TALENT Course 6: Theory for exploring nuclear reaction experiments

Eikonal methods – High-energy approximations – elastic scattering

GANIL, 1st - 19th July 2013

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Session aims: part 1

To discuss <u>approximate</u> solutions of the Schrodinger equation for states of **two, three** or more bodies at ‘high energies’ by introducing the eikonal (forward-scattering dominated) approximation of the reaction dynamics.

To bring out the importance of the eikonal S-matrix, a function if the impact parameter of the projectile, or of a component of the projectile, in this formulation of the reaction and scattering of the interacting systems.

To gain an impression of how accurate the eikonal approximation is as a function of the projectile energy, and to see how one can describe both point particle and composite projectile scattering using these methods.

Radioactive ion-beams: fragmentation facilities

RIKEN RI BEAM FACTORY

RIKEN

RIKEN NiSHiNA CENTER

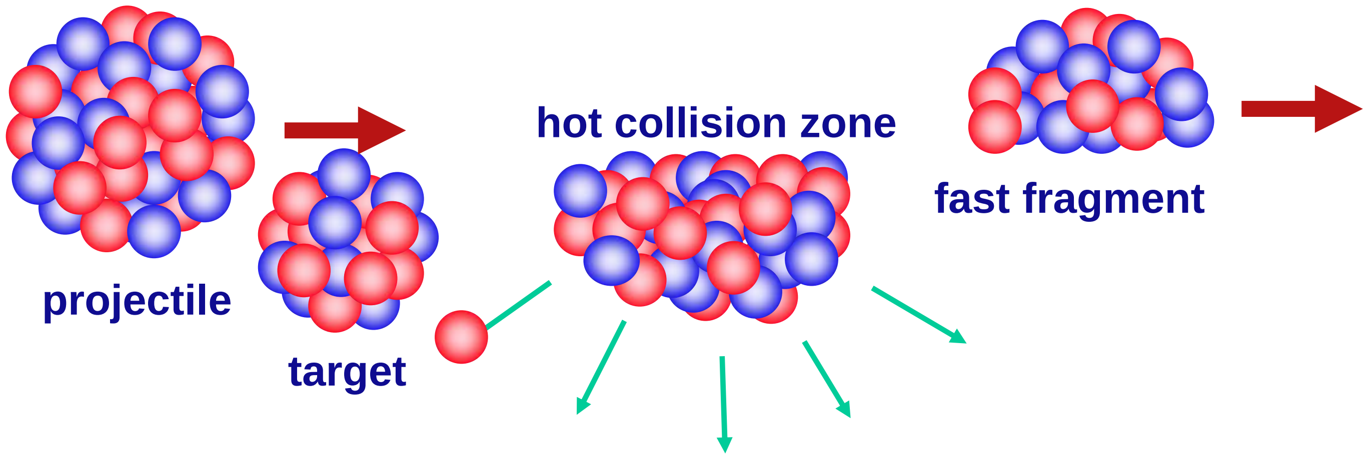
---A Dream Factory for Particle Beams---

NSCL

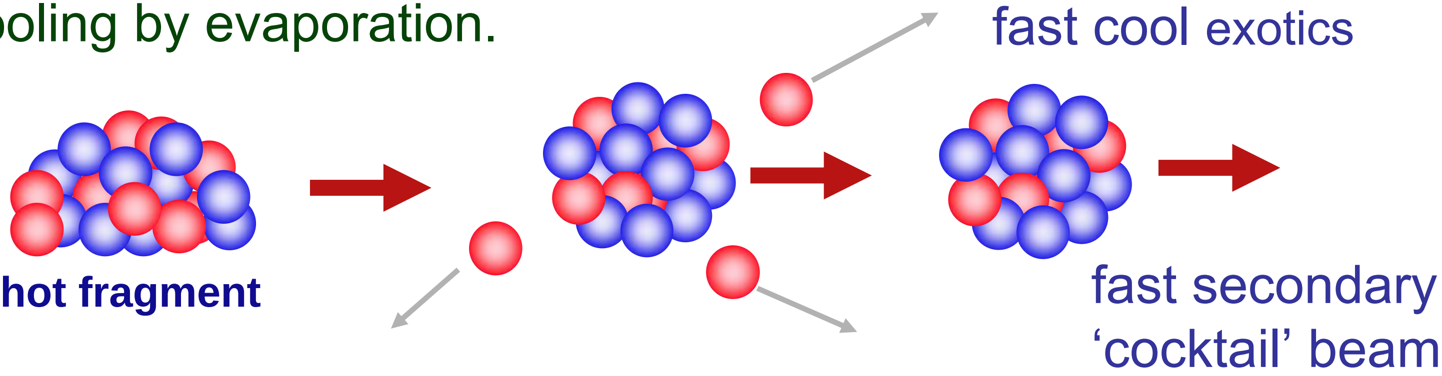
FRIB

Exotic nuclei production - projectile fragmentation

Random removal of protons and neutrons from heavy projectile in peripheral collisions at <u>high energy</u> - 100 MeV per nucleon or more



Cooling by evaporation.



Reaction timescales – in surface grazing collisions

For 100 and 250 MeV/u incident energy:

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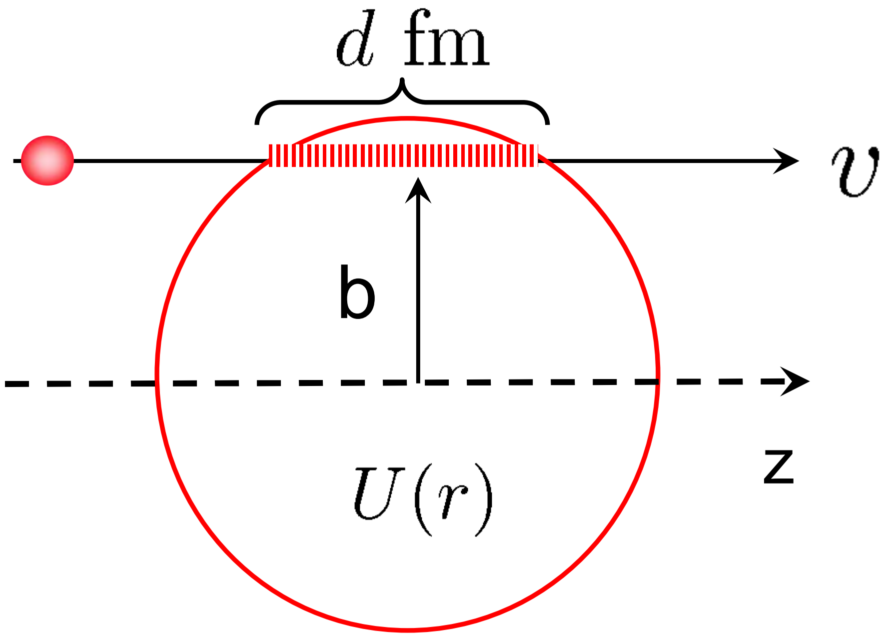
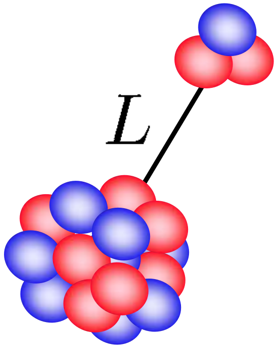
$$\gamma = 1.1, \quad v/c = 0.42, \qquad \gamma = 1.25, \quad v/c = 0.6,$$

$$\Delta t = 7.9 \times d \times 10^{-24} s, \qquad \Delta t = 5.6 \times d \times 10^{-24} s$$



The Schrodinger equation – differential approach



So, using usual notation

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi^{+}_{\vec{k}}(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

and for scattering states $E_{cm} > 0$

$$\chi^{+}_{\vec{k}}(\vec{r}) = \frac{4\pi}{k}\sum_{L\Lambda} i^L \frac{u_{kL}(r)}{r} Y_L^\Lambda(\hat{r}) Y_L^\Lambda(\hat{k})^*, \qquad k^2 = \frac{2\mu E_{cm}}{\hbar^2}$$

