TALENT Course 6: Theory for exploring nuclear reaction experiments

Eikonal methods – High-energy approximations – knockout reactions

GANIL, 1st - 19th July 2013

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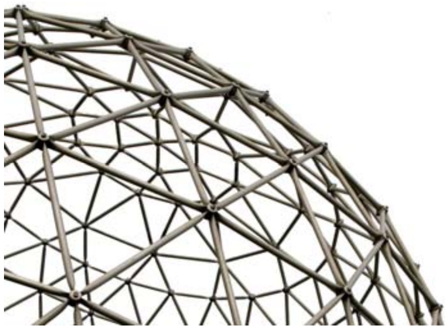


Session aims: part 2

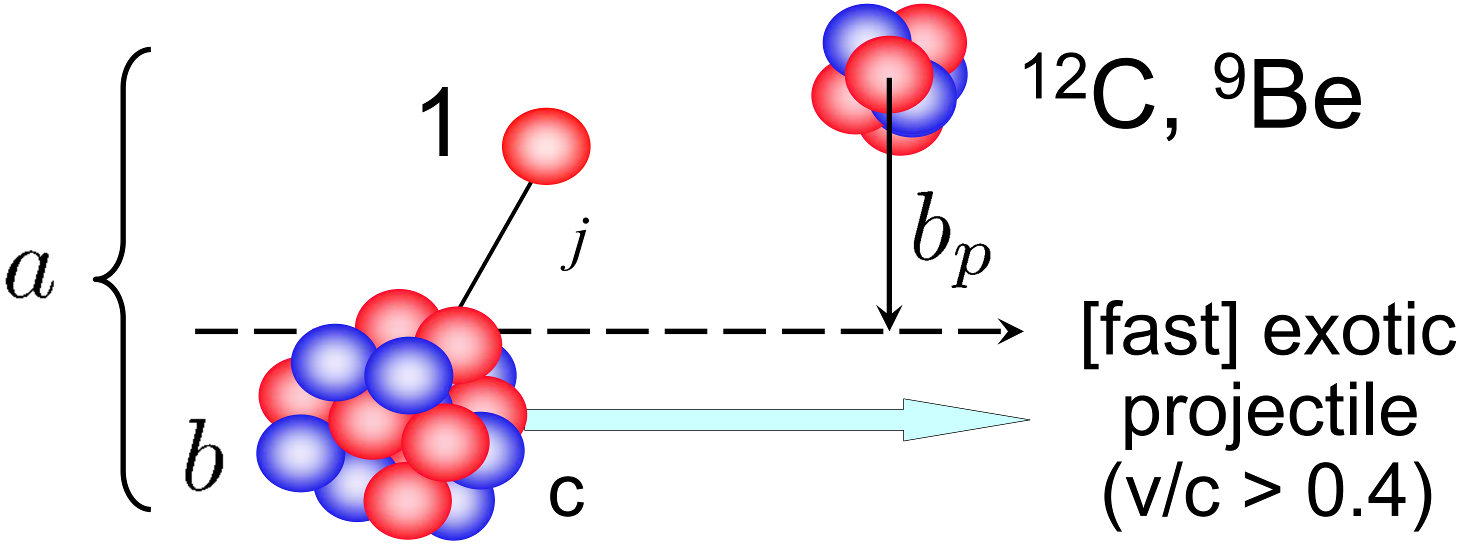
To introduce nucleon knockout reactions and show how these reactions can be calculated from an assumed bound state wave function (i.e. overlap) for the removed nucleon and the eikonal S-matrices of the part 1 lecture.

To discuss the momentum distributions of the reaction residues (the projectile less one nucleon) and to be able to calculate these for different assumed bound states of the removed nucleon and reasonable choices of the S(b).

To calculate realistic S-matrices for the core-target and nucleon-target systems and to compare predictions with neutron knockout data for $^{11}Be \rightarrow {}^{10}Be$(gs) and to different final states in the case of the $^{15}C \rightarrow {}^{14}C(J^{\pi})$ reaction.

Orientation – neutron removal – or knockout

A nuclear spectroscopy probe is one-nucleon removal – at energies ~100 MeV/nucleon and greater



Experiments do not measure target final states. Final state of core b can be measured – using decay gamma rays.

How to describe? what can we learn from these?

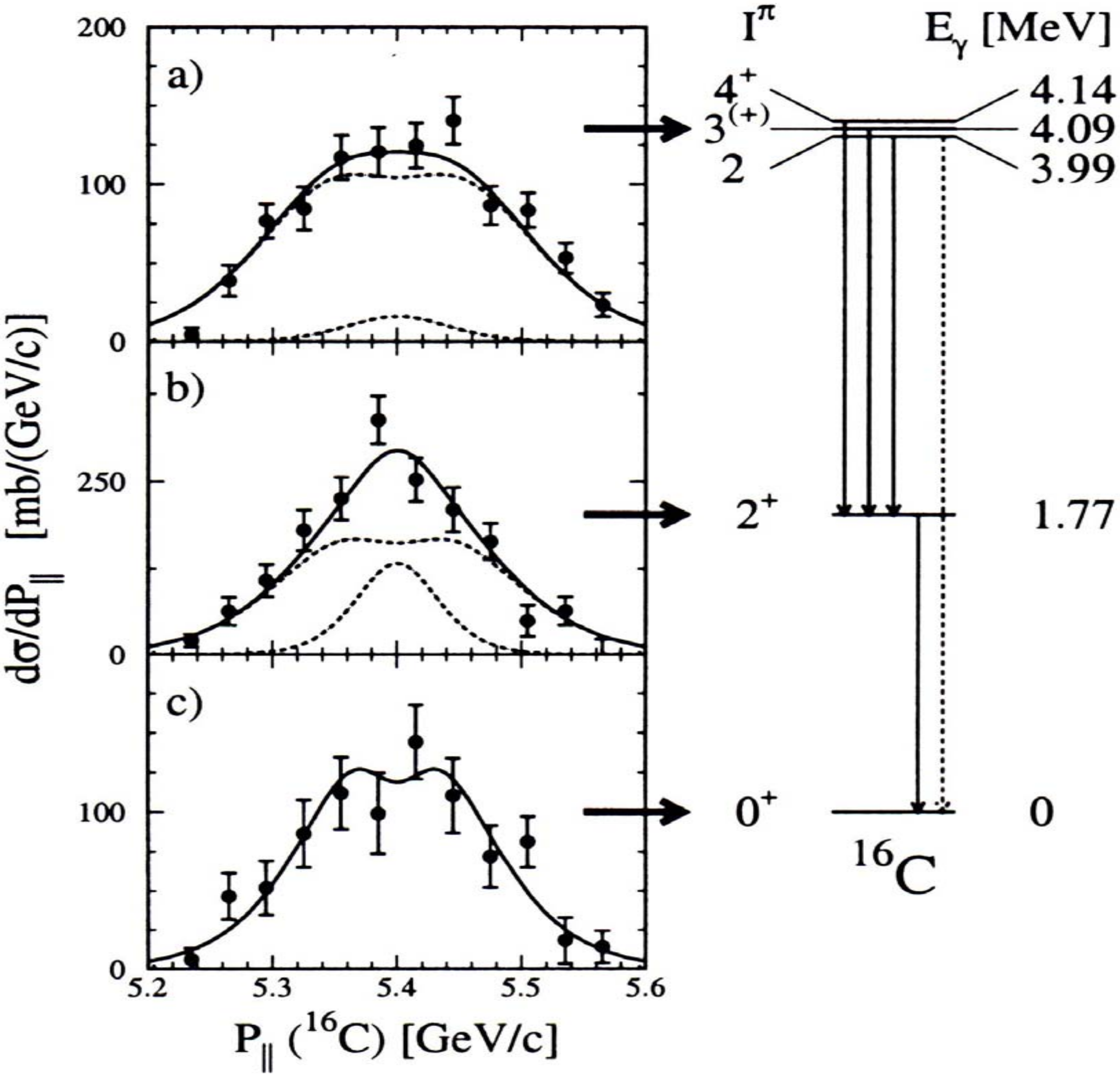
P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Single-neutron knockout from ^{17}C

ℓ=0,2 admixture

ℓ=0,2 admixture

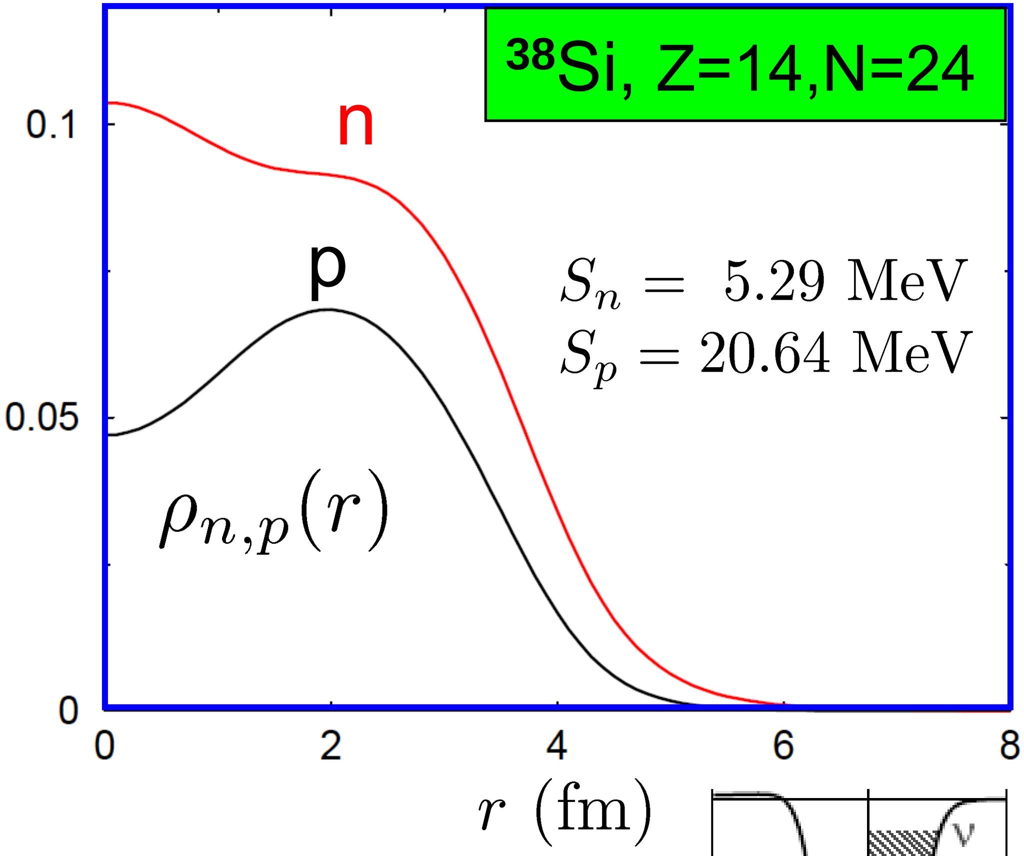
pure ℓ=2



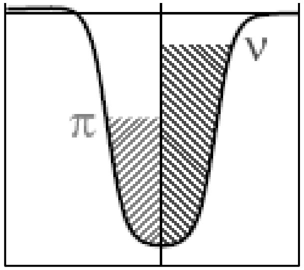
V. Maddalena et al. Phys. Rev. C **63** (2001) 024613

