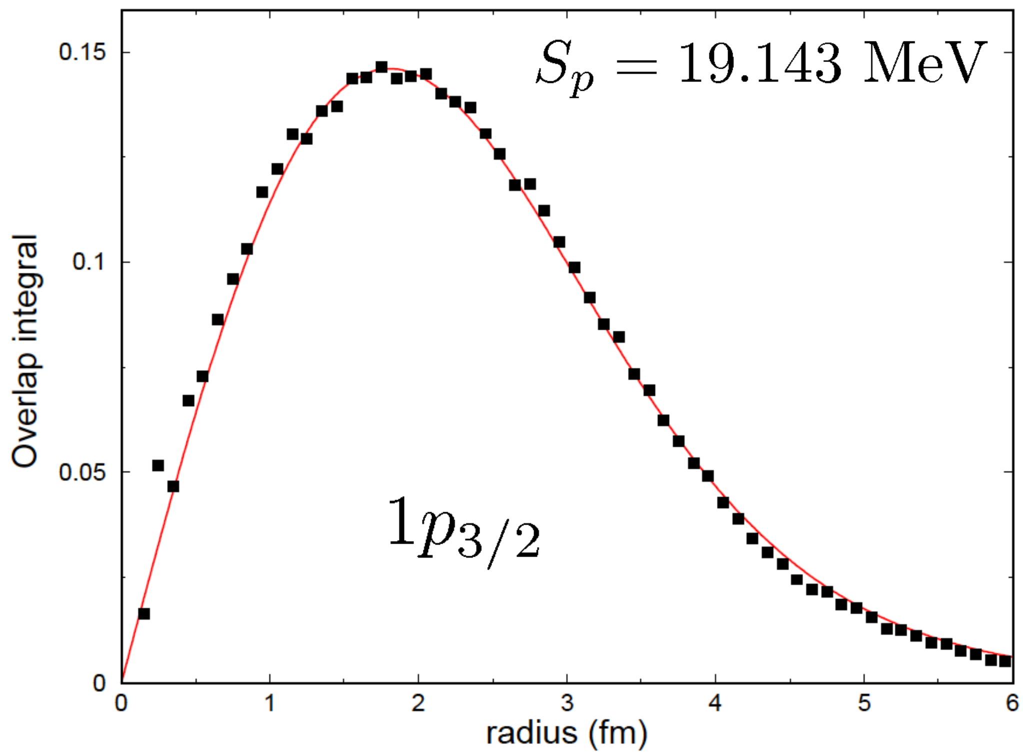
The potential model wave function approximates the overlap function of the A and A–1-body wave functions (Y and X in the case of an single nucleon) i.e. the overlap

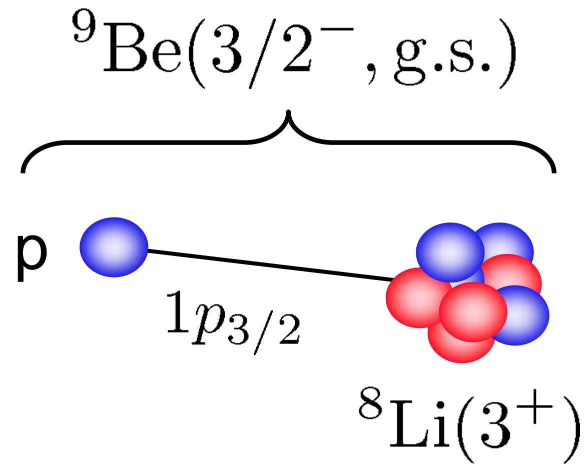
$$(\Phi^{b\beta}_X, A-1|\Phi^{a\alpha}_Y, A\rangle\rangle \longrightarrow F^{a\alpha b\beta}_{YX}(\vec{r})$$

Need to introduce spectroscopic factors, that relate these normalised single-particle wave functions and the overlaps to take account realistically of nuclear structure effects

Bound states – microscopic overlaps

$$\langle \vec{r}, {}^8\mathrm{Li}(3^+) | {}^9\mathrm{Be}(3/2^-, \mathrm{g.s.}) \rangle$$





