TALENT Course 6: Theory for exploring nuclear reaction experiments

Links to nuclear structure – Overlap functions

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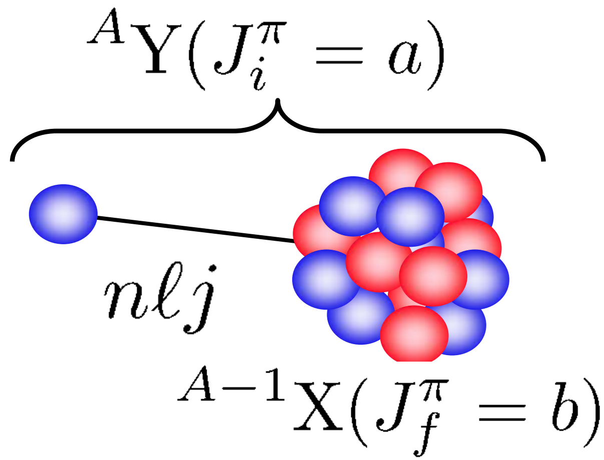
Bound states – Overlaps and spectroscopic factors

In a potential model it is natural to define normalised bound state wave functions.



$$\phi_{n\ell j}^{m}(\vec{r}) = \sum_{\lambda\sigma} (\ell\lambda s\sigma | jm) \frac{u_{n\ell j}(r)}{r} Y_\ell^\lambda(\hat{r}) \chi_s^\sigma$$

$$\int_0^\infty [u_{n\ell j}(r)]^2 dr = 1$$

The potential model wave function approximates the overlap function of the A and A–1-body wave functions (Y and X in the case of an single nucleon) i.e. the overlap

$$(\Phi_X^{b\beta}, A-1 | \Phi_Y^{a\alpha}, A\rangle\rangle \longrightarrow F_{YX}^{a\alpha b\beta}(\vec{r})$$

Need to introduce spectroscopic factors, that relate these normalised single-particle wave functions and the overlaps to take account realistically of nuclear structure effects

Session aims:

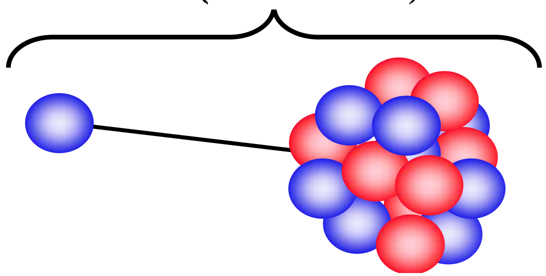
1. To discuss the link between two-body bound state solutions of the Schrodinger equation and the many-body wave functions of the nuclei, i.e. the interface between structure and reaction models. Spectroscopy.

2. Define carefully overlap functions and spectroscopic factors (SF), the expected forms of these overlaps, and IPM estimates of the SF in simple cases. Parentage.

3. The generalisations for overlaps and calculations for reactions involving two nucleons, e.g. (p,t) transfer or fast two-nucleon removal. Expectations in simple cases (e.g. independent particle models) and from the shell model. The transformations that enter such models.

Overlap functions (i) – Notation

$^{A}\mathrm{Y}(J_i = a)$



$^{A-1}\mathrm{X}(J_f = b)$

<u>Assumption</u>: there are reactions and/or processes that perturb/change the motion of just a single nucleon – but not the degrees of freedom of A-1. For example the (p,d) removal/transfer of a neutron <u>from</u> the nucleus Y.

So, here Y and X have antisymmetric many-body wave functions with A (Z, N) and A-1 (Z, N-1) (identical) nucleons.

$$|\Phi_Y^{a\alpha}, A\rangle\!\rangle \equiv |\Phi_Y^{a\alpha}\rangle\!\rangle_A \equiv |\Phi_Y^{a\alpha}\rangle\!\rangle, \;\; i = 1, 2, ..N, k = 1, ..Z$$

$$|\Phi_X^{b\beta}, A-1) \equiv |\Phi_X^{b\beta})_{A-1} \equiv |\Phi_X^{b\beta}), \;\; j = 2, ..N, k = 1, ..Z$$

$$H_Y|\Phi_Y^{a\alpha}\rangle\!\rangle = E_Y|\Phi_Y^{a\alpha}\rangle\!\rangle \qquad H_X|\Phi_X^{b\beta}) = E_X|\Phi_X^{b\beta})$$

$$|E_Y| > |E_X|, \;\; E_Y, E_X < 0, E_X - E_Y = S_n > 0.$$

Overlap functions (ii) – Definition

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) \equiv F_{YX}(1) = F(\vec{r})$$

is a function (wave function) only of the removed neutron coordinate. What are its properties?

$$(H_Y - E_Y)|\Phi_Y^{a\alpha}\rangle\rangle = (T_Y + V_Y - E_Y)|\Phi_Y^{a\alpha}\rangle\rangle = 0$$

$$(H_X - E_X)|\Phi_X^{b\beta}) = (T_X + V_X - E_X))|\Phi_X^{b\beta}) = 0$$

$$T_Y = T_X + T_1,\ V_Y = V_X + V_{1X},\ E_X - E_Y = S_n > 0.$$

$$(\Phi_X^{b\beta}|H_Y - E_Y|\Phi_Y^{a\alpha}\rangle\rangle = (\Phi_X^{b\beta}|\boxed{T_X + V_X} + T_1 + V_{1X} - E_Y|\Phi_Y^{a\alpha}\rangle\rangle = 0$$

$$(T_1 + Sn)(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle + (\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle = 0$$

overlap is a solution of an inhomogeneous equation:

$$(T_1 + Sn)F_{YX}(1) = -(\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle$$

Overlap functions (iii) – Asymptotic properties

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) \equiv F_{YX}(1) = F(\vec{r})$$

$$(T_1 + Sn)F_{YX}(1) = -(\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle$$

source term approach

as neutron moves large distances from the residual nucleus:

$$(\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle \to 0, \text{ as } |\vec{r}| \to \infty$$

and like our two-body model solutions, at large distances

$$T_1 F_{YX}(1) = -S_n F_{YX}(1)$$

$$\left\{F_{YX}(r), \frac{u_{n\ell j}(r)}{r}\right\} \longrightarrow h_\ell(\kappa_b r) \longrightarrow \frac{\exp(-\kappa_b r)}{\kappa_b r}$$

So, two-body calculations with the right neutron separation energy will automatically have the correct long-range forms.

What about their normalization and forms for smaller radii?