Asymmetric nuclei – two displaced Fermi surfaces

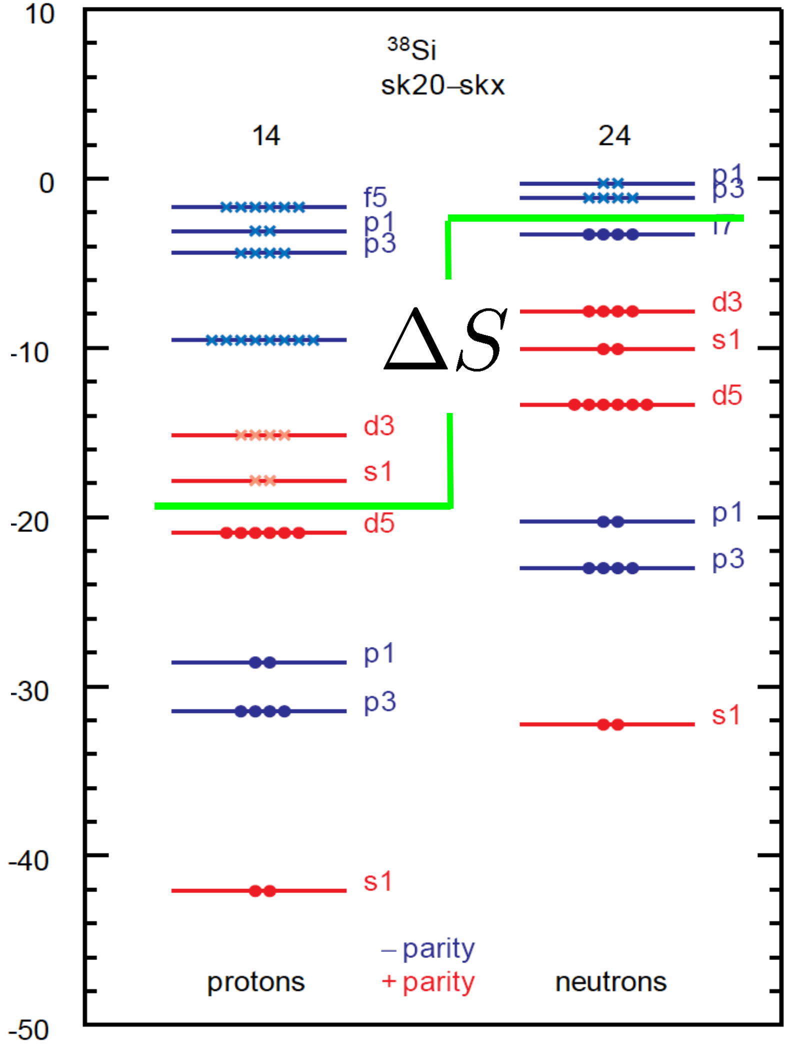
Spatial domain



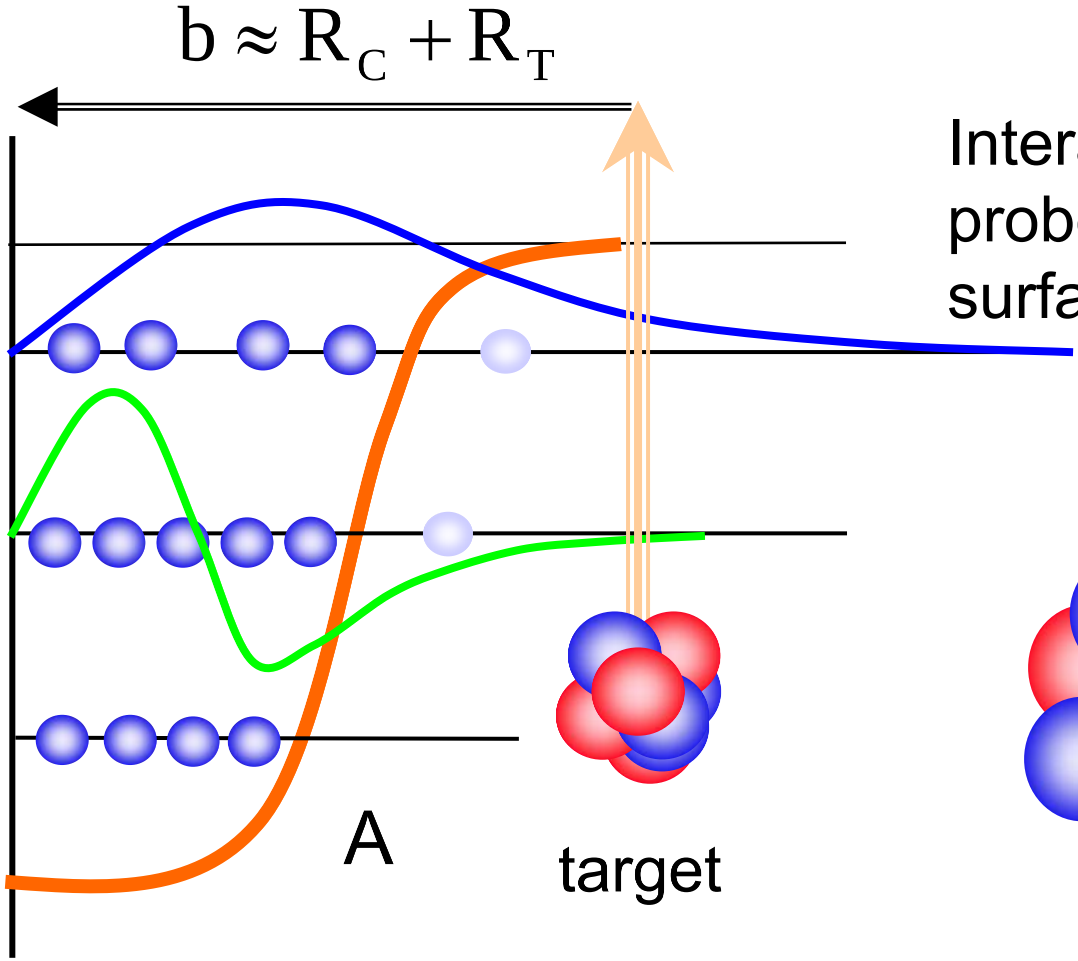
Spherical Hartree Fock density (SkX)



