TALENT Course 6: Theory for exploring nuclear reaction experiments

Fusion reactions and coupled channels and breakup effects

GANIL, 1st-19th July 2013.

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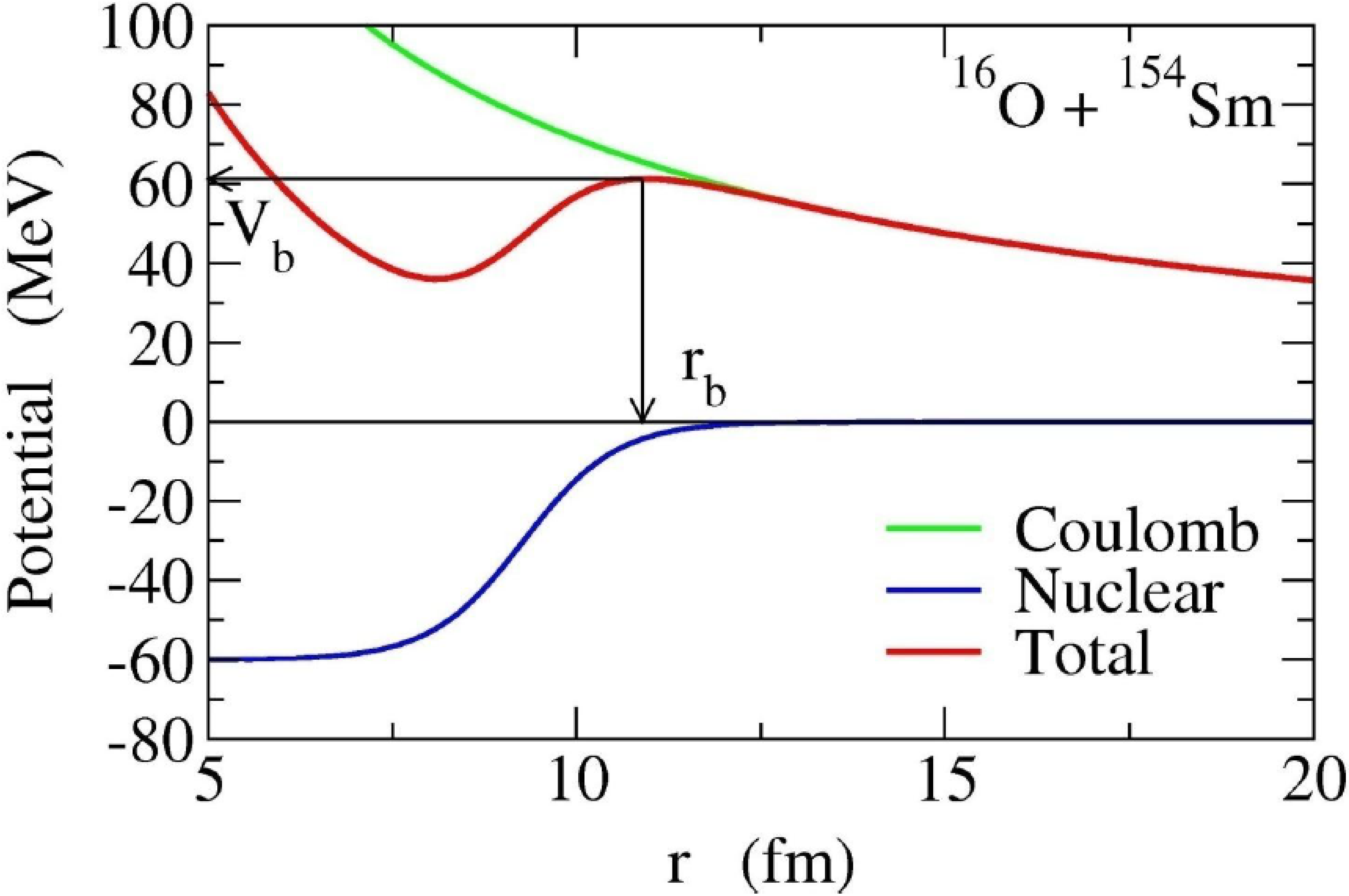
Session aims:

1. To discuss the physics of barrier-passing models of heavy-ion fusion reactions at energies near-to and well below the Coulomb barrier. Their theoretical description and what is expected and observed experimentally.

2. The importance and the treatment of collective-type channel coupling effects and the interrogation of such effects by use of the distribution-of-barriers concept.

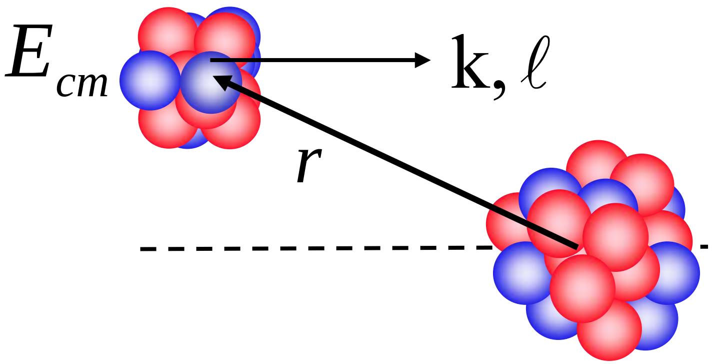
3. The possible roles of other degrees of freedom, such as breakup channels and transfer reactions, in the case of a neutron rich or weakly-bound projectile nucleus. To enable model estimates of these phenomena.

Tunnelling



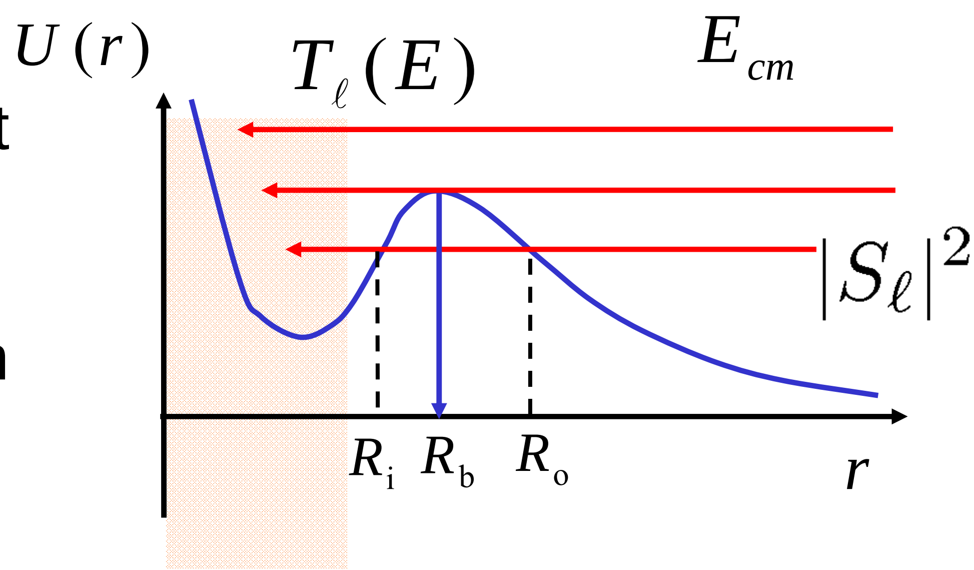
100
80
60
40
20
0
-20
-40
-60
-80
Potential (MeV)
5
10
15
20
r (fm)
16O + 154Sm
V b
r b
Coulomb
Nuclear
Total

Barrier passing models of fusion



Theoretical ideas for simple (barrier passing) models of nucleus-nucleus fusion reactions

an imaginary part in *U(r)*, at short distances, can be included to absorb all flux that passes over or through the barrier – assumed to result in fusion

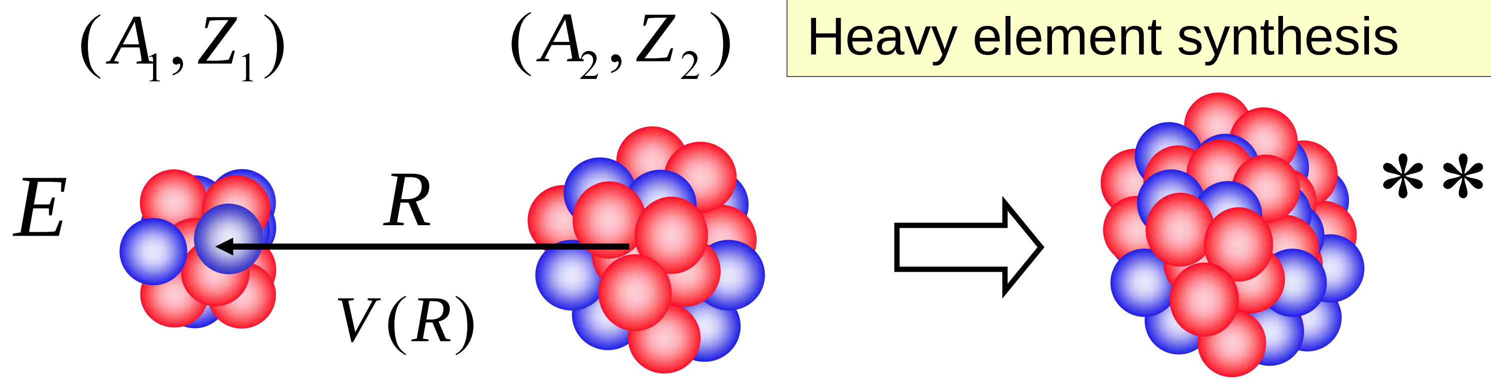


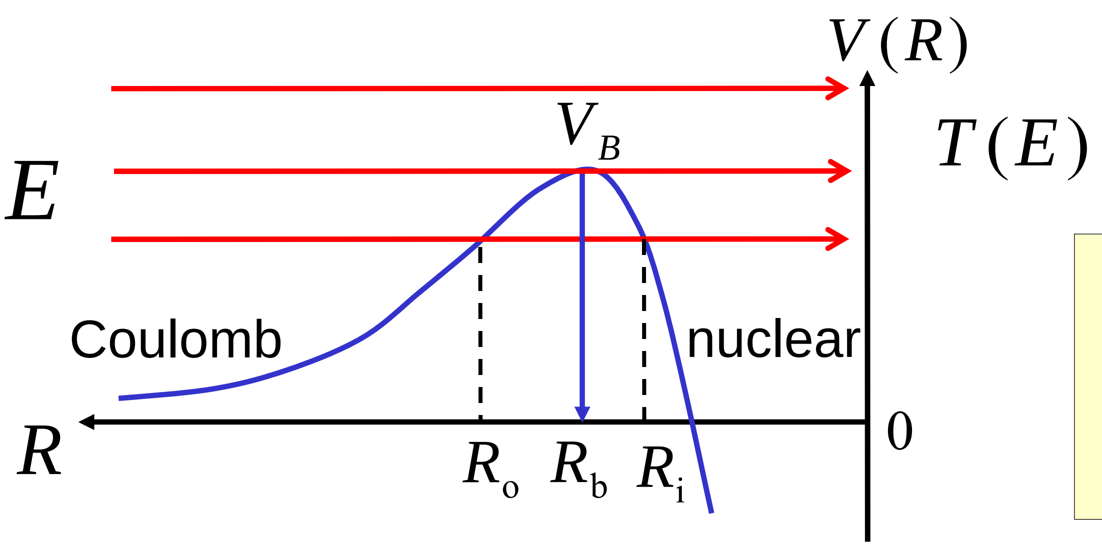
$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_\ell(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - |S_\ell|^2)$$

Complete fusion process – static picture

Nuclear astrophysics

Heavy element synthesis

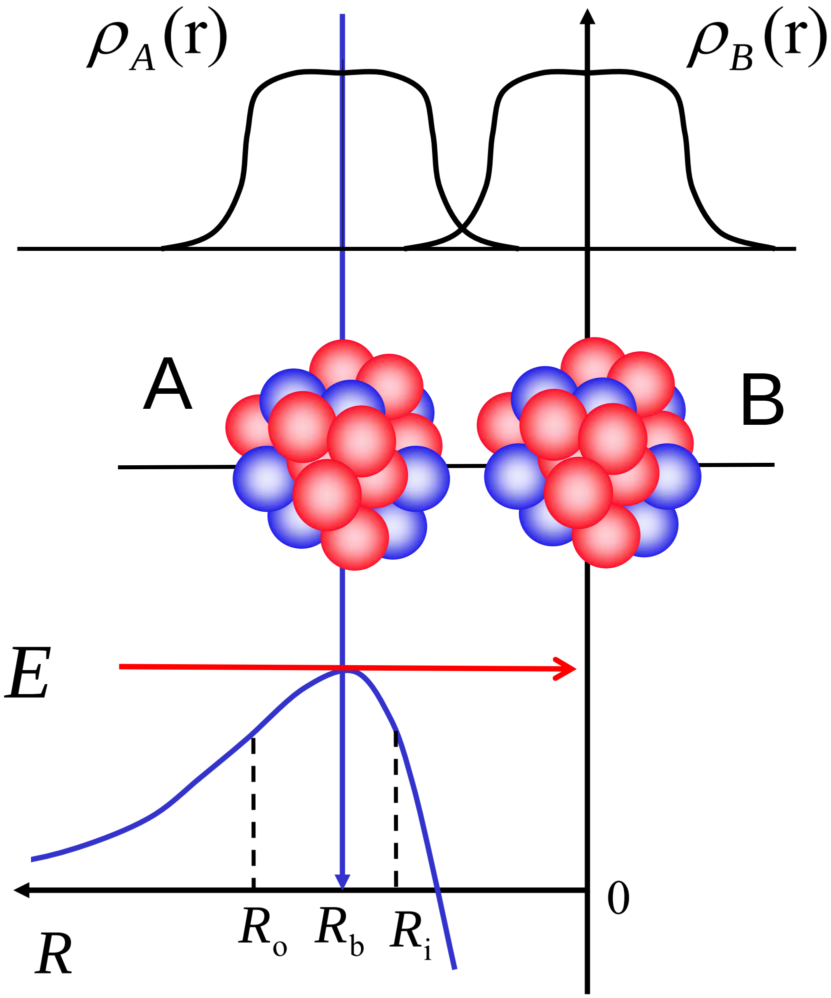




transmission probability:

Dominated by interaction *V(R)* near 'surface', R_b

Barrier radii and nuclear densities - surfaces



Fusion will be probe and be sensitive to:

nuclear binding (tails of the nuclear densities),
nuclear structure (tails of the single particle wave functions)

but also expect sensitivity and complications due to the reaction dynamics –

intrinsically surface dominated

From two-body asymptotic to massive overlap

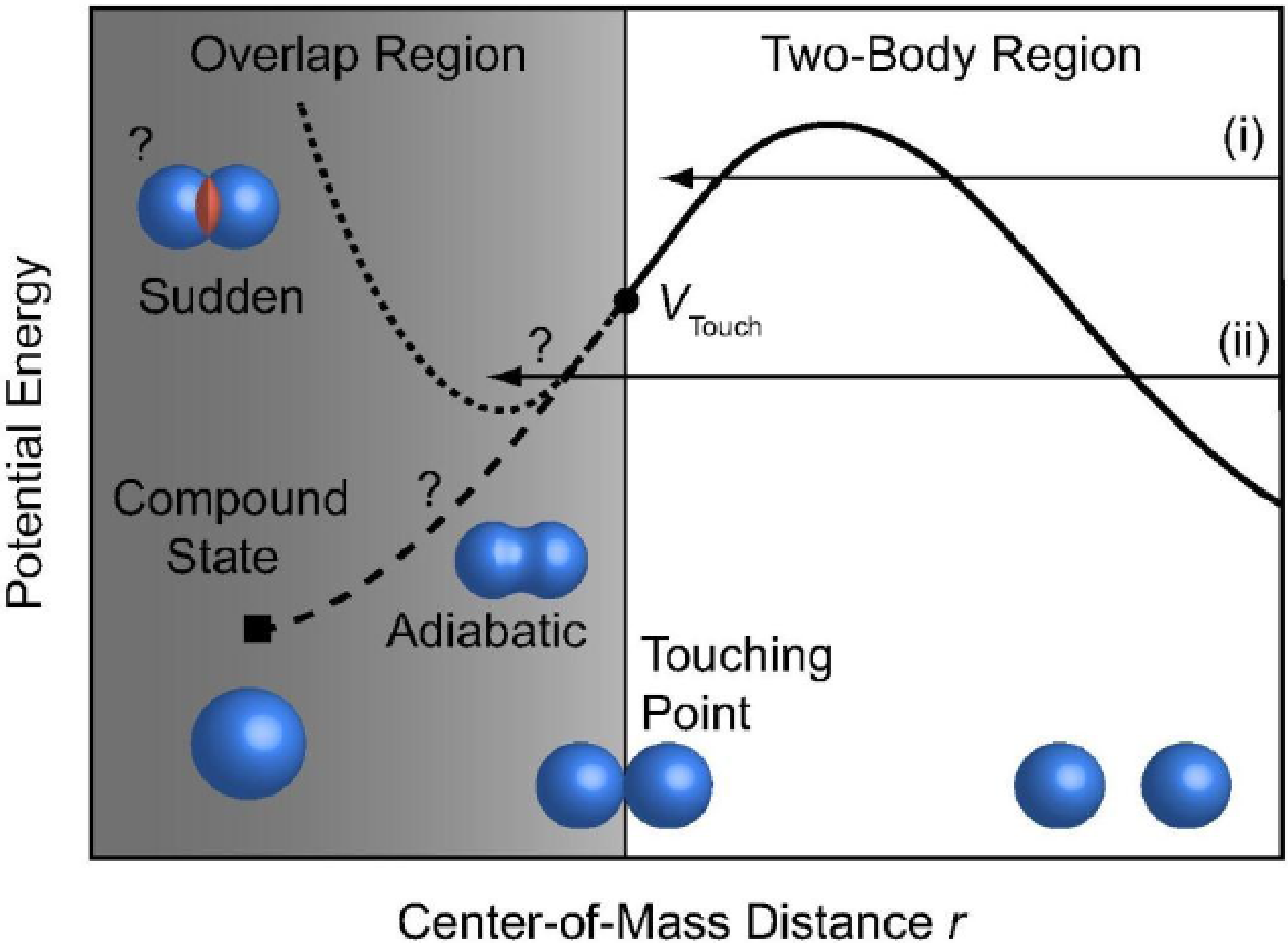
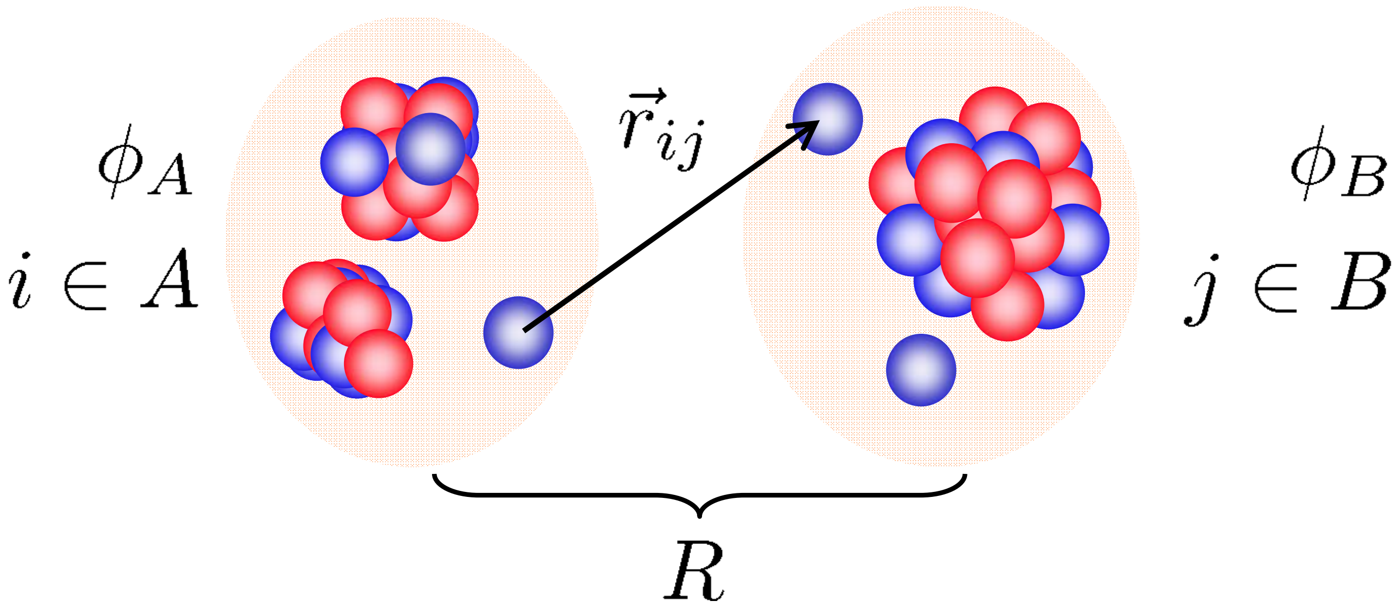


Figure from K. Hagino

Interactions (barriers) from folding models

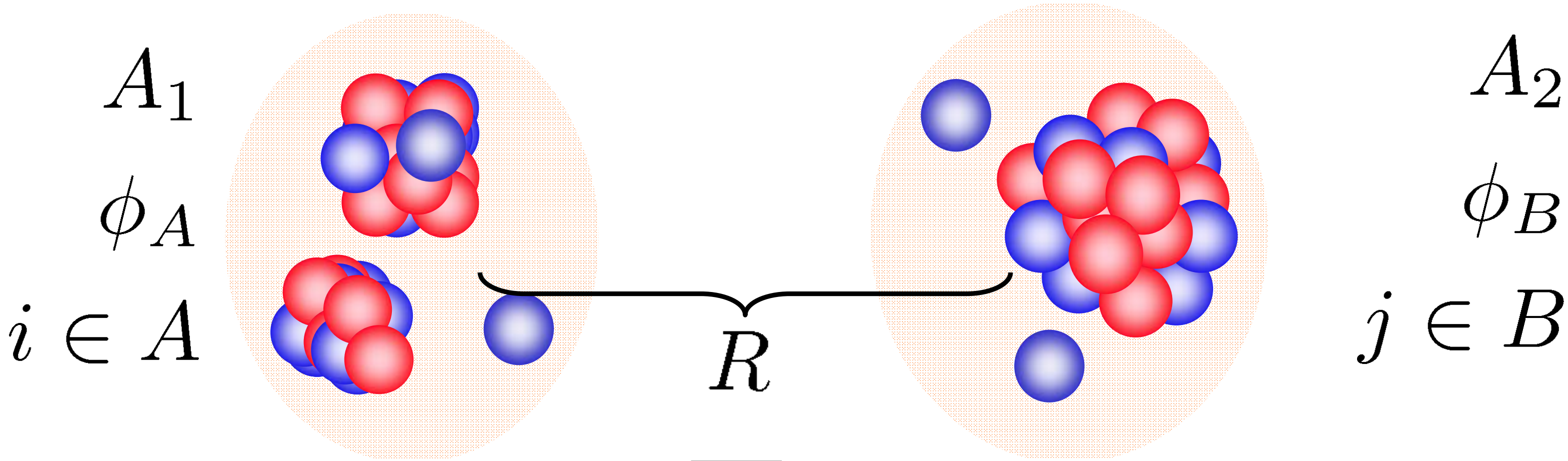
Diagonal interactions



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} V_{ij}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

Pair-wise interactions integrated (averaged) over the internal motions of the two composites – like interaction between two extended charge denstities

Double folding models – useful identities



$$V_F(R) = \langle \phi_A \phi_B | \sum_{ij} v_{NN}(\vec{r}_{ij}) | \phi_A \phi_B \rangle$$

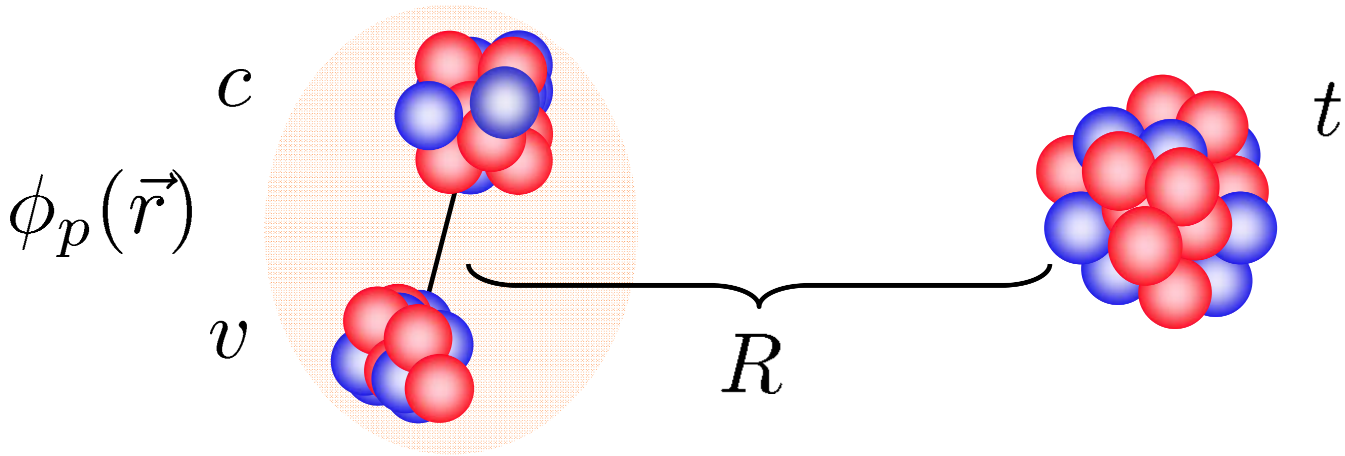
$$J_{V_F} = A_1 \, A_2 \, J_{v_{NN}}$$

$$\langle r^2 \rangle_{V_F} = \langle r^2 \rangle_A + \langle r^2 \rangle_B + \langle r^2 \rangle_{v_{NN}}$$

$$J_f = \int d\vec{r} \, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r} \, r^2 f(r) / J_f$$

proofs by taking Fourier transforms of each element

Cluster folding models – useful identities



$$V_F(R) = \langle \phi_p | V_{ct}(\vec{r}_{ct}) + V_{vt}(\vec{r}_{vt}) | \phi_p \rangle$$

$$J_{V_F} = J_{V_{ct}} + J_{V_{vt}}$$

$$\langle r^2 \rangle_{V_F} = \frac{A_c}{A_p} \langle r^2 \rangle_{V_{ct}} + \frac{A_v}{A_p} \langle r^2 \rangle_{V_{vt}} + \frac{A_c A_v}{A_p^2} \langle r^2 \rangle_{\phi_p}$$

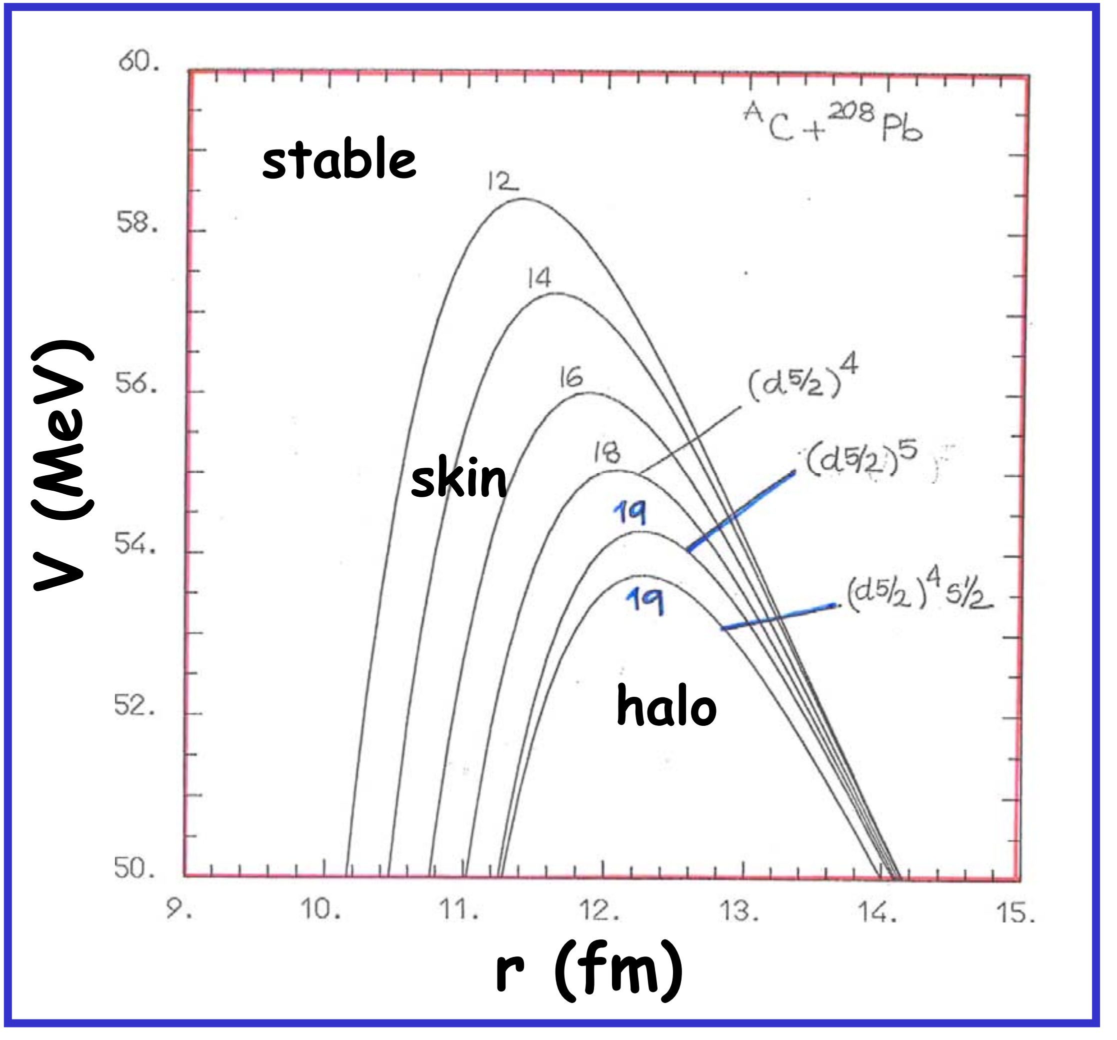
$$J_f = \int d\vec{r}\, f(r), \qquad \langle r^2 \rangle_f = \int d\vec{r}\, r^2 f(r) / J_f$$

