TALENT Course 6: Theory for exploring nuclear reaction experiments

Nuclear spectroscopy – nucleon transfer reactions for spectroscopy

GANIL, 1st - 19th July 2013

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Session aims:

1. To discuss in some detail the different physical inputs that are needed for calculations of single-nucleon transfer reactions, and the considerations needed when choosing these input parameters.

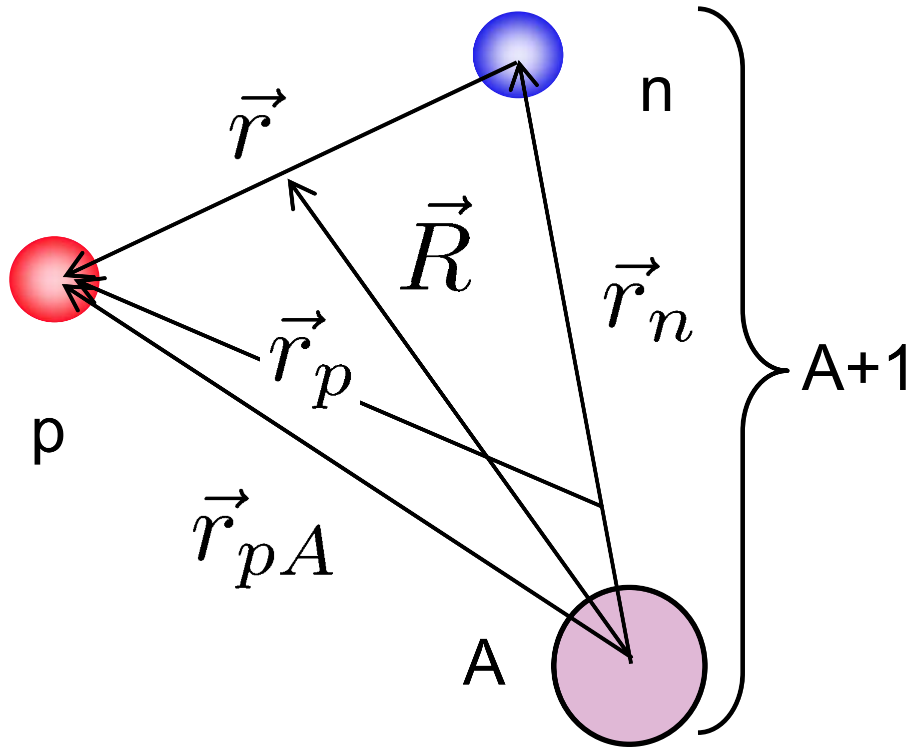
2. To remind you of the essential role of the overlap functions and their spectroscopic factors (SF) in determining reaction yields – and IPM estimates - using the same nuclear information as in knockout.

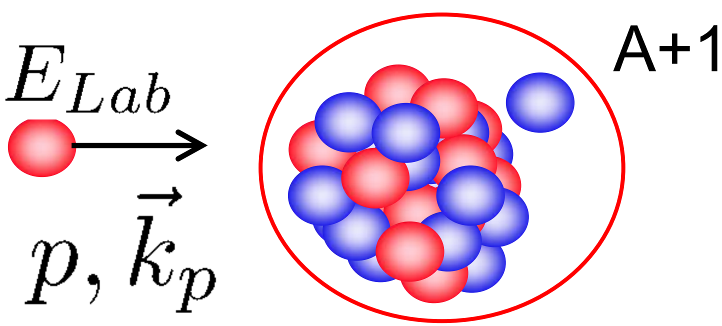
3. To try to make clear the essential physics probed by the transfer reaction angular distributions and the angular momentum matching present in such reactions and provide the means to explore the sensitivity of cross sections to the inputs used.

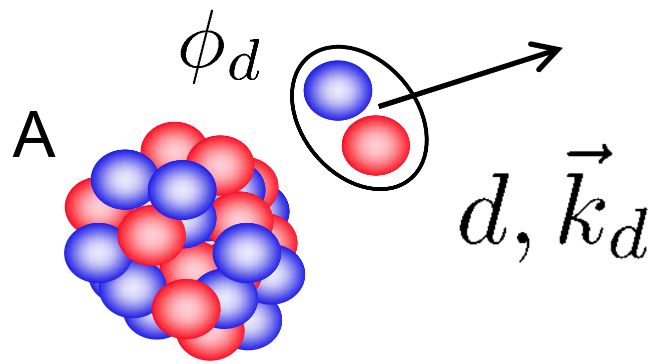
Transfer reactions e.g. (p,d) – coordinate systems

$$\vec{R} = [\vec{r}_n + \vec{r}_p]/2$$
$$\vec{r}_{pA} = \vec{R} + \vec{r}/2$$
$$\vec{r}_n = \vec{R} - \vec{r}/2$$

$$V_p(\vec{r}_{pA}) = V_p(\vec{R} + \vec{r}/2)$$
$$V_n(\vec{r}_n) = V_n(\vec{R} - \vec{r}/2)$$
$$V_{np}(\vec{r})$$





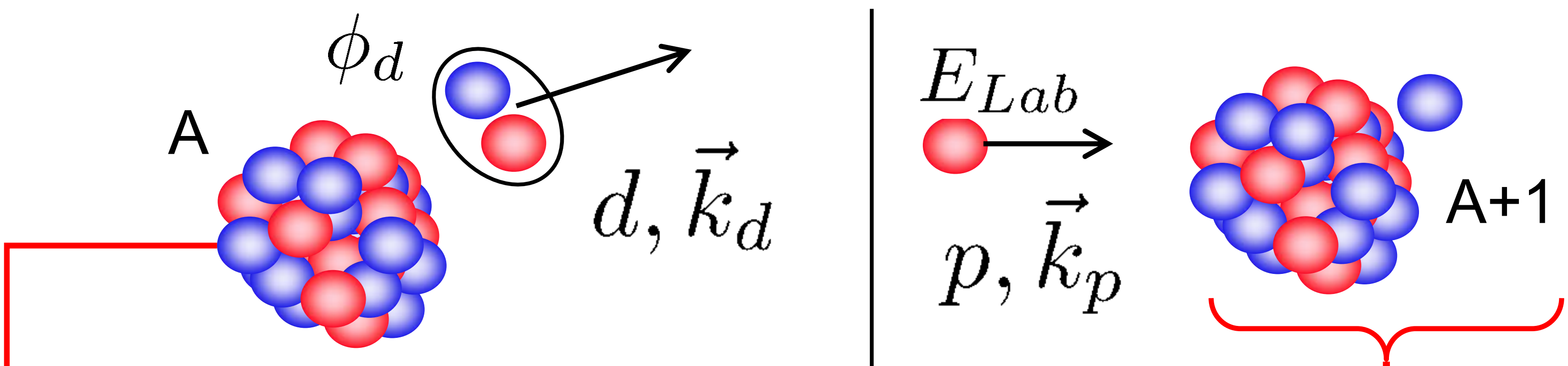


Transfer reaction transition amplitudes – ‘exact’

$$T(p,d) = \langle \underbrace{\psi^{(-)}_{d,\vec{k}_d}\ \Phi(A,J_f)}_{\text{exit channel}} | V_{np} | \underbrace{\chi^{(+)}_{p,\vec{k}_p}\ \Phi(A+1,J_i)}_{\text{entrance channel}} \rangle$$

$$[T_R + \mathcal{H}_{np} + V_p(r_{pA}) + V_n(r_n) - E]\psi^{(+)}_{d,\vec{k}_d}(\vec{r},\vec{R}) = 0$$

$V_{np}(r)$ - the n-p interaction – short but finite range



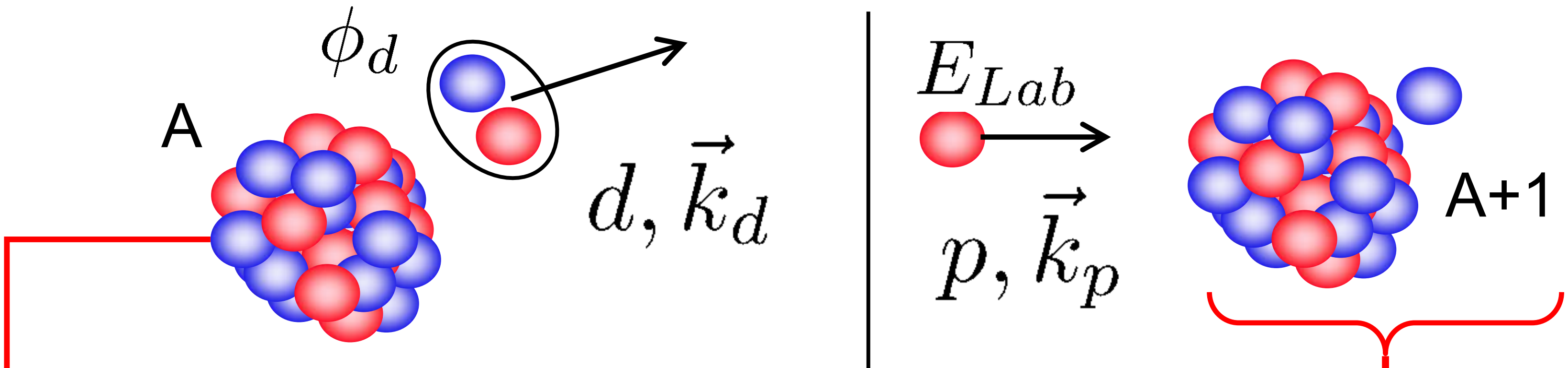
$$\phi_{n\ell j}(\vec{r}_n) \leftrightarrow (\Phi(A,J_f)|\Phi(A+1,J_i)\rangle\rangle$$

Transfer reaction transition amplitudes - DWBA

$$T(p,d) = \underbrace{\langle \chi^{(-)}_{d,\vec{k}_d}\, \phi_d\ \Phi(A, J_f)}_{\text{exit channel}} | V_{np} | \underbrace{\chi^{(+)}_{p,\vec{k}_p}\ \Phi(A+1, J_i)\rangle}_{\text{entrance channel}}$$

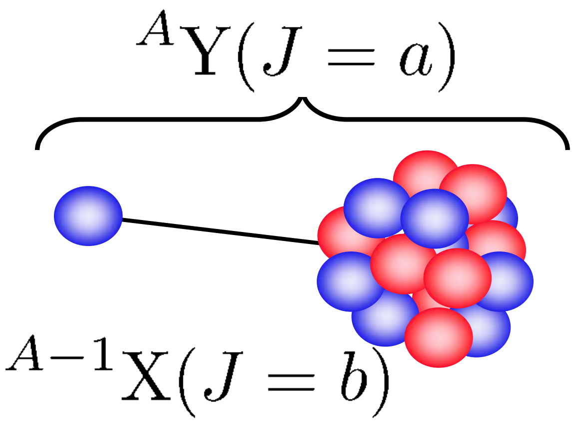
distorted wave $\quad [T_R + U_{dA}(R) - E_0]\chi^{(+)}_{d,\vec{k}_d}(\vec{R}) = 0$

$V_{np}\phi_d(\vec{r})$ - vertex function – of short but finite range



$$\phi_{n\ell j}(\vec{r}_n) \leftrightarrow (\Phi(A, J_f)|\Phi(A+1, J_i)\rangle\rangle$$

Overlap functions review - we have learned



<u>Assumption</u>: the reaction process acts on and perturbs/changes the motion of just a single nucleon (here a neutron, 1) – but not the degrees of freedom of A-1, or X.

Here: $\mathcal{O}(1) = V_{np}(r) = V_{np}(p,1)$

So, have $T_{if} = \langle final; \Phi_X^{b\beta} | \mathcal{O}(1) | initial; \Phi_Y^{a\alpha} \rangle_{A,\ldots}$

and there will be equal contributions to the <u>cross section</u> from each of the N (identical) neutrons in the target, thus

$$\sigma_{if} \propto N|T_{if}|^2 = |\tilde{T}_{if}|^2, \quad \tilde{T} = \sqrt{N} T_{if}$$

and the nuclear structure enters via the overlap function since

$$(\Phi_X^{b\beta} | \mathcal{O}(1) | \Phi_Y^{a\alpha} \rangle\!\rangle = \mathcal{O}(1) (\Phi_X^{b\beta} | \Phi_Y^{a\alpha} \rangle\!\rangle = \mathcal{O}(1) F_{YX}^{a\alpha b\beta}(1)$$

Overlap functions – Asymptotic properties

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) \equiv F_{YX}(1) = F(\vec{r})$$

$$\boxed{(T_1 + Sn)F_{YX}(1) = -(\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle}$$

source term approach

as neutron moves large distances from the residual nucleus:

$$(\Phi_X^{b\beta}|V_{1X}|\Phi_Y^{a\alpha}\rangle\rangle \to 0, \text{ as } |\vec{r}| \to \infty$$

and like our two-body model solutions, at large distances

$$T_1 F_{YX}(1) = -S_n F_{YX}(1)$$

$$\left\{F_{YX}(r), \frac{u_{n\ell j}(r)}{r}\right\} \longrightarrow h_\ell(\kappa_b r) \longrightarrow \frac{\exp(-\kappa_b r)}{\kappa_b r}$$

So, two-body calculations with the right neutron separation energy will automatically have the correct long-range forms.

Norm is vital for reaction calculations and astrophysical rates.