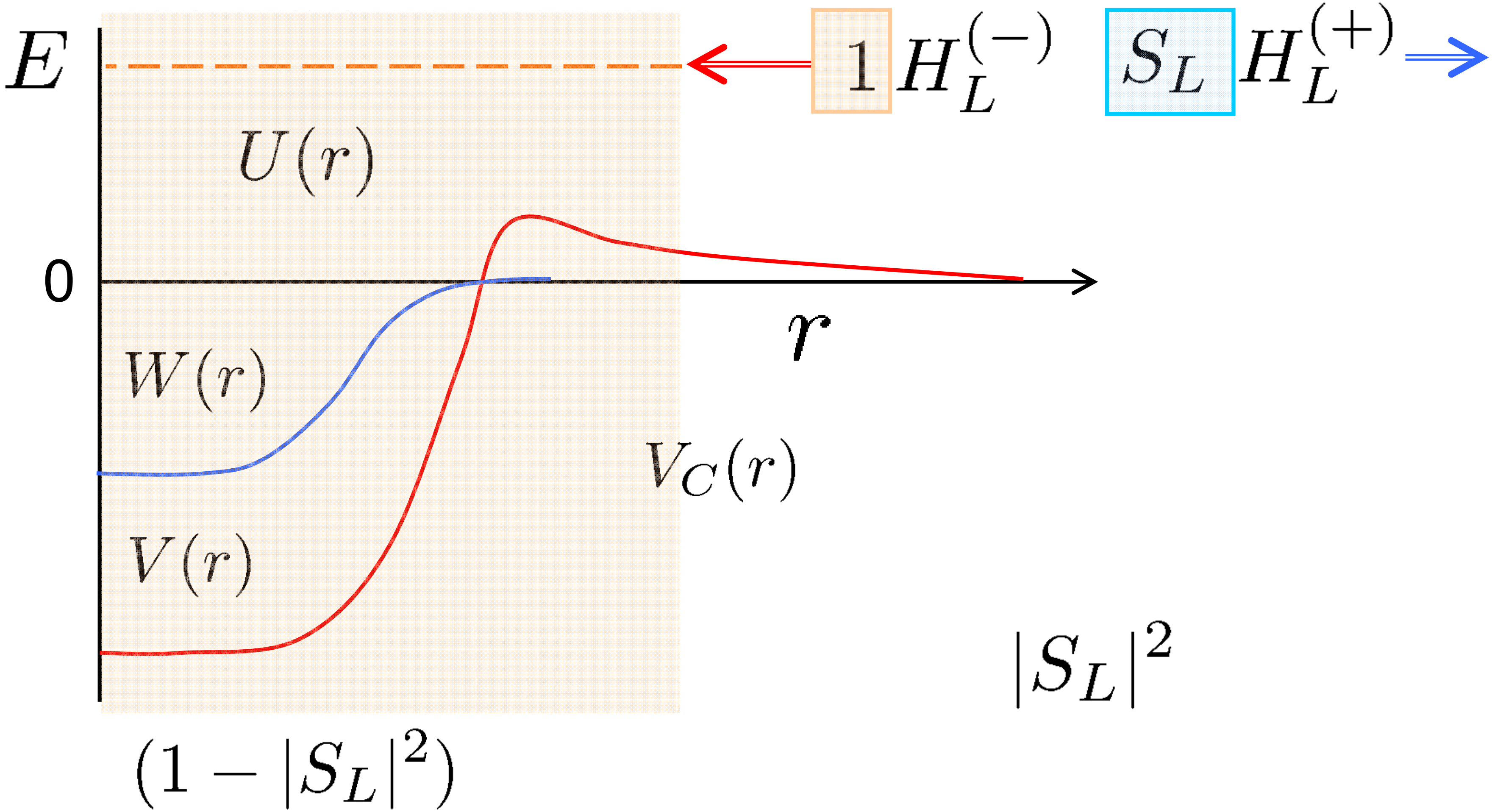
$$\boxed{\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} + k^2 - \frac{2\mu}{\hbar^2}U(r)\right) u_{kL}(r) = 0}$$

$$U(r) = V_C(r) + V(r) + iW(r)$$

$$u_{kL}(r) \rightarrow (i/2)[1\, H_L^{(-)} - S_L H_L^{(+)}]$$

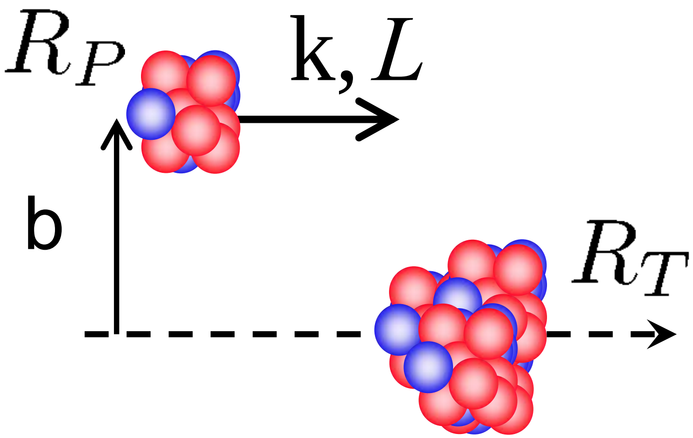
S-matrix - ingoing and outgoing waves amplitudes

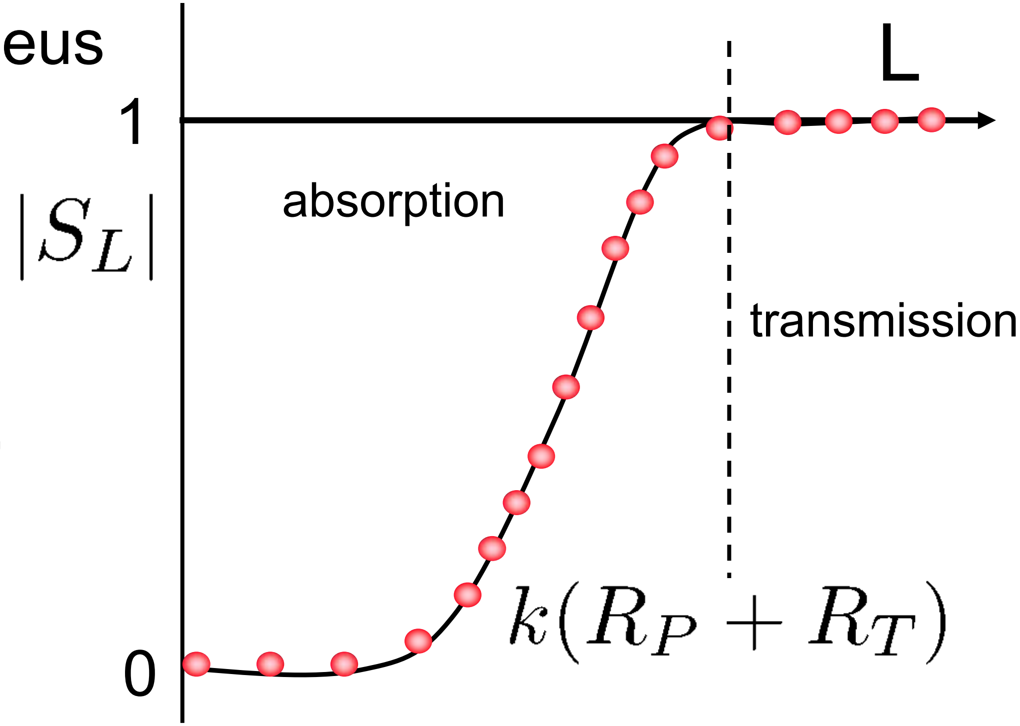
$$u_{kL}(r) \rightarrow (i/2)[\boxed{1}\, H_L^{(-)} - \boxed{S_L}\, H_L^{(+)}]$$



Semi-classical approaches – many L-values

large k – e.g. nucleus-nucleus





$$\mathrm{semi-classical:}\ S(b), \quad L + 1/2 = kb$$

$$\sum_{L=0}^{\infty}(2L+1)\ldots \rightarrow 2k^2\int_0^{\infty} db\, b\ldots \qquad \frac{L(L+1)}{r^2} \rightarrow \frac{k^2b^2 - 1/4}{r^2}$$

Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the semi-classical (impact parameter) representations, for example (spinless case):

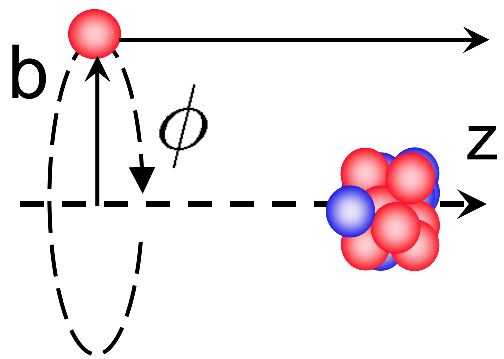
$$\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\;|1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\;(1-|S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el} + \sigma_R = 2\int d^2\vec{b}\;[1-\mathrm{Re}.S(b)] \quad \text{etc.}$$

and where (cylindrical coordinates)

$$\int d^2\vec{b} \equiv \int_0^{\infty} bdb \int_0^{2\pi} d\phi = 2\pi\int_0^{\infty} bdb$$



Eikonal approximation: for point particles (1)

Approximate (<u>semi-classical</u>) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0, \qquad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2}U(r) + k^2\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0$$

valid when $\boxed{|U|/E \ll 1, \quad ka \gg 1}$ → small wavelength high energy

Key steps are: (1) the distorted wave function is written

$$\chi_{\vec{k}}^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\boxed{\omega(\vec{r})}$$

all effects due to *U(r)*, modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

Eikonal approximation: point neutral particles (2)

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \cancel{\nabla^2\omega(\vec{r})}\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

The conditions $|U|/E \ll 1, \quad ka \gg 1$ → imply that

$$\boxed{2\vec{k}\cdot\nabla\omega(\vec{r}) \gg \nabla^2\omega(\vec{r})}$$

Slow spatial variation cf. k

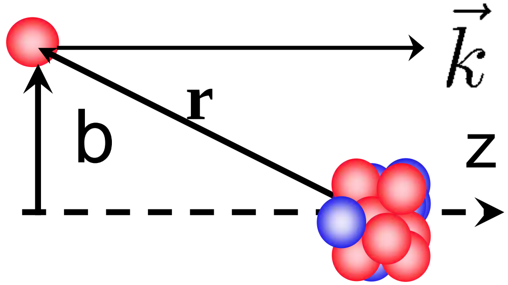
and choosing the z-axis in the beam direction $\vec{k}$

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r})$$

phase that develops with z

with solution

$$\boxed{\omega(\vec{r}) = \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r')dz'\right]}$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point neutral particles (3)

$$\chi^+_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \omega(\vec{r}) \approx \exp(i\vec{k}\cdot\vec{r})\ \boxed{\exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r')dz'\right]}$$

So, after the interaction and as $z\to\infty$

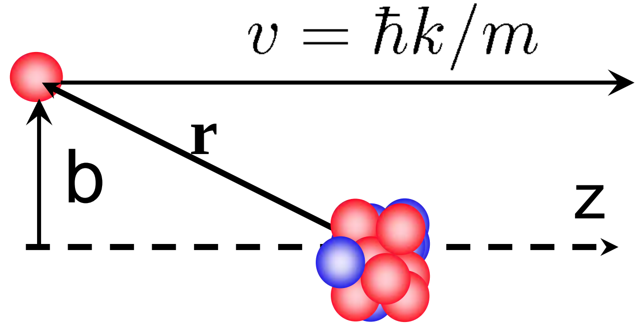
$$\chi^+_{\vec{k}}(\vec{r}) \to \exp(i\vec{k}\cdot\vec{r})\ \ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{\infty} U(r')dz'\right] = S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{\chi^+_{\vec{k}}(\vec{r}) \longrightarrow S(b)\exp(i\vec{k}\cdot\vec{r})}$$

Eikonal approximation to the S-matrix S(b)

S(b) is amplitude of the forward going outgoing waves from the scattering at impact parameter b

$$\boxed{S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r')dz'\right]}$$

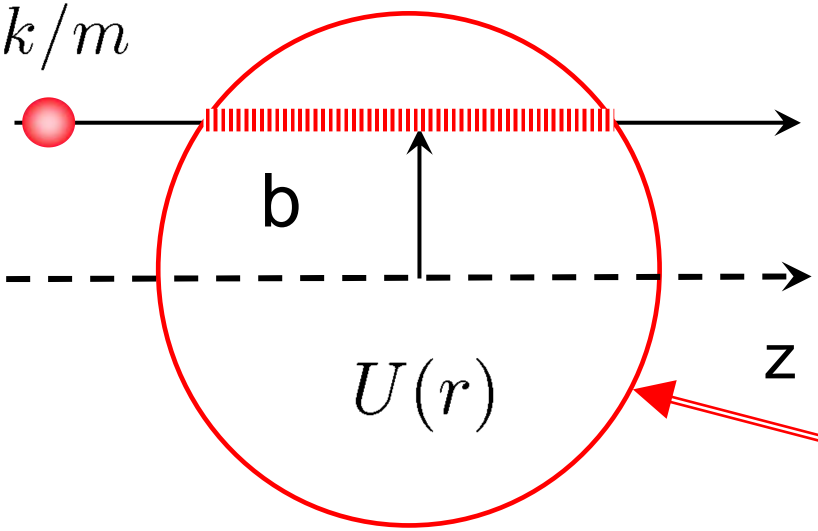


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal approximation: point particles - summary

$$\chi^{+}_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r}) \;\; \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r')dz'\right]$$

$$v = \hbar k/m$$



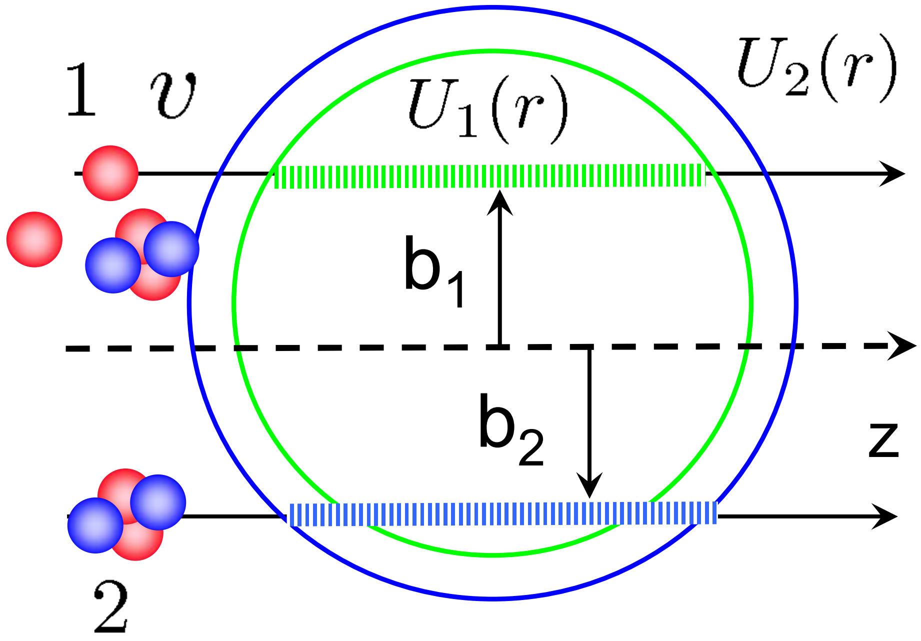
$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U(r')dz$$

limit of range of finite ranged potential

$$\chi^{+}_{\vec{k}}(\vec{r}) \rightarrow S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r')dz'\right]}$$

Eikonal approximation: several particles (preview)



$$\chi_i(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_i(r')dz$$

Total interaction energy

$$U(r_1,\ldots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp[i\chi_i(b_i)] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_i(r_i')dz'\right]$$

$$\chi(b_1,\ldots) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}\sum_i U_i(r_i')dz$$

with composite objects we will get products of the S-matrices

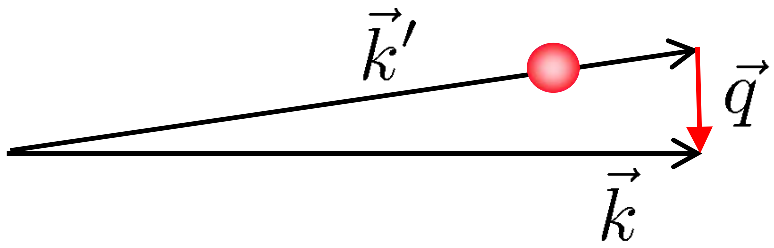
$$\exp[i\chi(b_1,\ldots)] = \prod_i S_i(b_i)$$