proofs by taking Fourier transforms of each element

Static effects – barriers for n-rich Carbon isotopes

$^{A}C + ^{208}Pb$

HF predictions



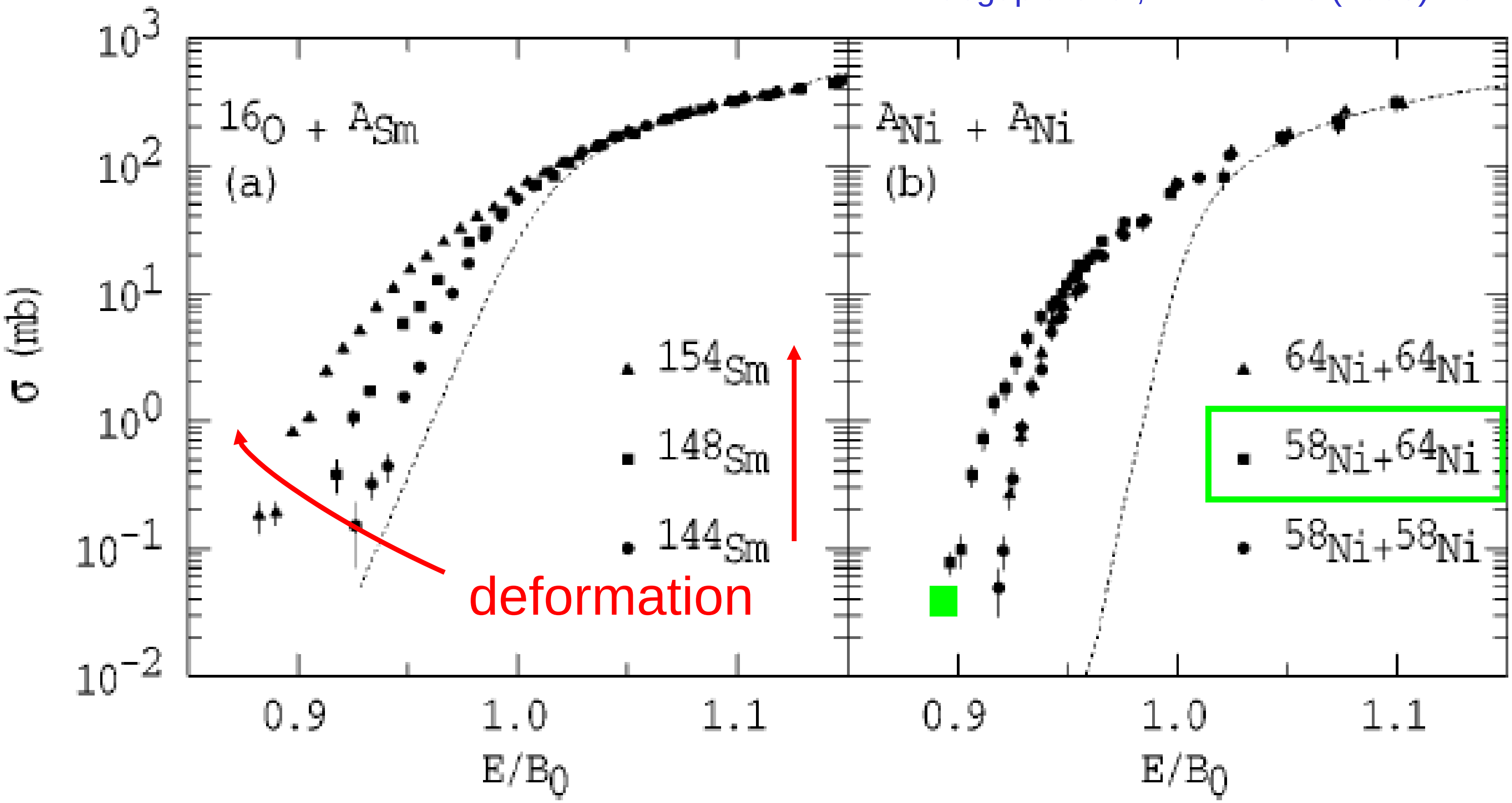
A. Vitturi, NUSTAR'05,

Challenges – potentials, thresholds and dynamics

- Expect a complex interplay of *static*, density driven, and surface, *dynamical* effects
- Far below the barrier, for normally bound nuclei, direct reaction channels switch off – have opportunity to study *threshold effects* as reaction channels open and evolve as a function of energy
- Fusion expected to be a severe test of our models of nuclear structures and of treatments of direct reaction dynamics – surface sensitivity of the reaction.
- Facilities available for sophisticated and very precise experiments - ANU (Canberra), USP, Legnaro, etc.
- Weakly bound systems are different – do break-up channels turn off below the barrier? What can we learn?

Channel coupling – indications of their importance

M.Dasgupta et al, ARNPS **48** (1998) 401



R.G. Stokstad et al, PRL **41** (1978) 465, PRC **21** (1980) 2427.

M. Beckerman et al, PRL **45** (1980) 1472, PRC **23** (1981) 1581, PRC **25** (1982) 837.

Changing the potential depth is not a solution

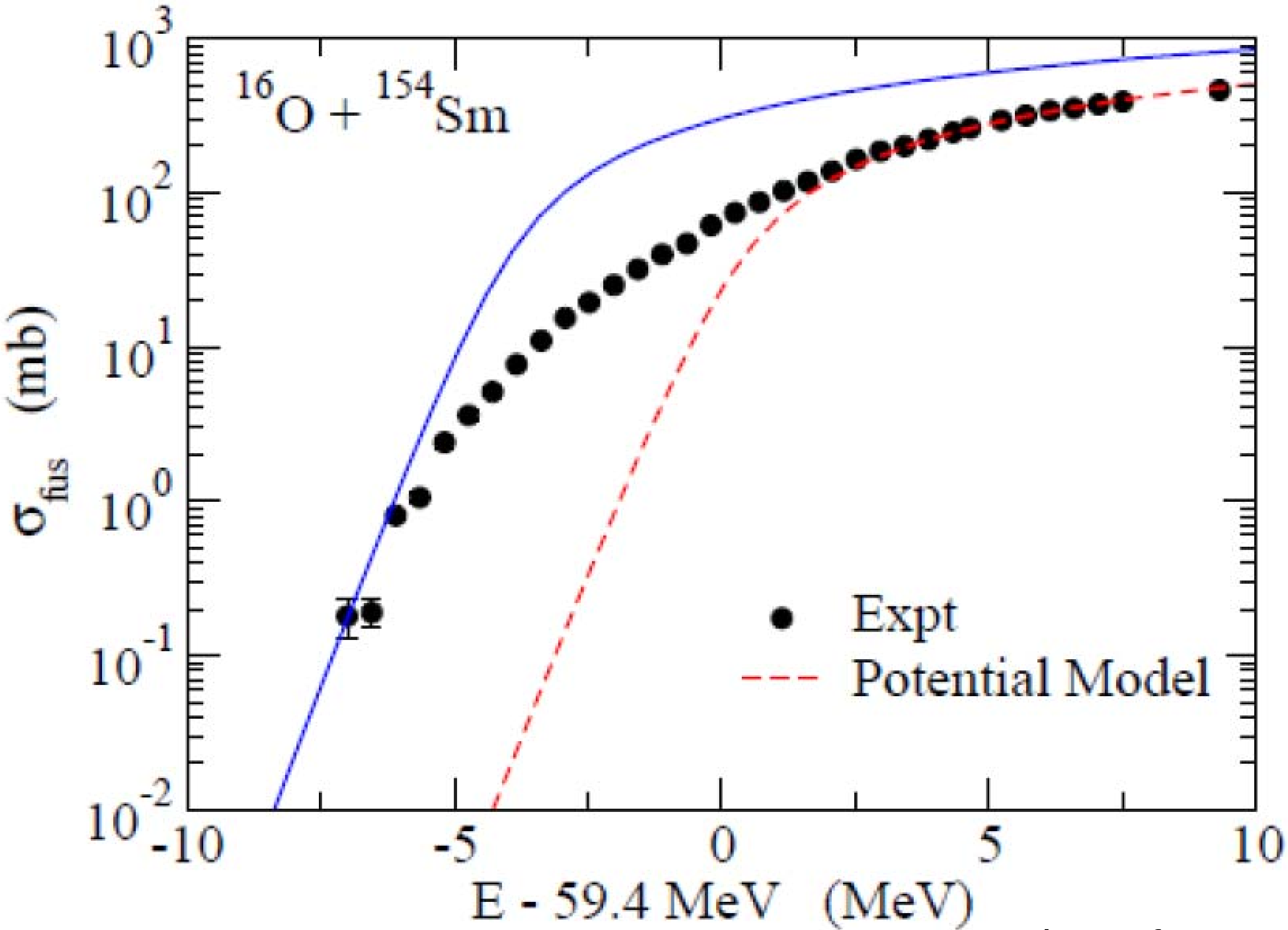
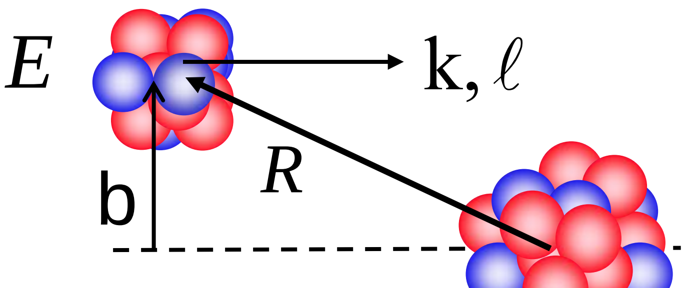


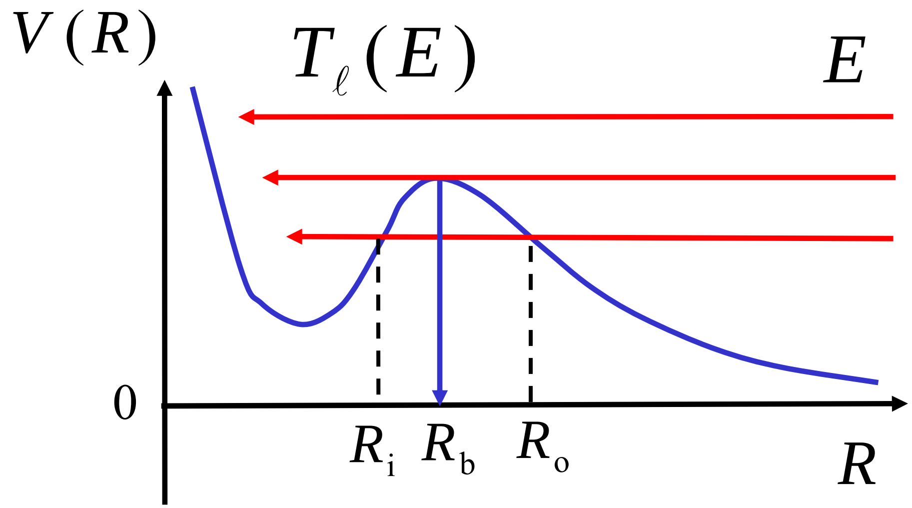
Figure from K. Hagino

Complete fusion - expectations – static model



$$kb \cong \ell$$

$$(A_1, Z_1) + (A_2, Z_2)$$



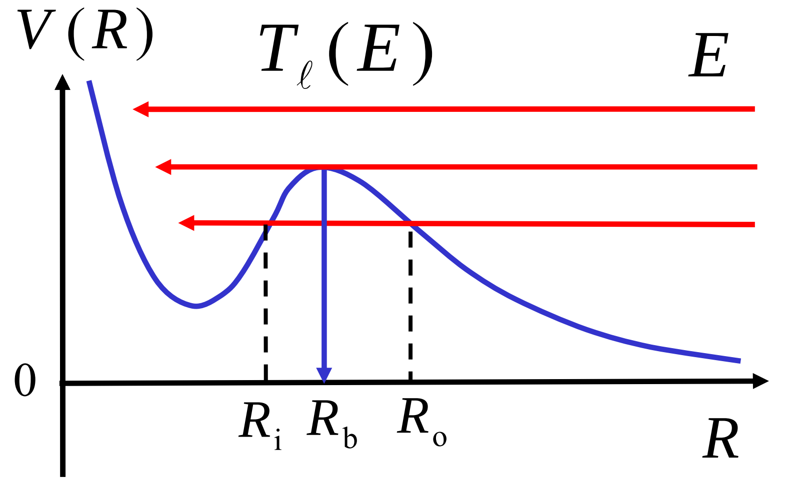
$$\sigma(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_\ell(E), \quad T_\ell(E) = [1 - |S_\ell|^2]$$

$$\frac{d^2 u_\ell(R)}{dR^2} + \frac{2\mu}{\hbar^2} \left[E - V(R) - \frac{\ell(\ell+1)}{R^2} \right] u_\ell(R) = 0$$

Quantum mechanical barrier penetration

$$\frac{d^2 u_\ell(R)}{dR^2} + \frac{2\mu}{\hbar^2}\left[E - V(R) - \frac{\ell(\ell+1)}{R^2}\right]u_\ell(R) = 0$$