Overlap functions – Spectroscopic factors

We now make the angular momentum couplings explicit :

$$(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}\rangle\rangle = F_{YX}^{a\alpha b\beta}(1) = \sum_{jm}(b\beta jm|a\alpha)\tilde{\varphi}_{jm}(1)$$

where the transferred j is unique if a or b is spin-zero

Relationship of

$$\boxed{\tilde{\varphi}_{jm}(1) \longleftrightarrow \phi_{n\ell j}^{m}(1)}$$

normalised sp wave function

Definition:

$$\boxed{\tilde{\varphi}_{jm}(1) = \sqrt{\frac{\mathcal{S}(\ell j)}{N}}\phi_{n\ell j}^{m}(1) = \frac{\mathcal{A}(\ell j)}{\sqrt{N}}\phi_{n\ell j}^{m}(1)}$$

Defined this way, the sqrt(N) factor cancels that in the cross section expression (below) and so the many-body (structure) information remains only through the SFs or S-amplitudes

$$\sigma_{if} \propto N|T_{if}|^2 = |\tilde{T}_{if}|^2, \quad \tilde{T} = \sqrt{N}T_{if}$$

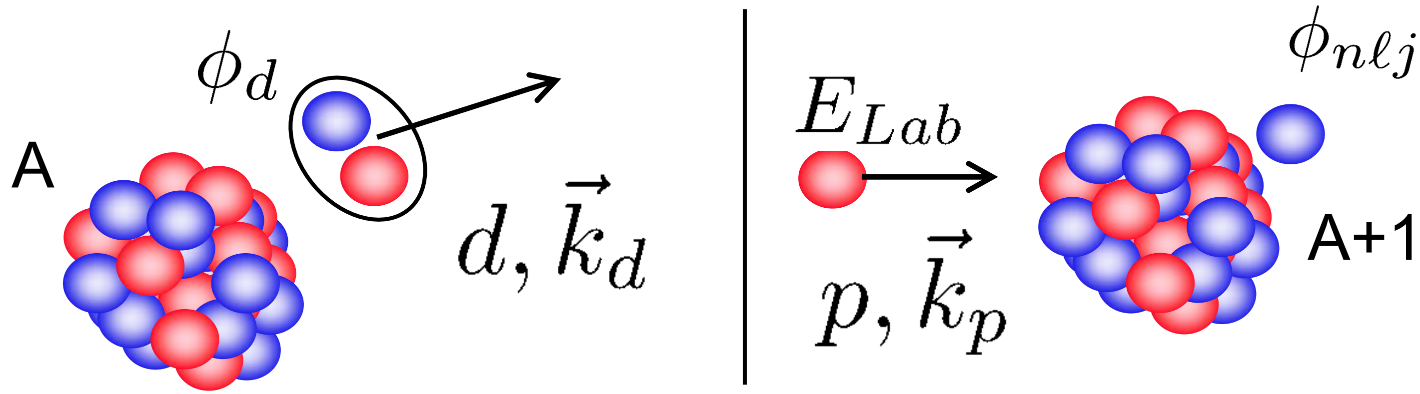
Transfer reaction – exact 3-body model T(p,d)

$$T(p,d) = \langle \psi^{(-)}_{d,\vec{k}_d} \Phi(A, J_f) | V_{np} | \chi^{(+)}_{p,\vec{k}_p} \Phi(A+1, J_i) \rangle$$

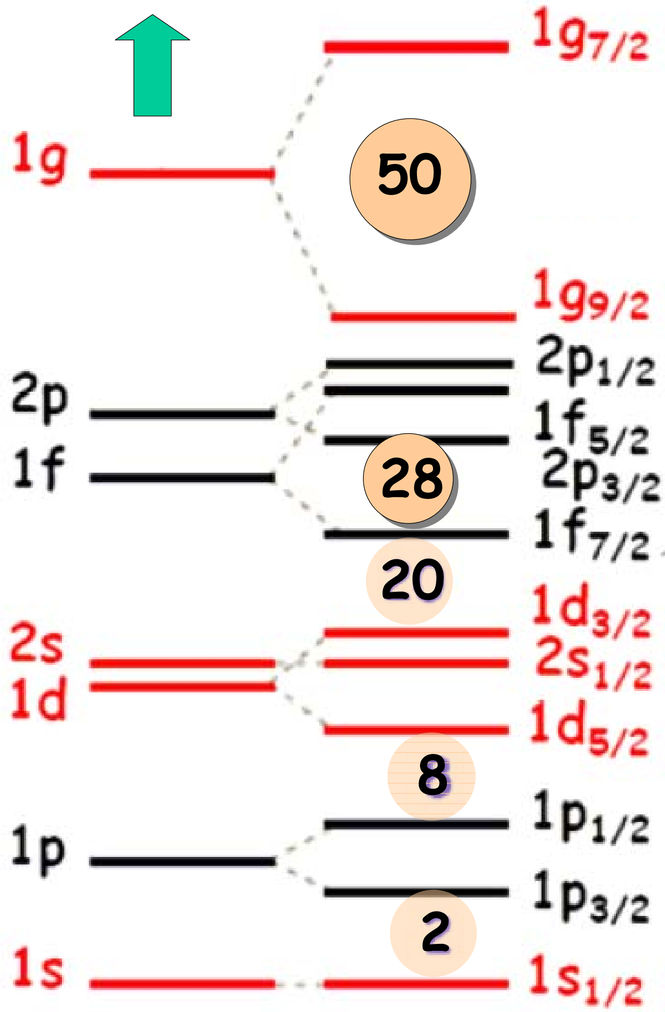
$$\sqrt{\mathcal{S}(\ell j)}\,\phi_{n\ell j}(\vec{r}_n) = (\Phi(A, J_f) | \Phi(A+1, J_i) \rangle\rangle$$

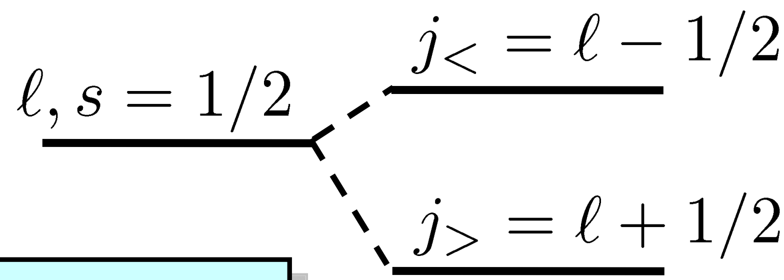
$$\bar{T}(p,d) = \langle \psi^{(-)}_{d,\vec{k}_d}(\vec{R}) | V_{np} | \chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\,\phi_{n\ell j}(\vec{r}_n) \rangle$$

$$\sigma(p,d) = [\text{phase space}] \times \mathcal{S}(\ell j) \times |\bar{T}(d,p)|^2$$



What is involved in realistic transfer calculations?

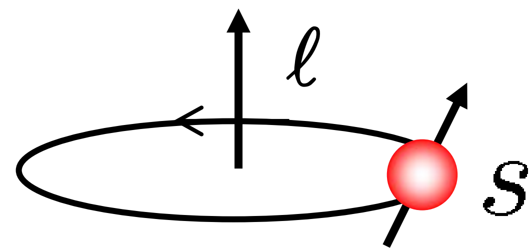




$^{23}O(p,d)$

Z=8

N=15

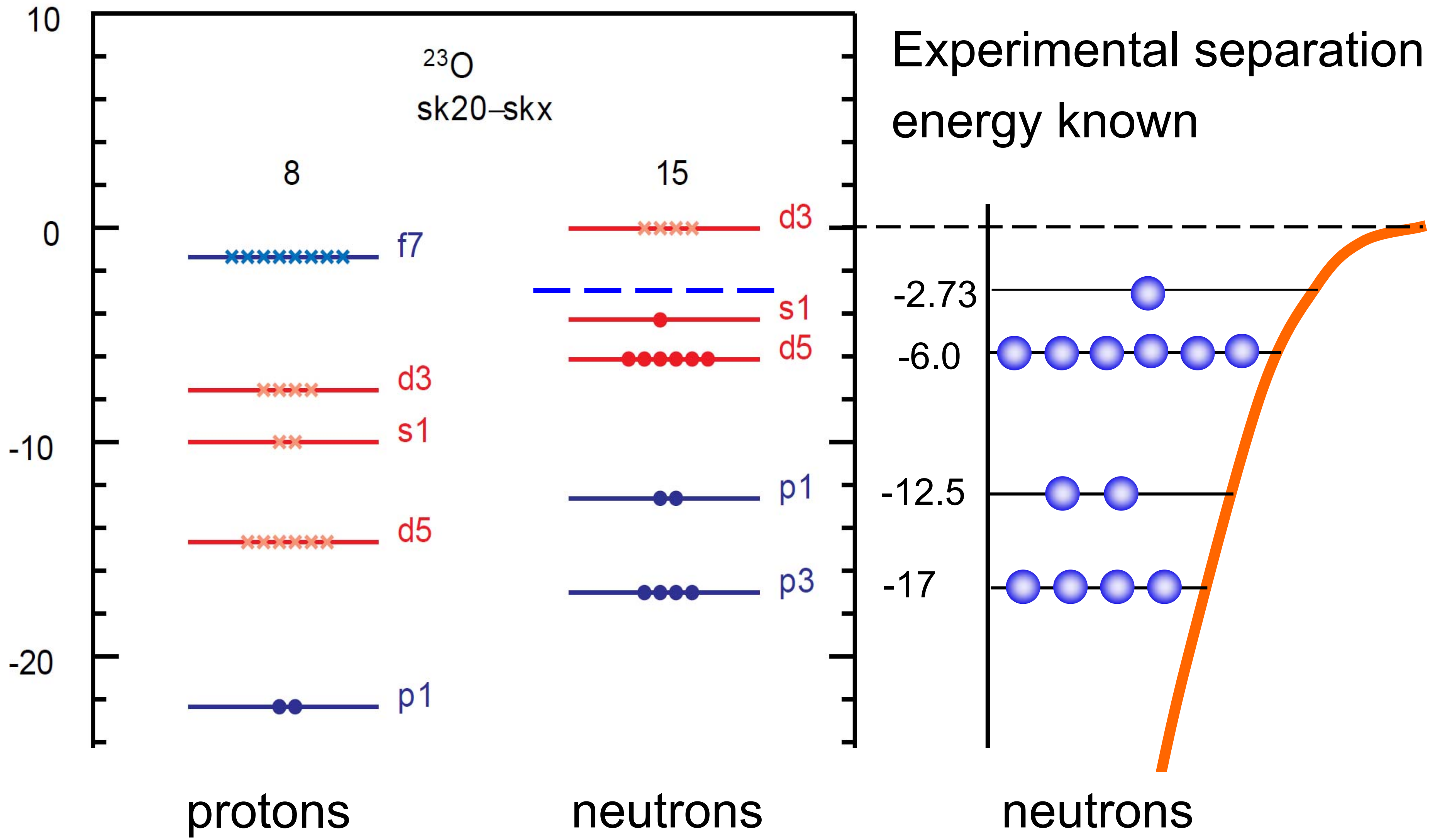


$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V_{so}(r) < 0$$

Example: What is involved – take neutron from ^{23}O



Experimental separation energy known

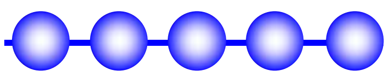
Hartree-Fock mean field calculation

Overlap functions – IPM Spectroscopic factors

$$\sqrt{\mathcal{S}(\ell j)} = \mathcal{A}(\ell j) = \sqrt{n}\langle\, [\Psi_X^b \otimes \phi_{\ell j}(1)]_a^\alpha \,|\, \Psi_Y^{a\alpha}\,\rangle$$

Other (independent particle model) cases for removal from a state of a given j are less simple, can be worked out, but are given by the coefficients of fractional parentage - cfps

j



n

$$\boxed{\begin{aligned}&\langle\, [\Psi_{n-1}^b \otimes \phi_{\ell j}(1)]_a^\alpha \,|\, \Psi_n^{a\alpha}\,\rangle \\ &\qquad = ((j^{n-1})b, j; a|\}(j^n)\, a)\end{aligned}}$$