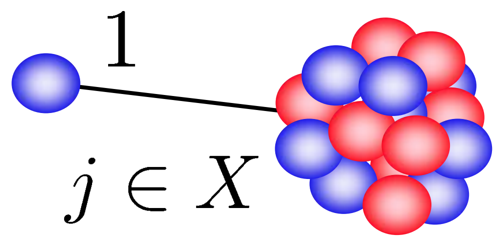
Overlap functions (iv) – Approximations

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) \equiv F_{YX}(1) = F(\vec{r})$$

$$(T_1 + Sn)F_{YX}(1) = -(\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle$$

source term approach

$$V_{1X} = \sum_{j=2,..N,1..Z} V_{NN}(1,j)$$



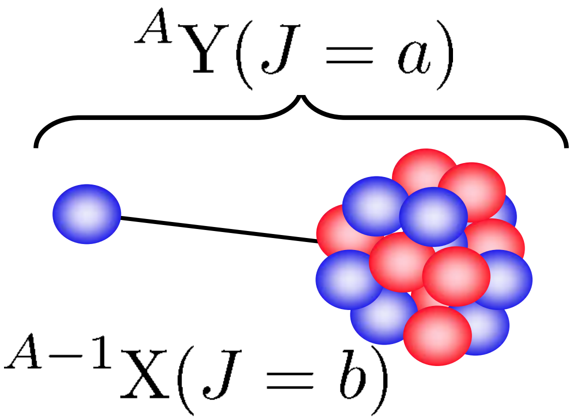
and, thus if $V_{1X} \approx V_{WS}(r)$

$$(T_1 + V_{WS}(r))F_{YX}(1) = -S_n F_{YX}(1)$$

Assuming we can made a reasonable choice for the parameters of a Woods-Saxon binding potential – related to density of X, etc – we can compute a radial overlap for all r.

What about their normalization? Spectroscopic factors

Overlap functions (v) – Spectroscopic factors I



<u>Assumption</u>: there are reactions and/or processes that perturb/change the motion of just a single nucleon – but not the degrees of freedom of A-1. To be specific consider that 1 neutron (of N) is involved.

So, for e.g. $T_{if} = \langle final; \Phi_X^{b\beta} | \mathcal{O}(1) | initial; \Phi_Y^{a\alpha} \rangle_{A,\ldots}$

and there will be equal contributions to the cross section from each of the N (identical) neutrons, thus

$$\sigma_{if} \propto N |T_{if}|^2 = |\tilde{T}_{if}|^2, \quad \tilde{T} = \sqrt{N} T_{if}$$

and the nuclear structure enters via the overlap function since

$$(\Phi_X^{b\beta} | \mathcal{O}(1) | \Phi_Y^{a\alpha} \rangle\!\rangle = \mathcal{O}(1) (\Phi_X^{b\beta} | \Phi_Y^{a\alpha} \rangle\!\rangle = \mathcal{O}(1) F_{YX}^{a\alpha b\beta}(1)$$

Overlap functions (v) – Spectroscopic factors II

We now make the angular momentum couplings explicit :

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) = \sum_{jm}(b\beta jm|a\alpha)\tilde{\varphi}_{jm}(1)$$

where the transferred j is unique only if a or b is spin-zero

Relationship of

$$\tilde{\varphi}_{jm}(1) \longleftrightarrow \phi_{n\ell j}^{m}(1)$$

normalised sp wave function

Definition:

$$\tilde{\varphi}_{jm}(1) = \sqrt{\frac{\mathcal{S}(\ell j)}{N}}\phi_{n\ell j}^{m}(1) = \frac{\mathcal{A}(\ell j)}{\sqrt{N}}\phi_{n\ell j}^{m}(1)$$

Defined this way, the sqrt(N) factor cancels that in the cross section expression (below) and so the many-body (structure) information remains only through the SFs or S-amplitudes

$$\sigma_{if} \propto N|T_{if}|^2 = |\tilde{T}_{if}|^2, \quad \tilde{T} = \sqrt{N}T_{if}$$

Overlap functions (v) – Spectroscopic factors III

So, using the definition of the SF in

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) = \sum_{jm}(b\beta jm|a\alpha)\tilde{\varphi}_{jm}(1)$$

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = \sum_{jm}(b\beta jm|a\alpha)\frac{\mathcal{A}(\ell j)}{\sqrt{N}}\phi_{n\ell j}^{m}(1)$$

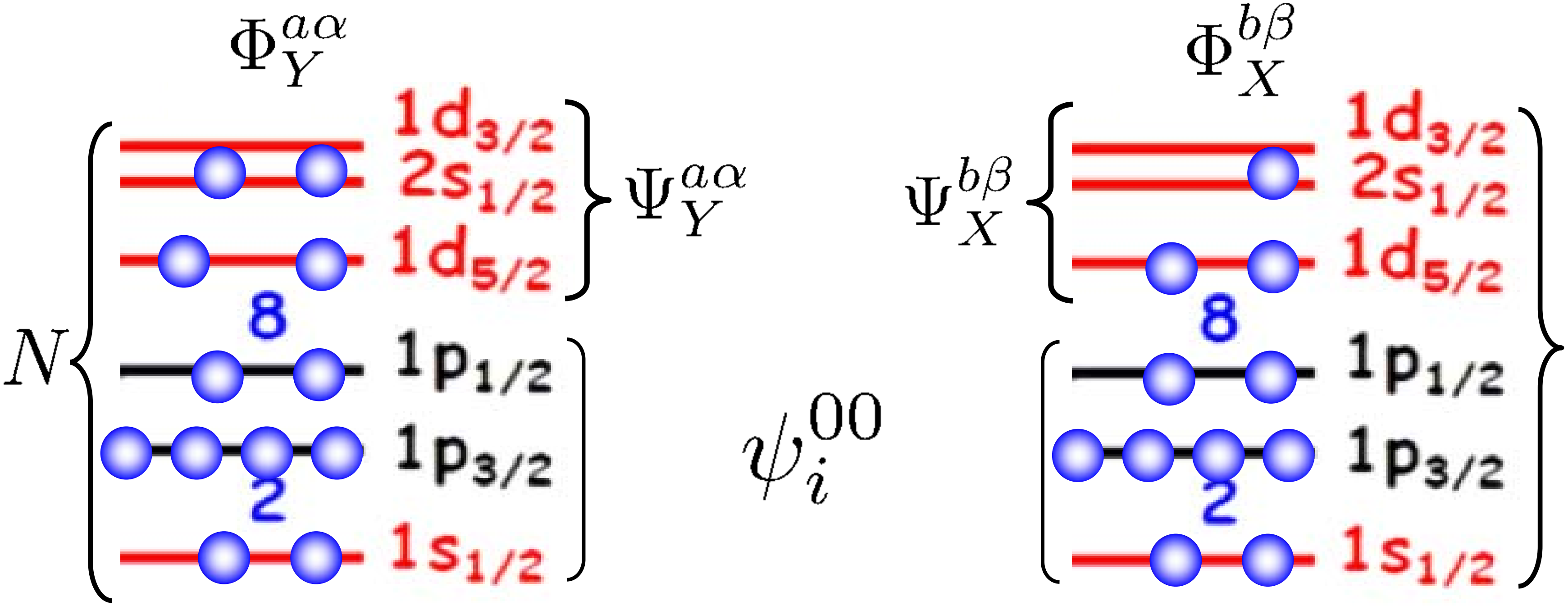
$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{N}\langle\langle\, [\Phi_X^b \otimes \phi_{n\ell j}(1)]_a^\alpha \,|\, \Phi_Y^{a\alpha}\,\rangle\rangle$$

and measures the extent to which Y (a,α) looks like a neutron in the normalised sp wave function (n ℓ j) moving about X (b,β), also referred to as the parentage coefficient.