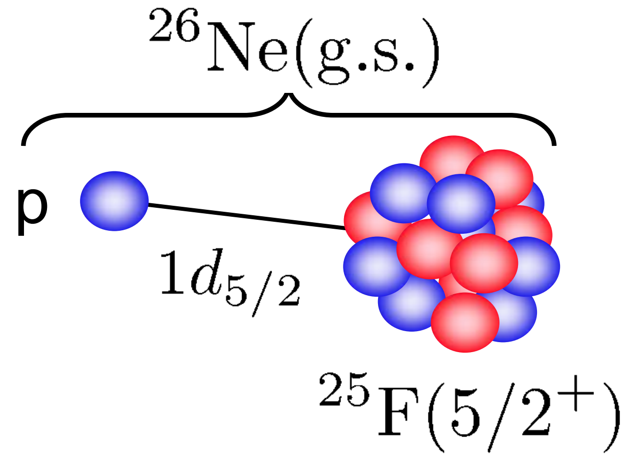
■ Microscopic overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a few cases

Normalised bound state in Woods-Saxon potential well x $(0.23)^{1/2}$ Spectroscopic factor

$$r_V = r_{so} = \mathrm{fitted},\ a_V = a_{so} = \mathrm{fitted},\ V_{so} = 6.0$$

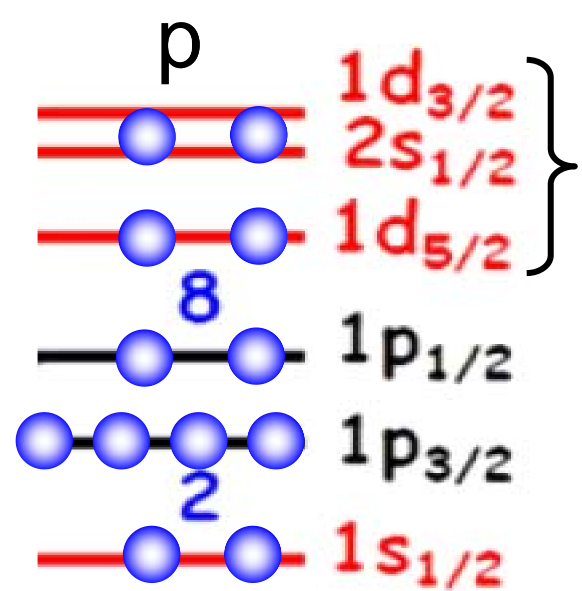
Bound states – shell model overlaps

$$\langle {}^{25}\mathrm{F}(5/2^+, E^*)|{}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.})\rangle\rangle$$



USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides <u>spectroscopic factors</u> but **<u>not</u>** the bound state radial wave function.

```
-- core state --- - overlap state -   (1 2  5)(
2j2t p   n      e  2j2t p   n      e     s
             -59.414             -81.625
 5 7 +   1    0.000 0 6 +   1    0.000  1.79039
 5 7 +   2    3.756 0 6 +   1    0.000  0.02316
 5 7 +   3    4.799 0 6 +   1    0.000  0.01084
 5 7 +   4    5.631 0 6 +   1    0.000  0.00012
 5 7 +   5    6.022 0 6 +   1    0.000  0.00589
 5 7 +   6    6.504 0 6 +   1    0.000  0.00044
 5 7 +   7    6.796 0 6 +   1    0.000  0.00002
 5 7 +   8    8.034 0 6 +   1    0.000  0.00006
 5 7 +   9    8.186 0 6 +   1    0.000  0.00097
 5 7 +  10    8.398 0 6 +   1    0.000  0.00006
total =                                 1.83196
centroid =    0.102     centroids =     0.000
centroid*= -22.313      centroids =   -22.211
```



Bound states – for clusters – conventions

How many nodes for cluster states ?