For low seniority states (where each pair couples to spin zero)

$$\boxed{\mathcal{S}(\ell j) = n((j^{n-1})b, j; a|\}(j^n)\, a)^2}$$

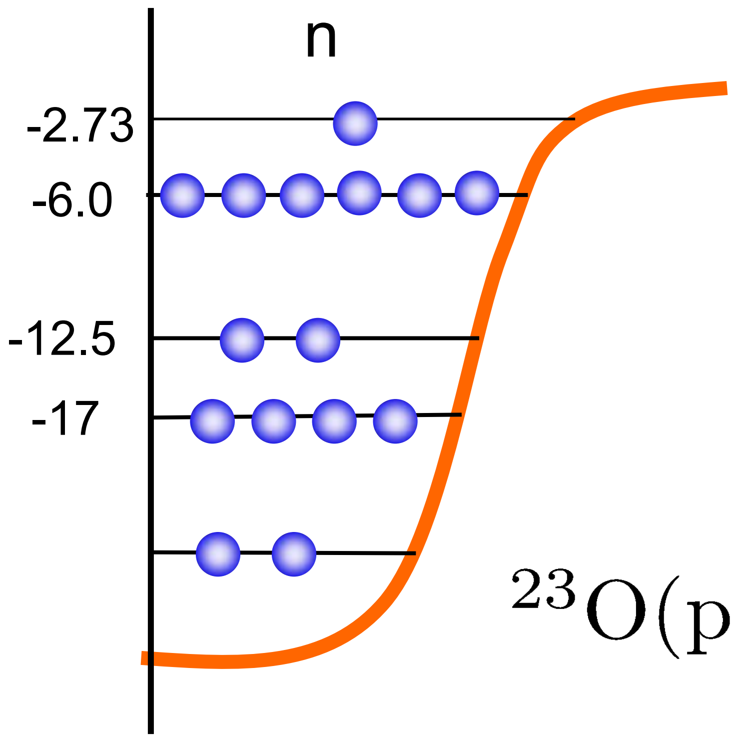
$$((j^{n-1})j, j; 0|\}(j^n)\, 0) = 1, \quad n = \text{even}, \ \text{seniority} = 0$$

$$((j^{n-1})0, j; j|\}(j^n)\, j) = \left(\frac{2j+1-(n-1)}{n(2j+1)}\right)^{\frac{1}{2}},$$

$$n = \text{odd}, \ \text{seniority} = 1$$

Independent particle model – (p,d) reaction

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$2s_{1/2}$ S_n=2.73 MeV

$$\mathcal{S}(2s_{1/2}) = 1$$

$1d_{5/2}$ S_n=6.0 MeV

$$\mathcal{S}(1d_{5/2}) = 6$$

$$^{23}\mathrm{O(p,d)}^{22}\mathrm{O}(J_f^\pi)$$

$J_f^\pi = 0^+$, g.s.
S_n=2.73 MeV

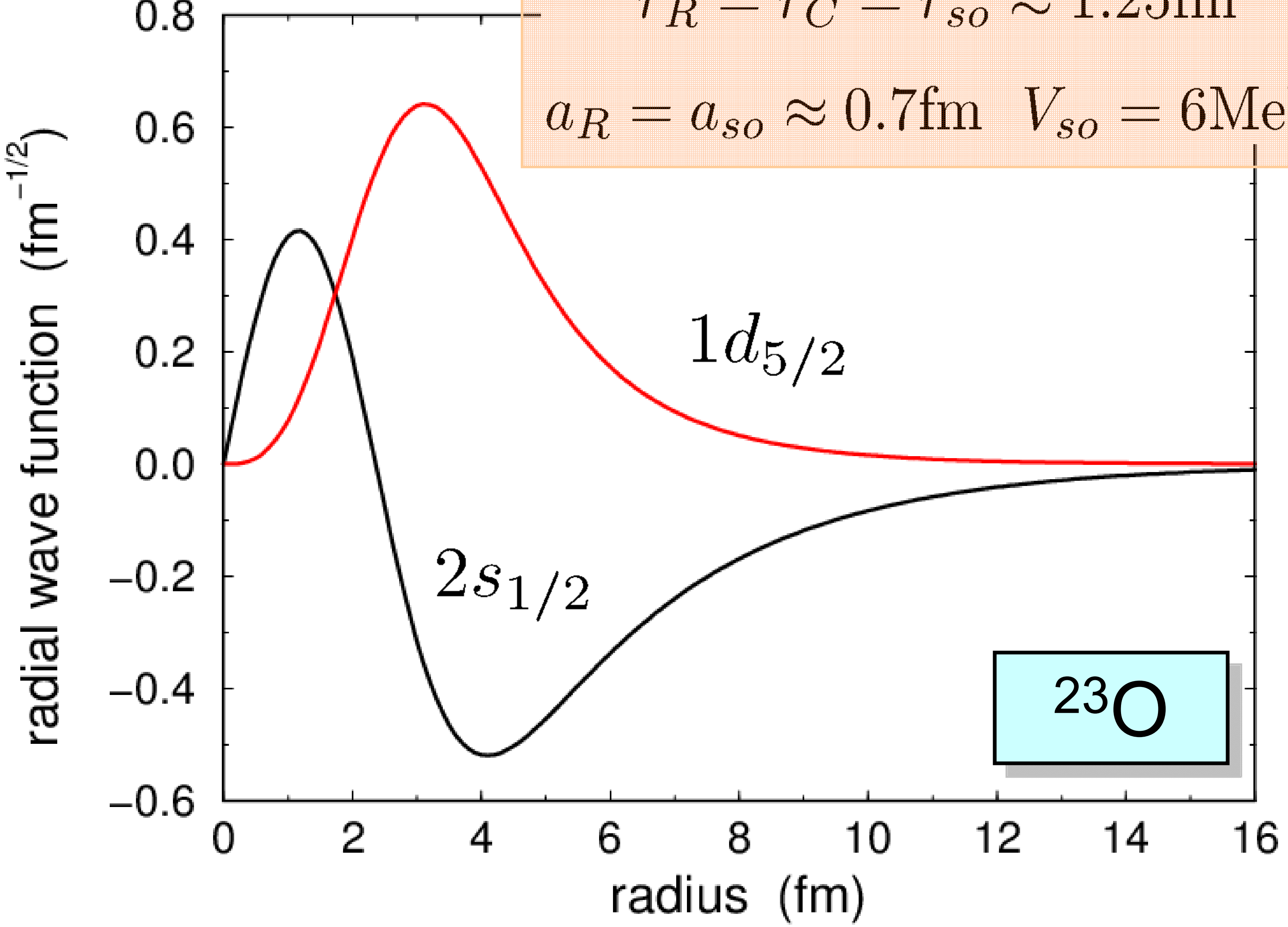
$J_f^\pi = 2^+$, 3.2 MeV
S_n=6.0 MeV

transfer reaction code(s) available at:
http://www.nucleartheory.net/NPG/code.htm

Neutron bound state wave functions

$$r_R = r_C = r_{so} \approx 1.25\text{fm}$$

$$a_R = a_{so} \approx 0.7\text{fm} \quad V_{so} = 6\text{MeV}$$

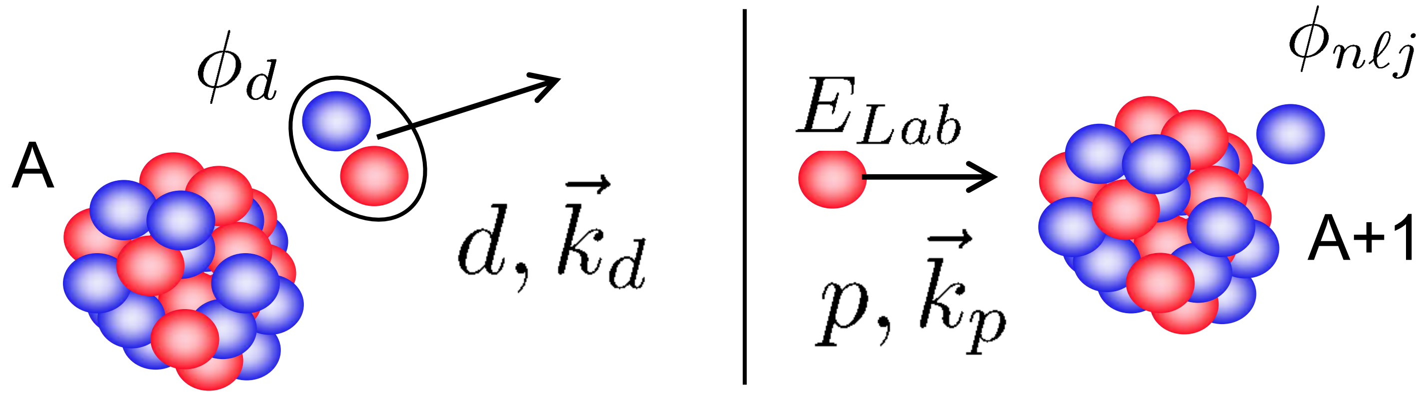


Transfer reaction – DWBA model T(p,d)

$$T(p,d) = \langle \chi^{(-)}_{d,\vec{k}_d} \Phi(A, J_f)\phi_d | V_{np} | \chi^{(+)}_{p,\vec{k}_p} \Phi(A+1, J_i) \rangle$$

$$\sqrt{\mathcal{S}(\ell j)}\,\phi_{n\ell j}(\vec{r}_n) = (\Phi(A, J_f) | \Phi(A+1, J_i) \rangle\rangle$$

$$\boxed{\begin{array}{c} \bar{T}(p,d) = \langle \chi^{(-)}_{d,\vec{k}_d}(\vec{R})\phi_d(r) | V_{np} | \chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n) \rangle \\[2ex] \sigma(p,d) = [\text{phase space}] \times \mathcal{S}(\ell j) \times |\bar{T}(d,p)|^2 \end{array}}$$

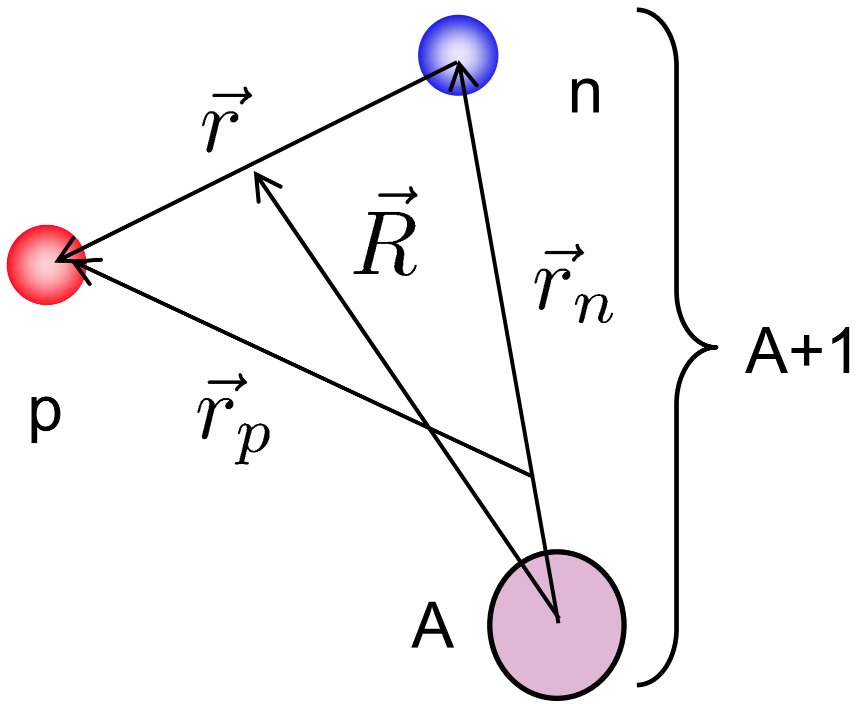


Transfer reaction – plane waves for insight (1)

$$T(p,d) = \langle \chi^{(-)}_{d,\vec{k}_d}(\vec{R})\phi_d(r)|V_{np}|\chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n)\rangle$$

when using plane waves for p and d – i.e. 'weak' distortions

$$T_{pw} \approx \int d\vec{r}_n \int d\vec{r}\, e^{-i\vec{k}_d\cdot\vec{R}} V_{np}\phi_d(r) e^{i\vec{k}_p\cdot\vec{r}_p}\phi_{n\ell j}(\vec{r}_n)$$



$$\vec{r}_p = \gamma\vec{r}_n + \vec{r}$$

$$\gamma = A/(A+1) \approx 1$$

$$\vec{R} = \vec{r}_n + \vec{r}/2$$

$$\vec{k}_p\cdot\vec{r}_p - \vec{k}_d\cdot\vec{R} =$$
$$[\gamma\vec{k}_p - \vec{k}_d]\cdot\vec{r}_n +$$
$$[\vec{k}_p - \vec{k}_d/2]\cdot\vec{r}$$

Transfer reaction – plane waves for insight (2)

$$T_{pw} \approx \int d\vec{r}_n \int d\vec{r}\, e^{-i\vec{k}_d\cdot\vec{R}} V_{np}\phi_d(r) e^{i\vec{k}_p\cdot\vec{r}_p}\phi_{n\ell j}(\vec{r}_n)$$

$$\begin{aligned} \vec{k}_p\cdot\vec{r}_p - \vec{k}_d\cdot\vec{R} &= [\gamma\vec{k}_p - \vec{k}_d]\cdot\vec{r}_n + [\vec{k}_p - \vec{k}_d/2]\cdot\vec{r} \\ &= \vec{q}_n\cdot\vec{r}_n + \vec{q}\cdot\vec{r} \end{aligned}$$

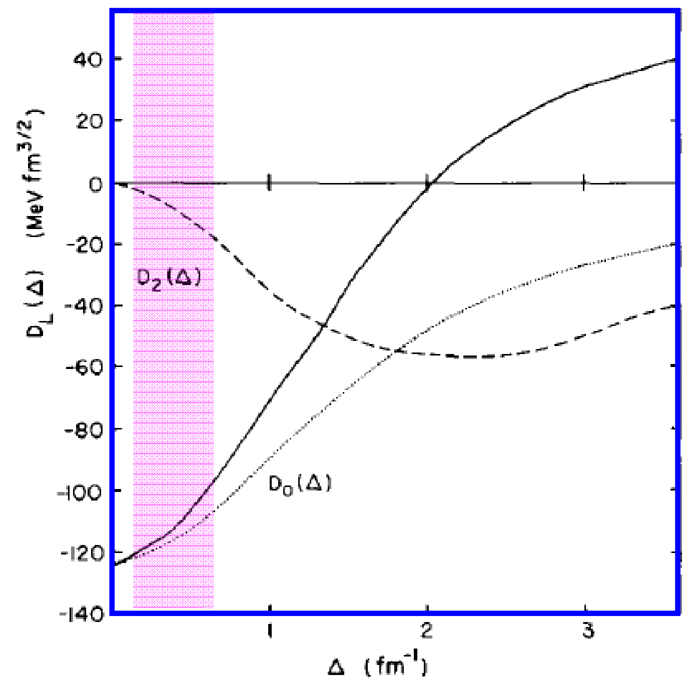
$$T_{pw} \approx \boxed{\int d\vec{r}_n e^{i\vec{q}_n\cdot\vec{r}_n}\phi_{n\ell j}(\vec{r}_n)} \times \boxed{\int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}} V_{np}\phi_d(r)}$$

$$\boxed{\sigma_{pw} \propto |F(\vec{q}_n)|^2 D^2(q)}$$

and expanding $e^{i\vec{q}\cdot\vec{r}}$ for small q

$$D(q) = D_0[1 - \beta^2 q^2 + \ldots]$$

$$\boxed{\begin{aligned} &D_0 = -122.5\ \mathrm{MeV\ fm}^{3/2} \\ &\beta \approx 0.75\ \mathrm{fm} \end{aligned}}$$