In this expression (Austern text, p289) inert groups of nucleons that couple to spin-zero – e.g. well bound closed shells – do not contribute to this integral expression for *A* (or *S*).

Overlap functions (v) – Spectroscopic factors IV

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{N} \langle\!\langle\, [\Phi^{b}_{X} \otimes \phi_{n\ell j}(1)]^{\alpha}_{a} \,|\, \Phi^{a\alpha}_{Y} \,\rangle\!\rangle$$



with the result that

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{\mathrm{n}} \langle\, [\Psi^{b}_{X} \otimes \phi_{n\ell j}(1)]^{\alpha}_{a} \,|\, \Psi^{a\alpha}_{Y} \,\rangle$$

Overlap functions (v) – Spectroscopic factors V

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{N} \langle\!\langle\, [\Phi_X^b \otimes \phi_{n\ell j}(1)]_a^\alpha \,|\, \Phi_Y^{a\alpha} \,\rangle\!\rangle$$

Limiting cases I: Extreme independent particle model with one neutron in state (nℓjm) outside a closed shell of $\phi_2, \phi_3 \ldots \phi_N$

$$\langle 1 \ldots N | \Phi_Y^{jm} \rangle \equiv \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_j^m(1) & \phi_j^m(2) & \ldots & \phi_j^m(N) \\ \phi_2(1) & \phi_2(2) & \ldots & \phi_2(N) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_N(1) & \phi_N(2) & \ldots & \phi_N(N) \end{vmatrix}$$

$$\langle 2 \ldots N | \Phi_X^{00} \rangle \equiv \frac{1}{\sqrt{(N-1)!}} \begin{vmatrix} \phi_2(2) & \phi_2(3) & \ldots & \phi_2(N) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_A(2) & \phi_A(3) & \ldots & \phi_N(N) \end{vmatrix}$$

with the result that $\boxed{\mathcal{S}(\ell j) = 1}$ or, more simply (n=1) using

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{\mathrm{n}} \langle\, [\Psi_X^b \otimes \phi_{n\ell j}(1)]_a^\alpha \,|\, \Psi_Y^{a\alpha} \,\rangle$$

Overlap functions (v) – Spectroscopic factors VI

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{\mathrm{n}}\langle\, [\Psi_X^b \otimes \phi_{n\ell j}(1)]_a^\alpha \,|\, \Psi_Y^{a\alpha}\,\rangle$$

<u>Limiting cases II</u>: Similarly, if the neutrons fill a closed subshell (of 2j+1 particles) then, for example, shown with final state (jm)

$$\langle 1\ldots N|\Psi_Y^{00}\rangle \equiv \frac{1}{\sqrt{(2j+1)!}}\begin{vmatrix} \phi_j^m(1) & \phi_j^m(2) & \ldots & \phi_j^m(N) \\ \phi_j^{m-1}(1) & \phi_j^{m-1}(2) & \ldots & \phi_J^{m-1}(N) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_j^{-m}(1) & \phi_j^{-m}(2) & \ldots & \phi_j^{-m}(N) \end{vmatrix}$$

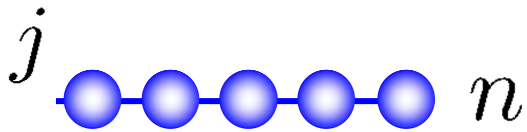
$$\langle 2\ldots N|\Psi_X^{jm}\rangle \equiv \frac{1}{\sqrt{(2j)!}}\begin{vmatrix} \phi_j^{m-1}(2) & \phi_j^{m-1}(3) & \ldots & \phi_j^{m-1}(N) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_j^{-m}(2) & \phi_j^{-m}(3) & \ldots & \phi_j^{-m}(N) \end{vmatrix}$$

with the result that $\boxed{\mathcal{S}(\ell j) = 2j+1}$

Overlap functions (v) – Spectroscopic factors VII

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{\mathrm{n}}\langle\, [\Psi_X^b \otimes \phi_{\ell j}(1)]_a^\alpha \,|\, \Psi_Y^{a\alpha}\,\rangle$$

Other (independent particle model) cases for removal from a state of a given j are less simple, can be worked out, but are given by the coefficients of fractional parentage - cfps



$$\langle\, [\Psi_{n-1}^b \otimes \phi_{\ell j}(1)]_a^\alpha \,|\, \Psi_n^{a\alpha}\,\rangle = ((j^{n-1})b, j; a|\}(j^n)\, a)$$

For low seniority states (where each pair couples to spin zero)

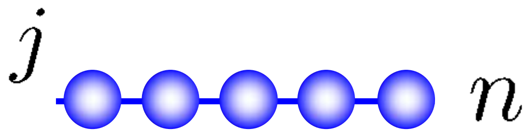
$$((j^{n-1})j, j; 0|\}(j^n)\, 0) = 1, \quad n = \text{even}, \ \text{seniority} = 0$$

$$((j^{n-1})0, j; j|\}(j^n)\, j) = \left(\frac{2j+1-(n-1)}{n(2j+1)}\right)^{\frac{1}{2}},$$

$$n = \text{odd}, \ \text{seniority} = 1$$

$$\mathcal{S}(\ell j) = n((j^{n-1})b, j; a|\}(j^n)\, a)^2$$

Overlap functions (v) – Spectroscopic factors VIII



$$\langle\, [\Psi^{b}_{n-1} \otimes \phi_{\ell j}(1)]^{\alpha}_{a} \,|\, \Psi^{a\alpha}_{n} \,\rangle = ((j^{n-1})b, j; a|\}(j^n)\, a)$$

For low seniority states (where each pair couples to spin zero)

$$((j^{n-1})j, j; 0|\}(j^n)\, 0) = 1, \quad n = \text{even}$$

$$((j^{n-1})0, j; j|\}(j^n)\, j) = \left(\frac{2j+1-(n-1)}{n(2j+1)}\right)^{\frac{1}{2}}, n = \text{odd}$$

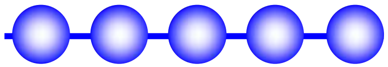
and spectroscopic factors for removal in such paired systems are:

$$\mathcal{S}(\ell j) = n((j^{n-1})j, j; 0|\}(j^n)\, 0)^2 = n, \quad n = \text{even}$$

$$\mathcal{S}(\ell j) = n((j^{n-1})0, j; j|\}(j^n)\, j)^2 = 1 - \frac{n-1}{2j+1}, \quad n = \text{odd}$$

that coincide with the earlier limiting cases for n=1 and n=2j+1

Overlap functions (v) – Spectroscopic factors IX

j  n $\quad \text{cfp} \equiv ((j^{n-1})b, j; a|\}(j^n)\, a)$

<u>Sum Rules:</u> the cfp satisfy $\sum_b ((j^{n-1})b, j; a|\}(j^n)\, a)^2 = 1$

The only cfp for a=0 and n=even is for b=j for which

$$((j^{n-1})j, j; 0|\}(j^n)\, 0) = 1, \quad n = \text{even}$$

but for n=odd several (higher seniority) states and b are possible

i.e. $((j^{n-1})b, j; j|\}(j^n)\, j), \quad n = \text{odd}$

$$\sum_b \mathcal{S}(\ell j) = n \sum_b ((j^{n-1})b, j; j|\}(j^n)\, j)^2 = n$$

i.e. the sum of the SF to all final states of X reached by the neutron (j) transfer/removal is equal to the occupancy of that (nℓj) subshell in the original nucleus Y – a SF sum rule.