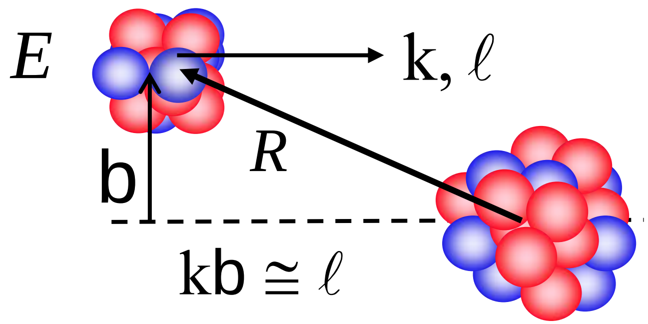
Numerical solutions of this QM barrier penetration problem, the solution of the radial equation for *u(R)* and the transmission prob. - and later, more complex (coupled channels) examples, account for fusion by one of two methods:

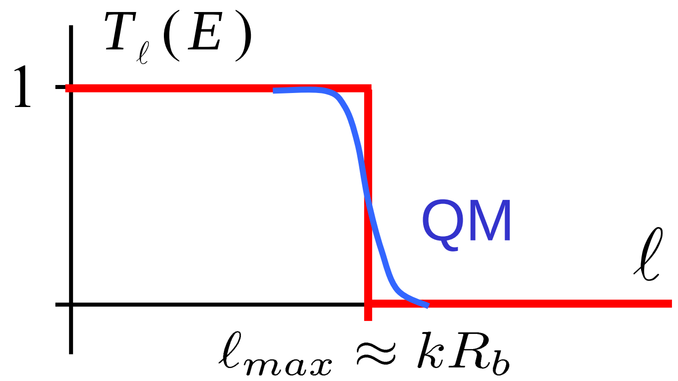


(i) the $u_\ell(R)$ have ingoing wave boundary conditions for an $R=R_0$
No flux transmitted through the barrier is reflected [exp(-ikR)]

(ii) including a strong absorptive (imaginary) part in $V(R)$ at short distances absorbs all flux transmitted through the barrier

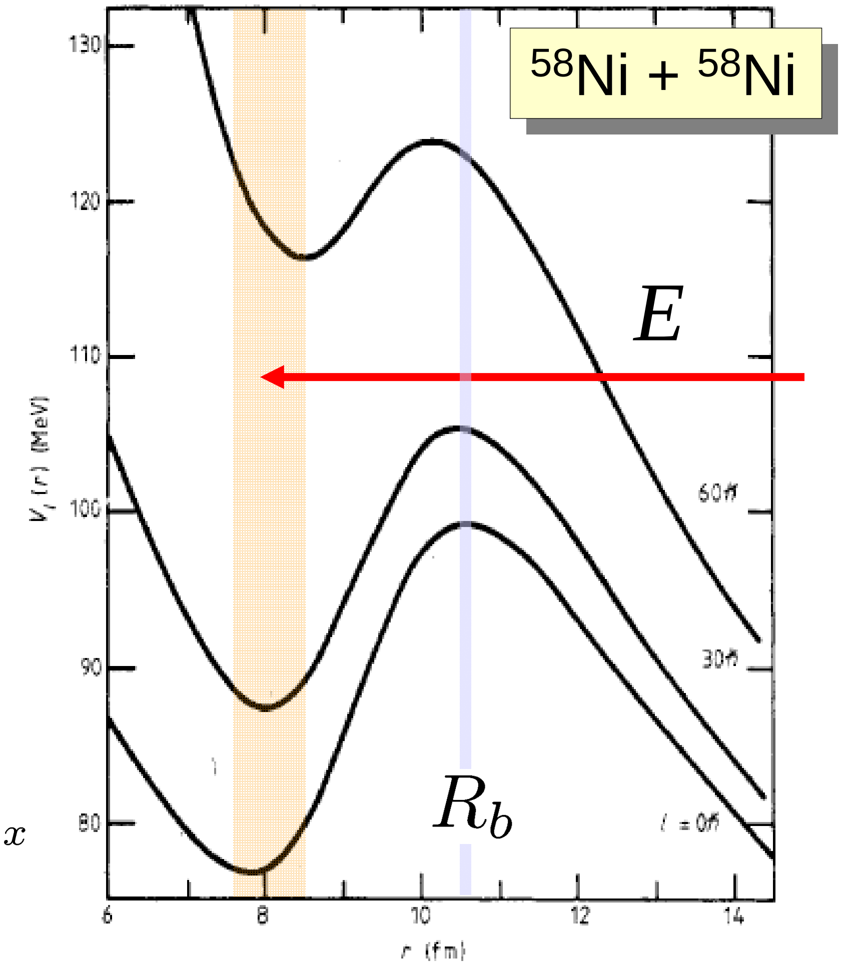
Angular momentum dependence of the barrier





$$\sigma_R(E) \approx \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell+1) \approx \frac{\pi}{k^2}\ell_{max}^2$$

$$\sigma^{cf}(E) \approx \sigma_R \approx \pi R_b^2$$

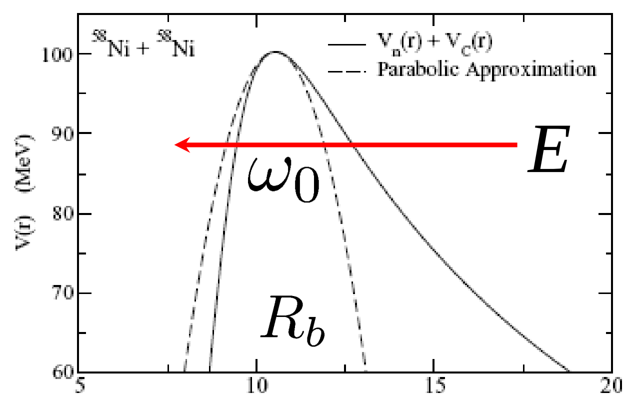


M. Beckerman, Rep. Prog. Phys. **51** (1988) 1047

Formula of Wong – quadratic form barrier

$$V_\ell(R) = V_b - \frac{1}{2}\mu\omega_0^2(R-R_b)^2 + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2}$$

$$T_\ell(E) = \{1+\exp\left[(2\pi/\hbar\omega_\ell)(V_\ell - E)\right]\}^{-1}$$



Assuming $\hbar\omega_\ell = \hbar\omega_0$

$$V_\ell = V_b + \ell(\ell+1)\hbar^2/2\mu R_b^2$$

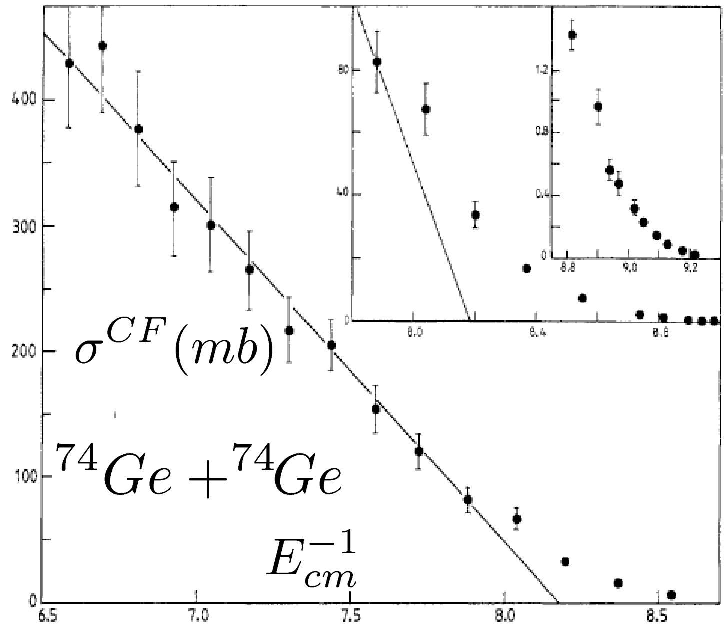
$$\sigma^{cf}(E) = \frac{R_b^2\hbar\omega_0}{2E}\ln(1+e^x)$$

$$x = (2\pi/\hbar\omega_0)(E - V_b)$$

and for $E \gg V_b$

$$\sigma^{cf}(E) = \pi R_b^2(1 - V_b/E)$$

C.Y. Wong, PRL **31** (1973) 766

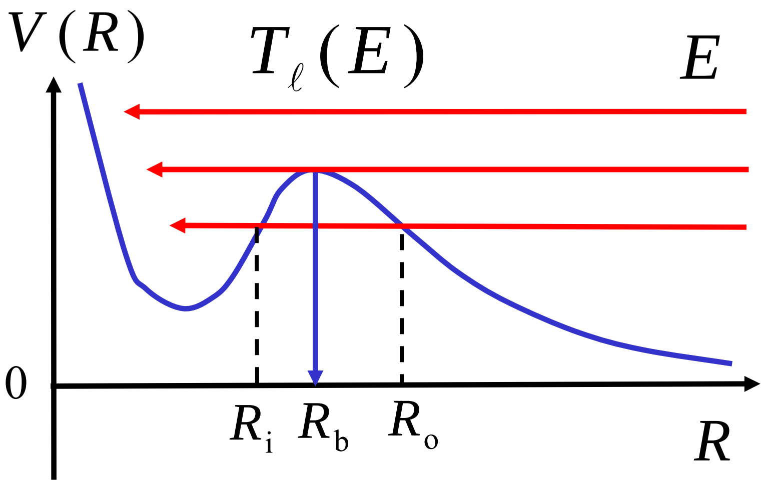


Making connection with empirical cross sections

$$T_\ell(E) \approx \left[1 + \exp \sqrt{\frac{8\mu}{\hbar^2}} \int_{R_i(\ell)}^{R_o(\ell)} dR \left\{ V(R) + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2} - E \right\}^{1/2} \right]^{-1}$$

Localised barrier of height (for ℓ=0) of $V_B = V(R_b)$

$$\frac{\ell(\ell+1)}{R^2} \approx \frac{\ell(\ell+1)}{R(E)^2} \rightarrow T_\ell(E) \approx T_0 \underbrace{\left(E - \frac{\ell(\ell+1)\hbar^2}{2\mu R(E)^2} \right)}_{E'}, \; R(E) \approx R_b$$



$$\sigma(E) = \sum_\ell \sigma_\ell(E) \rightarrow \int d\ell \; \sigma(\ell, E)$$

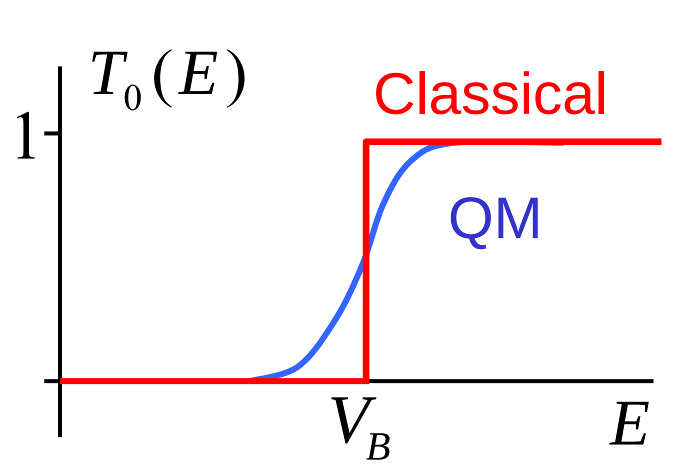
$$E\sigma(E) = \pi R(E)^2 \int_0^E dE' \; T_0(E')$$

$$\sigma(\ell, E) = \frac{\pi}{k^2}(2\ell+1)T_\ell(E) \; (= T_0(E'))$$

A.B. Balantekin, Rev. Mod. Phys. **70** (1998) 77

Distribution of barriers – directly from the data

$$E\sigma(E) = \pi R(E)^2 \int_0^E dE' \; T_0(E')$$

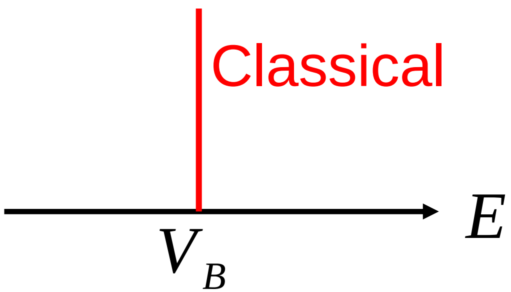


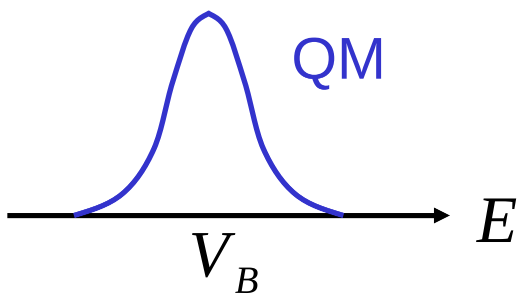
Classically

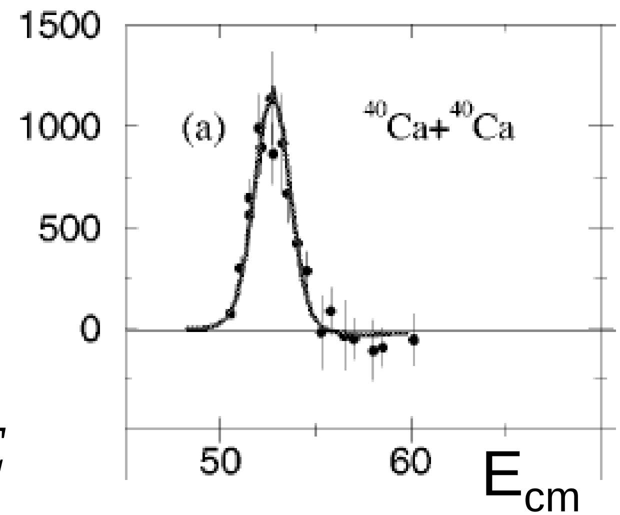
$$R(E) \equiv R_b$$

$$\begin{aligned} E\sigma(E) &= \pi R_b^2 (E - V_B), \; E > V_B \\ &= 0, \quad E < V_B \end{aligned}$$

$$\frac{d^2}{dE^2}[E\sigma(E)] = \pi R_b^2 \delta(E - V_B)$$







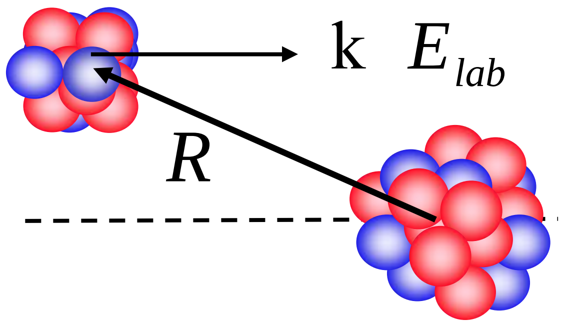
A.B. , Rev. Mod. Phys. **70** (1998) 77

M.Dasgupta et al, ARNPS **48** (1998) 401

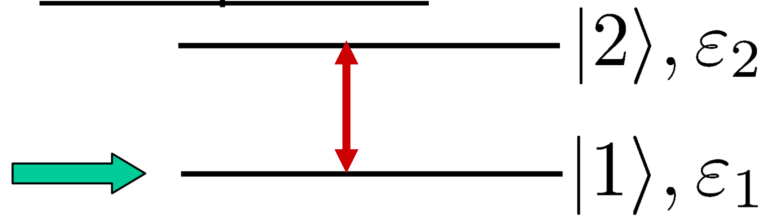
Coupled channels: for two-states problem

$$H = T_R + V(R) + h(\xi) + F(R,\xi)$$

$h(\xi)$



<u>Model problem</u>



Two channels 1,2 – incident waves in channel 1.