Point particle – the differential cross section

Using the standard result from scattering theory, the elastic scattering amplitude is

$$
\begin{aligned}
f(\theta) &= -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(-i\vec{k}'\cdot\vec{r})\,U(r)\,\chi^{+}_{\vec{k}}(\vec{r}) \\
&= -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(-i\vec{k}'\cdot\vec{r})\,U(r)\,\exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r}) \\
&= -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(i\vec{q}\cdot\vec{r})\,U(r)\,\omega(\vec{r})
\end{aligned}
$$

with $\vec{q} = \vec{k} - \vec{k}',\ q = 2k\sin(\theta/2)$ is the momentum transfer. Consistent with the earlier high energy (forward scattering) approximation



$$
\boxed{\begin{aligned} \vec{q}\cdot\vec{r} &\approx \vec{q}\cdot\vec{b} \\ \vec{q}\cdot\vec{k} &\approx 0 \end{aligned}}
$$

Point particles – the differential cross section

So, the elastic scattering amplitude

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(i\vec{q}\cdot\vec{r})\,U(r)\,\omega(\vec{r})$$

$$\begin{cases} \dfrac{d\omega}{dz} = -\dfrac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r}) \\ U(r)\omega(\vec{r}) = \dfrac{i\hbar^2 k}{\mu}\dfrac{d\omega}{dz} \end{cases}$$

is approximated by

$$f_{eik}(\theta) = -\frac{ik}{2\pi}\int d^2\vec{b}\,\exp(i\vec{q}\cdot\vec{b})\int_{-\infty}^{\infty}\frac{d\omega}{dz}dz$$

Performing the z- and azimuthal ϕ integrals

$$\boxed{f_{eik}(\theta) = -ik\int_0^{\infty} b\,db\,J_0(qb)\,[S(b)-1]}$$

$$\boxed{S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty}U(r')dz'\right]}$$

$J_0(qb)$

Bessel function

Point particle – the Coulomb interaction

Treatment of the Coulomb interaction (as in partial wave analysis) requires a little care. Problem is, eikonal phase integral due to Coulomb potential diverges logarithmically.

$$\chi_C(b) = -\frac{1}{\hbar v}\int_{-a}^{+a} V_C(r')dz$$

Must 'screen' the potential at some large screening radius

$$f_{eik}(\theta) = e^{i\chi_a}\left[f_{pt}(\theta) - ik\underbrace{\int_0^\infty b\,db\,J_0(qb)\,e^{i\chi_{pt}}[\bar{S}(b)-1]}_{\text{nuclear scattering in the presence of Coulomb}}\right]$$