Overlap functions (v) – Isospin formalism

Here I have written the formalism in neutron and proton (np) form with (N, Z). If one uses the isospin (indistinguishable nucleons) formalism then N → A but the results are unchanged, except for the need for an isospin Clebsch Gordan coefficient *C*

$$\mathcal{S}_{np} \to C^2 \mathcal{S}_{iso}, \quad C = (T_X \tau_X t \tau | T_Y \tau_Y)$$

with removal of a nucleon with $[t\tau]$, since we now have:

$$(\Phi_X^{b\beta}[T_X \tau_X] | \Phi_Y^{a\alpha}[T_Y \tau_Y] \rangle\rangle =$$

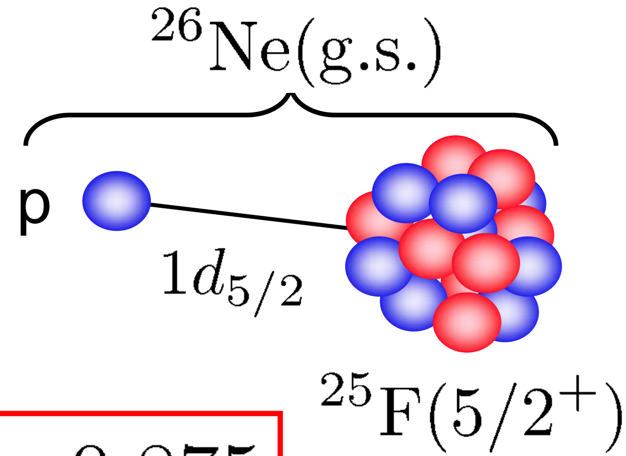
$$= \sum_{jm} (b\beta jm|a\alpha)(T_X \tau_X t\tau | T_Y \tau_Y) \tilde{\varphi}_{jm}(1) \chi_{t\tau}(1)$$

So, e.g. for $({}^{25}\mathrm{F}(5/2^+, E^*)|{}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.})\rangle\rangle$

$$C = (\tfrac{7}{2}\,\tfrac{7}{2}\,\tfrac{1}{2}\,{-\tfrac{1}{2}}|3\,3) = \sqrt{14}/4, \quad C^2 = 0.875$$

Overlap functions (v) – Spectroscopic factors X

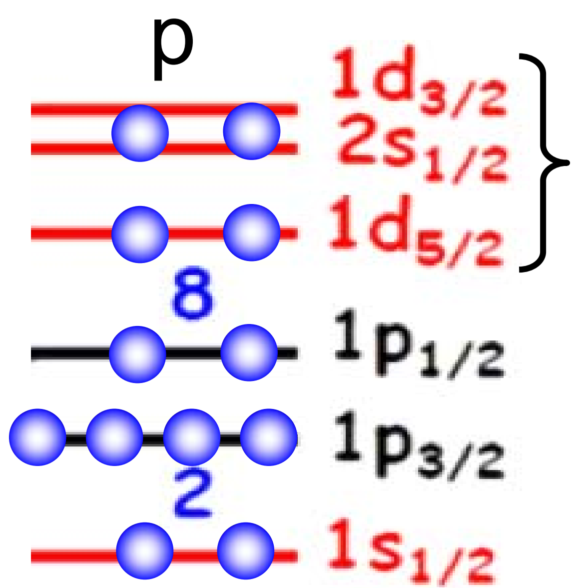
$$\langle {}^{25}\mathrm{F}(5/2^+, E^*)|{}^{26}\mathrm{Ne}(0^+, \mathrm{g.s.})\rangle\rangle$$



USDA sd-shell model overlap from e.g. OXBASH (*Alex Brown et al.).* Provides spectroscopic factors but **not** the bound state radial wave function.