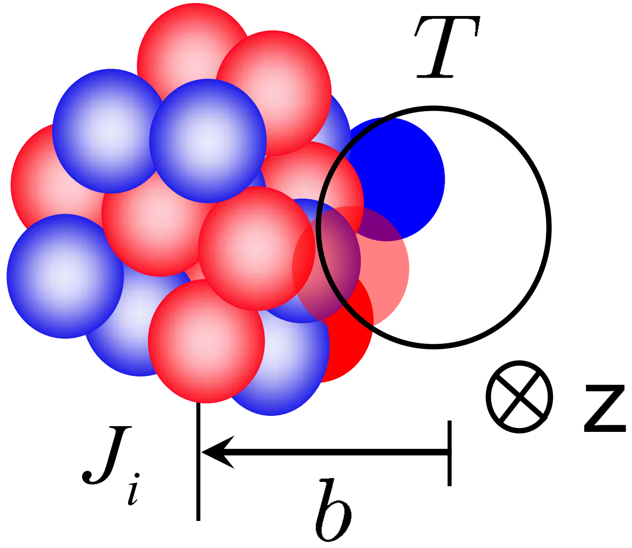
Energy domain



Viewed from the rest frame of the projectile

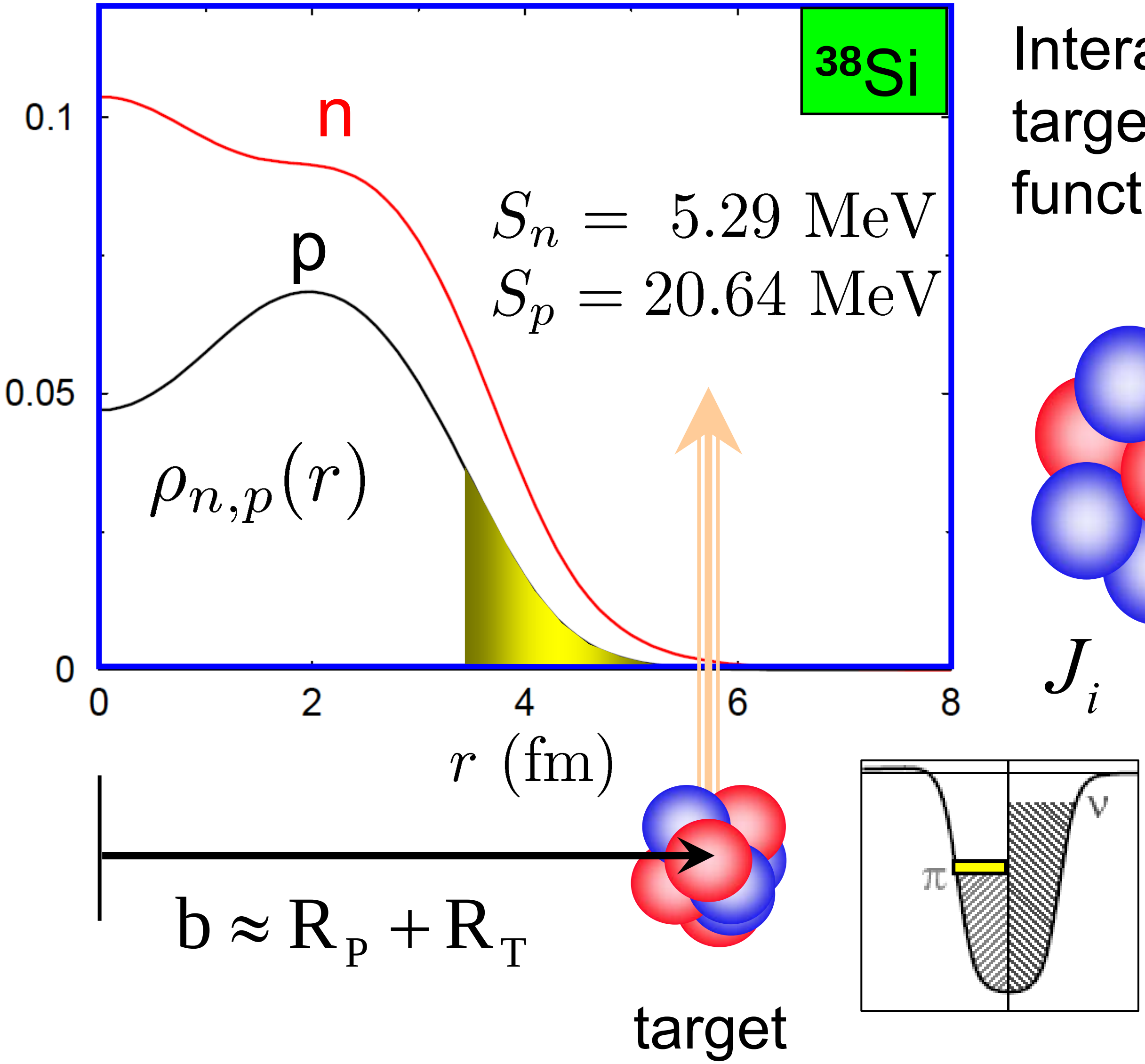


Interaction with the target probes wave functions at surface and beyond

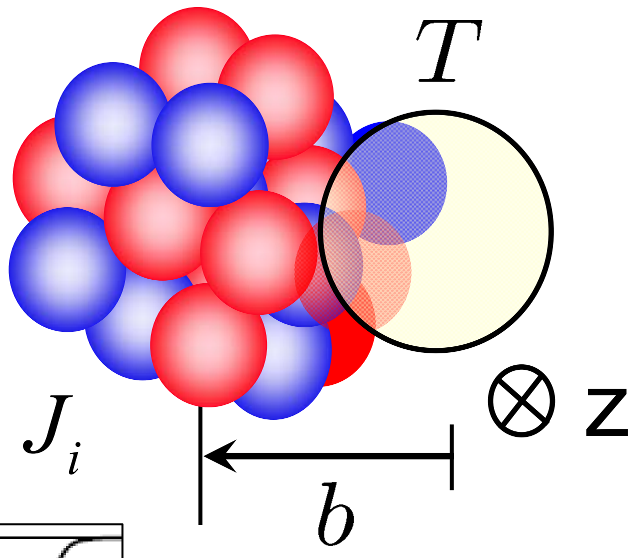


Mass A-1 residue will be left in the ground state or an excited state

Removal probes single-nucleon wave functions

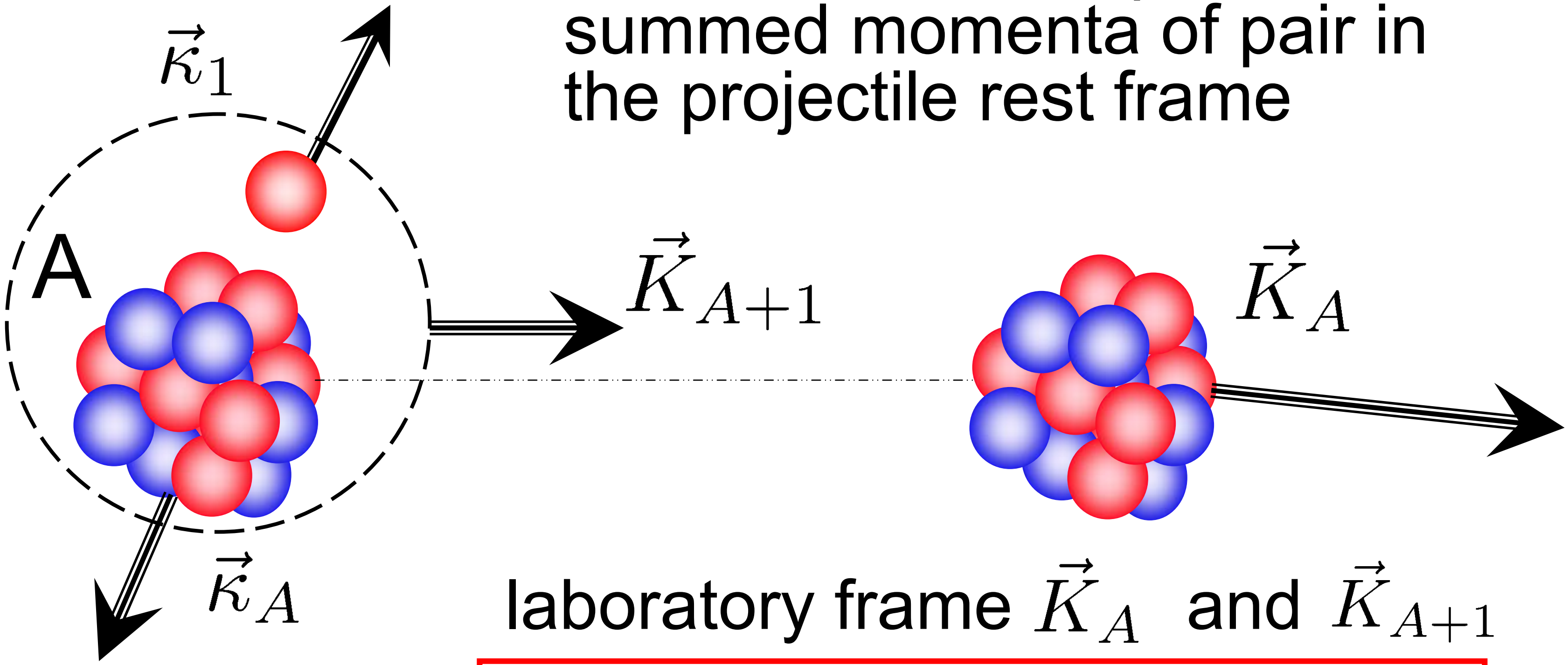


Interaction with the target probes wave functions at surface



Sudden 1N removal from the mass A projectile

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame



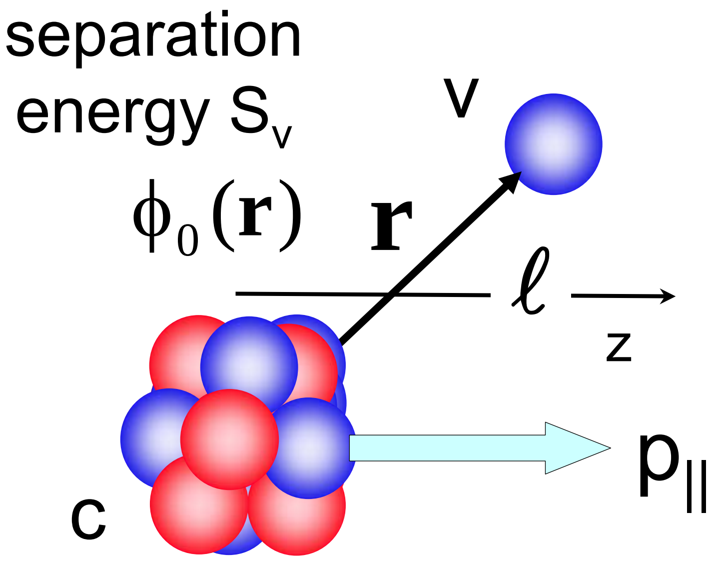
Projectile rest frame

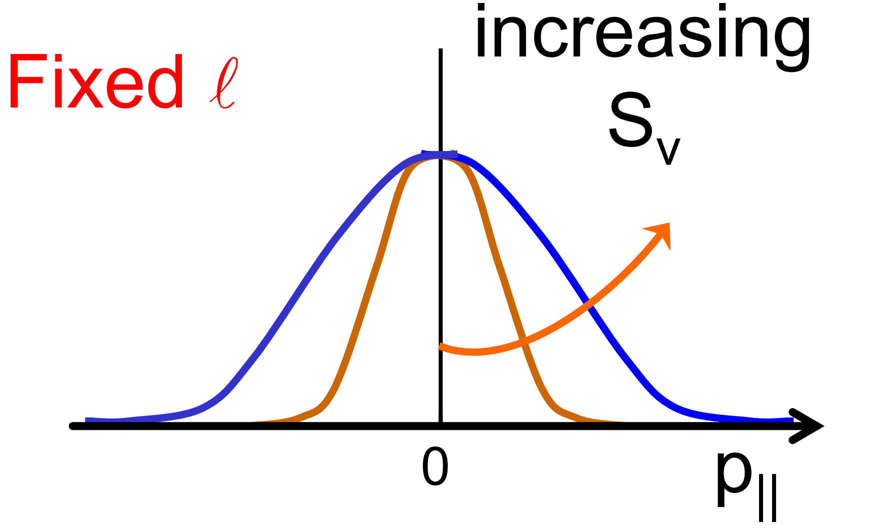
laboratory frame $\vec{K}_A$ and $\vec{K}_{A+1}$

$$\vec{K}_A = \frac{A}{A+1}\vec{K}_{A+1} - \vec{\kappa}_1$$

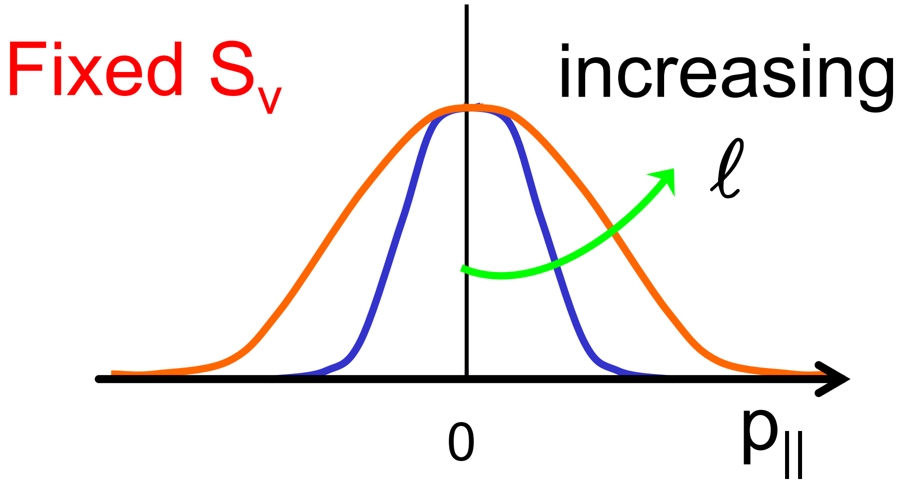
and component equations

Measurement of the residue's momentum





consider momentum components $p_{||}$ of the core parallel to the beam direction, in the projectile rest frame



$$\Delta p \Delta z > \hbar/2$$

Absorptive cross sections - target excitation

Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels

$$|S(b)|^2 \leq 1$$

$$\sigma_{abs} = \sigma_R - \sigma_{diff} = \int d\mathbf{b}\ \langle \phi_0 \,|\, 1 - |S_c S_v|^2 \,|\, \phi_0 \rangle$$

$$\begin{cases} |S_v|^2 (1 - |S_c|^2) + & \text{v survives, c absorbed} \\ |S_c|^2 (1 - |S_v|^2) + & \text{v absorbed, c survives} \\ (1 - |S_c|^2)(1 - |S_v|^2) & \text{v absorbed, c absorbed} \end{cases}$$

v survives, c absorbed

v absorbed, c survives

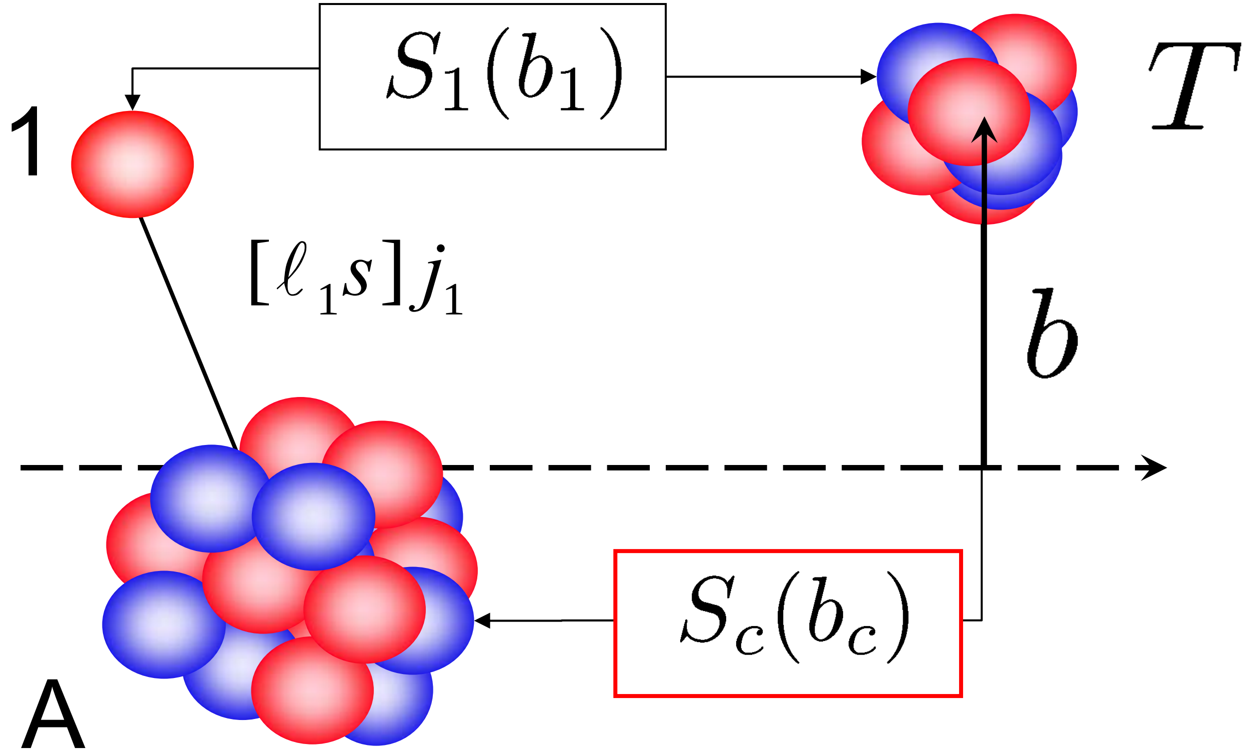
v absorbed, c absorbed

stripping of v from projectile exciting the target. c scatters elastically from the target

$$\sigma_{strip} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1 - |S_v|^2) \,|\, \phi_0 \rangle$$

Related equations exist for the differential cross sections, etc.

Stripping of a nucleon – nucleon ‘absorbed’



$$\sigma_{\rm strip} = \int d\mathbf{b} \ \langle \phi_0 \,||\, S_C \,|^2 \,(1 - |\, S_1 \,|^2)\,|\, \phi_0 \rangle$$

Diffractive dissociation of composite systems

The total cross section for removal of the valence particle from the projectile due to the break-up (also called diffractive dissociation) mechanism is the break-up amplitude, summed over all final continuum states, i.e.

$$\sigma_{\rm diff} = \int d\mathbf{k} \int d\mathbf{b} \, |\langle \phi_{\mathbf{k}} \,|\, S_c(b_c)\, S_v(b_v) \,|\, \phi_0 \rangle|^2$$

but, using completeness of the break-up states

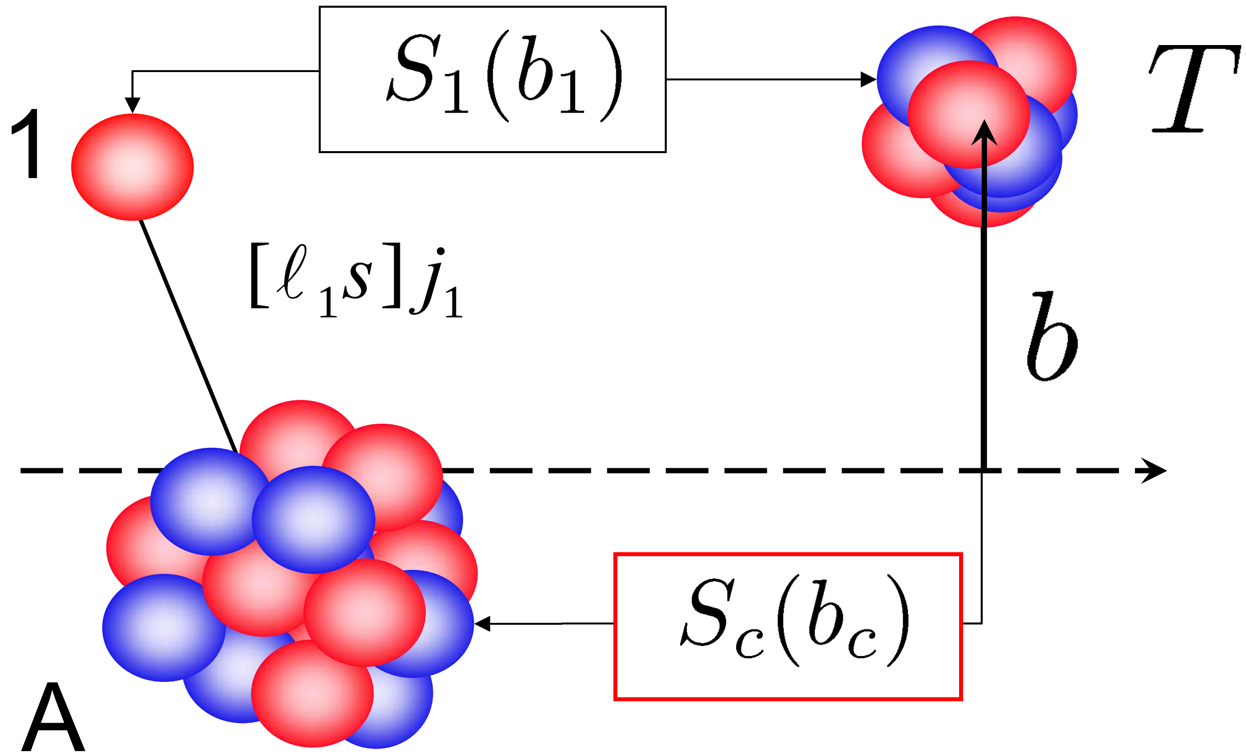
$$\int d\mathbf{k} \, |\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}| = 1 - |\phi_0\rangle\langle\phi_0| - |\phi_1\rangle\langle\phi_1| \; ......$$

If > 1 bound state

can (for a weakly bound system with a single bound state) be expressed in terms of only the projectile ground state wave function as:

$$\sigma_{\rm diff} = \int d\mathbf{b} \left\{ \langle\phi_0 \,||\, S_c\, S_v \,|^2 \,|\, \phi_0\rangle - |\langle\phi_0 \,|\, S_c\, S_v \,|\, \phi_0\rangle|^2 \right\}$$

Diffractive (breakup) removal of a nucleon



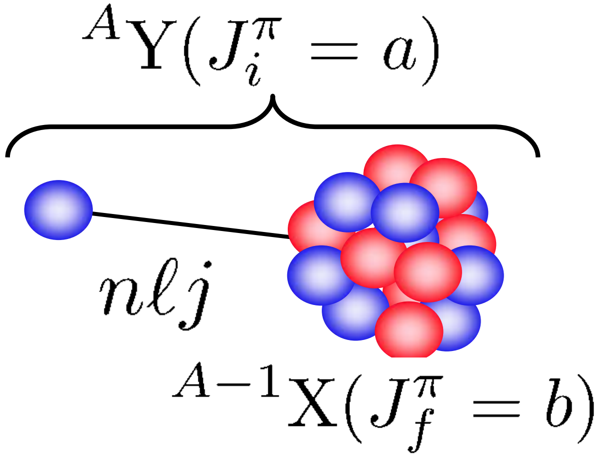
$$\sigma_{\mathrm{diff}} = \int d\mathbf{b} \left\{ \langle \phi_0 \,||\, S_c\, S_v \,|^2 \,|\, \phi_0 \rangle - |\langle \phi_0 \,|\, S_c\, S_v \,|\, \phi_0 \rangle|^2 \right\}$$

Bound states – Overlaps, spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.

$$\phi^{m}_{n\ell j}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma|jm)\frac{u_{n\ell j}(r)}{r} Y^{\lambda}_{\ell}(\hat{r})\chi^{\sigma}_{s}$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$



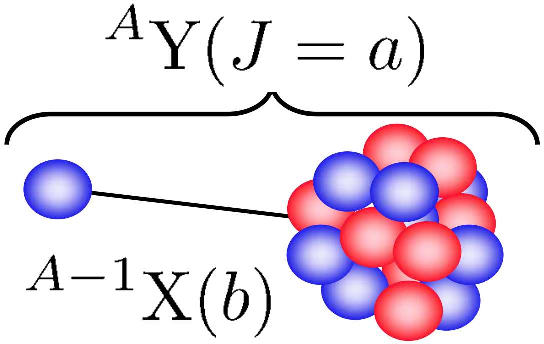
The potential model wave function approximates the overlap function of the A and A–1-body wave functions (Y and X in the case of an single nucleon) i.e. the overlap

$$(\Phi^{b\beta}_{X}, A-1|\Phi^{a\alpha}_{Y}, A\rangle\rangle \longrightarrow F^{a\alpha b\beta}_{YX}(\vec{r})$$

Need to introduce spectroscopic factors, that relate these normalised single-particle wave functions and the overlaps to take account realistically of nuclear structure effects

Overlap functions and spectroscopic factors (1)

Written equations as if projectile is a two-body bound state, but of course the nucleon (say a neutron) is removed from a many-body wave function as discussed in the overlaps lecture(s).