overall unobservable screening phase

usual Coulomb (Rutherford) point charge amplitude

nuclear scattering in the presence of Coulomb

$$\bar{\chi}(b) = \chi_N(b) + \chi_\rho(b) - \chi_{pt}(b)$$

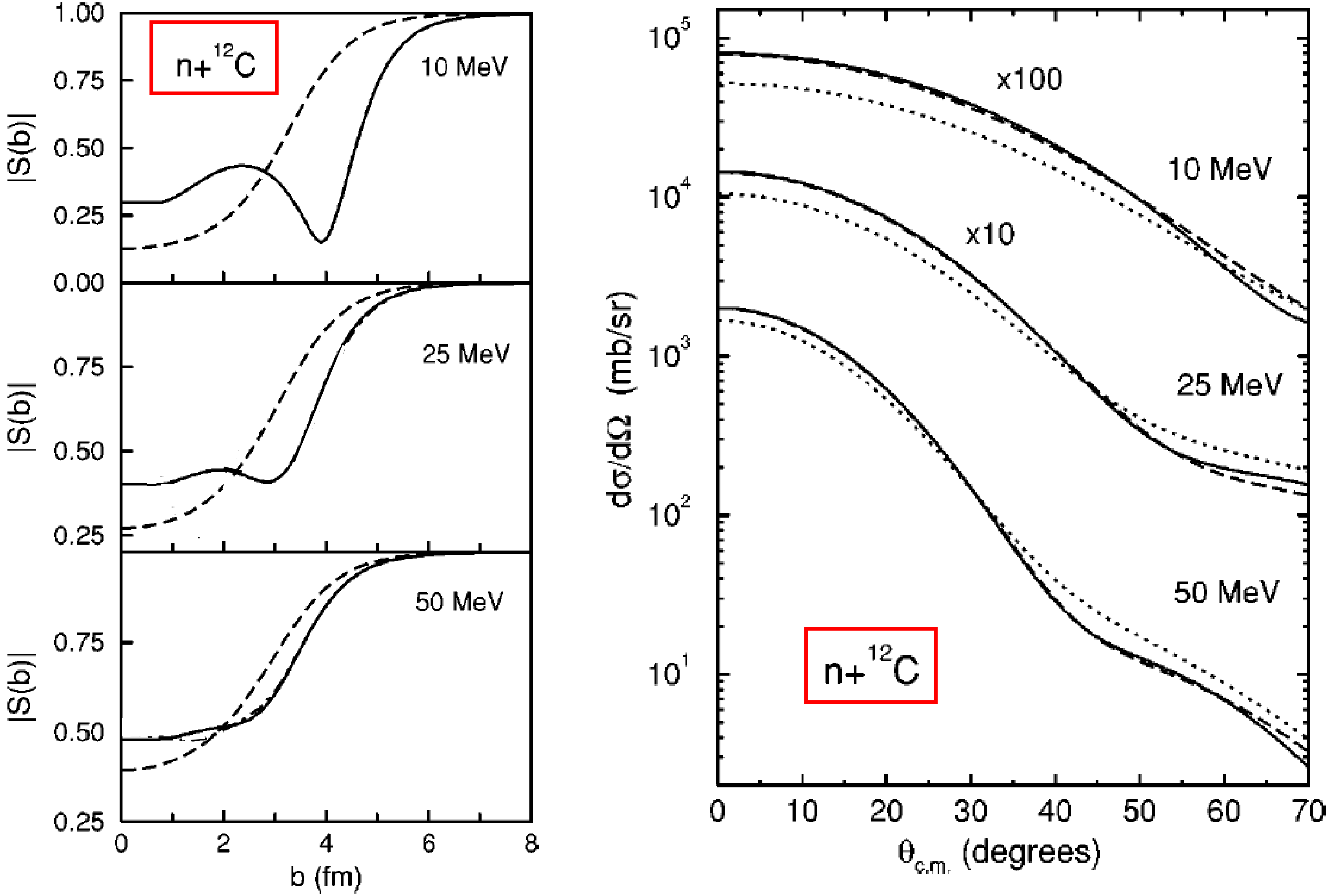
nuclear phase

Due to finite charge distribution

$$\chi_{pt}(b) = 2\eta\ln(kb)$$

See e.g. J.M. Brooke, J.S. Al-Khalili, and J.A. Tostevin PRC **59** 1560

Accuracy of the eikonal S(b) and cross sections



J.M. Brooke, J.S. Al-Khalili, and J.A. Tostevin PRC **59** 1560

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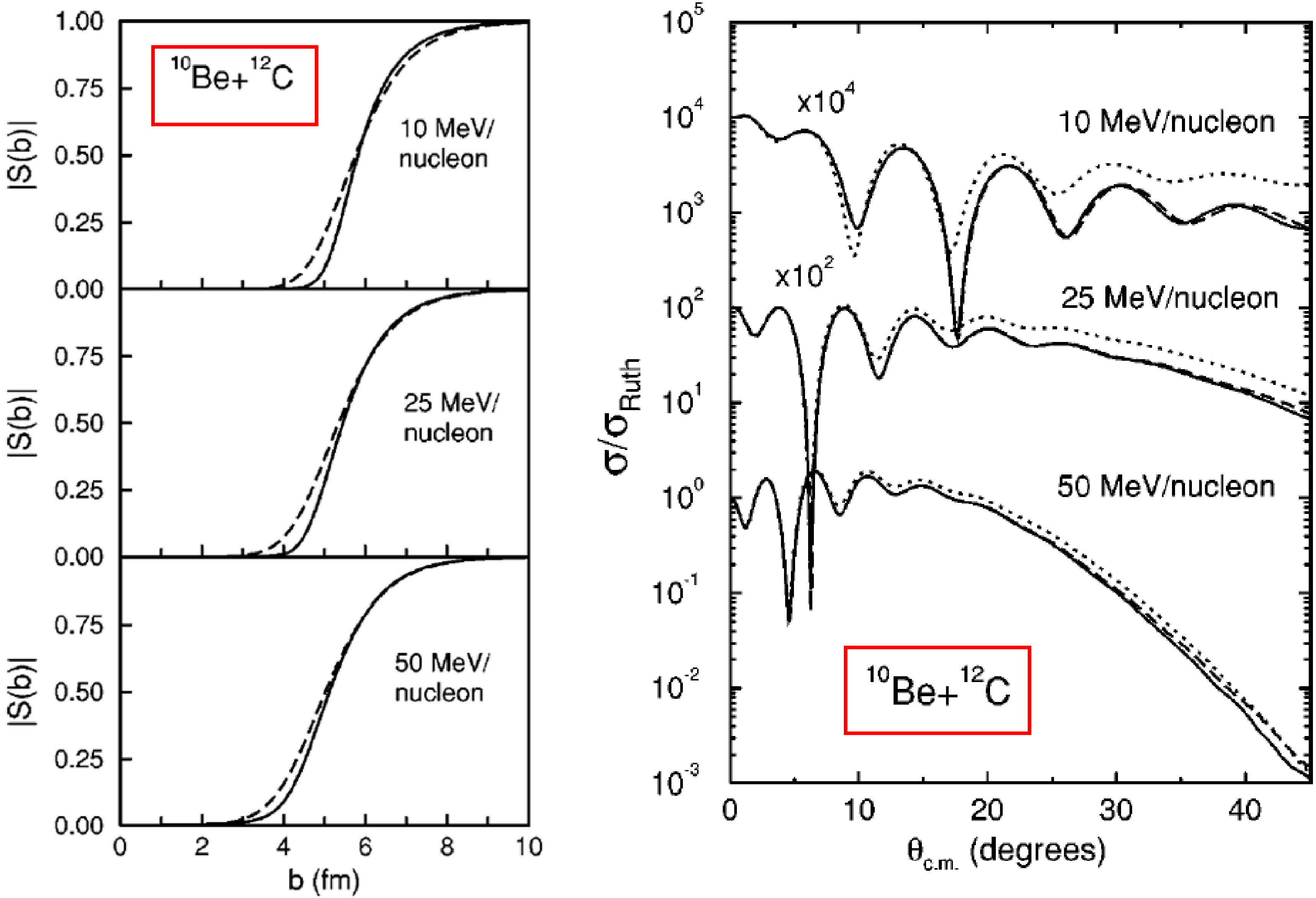
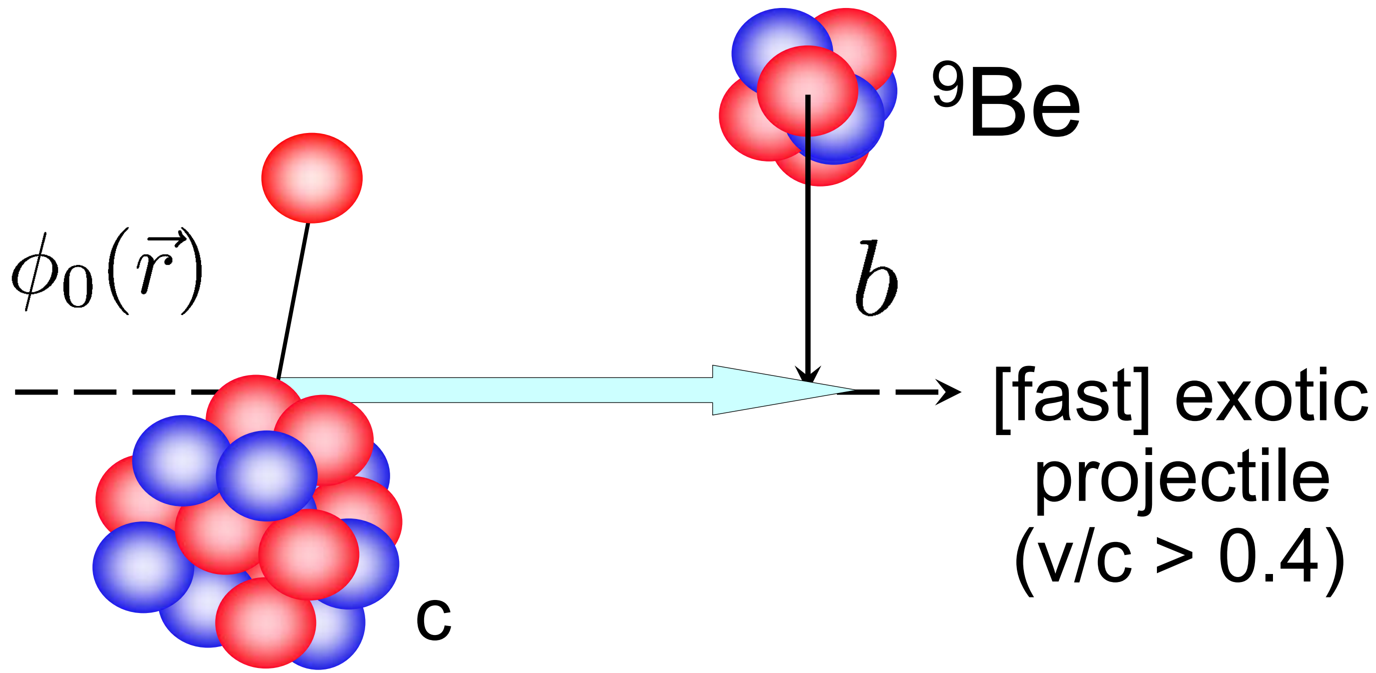


J.M. Brooke, J.S. Al-Khalili, and J.A. Tostevin PRC **59** 1560

Non-point particles: such as in knockout reactions

Elastic scattering of composite nuclei or description of one or two-nucleon removal – at ~100 MeV/nucleon



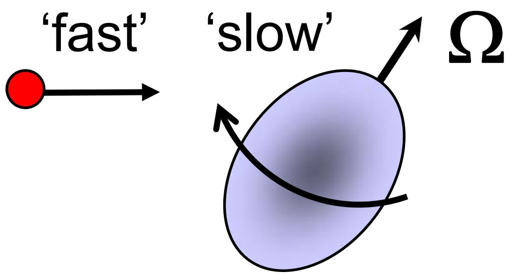
How to describe? - what can we learn from these?