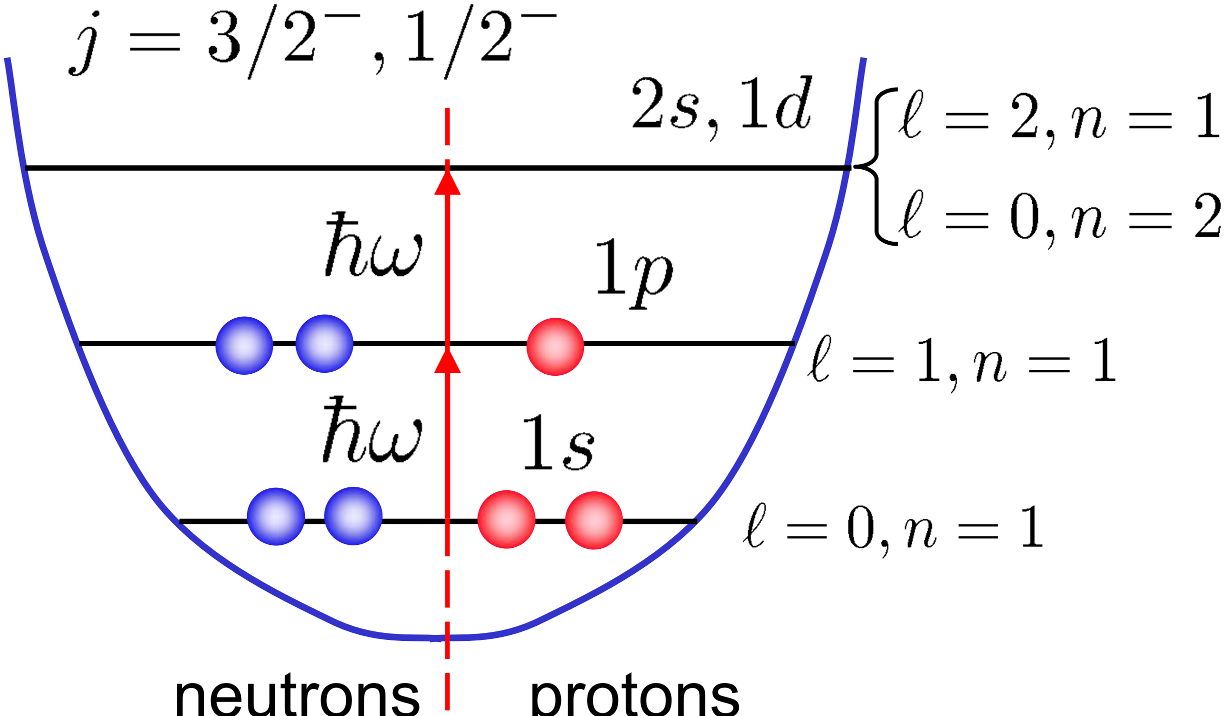
$$\phi^{m}_{n\ell j}(\vec{r})$$

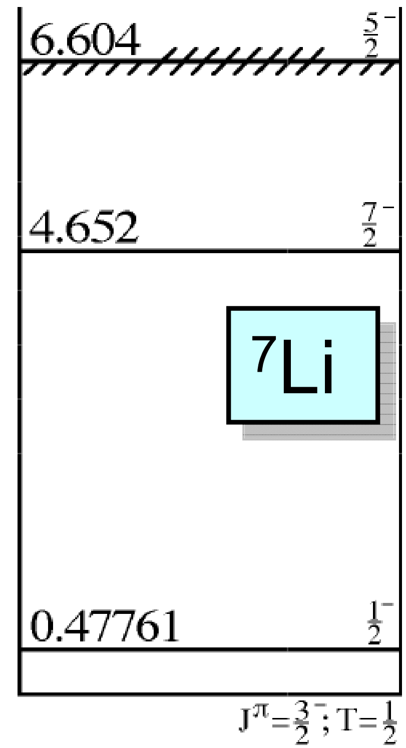
Usually guided by what the 3D harmonic oscillator potential requires - so as not to violate the Pauli Principle.