Assumption: the removal reactions perturbs the motion of just a single nucleon – but not the degrees of freedom of A-1.

and e.g. $\mathcal{O}(1) = (1 - |S_n(1)|^2)$

$$(\Phi_X^{b\beta}|\mathcal{O}(1)|\Phi_Y^{a\alpha}\rangle\rangle = \mathcal{O}(1)(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = \mathcal{O}(1)F_{YX}^{a\alpha b\beta}(1)$$

Starting point for the stripping term is therefore

$$\sigma_{str} = \frac{N}{2a+1}\sum_{\alpha}\int d\vec{b}\,\langle\langle\Phi_Y^{a\alpha}|(1-|S_n(1)|^2)|S_X|^2|\Phi_Y^{a\alpha}\rangle\rangle$$

$|S_X|^2 = |S_X(2,\ldots A)|^2$ determines the (survival probability) for the A –1 nucleons comprising X

Overlap functions and spectroscopic factors (2)

Manipulating $$\mathcal{M} = \langle\langle \Phi_Y^{a\alpha} | (1 - |S_n(1)|^2) |S_X|^2 | \Phi_Y^{a\alpha} \rangle\rangle$$

$$\mathcal{M} = \langle\langle \Phi_Y^{a\alpha} | (1 - |S_n(1)|^2) \, S_X \, S_X^* \, | \Phi_Y^{a\alpha} \rangle\rangle$$

on inserting complete sets of states of nucleus X at the points shown - and assuming in addition that S_X does not couple/excite the states of X

$$1 = \sum_{b\beta} |\Phi_X^{b\beta})(\Phi_X^{b\beta}|$$

$$(\Phi_X^{b\beta} | S_X | \Phi_X^{b'\beta'}) = S_b(b_X) \delta_{bb'} \delta_{\beta\beta'}$$

$$\mathcal{M} = \sum_{b\beta} \langle\langle \underline{\Phi_Y^{a\alpha} | \Phi_X^{b\beta}}) | (1 - |S_n(1)|^2) |S_b(b_X)|^2 | (\underline{\Phi_X^{b\beta} | \Phi_Y^{a\alpha}} \rangle\rangle$$

where we recognise the overlap functions and the eikonal S-matrix for the scattering of nucleus X in state b from the target

$$S_b(b_X) = (\Phi_X^{b\beta} | S_X | \Phi_X^{b\beta})$$

Overlap functions and spectroscopic factors (3)

$$\mathcal{M} = \sum_{b\beta} \langle\!\langle \underline{\Phi_Y^{a\alpha} | \Phi_X^{b\beta}}) | (1 - |S_n(1)|^2) | S_b(b_X)|^2 | (\underline{\Phi_X^{b\beta} | \Phi_Y^{a\alpha}} \rangle\!\rangle$$

Using our earlier definition for the spectroscopic factor, that

$$(\underline{\Phi_X^{b\beta} | \Phi_Y^{a\alpha}} \rangle\!\rangle = \sum_{jm} (b\beta jm | a\alpha) \sqrt{\frac{\mathcal{S}(\ell j : a \to b)}{N}} \phi_{n\ell j}^{m}(1)$$

and that $$\sigma_{str} = \frac{N}{2a+1} \sum_{\alpha} \int d\vec{b}\, \mathcal{M}(b)$$

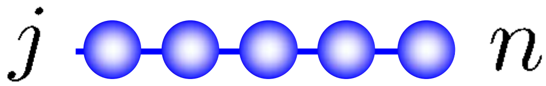
$$\sigma_{str} = \sum_{bj} \mathcal{S}(\ell j ; a \to b)\, \sigma_{str}^{sp}(bj)$$

$$\sigma_{str}^{sp}(bj) = \frac{1}{2j+1} \sum_{m} \int d\vec{b}\, \langle \phi_{n\ell j}^{m} \, | (1 - |S_n|^2) | S_b|^2 | \, \phi_{n\ell j}^{m} \rangle$$

Overlap functions – IPM – as previously

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{n} \langle\, [\Psi^{b}_{X} \otimes \phi_{\ell j}(1)]^{\alpha}_{a} \mid \Psi^{a\alpha}_{Y}\, \rangle$$

Other (independent particle model) cases for removal from a state of a given j are less simple, can be worked out, but are given by the <u>coefficients of fractional parentage</u> - cfps



$$\langle\, [\Psi^{b}_{n-1} \otimes \phi_{\ell j}(1)]^{\alpha}_{a} \mid \Psi^{a\alpha}_{n}\, \rangle = ((j^{n-1})b, j; a|\}(j^{n})\, a)$$

$$\boxed{\mathcal{S}(\ell j : a \to b) = n((j^{n-1})b, j; a|\}(j^{n})\, a)^{2}}$$

For low seniority states (where each pair couples to spin

$$((j^{n-1})j, j; 0|\}(j^{n})\, 0) = 1, \quad n = \text{even}, \;\; \text{seniority} = 0$$

$$((j^{n-1})0, j; j|\}(j^{n})\, j) = \left(\frac{2j + 1 - (n-1)}{n(2j+1)} \right)^{\frac{1}{2}},$$

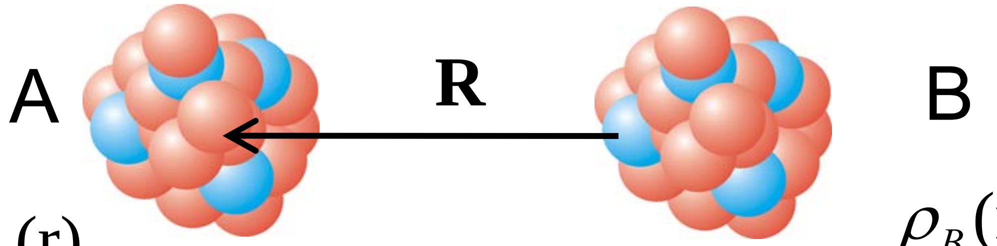
$$n = \text{odd}, \;\; \text{seniority} = 1$$

Core-target effective interactions – for $S_c(b_c)$

Double folding

$$U_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, t_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

U_{AB}



$\rho_A(r)$ $\rho_B(r)$

At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2}\sigma_{NN}(i + \alpha_{NN})\right] f(r), \quad \int d\vec{r} f(r) = 1$$

nucleon-nucleon cross section

e.g. $f(r) = \delta(r)$

$$f(r) = (\sqrt{\pi}t)^{-3}\exp(-r^2/t^2)$$

resulting in a COMPLEX nucleus-nucleus potential

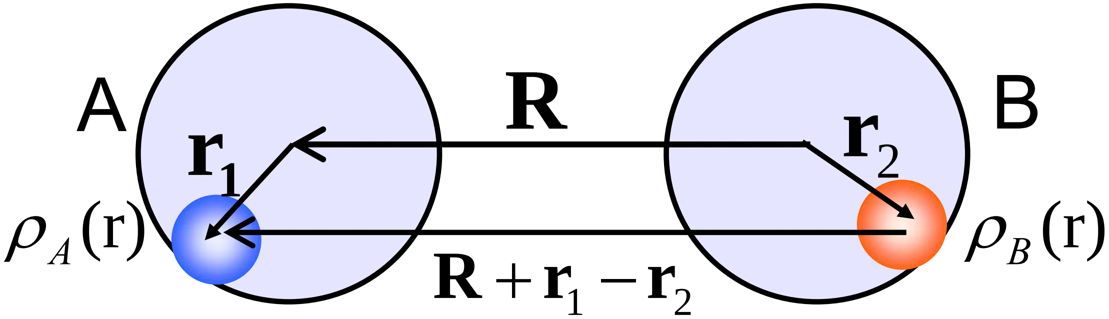
M.E. Brandan and G.R. Satchler, Phys. Rep. **285** (1997) 143-243.

Effective interactions – Folding models

Double folding

$$U_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$

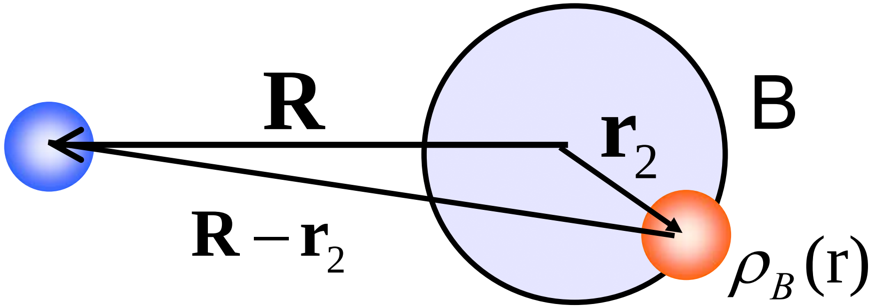
U_{AB}



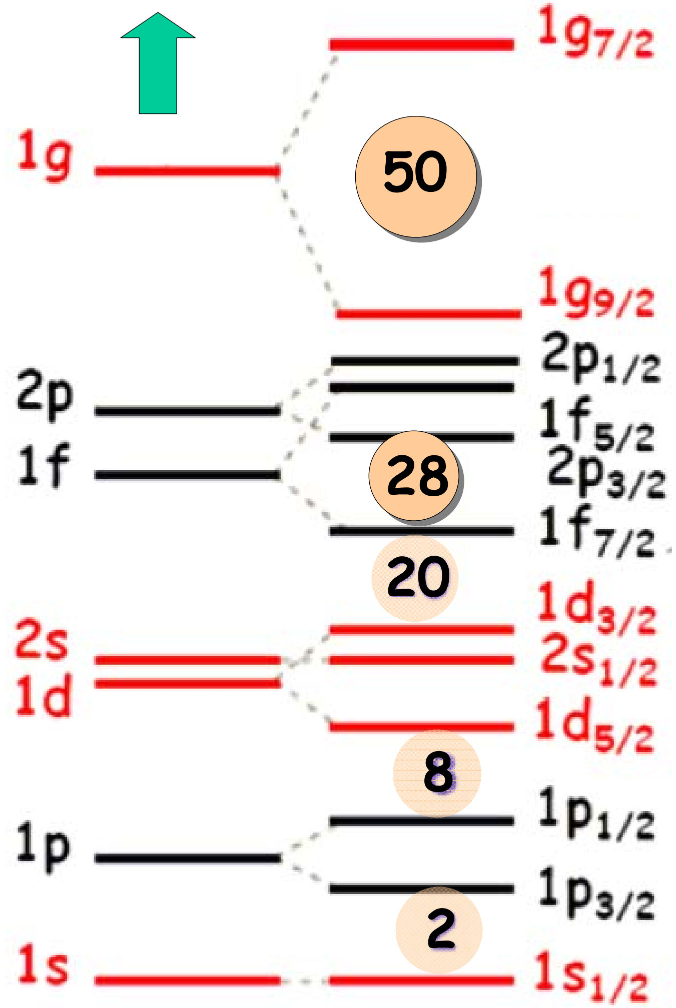
Single folding

$$U_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} - \mathbf{r}_2)$$

U_B



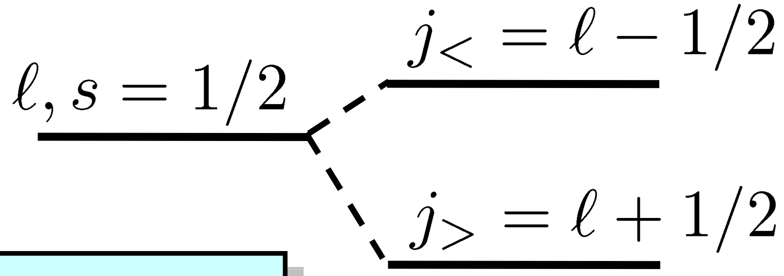
What is involved in realistic reaction calculations?