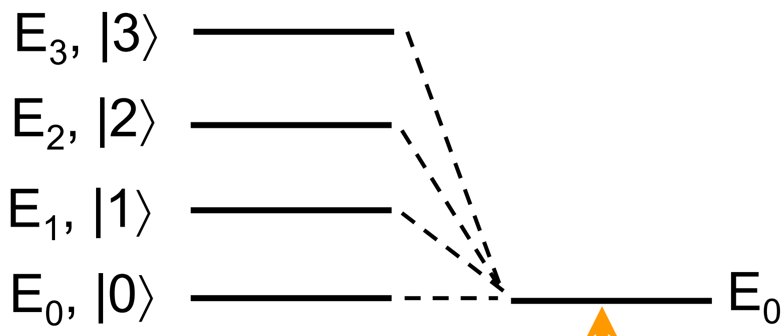
P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Adiabatic (sudden) approximations in physics

Can often identify motion as being of high energy/fast and low energy/slow in different degrees of freedom

Fast neutron scattering from a rotational nucleus



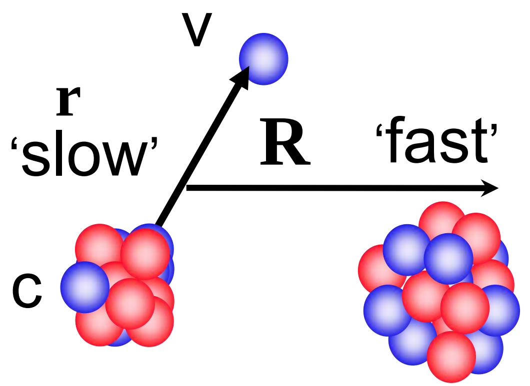


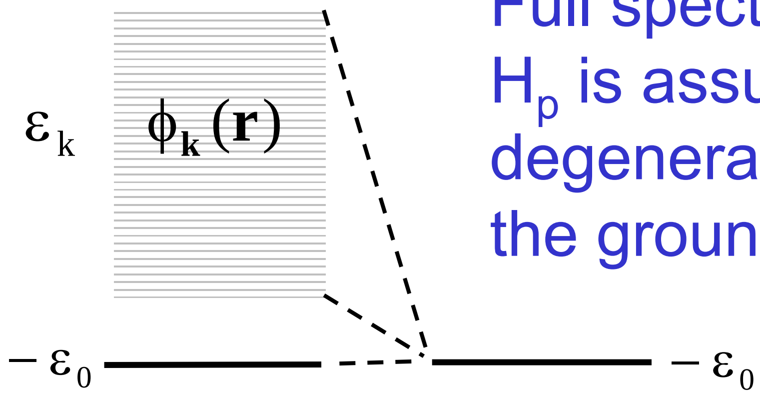
Fix Ω, calculate scattering amplitude $f(\theta, \Omega)$ for each (fixed) orientation Ω of the rotational nucleus

fixing the orientation, we assume moment of inertia $\to \infty$ and so states of the rotational spectrum are assumed degenerate

Transition amplitudes $f_{\alpha\beta}(\theta) = \langle \beta \,|\, f(\theta, \Omega) \,|\, \alpha \rangle_{\Omega}$

Few-body projectiles – the adiabatic model





Full spectrum of H_p is assumed degenerate with the ground state

Freeze internal co-ordinate $\mathbf{r}$, then scatter c+v from target and compute $f(\theta,\mathbf{r})$ for all required fixed values of $\mathbf{r}$

Physical amplitude for e.g. breakup to state $\phi_{\mathbf{k}}(\mathbf{r})$ is then,

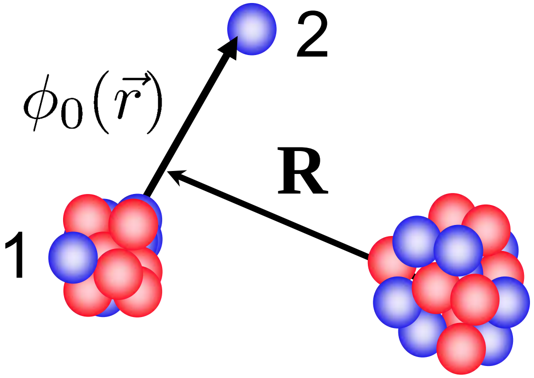
$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_{\mathbf{r}}$$

Achieved trivially by replacing $H_p \rightarrow -\varepsilon_0$ in the Schrödinger equation – the assumed degeneracy

Adiabatic approximation: its implementation

$$\mathcal{H} = T_R + U_1(r_1) + U_2(r_2) + H_{int}(\vec{r})$$

$$\vec{R} = [m_1\vec{r}_1 + m_2\vec{r}_2]/(m_1 + m_2)$$
$$\vec{r}_1 = \vec{R} - m_2\vec{r}/(m_1 + m_2)$$
$$\vec{r}_2 = \vec{R} + m_1\vec{r}/(m_1 + m_2)$$



$$\left(-\frac{\hbar^2}{2\mu}\nabla_R^2 + U_1(r_1) + U_2(r_2) + H_{int} - E\right)\Psi_{\vec{k}}^{+}(\vec{r},\vec{R}) = 0,$$

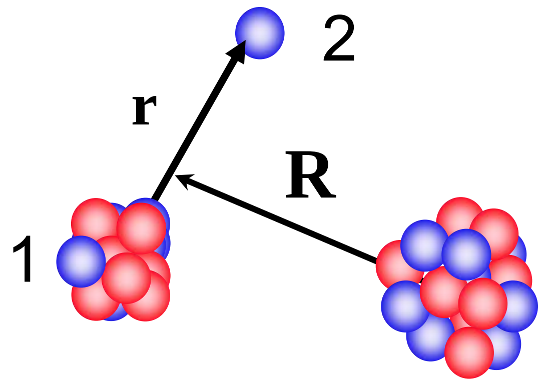
with $\Psi_{\vec{k}}^{+}(\vec{r},\vec{R}) = \exp(i\vec{k}\cdot\vec{R})\,\phi_0(\vec{r}) + \ldots$

The adiabatic approximation replaces $H_{int} \to -\varepsilon_0$

$$\boxed{\left(-\frac{\hbar^2}{2\mu}\nabla_R^2 + U_1(r_1) + U_2(r_2) - E_{cm}\right)\Psi_{\vec{k},Ad}^{+}(\vec{r},\vec{R}) = 0}$$

where $E_{cm} = E + \varepsilon_0$ is the projectile incident energy in the cm frame and <u>r is a parameter</u> in this adiabatic 3-body eqn.

Adiabatic approximation - time perspective



The time-dependent equation is

$$H\Psi(\mathbf{r},\mathbf{R},t) = i\hbar\frac{\partial\Psi}{\partial t}$$

and can be written

$$\Psi(\mathbf{r},\mathbf{R},t) = \Lambda\,\Phi(\mathbf{r}(t),\mathbf{R}), \quad \mathbf{r}(t) = \Lambda^{+}\,\mathbf{r}\,\Lambda$$

$\Lambda = \exp\{-i(H_p + \varepsilon_0)t/\hbar\}$ and where

$$[T_R + U(\mathbf{r}(t),\mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t),\mathbf{R}) = i\hbar\frac{\partial\Phi}{\partial t}$$

Adiabatic equation

$$[T_R + U(\mathbf{r},\mathbf{R})]\Phi(\mathbf{r},\mathbf{R}) = \left(E + \varepsilon_0\right)\Phi(\mathbf{r},\mathbf{R})$$

Adiabatic step <u>assumes</u>

$\mathbf{r}(t) \approx \mathbf{r}(0)=\mathbf{r}=$fixed or $\Lambda=1$ for the collision time t_{coll}

requires

$$(H_p + \varepsilon_0)t_{coll}/\hbar << 1$$

Reaction timescales – in surface grazing collisions

For 100 and 250 MeV/u incident energy:

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$$\gamma = 1.1, \quad v/c = 0.42, \qquad \gamma = 1.25, \quad v/c = 0.6,$$

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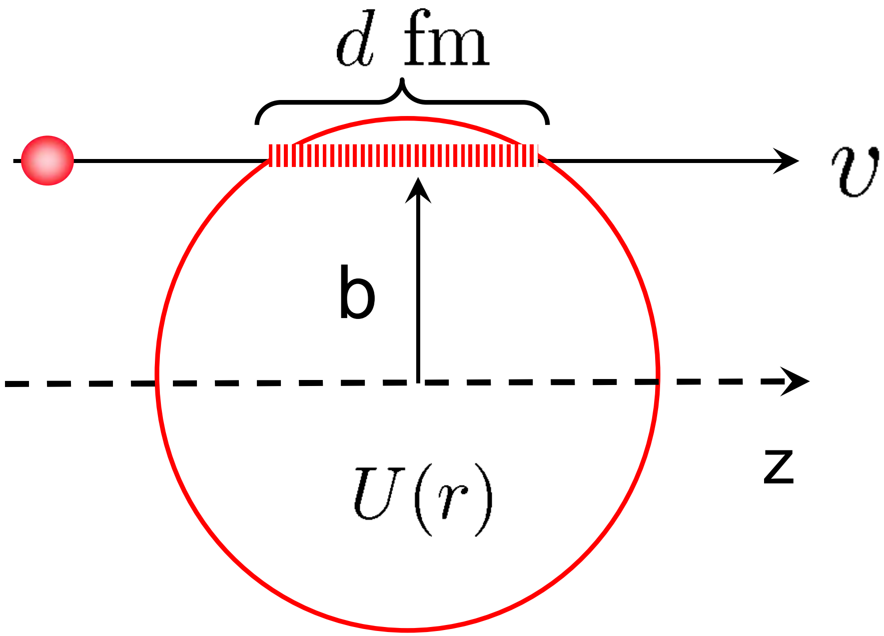