^{7}Li (α+t)

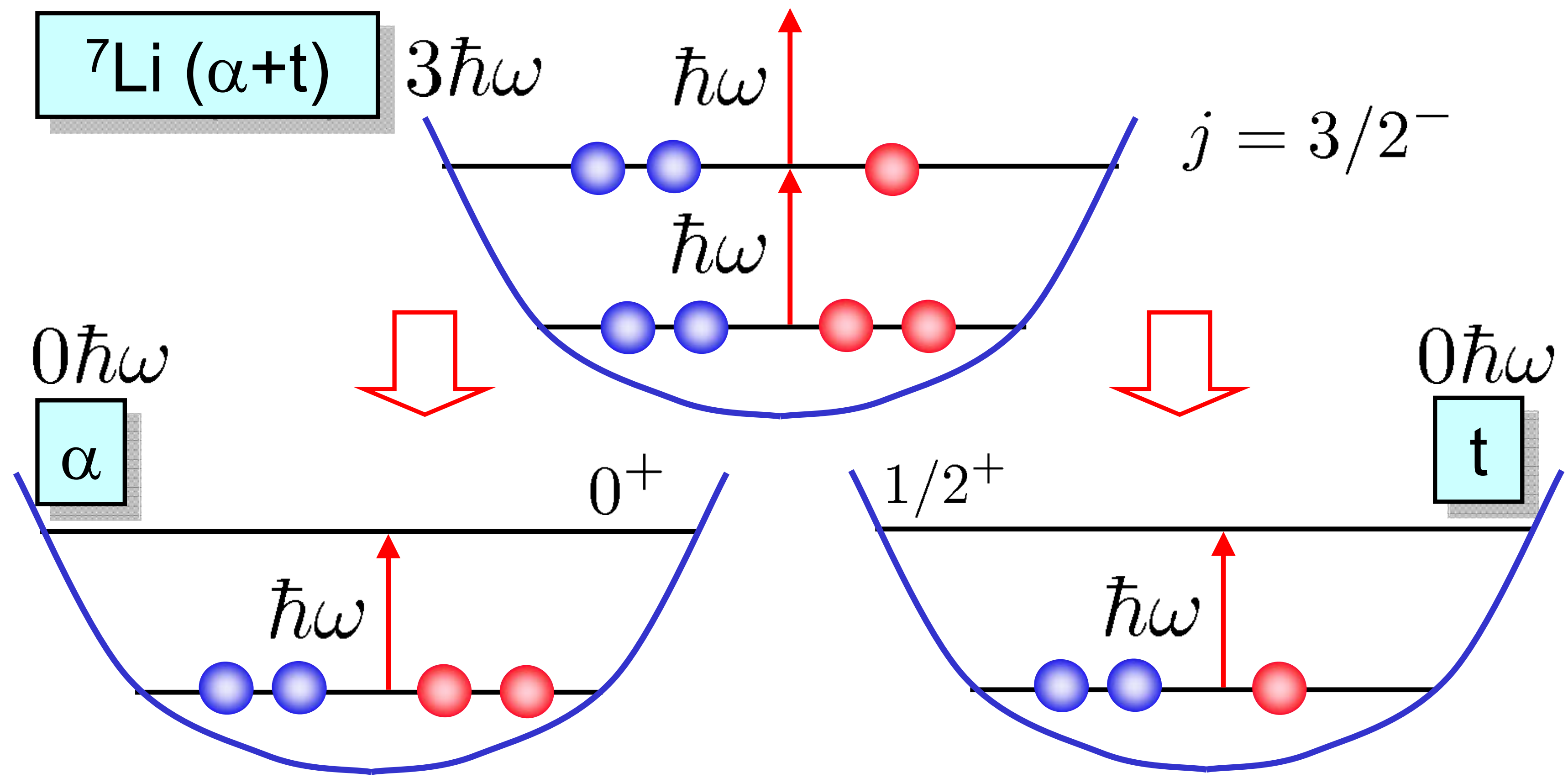
$$[2(n-1)+\ell\,]\hbar\omega$$

excitation due to a nucleon in a level