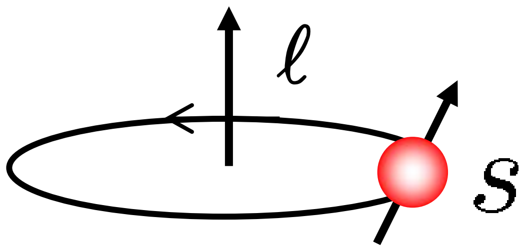


^{23}O(-1n)

Z=8

N=15



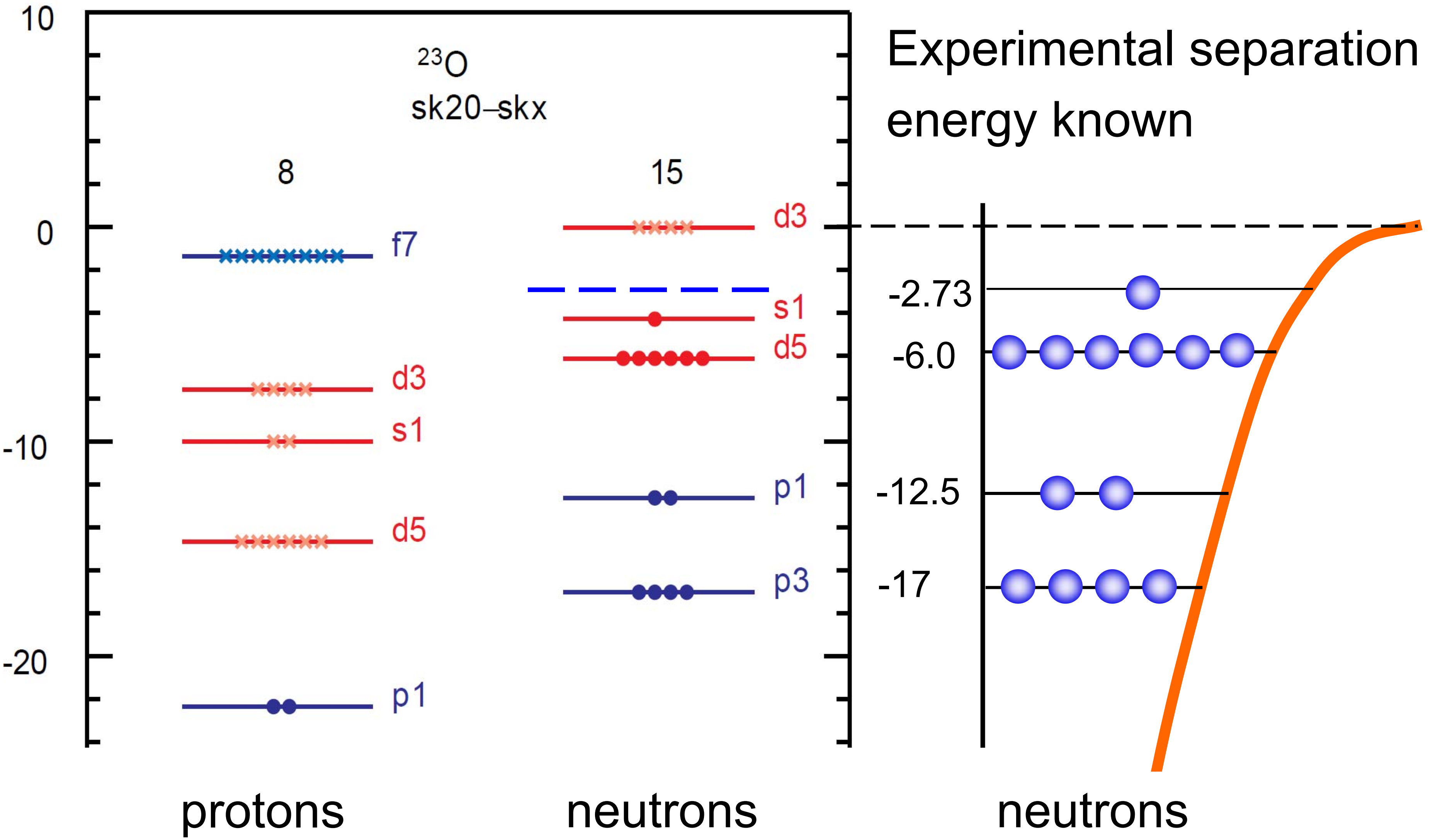


$$V_{\ell s}(r)\vec{\ell} \cdot \vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell} \cdot \vec{s}$$

$$V_{so}(r) < 0$$

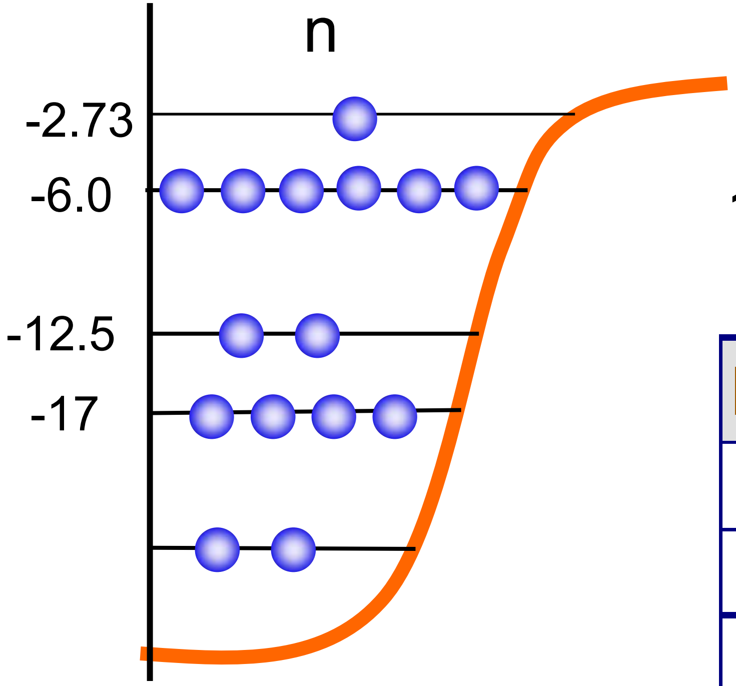
Example: What is involved – take neutron from ^{23}O



Hartree-Fock mean field calculation

Independent particle – neutron removal reaction

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



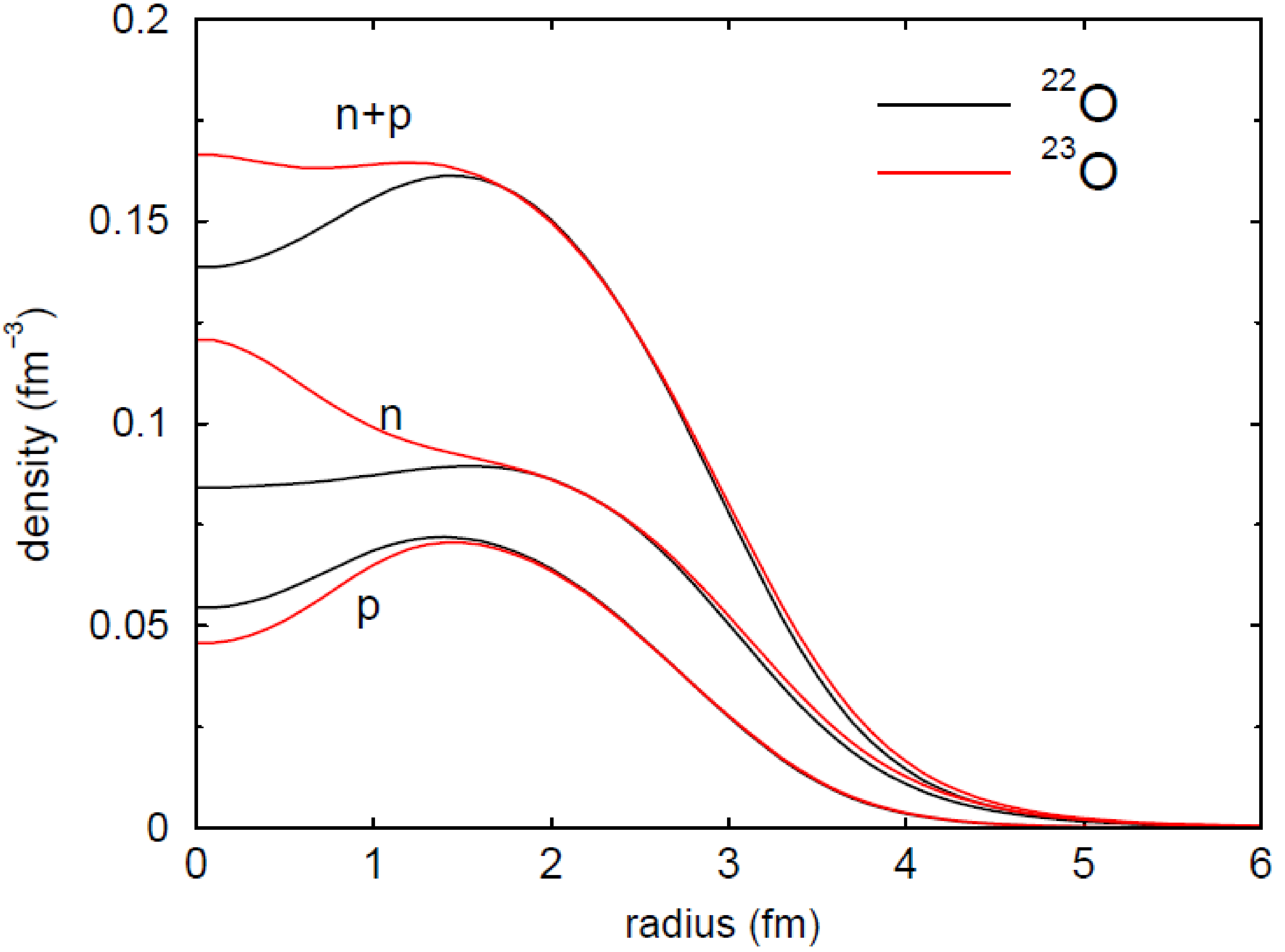
$2s_{1/2}$ S_n=2.73 MeV
$1d_{5/2}$ S_n=6.0 MeV
} SF are?

$J_f^\pi = 0^+$, g.s.
S_n=2.73 MeV

$J_f^\pi = 2^+$, 3.2 MeV
S_n=6.0 MeV

Nucleus	E_{level}(keV)	Jπ
22O	0	0+
22O	3199 *8*	(2+)
22O	4582 *9*	(3+)
22O	4909 *90*	(0+)
22O	5800	(1-,0-)
22O	6509 *90*	(2+)
22O	6936 *10*	(4+)

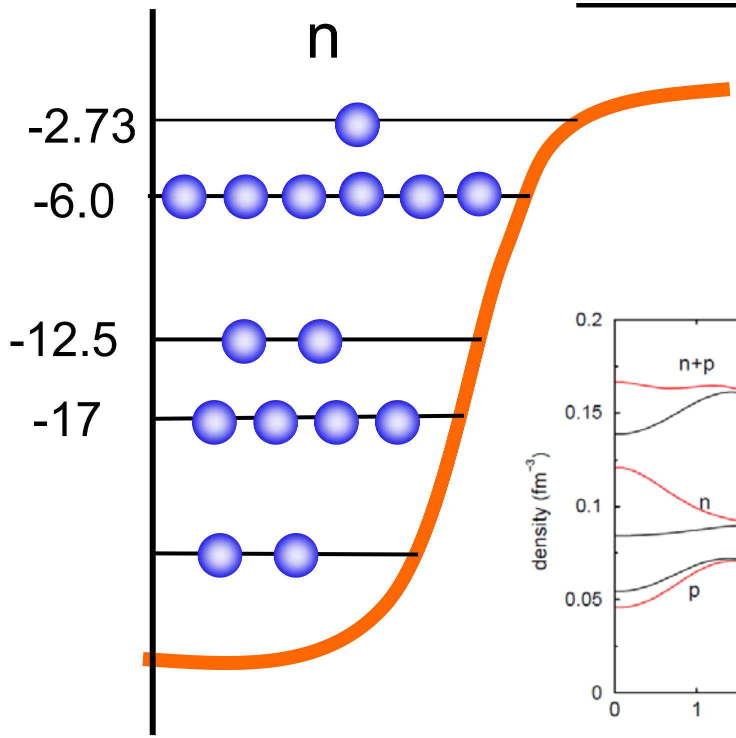
Neutron: proton: nucleon radial densities (HF)