Eikonal approximation: for composite particles (1)

Having made the adiabatic approximation, <u>for each fixed r</u>

$$\left(-\frac{\hbar^2}{2\mu}\nabla_R^2 + U_1(r_1) + U_2(r_2) - E_{cm}\right)\Psi_{\vec{k}}^{+}(\vec{r},\vec{R}) = 0,$$

where $\Psi_{\vec{k}}^{+}(\vec{r},\vec{R}) = \exp(i\vec{k}\cdot\vec{R})\,\phi_0(\vec{r}) + \ldots$

One now assumes, as before, a modulating function,

$$\Psi_{\vec{k}}^{+}(\vec{r},\vec{R}) = \exp(i\vec{k}\cdot\vec{R})\,\omega(\vec{r},\vec{R})\,\phi_0(\vec{r})$$

and, as $\vec{R} \to -\infty\hat{Z}, \quad \omega(\vec{r},\vec{R}) = 1$

As before, with this product form, the equation for ω is

$$\left[2i\vec{k}\cdot\nabla_R\omega - \frac{2\mu}{\hbar^2}[U_1(r_1) + U_2(r_2)]\omega + \cancel{\nabla_R^2\omega}\right]\exp(i\vec{k}\cdot\vec{R}) = 0$$

in which r is just a parameter (a frozen value)

Eikonal approximation: for composite particles (2)

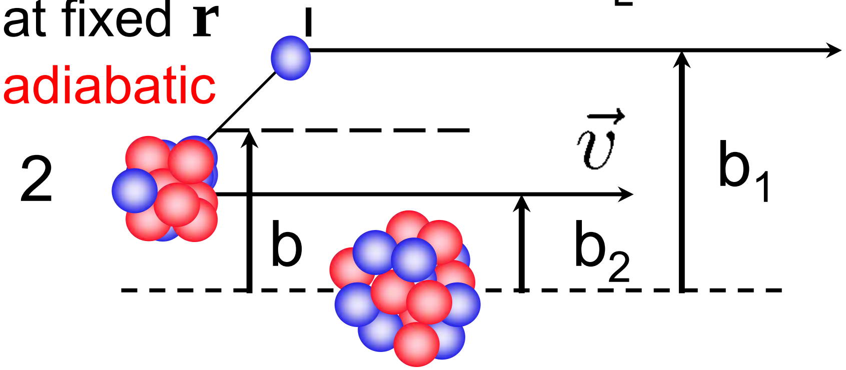
With the z-axis in the beam direction $\hat{k}$

$$\frac{d\omega}{dZ} \approx -\frac{i}{\hbar v}[U_1(r_1) + U_2(r_2)]\omega(\vec{r}, \vec{R})$$ with solution

$$\omega(\vec{r}, \vec{R}) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{Z}[U_1(r_1') + U_2(r_2')]\, dZ'\right]$$

at fixed $\mathbf{r}$

adiabatic



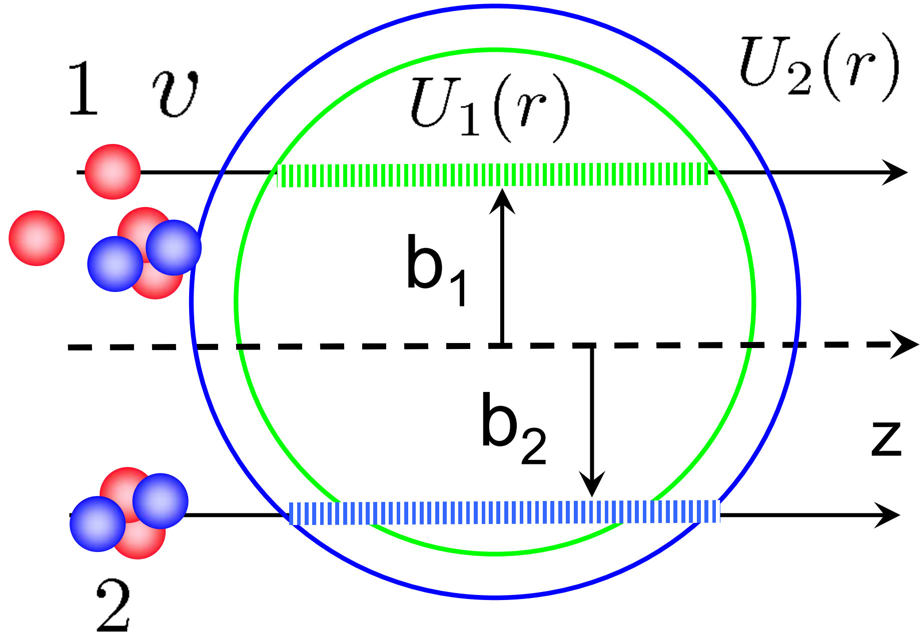
1D integrals over straight line paths through U's at impact parameters b of the 2 bodies

So, after the collision, as Z $\rightarrow \infty$

$$\omega(\vec{r}, \vec{R}) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_1(r_1')\, dZ'\right] \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_2(r_2')]\, dZ'\right]$$

$$\boxed{\omega(\vec{r}, \vec{R}) = S_1(b_1)S_2(b_2)}$$

Eikonal approximation: for composite particles (3)



$$\chi_i(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_i(r')dz$$

Total interaction energy

$$U(r_1,\ldots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp\left[i\chi_i(b_i)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_i(r_i')dz'\right]$$

$$\chi(b_1,\ldots) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}\sum_i U_i(r_i')dz$$

with composite systems: get products of the S-matrices

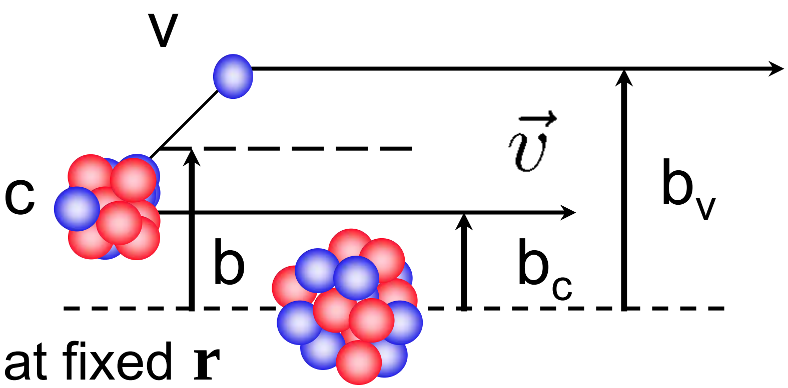
$$\exp[i\chi(b_1,\ldots)] = \prod_i S_i(b_i)$$

Few-body eikonal model amplitudes

So, after the collision, as $Z \to \infty$ $\quad \omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\, S_v(b_v)$

$$\boxed{\Psi_{\mathbf{K}}^{\text{Eik}}(\mathbf{r},\mathbf{R}) \to [S_c(b_c)\, S_v(b_v)]\, e^{i\mathbf{K}\cdot\mathbf{R}}\, \phi_0(\mathbf{r})}$$

with S_c and S_v the eikonal approximations to the S-matrices for independent scattering of c and v by the target - dynamics



at fixed $\mathbf{r}$

adiabatic

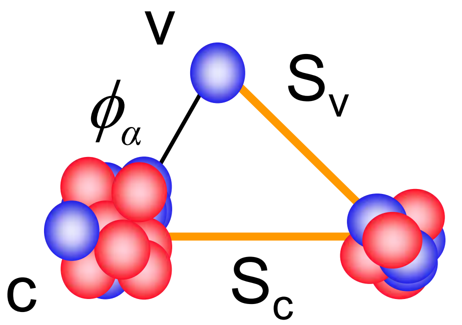
So, elastic S-matrix, $S_p(b)$ for the scattering of the projectile, at an impact parameter b - i.e. the amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\, S_v(b_v)} \,|\, \phi_0 \rangle_{\mathbf{r}}$$

averaged over position probabilities of c and v ← amplitude that c,v survive interaction with fixed b_c and b_v