Bound states – for clusters – count quanta



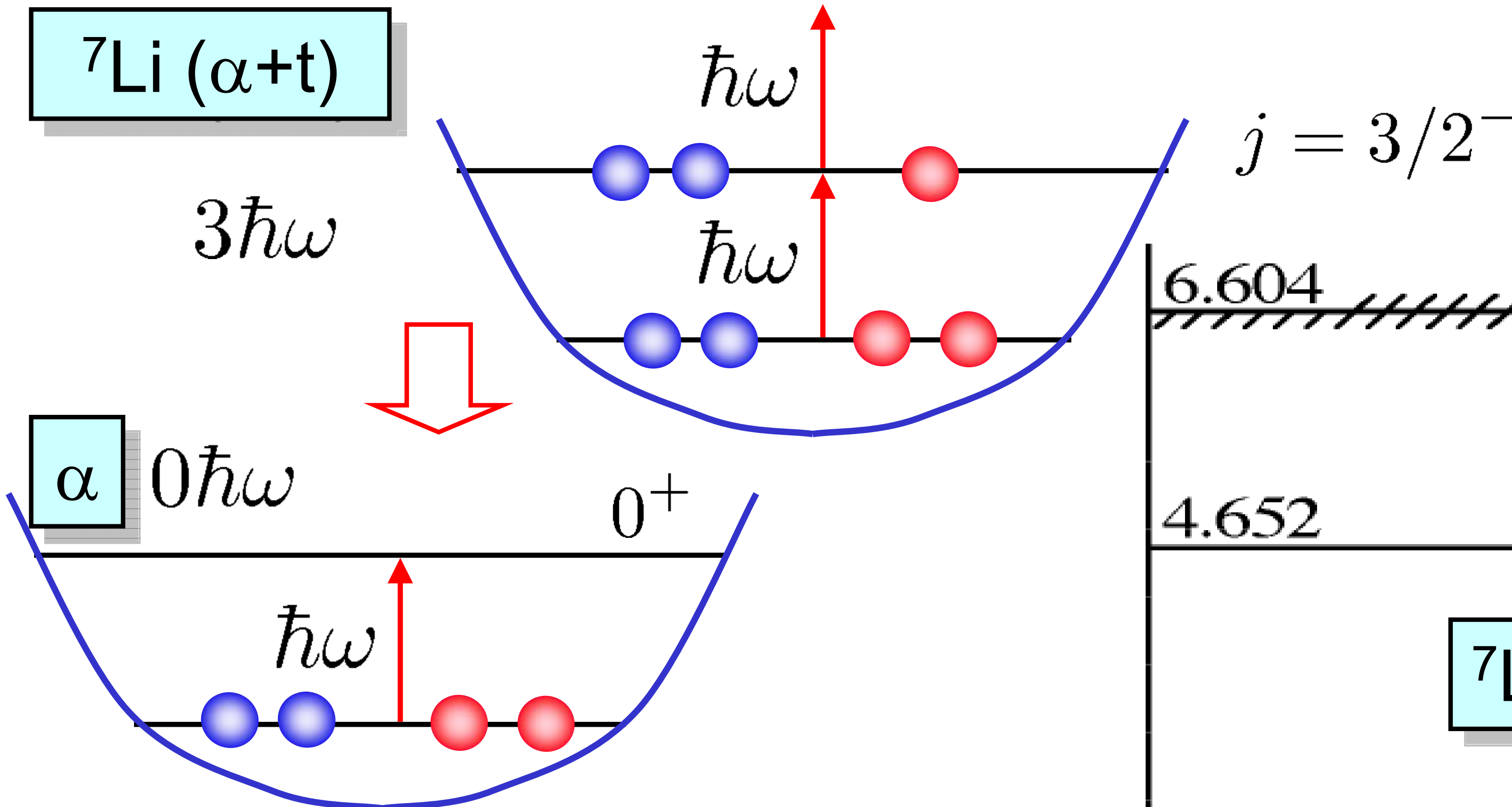
$$[2(N-1)+L\,]\hbar\omega = 3\hbar\omega$$

