χ^2 fits (from F. Nunes slides)

Fitting data (ch15)

Comparing theory and experiment
$\{p_j\}$ inputs parameter set
$\sigma^{exp}(i)$ experimental data ($\Delta\sigma$ standard deviation)

Measure of discrepancy

$$\chi^2 = \sum_{i=1}^{N} \frac{(\sigma^{th}(i) - \sigma^{exp}(i))^2}{\Delta\sigma(i)^2}.$$

If theory agrees exactly with experiment $\chi^2=0$ (very unlikely!)

What is statistically reasonable $\sigma^{th}(i) - \sigma^{exp}(i) \sim \Delta\sigma(i)$ so $\chi^2 \sim N$ (or $\chi^2/N \sim 1$)

If $\chi^2/N >> 1$ then theory needs improvement
If $\chi^2/N << 1$ errors have been overestimated

Ex: optical potential fits (optical model)

- Strongly non linear (fitting is done by iteration only)
- Need data at large scattering angles
- Spin orbit affect T11 and not so much elastic cross sections
  - Ambiguities:
  - ❑ low energy (phase equivalent potentials)

  - ❑ medium energy (volume integral $V_{ws}=R_{ws}^2$)

$$\mathcal{J} = \int V(\mathbf{r})\mathrm{d}\mathbf{r} = 4\pi \int_0^\infty V(r) r^2 \mathrm{d}r$$

  - ❑ heavy nuclei (governed by tail of V)

$$V(R) \approx -V_{ws} e^{-(R-R_{ws})/a_{ws}} = -V_{ws} e^{R_{ws}/a_{ws}} e^{-R/a_{ws}}$$

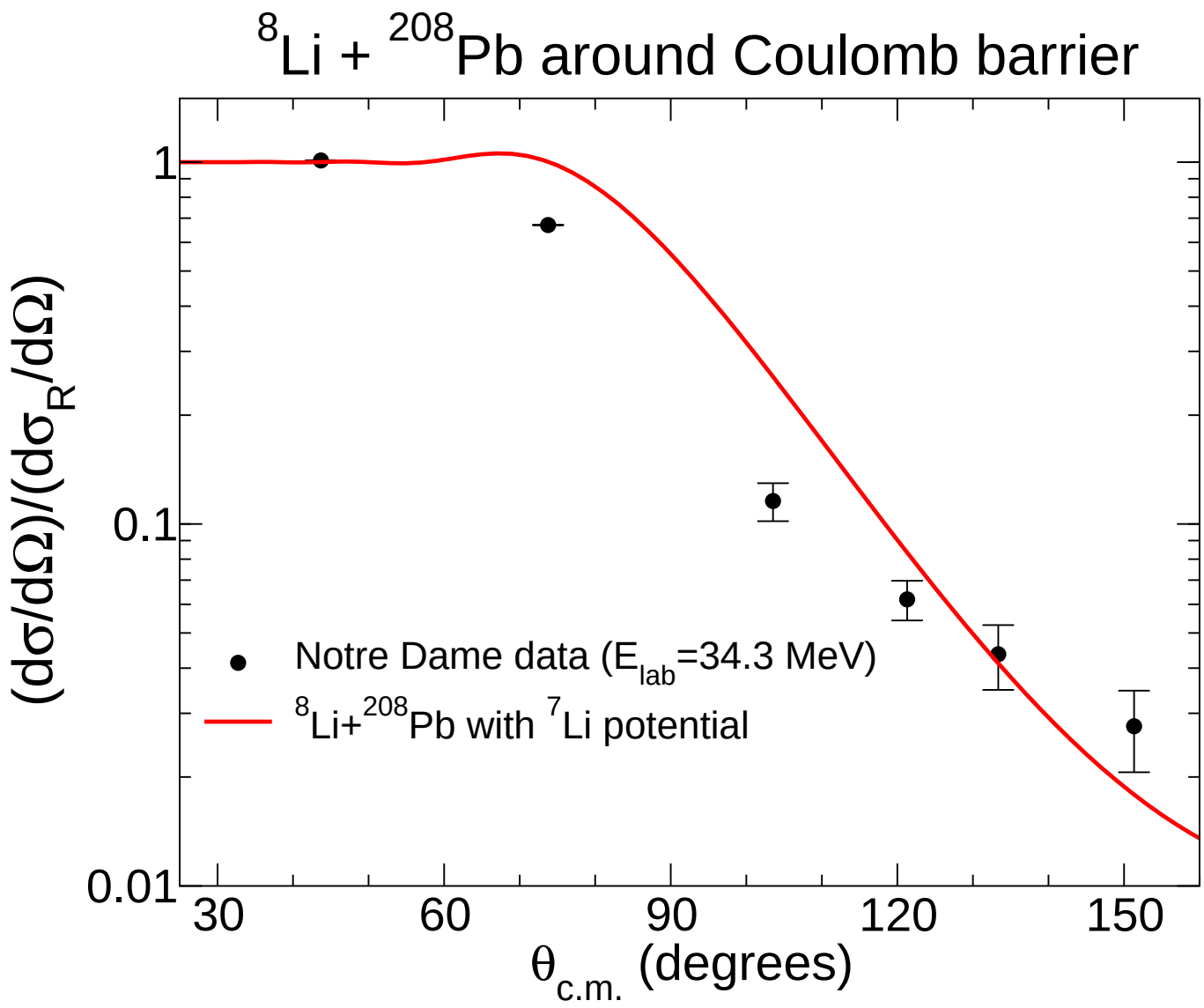
SFRESCO: Can be used together with FRESCO to do determine automatically optical model parameters by means of a χ^2 analysis of experimental angular distribution.