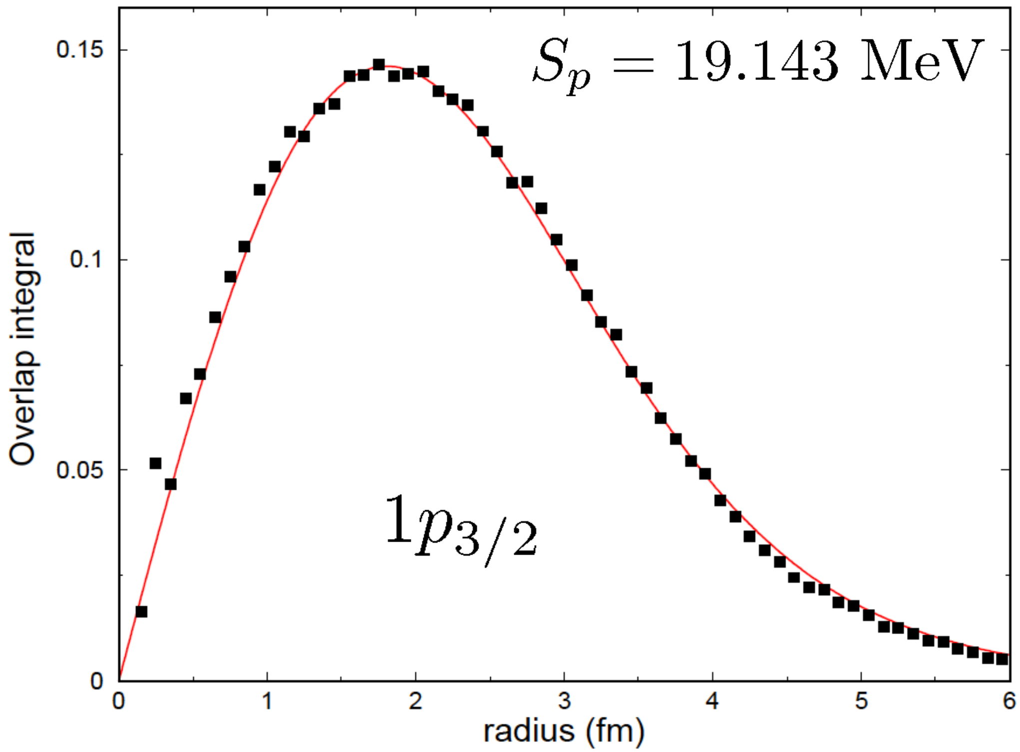
$$C^2 = 0.875$$

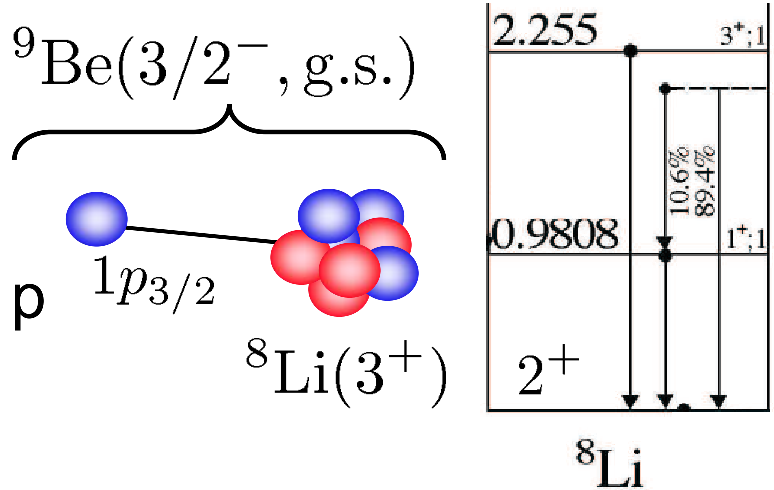
```
 -- core state --- - overlap state -  (1 2  5)(
 2j2t p   n      e  2j2t p   n      e     s
            -59.414              -81.625
  5 7 +   1   0.000 0 6 +    1   0.000  1.79039
  5 7 +   2   3.756 0 6 +    1   0.000  0.02316
  5 7 +   3   4.799 0 6 +    1   0.000  0.01084
  5 7 +   4   5.631 0 6 +    1   0.000  0.00012
  5 7 +   5   6.022 0 6 +    1   0.000  0.00589
  5 7 +   6   6.504 0 6 +    1   0.000  0.00044
  5 7 +   7   6.796 0 6 +    1   0.000  0.00002
  5 7 +   8   8.034 0 6 +    1   0.000  0.00006
  5 7 +   9   8.186 0 6 +    1   0.000  0.00097
  5 7 +  10   8.398 0 6 +    1   0.000  0.00006
 total  =                               1.83196
 centroid =   0.102     centroids =    0.000
 centroid*= -22.313     centroids =  -22.211
```



Bound states – microscopic overlaps for light nuclei

$$\langle {}^8\mathrm{Li}(3^+, 2.225) | {}^9\mathrm{Be}(3/2^-, \mathrm{g.s.})\rangle$$





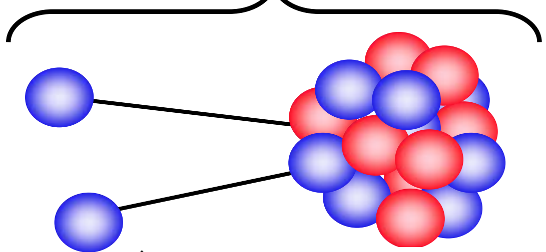
- Microscopic overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a several cases: at

http://www.phy.anl.gov/theory/research/overlap/

Normalised bound state in Woods-Saxon potential well x $(0.23)^{1/2}$ Spectroscopic factor

$$r_V = r_{so} = \text{fitted},\ a_V = a_{so} = \text{fitted},\ V_{so} = 6.0$$

Two-nucleon (neutron) overlap functions



Assumption: there are reactions and/or processes that perturb/change the motion of just a two nucleons – but not the degrees of freedom of A-2. For example the (p,t) removal/transfer of neutrons from Y.

So, here Y and X have antisymmetric many-body wave functions with A (Z, N) and A-1 (Z, N-1) (identical) nucleons.

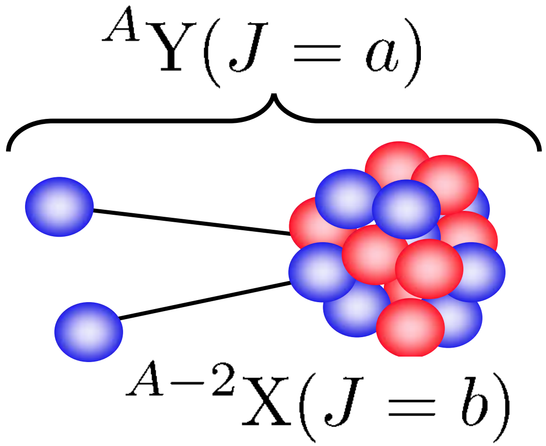
$$|\Phi_Y^{a\alpha}, A\rangle\rangle \equiv |\Phi_Y^{a\alpha}\rangle\rangle_A \equiv |\Phi_Y^{a\alpha}\rangle\rangle, \;\; i = 1, 2, ..N, k = 1, ..Z$$

$$|\Phi_X^{b\beta}, A-2) \equiv |\Phi_X^{b\beta})_{A-2} \equiv |\Phi_X^{b\beta}), \;\; j = 3, ..N, k = 1, ..Z$$

$$H_Y|\Phi_Y^{a\alpha}\rangle\rangle = E_Y|\Phi_Y^{a\alpha}\rangle\rangle \qquad H_X|\Phi_X^{b\beta}) = E_X|\Phi_X^{b\beta})$$

$$|E_Y| > |E_X|, \;\; E_Y, E_X < 0, E_X - E_Y = S_{2n} > 0.$$

Overlap functions – Two-nucleon amplitudes I



Assumption: there are reactions and/or processes that perturb/change the motion of just a two nucleons – but not the degrees of freedom of A-2. To be specific consider that 2 neutrons (from N) are involved.

So, for e.g. $T_{if} = \langle final; \Phi_X^{b\beta} | \mathcal{O}(1,2) | initial; \Phi_Y^{a\alpha} \rangle_{A,...}$

and there will be equal contributions to the cross section from each pair of the N (identical) neutrons, thus

$$\sigma_{if} \propto \frac{N(N-1)}{2} |T_{if}|^2 = |\tilde{T}_{if}|^2, \quad \tilde{T} = \sqrt{\frac{N(N-1)}{2}} T_{if}$$

$$(\Phi_X^{b\beta} | \mathcal{O}(1,2) | \Phi_Y^{a\alpha} \rangle\rangle = \mathcal{O}(1,2)(\Phi_X^{b\beta} | \Phi_Y^{a\alpha} \rangle\rangle = \mathcal{O}(1) F_{YX}^{a\alpha b\beta}(1,2)$$

Overlap functions – Two-nucleon amplitudes II

We now make the angular momentum couplings explicit :

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = \sum_{IM}(b\beta IM|a\alpha)[\overline{\tilde{\varphi}_{j_1}(1)\otimes\tilde{\varphi}_{j_2}(2)}]_I^M$$

Definition:

$$= \sum_{IM}(b\beta IM|a\alpha)\frac{\mathcal{C}(I,j_1,j_2)}{\sqrt{\frac{1}{2}N(N-1)}}[\overline{\phi_{\ell_1 j_1}(1)\otimes\phi_{\ell_2 j_2}(2)}]_I^M$$