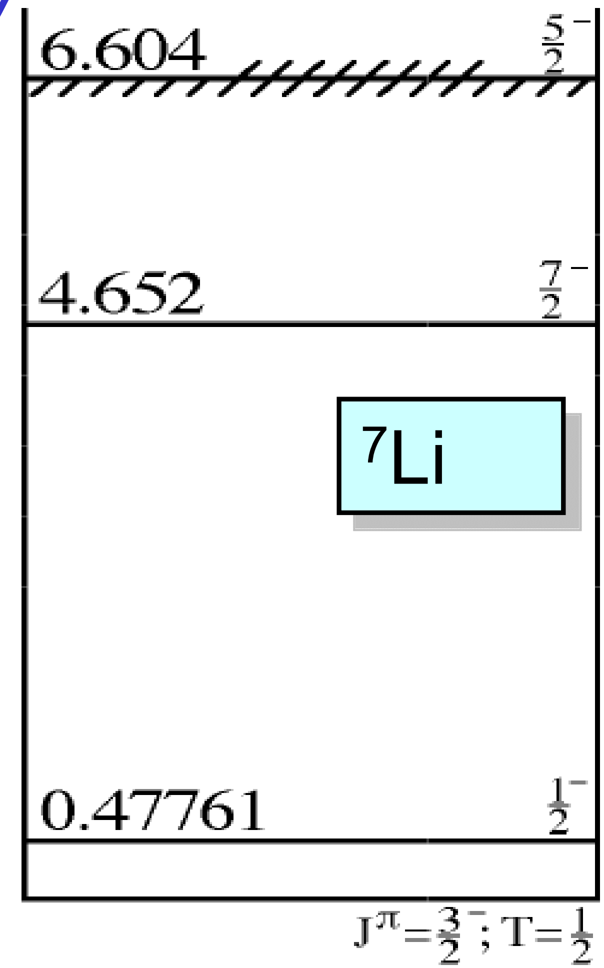
⇒ must be associated with the α+t relative motion

$L=1, N=2$ ⟶ $j = 3/2^-, 1/2^-,$

$L=3, N=1$ ⟶ $j = 7/2^-, 5/2^-$

Bound states – for clusters – relative motion





$$[2(N-1)+L\,]\hbar\omega = 3\hbar\omega$$

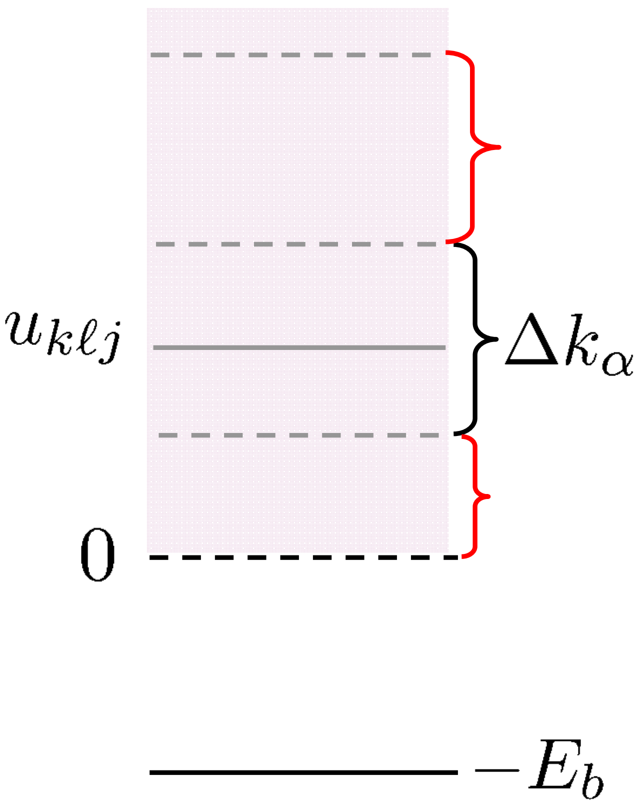
$$L=1, N=2 \qquad L=3, N=1$$

$$j = 3/2^-, 1/2^-, \quad j = 7/2^-, 5/2^-$$

Neither bound nor scattering – continuum bins

Scattering states

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}}[\cos\delta_{\ell j}F_\ell(\eta,kr)+\sin\delta_{\ell j}G_\ell(\eta,kr)]$$



$$\int_0^\infty dr\, u_{k\ell j}(r)\, u^*_{k'\ell j}(r) = \frac{\pi}{2}\delta(k-k')$$