Orientation – neutron removal – cross sections

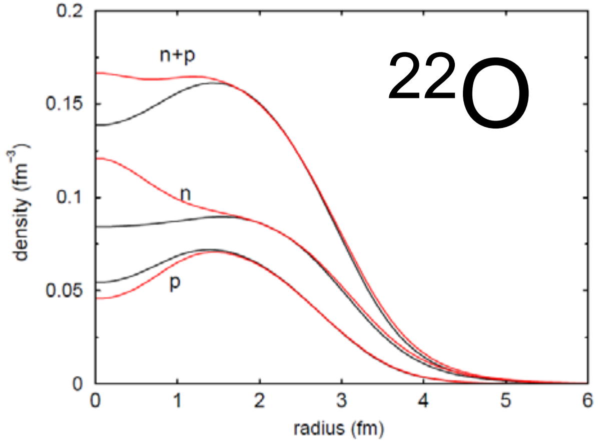
Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$

at <u>72 MeV/nucleon</u> on a ^{12}C target

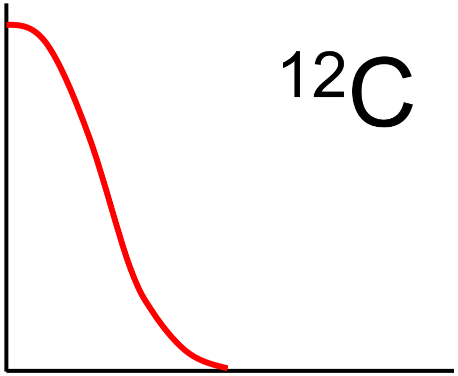


$2s_{1/2}$ 2.73 MeV

$1d_{5/2}$ 6.0 MeV

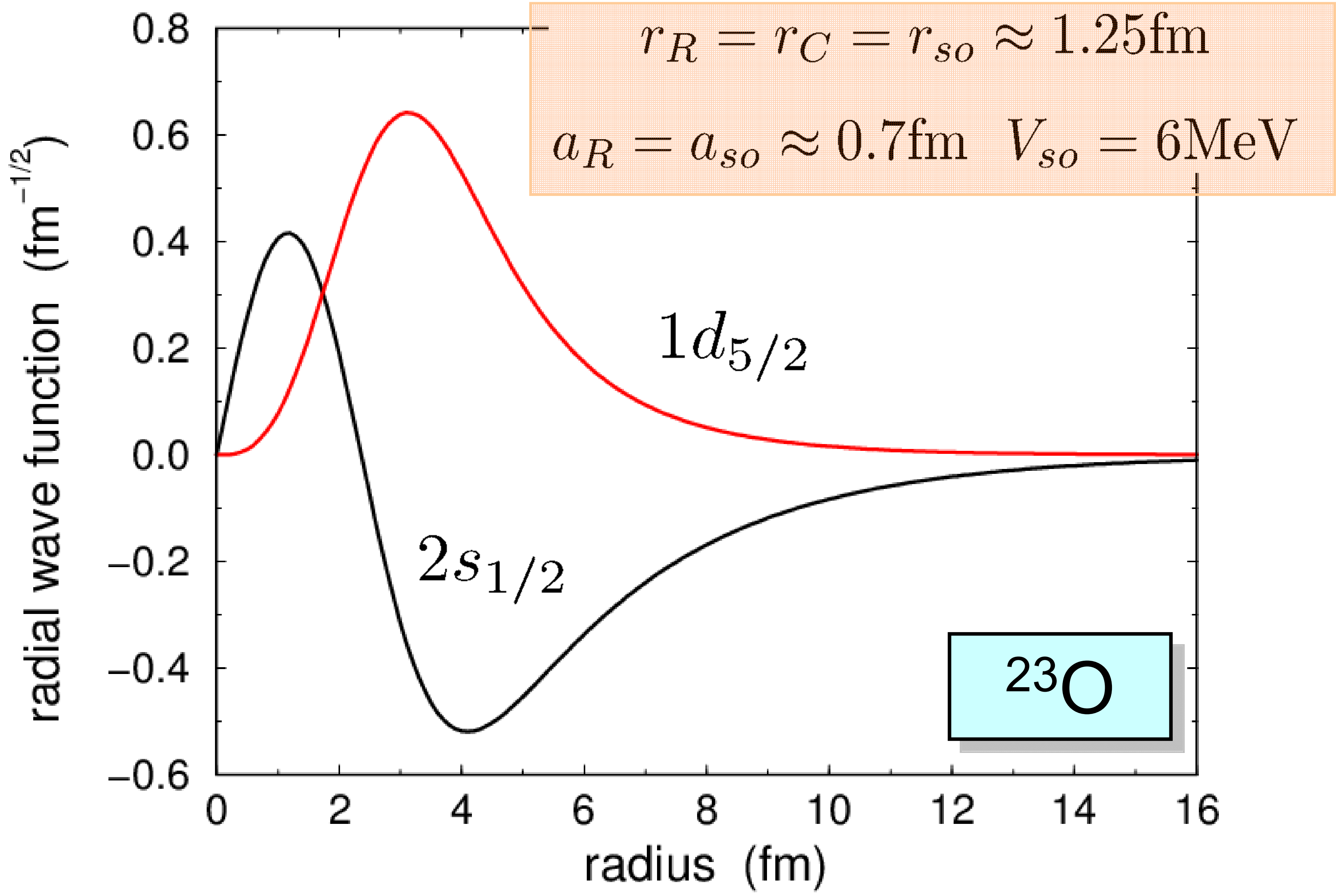


Gaussian density
rms radius 2.32 fm

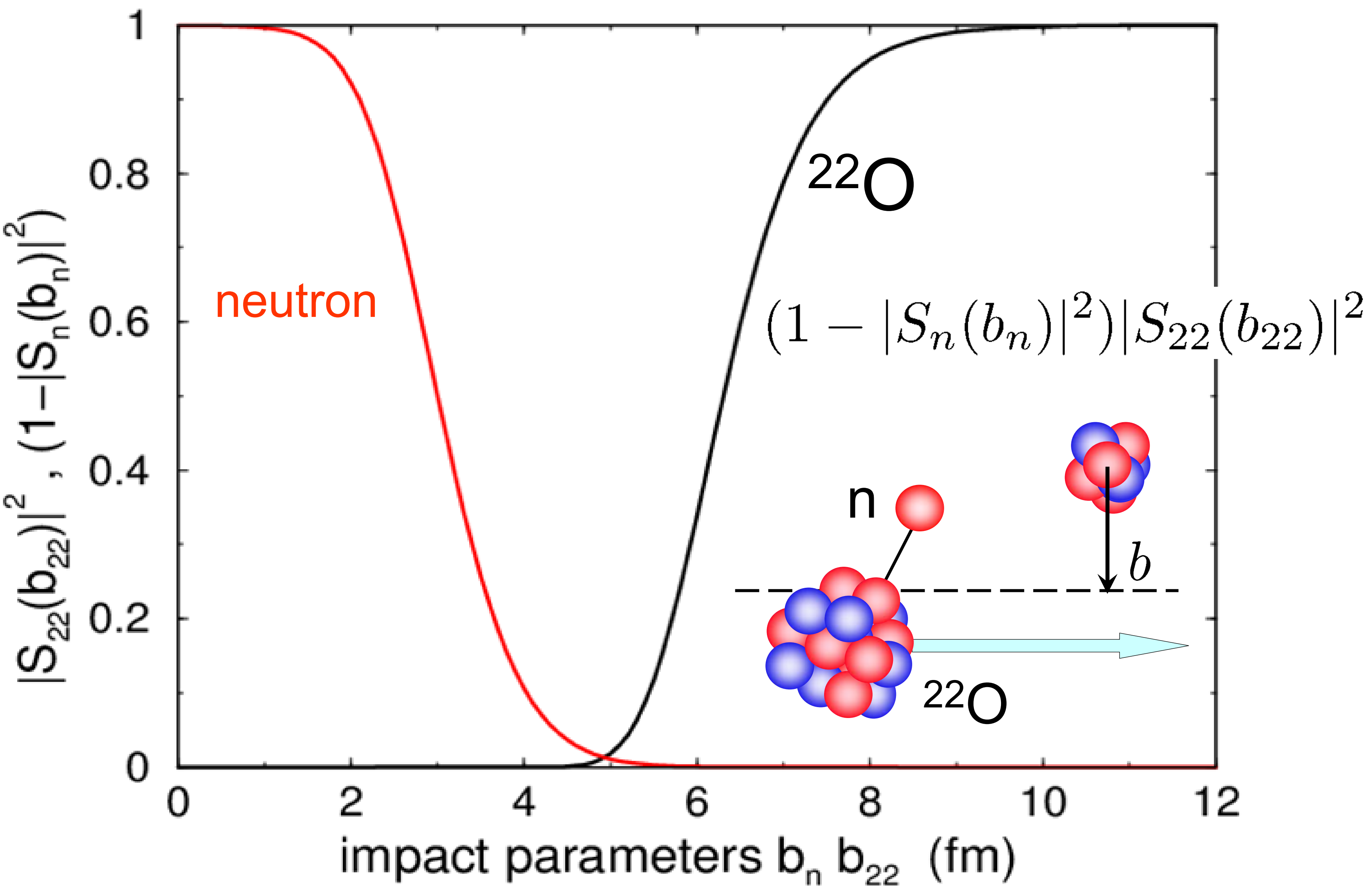


$$t_{NN}(r) = \left[-\frac{\hbar v}{2}\sigma_{NN}(i+\alpha_{NN})\right] f(r), \quad \int d\vec{r} f(r) = 1$$

Neutron bound state wave functions

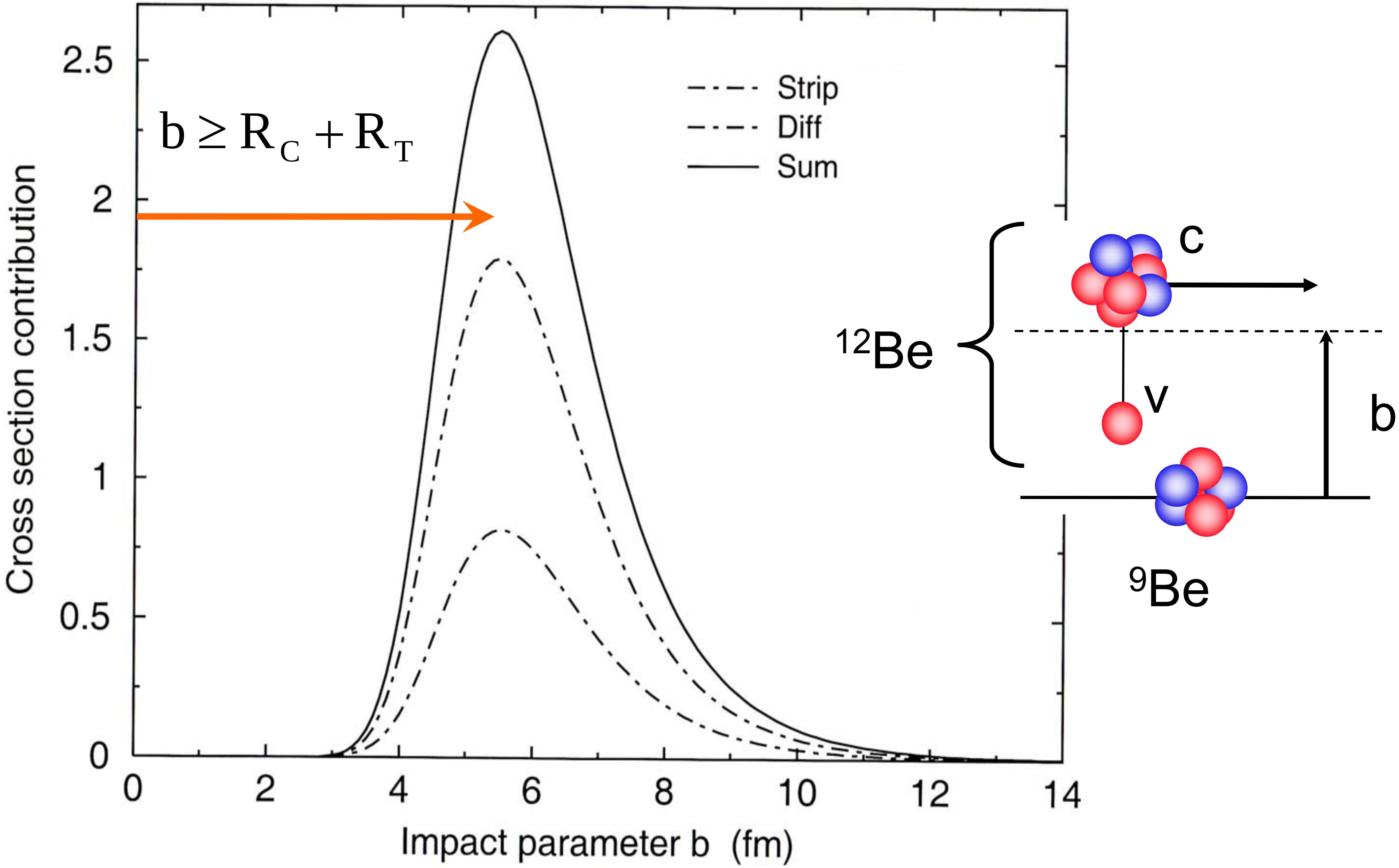


Eikonal S-matrix spatial selectivity



Contributions are from nuclear surface and beyond

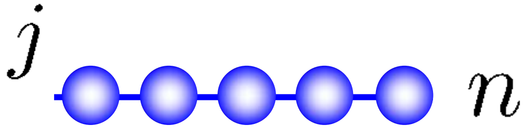
$^{12}Be + {}^{9}Be \rightarrow {}^{11}Be(gs)+X$, 80A MeV



Overlap functions – Spectroscopic factors

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{\mathrm{n}}\langle\, [\Psi_X^b \otimes \phi_{\ell j}(1)]_a^\alpha \,|\, \Psi_Y^{a\alpha}\,\rangle$$

Other (independent particle model) cases for removal from a state of a given j are less simple, can be worked out, but are given by the <u>coefficients of fractional parentage</u> - cfps



$$\langle\, [\Psi_{n-1}^b \otimes \phi_{\ell j}(1)]_a^\alpha \,|\, \Psi_n^{a\alpha}\,\rangle = ((j^{n-1})b, j; a|\}(j^n)\, a)$$

For low seniority states (where each pair couples to spin zero)

$$((j^{n-1})j, j; 0|\}(j^n)\, 0) = 1, \quad n = \text{even}, \;\; \text{seniority} = 0$$