$$H|\Psi\rangle = E|\Psi\rangle \qquad \langle \vec{R}, \xi|\Psi\rangle = \phi_1(\vec{R})\langle\xi|1\rangle + \phi_2(\vec{R})\langle\xi|2\rangle$$

$$\begin{aligned}\langle 1|H|\Psi\rangle &= [T_R + V(R) + \varepsilon_1 + F_{11}(R)]\phi_1(\vec{R}) + F_{12}(R)\phi_2(\vec{R}) \\ &= E\langle 1|\Psi\rangle = E\phi_1(\vec{R}), \qquad F_{ij}(R) = \langle i|F(R,\xi)|j\rangle\end{aligned}$$

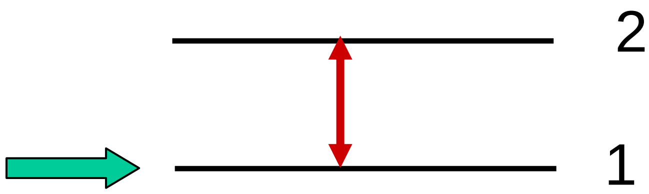
and similarly for the overlap with state 2, gives <u>coupled</u> equations

$$[E - \varepsilon_1 - T_R - V(R) - F_{11}(R)]\phi_1(\vec{R}) = F_{12}(R)\phi_2(\vec{R})$$

$$[E - \varepsilon_2 - T_R - V(R) - F_{22}(R)]\phi_2(\vec{R}) = F_{21}(R)\phi_1(\vec{R})$$

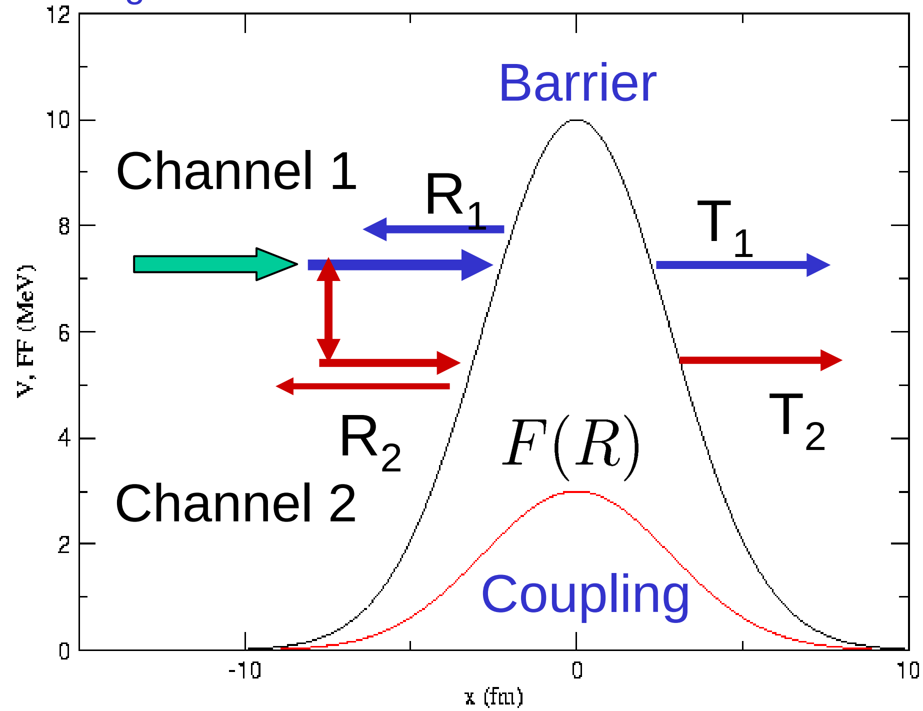
Coupled channels effects on barrier distribution

Model problem



Coupling of two channels 1,2 assumed degenerate for simplicity - coupling *F(R)* – incident waves in channel 1.

Figure from to A.Vitturi



$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + V(R) - E\right]\phi_1(R) = -F(R)\phi_2(R)$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + V(R) - E\right]\phi_2(R) = -F(R)\phi_1(R)$$

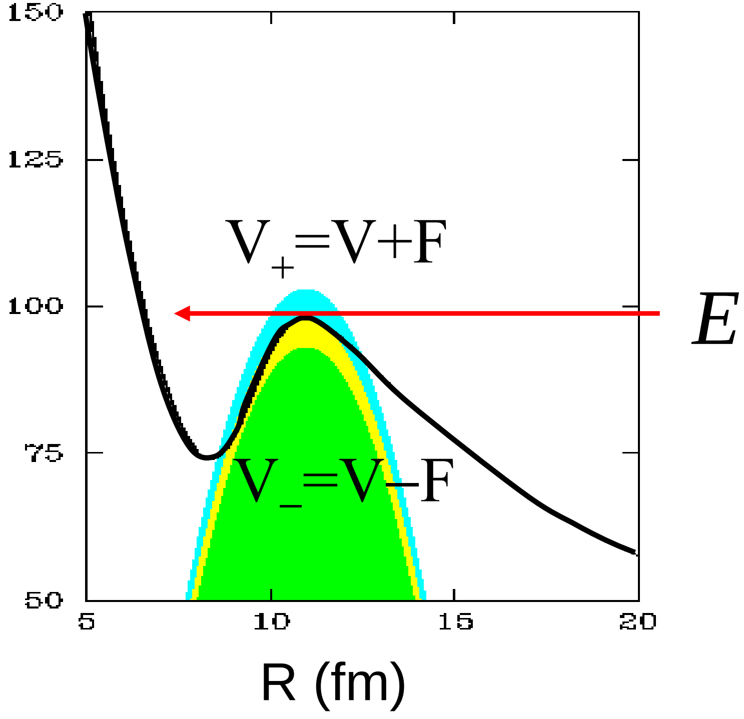
Decoupled by addition and subtraction

Decoupled, two barriers problem

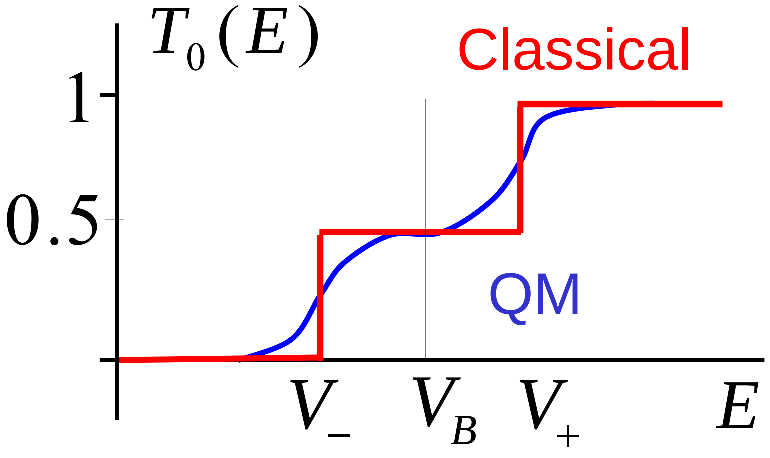
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\{V(R)\pm F(R)\}-E\right]\mathcal{X}_\pm(R)=0$$

two barriers

$$\mathcal{X}_\pm(R)=[\phi_1(R)\pm\phi_2(R)]/\sqrt{2} \qquad |\langle\mathcal{X}_\pm|\phi_1\rangle|^2=1/2$$

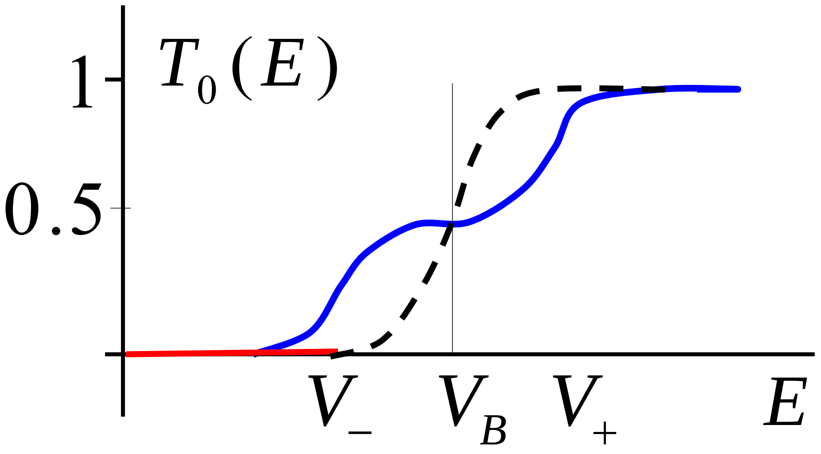


$$T_0(E)=\frac{1}{2}\left[T_+(E)+T_-(E)\right]$$



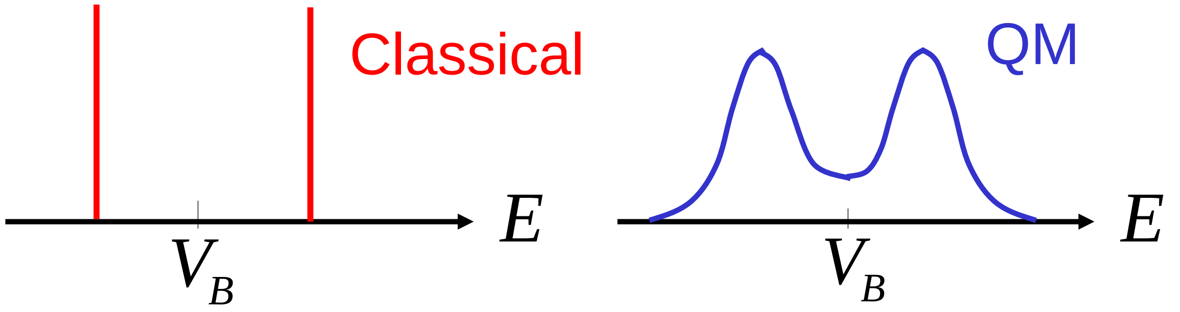
Barrier distributions will reflect channel couplings

In this simple model, channel coupling (no matter what the sign of the coupling potential) enhances fusion below and hinders fusion above the barrier – quite general result



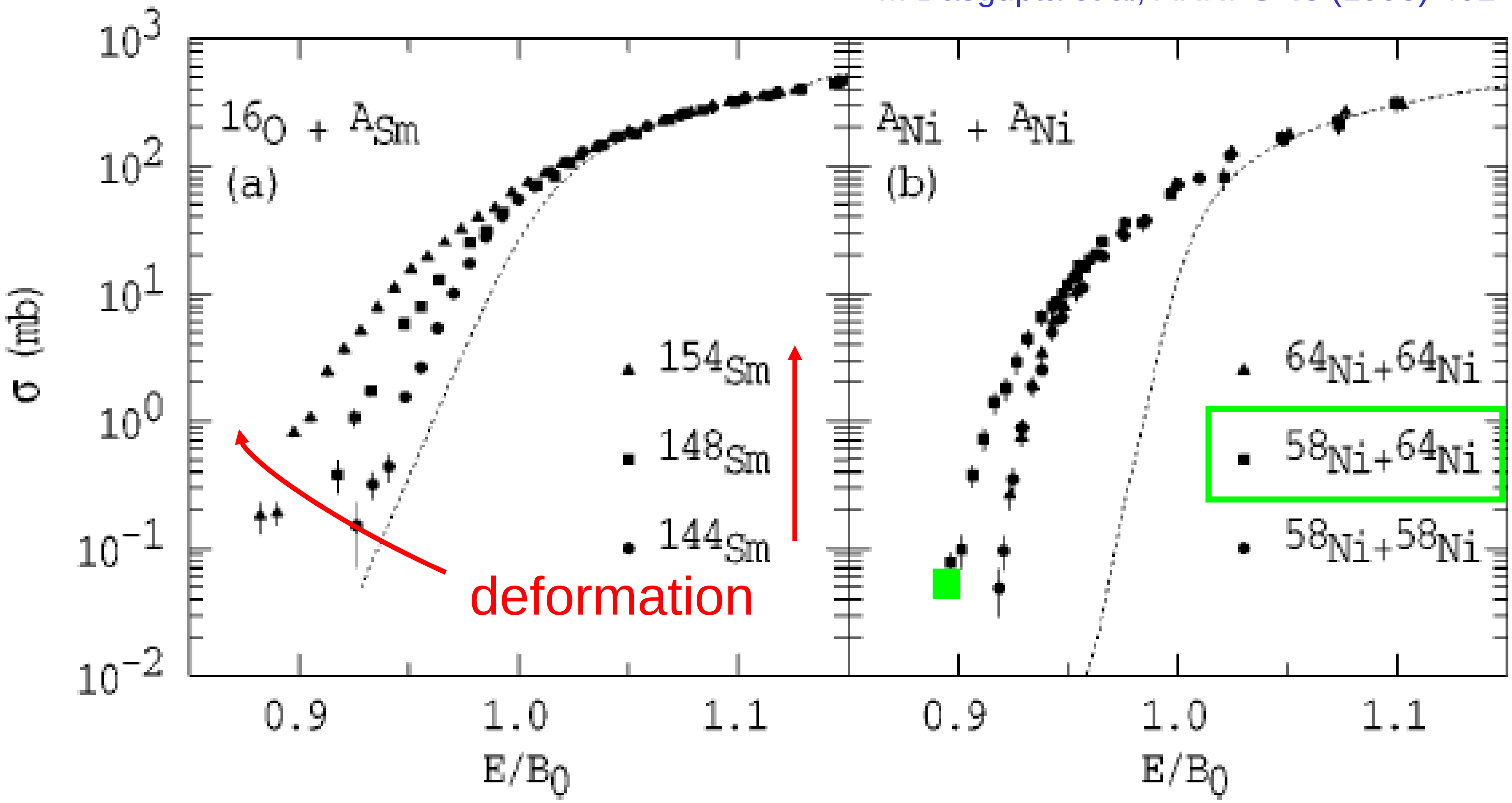
Non-degeneracy of the channels divides the flux incident on the barriers in a more complex way in the different channels (e.g. Beckerman, Rep. Prog. Phys. **51** (1988) 1047)

$$\frac{d^2}{dE^2}[E\sigma(E)] = \frac{\pi R_b^2}{2}\left[\delta(E - V_-) + \delta(E - V_+)\right]$$



Channel coupling – classic examples

M.Dasgupta et al, ARNPS **48** (1998) 401



R.G. Stokstad et al, PRL **41** (1978) 465, PRC **21** (1980) 2427.