$$\hat{u}_{\alpha\ell j}(r) = \sqrt{\frac{2}{\pi N_\alpha}}\int_{\Delta k_\alpha} dk\, g(k)\, u_{k\ell j}(r)$$

$$N_\alpha = \int_{\Delta k_\alpha} dk\, [g(k)]^2$$

weight function

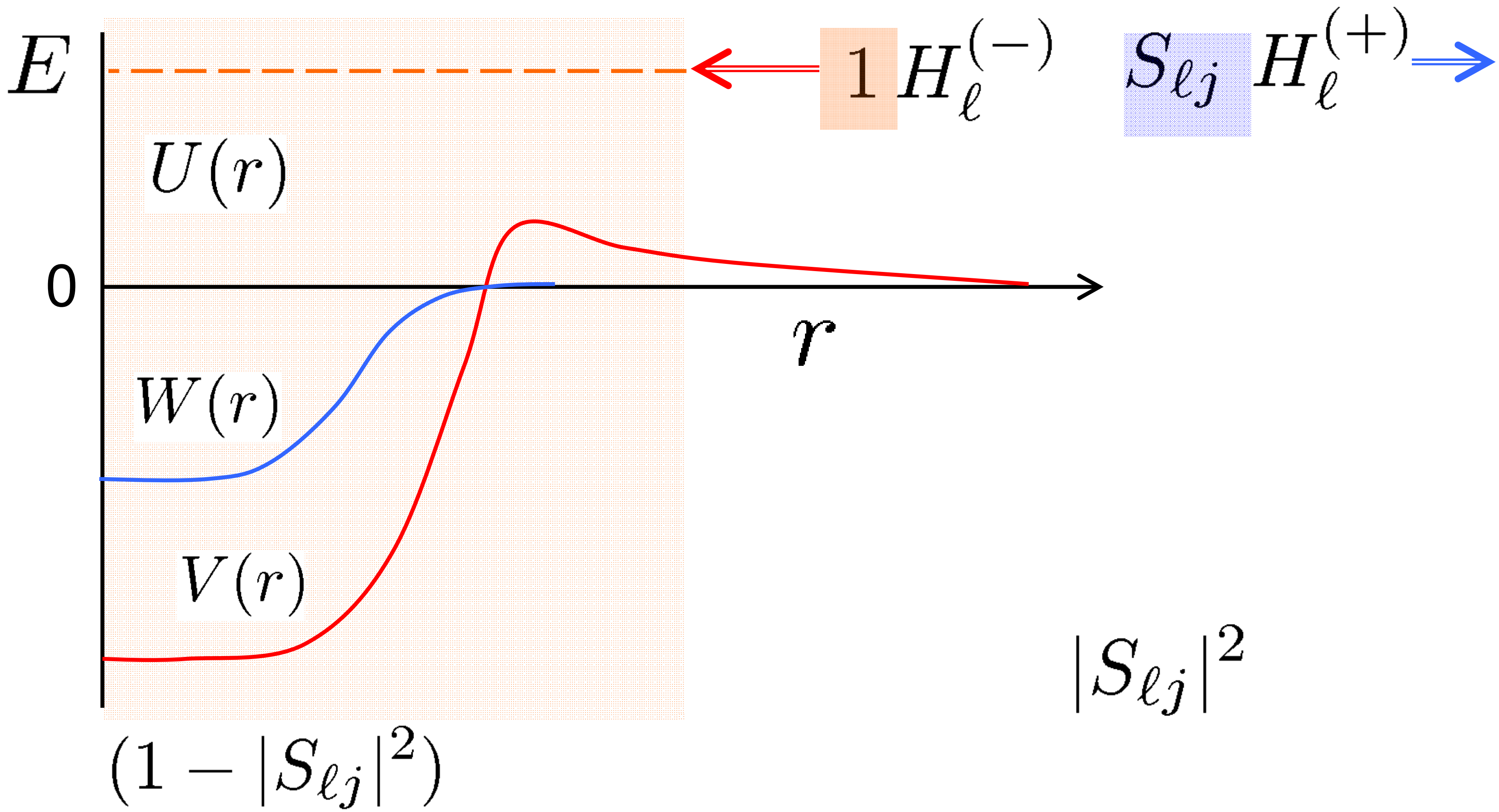
orthonormal set

$$\int_0^\infty dr\, \hat{u}^*_{\alpha\ell j}(r)\, \hat{u}_{\beta\ell j}(r) = \delta_{\alpha\beta}$$