Transfer reactions – “matching” considerations

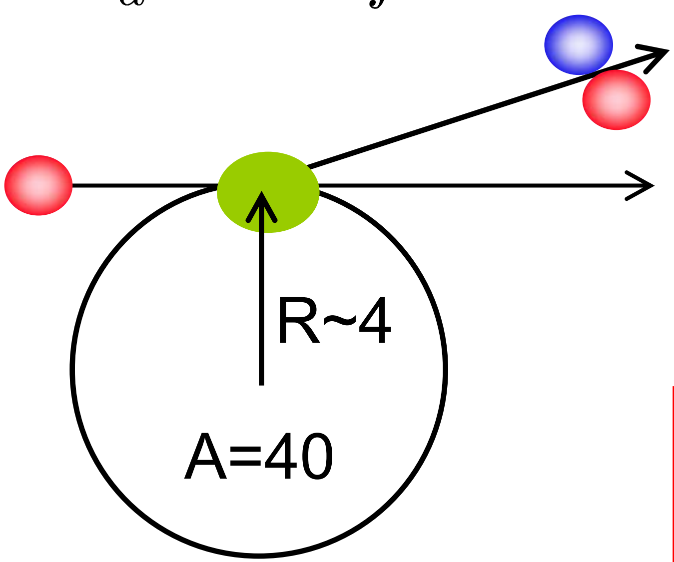
$$\vec{q}_n = [\gamma \vec{k}_p - \vec{k}_d] \qquad \vec{q} = [\vec{k}_p - \vec{k}_d/2] \cdot \vec{r}$$

$$T_{pw} \approx \boxed{\int d\vec{r}_n e^{i\vec{q}_n \cdot \vec{r}_n} \phi_{n\ell j}(\vec{r}_n)} \times \boxed{\int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}} V_{np} \phi_d(r)}$$

e.g. (p,d) on A=40 at E_p=10 MeV, say S_n=4 MeV, R_{40}~4 fm

$$k_p = 0.68 fm^{-1} \qquad \Delta L = |L_d - L_p| \approx 0.7$$

$$k_d = 0.86 fm^{-1}$$



$\vec{k}_d$

$L_d \approx k_d R$

$\vec{k}_p$

$L_p \approx k_p R$

$$\boxed{\begin{aligned} L_p &= 2.8 \\ L_d &= 3.5 \end{aligned}}$$

with E_p=20 MeV

$$k_p = 0.96 fm^{-1}$$

$$k_d = 1.27 fm^{-1}$$

$$L_p = 3.9$$

$$L_d = 5.2$$

$$\Delta L = |L_d - L_p| \approx 1.3$$

Wisdom on the choice of distorting potentials

The DWBA analysis of single-nucleon transfer is a semi empirical procedure which describes gross dependences of the cross sections on energy, the Coulomb field, and the atomic weight of the target. This still leaves the novice experimental physicist with the task of choosing the imposing array of parameters needed in such a calculation. **It used to be thought that the best procedure is to measure the elastic scattering by the target nucleus of the incident projectiles and that by the final nucleus of the outgoing particles, all at the proper energies, and then to fit the elastic data as well as possible with optical model potentials.** These potentials were then to be used as input to DWBA calculations.

Experience has shown that **a more sensible procedure is to use distorting parameters which are appropriate for a wider range of target nuclei and energies.** Emphasis on accurate fitting of data on one or two nuclei tends to optimize the fit by selecting a peculiar (and perhaps unphysical) set of parameters. In any case, the basic purpose of an optical potential is to describe the average interaction between a projectile and target, and **if this interaction turns out to be sharply dependent on the precise energy or target, then the approximations made in assuming an average potential in the first place are likely to be wrong.**

M.H. Macfarlane and J.P. Schiffer, Nucl. Spectroscopy and Reactions, Vol B, pp 169

Phenomenological optical potentials – where?

C.M. Perey and F.G. Perey, At. Data Nucl. Data Tables 17, 1 (1976) Compilation for many systems

J.J.H. Menet, E.E. Gross, J.J. Malanify, and A. Zucker, Phys. Rev. C 4, 1114 (1971) – for nucleons

R.L. Varner, W.J. Thompson, T.L. McAbee, E.J. Ludwig, and T.B. Clegg, Phys. Rep. 201, 57 (1991) – Chapel Hill 89 potential – for nucleons

F.D. Becchetti, Jr. and G.W. Greenlees, Phys. Rev. 182, 1190 (1969) – 'old faithful' parameterisation – for nucleons

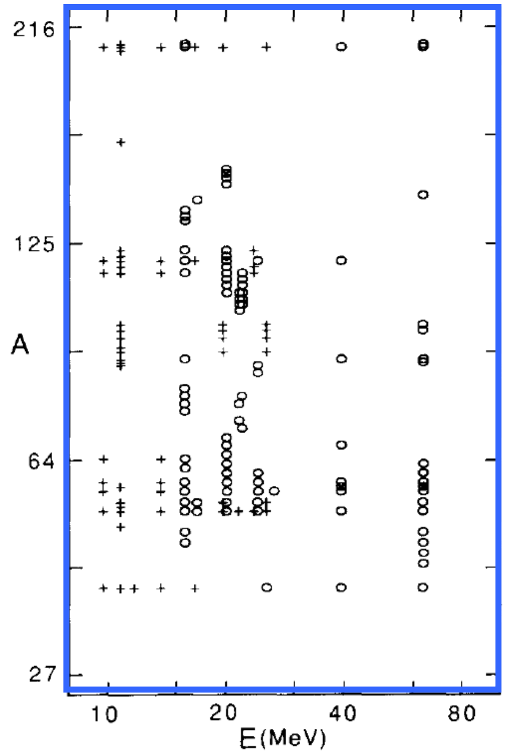
W.W. Daehnick, J.D. Childs, and Z. Vrcelj, Phys. Rev. C 21, 2253 (1980) – good parameter set – for deuterons

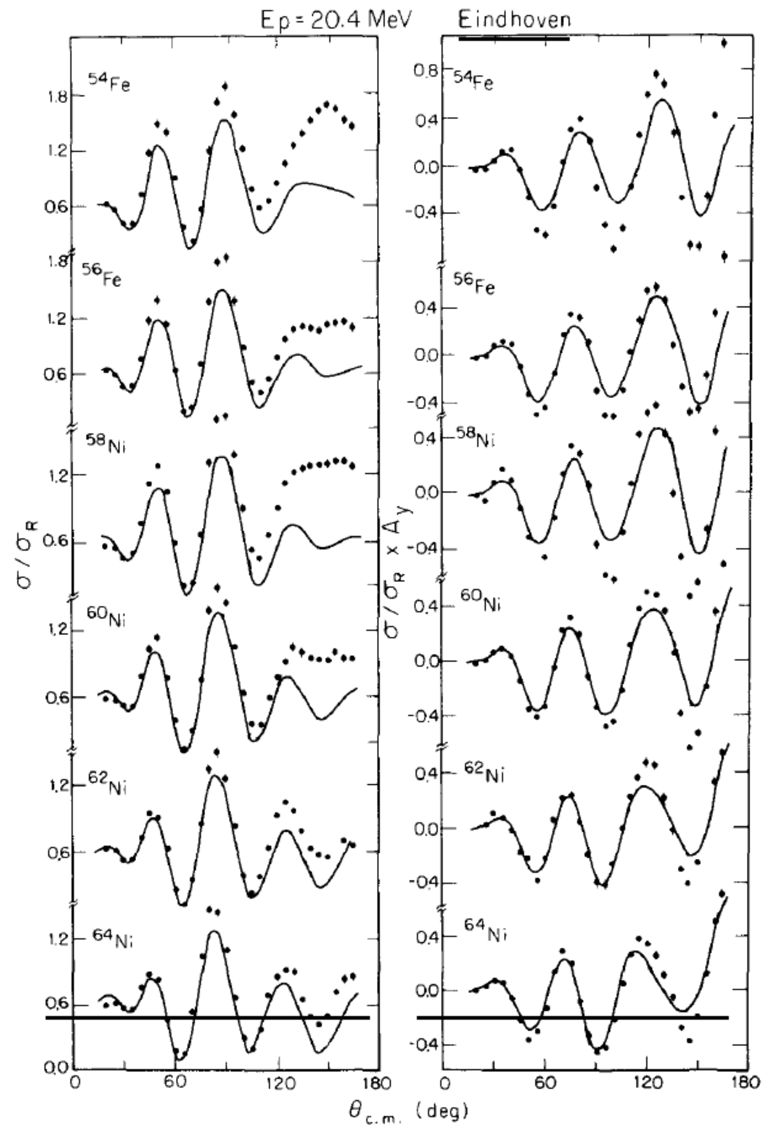
J.M. Lohr and W. Haeberli, Nucl. Phys. A232, 381 (1974) – for low energy deuterons

..... and many many more ... but many many gaps … if data, can fit !!

http://www-nds.iaea.org/RIPL-3/

Global optical potentials – e.g. CH91 for nucleons





A GLOBAL NUCLEON OPTICAL MODEL POTENTIAL*

R.L. VARNER

Oak Ridge National Laboratory, Oak Ridge, TN 37831-6368, USA
and Triangle Universities Nuclear Laboratory, Duke University, Durham, NC 27706, USA

and

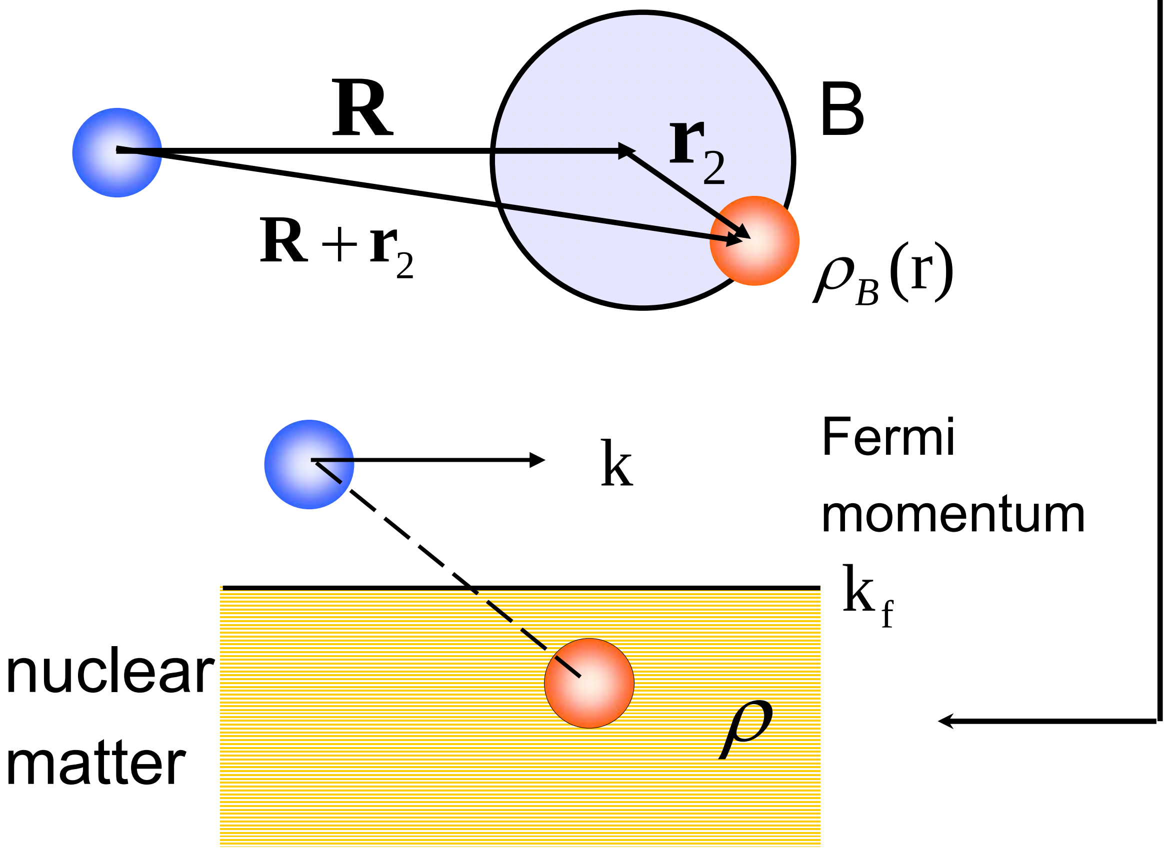
W.J. THOMPSON, T.L. McABEE**, E.J. LUDWIG and T.B. CLEGG