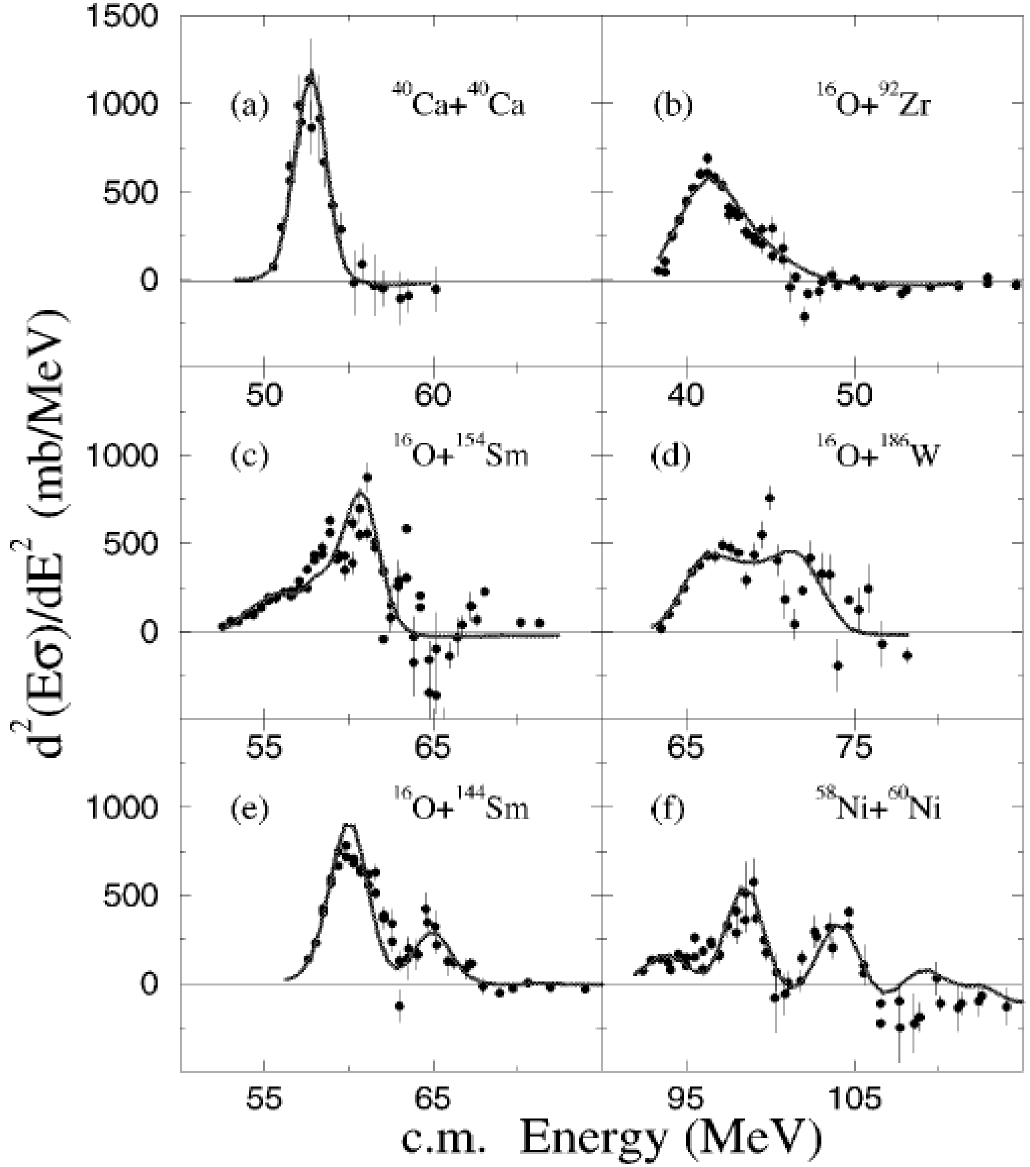
M. Beckerman et al, PRL **45** (1980) 1472, PRC **23** (1981) 1581, PRC **25** (1982) 837.

Empirical and calculated barrier distributions

For data of sufficiently high accuracy and precision, one can compare the values of

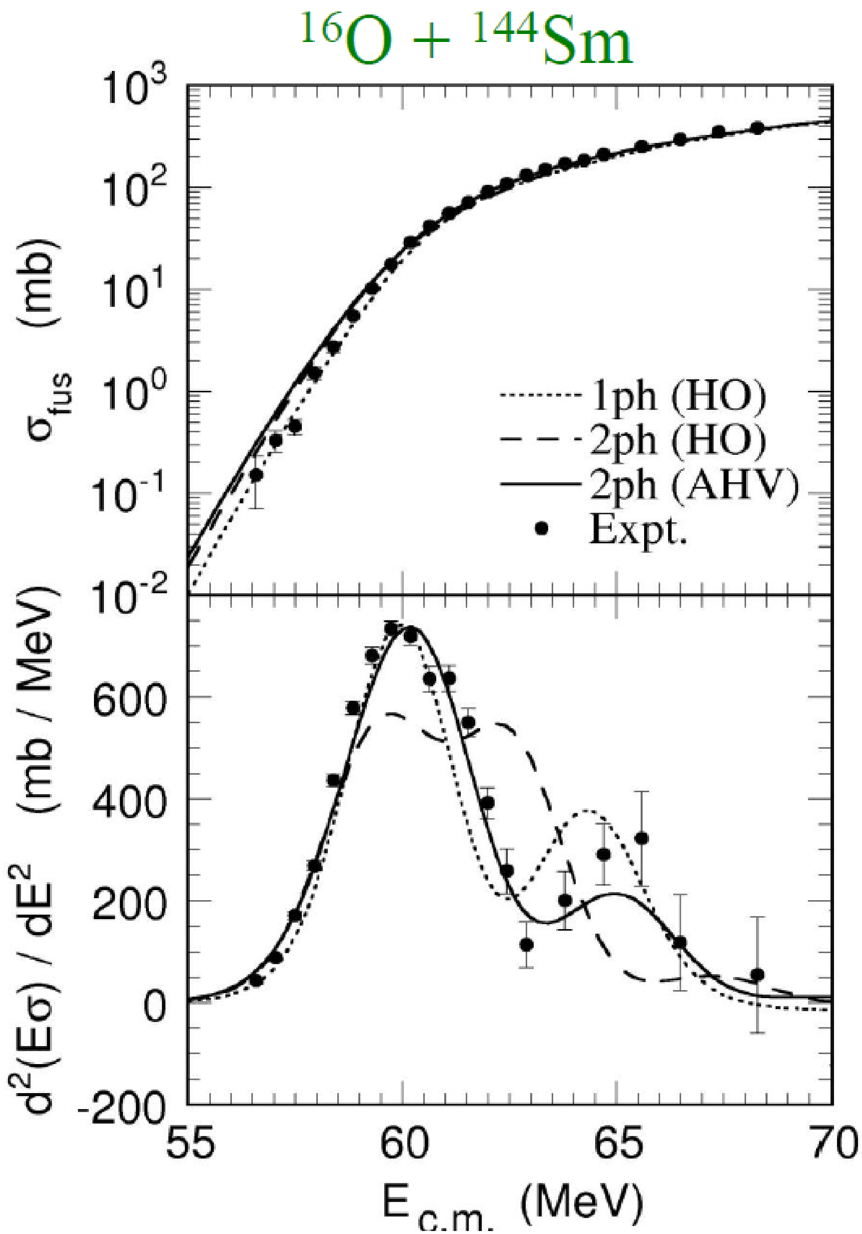
$$\frac{d^2}{dE^2}[E\sigma(E)]$$

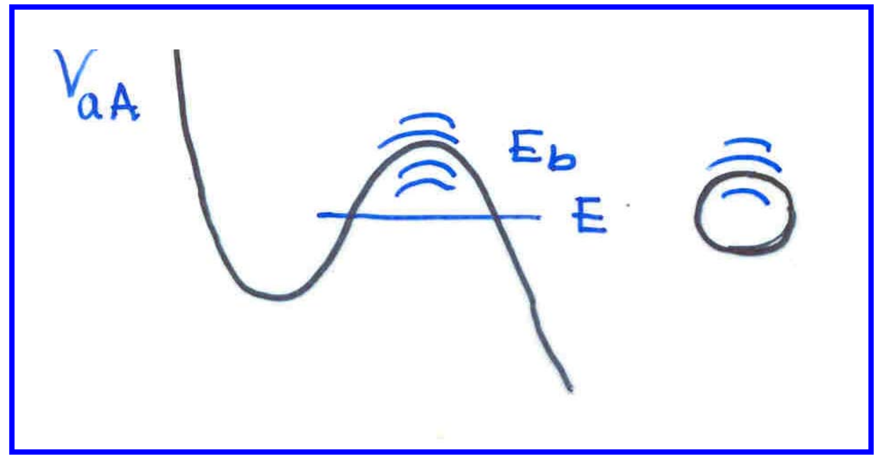
deduced from the data and from detailed coupled channels calculations, including rotational, vibrational single particle or transfer couplings

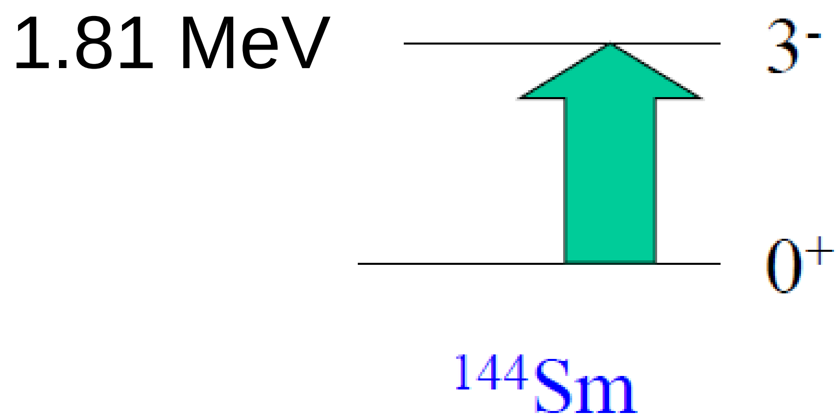


M.Dasgupta et al, ARNPS **48** (1998) 401

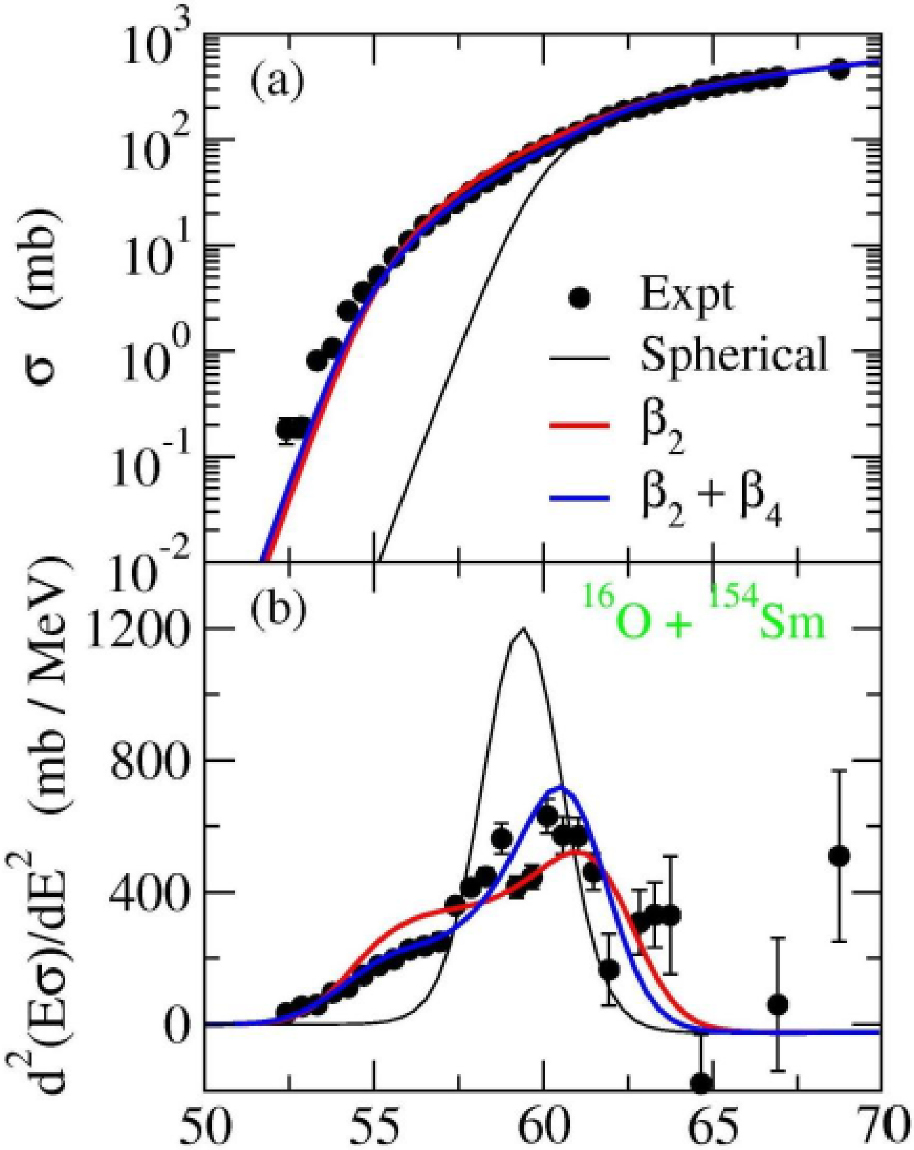
Homework problem I – Vibrational excitations