Again, defined this way, the sqrt(N[N-1]/2) factor cancels that in the cross section expression (below) and so the many-body (structure) information remains only through the two nucleon amplitudes, the C's in the overlap above

$$\sigma_{if} \propto \frac{N(N-1)}{2}|T_{if}|^2 = |\tilde{T}_{if}|^2, \quad \tilde{T} = \sqrt{\frac{N(N-1)}{2}}T_{if}$$

Overlap functions – Two-nucleon amplitudes III

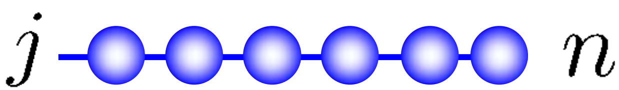
So, using these definitions the TNA are:

$$\mathcal{C}(I,j_1,j_2) = \sqrt{\frac{N(N-1)}{2}} \langle\!\langle\, [\Phi_X^b \otimes [\ldots]_I^M]_a^\alpha \,|\, \Phi_Y^{a\alpha} \,\rangle\!\rangle$$

with $$[\ldots]_I^M = [\overline{\phi_{\ell_1 j_1}(1) \otimes \phi_{\ell_2 j_2}(2)}]_I^M$$

and measure the extent to which Y (a,α) looks like two neutrons in normalised sp wave functions ($n_1\ \ell_1\ j_1$) and ($n_2\ \ell_2\ j_2$) coupled to total angular momentum I, moving about X (b,β), again related to fractional parentage coefficients of the two-particle type.

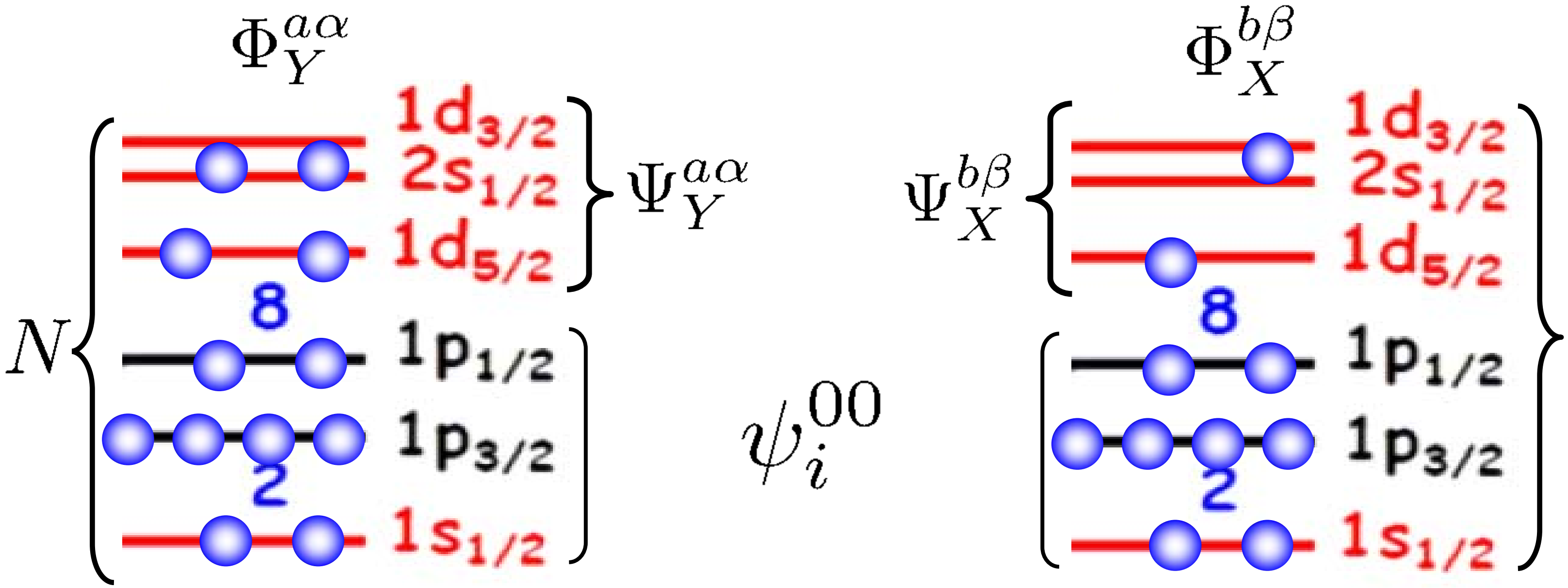
E.g., for pair removal from a single j orbital



$$\text{cfp} \equiv ((j^{n-2})b,(j^2)I;a|\}(j^n)\,a)$$

Overlap functions – Two-nucleon amplitudes IV

$$\mathcal{C}(I,j_1,j_2) = \sqrt{½N(N-1)}\langle\!\langle\, [\Phi^b_X \otimes [\ldots]^M_I]^\alpha_a \,|\, \Phi^{a\alpha}_Y \,\rangle\!\rangle$$

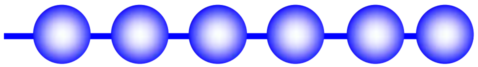


with the result that

$$\mathcal{C}(I,j_1,j_2) = \sqrt{n_1 n_2}\langle\!\langle\, [\Phi^b_X \otimes [\ldots]^M_I]^\alpha_a \,|\, \Phi^{a\alpha}_Y \,\rangle\!\rangle$$

$$\mathcal{C}(I,j,j) = \sqrt{½\mathrm{n}(\mathrm{n}-1)}\langle\!\langle\, [\Phi^b_X \otimes [\ldots]^M_I]^\alpha_a \,|\, \Phi^{a\alpha}_Y \,\rangle\!\rangle$$

Overlap functions – Two-nucleon amplitudes V

j  n $\langle\, [\Psi^{b}_{n-2} \otimes [\ldots]^{M}_{I})]^{\alpha}_{a} \,|\, \Psi^{a\alpha}_{n} \,\rangle$

$$= ((j^{n-2})b, (j^2)I; a|\}(j^n)\, a)$$

For low seniority states (where each pair couples to spin zero)

For even n

$$((j^{n-2})v = 0\,0, (j^2)0|(j^n)0) = \left[\frac{2j+3-n}{(n-1)(2j+1)}\right]^{1/2}$$

$$((j^{n-2})v = 2\;\, J, (j^2)J|(j^n)0) = \left[\frac{2(n-2)}{(n-1)}\,\frac{(2J+1)}{(2j-1)(2j+1)}\right]^{1/2}$$

For odd n, see for e.g. N. Glendenning, Phys Rev 137 (1965) B106

$$((j^{n-2})v = 1\, j, (j^2)I|(j^n)j) = \ldots$$

$$\mathcal{C}(I, j, j) = \sqrt{\tfrac{1}{2}\mathrm{n}(\mathrm{n}-1)}\,((j^{n-2})b, (j^2)I; a|\}(j^n)\, a)$$