We need 3 input files:

1. FRESCO input file: `li8pb_e34.in`
2. MINUIT input file: `sfresco.in`
3. SEARCH input file: `search.in`

sfresco.in $\Longrightarrow$ search.in $\Longrightarrow$ `li8pb_e34.in`

^{8}Li+^{208}Pb OM before fit



FRESCO input file (before fit)

```
li8pb_e34.in
NAMELIST
&FRESCO hcm=0.05 rmatch=40.0 jtmax=60
       thmin=5.00 thmax=160.00 thinc=2.00
       smats=2 xstabl=1
       elab= 34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
       masst=208 zt=82 qval=0.0000 pwf=T nex=1 /
&STATES jp=2.0 bandp=1 ep=0.000 cpot=1 jt=0.0
       bandt=1 et=0.000 fexch=F /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=13.2 p5=1.3 p6=0.65 /
&pot /
```

Performing fits with SFRESCO:

1.- FRESCO input file: `li8pb_e34.in` (previous slide)

2.- MUNUIT input file: `sfresco.in`

```
search.in <---- file with search parameters
min
fix
migrad
end
q
show
plot
```

3.- SEARCH input file: `search.in`

```
'li8pb_e34.in' 'li8pb_e34.out' 2 1

&variable kind=1 name='V' kp=1 pline=2 col=1 valmin=5.0 valmax=150.0 step=0.2/
&variable kind=1 name='W' kp=1 pline=2 col=4 valmin=5.0 valmax=100.0 step=0.2 /

&data type=0 iscale=2 idir=1 lab=F abserr=T/
43.7     1.01026  0.014
73.76    0.67003  0.014
103.537  0.11577  0.01394
121.296  0.06194  0.00778
133.351  0.04369  0.00888
151.332  0.02763  0.00701
&
```

```
sfresco < sfresco.in > sfresco.out
```

SEARCH input file (continued):

```
'li8pb_e34.in' 'li8pb_e34.out' 2 1
```

`input_file, output_file, nvariables, ndatasets`

```
&variable kind=1 name='V' kp=1 pline=2 col=1 valmin=5.0 valmax=150.0 /
&variable kind=1 name='W' kp=1 pline=2 col=4 valmin=5.0 valmax=100.0 /
```

- `kind`: type of variable (1=potential)
- `kp`: potential index
- `pline=2`: potential component
- `col`: column (identifies parameter within component)
- `valmin-valmax`: constraints for this parameter

SEARCH input file (continued):

```
&data type=0 iscale=2 idir=1 abserr=T/
43.7     1.01026  0.014
73.76    0.67003  0.014
(...)
 &
```

- `type`: type of observable (0= angular distribution for fixed energy)
- `iscale`: data units for absolute scale (2=mb/sr)
- `idir`: scale (1=ratio to Rutherford)
- `abserr`: specified errors are absolute (T) or relative (F).

sfresco.out

```
 Var   1=V         value   15.400000
 Var   2=W         value   13.200000

Total ChiSq/N =  78.8745 from   78.874
 (...)

PARAMETER CORRELATION COEFFICIENTS
    NO.  GLOBAL    1     2
     1  0.62638 1.000-0.626
     2  0.62638 -0.626 1.000
 (...)

 Var   1=V         value   12.440562, step 0.2000, error 4.4317
 Var   2=W         value   60.305833, step 0.2000, error 4.9913

  Angle  Datum     Abs. error Theory       Chi
  43.700 1.0103    0.14000E-01 0.99683      0.9199
  73.760 0.67003   0.14000E-01 0.66383      0.1962
 103.537 0.11577   0.13940E-01 0.14595      4.6878
 121.296 0.61940E-01 0.77800E-02 0.59023E-01 0.1406
 133.351 0.43690E-01 0.88800E-02 0.34596E-01 1.0489
 151.332 0.27630E-01 0.70100E-02 0.18740E-01 1.6082

Total ChiSq/N =  1.4336 from    1.434
(...)
```

Correlation matrix

FRESCO input file (after fit)