PHYSICS REPORTS (Review Section of Physics Letters) 201, No. 2 (1991) 57–119. North-Holland

Theoretical nucleon potential – based on density

G-matrix effective interaction – the JLM approach

$$V_{NB}(R) = \int d\mathbf{r}_2\ \rho_B(r_2)\ v_{NN}(\mathbf{R}+\mathbf{r}_2)$$



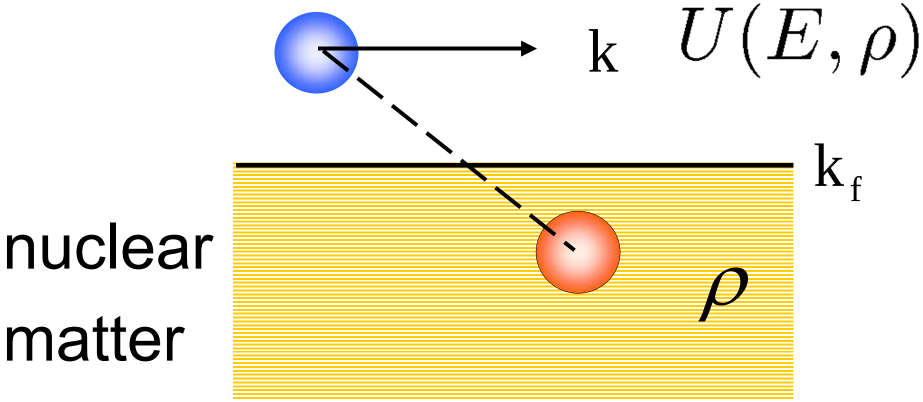
include the effect of NN interaction in the “nuclear medium” – Pauli blocking of pair scattering into occupied states

$$\rightarrow v_{NN}(\rho, \mathbf{r})$$

and as E → high

$$v_{NN} \rightarrow v_{NN}^{free}$$

JLM interaction + the local density approximation

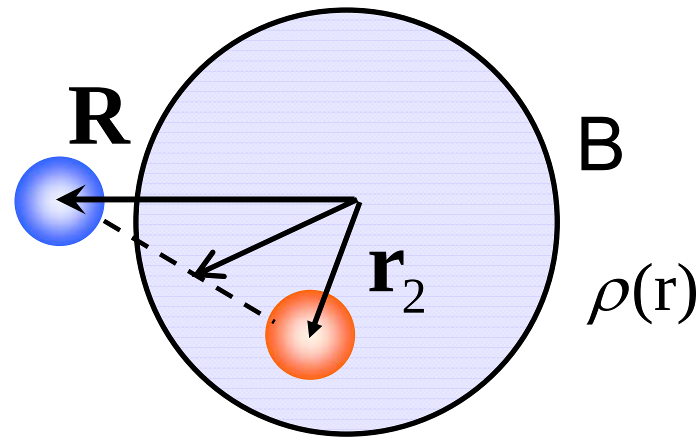


complex and density dependent interaction

$$v_{NN}(r) = \frac{U(E,\rho)}{\rho} f(r)$$

$$f(r) = (\sqrt{\pi}t)^{-3} \exp(-r^2/t^2)$$

For finite nuclei, what value of density should be used to calculate the nucleon-nucleus potential? Usually the <u>local density</u> at the mid-point of the two nucleon positions $\mathbf{r}_x$



$$\boxed{U_B(R) = V_B(R) + iW_B(R) = \int d\vec{r}_2\, \rho_B(r_2) \frac{U(E,\rho(r_x))}{\rho(r_x)} f(r)}$$

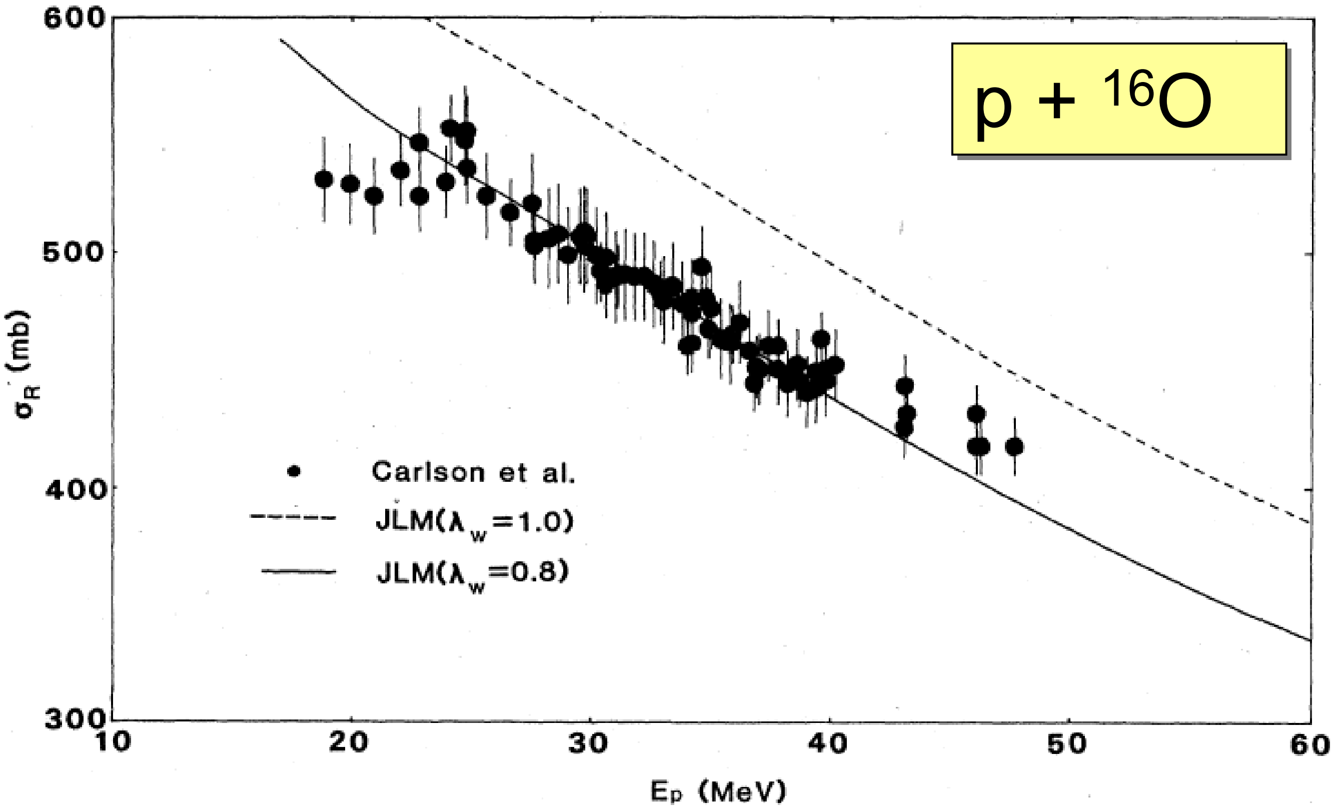
JLM interaction – fine tuning the theory to data

Strengths of the real and imaginary parts of the potential can be adjusted based on experience of fitting data.

$$\boxed{U_B(R) = \lambda_v V_B(R) + i\lambda_w W_B(R)}$$

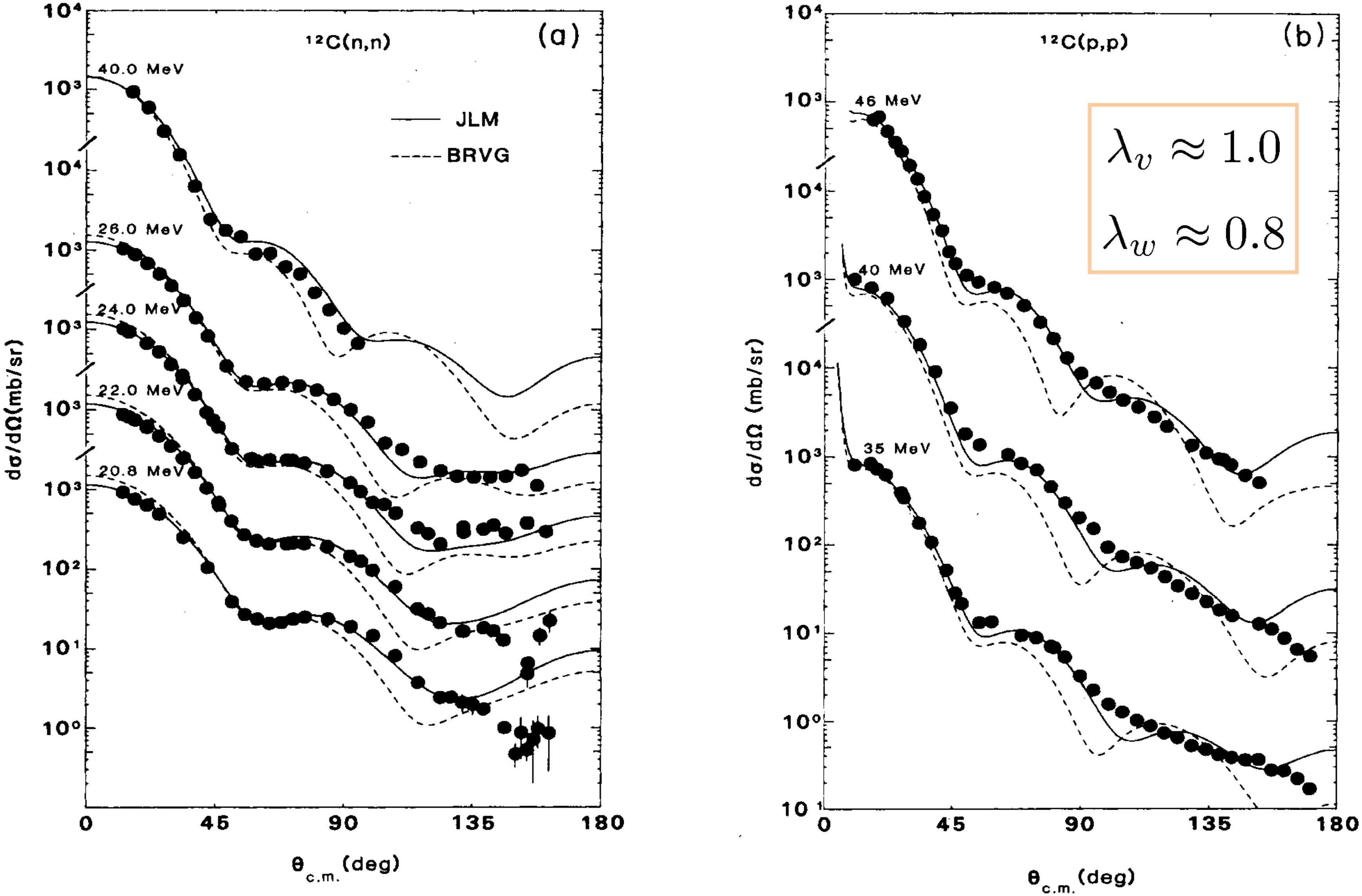
$$\lambda_v \approx 1.0$$

$$\lambda_w \approx 0.8$$



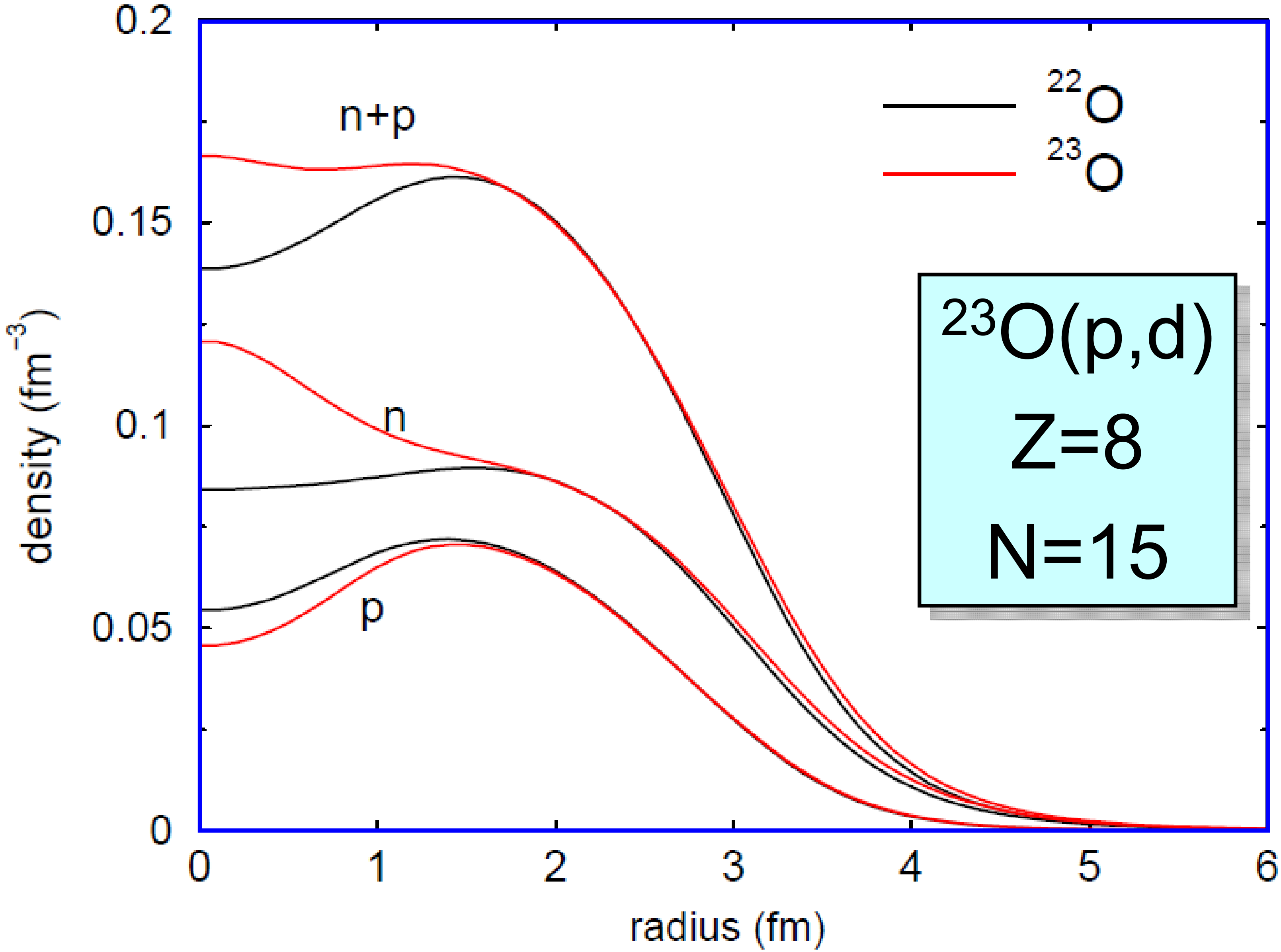
J.S. Petler et al. Phys. Rev. C **32** (1985), 673