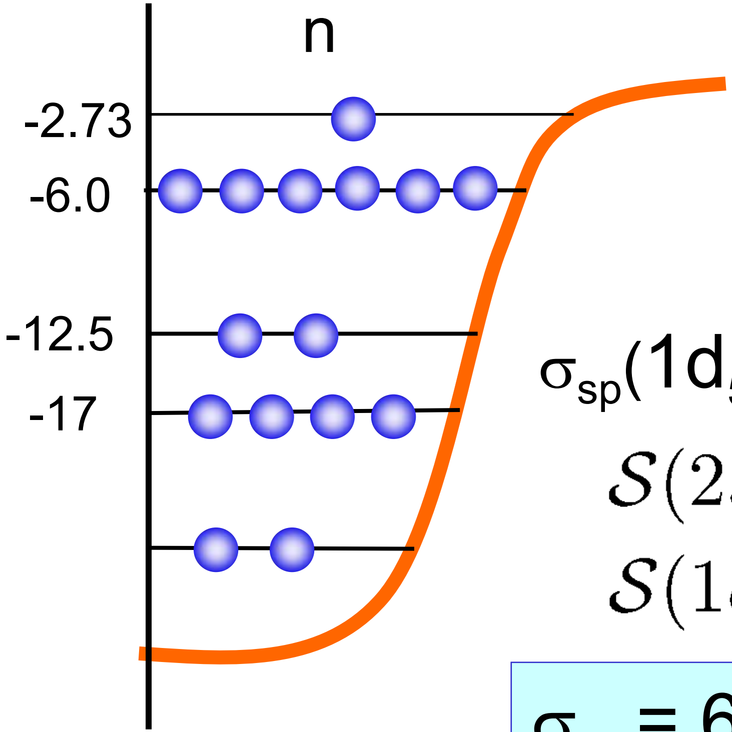
$$((j^{n-1})0, j; j|\}(j^n)\, j) = \left(\frac{2j+1-(n-1)}{n(2j+1)}\right)^{\frac{1}{2}},$$

$$n = \text{odd}, \;\; \text{seniority} = 1$$

$$\mathcal{S}(\ell j) = n((j^{n-1})b, j; a|\}(j^n)\, a)^2$$

Orientation – neutron knockout cross sections

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$2s_{1/2}$ S_n=2.73 MeV

$1d_{5/2}$ S_n=6.0 MeV

$\sigma_{sp}(1d_{5/2})$=28.3 mb $\sigma_{sp}(2s_{1/2})$=72.7 mb

$$\mathcal{S}(2s_{1/2}) = 1$$

$$\mathcal{S}(1d_{5/2}) = n(1d_{5/2}) = 6$$

$$\sigma_{-n} = 6\,\sigma_{sp}(1d_{5/2}) + \sigma_{sp}(2s_{1/2}) = 242.5 \text{ mb}$$

Measurement at RIKEN [Kanungo *et al.* PRL **88** ('02) 142502] at 72 MeV/nucleon on a ^{12}C target; σ_{-n} = 233(37)mb

Residue momentum distributions after knockout

$$\sigma_{str} = \frac{1}{2l+1} \sum_m \int d^2b \, \langle \psi_{lm} | |S_c(b_c)|^2 (1 - |S_n(b_n)|^2) | \psi_{lm} \rangle$$

$$= \frac{1}{2l+1} \sum_m \int d^2b_n \, (1 - |S_n(b_n)|^2) \langle \psi_{lm} | S_c^* \, S_c | \psi_{lm} \rangle$$

$$\frac{1}{(2\pi)^3} \int d\vec{k}_c |\vec{k}_c\rangle\langle\vec{k}_c| = 1$$

In projectile rest frame:

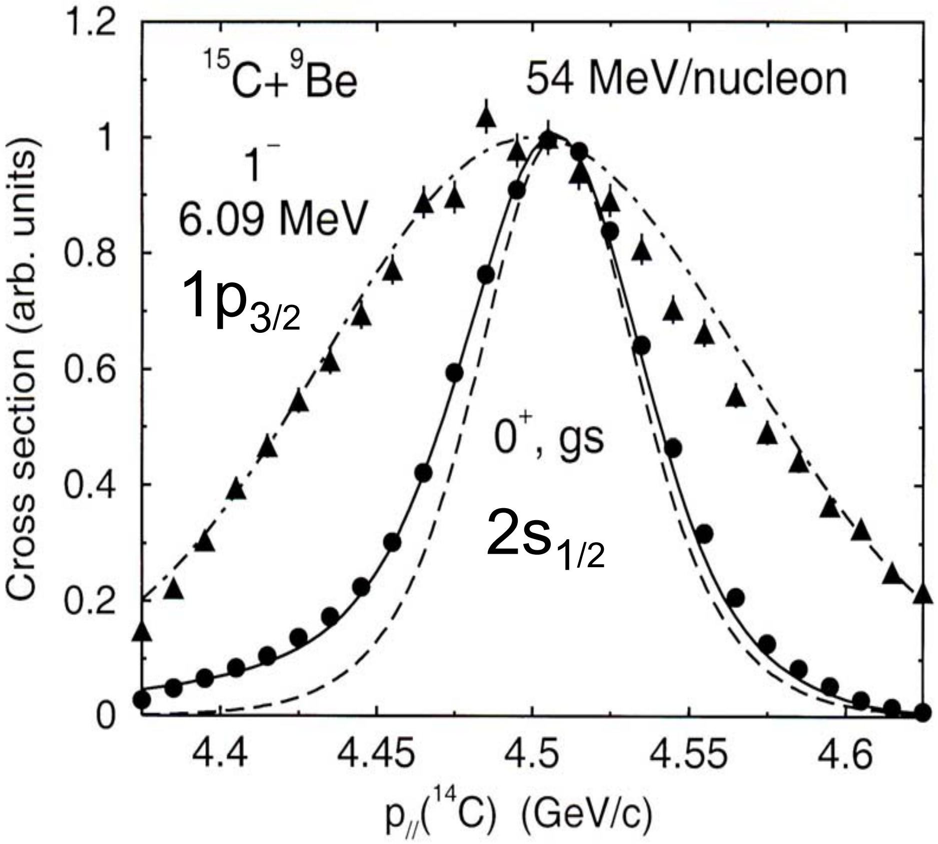
$$\frac{d\sigma_{str}}{d^3k_c} = \frac{1}{(2\pi)^3} \frac{1}{2l+1} \sum_m \int d^2b_n [1 - |S_n(b_n)|^2] \times \left| \int d^3r e^{-i\mathbf{k}_c \cdot \mathbf{r}} S_c(b_c) \psi_{lm}(\mathbf{r}) \right|^2$$

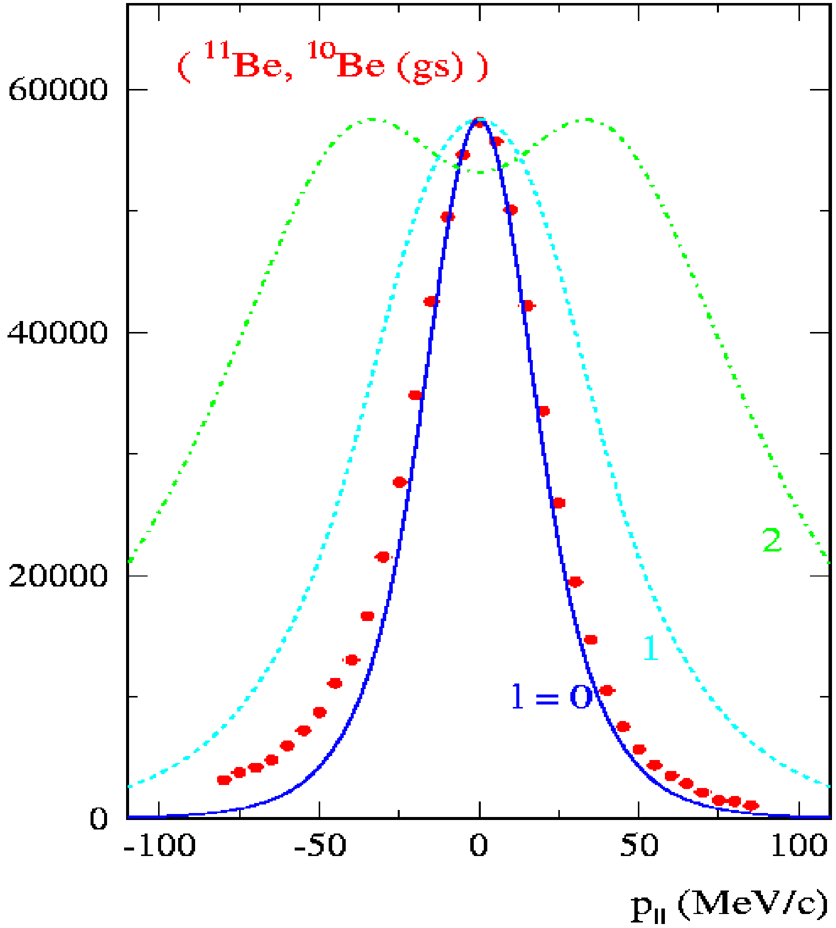
Residue parallel momentum distribution

$$\frac{d\sigma_{str}}{dk_z} = \frac{1}{2\pi}\frac{1}{2l+1}\sum_m \int_0^\infty d^2b_n[1-|S_n(b_n)|^2]\int_0^\infty d^2\rho|S_c(b_c)|^2$$

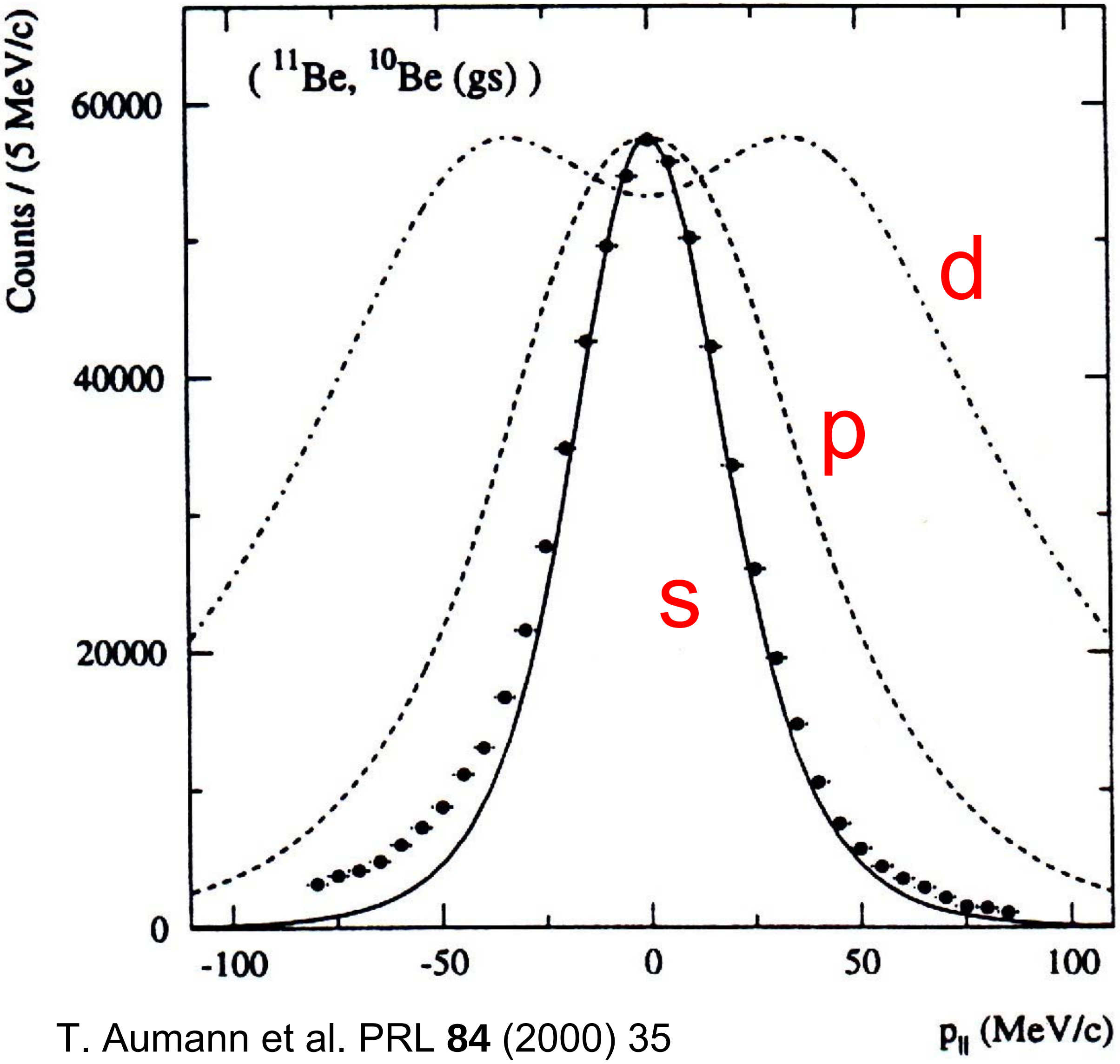
$$\vec{r} \equiv (\vec{\rho}, z)$$

$$\times \left| \int_{-\infty}^{\infty} dz \exp[-ik_z z]\psi_{lm}(\mathbf{r}) \right|^2$$