Overlap functions – Independent particle model

e.g. $\langle {}^{26}\mathrm{Ne}(0^+)|{}^{28}\mathrm{Mg}(0^+,\mathrm{g.s.})\rangle\rangle$

$$z = 4,\;\; j = {}^5\!/\!_2$$

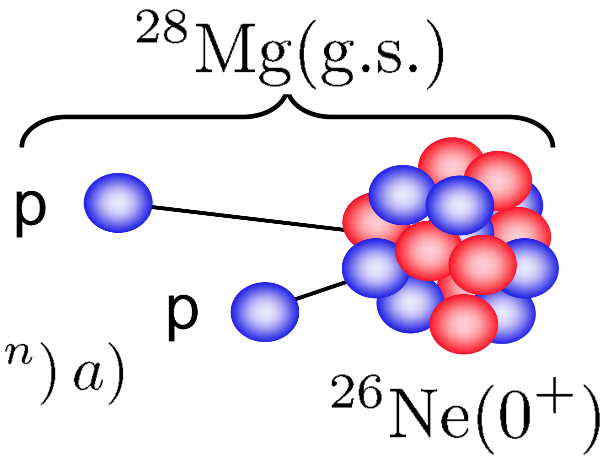
$$\mathcal{C}(I,j,j) = \sqrt{\frac{\mathrm{n(n-1)}}{2}}((j^{n-2})b,(j^2)I;a|\}(j^n)\,a)$$

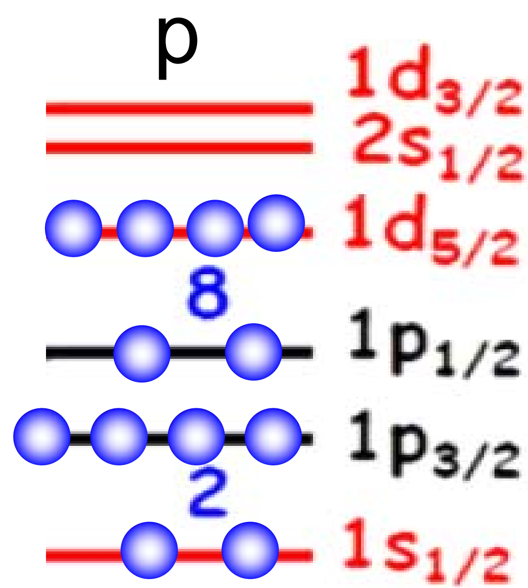
$$a = b = 0, I = 0, n = 4$$

$$((j^{n-2})v = 0\,0,(j^2)0|(j^n)0) =$$

$$\left[\frac{2j+3-n}{(n-1)(2j+1)}\right]^{1/2} = \sqrt{\frac{4}{15}}$$

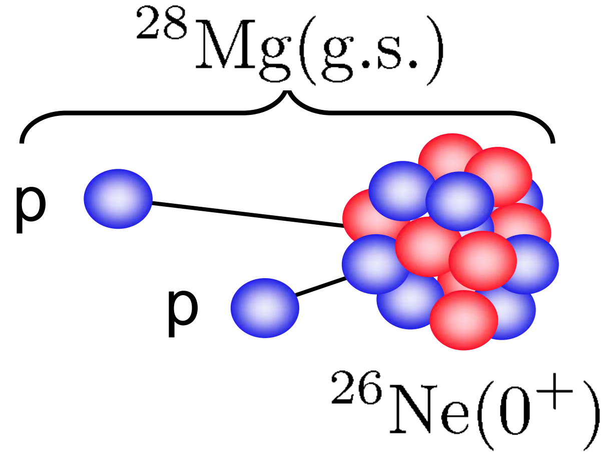
$$\mathcal{C}(0,{}^5\!/\!_2,{}^5\!/\!_2) = \sqrt{\frac{4\times 3}{2}\frac{4}{15}} = 1.2649$$





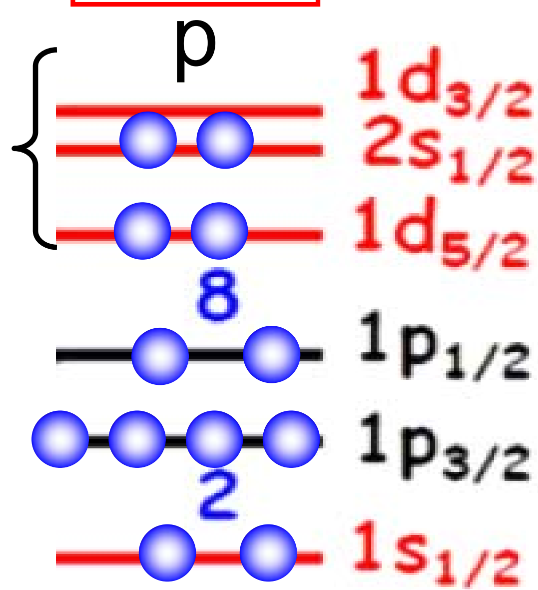
Overlap functions – Shell model TNA

$$({}^{26}\mathrm{Ne}(0^+, E^*)|{}^{28}\mathrm{Mg}(0^+, \mathrm{g.s.})\rangle\rangle$$



sd-shell model overlap - oxbash

$$z = 4$$




```
! Two-nucleon spectroscopic amplitudes A(DJ,DT) =
! = -<f||| [a+(k1)a+(k2)]^(DJ,DT) |||i>/SQRT{(2Jf+1)(
! with Edmonds (de-Shalit Talmi) reduced matrix eleme
!
! For n,l,j =   1.0  2.0  2.5  label  k =   4
! For n,l,j =   1.0  2.0  1.5  label  k =   5
! For n,l,j =   2.0  0.0  0.5  label  k =   6
!
! Ji,   Jf,   Ti,   Tf,
! DJ,   Ni,   Nf,        Ef,        Ei,       Exi,       Exf,
!
 0.0,  0.0,  3.0,  2.0,  0.0,  0.0,
 0.0,   1.,   1., -120.533,  -81.625,    0.000,     0.000,
  5,  5,   0.00000,  -0.30156, ! k1,k2,A(DT=0),A(DT=1)
  4,  4,   0.00000,  -1.04698,
  6,  6,   0.00000,  -0.30495,
  0,
 0.0,   2.,   1., -120.533,  -77.813,    3.812,     0.000,
  5,  5,   0.00000,  -0.06686,
  4,  4,   0.00000,  -0.25604,
  6,  6,   0.00000,   0.00840,
  0,
```

Overlap functions – additional needs (p,t) reactions

$$T_{if} = \langle \phi_t \ldots \Phi_X^{b\beta} | \mathcal{O}(1,2) | \phi_p \ldots \Phi_Y^{a\alpha} \rangle$$ e.g. DWBA

and the proton couples to that part of the wave function (overlap) with the two neutrons in a relative s-state (ℓ=0) with total spin S=0

$$(\Phi_X^{b\beta} | \Phi_Y^{a\alpha} \rangle\rangle = \sum_{IM} (b\beta IM | a\alpha) \frac{\mathcal{C}(I, j_1, j_2)}{\sqrt{\frac{1}{2}N(N-1)}} [\overline{\phi_{\ell_1 j_1}(1) \otimes \phi_{\ell_2 j_2}(2)}]_I^M$$

needs two extra considerations (transformations)