JLM folded nucleon-nucleus optical potentials



J.S. Petler et al. Phys. Rev. C **32** (1985), 673

Neutron: proton: nucleon radial densities (HF)

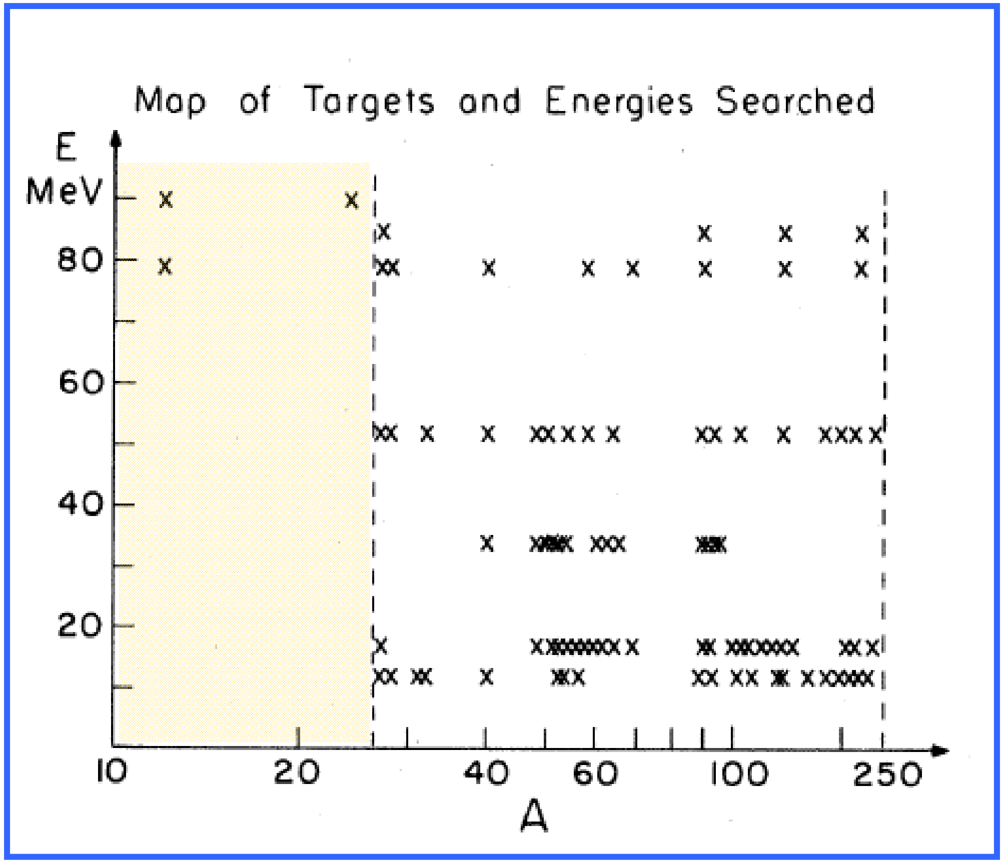


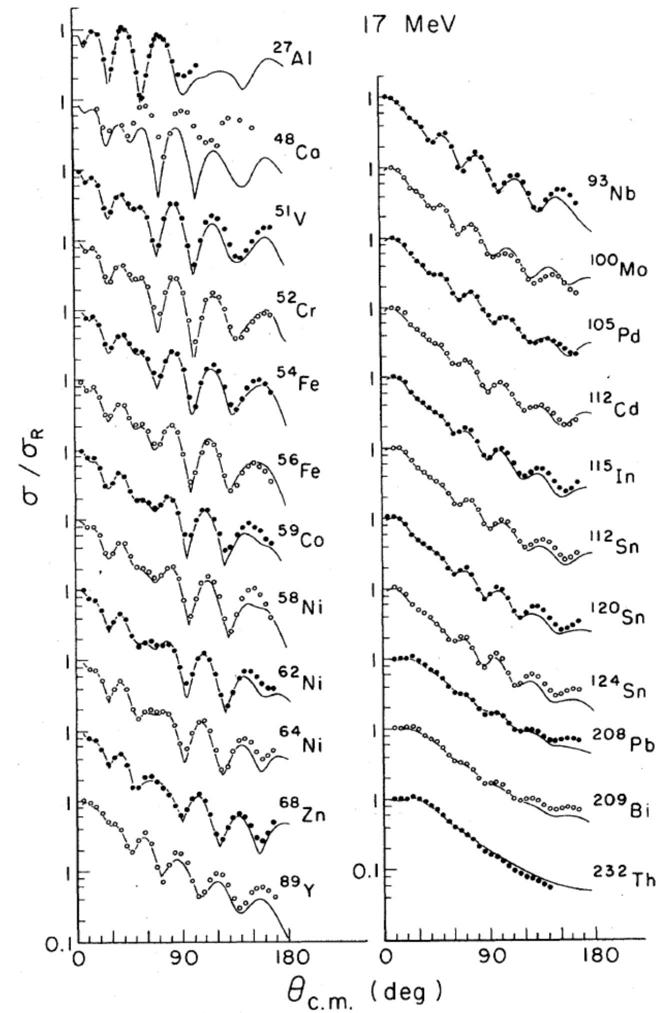
Global optical potentials – now for the deuterons

PHYSICAL REVIEW C VOLUME 21, NUMBER 6 JUNE 1980

Global optical model potential for elastic deuteron scattering from 12 to 90 MeV

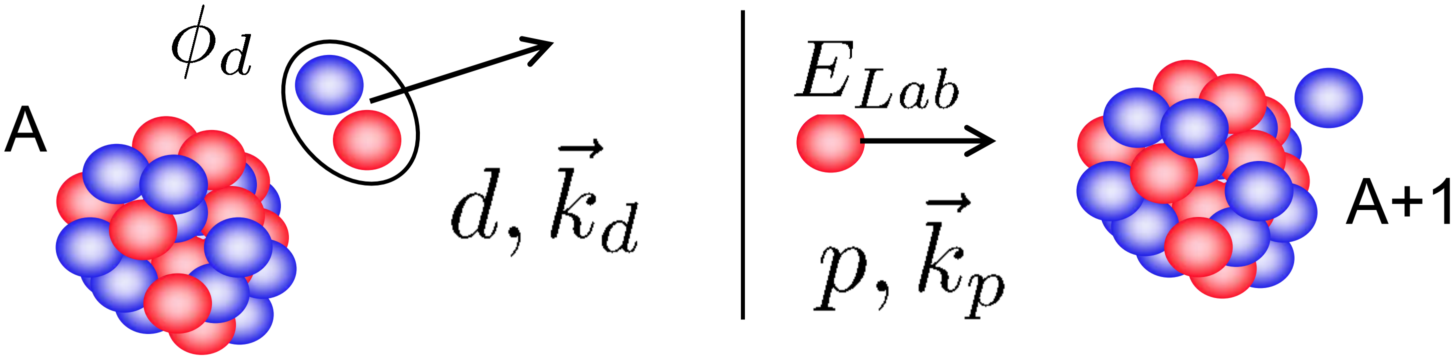
W. W. Daehnick, J. D. Childs,* and Z. Vrcelj





Zero-range approximation to the T(p,d) of DWBA

$$T(p,d) = \langle \chi^{(-)}_{d,\vec{k}_d}(\vec{R})\phi_d(r)|V_{np}|\chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n)\rangle$$



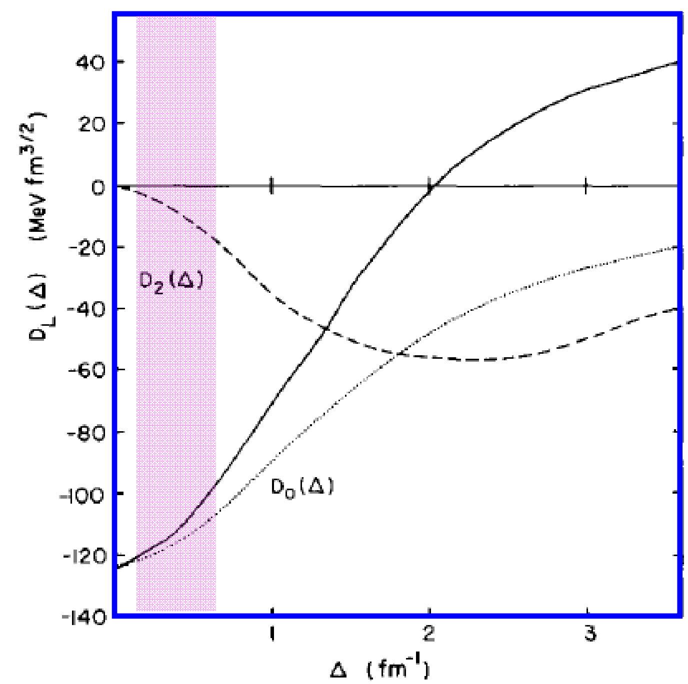
short range $V_{np}\phi_d(\vec{r}) \approx D_0\delta(\vec{r})$

$$D(q) = \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\, V_{np}\phi_d(r)$$

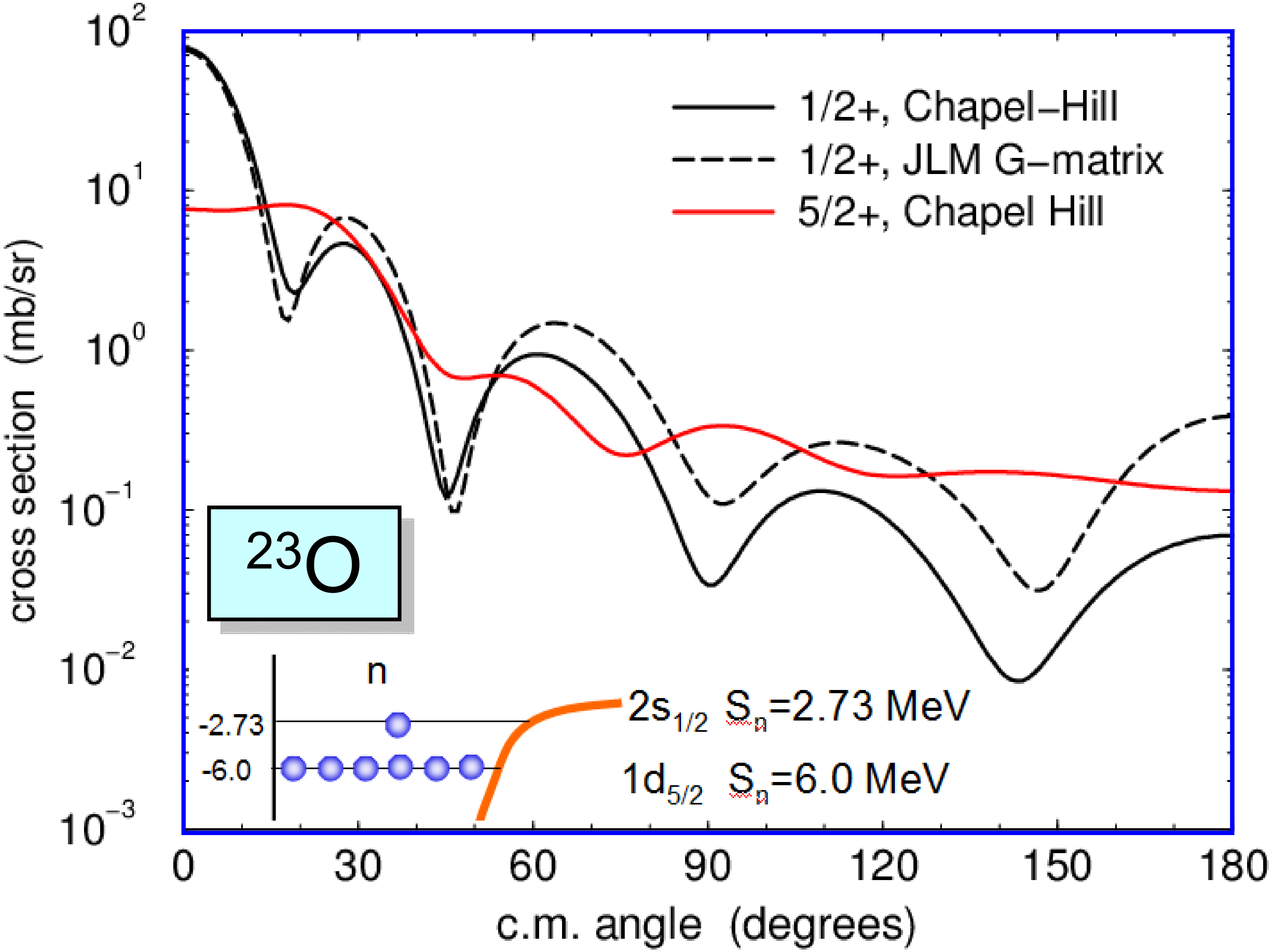
$$D(q) \approx D_0[1 - \beta^2 q^2 + \ldots]$$