Eikonal theory - dynamics and structure separation

Independent scattering information of c and v from target



$$S_{\alpha\beta}(b) = \langle \phi_\beta \,|\, \overbrace{S_c(b_c)\, S_v(b_v)}^{\text{scattering}} \,|\, \phi_\alpha \rangle$$

structure

Use the best available few- or many-body wave functions

More generally,

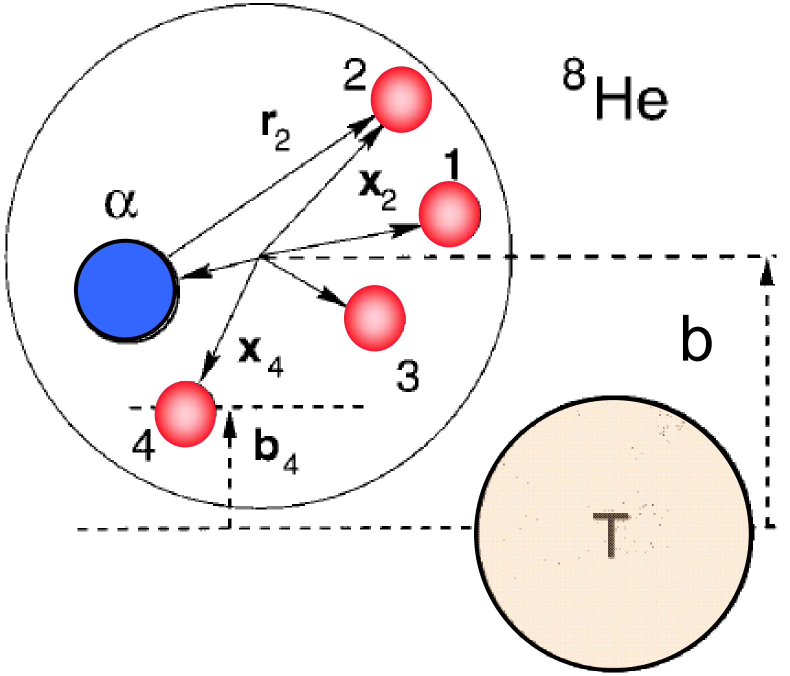
$$S_{\alpha\beta}(b) = \langle \Phi_\beta \,|\, S_1(b_1)\, S_2(b_2) \ldots\ldots\; S_n(b_n) \,|\, \Phi_\alpha \rangle$$

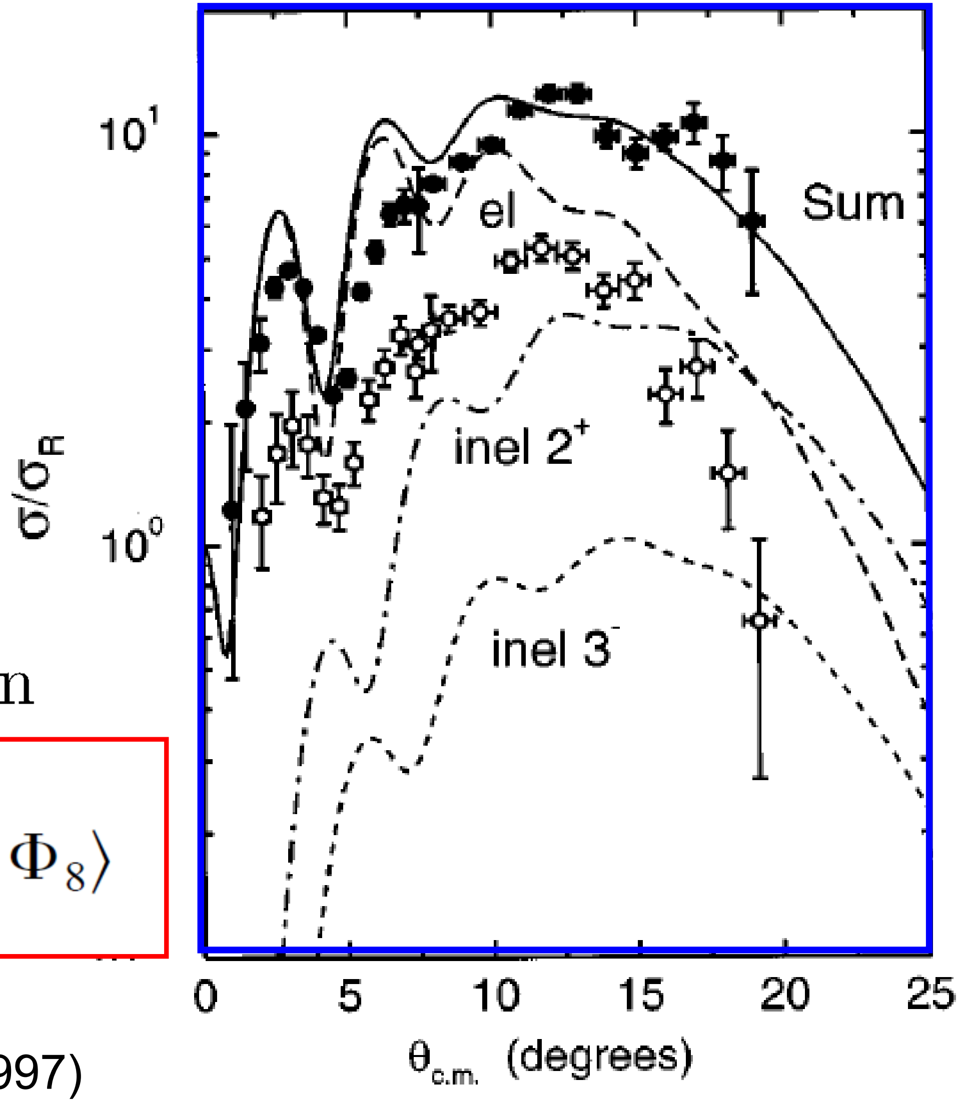
for any choice of 1,2 ,3, ….. n clusters or nucleons.

These S are for a fixed projectile c.m. impact parameter and observables will be integrals over all b

$$\sigma_{\alpha\beta} \propto \int d\vec{b}\, |S_{\alpha\beta}(b)|^2$$

Practical calculations for several-body projectiles





$^8\mathrm{He}+^{12}\mathrm{C}$ at 60 MeV/nucleon

$$S_8(b)=\langle\Phi_8|S_\alpha(b_\alpha)\prod_{i=1}^{4} S_i(b_i)|\Phi_8\rangle$$

J.A. Tostevin et al. PRC **56,** R2929 (1997)

Use an old friend – and make some new ones

`bound` (bound states solver – see **`bound.outline`**)

`eikonal_s` (for eikonal S-matrix from a specified interaction potential - **`eikonal_s.outline`**)

`glauber` (elastic scattering calculation from a specified eikonal S-matrix - **`glauber.outline`**)

`knockout` (composite two-body projectile S-matrix from a bound state wave function and component S-matrices – **`knockout.outline`**)

You can now calculate bound states (**`bound`**) and eikonal S-matrices (**`eikonal_s`**) and can calculate this composite S-matrix (using **`knockout`**). The elastic scattering of c, v or the composite can then be calculated (using **`glauber`**). So you can now calculate the elastic scattering of the neutron, ^{10}Be, and the composite halo system ^{11}Be

This first session discussed:

Approximate solutions of the Schrodinger equation for states of **two, three** or more bodies at ‘high energies’ using the eikonal (scattering is forward-dominated) and adiabatic approximations for the reaction dynamics.

Highlighted the importance of the eikonal S-matrices, functions of the impact parameters of the projectile, or components of the projectile, in this formulation of the reaction and scattering of the interacting systems.

Discussed examples to assess the accuracy of the eikonal approximation with the projectile energy, and clarified how both point particle and composite projectile scattering can be calculated using these methods.


COFFEE