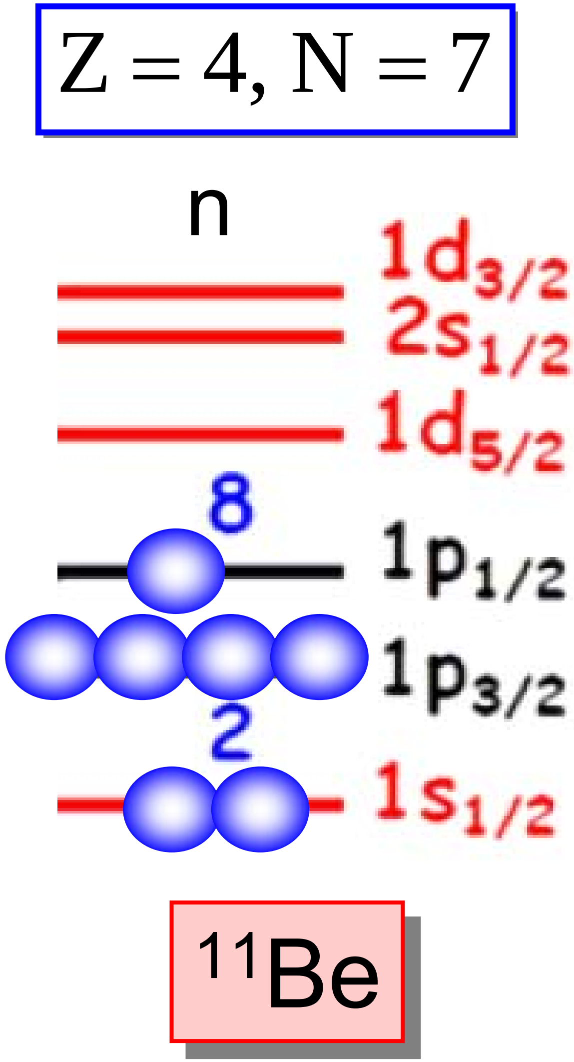




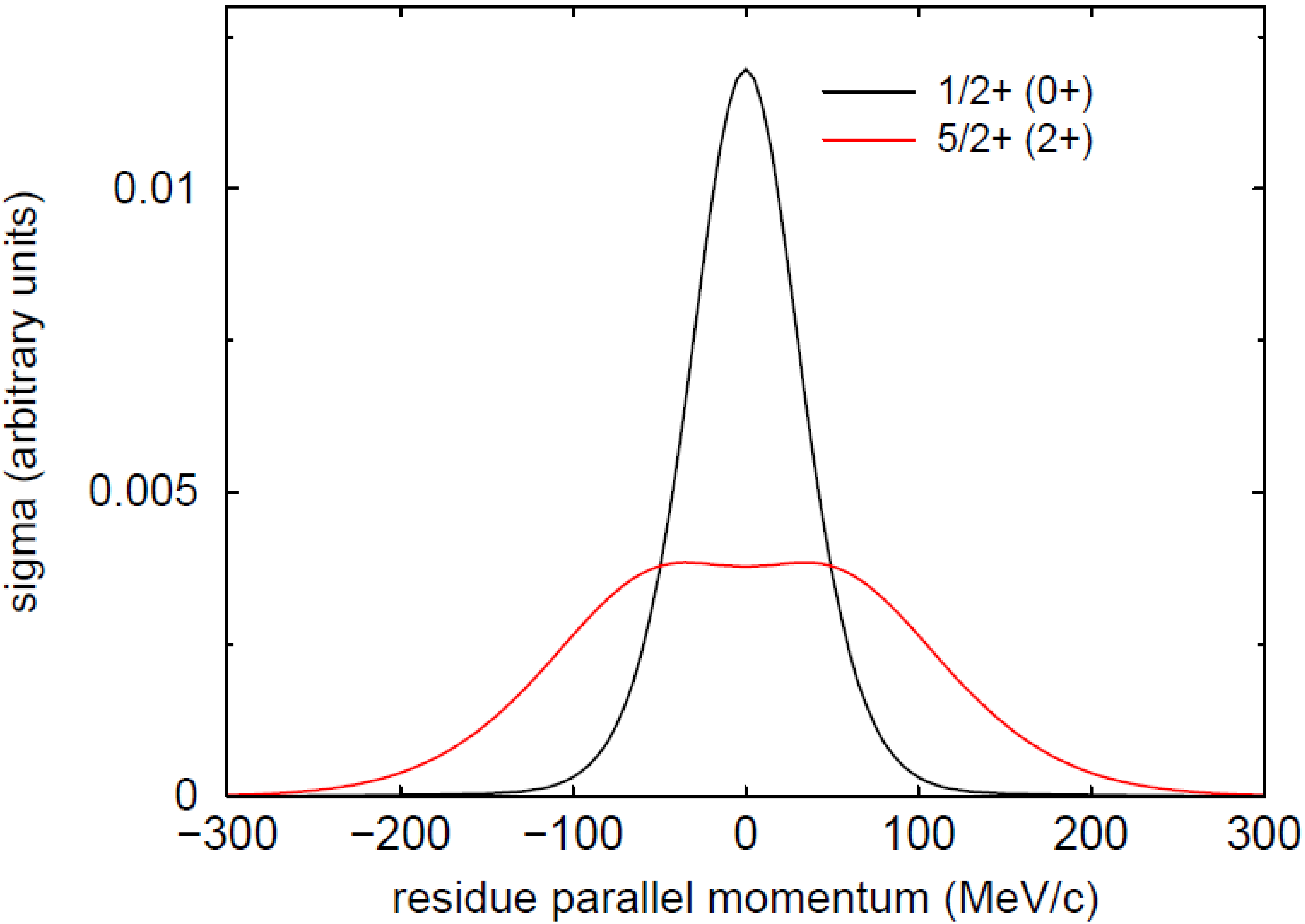
Residue momentum $^{11}Be \rightarrow ^{10}Be$ – halo case



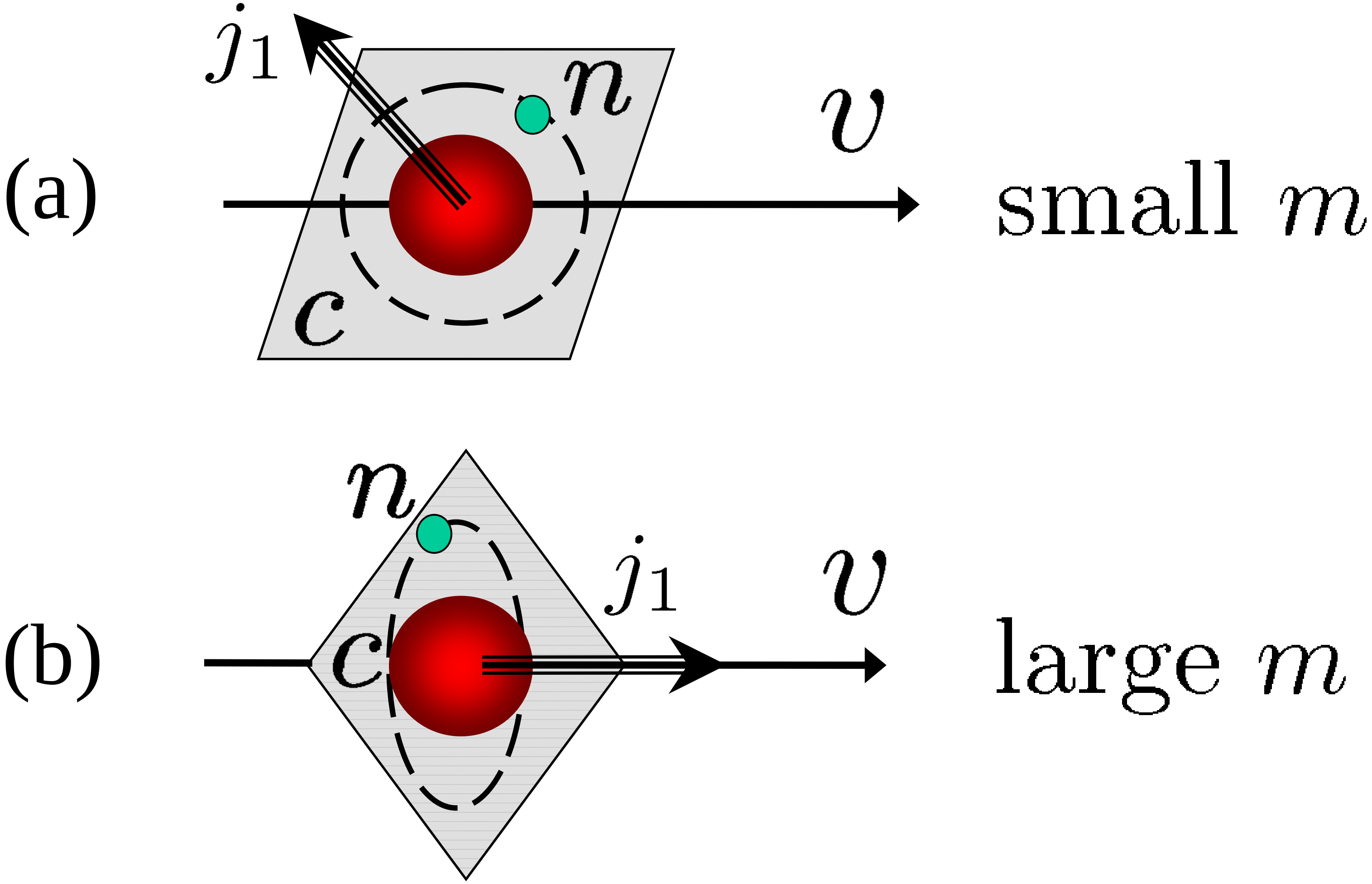
T. Aumann et al. PRL **84** (2000) 35



Forward momentum distributions of ^{22}O residues

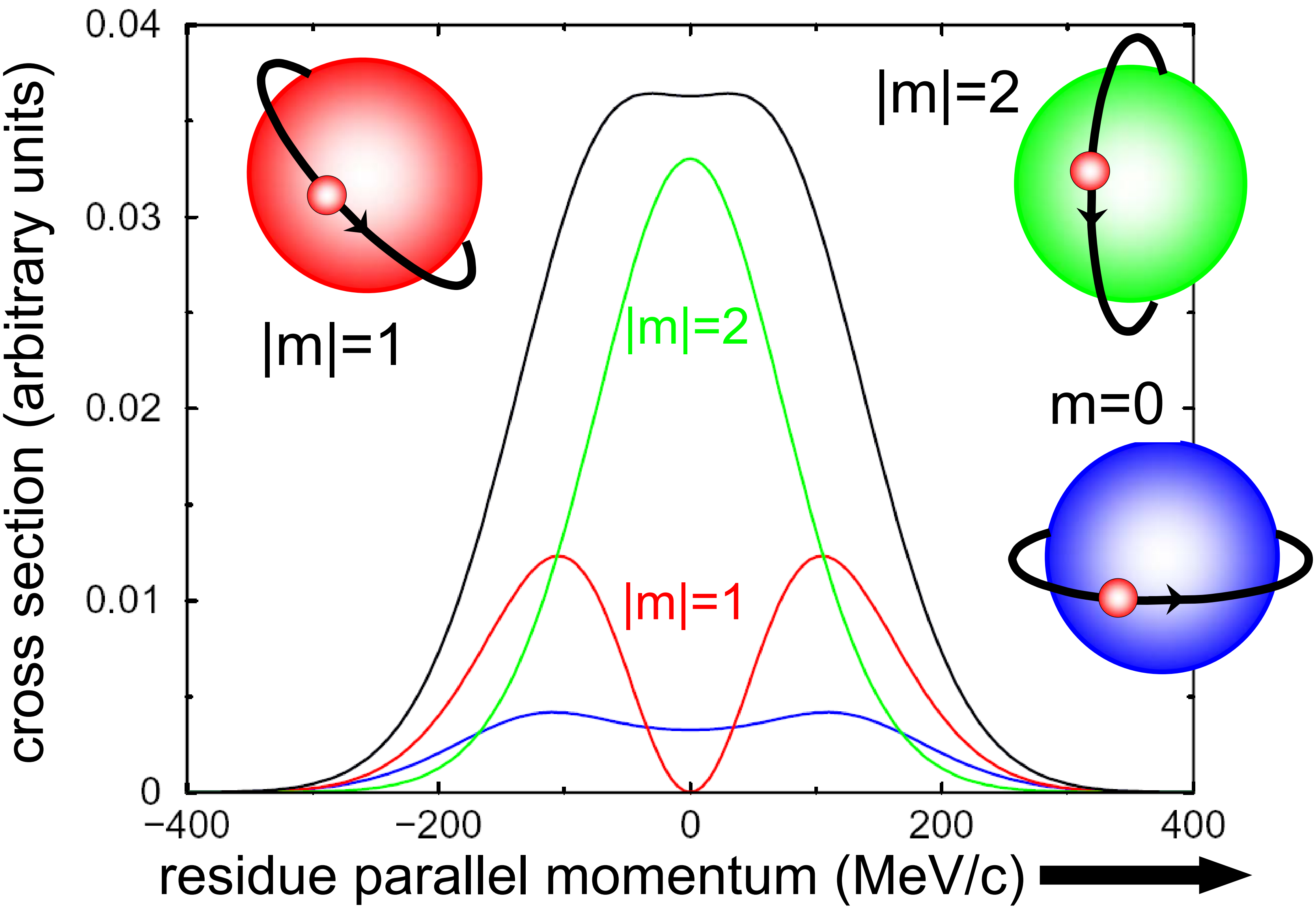


Projection dependence … what do we expect?



knockout calculates to **sigl.0,sigl.1,sigl.2 etc**

One nucleon knockout – ^{28}Mg (–p,ℓ=2) 82A MeV



Some further tools to use …

dfold_front (data set generator for folding model calculations of optical potentials)

dfold (single and double-folding model optical potential generator)

knockout (eikonal model code to calculate the cross sections for single nucleon removal from a fast nuclear beam - **knockout.outline**)

momentum (eikonal model code to calculate the momentum distributions of the heavy residue after single nucleon removal from a fast nuclear beam – see **momentum.outline**)

outlines at the website

Part 2 discussed:

How nucleon knockout cross sections are calculated from an assumed bound state wave function (i.e. an overlap) for the removed nucleon and eikonal S-matrices of part 1 of this lecture. The adiabatic approximation is used.

That the momentum distributions of the reaction residues (the projectiles less one nucleon) can be calculated. They have sensitivity to the ℓ values of the removed nucleon's overlaps – and so are of value for nuclear spectroscopy.

That we need to calculate realistic S-matrices for the core-target and nucleon-target systems. These can use theoretical nuclear (e.g. HF) densities and effective NN interactions at the >100 MeV/u beam energies of interest.