$$D_0 = -122.5 \text{ MeV fm}^{3/2}$$

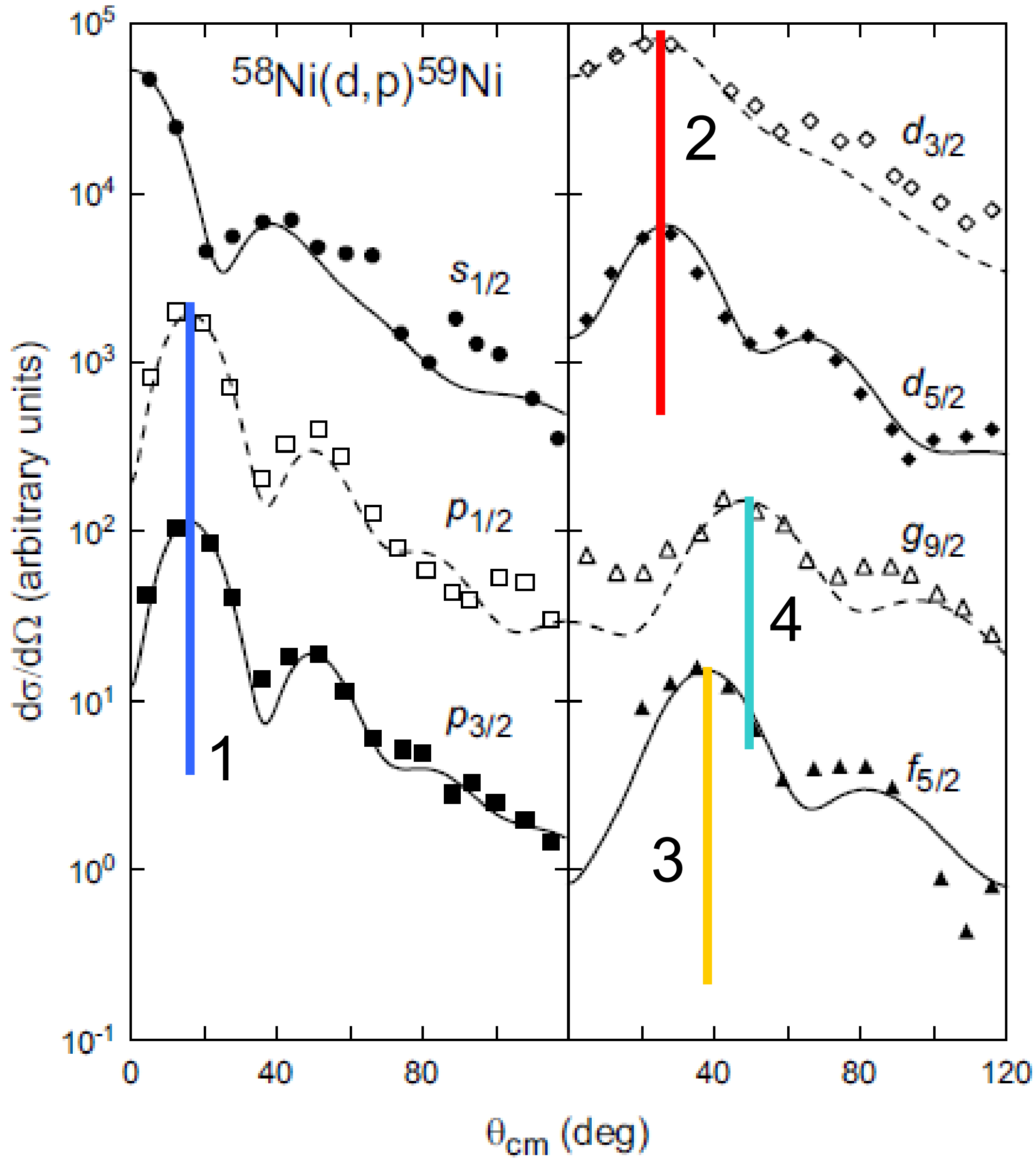
$$\beta \approx 0.75 \text{ fm}$$



Calculated (p,d) transfer (pick-up) cross sections



Single-particle spectroscopy – angular distributions

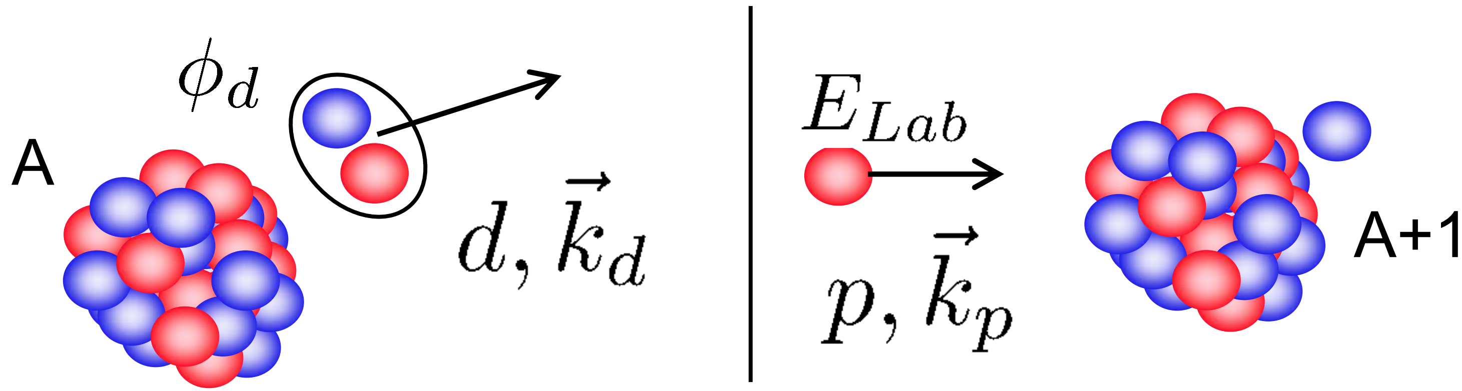


Data: M.S. Chowdhury and H.M. Sen Gupta Nucl. Phys. **A205**, 454 (2005)

Figure: Isotope Science Facility (ISF) White Paper, NSCL (2007)

Transfer – beyond DWBA – deuteron breakup

$$T(p,d) = \langle \psi^{(-)}_{d,\vec{k}_d}(\vec{r},\vec{R}) | V_{np} | \chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n)\rangle$$



$$[T_R + \mathcal{H}_{np} + V_p(r_p) + V_n(r_n) - E]\psi^{(+)}_{d,\vec{k}_d} = 0$$

$$\psi^{(+)}_{d,\vec{k}_d} = \exp(i\vec{k}_d \cdot \vec{R})\phi_d(r) + \text{outgoing waves}$$

$$\mathcal{H}_{np}\phi_d = -\varepsilon_0\phi_d, \quad \mathcal{H}_{np}\hat{\phi}_i = \hat{\varepsilon}_i\hat{\phi}_i$$

$$\psi^{(+)}_{d,\vec{k}_d}(\vec{r},\vec{R}) = \chi_{d,\vec{k}_d}(\vec{R})\phi_d(r) + \sum_i \chi_i(\vec{R})\hat{\phi}_i(\vec{r})$$

Transfer reaction – three-body models - breakup

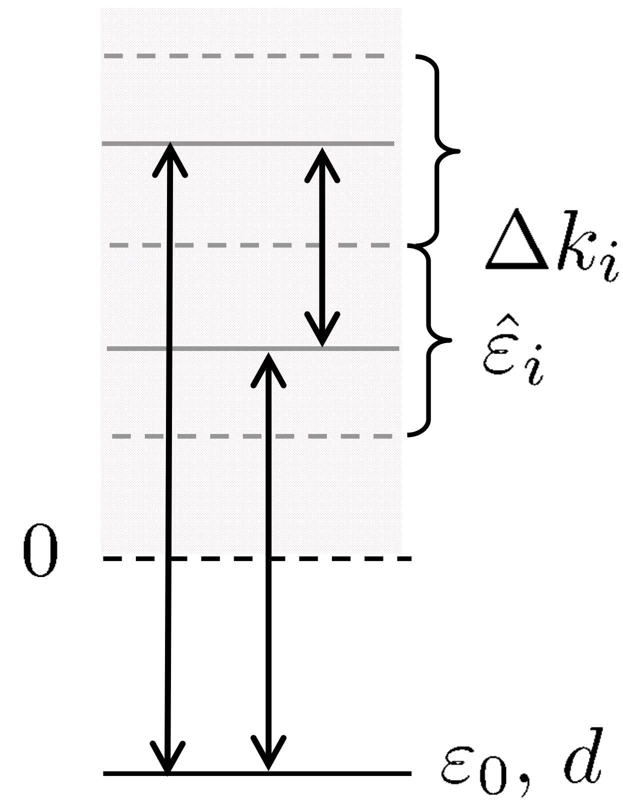
$$T(p,d) = \langle \psi^{(-)}_{d,\vec{k}_d}(\vec{r},\vec{R}) | V_{np} | \chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n)\rangle$$

$$\mathcal{H}_{np}\phi_d = \varepsilon_0\phi_d, \quad \mathcal{H}_{np}\hat{\phi}_i = \hat{\varepsilon}_i\hat{\phi}_i$$

neutron is also transferred to unbound states (d*) of the n-p system – represented by continuum bins – that are coupled to the deuteron g.s. for as long as the two nucleons remain within the range of

$$\boxed{V_p(r_{pA}),\ V_n(r_n)}$$

These <u>higher-order effects</u> can be important in slower (lower-energy) reactions

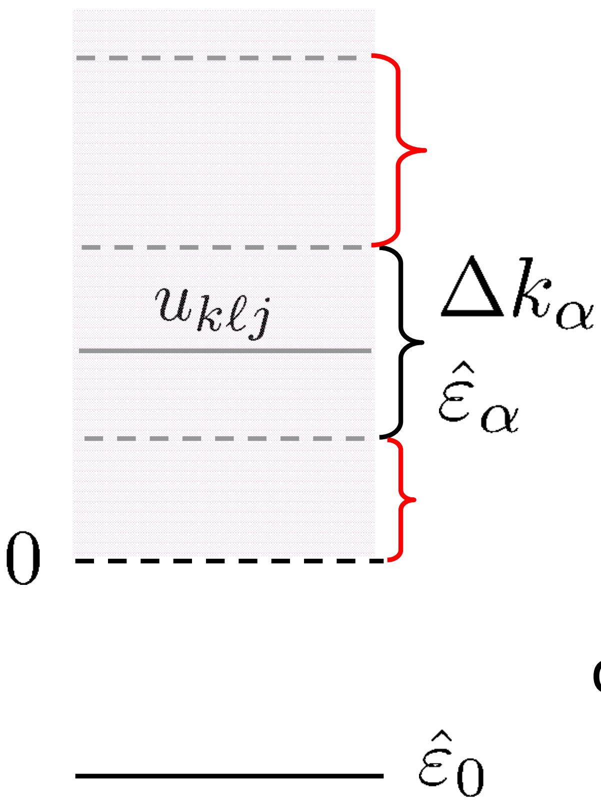


Treating breakup effects with continuum bins

<u>Scattering states</u>

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right]$$

$$\int_0^\infty dr\, u_{k\ell j}(r)\, u^*_{k'\ell j}(r) = \frac{\pi}{2}\delta(k-k')$$



$$\hat{u}_{\alpha\ell j}(r) = \sqrt{\frac{2}{\pi N_\alpha}} \int_{\Delta k_\alpha} dk\, g(k)\, u_{k\ell j}(r)$$