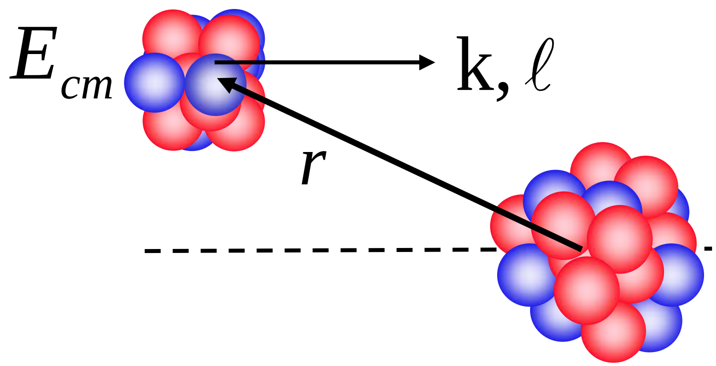
$$g(k) = 1 \qquad g(k) = \sin\delta_{\ell j}$$

Recall - ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[1\,H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}]$$

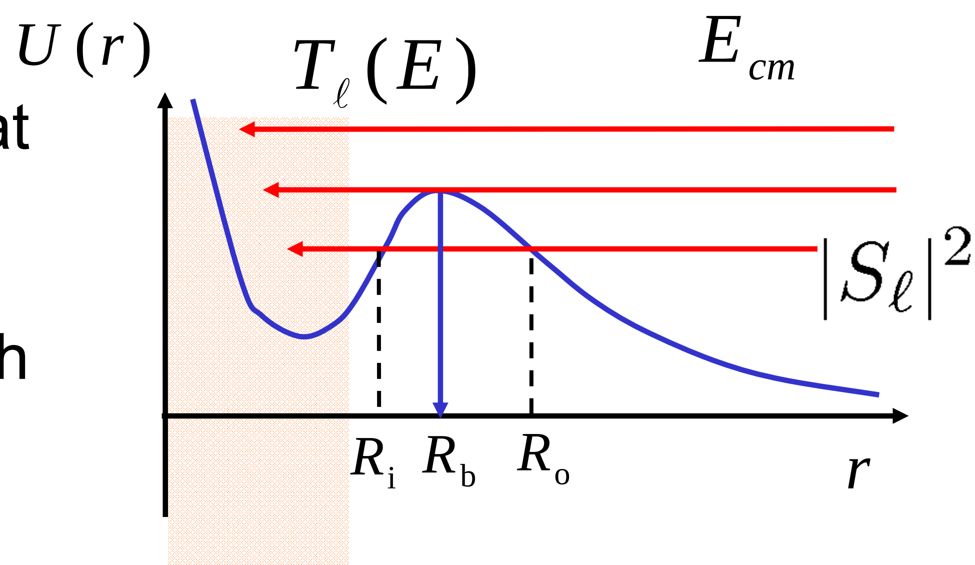


Barrier passing models of fusion



Gives basis also for simple (barrier passing) models of nucleus-nucleus fusion reactions

an imaginary part in *U(r)*, at short distances, can be included to absorb all flux that passes over or through the barrier – assumed to result in fusion



$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - |S_\ell|^2)$$

Homeworks: to become familiar with these items

`bound` (bound states solver – see **`bound.outline`**)

`scatter` (spin-independent scattering solver, range of ℓ or energy - see **`scatter.outline`**)

`scat` (single ℓ , j, scattering as a function of energy – see **`scat.outline`**)

`scat_one` (single ℓ , j, and single energy scattering)

`bins` (single ℓ , j, continuum bin state constructor – see **`bins.outline`**)

`dens` (Alex Brown Hartree Fock solver– for mean field [Hartree-Fock] wave functions and densities)