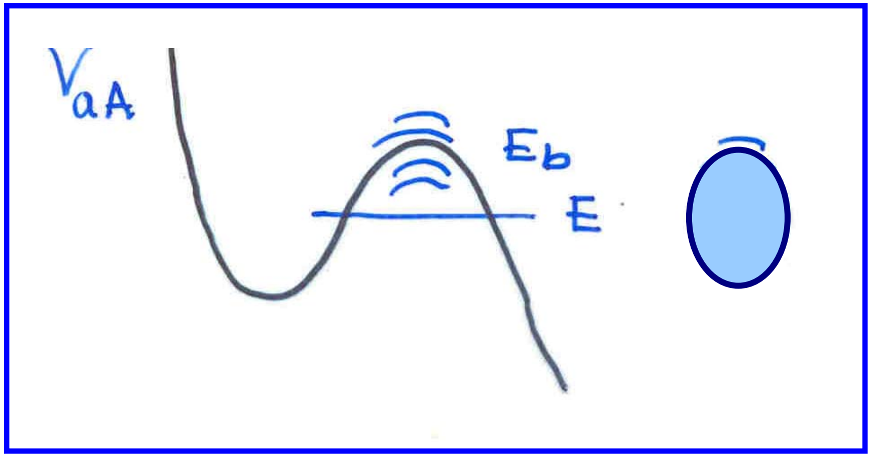


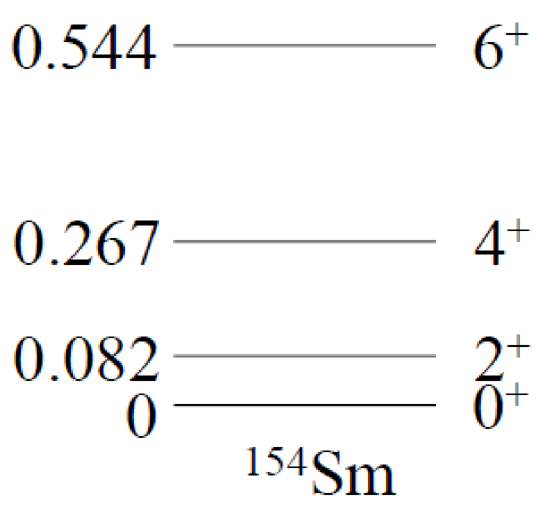




Homework problem II – Rotational excitations

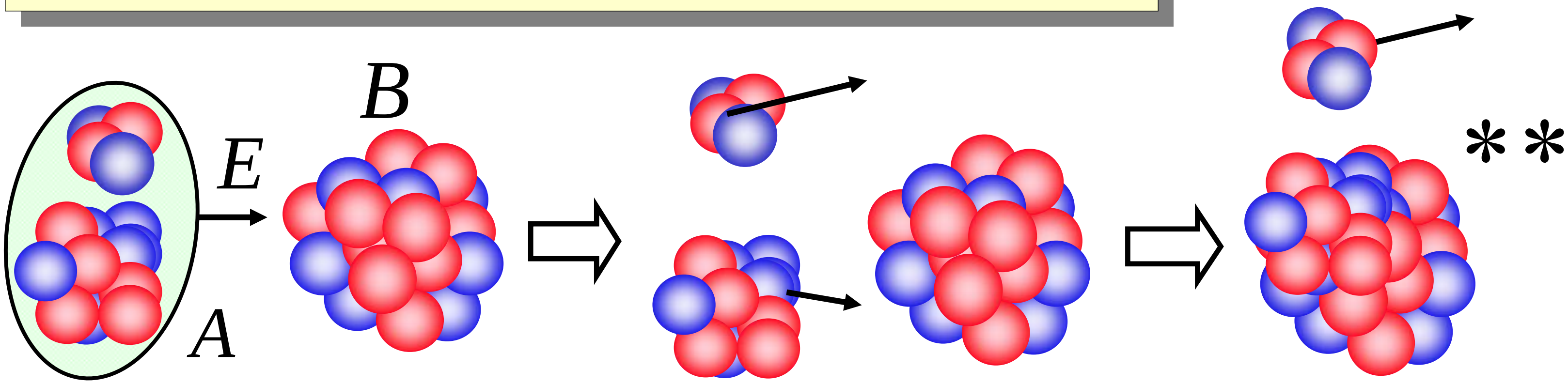




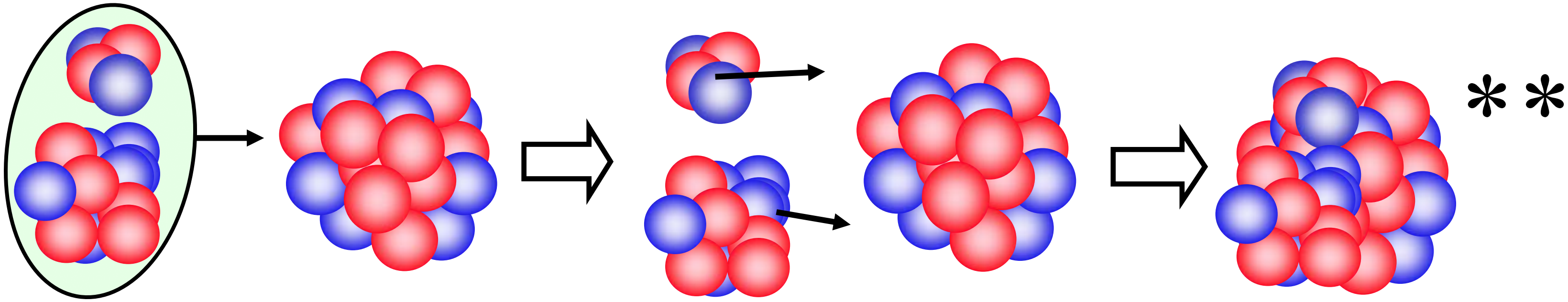


New challenge is presented by weak binding

Break-up, followed by incomplete fusion



Break-up, followed by complete fusion



Weakly-bound and exotic nuclear systems

Stable systems

^{6}Li $\rightarrow$	$^{4}He + d$	$S_{\alpha} = 1.48$ MeV
^{7}Li $\rightarrow$	$^{4}He + t$	$S_{\alpha} = 2.45$ MeV
$^{9}Be \rightarrow {}^{8}Be + n \rightarrow$	$^{4}He + {}^{4}He + n$	$S_{n} = 1.67$ MeV

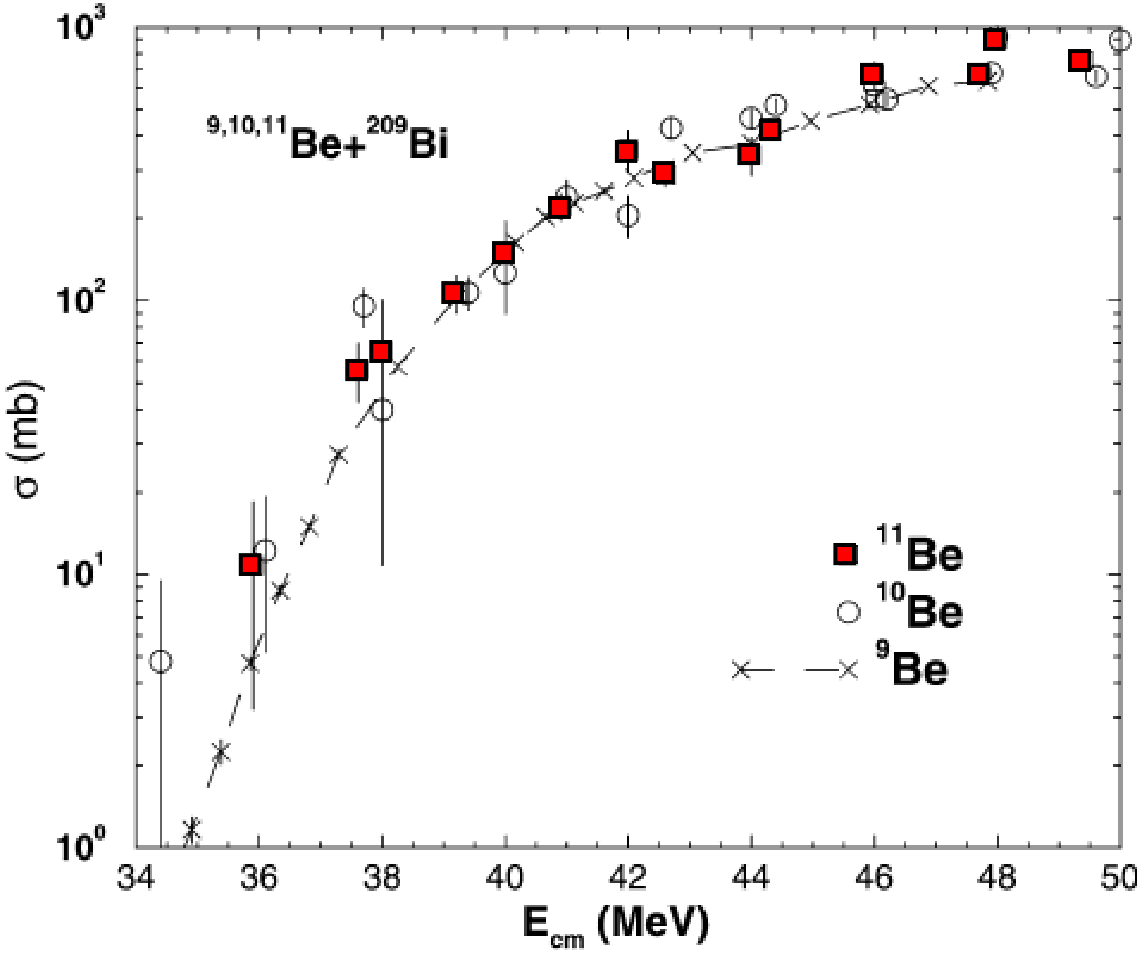
Unstable (exotic) systems

^{6}He $\rightarrow$	$^{4}He + 2n$	$S_{2n} = 0.98$ MeV
^{11}Be $\rightarrow$	$^{10}Be + n$	$S_{n} = 0.50$ MeV
^{11}Li $\rightarrow$	$^{9}Li + 2n$	$S_{2n} = 0.33$ MeV

What are considerations for weakly-bound nuclei

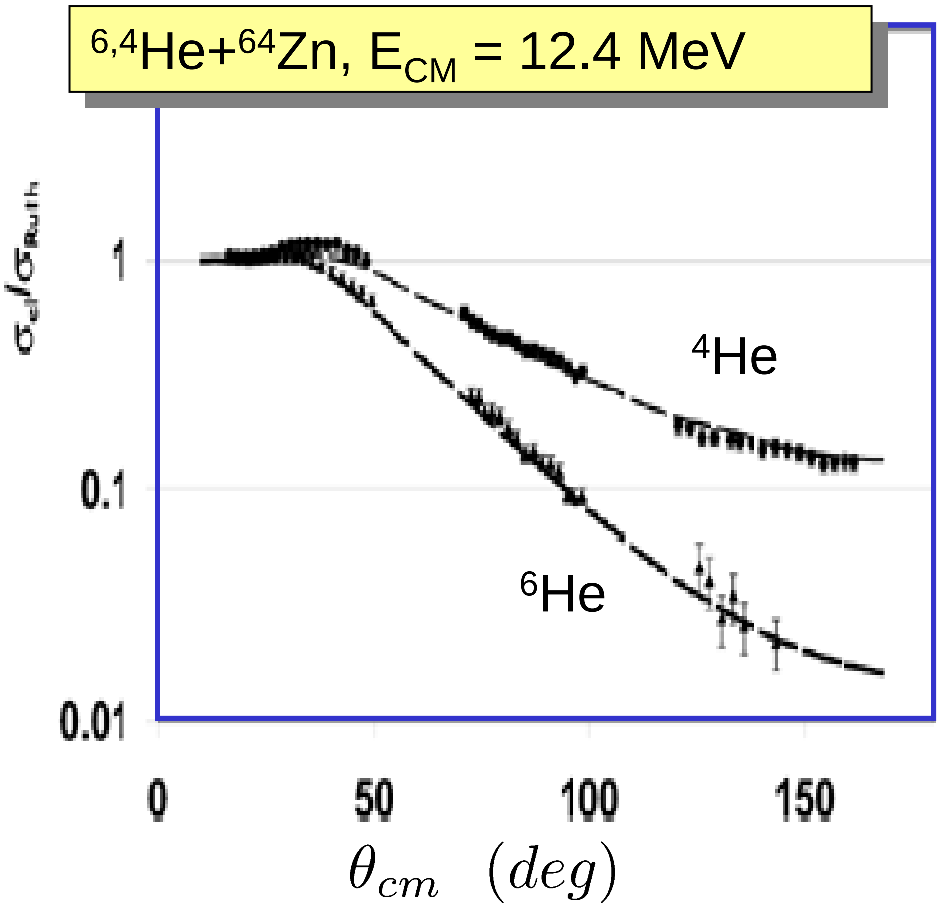
- Static effects due to tails in density distribution - longer tails in ion-ion potential, lowering of Coulomb barrier – might expect larger sub-barrier fusion probabilities
- Dynamical effects due to coupling to states in the continuum (break-up processes), polarization term in optical potential – and expect larger sub-barrier fusion
- Breakup is due to the different forces acting on the fragments, that then separate – and so a reduced expectation of total fusion
- Weak binding leads typically to large +ve Q-values for nucleon transfers
- But experiments with weak beams are very challenging at the sub-barrier energies where the sensitivity is.