$$N_\alpha = \int_{\Delta k_\alpha} dk\, [g(k)]^2$$

weight function

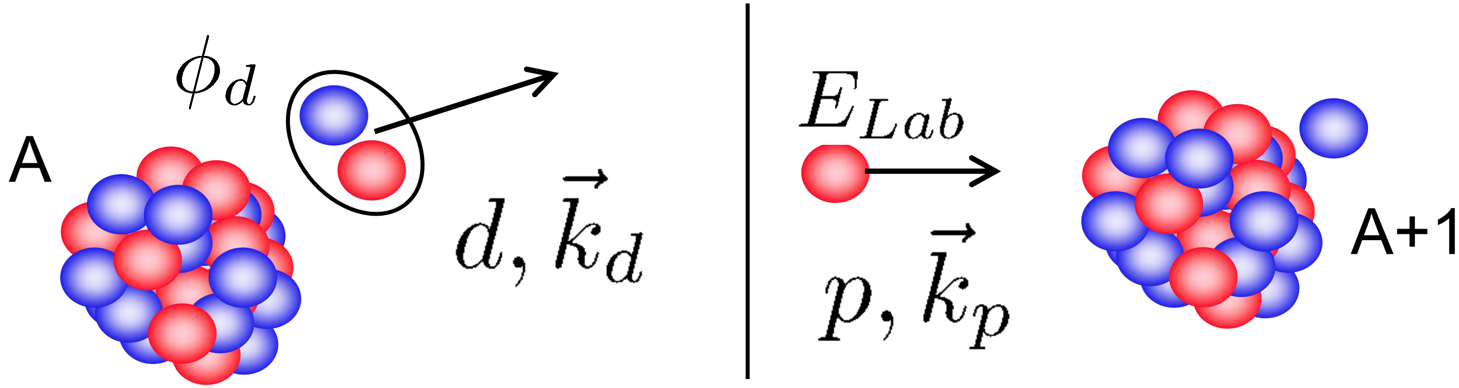
orthonormal set

$$\int_0^\infty dr\, \hat{u}^*_{\alpha\ell j}(r)\, \hat{u}_{\beta\ell j}(r) = \delta_{\alpha\beta}$$

$$g(k) = 1 \qquad g(k) = \sin\delta_{\ell j}$$

Transfer – beyond DWBA – Adiabatic breakup

$$T(p,d) = \langle \psi^{(-)}_{d,\vec{k}_d}(\vec{r},\vec{R})|V_{np}|\chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n)\rangle$$



$$[T_R + \mathcal{H}_{np} + V_p(r_p) + V_n(r_n) - E]\psi^{(+)}_{d,\vec{k}_d} = 0$$

$$\psi^{(+)}_{d,\vec{k}_d} = \exp(i\vec{k}_d \cdot \vec{R})\phi_d(r) + \text{outgoing waves}$$

$$\mathcal{H}_{np}\phi_d = -\varepsilon_0\phi_d$$

The adiabatic approximation replaces $\mathcal{H}_{np} \rightarrow -\varepsilon_0$

Adiabatic three-body model – breakup made simple

$$T(p,d) = \langle \psi^{(-)}_{d,\vec{k}_d}(\vec{r},\vec{R})|V_{np}|\chi^{(+)}_{p,\vec{k}_p}(\vec{r}_p)\phi_{n\ell j}(\vec{r}_n)\rangle$$

$$[T_R + \mathcal{H}_{np} + V_p(r_{pA}) + V_n(r_n) - E]\psi^{(+)}_{d,\vec{k}_d}(\vec{r},\vec{R}) = 0$$

Since to calculate the transfer amplitude we need the three body wave function only in regions where $V_{np} \neq 0$, $r \approx 0$

$$\mathcal{H}_{np} \rightarrow -\varepsilon_0,\ V_p(r_{pA}) \rightarrow V_p(R), \quad V_n(r_n) \rightarrow V_n(R)$$

So, with $E_0 = E + \varepsilon_0$

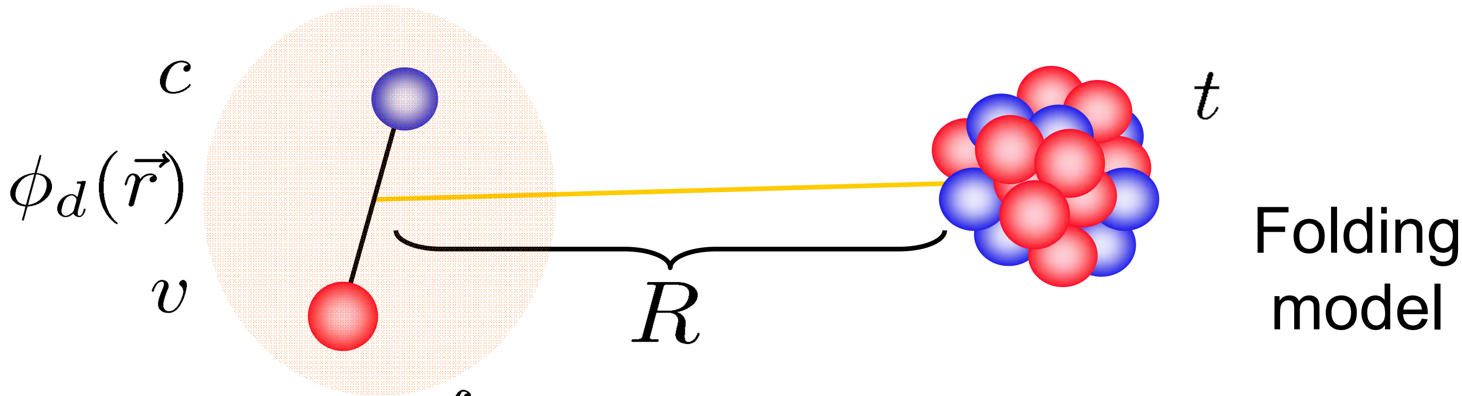
"ADWA"

$$\psi^{(+)}_{d,\vec{k}_d}(\vec{r},\vec{R}) = \chi^{Ad}_{d,\vec{k}_d}(\vec{R})\phi_d(r),\ \vec{r} \approx 0$$

$$[T_R + V_p(R) + V_n(R) - E_0]\chi^{Ad}_{d,\vec{k}_d}(\vec{R}) = 0$$

Compared with $[T_R + U_{dA}(R) - E_0]\chi^{(+)}_{d,\vec{k}_d}(\vec{R}) = 0$

Adiabatic potential versus distorting potential



$$V_{FM}(R) = \int d\vec{r}\,[V_n(r_n) + V_p(r_{pA})]|\phi_d(\vec{r})|^2$$

$$[T_R + \underbrace{V_p(R) + V_n(R)}_{} - E_0]\chi^{Ad}_{d,\vec{k}_d}(\vec{R}) = 0$$

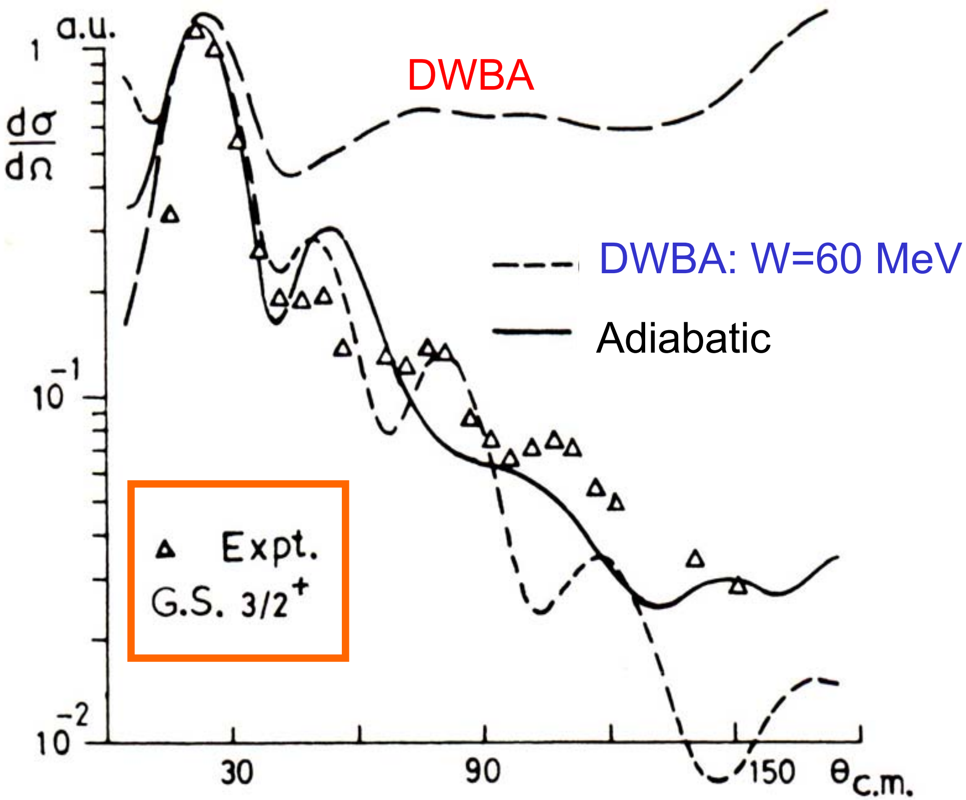
$$V_{Ad}(R) = V_n(R) + V_p(R)$$

$$V_{Ad}(R) = \int d\vec{r}\,[V_n(r_n) + V_p(r_{pA})]\,\delta(\vec{r})$$

this is NOT an optical potential and is not meant to and DOES NOT describe deuteron elastic scattering

Key features for transfer reactions - spectroscopy

^{40}Ca(p,d)^{39}Ca, 30.5 MeV



Increased reflection at nuclear surface - less diffuse deuteron channel potential

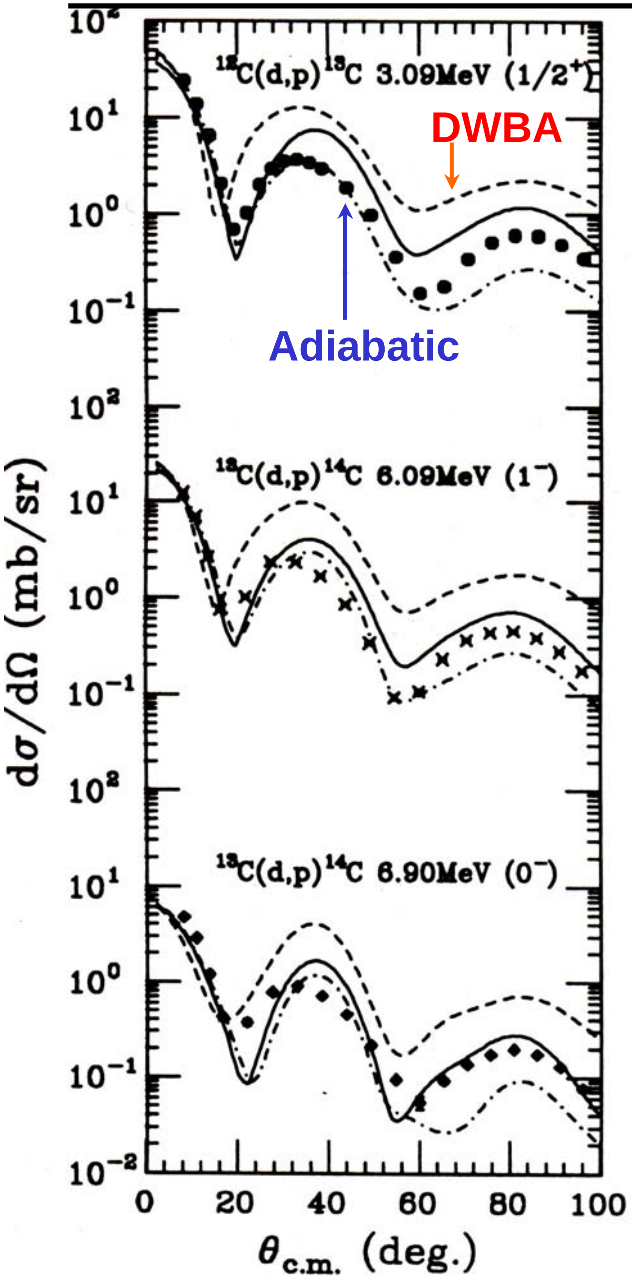
Greater surface localisation - L-space localisation

Less nuclear volume contribution and less sensitivity to optical model parameters

More consistent sets of deduced spectroscopic factors

J.D. Harvey and R.C. Johnson, Phys. Rev.C **3** (1971) 636

Spectroscopic factors in the adiabatic limit



Excitation energy (MeV)	J^π	Transferred nl_j	Spectroscopic factor DWBA	ADBA	CCBA	Shell Model
0.00	$1/2^-$	$0p_{1/2}$	1.0	0.7	0.8	0.61
3.09	$1/2^+$	$1s_{1/2}$	1.8	0.8	0.9	—
3.68	$3/2^-$	$0p_{3/2}$	0.14	0.14	0.14	0.19
3.85	$5/2^+$	$0d_{5/2}$	0.7	0.6	0.6	—

Table 16. Spectroscopic factors obtained from the $^{12}C(d,p)^{13}C$ reactions and the shell model calculations.

Way to systematically improve the adiabatic approximation to transfer reactions (Weinberg states)

R.C. Johnson and P.C. Tandy, Nucl. Phys. A 235 (1974) 56

implemented for practical calculations

A. Laid, J.A. Tostevin and R.C. Johnson, Phys. Rev. C **48** (1993), 1307

H. Toyokawa, PhD Thesis, RCNP, Osaka University 1995

Session discussed:

1. The details of the different physical inputs needed for calculations of single-nucleon transfer reactions, and some considerations/choices that can be made for these parameters. Use of DWBA and beyond.

2. The essential role played by the overlap functions and the primary sensitivity of the cross sections to these bound states and their spectroscopic factors

3. The essential physics and treatment of breakup effects on transfer reaction calculations and the need to assess the sensitivity of calculated cross sections to these model assumptions and to the parameter sets used - appreciation of reliability.