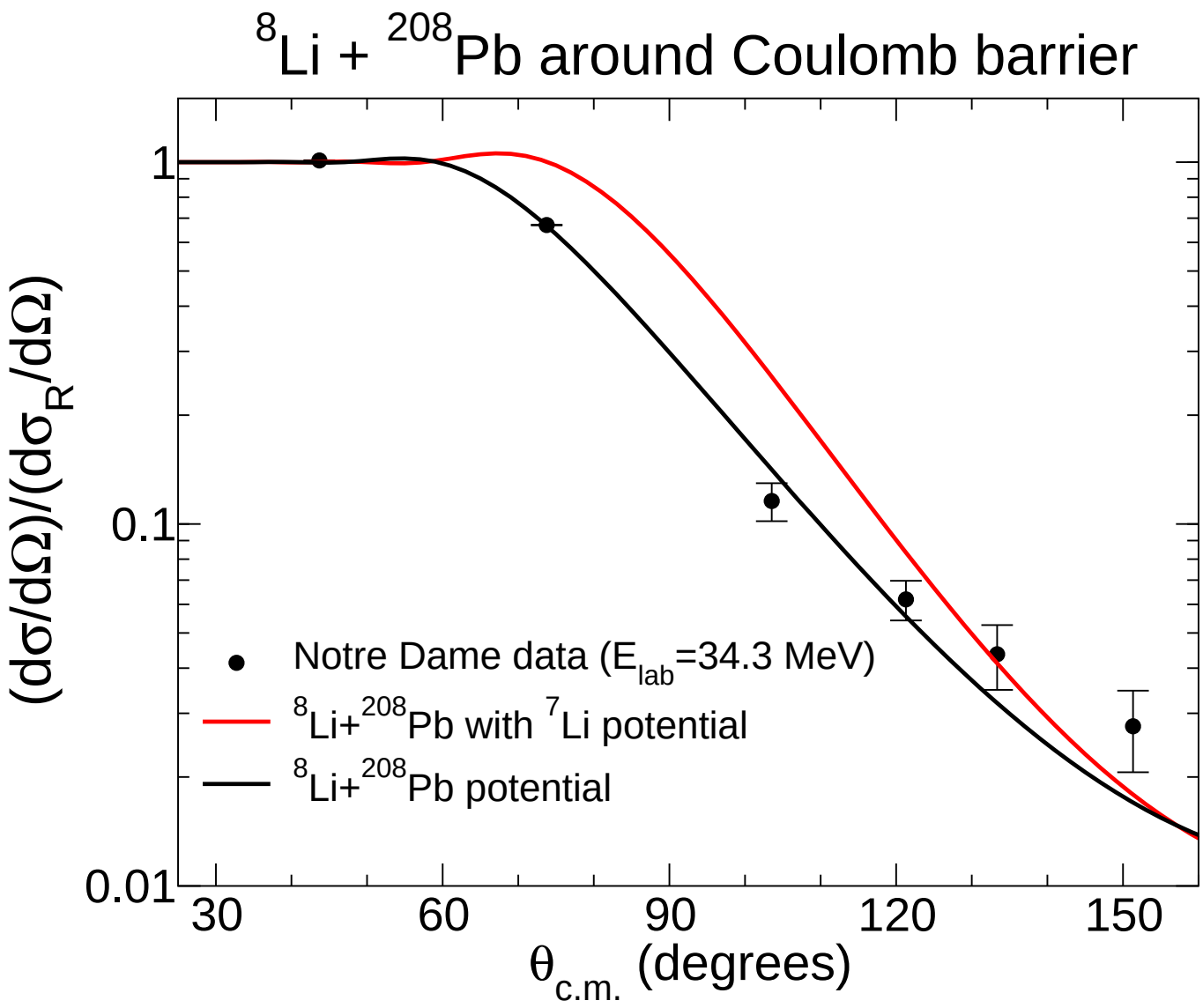
```
li8pb_e34.in
NAMELIST
&FRESCO hcm=0.05 rmatch=40.0 jtmax=60
       thmin=5.00 thmax=160.00 thinc=2.00
       smats=2 xstabl=1
       elab= 34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
       masst=208 zt=82 qval=0.0000 pwf=T nex=1 /
&STATES jp=2.0 bandp=1 ep=0.000 cpot=1 jt=0.0
       bandt=1 et=0.000 fexch=F /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=12.4 p2=1.3 p3=0.65 p4=60.3 p5=1.3 p6=0.65 /
&pot /
```

^{8}Li+^{208}Pb OM after fit



Stategies for chi2 fitting

- Start with simplest data and simplest reaction model (for example elastic and optical model)
- Restart from any intermediate stage
- If there are ambiguities, do grid searches and look at correlations in errors
- Artificially reduce error in data points if theory is having a hard time to get close in some region
- If minimum is found near the end of the range of a parameter, this is spurious – repeat with wider range
- Constrain with other experiments
- Two correlated variables : combine into one

Progressive improvement policy

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