Beryllium isotopes – 11Be a halo nucleus case

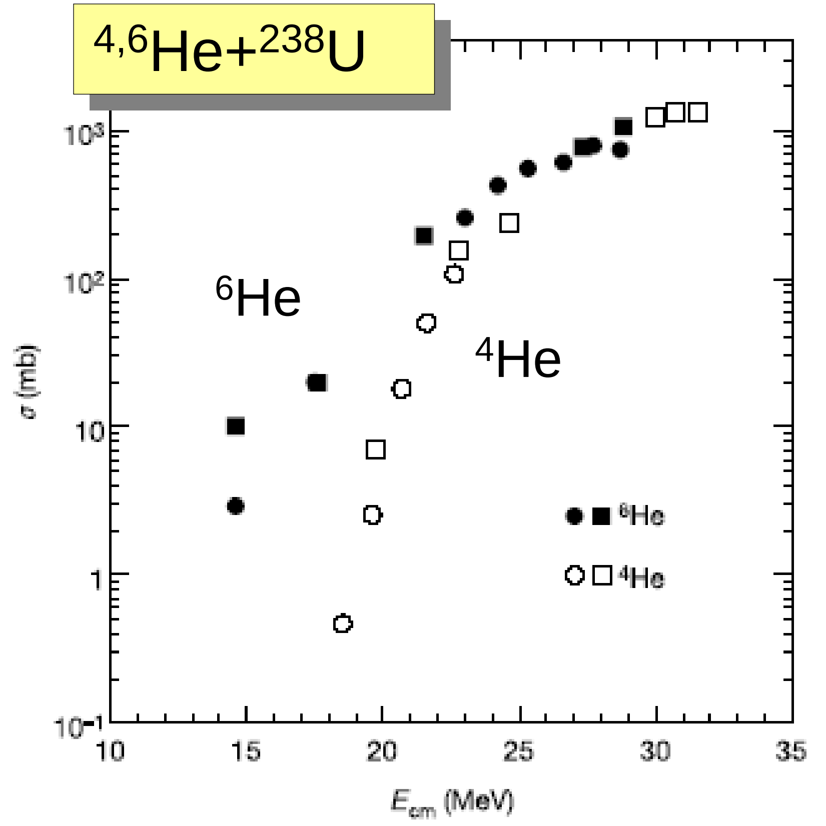


C. Signorini, Nucl.Phys. **A735** (2004) 329

Elastic scattering reflects loose binding



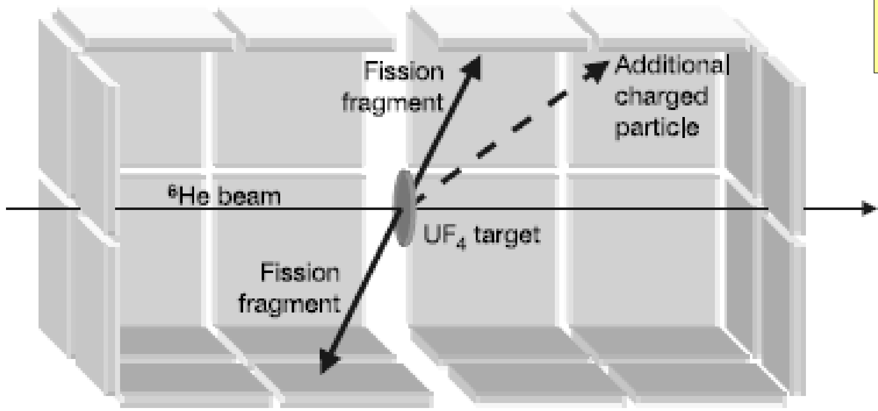
A. Di Pietro et al., Europhys. Lett. **64** (2003) 309



M. Trotta et al., PRL **84** (2000) 2342

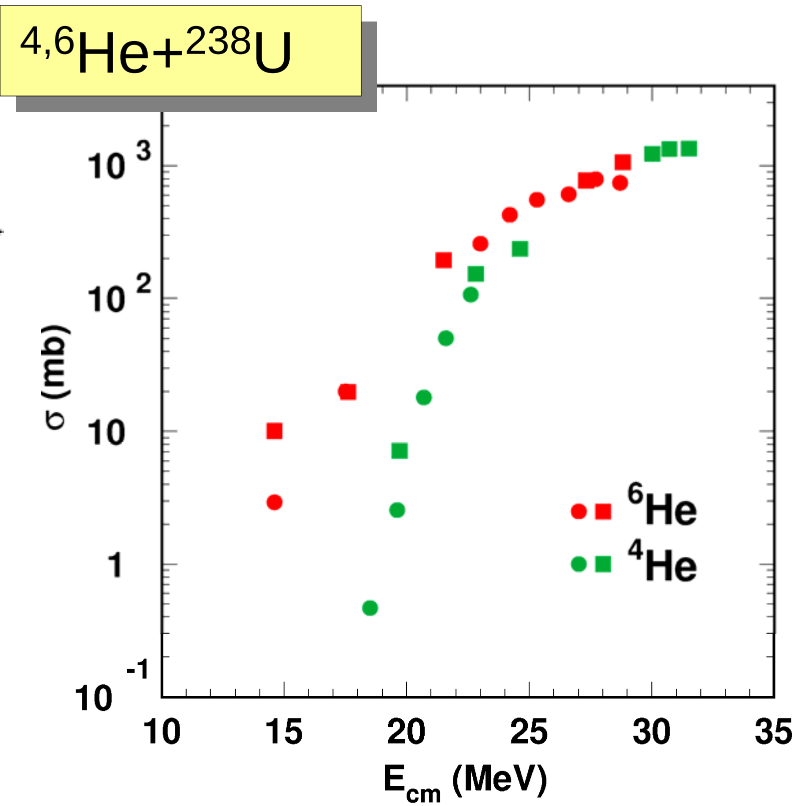
R. Raabe et al., Nature **431** (2004) 823

Exclusive measurements – transfer channels



No enhancement of fusion probability by the neutron halo of ^{6}He

R. Raabe[1,2], J. L. Sida[1*], J. L. Charvet[1], N. Alamanos[1], C. Angulo[3], J. M. Casandjian[4], S. Courtin[5], A. Drouart[1], D. J. C. Durand[1], P. Figuera[6], A. Gillibert[1], S. Heinrich[1], C. Jouanne[1], V. Lapoux[1], A. Lepine-Szily[7], A. Musumarra[6], L. Nalpas[1], D. Pierroutsakou[8], M. Romoli[8], K. Rusek[9] & M. Trotta[8]



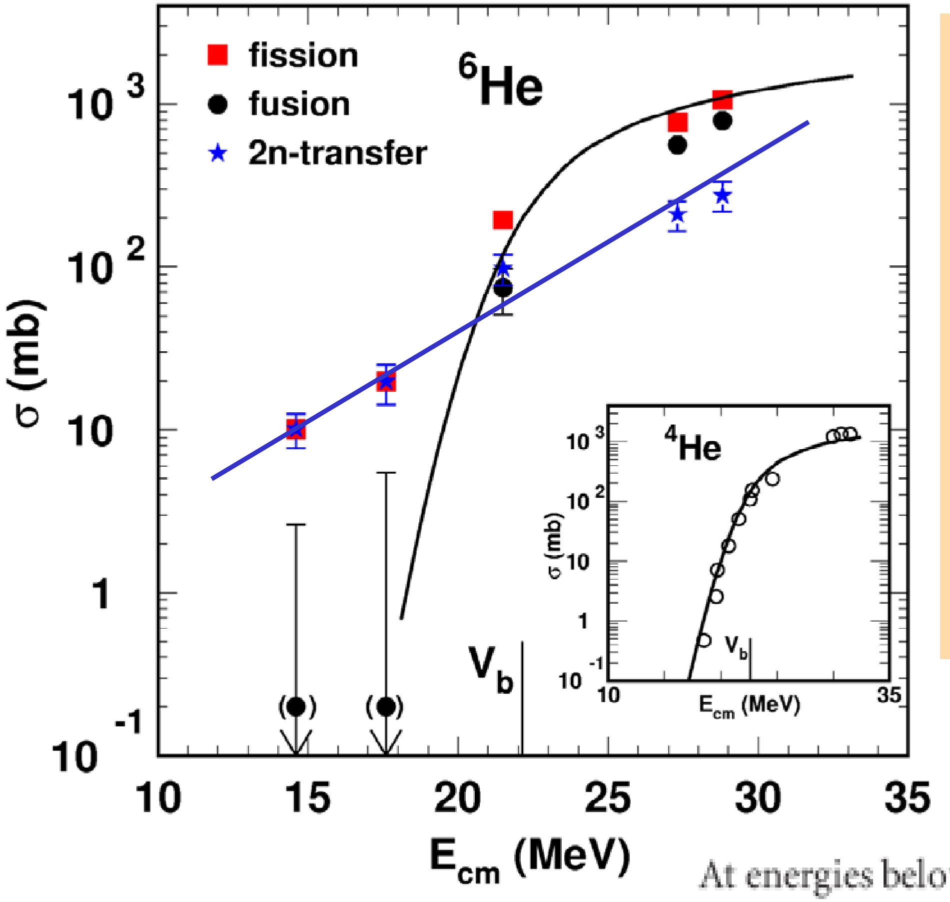
Transfer effects found to be larger than break-up for ^{6}He+^{65}Cu reactions

A. Navin et al., Phys Rev C **70**, 044601

M. Trotta et al., PRL **84** (2000) 2342

R. Raabe et al., Nature **431** (2004) 823

Two-neutron transfer and (no) enhancement



$^{4,6}He+^{238}U$

Measurement of coincidences with alpha-particles to clarify the role of 2n transfer (incomplete fusion)

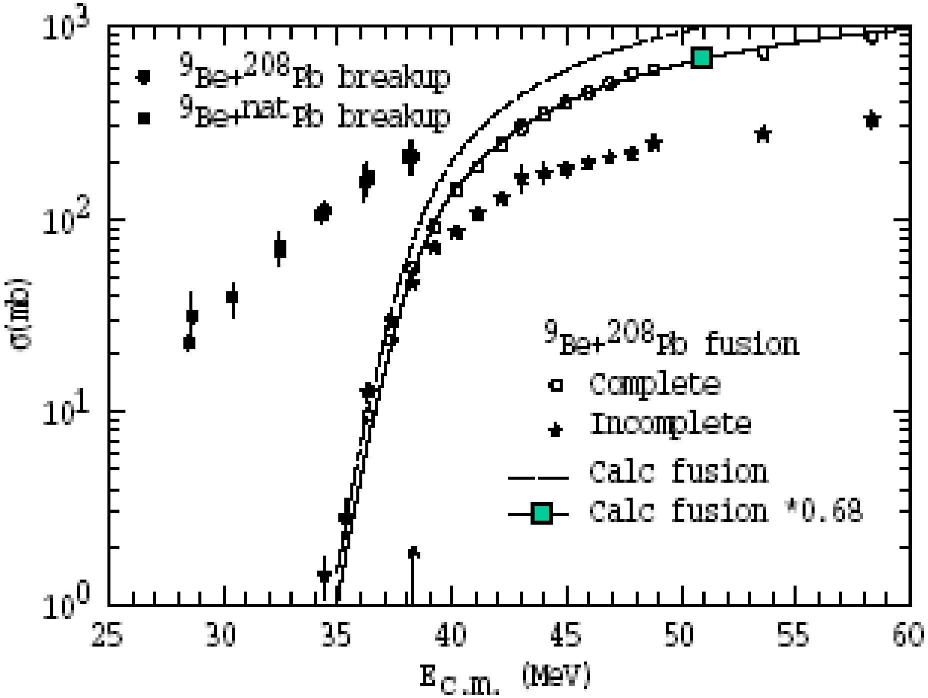
No enhancement
of the fusion cross section

Below the barrier, the two-neutron transfer dominates

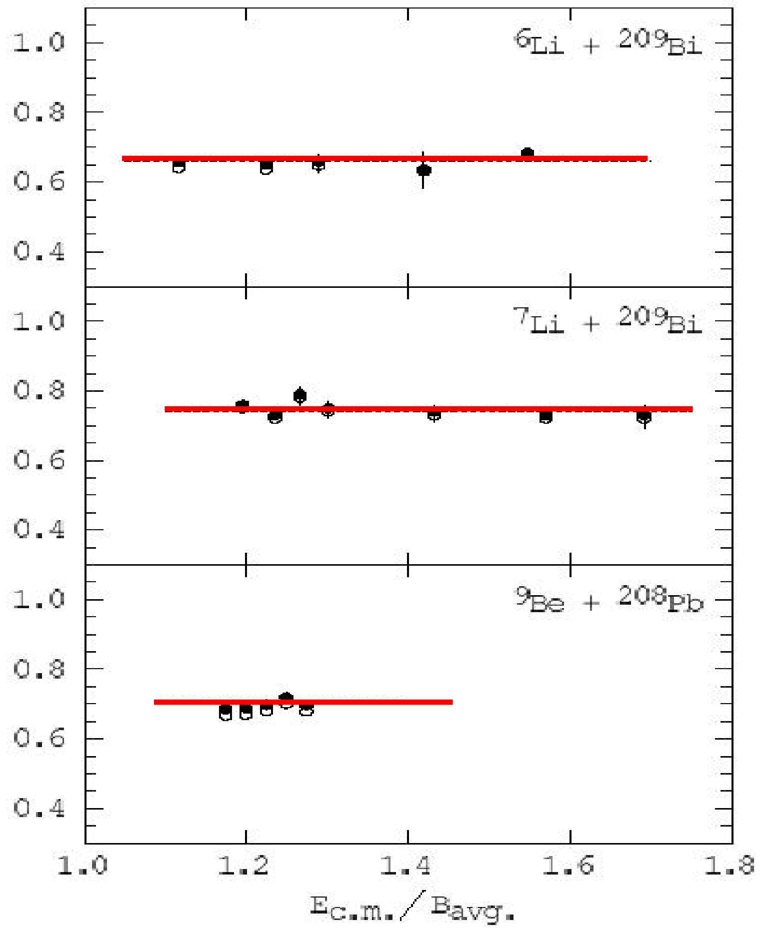
R. Raabe et al., et al, Nature **431** (2004) 823

At energies below the barrier, we find experimentally that there is no substantial enhancement of the fusion cross-section for the halo nucleus ^{6}He. The large observed yield for fission is entirely due to a direct process, the two-neutron transfer to the target nucleus.

Break-up suppressing fusion above the barrier?



several examples suggesting break-up channels suppress the expected complete fusion cross section above the barrier



D.J. Hinde et al., PRL **89** (2002), 272701

M. Dasgupta et al., PRC **70** (2004), 024606

Useful papers/reviews and conferences

- Fusion Conference series: for example
- Fusion03: *From a Tunnelling Nuclear Microscope to Nuclear Processes in Matter,* Progress of Theoretical Physics Supplement **154**, 2004.
- A.B. Balantekin and N. Takigawa, *Quantum Tunnelling in Nuclear Fusion,* Rev. Mod. Phys. 70 (1998) 77-100.
- M. Dasgupta et al., *Measuring Barriers to Fusion,* Ann. Rev. Nucl. Part. Phys. 48 (1998) 401-461
- Workshop: *Heavy-ion Collisions at Energies Near the Coulomb Barrier* 1990, IoP Conference Series, Vol 110 (1990).
- S.G. Steadman et al., ed. *Fusion Reactions Below the Coulomb Barrier*, Springer Verlag (1984)
- M.E. Brandan and G.R. Satchler, *The Interaction between Light Heavy-ions and what it tells us,* Phys. Rep. **285** (1997) 143-243.
- M. Beckerman, *Sub-barrier Fusion of Two Nuclei,* Rep. Prog. Phys. **51** (1988) 1047-1103.
- M.S. Hussein and K.W. McVoy, *Inclusive Projectile Fragmentation in the Spectator Model,* Nucl. Phys. **A445** (1985) 124-139.
- M. Ichimura, *Theory of Inclusive Break-up Reactions,* Conf on Nucl. React. Mechanism, World Scientific (Singapore), 1989, 374-381.
- plus enormous volume of relevant literature – much of which is cited in the above

Session discussed:

1. The physics of barrier-passing models of heavy-ion fusion reactions at energies near-to and well below the Coulomb barrier. The boundary conditions used and the form of the numerical solutions with energy.

2. The importance and the treatment of collective-type channel coupling effects and the distribution-of-barriers method for identifying the strongly coupled channels.

3. The (still ambiguous) role of breakup channels on sub-barrier fusion yields and the importance of neutron transfer channels in the case of neutron rich and weakly-bound (halo-like) projectile nuclei.