I. recoupling of angular momenta of overlap from jj → LS

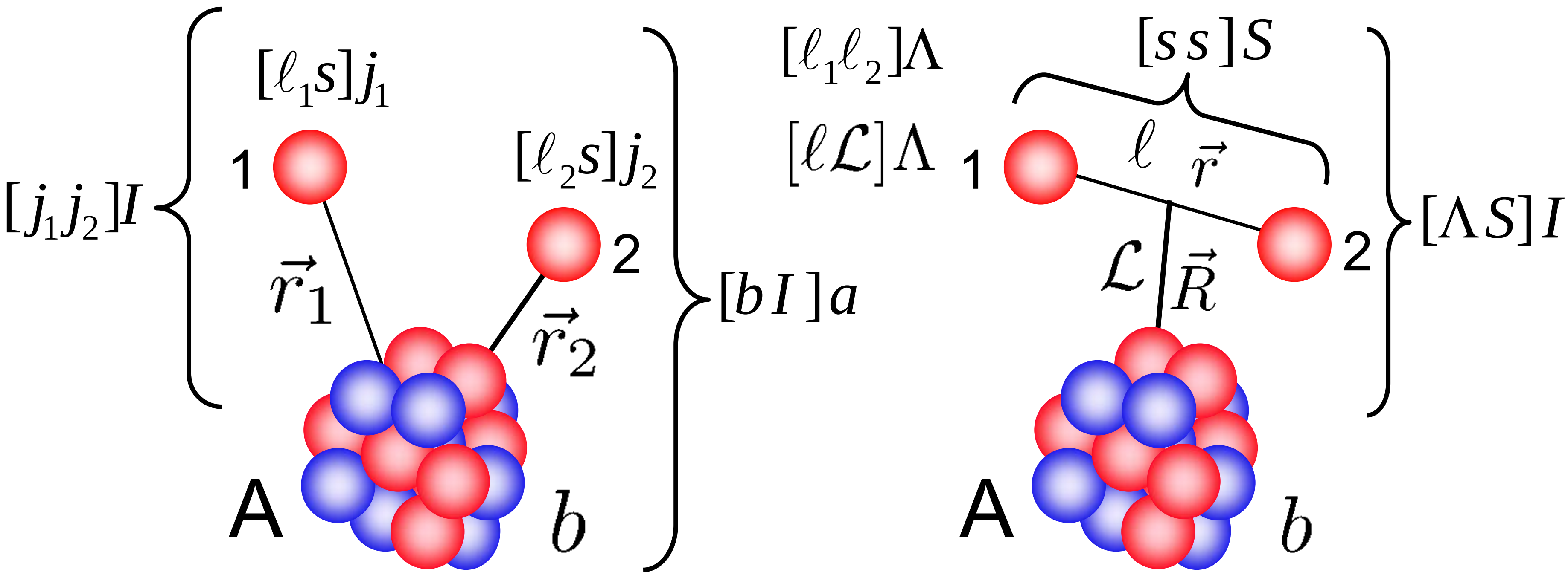
II. transformation of the single-particle wave functions from the individual particle coordinates 1,2, $(\vec{r}_1, \vec{r}_2)$ to relative and c.m. coordinates $(\vec{r} = \vec{r}_1 - \vec{r}_2, \vec{R} = (\vec{r}_1 + \vec{r}_2)/2)$.

to extract the amplitude for the S=0 and relative s-wave terms.

Overlap functions – additional needs (p,t) reactions

$$T_{if} = \langle \phi_t \ldots \Phi_X^{b\beta} | \mathcal{O}(1,2) | \phi_p \ldots \Phi_Y^{a\alpha} \rangle$$ e.g. DWBA

$$(\Phi_X^{b\beta} | \Phi_Y^{a\alpha} \rangle\rangle = \sum_{IM} (b\beta IM | a\alpha) \frac{\mathcal{C}(I, j_1, j_2)}{\sqrt{\frac{1}{2}N(N-1)}} [\overline{\phi_{\ell_1 j_1}(1) \otimes \phi_{\ell_2 j_2}(2)}]_I^M$$



(p,t) - jj to LS and sp wave function recoupling

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = \sum_{IM}(b\beta IM|a\alpha)\frac{\mathcal{C}(I,j_1,j_2)}{\sqrt{\frac{1}{2}N(N-1)}}\underbrace{\overline{[\phi_{\ell_1 j_1}(1)\otimes\phi_{\ell_2 j_2}(2)]}_I^M}$$

I. $\langle(\ell_1\ell_2)\Lambda\,(\frac{1}{2}\frac{1}{2})S;I|(\ell_1\frac{1}{2})j_1\,(\ell_2\frac{1}{2})j_2;I\rangle =$

$$= \sum_{\Lambda\,S}\sqrt{(2j_1+1)(2j_2+1)(2\Lambda+1)(2S+1)}\begin{Bmatrix}\ell_1 & \frac{1}{2} & j_1\\ \ell_2 & \frac{1}{2} & j_2\\ \Lambda & S & I\end{Bmatrix}$$

II. for harmonic oscillator single particle wave functions, the transformation to relative and c.m. coordinates is achieved (analytically) by use of Moshinsky brackets, written as

$$[\phi_{n_1\ell_1}(1)\otimes\phi_{n_2\ell_2}(2)]_\Lambda =$$

$$\sum_{\mathcal{N}\mathcal{L}\,n\ell}\langle n\ell,\mathcal{N}\mathcal{L};\Lambda|n_1\ell_1,n_2\ell_2;\Lambda\rangle\times[\phi_{n\ell}(\vec{r})\otimes\phi_{\mathcal{N}\mathcal{L}}(\vec{R})]_\Lambda$$

Hence we achieve the decomposition of the overlap with n,ℓ and S

Session discussed:

1. The link between two-body bound state solutions of the Schrodinger equation and the many-body wave functions of the nuclei, that is. the interface between structure and reaction models used for spectroscopy.

2. Defined carefully overlap functions and spectroscopic factors (SF), the expected forms of these overlaps, and estimates of the SF in simple cases. Parentage coeffs.

3. Overlaps involving two nucleons, e.g. for (p,t) transfer and fast two-nucleon removal reactions. Discussed the expectations in simple cases (independent particle models). The jj → LS and Moshinsky (oscillator state) transformations that enter such models were introduced.

Homework: to consolidate/use these ideas

1. SF for neutron removal along the Ca isotopic chain

2. Predict the relative magnitudes of the (p,t) reaction cross sections for $^{20}C(0^+) \rightarrow {}^{18}C(0^+)$ with different assumed ^{20}C ground state configurations

moshinsky calculates the individual particle to cm and relative motion transformation brackets